Computer algebra independent integration tests

6-Hyperbolic-functions/6.1-Hyperbolic-sine/6.1.7-hyper^m-a+b-sinh^n-^p

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [525]. This is test number [164].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.1 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12 on windows 10.
3. Maple 2020 (64 bit) on windows 10.
4. Maxima 5.43 on Linux. (via sagemath 8.9)
5. Fricas 1.3.6 on Linux (via sagemath 9.0)
6. Sympy 1.5 under Python 3.7.3 using Anaconda distribution.
7. Giac/Xcas 1.5 on Linux. (via sagemath 8.9)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 100. (525)	% 0. (0)
Mathematica	% 95.43 (501)	% 4.57 (24)
Maple	% 92.95 (488)	% 7.05 (37)
Maxima	% 37.33 (196)	% 62.67 (329)
Fricas	% 69.71 (366)	% 30.29 (159)
Sympy	% 12.95 (68)	% 87.05 (457)
Giac	% 51.24 (269)	% 48.76 (256)

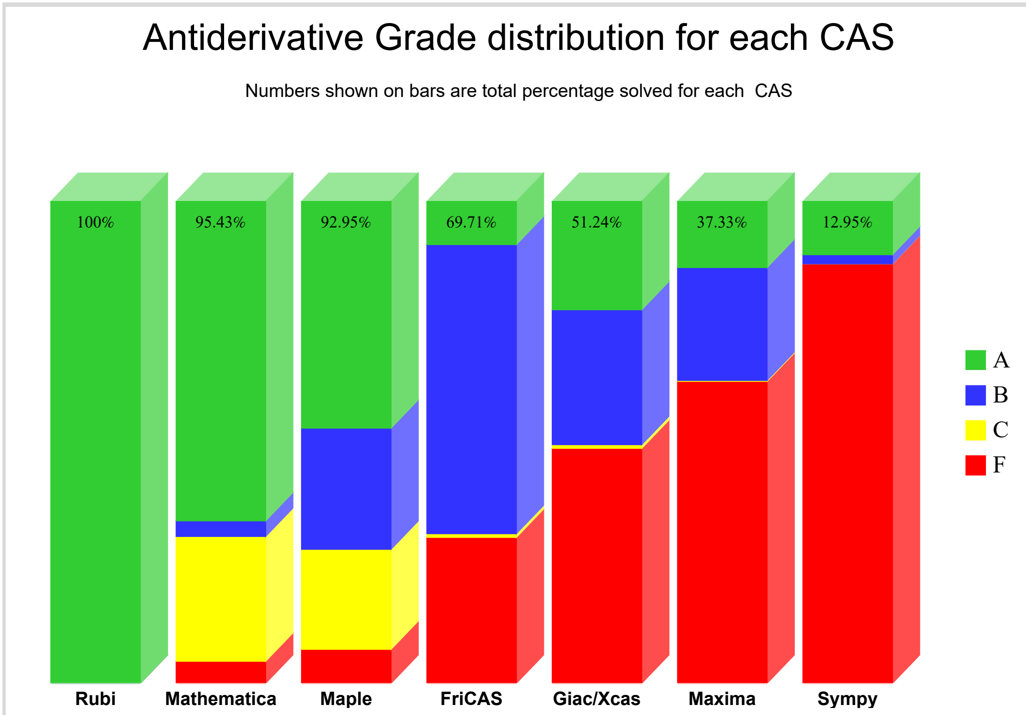
The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

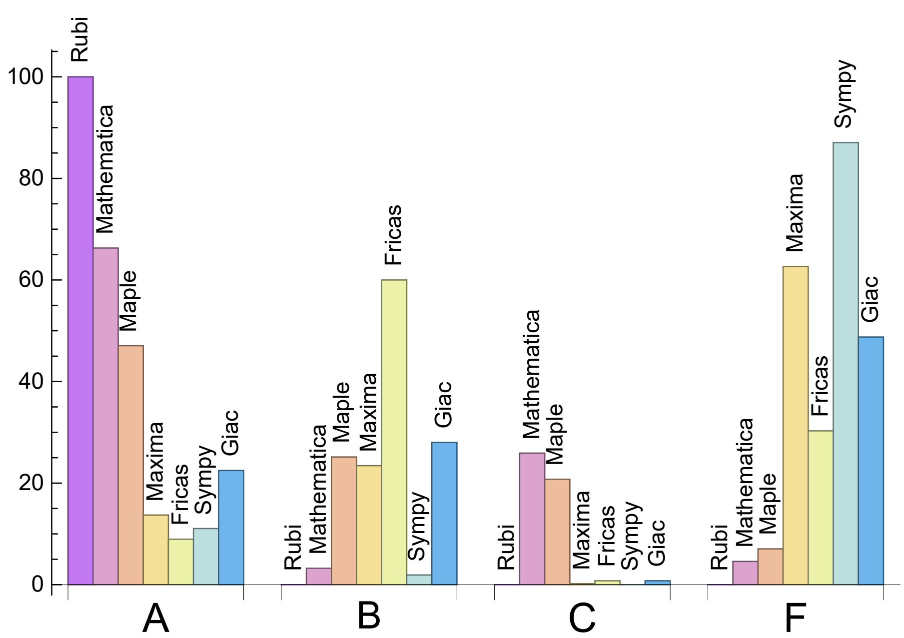
Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

System	% A grade	% B grade	% C grade	% F grade
Rubi	100.	0.	0.	0.
Mathematica	66.29	3.24	25.9	4.57
Maple	47.05	25.14	20.76	7.05
Maxima	13.71	23.43	0.19	62.67
Fricas	8.95	60.	0.76	30.29
Sympy	11.05	1.9	0.	87.05
Giac	22.48	28.	0.76	48.76

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



1.3 Performance

The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.17	133.48	1.	114.	1.
Mathematica	0.97	155.03	1.19	117.	0.97
Maple	0.08	344.59	2.36	130.	1.23
Maxima	1.32	448.83	4.89	254.5	3.16
Fricas	2.58	5148.96	43.06	2792.	34.94
Sympy	24.51	371.74	5.11	205.	2.54
Giac	1.8	295.92	2.9	209.	2.57

1.4 list of integrals that has no closed form antiderivative

{}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {76, 116, 138, 247, 299, 301, 303, 310, 312, 314, 356, 367, 376, 384, 385, 394}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 30 such integrals out of total 705, or about 4 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. If the output was an exception `ValueError` then this is most likely due to this reason.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size is determined as follows.

For Fricas, Giac and Maxima (all called via sagemath) the following code is used

```
#see https://stackoverflow.com/questions/25202346/how-to-obtain-leaf-count-expression-size-in-

def tree(expr):
    if expr.operator() is None:
       return expr
    else:
       return [expr.operator()]+map(tree, expr.operands())

try:
    # 1.35 is a fudge factor since this estimate of leaf count is bit lower than
    #what it should be compared to Mathematica's
    leafCount = round(1.35*len(flatten(tree(anti))))
except Exception as ee:
    leafCount =1
```

For Sympy, called directly from Python, the following code is used

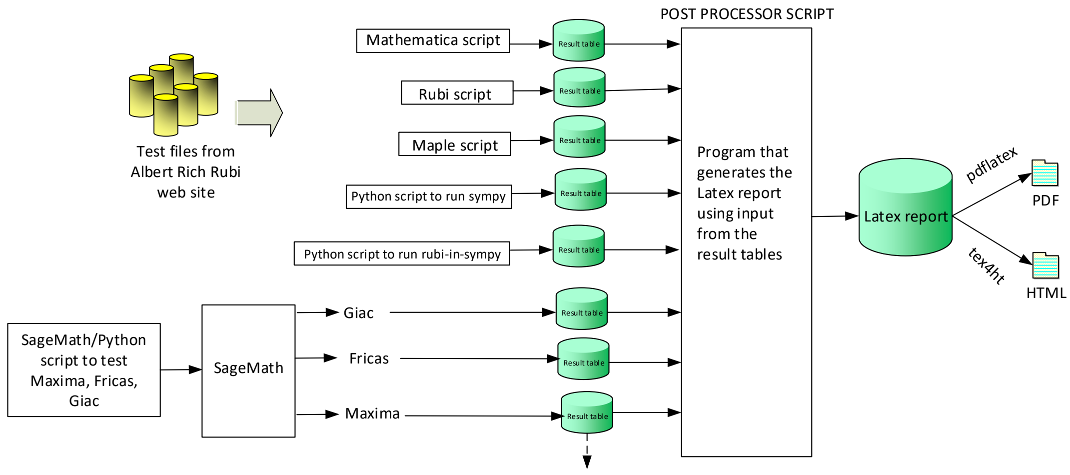
```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
         leafCount =1
```

When these cas systems have a buildin function to find the leaf size of expressions, it will be used instead, and these tests run again.

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. It contains 13 fields. This is description of each record (line)

1. integer, the problem number.
2. integer. 0 or 1 for failed or passed. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntx.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. The optimal antiderivative in CAS own syntax.

High level overview of the CAS independent integration test build system

Nasser M. Abbasi
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Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 7, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 27, 29, 31, 33, 35, 37, 39, 41, 42, 44, 46, 48, 50, 51, 53, 55, 57, 59, 60, 61, 62, 64, 65, 66, 67, 68, 69, 70, 72, 73, 76, 77, 78, 79, 80, 81, 83, 84, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 103, 104, 107, 108, 109, 110, 113, 114, 116, 117, 118, 119, 122, 123, 125, 126, 127, 128, 129, 133, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 194, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 265, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 300, 302, 304, 305, 306, 307, 308, 309, 311, 313, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 359, 362, 363, 364, 365, 370, 373, 374, 375, 379, 382, 383, 389, 391, 392, 393, 398, 402, 403, 411, 412, 413, 415, 416, 417, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 454, 455, 456, 457, 458, 459, 460, 461, 462, 465, 468, 469, 470, 471, 472, 473, 476, 479, 480, 481, 482, 483, 484, 487, 498, 509, 513, 514, 515, 516, 521, 522, 523, 524, 525 }

B grade: { 6, 8, 17, 18, 26, 138, 159, 193, 195, 206, 225, 226, 227, 228, 315, 345, 355 }

C grade: { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 71, 74, 75, 82, 85, 86, 102, 105, 106, 111, 112, 115, 120, 121, 124, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 229, 230, 231, 232, 233, 234, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 266, 267, 268, 269, 270, 271, 272, 273, 274, 299, 301, 303, 310, 312, 314, 356, 357, 358, 360, 361, 366, 367, 368, 369, 371, 372, 376, 377, 378, 380, 381, 384, 385, 386, 387, 388, 390, 394, 395, 396, 397, 399, 414, 418, 453, 463, 464, 466, 467, 474, 475, 477, 478, 485, 486, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 510, 511 }

F grade: { 63, 130, 131, 132, 134, 135, 136, 137, 139, 140, 400, 401, 404, 405, 406, 407, 408, 409, 410, 512, 517, 518, 519, 520}

2.1.3 Maple

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 34, 36, 38, 40, 71, 72, 73, 74, 75, 82, 83, 84, 85, 86, 88, 89, 91, 93, 94, 95, 96, 102, 103, 104, 105, 106, 107, 108, 111, 112, 113, 114, 115, 116, 117, 118, 119, 123, 124, 125, 126, 127, 129, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 230, 231, 232, 233, 234, 278, 280, 283, 284, 285, 286, 287, 288, 291, 292, 293, 294, 295, 296, 297, 298, 304, 306, 307, 308, 309, 311, 322, 334, 344, 353, 357, 358, 359, 360, 361, 363, 368, 369, 370, 371, 372, 374, 377, 378, 379, 380, 381, 383, 386, 387, 388, 389, 390, 393, 398, 428, 431, 432, 433, 434, 435, 436, 439, 442, 443, 444, 445, 446, 449, 452, 453, 454, 455, 456, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 485, 486, 487, 488, 489, 496, 497, 498, 499, 500, 507, 509, 510, 522, 523, 524, 525 }

B grade: { 28, 29, 30, 31, 32, 33, 35, 37, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 76, 77, 78, 79, 80, 81, 87, 90, 92, 97, 98, 99, 100, 101, 109, 110, 120, 121, 122, 128, 229, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 265, 276, 277, 281, 282, 289, 290, 299, 300, 301, 302, 303, 305, 310, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 350, 351, 395, 396, 397, 399, 413, 417, 425, 508, 511 }

C grade: { 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 235, 236, 237, 238, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 279, 352, 354, 355, 356, 362, 364, 365, 366, 367, 373, 375, 376, 382, 384, 385, 391, 392, 394, 411, 412, 414, 415, 416, 418, 426, 427, 429, 430, 437, 438, 440, 441, 447, 448, 450, 451, 457, 458, 459, 460, 461, 462, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 490, 491, 492, 493, 494, 495, 501, 502, 503, 504, 505, 506, 521 }

F grade: { 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 419, 420, 421, 422, 423, 424, 512, 513, 514, 515, 516, 517, 518, 519, 520 }

2.1.4 Maxima

A grade: { 1, 3, 5, 6, 7, 10, 12, 14, 16, 19, 21, 23, 25, 27, 88, 93, 125, 141, 142, 143, 144, 145, 146, 147, 150, 151, 152, 153, 154, 155, 161, 162, 163, 164, 165, 166, 168, 184, 186, 188, 189, 190, 197, 199, 201, 203, 217, 218, 219, 220, 277, 279, 280, 283, 285, 286, 287, 293, 295, 296, 304, 306, 307, 428, 429, 430, 433, 439, 440, 441, 450, 451

B grade: { 2, 4, 8, 9, 11, 13, 15, 17, 18, 20, 22, 24, 26, 60, 61, 62, 63, 64, 65, 89, 94, 108, 117, 118, 148, 149, 156, 157, 158, 159, 160, 167, 169, 170, 185, 187, 191, 192, 193, 194, 195, 196, 198, 200, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 221, 222, 223, 224, 225, 226, 227, 228, 265, 276, 278, 281, 282, 284, 288, 289, 290, 291, 292, 294, 297, 298, 299, 300, 301, 302, 303, 305, 308, 309, 310, 311, 312, 313, 314, 315, 349, 350, 351, 383, 392, 393, 425, 426, 427, 431, 432, 434, 435, 436, 437, 438, 442, 443, 444, 445, 446, 447, 448, 449, 452, 453, 454, 455, 456 }

C grade: { 128 }

F grade: { 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 109, 110, 111, 112, 113, 114, 115, 116, 119, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 384, 385, 386, 387, 388, 389, 390, 391, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

2.1.5 FriCAS

A grade: { 1, 3, 4, 5, 10, 12, 14, 16, 19, 21, 23, 25, 88, 93, 125, 141, 142, 143, 144, 145, 147, 151, 152, 153, 155, 162, 184, 186, 188, 190, 199, 276, 277, 278, 279, 280, 283, 284, 285, 286, 293, 295, 298, 304, 306, 524, 525 }

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C grade: { 89, 94, 128, 521 }

F grade: { 58, 59, 71, 72, 73, 74, 75, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 102, 103, 104, 105, 106, 111, 112, 113, 114, 115, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 215, 216, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 267, 268, 269, 347, 348, 357, 358, 359, 360, 361, 368, 369, 370, 371, 372, 377, 378, 379, 380, 381, 386, 387, 388, 389, 390, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 414, 418, 419, 420, 421, 422, 423, 424, 463, 464, 465, 466, 467, 474, 475, 476, 477, 478, 485, 486, 487, 488, 489, 496, 497, 498, 499, 500, 501, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522 }

2.1.6 Sympy

A grade: { 1, 2, 3, 4, 5, 10, 11, 12, 13, 14, 19, 20, 21, 22, 23, 35, 141, 142, 143, 144, 145, 150, 151, 152, 153, 161, 162, 163, 184, 185, 186, 187, 188, 196, 197, 198, 199, 209, 218, 279, 280, 283, 284, 285, 286, 293, 294, 295, 296, 304, 305, 306, 307, 322, 334, 344, 413, 417 }

B grade: { 60, 61, 62, 63, 182, 183, 276, 277, 278, 350 }

C grade: { }

F grade: { 6, 7, 8, 9, 15, 16, 17, 18, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 146, 147, 148, 149, 154, 155, 156, 157, 158, 159, 160, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 189, 190, 191, 192, 193, 194, 195, 200, 201, 202, 203, 204, 205, 206, 207, 208, 210, 211, 212, 213, 214, 215, 216, 217, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 281, 282, 287, 288, 289, 290, 291, 292, 297, 298, 299, 300, 301, 302, 303, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 345, 346, 347, 348, 349, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525 }

2.1.7 Giac

A grade: { 7, 9, 25, 27, 33, 35, 37, 39, 42, 44, 46, 48, 50, 57, 59, 61, 62, 65, 88, 93, 125, 141, 142, 143, 144, 145, 146, 149, 150, 151, 152, 153, 159, 160, 161, 162, 163, 164, 182, 183, 184, 186, 188, 192, 203, 235, 236, 237, 239, 240, 247, 248, 249, 250, 251, 252, 259, 260, 261, 262, 263, 264, 268, 269, 271, 272, 275, 276, 277, 279, 280, 285, 287, 288, 290, 292, 293, 295, 304, 306, 321, 324, 326, 333, 336, 413, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 452, 453, 454, 455, 456, 521 }

B grade: { 1, 2, 3, 4, 5, 6, 8, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 26, 29, 31, 41, 51, 53, 55, 60, 63, 64, 69, 70, 100, 108, 109, 117, 118, 119, 147, 148, 154, 155, 156, 157, 158, 165, 166, 167, 168, 169, 170, 185, 187, 189, 190, 191, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 265, 274, 278, 281, 282, 283, 284, 286, 289, 291, 294, 296, 297, 298, 299, 300, 301, 302, 303, 305, 307, 308, 309, 310, 311, 312, 313, 314, 315, 317, 319, 323, 328, 329, 331, 337, 338, 339, 341, 343, 346, 348, 349, 350, 351, 355, 356, 375, 376, 383, 384, 392, 393, 394 }

C grade: { 89, 94, 128, 266 }

F grade: { 28, 30, 32, 34, 36, 38, 40, 43, 45, 47, 49, 52, 54, 56, 58, 66, 67, 68, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 90, 91, 92, 95, 96, 97, 98, 99, 101, 102, 103, 104, 105, 106, 107, 110, 111, 112, 113, 114, 115, 116, 120, 121, 122, 123, 124, 126, 127, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 229, 230, 231, 232, 233, 234, 238, 241, 242, 243, 244, 245, 246, 253, 254, 255, 256, 257, 258, 267, 270, 273, 316, 318, 320, 322, 325, 327, 330, 332, 334, 335, 340, 342, 344, 345, 347, 352, 353, 354, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 377, 378, 379, 380, 381, 382, 385, 386, 387, 388, 389, 390, 391, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 451, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 522, 523, 524, 525 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	68	88	203	319	258	221
normalized size	1	1.	0.76	0.99	2.28	3.58	2.9	2.48
time (sec)	N/A	0.055	0.108	0.014	1.034	1.952	4.778	1.323

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	77	56	190	273	105	166
normalized size	1	1.	1.45	1.06	3.58	5.15	1.98	3.13
time (sec)	N/A	0.056	0.026	0.013	1.041	1.823	2.48	1.255

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	47	66	131	163	158	161
normalized size	1	1.	0.77	1.08	2.15	2.67	2.59	2.64
time (sec)	N/A	0.043	0.085	0.013	1.059	1.909	1.428	1.369

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	53	34	90	127	56	96
normalized size	1	1.	1.66	1.06	2.81	3.97	1.75	3.
time (sec)	N/A	0.029	0.024	0.013	1.015	1.875	0.769	1.267

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	36	32	51	74	51	72
normalized size	1	1.	1.2	1.07	1.7	2.47	1.7	2.4
time (sec)	N/A	0.016	0.032	0.007	1.044	1.841	0.409	1.308

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	62	24	58	374	0	78
normalized size	1	1.	2.48	0.96	2.32	14.96	0.	3.12
time (sec)	N/A	0.035	0.032	0.029	1.041	1.974	0.	1.214

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	31	89	0	41
normalized size	1	1.	1.	1.38	1.94	5.56	0.	2.56
time (sec)	N/A	0.027	0.02	0.027	1.024	1.85	0.	1.293

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	99	40	169	1323	0	136
normalized size	1	1.	2.48	1.	4.22	33.08	0.	3.4
time (sec)	N/A	0.041	0.032	0.034	1.055	1.936	0.	1.253

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	49	35	153	425	0	82
normalized size	1	1.	1.14	0.81	3.56	9.88	0.	1.91
time (sec)	N/A	0.039	0.031	0.034	1.025	1.807	0.	1.173

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	146	146	133	150	360	589	490	429
normalized size	1	1.	0.91	1.03	2.47	4.03	3.36	2.94
time (sec)	N/A	0.18	0.204	0.048	1.044	1.973	15.386	1.363

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	154	102	333	554	204	332
normalized size	1	1.	1.81	1.2	3.92	6.52	2.4	3.91
time (sec)	N/A	0.097	0.038	0.017	1.06	2.249	8.167	1.325

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	117	99	118	255	373	332	316
normalized size	1	1.06	0.9	1.07	2.32	3.39	3.02	2.87
time (sec)	N/A	0.11	0.192	0.017	1.043	2.145	4.88	1.356

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	111	70	212	309	128	220
normalized size	1	1.	1.95	1.23	3.72	5.42	2.25	3.86
time (sec)	N/A	0.06	0.036	0.015	1.046	2.018	2.35	1.25

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	60	79	142	193	168	204
normalized size	1	1.	0.83	1.1	1.97	2.68	2.33	2.83
time (sec)	N/A	0.022	0.127	0.013	1.008	1.878	1.277	1.226

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	104	50	138	1280	0	174
normalized size	1	1.	2.	0.96	2.65	24.62	0.	3.35
time (sec)	N/A	0.066	0.034	0.032	1.049	1.851	0.	1.28

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	64	56	52	85	215	0	189
normalized size	1	1.28	1.12	1.04	1.7	4.3	0.	3.78
time (sec)	N/A	0.081	0.163	0.033	1.014	1.87	0.	1.228

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	134	53	212	2310	0	180
normalized size	1	1.	2.39	0.95	3.79	41.25	0.	3.21
time (sec)	N/A	0.088	0.064	0.041	1.075	1.952	0.	1.359

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	85	47	163	428	0	109
normalized size	1	1.	2.12	1.18	4.08	10.7	0.	2.72
time (sec)	N/A	0.074	0.712	0.04	1.063	1.893	0.	1.345

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	261	261	162	222	547	1013	777	679
normalized size	1	1.	0.62	0.85	2.1	3.88	2.98	2.6
time (sec)	N/A	0.43	0.422	0.052	1.077	1.81	37.062	1.399

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	127	158	508	940	330	544
normalized size	1	1.	1.1	1.37	4.42	8.17	2.87	4.73
time (sec)	N/A	0.129	0.797	0.049	1.083	1.82	21.724	1.414

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	130	180	413	667	561	521
normalized size	1	1.	0.72	0.99	2.28	3.69	3.1	2.88
time (sec)	N/A	0.193	0.291	0.014	1.105	1.817	14.363	1.408

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	94	116	355	589	221	386
normalized size	1	1.	1.19	1.47	4.49	7.46	2.8	4.89
time (sec)	N/A	0.087	0.304	0.016	1.053	1.774	7.788	1.32

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	95	131	266	398	350	362
normalized size	1	1.	0.74	1.02	2.08	3.11	2.73	2.83
time (sec)	N/A	0.1	0.259	0.013	1.045	1.823	5.044	1.322

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	83	86	261	2889	0	311
normalized size	1	1.	1.	1.04	3.14	34.81	0.	3.75
time (sec)	N/A	0.087	0.215	0.03	1.065	2.065	0.	1.322

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	137	137	113	94	176	409	0	257
normalized size	1	1.	0.82	0.69	1.28	2.99	0.	1.88
time (sec)	N/A	0.192	1.917	0.03	1.065	1.841	0.	1.363

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	210	79	293	4575	0	261
normalized size	1	1.	2.53	0.95	3.53	55.12	0.	3.14
time (sec)	N/A	0.107	4.7	0.042	1.075	2.084	0.	1.363

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	113	113	107	77	217	668	0	217
normalized size	1	1.	0.95	0.68	1.92	5.91	0.	1.92
time (sec)	N/A	0.141	2.504	0.039	1.03	1.764	0.	1.368

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	109	109	165	448	0	7640	0	0
normalized size	1	1.	1.51	4.11	0.	70.09	0.	0.
time (sec)	N/A	0.149	0.868	0.044	0.	2.284	0.	0.

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	97	670	0	4313	0	296
normalized size	1	1.	0.8	5.54	0.	35.64	0.	2.45
time (sec)	N/A	0.235	0.475	0.08	0.	2.054	0.	1.209

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	134	227	0	3993	0	0
normalized size	1	1.	1.7	2.87	0.	50.54	0.	0.
time (sec)	N/A	0.108	0.434	0.033	0.	1.97	0.	0.

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	71	454	0	2191	0	181
normalized size	1	1.	0.9	5.75	0.	27.73	0.	2.29
time (sec)	N/A	0.121	0.25	0.043	0.	2.129	0.	1.238

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	107	98	0	1894	0	0
normalized size	1	1.	1.91	1.75	0.	33.82	0.	0.
time (sec)	N/A	0.085	0.234	0.03	0.	1.916	0.	0.

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	312	0	1122	0	88
normalized size	1	1.	1.	6.24	0.	22.44	0.	1.76
time (sec)	N/A	0.087	0.131	0.033	0.	2.02	0.	1.265

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	91	51	0	1293	0	0
normalized size	1	1.	2.28	1.27	0.	32.32	0.	0.
time (sec)	N/A	0.047	0.123	0.016	0.	2.016	0.	0.

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	40	267	0	1068	3859	63
normalized size	1	1.	1.	6.68	0.	26.7	96.48	1.58
time (sec)	N/A	0.027	0.071	0.042	0.	1.928	118.484	1.258

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	124	74	0	1544	0	0
normalized size	1	1.	2.07	1.23	0.	25.73	0.	0.
time (sec)	N/A	0.074	0.21	0.049	0.	2.014	0.	0.

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	57	319	0	1661	0	100
normalized size	1	1.	1.	5.6	0.	29.14	0.	1.75
time (sec)	N/A	0.081	0.292	0.061	0.	2.214	0.	1.308

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	201	133	0	4867	0	0
normalized size	1	1.	2.28	1.51	0.	55.31	0.	0.
time (sec)	N/A	0.123	0.694	0.062	0.	2.819	0.	0.

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	126	401	0	4710	0	159
normalized size	1	1.	1.62	5.14	0.	60.38	0.	2.04
time (sec)	N/A	0.12	0.693	0.071	0.	2.378	0.	1.425

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	574	232	0	14357	0	0
normalized size	1	1.	4.42	1.78	0.	110.44	0.	0.
time (sec)	N/A	0.204	6.283	0.066	0.	3.037	0.	0.

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	155	519	0	10761	0	289
normalized size	1	1.	1.41	4.72	0.	97.83	0.	2.63
time (sec)	N/A	0.136	1.458	0.074	0.	2.781	0.	1.493

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	102	102	99	798	0	4199	0	228
normalized size	1	1.	0.97	7.82	0.	41.17	0.	2.24
time (sec)	N/A	0.169	0.833	0.051	0.	2.56	0.	1.283

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	141	341	0	4536	0	0
normalized size	1	1.	1.57	3.79	0.	50.4	0.	0.
time (sec)	N/A	0.113	0.597	0.032	0.	2.432	0.	0.

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	81	428	0	3564	0	184
normalized size	1	1.	0.96	5.1	0.	42.43	0.	2.19
time (sec)	N/A	0.086	0.444	0.041	0.	2.392	0.	1.402

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	130	256	0	3999	0	0
normalized size	1	1.	1.6	3.16	0.	49.37	0.	0.
time (sec)	N/A	0.065	0.348	0.027	0.	2.395	0.	0.

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	95	95	96	749	0	3767	0	196
normalized size	1	1.	1.01	7.88	0.	39.65	0.	2.06
time (sec)	N/A	0.069	0.303	0.046	0.	2.437	0.	1.213

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	176	350	0	6178	0	0
normalized size	1	1.	1.6	3.18	0.	56.16	0.	0.
time (sec)	N/A	0.149	0.644	0.062	0.	2.911	0.	0.

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	170	810	0	6962	0	312
normalized size	1	1.	1.2	5.7	0.	49.03	0.	2.2
time (sec)	N/A	0.153	0.811	0.076	0.	2.621	0.	1.485

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	350	415	0	19038	0	0
normalized size	1	1.	2.17	2.58	0.	118.25	0.	0.
time (sec)	N/A	0.273	1.406	0.079	0.	3.669	0.	0.

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	210	890	0	16662	0	301
normalized size	1	1.	1.21	5.11	0.	95.76	0.	1.73
time (sec)	N/A	0.206	1.259	0.089	0.	3.163	0.	1.508

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	104	768	0	11786	0	386
normalized size	1	1.	0.84	6.19	0.	95.05	0.	3.11
time (sec)	N/A	0.123	1.369	0.052	0.	2.942	0.	1.565

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	170	961	0	14074	0	0
normalized size	1	1.	1.26	7.12	0.	104.25	0.	0.
time (sec)	N/A	0.141	1.222	0.042	0.	2.963	0.	0.

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	121	1408	0	12358	0	378
normalized size	1	1.	0.87	10.13	0.	88.91	0.	2.72
time (sec)	N/A	0.157	1.346	0.049	0.	3.003	0.	1.526

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	118	118	149	964	0	12081	0	0
normalized size	1	1.	1.26	8.17	0.	102.38	0.	0.
time (sec)	N/A	0.083	0.747	0.034	0.	2.862	0.	0.

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	154	154	132	1768	0	13509	0	413
normalized size	1	1.	0.86	11.48	0.	87.72	0.	2.68
time (sec)	N/A	0.155	1.219	0.053	0.	2.969	0.	1.462

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	237	1145	0	22814	0	0
normalized size	1	1.	1.43	6.9	0.	137.43	0.	0.
time (sec)	N/A	0.267	3.426	0.069	0.	4.056	0.	0.

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	215	215	225	1850	0	21173	0	455
normalized size	1	1.	1.05	8.6	0.	98.48	0.	2.12
time (sec)	N/A	0.287	1.813	0.089	0.	3.586	0.	1.6

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	224	224	419	1225	0	0	0	0
normalized size	1	1.	1.87	5.47	0.	0.	0.	0.
time (sec)	N/A	0.439	2.943	0.091	0.	0.	0.	0.

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	167	1930	0	0	0	518
normalized size	1	1.	0.64	7.45	0.	0.	0.	2.
time (sec)	N/A	0.338	2.762	0.111	0.	0.	0.	1.609

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	2	2	2	17	14	70	14	14
normalized size	1	1.	1.	8.5	7.	35.	7.	7.
time (sec)	N/A	0.016	0.004	0.016	1.026	1.758	0.895	1.298

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	17	36	66	286	104	24
normalized size	1	1.	1.55	3.27	6.	26.	9.45	2.18
time (sec)	N/A	0.019	0.003	0.015	1.039	1.793	4.906	1.275

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	27	52	150	628	260	32
normalized size	1	1.	1.42	2.74	7.89	33.05	13.68	1.68
time (sec)	N/A	0.02	0.003	0.019	1.045	1.78	18.192	1.275

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	B	B	B	B	B
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	15	15	0	40	82	215	209	50
normalized size	1	1.	0.	2.67	5.47	14.33	13.93	3.33
time (sec)	N/A	0.013	0.022	0.016	1.53	1.843	3.567	1.266

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	35	92	117	729	0	84
normalized size	1	1.	0.95	2.49	3.16	19.7	0.	2.27
time (sec)	N/A	0.027	0.128	0.02	1.549	1.854	0.	1.277

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	51	124	150	1936	0	100
normalized size	1	1.	0.93	2.25	2.73	35.2	0.	1.82
time (sec)	N/A	0.058	0.178	0.023	1.566	2.069	0.	1.277

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	114	339	0	7794	0	0
normalized size	1	1.	0.88	2.61	0.	59.95	0.	0.
time (sec)	N/A	0.148	0.651	0.114	0.	3.896	0.	0.

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	97	200	0	5562	0	0
normalized size	1	1.	1.18	2.44	0.	67.83	0.	0.
time (sec)	N/A	0.07	0.162	0.066	0.	3.464	0.	0.

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	97	174	0	11636	0	0
normalized size	1	1.	1.15	2.07	0.	138.52	0.	0.
time (sec)	N/A	0.105	0.124	0.098	0.	3.265	0.	0.

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	104	230	0	3421	0	938
normalized size	1	1.	1.18	2.61	0.	38.88	0.	10.66
time (sec)	N/A	0.118	0.293	0.09	0.	2.279	0.	1.373

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	129	381	0	8303	0	3086
normalized size	1	1.	0.9	2.65	0.	57.66	0.	21.43
time (sec)	N/A	0.15	0.538	0.099	0.	4.097	0.	1.574

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	300	300	210	512	0	0	0	0
normalized size	1	1.	0.7	1.71	0.	0.	0.	0.
time (sec)	N/A	0.327	1.369	0.086	0.	0.	0.	0.

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	170	343	0	0	0	0
normalized size	1	1.	0.96	1.94	0.	0.	0.	0.
time (sec)	N/A	0.213	0.841	0.065	0.	0.	0.	0.

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0
normalized size	1	1.	1.15	2.33	0.	0.	0.	0.
time (sec)	N/A	0.036	0.096	0.065	0.	0.	0.	0.

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	151	160	0	0	0	0
normalized size	1	1.	0.76	0.8	0.	0.	0.	0.
time (sec)	N/A	0.182	0.594	0.072	0.	0.	0.	0.

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	276	276	208	436	0	0	0	0
normalized size	1	1.	0.75	1.58	0.	0.	0.	0.
time (sec)	N/A	0.282	3.028	0.121	0.	0.	0.	0.

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	177	177	151	483	0	11580	0	0
normalized size	1	1.	0.85	2.73	0.	65.42	0.	0.
time (sec)	N/A	0.181	0.582	0.091	0.	3.807	0.	0.

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	111	336	0	7737	0	0
normalized size	1	1.	0.92	2.78	0.	63.94	0.	0.
time (sec)	N/A	0.097	0.312	0.071	0.	2.926	0.	0.

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	136	268	0	14642	0	0
normalized size	1	1.	1.07	2.11	0.	115.29	0.	0.
time (sec)	N/A	0.165	0.554	0.086	0.	3.685	0.	0.

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	143	297	0	17554	0	0
normalized size	1	1.	1.1	2.28	0.	135.03	0.	0.
time (sec)	N/A	0.168	0.711	0.097	0.	4.753	0.	0.

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	135	135	123	379	0	8128	0	0
normalized size	1	1.	0.91	2.81	0.	60.21	0.	0.
time (sec)	N/A	0.141	0.63	0.095	0.	4.112	0.	0.

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	199	199	174	569	0	18294	0	0
normalized size	1	1.	0.87	2.86	0.	91.93	0.	0.
time (sec)	N/A	0.199	1.012	0.122	0.	10.174	0.	0.

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	367	367	262	743	0	0	0	0
normalized size	1	1.	0.71	2.02	0.	0.	0.	0.
time (sec)	N/A	0.471	2.873	0.074	0.	0.	0.	0.

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	236	236	213	535	0	0	0	0
normalized size	1	1.	0.9	2.27	0.	0.	0.	0.
time (sec)	N/A	0.325	1.363	0.071	0.	0.	0.	0.

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0
normalized size	1	1.	0.97	2.39	0.	0.	0.	0.
time (sec)	N/A	0.194	0.679	0.069	0.	0.	0.	0.

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	155	243	0	0	0	0
normalized size	1	1.	0.76	1.19	0.	0.	0.	0.
time (sec)	N/A	0.211	1.047	0.079	0.	0.	0.	0.

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	213	454	0	0	0	0
normalized size	1	1.	0.8	1.7	0.	0.	0.	0.
time (sec)	N/A	0.306	4.061	0.087	0.	0.	0.	0.

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	208	609	0	0	0	0
normalized size	1	1.	0.9	2.62	0.	0.	0.	0.
time (sec)	N/A	0.304	1.394	0.085	0.	0.	0.	0.

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	14	15	12	0	15
normalized size	1	1.	1.	1.27	1.36	1.09	0.	1.36
time (sec)	N/A	0.024	0.007	0.049	1.558	1.874	0.	1.257

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	34	38	0	15
normalized size	1	1.	1.	1.15	2.62	2.92	0.	1.15
time (sec)	N/A	0.026	0.005	0.033	1.553	1.768	0.	1.301

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	51	0	0	0	0
normalized size	1	1.	1.	4.64	0.	0.	0.	0.
time (sec)	N/A	0.01	0.024	0.07	0.	0.	0.	0.

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	0	0	0
normalized size	1	1.	1.	1.85	0.	0.	0.	0.
time (sec)	N/A	0.02	0.033	0.064	0.	0.	0.	0.

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	54	109	0	0	0	0
normalized size	1	1.	1.29	2.6	0.	0.	0.	0.
time (sec)	N/A	0.032	0.044	0.067	0.	0.	0.	0.

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	23	21	31	62	0	34
normalized size	1	1.	0.79	0.72	1.07	2.14	0.	1.17
time (sec)	N/A	0.026	0.02	0.041	1.558	1.788	0.	1.227

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	C	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	25	21	72	81	0	34
normalized size	1	1.	0.76	0.64	2.18	2.45	0.	1.03
time (sec)	N/A	0.029	0.006	0.043	1.55	1.806	0.	1.199

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	103	0	0	0	0
normalized size	1	1.	1.	2.29	0.	0.	0.	0.
time (sec)	N/A	0.064	0.069	0.077	0.	0.	0.	0.

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	78	106	0	0	0	0
normalized size	1	1.	0.9	1.22	0.	0.	0.	0.
time (sec)	N/A	0.086	0.122	0.079	0.	0.	0.	0.

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	123	123	132	329	0	0	0	0
normalized size	1	1.	1.07	2.67	0.	0.	0.	0.
time (sec)	N/A	0.163	0.379	0.07	0.	0.	0.	0.

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	98	204	0	5576	0	0
normalized size	1	1.	1.18	2.46	0.	67.18	0.	0.
time (sec)	N/A	0.099	0.269	0.109	0.	2.562	0.	0.

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	49	108	0	4286	0	0
normalized size	1	1.	1.2	2.63	0.	104.54	0.	0.
time (sec)	N/A	0.051	0.109	0.054	0.	2.362	0.	0.

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	49	113	0	1561	0	143
normalized size	1	1.	1.17	2.69	0.	37.17	0.	3.4
time (sec)	N/A	0.08	0.178	0.069	0.	2.077	0.	1.401

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	102	232	0	3468	0	0
normalized size	1	1.	1.15	2.61	0.	38.97	0.	0.
time (sec)	N/A	0.115	0.319	0.091	0.	2.567	0.	0.

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	229	229	168	344	0	0	0	0
normalized size	1	1.	0.73	1.5	0.	0.	0.	0.
time (sec)	N/A	0.218	0.952	0.068	0.	0.	0.	0.

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	128	128	89	113	0	0	0	0
normalized size	1	1.	0.7	0.88	0.	0.	0.	0.
time (sec)	N/A	0.136	0.257	0.066	0.	0.	0.	0.

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0
normalized size	1	1.	1.13	1.43	0.	0.	0.	0.
time (sec)	N/A	0.036	0.081	0.047	0.	0.	0.	0.

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	150	189	0	0	0	0
normalized size	1	1.	1.12	1.41	0.	0.	0.	0.
time (sec)	N/A	0.149	0.515	0.079	0.	0.	0.	0.

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	201	456	0	0	0	0
normalized size	1	1.	0.75	1.71	0.	0.	0.	0.
time (sec)	N/A	0.284	3.795	0.098	0.	0.	0.	0.

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	98	146	0	7602	0	0
normalized size	1	1.	1.18	1.76	0.	91.59	0.	0.
time (sec)	N/A	0.112	0.422	0.109	0.	3.168	0.	0.

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	36	36	43	32	319	738	0	207
normalized size	1	1.	1.19	0.89	8.86	20.5	0.	5.75
time (sec)	N/A	0.053	0.155	0.051	1.606	2.172	0.	1.229

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	98	154	0	4082	0	331
normalized size	1	1.	1.17	1.83	0.	48.6	0.	3.94
time (sec)	N/A	0.108	0.401	0.123	0.	2.585	0.	1.424

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	134	251	0	10539	0	0
normalized size	1	1.	0.96	1.81	0.	75.82	0.	0.
time (sec)	N/A	0.182	0.712	0.132	0.	4.792	0.	0.

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	211	500	0	0	0	0
normalized size	1	1.	0.62	1.47	0.	0.	0.	0.
time (sec)	N/A	0.341	1.252	0.102	0.	0.	0.	0.

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	156	313	0	0	0	0
normalized size	1	1.	0.61	1.22	0.	0.	0.	0.
time (sec)	N/A	0.235	1.05	0.093	0.	0.	0.	0.

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	173	173	151	127	0	0	0	0
normalized size	1	1.	0.87	0.73	0.	0.	0.	0.
time (sec)	N/A	0.212	0.481	0.084	0.	0.	0.	0.

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	252	0	0	0	0
normalized size	1	1.	0.87	2.19	0.	0.	0.	0.
time (sec)	N/A	0.062	0.159	0.078	0.	0.	0.	0.

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	185	284	0	0	0	0
normalized size	1	1.	0.64	0.98	0.	0.	0.	0.
time (sec)	N/A	0.31	1.262	0.104	0.	0.	0.	0.

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	143	143	130	230	0	18590	0	0
normalized size	1	1.	0.91	1.61	0.	130.	0.	0.
time (sec)	N/A	0.166	0.871	0.16	0.	6.888	0.	0.

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	67	58	1251	2873	0	699
normalized size	1	1.	0.77	0.67	14.38	33.02	0.	8.03
time (sec)	N/A	0.107	0.329	0.077	1.774	3.411	0.	1.339

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	63	57	655	2789	0	622
normalized size	1	1.	0.8	0.72	8.29	35.3	0.	7.87
time (sec)	N/A	0.068	0.18	0.066	1.675	3.325	0.	1.378

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	130	236	0	12269	0	857
normalized size	1	1.	0.96	1.74	0.	90.21	0.	6.3
time (sec)	N/A	0.171	0.798	0.176	0.	5.343	0.	1.518

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	344	344	207	868	0	0	0	0
normalized size	1	1.	0.6	2.52	0.	0.	0.	0.
time (sec)	N/A	0.366	2.035	0.13	0.	0.	0.	0.

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	198	659	0	0	0	0
normalized size	1	1.	0.81	2.7	0.	0.	0.	0.
time (sec)	N/A	0.249	1.609	0.107	0.	0.	0.	0.

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	187	598	0	0	0	0
normalized size	1	1.	0.78	2.48	0.	0.	0.	0.
time (sec)	N/A	0.326	1.403	0.114	0.	0.	0.	0.

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0
normalized size	1	1.	0.76	1.62	0.	0.	0.	0.
time (sec)	N/A	0.28	1.32	0.198	0.	0.	0.	0.

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	234	747	0	0	0	0
normalized size	1	1.	0.61	1.94	0.	0.	0.	0.
time (sec)	N/A	0.456	2.311	0.128	0.	0.	0.	0.

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	14	14	19	15	7	39	0	7
normalized size	1	1.	1.36	1.07	0.5	2.79	0.	0.5
time (sec)	N/A	0.024	0.011	0.05	1.657	2.036	0.	1.146

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	11	11	11	41	0	0	0	0
normalized size	1	1.	1.	3.73	0.	0.	0.	0.
time (sec)	N/A	0.011	0.04	0.147	0.	0.	0.	0.

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	61	0	0	0	0
normalized size	1	1.	1.	1.85	0.	0.	0.	0.
time (sec)	N/A	0.022	0.041	0.068	0.	0.	0.	0.

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	C	C	F	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	16	16	21	34	7	39	0	7
normalized size	1	1.	1.31	2.12	0.44	2.44	0.	0.44
time (sec)	N/A	0.024	0.008	0.04	1.925	1.775	0.	1.15

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	53	63	0	0	0	0
normalized size	1	1.	1.26	1.5	0.	0.	0.	0.
time (sec)	N/A	0.033	0.051	0.059	0.	0.	0.	0.

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	128	128	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.116	8.657	0.641	0.	0.	0.	0.

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	226	226	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.259	11.434	0.629	0.	0.	0.	0.

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	137	137	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.123	15.097	0.436	0.	0.	0.	0.

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	78	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.058	0.207	0.331	0.	0.	0.	0.

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.088	4.189	0.264	0.	0.	0.	0.

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	87	87	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	7.895	0.294	0.	0.	0.	0.

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	88	88	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.097	15.005	0.273	0.	0.	0.	0.

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.11	10.263	0.497	0.	0.	0.	0.

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	F	F	F	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	101	101	250	0	0	0	0	0
normalized size	1	1.	2.48	0.	0.	0.	0.	0.
time (sec)	N/A	0.192	0.745	0.356	0.	0.	0.	0.

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	5.551	0.26	0.	0.	0.	0.

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.105	10.55	0.249	0.	0.	0.	0.

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	81	82	221	539	192	213
normalized size	1	1.	0.76	0.77	2.08	5.08	1.81	2.01
time (sec)	N/A	0.094	0.103	0.016	1.197	1.931	8.107	1.212

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	66	72	193	375	194	177
normalized size	1	1.	0.67	0.73	1.95	3.79	1.96	1.79
time (sec)	N/A	0.108	0.129	0.015	1.116	1.884	4.725	1.153

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	79	60	162	302	117	149
normalized size	1	1.	1.13	0.86	2.31	4.31	1.67	2.13
time (sec)	N/A	0.071	0.095	0.013	1.118	2.054	2.307	1.165

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	45	50	100	171	121	113
normalized size	1	1.	0.75	0.83	1.67	2.85	2.02	1.88
time (sec)	N/A	0.068	0.126	0.014	1.151	1.802	1.248	1.172

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	34	28	80	128	41	72
normalized size	1	1.	1.06	0.88	2.5	4.	1.28	2.25
time (sec)	N/A	0.021	0.011	0.005	1.155	1.905	0.55	1.124

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	40	40	72	40	68	721	0	97
normalized size	1	1.	1.8	1.	1.7	18.02	0.	2.42
time (sec)	N/A	0.056	0.054	0.036	1.145	1.998	0.	1.139

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	24	24	35	23	63	103	0	84
normalized size	1	1.	1.46	0.96	2.62	4.29	0.	3.5
time (sec)	N/A	0.05	0.029	0.036	1.135	1.951	0.	1.204

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	63	37	123	1413	0	108
normalized size	1	1.	1.62	0.95	3.15	36.23	0.	2.77
time (sec)	N/A	0.063	0.013	0.04	1.099	2.126	0.	1.161

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	76	36	177	1773	0	88
normalized size	1	1.	1.85	0.88	4.32	43.24	0.	2.15
time (sec)	N/A	0.057	0.03	0.043	1.162	2.029	0.	1.194

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	192	192	125	128	367	973	325	369
normalized size	1	1.	0.65	0.67	1.91	5.07	1.69	1.92
time (sec)	N/A	0.179	0.683	0.02	1.189	1.918	31.015	1.269

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	180	180	133	122	320	734	340	317
normalized size	1	1.	0.74	0.68	1.78	4.08	1.89	1.76
time (sec)	N/A	0.154	0.13	0.021	1.152	1.909	15.338	1.251

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	92	96	243	594	219	266
normalized size	1	1.	0.71	0.74	1.87	4.57	1.68	2.05
time (sec)	N/A	0.119	0.41	0.019	1.073	1.903	8.429	1.203

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	94	85	204	421	212	212
normalized size	1	1.	0.82	0.75	1.79	3.69	1.86	1.86
time (sec)	N/A	0.083	0.146	0.017	1.142	1.987	5.482	1.128

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	96	76	189	2851	0	240
normalized size	1	1.	1.09	0.86	2.15	32.4	0.	2.73
time (sec)	N/A	0.102	0.302	0.056	1.176	2.16	0.	1.254

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	92	65	153	360	0	224
normalized size	1	1.	1.12	0.79	1.87	4.39	0.	2.73
time (sec)	N/A	0.098	0.285	0.046	1.076	1.912	0.	1.229

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	105	63	205	4246	0	244
normalized size	1	1.	1.36	0.82	2.66	55.14	0.	3.17
time (sec)	N/A	0.105	0.031	0.059	1.22	2.207	0.	1.229

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	76	76	81	65	230	4551	0	219
normalized size	1	1.	1.07	0.86	3.03	59.88	0.	2.88
time (sec)	N/A	0.091	0.399	0.069	1.26	2.081	0.	1.253

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	149	66	254	5495	0	246
normalized size	1	1.	1.66	0.73	2.82	61.06	0.	2.73
time (sec)	N/A	0.132	0.038	0.109	1.122	2.27	0.	1.267

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	197	73	409	6268	0	197
normalized size	1	1.	2.24	0.83	4.65	71.23	0.	2.24
time (sec)	N/A	0.103	0.941	0.108	1.153	2.312	0.	1.248

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	235	90	427	9393	0	281
normalized size	1	1.	1.77	0.68	3.21	70.62	0.	2.11
time (sec)	N/A	0.175	0.053	0.099	1.197	2.284	0.	1.215

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	291	291	194	188	522	1546	498	545
normalized size	1	1.	0.67	0.65	1.79	5.31	1.71	1.87
time (sec)	N/A	0.209	0.477	0.06	1.09	1.931	83.459	1.442

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	267	267	184	168	429	1193	496	474
normalized size	1	1.	0.69	0.63	1.61	4.47	1.86	1.78
time (sec)	N/A	0.222	0.459	0.023	1.147	2.006	97.203	1.476

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	204	204	159	141	378	1017	340	405
normalized size	1	1.	0.78	0.69	1.85	4.99	1.67	1.99
time (sec)	N/A	0.128	0.251	0.02	1.177	1.813	33.478	1.163

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	201	201	158	138	347	7191	0	427
normalized size	1	1.	0.79	0.69	1.73	35.78	0.	2.12
time (sec)	N/A	0.189	0.219	0.072	1.184	2.071	0.	1.407

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	140	111	297	809	0	410
normalized size	1	1.	0.92	0.73	1.95	5.32	0.	2.7
time (sec)	N/A	0.138	1.144	0.06	1.201	1.871	0.	1.448

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	150	115	329	9546	0	428
normalized size	1	1.	0.96	0.74	2.11	61.19	0.	2.74
time (sec)	N/A	0.169	3.702	0.066	1.173	2.618	0.	1.452

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	169	101	351	10166	0	419
normalized size	1	1.	1.31	0.78	2.72	78.81	0.	3.25
time (sec)	N/A	0.12	0.445	0.077	1.208	2.437	0.	1.437

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	218	108	344	11867	0	475
normalized size	1	1.	1.47	0.73	2.32	80.18	0.	3.21
time (sec)	N/A	0.173	6.132	0.078	1.161	2.602	0.	1.409

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	131	131	225	99	493	12388	0	389
normalized size	1	1.	1.72	0.76	3.76	94.56	0.	2.97
time (sec)	N/A	0.125	1.846	0.085	1.087	2.366	0.	1.442

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	166	166	236	119	479	16188	0	455
normalized size	1	1.	1.42	0.72	2.89	97.52	0.	2.74
time (sec)	N/A	0.197	1.552	0.083	1.084	2.782	0.	1.461

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	328	328	168	259	0	0	0	0
normalized size	1	1.	0.51	0.79	0.	0.	0.	0.
time (sec)	N/A	0.739	0.349	0.066	0.	0.	0.	0.

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	295	295	299	207	0	0	0	0
normalized size	1	1.	1.01	0.7	0.	0.	0.	0.
time (sec)	N/A	0.559	0.316	0.06	0.	0.	0.	0.

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	303	303	214	128	0	0	0	0
normalized size	1	1.	0.71	0.42	0.	0.	0.	0.
time (sec)	N/A	0.545	0.345	0.052	0.	0.	0.	0.

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	294	294	145	129	0	0	0	0
normalized size	1	1.	0.49	0.44	0.	0.	0.	0.
time (sec)	N/A	0.44	0.231	0.043	0.	0.	0.	0.

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	262	262	275	78	0	0	0	0
normalized size	1	1.	1.05	0.3	0.	0.	0.	0.
time (sec)	N/A	0.271	0.178	0.042	0.	0.	0.	0.

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	199	82	0	0	0	0
normalized size	1	1.	0.69	0.28	0.	0.	0.	0.
time (sec)	N/A	0.336	0.236	0.038	0.	0.	0.	0.

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	280	280	131	87	0	0	0	0
normalized size	1	1.	0.47	0.31	0.	0.	0.	0.
time (sec)	N/A	0.251	0.174	0.04	0.	0.	0.	0.

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	295	100	0	0	0	0
normalized size	1	1.	1.03	0.35	0.	0.	0.	0.
time (sec)	N/A	0.432	0.254	0.072	0.	0.	0.	0.

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	304	304	230	123	0	0	0	0
normalized size	1	1.	0.76	0.4	0.	0.	0.	0.
time (sec)	N/A	0.478	0.39	0.075	0.	0.	0.	0.

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	322	322	178	146	0	0	0	0
normalized size	1	1.	0.55	0.45	0.	0.	0.	0.
time (sec)	N/A	0.457	0.497	0.089	0.	0.	0.	0.

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	317	317	370	178	0	0	0	0
normalized size	1	1.	1.17	0.56	0.	0.	0.	0.
time (sec)	N/A	0.433	6.062	0.087	0.	0.	0.	0.

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	156	82	0	591	1423	140
normalized size	1	1.	1.12	0.59	0.	4.25	10.24	1.01
time (sec)	N/A	0.188	1.399	0.029	0.	1.935	174.633	1.163

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	156	80	0	590	3742	146
normalized size	1	1.	1.17	0.6	0.	4.44	28.14	1.1
time (sec)	N/A	0.189	1.219	0.029	0.	1.851	72.418	1.18

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	82	98	236	467	306	258
normalized size	1	1.	0.74	0.88	2.13	4.21	2.76	2.32
time (sec)	N/A	0.143	0.142	0.014	1.141	1.63	14.166	1.203

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	93	66	212	439	128	198
normalized size	1	1.	1.39	0.99	3.16	6.55	1.91	2.96
time (sec)	N/A	0.066	0.029	0.014	1.129	1.635	8.487	1.162

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	63	76	165	292	206	193
normalized size	1	1.	0.76	0.92	1.99	3.52	2.48	2.33
time (sec)	N/A	0.098	0.077	0.014	1.114	1.604	4.94	1.166

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	69	44	131	250	80	132
normalized size	1	1.	1.5	0.96	2.85	5.43	1.74	2.87
time (sec)	N/A	0.034	0.022	0.013	1.056	1.67	2.538	1.186

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	49	44	89	155	100	99
normalized size	1	1.	0.94	0.85	1.71	2.98	1.92	1.9
time (sec)	N/A	0.034	0.054	0.005	1.164	1.619	1.248	1.129

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	70	36	96	1100	0	126
normalized size	1	1.	1.67	0.86	2.29	26.19	0.	3.
time (sec)	N/A	0.043	0.024	0.029	1.161	1.69	0.	1.181

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	39	39	45	39	73	185	0	124
normalized size	1	1.	1.15	1.	1.87	4.74	0.	3.18
time (sec)	N/A	0.052	0.119	0.03	1.16	1.672	0.	1.154

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	82	38	155	1867	0	155
normalized size	1	1.	1.74	0.81	3.3	39.72	0.	3.3
time (sec)	N/A	0.054	0.026	0.04	1.237	1.742	0.	1.199

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	40	33	131	333	0	61
normalized size	1	1.	1.29	1.06	4.23	10.74	0.	1.97
time (sec)	N/A	0.05	0.015	0.039	1.165	1.673	0.	1.162

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	139	54	235	3906	0	174
normalized size	1	1.	2.17	0.84	3.67	61.03	0.	2.72
time (sec)	N/A	0.063	0.029	0.039	1.04	1.86	0.	1.165

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	71	45	308	903	0	131
normalized size	1	1.	1.51	0.96	6.55	19.21	0.	2.79
time (sec)	N/A	0.043	0.034	0.036	1.043	1.661	0.	1.149

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	199	78	362	8357	0	285
normalized size	1	1.	2.16	0.85	3.93	90.84	0.	3.1
time (sec)	N/A	0.084	0.035	0.041	1.056	1.881	0.	1.174

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	207	132	414	1111	280	443
normalized size	1	1.	1.72	1.1	3.45	9.26	2.33	3.69
time (sec)	N/A	0.129	0.064	0.023	1.046	1.632	74.891	1.32

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	139	148	351	802	484	428
normalized size	1	1.	0.86	0.92	2.18	4.98	3.01	2.66
time (sec)	N/A	0.281	0.352	0.022	1.102	1.651	60.839	1.348

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	164	100	305	740	204	331
normalized size	1	1.	1.78	1.09	3.32	8.04	2.22	3.6
time (sec)	N/A	0.086	0.044	0.021	1.047	1.632	43.727	1.277

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	92	111	247	522	332	319
normalized size	1	1.	0.74	0.89	1.98	4.18	2.66	2.55
time (sec)	N/A	0.162	0.166	0.017	1.117	1.629	19.201	1.19

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	146	82	239	4263	0	301
normalized size	1	1.	1.59	0.89	2.6	46.34	0.	3.27
time (sec)	N/A	0.093	0.041	0.039	1.051	1.921	0.	1.303

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	103	103	77	91	197	562	0	273
normalized size	1	1.	0.75	0.88	1.91	5.46	0.	2.65
time (sec)	N/A	0.194	0.306	0.038	1.088	1.708	0.	1.293

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	144	74	275	6047	0	275
normalized size	1	1.	1.57	0.8	2.99	65.73	0.	2.99
time (sec)	N/A	0.135	0.046	0.048	1.118	1.982	0.	1.284

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	68	75	223	761	0	207
normalized size	1	1.	0.75	0.82	2.45	8.36	0.	2.27
time (sec)	N/A	0.165	0.325	0.044	1.048	1.748	0.	1.317

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	101	101	186	79	316	8741	0	262
normalized size	1	1.	1.84	0.78	3.13	86.54	0.	2.59
time (sec)	N/A	0.151	0.043	0.057	1.126	2.072	0.	1.318

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	67	74	360	1185	0	234
normalized size	1	1.	0.8	0.88	4.29	14.11	0.	2.79
time (sec)	N/A	0.146	0.635	0.049	1.065	1.714	0.	1.279

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	111	111	240	92	404	11957	0	338
normalized size	1	1.	2.16	0.83	3.64	107.72	0.	3.05
time (sec)	N/A	0.174	0.036	0.048	1.041	2.119	0.	1.278

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	288	258	810	2898	0	934
normalized size	1	1.	1.31	1.17	3.68	13.17	0.	4.25
time (sec)	N/A	0.218	2.266	0.096	1.076	1.787	0.	1.794

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	183	183	185	218	676	2207	0	779
normalized size	1	1.	1.01	1.19	3.69	12.06	0.	4.26
time (sec)	N/A	0.175	2.47	0.059	1.08	1.731	0.	1.721

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	157	176	539	1620	377	621
normalized size	1	1.	1.1	1.23	3.77	11.33	2.64	4.34
time (sec)	N/A	0.157	0.901	0.02	1.045	1.736	169.693	1.678

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	139	148	441	10637	0	570
normalized size	1	1.	0.88	0.94	2.79	67.32	0.	3.61
time (sec)	N/A	0.135	0.381	0.039	1.066	2.04	0.	1.65

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	155	130	451	13697	0	455
normalized size	1	1.	1.05	0.88	3.05	92.55	0.	3.07
time (sec)	N/A	0.209	0.34	0.049	1.079	2.122	0.	1.669

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	173	125	459	17403	0	404
normalized size	1	1.	1.22	0.88	3.23	122.56	0.	2.85
time (sec)	N/A	0.273	0.383	0.046	1.058	2.428	0.	1.743

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	223	128	527	22631	0	463
normalized size	1	1.	1.43	0.82	3.38	145.07	0.	2.97
time (sec)	N/A	0.304	0.703	0.047	1.114	2.397	0.	1.684

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	171	171	219	143	625	30027	0	473
normalized size	1	1.	1.28	0.84	3.65	175.6	0.	2.77
time (sec)	N/A	0.332	1.694	0.083	1.132	2.77	0.	1.703

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	189	189	265	166	774	0	0	653
normalized size	1	1.	1.4	0.88	4.1	0.	0.	3.46
time (sec)	N/A	0.369	2.473	0.081	1.148	0.	0.	1.704

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	246	202	972	0	0	730
normalized size	1	1.	1.12	0.92	4.42	0.	0.	3.32
time (sec)	N/A	0.398	2.069	0.085	1.152	0.	0.	1.651

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	255	255	189	240	597	1688	0	756
normalized size	1	1.	0.74	0.94	2.34	6.62	0.	2.96
time (sec)	N/A	0.557	0.68	0.054	1.218	1.405	0.	1.703

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	211	211	156	193	464	1200	666	601
normalized size	1	1.	0.74	0.91	2.2	5.69	3.16	2.85
time (sec)	N/A	0.387	0.39	0.056	1.112	1.484	103.123	1.141

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	181	181	134	163	383	1247	0	531
normalized size	1	1.	0.74	0.9	2.12	6.89	0.	2.93
time (sec)	N/A	0.438	0.771	0.039	1.092	1.445	0.	1.704

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	161	161	131	137	381	1476	0	423
normalized size	1	1.	0.81	0.85	2.37	9.17	0.	2.63
time (sec)	N/A	0.384	0.782	0.048	1.108	1.368	0.	1.704

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	110	126	485	2006	0	414
normalized size	1	1.	0.74	0.85	3.28	13.55	0.	2.8
time (sec)	N/A	0.355	1.16	0.046	1.066	1.523	0.	1.707

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	106	121	725	2464	0	356
normalized size	1	1.	0.8	0.91	5.45	18.53	0.	2.68
time (sec)	N/A	0.289	0.692	0.089	1.148	1.493	0.	1.707

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	115	130	1137	3646	0	495
normalized size	1	1.	0.82	0.93	8.12	26.04	0.	3.54
time (sec)	N/A	0.223	0.579	0.084	1.217	1.567	0.	1.715

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	147	147	239	145	1743	4510	0	485
normalized size	1	1.	1.63	0.99	11.86	30.68	0.	3.3
time (sec)	N/A	0.149	6.105	0.086	1.24	1.721	0.	1.699

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	144	144	350	177	2587	6778	0	760
normalized size	1	1.	2.43	1.23	17.97	47.07	0.	5.28
time (sec)	N/A	0.131	3.207	0.098	1.304	1.803	0.	1.667

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	182	182	404	218	3687	8951	0	838
normalized size	1	1.	2.22	1.2	20.26	49.18	0.	4.6
time (sec)	N/A	0.159	4.51	0.085	1.381	1.89	0.	1.707

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	494	258	5021	11491	0	917
normalized size	1	1.	2.24	1.17	22.72	52.	0.	4.15
time (sec)	N/A	0.192	6.181	0.085	1.297	1.909	0.	1.654

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	248	248	548	298	6592	13925	0	995
normalized size	1	1.	2.21	1.2	26.58	56.15	0.	4.01
time (sec)	N/A	0.223	6.168	0.09	1.29	2.261	0.	1.664

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	390	270	0	3729	0	0
normalized size	1	1.	2.64	1.82	0.	25.2	0.	0.
time (sec)	N/A	0.254	0.336	0.082	0.	2.639	0.	0.

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	235	175	0	2755	0	0
normalized size	1	1.	1.69	1.26	0.	19.82	0.	0.
time (sec)	N/A	0.198	0.26	0.048	0.	2.111	0.	0.

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	365	142	0	2202	0	0
normalized size	1	1.	3.17	1.23	0.	19.15	0.	0.
time (sec)	N/A	0.115	0.177	0.029	0.	1.902	0.	0.

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	221	126	0	2256	0	0
normalized size	1	1.	1.77	1.01	0.	18.05	0.	0.
time (sec)	N/A	0.104	0.168	0.04	0.	1.981	0.	0.

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	385	159	0	2431	0	0
normalized size	1	1.	2.83	1.17	0.	17.88	0.	0.
time (sec)	N/A	0.163	0.249	0.053	0.	2.261	0.	0.

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	184	184	265	190	0	4716	0	0
normalized size	1	1.	1.44	1.03	0.	25.63	0.	0.
time (sec)	N/A	0.201	0.379	0.066	0.	2.586	0.	0.

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	158	223	0	3123	0	82
normalized size	1	1.	0.9	1.27	0.	17.85	0.	0.47
time (sec)	N/A	0.258	0.867	0.051	0.	2.398	0.	1.308

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	127	127	143	144	0	2128	0	18
normalized size	1	1.	1.13	1.13	0.	16.76	0.	0.14
time (sec)	N/A	0.206	0.45	0.033	0.	2.164	0.	1.323

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	127	94	0	2130	0	1
normalized size	1	1.	1.02	0.75	0.	17.04	0.	0.01
time (sec)	N/A	0.121	0.349	0.031	0.	2.213	0.	1.616

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	128	102	0	2130	0	0
normalized size	1	1.	1.11	0.89	0.	18.52	0.	0.
time (sec)	N/A	0.098	0.239	0.033	0.	2.163	0.	0.

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	139	139	143	135	0	2840	0	28
normalized size	1	1.	1.03	0.97	0.	20.43	0.	0.2
time (sec)	N/A	0.181	0.814	0.059	0.	2.286	0.	2.003

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	148	148	165	179	0	5195	0	46
normalized size	1	1.	1.11	1.21	0.	35.1	0.	0.31
time (sec)	N/A	0.211	2.193	0.075	0.	2.219	0.	2.853

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	235	235	615	1191	0	17667	0	0
normalized size	1	1.	2.62	5.07	0.	75.18	0.	0.
time (sec)	N/A	0.486	0.949	0.082	0.	3.045	0.	0.

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	737	1200	0	14307	0	0
normalized size	1	1.	3.51	5.71	0.	68.13	0.	0.
time (sec)	N/A	0.37	0.663	0.184	0.	2.538	0.	0.

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	597	1116	0	13905	0	0
normalized size	1	1.	2.75	5.14	0.	64.08	0.	0.
time (sec)	N/A	0.294	0.551	0.077	0.	2.66	0.	0.

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	186	186	422	767	0	11888	0	0
normalized size	1	1.	2.27	4.12	0.	63.91	0.	0.
time (sec)	N/A	0.205	0.485	0.058	0.	2.364	0.	0.

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	221	221	597	1112	0	13570	0	0
normalized size	1	1.	2.7	5.03	0.	61.4	0.	0.
time (sec)	N/A	0.299	0.354	0.072	0.	2.731	0.	0.

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	761	966	0	17785	0	0
normalized size	1	1.	2.34	2.97	0.	54.72	0.	0.
time (sec)	N/A	0.361	0.838	0.096	0.	4.222	0.	0.

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	320	320	262	574	0	15757	0	201
normalized size	1	1.	0.82	1.79	0.	49.24	0.	0.63
time (sec)	N/A	0.461	4.637	0.063	0.	3.88	0.	1.193

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	233	233	238	716	0	13423	0	205
normalized size	1	1.	1.02	3.07	0.	57.61	0.	0.88
time (sec)	N/A	0.338	2.59	0.054	0.	3.555	0.	4.439

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	195	195	225	490	0	12467	0	171
normalized size	1	1.	1.15	2.51	0.	63.93	0.	0.88
time (sec)	N/A	0.249	4.137	0.046	0.	2.729	0.	4.337

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	220	220	253	708	0	14390	0	205
normalized size	1	1.	1.15	3.22	0.	65.41	0.	0.93
time (sec)	N/A	0.298	2.064	0.069	0.	3.52	0.	4.23

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	230	534	0	14862	0	171
normalized size	1	1.	1.1	2.54	0.	70.77	0.	0.81
time (sec)	N/A	0.246	2.932	0.062	0.	3.535	0.	4.289

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	272	765	0	20254	0	321
normalized size	1	1.	1.15	3.23	0.	85.46	0.	1.35
time (sec)	N/A	0.534	1.873	0.09	0.	4.285	0.	5.799

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F(-1)	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	315	315	1021	3542	0	0	0	0
normalized size	1	1.	3.24	11.24	0.	0.	0.	0.
time (sec)	N/A	0.577	1.689	0.099	0.	0.	0.	0.

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	290	290	802	2563	0	0	0	0
normalized size	1	1.	2.77	8.84	0.	0.	0.	0.
time (sec)	N/A	0.467	1.295	0.088	0.	0.	0.	0.

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	1019	3511	0	0	0	0
normalized size	1	1.	3.26	11.22	0.	0.	0.	0.
time (sec)	N/A	0.498	1.91	0.096	0.	0.	0.	0.

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	288	288	802	2916	0	0	0	0
normalized size	1	1.	2.78	10.12	0.	0.	0.	0.
time (sec)	N/A	0.519	1.3	0.082	0.	0.	0.	0.

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	313	313	1018	3512	0	0	0	0
normalized size	1	1.	3.25	11.22	0.	0.	0.	0.
time (sec)	N/A	0.463	1.367	0.106	0.	0.	0.	0.

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	617	617	1189	3159	0	0	0	0
normalized size	1	1.	1.93	5.12	0.	0.	0.	0.
time (sec)	N/A	0.819	5.757	0.138	0.	0.	0.	0.

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	319	319	331	2236	0	0	0	527
normalized size	1	1.	1.04	7.01	0.	0.	0.	1.65
time (sec)	N/A	0.485	3.953	0.092	0.	0.	0.	17.417

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	345	345	351	2681	0	0	0	609
normalized size	1	1.	1.02	7.77	0.	0.	0.	1.77
time (sec)	N/A	0.733	3.307	0.088	0.	0.	0.	18.204

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	314	314	316	2214	0	0	0	489
normalized size	1	1.	1.01	7.05	0.	0.	0.	1.56
time (sec)	N/A	0.648	4.751	0.083	0.	0.	0.	17.54

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	348	348	343	2670	0	0	0	608
normalized size	1	1.	0.99	7.67	0.	0.	0.	1.75
time (sec)	N/A	0.664	4.873	0.095	0.	0.	0.	17.205

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	320	320	333	2290	0	0	0	529
normalized size	1	1.	1.04	7.16	0.	0.	0.	1.65
time (sec)	N/A	0.609	2.988	0.103	0.	0.	0.	17.829

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	359	359	357	2747	0	0	0	660
normalized size	1	1.	0.99	7.65	0.	0.	0.	1.84
time (sec)	N/A	1.158	3.456	0.158	0.	0.	0.	23.584

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	24	55	93	382	0	65
normalized size	1	1.	0.96	2.2	3.72	15.28	0.	2.6
time (sec)	N/A	0.018	0.1	0.02	1.541	2.041	0.	1.149

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	C
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	176	176	45	44	0	1874	0	293
normalized size	1	1.	0.26	0.25	0.	10.65	0.	1.66
time (sec)	N/A	0.159	0.072	0.024	0.	2.257	0.	1.242

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	435	435	141	113	0	0	0	0
normalized size	1	1.	0.32	0.26	0.	0.	0.	0.
time (sec)	N/A	0.975	0.326	0.032	0.	0.	0.	0.

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	175	175	134	128	0	0	0	1
normalized size	1	1.	0.77	0.73	0.	0.	0.	0.01
time (sec)	N/A	0.279	0.169	0.03	0.	0.	0.	1.379

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	245	245	160	162	0	0	0	1
normalized size	1	1.	0.65	0.66	0.	0.	0.	0.
time (sec)	N/A	0.517	0.246	0.033	0.	0.	0.	1.904

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	242	242	439	124	0	11367	0	0
normalized size	1	1.	1.81	0.51	0.	46.97	0.	0.
time (sec)	N/A	0.518	0.916	0.033	0.	3.486	0.	0.

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	71	71	87	61	0	2303	0	14
normalized size	1	1.	1.23	0.86	0.	32.44	0.	0.2
time (sec)	N/A	0.138	0.201	0.033	0.	1.847	0.	1.274

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	129	129	127	64	0	11979	0	1
normalized size	1	1.	0.98	0.5	0.	92.86	0.	0.01
time (sec)	N/A	0.198	0.123	0.036	0.	2.875	0.	1.279

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	437	124	0	11359	0	0
normalized size	1	1.	1.92	0.54	0.	49.82	0.	0.
time (sec)	N/A	0.417	0.868	0.036	0.	4.086	0.	0.

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	70	160	0	505	0	193
normalized size	1	1.	0.84	1.93	0.	6.08	0.	2.33
time (sec)	N/A	0.1	0.431	0.032	0.	1.744	0.	1.175

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	64	99	0	2198	0	65
normalized size	1	1.	0.93	1.43	0.	31.86	0.	0.94
time (sec)	N/A	0.077	0.483	0.033	0.	1.858	0.	1.389

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	19	67	46	65	124	39
normalized size	1	1.	1.06	3.72	2.56	3.61	6.89	2.17
time (sec)	N/A	0.052	0.003	0.025	1.031	1.476	10.048	1.141

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	A	A	B	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	20	20	18	78	34	39	153	38
normalized size	1	1.	0.9	3.9	1.7	1.95	7.65	1.9
time (sec)	N/A	0.05	0.002	0.022	1.053	1.462	6.117	1.131

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	23	15	17	19
normalized size	1	1.	1.	1.17	3.83	2.5	2.83	3.17
time (sec)	N/A	0.044	0.002	0.012	1.171	1.441	3.295	1.11

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	5	5	5	11	7	7	2	7
normalized size	1	1.	1.	2.2	1.4	1.4	0.4	1.4
time (sec)	N/A	0.042	0.	0.012	1.114	1.423	1.772	1.155

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	7	7	12	8	14	42	5	11
normalized size	1	1.	1.71	1.14	2.	6.	0.71	1.57
time (sec)	N/A	0.028	0.003	0.008	1.576	1.446	0.251	1.108

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	22	22	20	50	54	522	0	70
normalized size	1	1.	0.91	2.27	2.45	23.73	0.	3.18
time (sec)	N/A	0.045	0.004	0.021	1.523	1.487	0.	1.137

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	35	35	34	94	93	1607	0	90
normalized size	1	1.	0.97	2.69	2.66	45.91	0.	2.57
time (sec)	N/A	0.06	0.004	0.026	1.517	1.491	0.	1.158

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	63	95	205	306	250	221
normalized size	1	1.	0.71	1.07	2.3	3.44	2.81	2.48
time (sec)	N/A	0.057	0.168	0.059	1.013	1.488	4.286	1.2

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	48	65	184	220	85	166
normalized size	1	1.	1.04	1.41	4.	4.78	1.85	3.61
time (sec)	N/A	0.042	0.139	0.053	1.013	1.482	2.047	1.174

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	43	70	103	153	150	124
normalized size	1	1.	0.7	1.15	1.69	2.51	2.46	2.03
time (sec)	N/A	0.049	0.07	0.026	1.007	1.459	1.14	1.126

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	39	25	35	103	36	97
normalized size	1	1.	1.39	0.89	1.25	3.68	1.29	3.46
time (sec)	N/A	0.022	0.012	0.01	1.033	1.449	0.487	1.133

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	37	39	76	282	0	61
normalized size	1	1.	1.32	1.39	2.71	10.07	0.	2.18
time (sec)	N/A	0.032	0.014	0.028	1.519	1.515	0.	1.128

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	36	29	63	101	0	46
normalized size	1	1.	1.89	1.53	3.32	5.32	0.	2.42
time (sec)	N/A	0.034	0.014	0.056	1.05	1.51	0.	1.151

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	71	82	184	903	0	143
normalized size	1	1.	1.69	1.95	4.38	21.5	0.	3.4
time (sec)	N/A	0.04	0.023	0.069	1.525	1.523	0.	1.184

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	44	65	250	427	0	63
normalized size	1	1.	1.38	2.03	7.81	13.34	0.	1.97
time (sec)	N/A	0.034	0.007	0.062	1.045	1.387	0.	1.186

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	60	124	308	2838	0	209
normalized size	1	1.	0.86	1.77	4.4	40.54	0.	2.99
time (sec)	N/A	0.047	0.149	0.072	1.537	1.6	0.	1.152

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	102	85	656	930	0	112
normalized size	1	1.	1.89	1.57	12.15	17.22	0.	2.07
time (sec)	N/A	0.047	0.05	0.066	1.07	1.458	0.	1.155

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	98	172	304	532	481	354
normalized size	1	1.	0.62	1.08	1.91	3.35	3.03	2.23
time (sec)	N/A	0.174	0.309	0.031	1.11	1.49	12.886	1.234

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	64	128	327	463	136	332
normalized size	1	1.	0.86	1.73	4.42	6.26	1.84	4.49
time (sec)	N/A	0.072	0.115	0.031	1.082	1.464	6.897	1.221

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	119	119	79	134	231	347	314	277
normalized size	1	1.	0.66	1.13	1.94	2.92	2.64	2.33
time (sec)	N/A	0.143	0.295	0.032	1.152	1.447	4.585	1.186

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	44	41	61	261	58	221
normalized size	1	1.	0.9	0.84	1.24	5.33	1.18	4.51
time (sec)	N/A	0.037	0.056	0.011	1.165	1.478	2.017	1.173

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	55	55	70	89	180	1143	0	159
normalized size	1	1.	1.27	1.62	3.27	20.78	0.	2.89
time (sec)	N/A	0.058	0.228	0.038	1.724	1.604	0.	1.182

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	A	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	50	71	161	235	0	184
normalized size	1	1.	0.94	1.34	3.04	4.43	0.	3.47
time (sec)	N/A	0.095	0.292	0.037	1.216	1.535	0.	1.189

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	64	64	253	169	316	1916	0	225
normalized size	1	1.	3.95	2.64	4.94	29.94	0.	3.52
time (sec)	N/A	0.08	7.367	0.047	1.893	1.626	0.	1.204

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	57	96	360	495	0	132
normalized size	1	1.	1.21	2.04	7.66	10.53	0.	2.81
time (sec)	N/A	0.063	0.332	0.046	1.116	1.482	0.	1.166

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	96	96	303	237	468	3672	0	297
normalized size	1	1.	3.16	2.47	4.88	38.25	0.	3.09
time (sec)	N/A	0.093	5.946	0.051	1.725	1.592	0.	1.16

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	69	158	942	1040	0	173
normalized size	1	1.	1.21	2.77	16.53	18.25	0.	3.04
time (sec)	N/A	0.06	0.373	0.048	1.127	1.46	0.	1.222

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	131	131	715	302	652	7356	0	394
normalized size	1	1.	5.46	2.31	4.98	56.15	0.	3.01
time (sec)	N/A	0.133	10.168	0.082	1.791	1.714	0.	1.179

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	238	238	144	267	490	906	774	598
normalized size	1	1.	0.61	1.12	2.06	3.81	3.25	2.51
time (sec)	N/A	0.332	0.521	0.032	1.104	1.497	34.767	1.262

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	98	98	83	209	471	787	182	506
normalized size	1	1.	0.85	2.13	4.81	8.03	1.86	5.16
time (sec)	N/A	0.09	0.234	0.035	1.099	1.52	19.494	1.26

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	A	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	120	216	387	626	559	482
normalized size	1	1.	0.59	1.06	1.91	3.08	2.75	2.37
time (sec)	N/A	0.268	0.302	0.032	1.12	1.481	13.704	1.279

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	59	56	85	512	75	387
normalized size	1	1.	0.88	0.84	1.27	7.64	1.12	5.78
time (sec)	N/A	0.045	0.116	0.011	1.054	1.548	6.669	1.284

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	100	155	315	2795	0	309
normalized size	1	1.	1.16	1.8	3.66	32.5	0.	3.59
time (sec)	N/A	0.076	0.531	0.056	1.695	1.622	0.	1.279

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	78	131	290	435	0	284
normalized size	1	1.	0.85	1.42	3.15	4.73	0.	3.09
time (sec)	N/A	0.129	0.511	0.039	1.091	1.551	0.	1.308

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	91	91	347	286	482	4188	0	354
normalized size	1	1.	3.81	3.14	5.3	46.02	0.	3.89
time (sec)	N/A	0.096	7.326	0.055	1.592	1.812	0.	1.43

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	82	82	84	148	516	788	0	290
normalized size	1	1.	1.02	1.8	6.29	9.61	0.	3.54
time (sec)	N/A	0.108	0.738	0.054	1.144	1.598	0.	1.372

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	103	103	472	376	660	5520	0	412
normalized size	1	1.	4.58	3.65	6.41	53.59	0.	4.
time (sec)	N/A	0.132	9.938	0.062	1.71	1.765	0.	1.358

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	74	74	86	199	1112	1287	0	288
normalized size	1	1.	1.16	2.69	15.03	17.39	0.	3.89
time (sec)	N/A	0.08	0.732	0.058	1.242	1.568	0.	1.38

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	B	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	154	154	1192	467	872	9044	0	518
normalized size	1	1.	7.74	3.03	5.66	58.73	0.	3.36
time (sec)	N/A	0.153	14.262	0.062	1.832	1.814	0.	1.296

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	80	80	163	289	2368	2063	0	351
normalized size	1	1.	2.04	3.61	29.6	25.79	0.	4.39
time (sec)	N/A	0.074	0.569	0.089	1.281	1.54	0.	1.267

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	108	108	117	1656	0	7526	0	0
normalized size	1	1.	1.08	15.33	0.	69.69	0.	0.
time (sec)	N/A	0.113	0.477	0.081	0.	1.859	0.	0.

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	121	121	106	1497	0	4566	0	320
normalized size	1	1.	0.88	12.37	0.	37.74	0.	2.64
time (sec)	N/A	0.221	0.317	0.063	0.	1.759	0.	1.301

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	79	1148	0	3804	0	0
normalized size	1	1.	1.03	14.91	0.	49.4	0.	0.
time (sec)	N/A	0.093	0.245	0.059	0.	1.72	0.	0.

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	81	81	80	993	0	2222	0	194
normalized size	1	1.	0.99	12.26	0.	27.43	0.	2.4
time (sec)	N/A	0.133	0.15	0.054	0.	1.709	0.	1.33

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	52	52	50	732	0	1754	0	0
normalized size	1	1.	0.96	14.08	0.	33.73	0.	0.
time (sec)	N/A	0.07	0.043	0.049	0.	1.613	0.	0.

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	50	577	0	1088	0	93
normalized size	1	1.	1.	11.54	0.	21.76	0.	1.86
time (sec)	N/A	0.082	0.082	0.04	0.	1.625	0.	1.281

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	32	32	32	24	0	1207	128	0
normalized size	1	1.	1.	0.75	0.	37.72	4.	0.
time (sec)	N/A	0.037	0.01	0.01	0.	1.573	5.011	0.

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	59	59	54	481	0	1341	0	289
normalized size	1	1.	0.92	8.15	0.	22.73	0.	4.9
time (sec)	N/A	0.067	0.126	0.058	0.	1.661	0.	1.431

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	60	335	0	1770	0	111
normalized size	1	1.	1.	5.58	0.	29.5	0.	1.85
time (sec)	N/A	0.079	0.154	0.06	0.	1.631	0.	1.225

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	92	92	91	662	0	4370	0	0
normalized size	1	1.	0.99	7.2	0.	47.5	0.	0.
time (sec)	N/A	0.105	0.222	0.068	0.	1.923	0.	0.

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	88	88	84	535	0	5762	0	192
normalized size	1	1.	0.95	6.08	0.	65.48	0.	2.18
time (sec)	N/A	0.105	0.491	0.07	0.	1.82	0.	1.411

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	138	138	139	1023	0	13526	0	0
normalized size	1	1.	1.01	7.41	0.	98.01	0.	0.
time (sec)	N/A	0.162	0.507	0.073	0.	2.41	0.	0.

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	119	907	0	14236	0	348
normalized size	1	1.	0.94	7.2	0.	112.98	0.	2.76
time (sec)	N/A	0.147	0.894	0.075	0.	2.133	0.	1.42

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	158	158	118	1659	0	8500	0	421
normalized size	1	1.	0.75	10.5	0.	53.8	0.	2.66
time (sec)	N/A	0.258	0.577	0.073	0.	1.906	0.	1.17

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	106	1403	0	6456	0	0
normalized size	1	1.	1.02	13.49	0.	62.08	0.	0.
time (sec)	N/A	0.138	0.283	0.065	0.	1.822	0.	0.

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	100	100	108	1116	0	3749	0	243
normalized size	1	1.	1.08	11.16	0.	37.49	0.	2.43
time (sec)	N/A	0.138	0.591	0.062	0.	1.752	0.	1.193

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	75	808	0	3937	0	0
normalized size	1	1.	0.97	10.49	0.	51.13	0.	0.
time (sec)	N/A	0.078	0.327	0.057	0.	1.656	0.	0.

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	78	404	0	3321	0	173
normalized size	1	1.	0.99	5.11	0.	42.04	0.	2.19
time (sec)	N/A	0.077	0.206	0.053	0.	1.68	0.	1.299

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	64	57	0	3372	428	0
normalized size	1	1.	0.97	0.86	0.	51.09	6.48	0.
time (sec)	N/A	0.047	0.046	0.013	0.	1.632	31.67	0.

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	174	1080	0	5192	0	0
normalized size	1	1.	1.64	10.19	0.	48.98	0.	0.
time (sec)	N/A	0.108	0.377	0.082	0.	2.003	0.	0.

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	105	798	0	7185	0	200
normalized size	1	1.	0.92	7.	0.	63.03	0.	1.75
time (sec)	N/A	0.179	0.855	0.075	0.	2.089	0.	2.441

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	230	1265	0	15107	0	1322
normalized size	1	1.	1.46	8.06	0.	96.22	0.	8.42
time (sec)	N/A	0.193	0.875	0.089	0.	3.007	0.	1.613

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	130	998	0	17920	0	377
normalized size	1	1.	0.91	6.98	0.	125.31	0.	2.64
time (sec)	N/A	0.207	1.894	0.092	0.	2.667	0.	1.38

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	164	2048	0	12741	0	481
normalized size	1	1.	1.02	12.8	0.	79.63	0.	3.01
time (sec)	N/A	0.236	1.34	0.076	0.	2.557	0.	1.194

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	149	1884	0	13445	0	0
normalized size	1	1.	1.12	14.17	0.	101.09	0.	0.
time (sec)	N/A	0.125	0.352	0.062	0.	2.451	0.	0.

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	114	114	102	664	0	10175	0	331
normalized size	1	1.	0.89	5.82	0.	89.25	0.	2.9
time (sec)	N/A	0.095	0.654	0.064	0.	2.288	0.	1.361

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	114	1348	0	11478	0	0
normalized size	1	1.	0.97	11.52	0.	98.1	0.	0.
time (sec)	N/A	0.093	0.666	0.068	0.	2.331	0.	0.

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	124	1322	0	11760	0	365
normalized size	1	1.	0.87	9.24	0.	82.24	0.	2.55
time (sec)	N/A	0.126	0.888	0.06	0.	2.353	0.	1.398

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	79	85	0	9451	915	0
normalized size	1	1.	0.82	0.89	0.	98.45	9.53	0.
time (sec)	N/A	0.057	0.171	0.013	0.	2.188	175.048	0.

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	B	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	159	159	321	2118	0	18737	0	0
normalized size	1	1.	2.02	13.32	0.	117.84	0.	0.
time (sec)	N/A	0.18	0.731	0.092	0.	3.158	0.	0.

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	172	172	165	1694	0	21442	0	509
normalized size	1	1.	0.96	9.85	0.	124.66	0.	2.96
time (sec)	N/A	0.265	1.079	0.084	0.	2.843	0.	1.398

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	F(-1)	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	222	2307	0	0	0	0
normalized size	1	1.	1.02	10.63	0.	0.	0.	0.
time (sec)	N/A	0.303	1.791	0.104	0.	0.	0.	0.

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F(-2)	F(-1)	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	203	169	1892	0	0	0	608
normalized size	1	1.	0.83	9.32	0.	0.	0.	3.
time (sec)	N/A	0.341	2.2	0.108	0.	0.	0.	1.498

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	24	54	86	220	0	55
normalized size	1	1.	1.26	2.84	4.53	11.58	0.	2.89
time (sec)	N/A	0.045	0.076	0.023	1.703	1.569	0.	1.175

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	10	10	14	50	53	275	129	50
normalized size	1	1.	1.4	5.	5.3	27.5	12.9	5.
time (sec)	N/A	0.039	0.01	0.028	1.106	1.495	2.309	1.171

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	30	30	32	98	101	552	0	82
normalized size	1	1.	1.07	3.27	3.37	18.4	0.	2.73
time (sec)	N/A	0.059	0.069	0.032	1.532	1.513	0.	1.154

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	117	117	124	52	0	8343	0	0
normalized size	1	1.	1.06	0.44	0.	71.31	0.	0.
time (sec)	N/A	0.114	0.663	0.09	0.	2.459	0.	0.

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	72	72	96	62	0	6229	0	0
normalized size	1	1.	1.33	0.86	0.	86.51	0.	0.
time (sec)	N/A	0.056	0.245	0.013	0.	2.091	0.	0.

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	85	85	130	51	0	13256	0	0
normalized size	1	1.	1.53	0.6	0.	155.95	0.	0.
time (sec)	N/A	0.092	0.71	0.091	0.	2.644	0.	0.

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	B	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	86	86	175	35	0	3534	0	938
normalized size	1	1.	2.03	0.41	0.	41.09	0.	10.91
time (sec)	N/A	0.096	1.514	0.085	0.	1.971	0.	1.298

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	151	151	684	35	0	9495	0	3009
normalized size	1	1.	4.53	0.23	0.	62.88	0.	19.93
time (sec)	N/A	0.143	12.423	0.104	0.	3.429	0.	1.417

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	301	301	211	521	0	0	0	0
normalized size	1	1.	0.7	1.73	0.	0.	0.	0.
time (sec)	N/A	0.298	1.376	0.106	0.	0.	0.	0.

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	168	339	0	0	0	0
normalized size	1	1.	0.75	1.52	0.	0.	0.	0.
time (sec)	N/A	0.203	0.698	0.085	0.	0.	0.	0.

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0
normalized size	1	1.	1.15	2.33	0.	0.	0.	0.
time (sec)	N/A	0.036	0.079	0.	0.	0.	0.	0.

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	70	70	148	177	0	0	0	0
normalized size	1	1.	2.11	2.53	0.	0.	0.	0.
time (sec)	N/A	0.084	0.484	0.101	0.	0.	0.	0.

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	206	206	204	318	0	0	0	0
normalized size	1	1.	0.99	1.54	0.	0.	0.	0.
time (sec)	N/A	0.167	2.837	0.132	0.	0.	0.	0.

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	157	157	149	77	0	11570	0	0
normalized size	1	1.	0.95	0.49	0.	73.69	0.	0.
time (sec)	N/A	0.129	1.2	0.085	0.	3.632	0.	0.

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	104	104	93	90	0	8070	0	0
normalized size	1	1.	0.89	0.87	0.	77.6	0.	0.
time (sec)	N/A	0.069	0.485	0.011	0.	2.758	0.	0.

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	125	142	63	0	16424	0	0
normalized size	1	1.	1.14	0.5	0.	131.39	0.	0.
time (sec)	N/A	0.139	0.453	0.085	0.	4.022	0.	0.

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	133	133	150	63	0	19290	0	0
normalized size	1	1.	1.13	0.47	0.	145.04	0.	0.
time (sec)	N/A	0.146	0.728	0.091	0.	4.642	0.	0.

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	66	63	0	7839	0	0
normalized size	1	1.	0.52	0.5	0.	62.21	0.	0.
time (sec)	N/A	0.121	0.109	0.107	0.	3.801	0.	0.

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	205	205	959	63	0	19302	0	0
normalized size	1	1.	4.68	0.31	0.	94.16	0.	0.
time (sec)	N/A	0.172	15.325	0.154	0.	10.114	0.	0.

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	357	357	256	730	0	0	0	0
normalized size	1	1.	0.72	2.04	0.	0.	0.	0.
time (sec)	N/A	0.393	2.531	0.167	0.	0.	0.	0.

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	299	299	213	535	0	0	0	0
normalized size	1	1.	0.71	1.79	0.	0.	0.	0.
time (sec)	N/A	0.291	1.327	0.12	0.	0.	0.	0.

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0
normalized size	1	1.	0.97	2.39	0.	0.	0.	0.
time (sec)	N/A	0.19	0.633	0.	0.	0.	0.	0.

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	210	210	160	334	0	0	0	0
normalized size	1	1.	0.76	1.59	0.	0.	0.	0.
time (sec)	N/A	0.201	0.939	0.115	0.	0.	0.	0.

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	193	193	197	324	0	0	0	0
normalized size	1	1.	1.02	1.68	0.	0.	0.	0.
time (sec)	N/A	0.188	1.965	0.116	0.	0.	0.	0.

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	79	79	77	35	0	6375	0	0
normalized size	1	1.	0.97	0.44	0.	80.7	0.	0.
time (sec)	N/A	0.09	0.103	0.073	0.	2.482	0.	0.

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	0	5052	0	0
normalized size	1	1.	1.	0.89	0.	132.95	0.	0.
time (sec)	N/A	0.047	0.016	0.013	0.	2.105	0.	0.

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	46	46	46	35	0	1615	0	113
normalized size	1	1.	1.	0.76	0.	35.11	0.	2.46
time (sec)	N/A	0.069	0.031	0.079	0.	1.874	0.	1.439

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	97	97	443	35	0	4010	0	976
normalized size	1	1.	4.57	0.36	0.	41.34	0.	10.06
time (sec)	N/A	0.107	9.479	0.096	0.	2.363	0.	2.01

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	241	241	179	344	0	0	0	0
normalized size	1	1.	0.74	1.43	0.	0.	0.	0.
time (sec)	N/A	0.219	0.822	0.109	0.	0.	0.	0.

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	177	177	95	86	0	0	0	0
normalized size	1	1.	0.54	0.49	0.	0.	0.	0.
time (sec)	N/A	0.148	0.241	0.082	0.	0.	0.	0.

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0
normalized size	1	1.	1.13	1.43	0.	0.	0.	0.
time (sec)	N/A	0.038	0.069	0.	0.	0.	0.	0.

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	160	160	159	133	0	0	0	0
normalized size	1	1.	0.99	0.83	0.	0.	0.	0.
time (sec)	N/A	0.181	0.622	0.138	0.	0.	0.	0.

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	219	343	0	0	0	0
normalized size	1	1.	1.	1.57	0.	0.	0.	0.
time (sec)	N/A	0.195	2.152	0.22	0.	0.	0.	0.

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	89	35	0	7876	0	0
normalized size	1	1.	1.16	0.45	0.	102.29	0.	0.
time (sec)	N/A	0.099	0.17	0.065	0.	2.726	0.	0.

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	319	641	0	211
normalized size	1	1.	1.	0.97	11.	22.1	0.	7.28
time (sec)	N/A	0.044	0.028	0.012	1.656	1.829	0.	1.245

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	85	85	315	101	0	4262	0	385
normalized size	1	1.	3.71	1.19	0.	50.14	0.	4.53
time (sec)	N/A	0.097	7.552	0.115	0.	2.482	0.	1.474

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	142	142	231	95	0	11344	0	0
normalized size	1	1.	1.63	0.67	0.	79.89	0.	0.
time (sec)	N/A	0.165	5.364	0.149	0.	4.657	0.	0.

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	325	325	196	498	0	0	0	0
normalized size	1	1.	0.6	1.53	0.	0.	0.	0.
time (sec)	N/A	0.301	1.07	0.118	0.	0.	0.	0.

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	244	244	155	322	0	0	0	0
normalized size	1	1.	0.64	1.32	0.	0.	0.	0.
time (sec)	N/A	0.218	0.602	0.116	0.	0.	0.	0.

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	143	181	0	0	0	0
normalized size	1	1.	1.57	1.99	0.	0.	0.	0.
time (sec)	N/A	0.101	0.307	0.097	0.	0.	0.	0.

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	252	0	0	0	0
normalized size	1	1.	0.87	2.19	0.	0.	0.	0.
time (sec)	N/A	0.061	0.143	0.001	0.	0.	0.	0.

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	178	345	0	0	0	0
normalized size	1	1.	0.82	1.59	0.	0.	0.	0.
time (sec)	N/A	0.203	1.177	0.148	0.	0.	0.	0.

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	134	134	126	65	0	15815	0	0
normalized size	1	1.	0.94	0.49	0.	118.02	0.	0.
time (sec)	N/A	0.141	0.864	0.082	0.	5.854	0.	0.

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	73	73	50	65	1251	2267	0	801
normalized size	1	1.	0.68	0.89	17.14	31.05	0.	10.97
time (sec)	N/A	0.093	0.105	0.082	1.757	3.102	0.	1.638

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	65	65	47	56	655	2161	0	702
normalized size	1	1.	0.72	0.86	10.08	33.25	0.	10.8
time (sec)	N/A	0.058	0.045	0.011	1.655	3.099	0.	1.695

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD
size	134	134	1331	169	0	12320	0	1831
normalized size	1	1.	9.93	1.26	0.	91.94	0.	13.66
time (sec)	N/A	0.149	9.303	0.133	0.	5.135	0.	3.258

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	330	330	206	812	0	0	0	0
normalized size	1	1.	0.62	2.46	0.	0.	0.	0.
time (sec)	N/A	0.331	2.133	0.133	0.	0.	0.	0.

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	223	223	178	597	0	0	0	0
normalized size	1	1.	0.8	2.68	0.	0.	0.	0.
time (sec)	N/A	0.211	1.378	0.253	0.	0.	0.	0.

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	228	228	193	662	0	0	0	0
normalized size	1	1.	0.85	2.9	0.	0.	0.	0.
time (sec)	N/A	0.204	1.402	0.151	0.	0.	0.	0.

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0
normalized size	1	1.	0.76	1.62	0.	0.	0.	0.
time (sec)	N/A	0.281	1.264	0.	0.	0.	0.	0.

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	260	1002	0	0	0	0
normalized size	1	1.	0.89	3.43	0.	0.	0.	0.
time (sec)	N/A	0.314	3.46	0.166	0.	0.	0.	0.

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	117	117	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.111	8.19	0.51	0.	0.	0.	0.

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	214	214	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.204	10.606	0.645	0.	0.	0.	0.

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	125	119	120	0	0	0	0	0
normalized size	1	0.95	0.96	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.202	0.453	0.	0.	0.	0.

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.046	0.026	0.341	0.	0.	0.	0.

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.076	3.354	0.255	0.	0.	0.	0.

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	78	78	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	5.454	0.295	0.	0.	0.	0.

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.086	9.884	0.482	0.	0.	0.	0.

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	10.074	0.365	0.	0.	0.	0.

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.052	2.137	0.145	0.	0.	0.	0.

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.083	4.189	0.257	0.	0.	0.	0.

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	92	92	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.082	7.957	0.256	0.	0.	0.	0.

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	259	259	220	780	0	6338	0	0
normalized size	1	1.	0.85	3.01	0.	24.47	0.	0.
time (sec)	N/A	0.296	0.444	0.105	0.	8.191	0.	0.

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	136	136	117	359	0	2182	0	0
normalized size	1	1.	0.86	2.64	0.	16.04	0.	0.
time (sec)	N/A	0.157	0.151	0.1	0.	7.326	0.	0.

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	43	43	41	89	0	563	68	92
normalized size	1	1.	0.95	2.07	0.	13.09	1.58	2.14
time (sec)	N/A	0.05	0.027	0.013	0.	6.355	1.9	1.237

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	286	286	229	206	0	0	0	0
normalized size	1	1.	0.8	0.72	0.	0.	0.	0.
time (sec)	N/A	0.482	0.248	0.108	0.	0.	0.	0.

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	288	955	0	12573	0	0
normalized size	1	1.	1.07	3.54	0.	46.57	0.	0.
time (sec)	N/A	0.323	0.619	0.218	0.	8.894	0.	0.

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	123	481	0	5027	0	0
normalized size	1	1.	0.87	3.39	0.	35.4	0.	0.
time (sec)	N/A	0.164	0.464	0.207	0.	7.246	0.	0.

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	F	B	A	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	49	49	42	144	0	1370	151	0
normalized size	1	1.	0.86	2.94	0.	27.96	3.08	0.
time (sec)	N/A	0.055	0.061	0.021	0.	1.776	4.926	0.

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	384	384	280	567	0	0	0	0
normalized size	1	1.	0.73	1.48	0.	0.	0.	0.
time (sec)	N/A	0.631	0.733	0.234	0.	0.	0.	0.

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.14	0.142	0.254	0.	0.	0.	0.

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.104	0.055	0.22	0.	0.	0.	0.

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.043	0.009	0.224	0.	0.	0.	0.

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	130	130	119	0	0	0	0	0
normalized size	1	1.	0.92	0.	0.	0.	0.	0.
time (sec)	N/A	0.133	0.111	5.743	0.	0.	0.	0.

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	84	84	82	0	0	0	0	0
normalized size	1	1.	0.98	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	0.052	5.335	0.	0.	0.	0.

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	37	37	37	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.044	0.008	5.571	0.	0.	0.	0.

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	B	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	17	17	23	41	61	165	0	34
normalized size	1	1.	1.35	2.41	3.59	9.71	0.	2.
time (sec)	N/A	0.032	0.014	0.029	1.109	1.782	0.	1.17

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	51	42	394	2361	0	108
normalized size	1	1.	0.81	0.67	6.25	37.48	0.	1.71
time (sec)	N/A	0.128	0.1	0.145	1.799	1.859	0.	1.301

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	42	143	829	0	72
normalized size	1	1.	0.76	1.11	3.76	21.82	0.	1.89
time (sec)	N/A	0.113	0.087	0.105	1.791	1.893	0.	1.255

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	18	18	18	19	43	365	0	35
normalized size	1	1.	1.	1.06	2.39	20.28	0.	1.94
time (sec)	N/A	0.069	0.04	0.026	1.758	1.806	0.	1.159

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	50	50	42	42	92	549	0	69
normalized size	1	1.	0.84	0.84	1.84	10.98	0.	1.38
time (sec)	N/A	0.098	0.059	0.089	1.781	1.853	0.	1.206

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	87	87	77	54	170	2056	0	120
normalized size	1	1.	0.89	0.62	1.95	23.63	0.	1.38
time (sec)	N/A	0.142	0.261	0.092	1.947	1.976	0.	1.259

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	120	120	75	85	1203	4517	0	134
normalized size	1	1.	0.62	0.71	10.02	37.64	0.	1.12
time (sec)	N/A	0.13	0.343	0.109	1.904	2.045	0.	1.403

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	55	69	522	2003	0	100
normalized size	1	1.	0.6	0.76	5.74	22.01	0.	1.1
time (sec)	N/A	0.125	0.194	0.098	1.675	1.962	0.	1.285

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	40	41	68	497	0	51
normalized size	1	1.	0.7	0.72	1.19	8.72	0.	0.89
time (sec)	N/A	0.109	0.053	0.088	1.761	1.971	0.	1.267

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	56	56	35	42	169	829	0	77
normalized size	1	1.	0.62	0.75	3.02	14.8	0.	1.38
time (sec)	N/A	0.114	0.075	0.085	1.734	1.788	0.	1.292

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	47	55	657	2361	0	108
normalized size	1	1.	0.52	0.6	7.22	25.95	0.	1.19
time (sec)	N/A	0.122	0.073	0.092	1.783	1.97	0.	1.272

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	124	124	67	65	1418	4725	0	140
normalized size	1	1.	0.54	0.52	11.44	38.1	0.	1.13
time (sec)	N/A	0.125	0.193	0.092	1.784	2.114	0.	1.299

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	43	41	602	3776	0	128
normalized size	1	1.	0.65	0.62	9.12	57.21	0.	1.94
time (sec)	N/A	0.129	0.09	0.102	2.084	1.895	0.	1.399

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	248	1692	0	88
normalized size	1	1.	0.74	0.98	5.9	40.29	0.	2.1
time (sec)	N/A	0.118	0.07	0.098	1.899	1.761	0.	1.344

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	19	19	19	20	45	428	0	39
normalized size	1	1.	1.	1.05	2.37	22.53	0.	2.05
time (sec)	N/A	0.07	0.032	0.024	1.774	1.754	0.	1.26

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	31	31	49	33	54	455	0	49
normalized size	1	1.	1.58	1.06	1.74	14.68	0.	1.58
time (sec)	N/A	0.086	0.053	0.07	1.866	1.842	0.	1.322

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	65	42	135	1409	0	109
normalized size	1	1.	0.98	0.64	2.05	21.35	0.	1.65
time (sec)	N/A	0.129	0.122	0.086	1.904	1.949	0.	1.367

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	91	91	66	68	848	3584	0	130
normalized size	1	1.	0.73	0.75	9.32	39.38	0.	1.43
time (sec)	N/A	0.144	0.133	0.108	1.955	2.048	0.	1.37

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	44	51	293	1343	0	85
normalized size	1	1.	0.71	0.82	4.73	21.66	0.	1.37
time (sec)	N/A	0.121	0.057	0.224	1.808	1.841	0.	1.314

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	136	428	0	39
normalized size	1	1.	1.	1.28	5.44	17.12	0.	1.56
time (sec)	N/A	0.104	0.037	0.07	1.849	1.874	0.	1.379

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	61	61	37	44	751	1692	0	88
normalized size	1	1.	0.61	0.72	12.31	27.74	0.	1.44
time (sec)	N/A	0.115	0.061	0.098	2.	1.75	0.	1.497

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	96	96	49	54	1662	3776	0	128
normalized size	1	1.	0.51	0.56	17.31	39.33	0.	1.33
time (sec)	N/A	0.123	0.082	0.113	2.242	1.895	0.	1.561

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	68	68	51	44	791	6649	0	132
normalized size	1	1.	0.75	0.65	11.63	97.78	0.	1.94
time (sec)	N/A	0.141	0.112	0.107	2.218	2.141	0.	1.733

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	44	44	34	44	362	3571	0	92
normalized size	1	1.	0.77	1.	8.23	81.16	0.	2.09
time (sec)	N/A	0.126	0.123	0.098	1.88	1.892	0.	1.536

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	21	21	21	20	82	1511	0	43
normalized size	1	1.	1.	0.95	3.9	71.95	0.	2.05
time (sec)	N/A	0.077	0.039	0.018	1.898	1.847	0.	1.316

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	53	53	41	44	103	706	0	84
normalized size	1	1.	0.77	0.83	1.94	13.32	0.	1.58
time (sec)	N/A	0.108	0.062	0.066	1.793	1.853	0.	1.351

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	A	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	66	66	67	36	135	1458	0	0
normalized size	1	1.	1.02	0.55	2.05	22.09	0.	0.
time (sec)	N/A	0.139	0.132	0.069	2.112	1.875	0.	0.

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	58	69	498	3698	0	126
normalized size	1	1.	0.55	0.65	4.7	34.89	0.	1.19
time (sec)	N/A	0.16	0.094	0.142	1.899	2.001	0.	1.422

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	B	B	F	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	64	64	46	51	433	652	0	58
normalized size	1	1.	0.72	0.8	6.77	10.19	0.	0.91
time (sec)	N/A	0.132	0.079	0.093	1.718	1.806	0.	1.43

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	1111	1511	0	43
normalized size	1	1.	0.76	0.92	29.24	39.76	0.	1.13
time (sec)	N/A	0.126	0.055	0.078	1.986	1.781	0.	1.652

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	41	67	2067	3571	0	92
normalized size	1	1.	0.53	0.87	26.84	46.38	0.	1.19
time (sec)	N/A	0.144	0.109	0.132	2.326	1.944	0.	1.803

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	B	B	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	51	57	2992	6649	0	132
normalized size	1	1.	0.44	0.5	26.02	57.82	0.	1.15
time (sec)	N/A	0.15	0.136	0.105	2.601	2.143	0.	1.943

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	151	43	0	12141	0	0
normalized size	1	1.	0.81	0.23	0.	64.93	0.	0.
time (sec)	N/A	0.225	0.572	0.341	0.	9.015	0.	0.

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	88	43	0	4352	0	0
normalized size	1	1.	0.7	0.34	0.	34.54	0.	0.
time (sec)	N/A	0.125	0.406	0.102	0.	7.675	0.	0.

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	62	62	65	41	0	1705	0	0
normalized size	1	1.	1.05	0.66	0.	27.5	0.	0.
time (sec)	N/A	0.064	0.052	0.079	0.	7.164	0.	0.

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	53	46	0	1656	0	0
normalized size	1	1.	0.98	0.85	0.	30.67	0.	0.
time (sec)	N/A	0.074	0.049	0.085	0.	3.907	0.	0.

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	106	106	69	58	0	3903	0	0
normalized size	1	1.	0.65	0.55	0.	36.82	0.	0.
time (sec)	N/A	0.122	0.363	0.214	0.	4.977	0.	0.

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	102	80	0	9559	0	0
normalized size	1	1.	0.61	0.48	0.	57.24	0.	0.
time (sec)	N/A	0.18	0.564	0.131	0.	5.962	0.	0.

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	292	292	214	369	0	0	0	0
normalized size	1	1.	0.73	1.26	0.	0.	0.	0.
time (sec)	N/A	0.306	1.947	0.177	0.	0.	0.	0.

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	168	168	150	233	0	0	0	0
normalized size	1	1.	0.89	1.39	0.	0.	0.	0.
time (sec)	N/A	0.177	0.494	0.135	0.	0.	0.	0.

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	69	140	0	0	0	0
normalized size	1	1.	1.15	2.33	0.	0.	0.	0.
time (sec)	N/A	0.038	0.084	0.	0.	0.	0.	0.

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	202	202	154	215	0	0	0	0
normalized size	1	1.	0.76	1.06	0.	0.	0.	0.
time (sec)	N/A	0.199	0.57	0.142	0.	0.	0.	0.

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	270	270	210	519	0	0	0	0
normalized size	1	1.	0.78	1.92	0.	0.	0.	0.
time (sec)	N/A	0.296	3.067	0.17	0.	0.	0.	0.

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	169	71	0	16471	0	0
normalized size	1	1.	0.73	0.31	0.	71.	0.	0.
time (sec)	N/A	0.29	1.636	0.122	0.	9.431	0.	0.

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	122	71	0	6527	0	0
normalized size	1	1.	0.78	0.46	0.	41.84	0.	0.
time (sec)	N/A	0.159	0.542	0.125	0.	7.387	0.	0.

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	90	90	86	69	0	2827	0	0
normalized size	1	1.	0.96	0.77	0.	31.41	0.	0.
time (sec)	N/A	0.09	0.146	0.296	0.	6.381	0.	0.

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	78	78	69	62	0	2696	0	0
normalized size	1	1.	0.88	0.79	0.	34.56	0.	0.
time (sec)	N/A	0.092	0.122	0.078	0.	3.797	0.	0.

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	140	140	90	84	0	6418	0	0
normalized size	1	1.	0.64	0.6	0.	45.84	0.	0.
time (sec)	N/A	0.148	0.446	0.124	0.	5.472	0.	0.

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	203	199	123	113	0	14438	0	0
normalized size	1	0.98	0.61	0.56	0.	71.12	0.	0.
time (sec)	N/A	0.232	0.79	0.254	0.	6.137	0.	0.

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	305	305	224	389	0	0	0	0
normalized size	1	1.	0.73	1.28	0.	0.	0.	0.
time (sec)	N/A	0.38	2.796	0.167	0.	0.	0.	0.

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	260	260	188	413	0	0	0	0
normalized size	1	1.	0.72	1.59	0.	0.	0.	0.
time (sec)	N/A	0.245	2.887	0.148	0.	0.	0.	0.

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	174	174	169	416	0	0	0	0
normalized size	1	1.	0.97	2.39	0.	0.	0.	0.
time (sec)	N/A	0.182	0.641	0.	0.	0.	0.	0.

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	256	256	184	327	0	0	0	0
normalized size	1	1.	0.72	1.28	0.	0.	0.	0.
time (sec)	N/A	0.272	2.126	0.156	0.	0.	0.	0.

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	306	306	229	540	0	0	0	0
normalized size	1	1.	0.75	1.76	0.	0.	0.	0.
time (sec)	N/A	0.359	4.701	0.288	0.	0.	0.	0.

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	142	142	116	43	0	9879	0	0
normalized size	1	1.	0.82	0.3	0.	69.57	0.	0.
time (sec)	N/A	0.193	0.463	0.125	0.	3.74	0.	0.

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	89	89	85	43	0	3425	0	0
normalized size	1	1.	0.96	0.48	0.	38.48	0.	0.
time (sec)	N/A	0.112	0.117	0.129	0.	2.747	0.	0.

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	41	41	44	41	0	1168	0	0
normalized size	1	1.	1.07	1.	0.	28.49	0.	0.
time (sec)	N/A	0.058	0.042	0.093	0.	2.354	0.	0.

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	33	33	33	35	0	1111	0	0
normalized size	1	1.	1.	1.06	0.	33.67	0.	0.
time (sec)	N/A	0.071	0.035	0.073	0.	2.228	0.	0.

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	77	77	72	44	0	2957	0	0
normalized size	1	1.	0.94	0.57	0.	38.4	0.	0.
time (sec)	N/A	0.102	0.202	0.107	0.	2.492	0.	0.

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	126	126	100	54	0	7682	0	0
normalized size	1	1.	0.79	0.43	0.	60.97	0.	0.
time (sec)	N/A	0.147	0.362	0.102	0.	2.846	0.	0.

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	219	219	206	366	0	0	0	0
normalized size	1	1.	0.94	1.67	0.	0.	0.	0.
time (sec)	N/A	0.196	2.105	0.289	0.	0.	0.	0.

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	156	156	109	239	0	0	0	0
normalized size	1	1.	0.7	1.53	0.	0.	0.	0.
time (sec)	N/A	0.181	0.382	0.161	0.	0.	0.	0.

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	60	60	68	86	0	0	0	0
normalized size	1	1.	1.13	1.43	0.	0.	0.	0.
time (sec)	N/A	0.037	0.071	0.001	0.	0.	0.	0.

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	207	207	105	217	0	0	0	0
normalized size	1	1.	0.51	1.05	0.	0.	0.	0.
time (sec)	N/A	0.197	0.389	0.179	0.	0.	0.	0.

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	285	285	208	522	0	0	0	0
normalized size	1	1.	0.73	1.83	0.	0.	0.	0.
time (sec)	N/A	0.305	3.549	0.187	0.	0.	0.	0.

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	187	187	113	103	0	24184	0	0
normalized size	1	1.	0.6	0.55	0.	129.33	0.	0.
time (sec)	N/A	0.251	0.468	0.225	0.	6.766	0.	0.

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	122	122	79	103	0	9603	0	0
normalized size	1	1.	0.65	0.84	0.	78.71	0.	0.
time (sec)	N/A	0.14	0.118	0.195	0.	3.543	0.	0.

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	69	69	58	93	0	3475	0	0
normalized size	1	1.	0.84	1.35	0.	50.36	0.	0.
time (sec)	N/A	0.074	0.078	0.123	0.	2.672	0.	0.

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	57	57	46	35	0	2955	0	0
normalized size	1	1.	0.81	0.61	0.	51.84	0.	0.
time (sec)	N/A	0.084	0.06	0.063	0.	2.386	0.	0.

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	69	43	0	7960	0	0
normalized size	1	1.	0.63	0.39	0.	72.36	0.	0.
time (sec)	N/A	0.136	0.103	0.104	0.	3.056	0.	0.

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	167	167	94	43	0	19003	0	0
normalized size	1	1.	0.56	0.26	0.	113.79	0.	0.
time (sec)	N/A	0.201	0.327	0.09	0.	4.582	0.	0.

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	275	275	212	354	0	0	0	0
normalized size	1	1.	0.77	1.29	0.	0.	0.	0.
time (sec)	N/A	0.276	2.184	0.204	0.	0.	0.	0.

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	217	217	158	256	0	0	0	0
normalized size	1	1.	0.73	1.18	0.	0.	0.	0.
time (sec)	N/A	0.213	1.282	0.185	0.	0.	0.	0.

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	115	115	100	252	0	0	0	0
normalized size	1	1.	0.87	2.19	0.	0.	0.	0.
time (sec)	N/A	0.068	0.15	0.	0.	0.	0.	0.

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	237	237	153	219	0	0	0	0
normalized size	1	1.	0.65	0.92	0.	0.	0.	0.
time (sec)	N/A	0.27	0.836	0.282	0.	0.	0.	0.

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	341	341	214	522	0	0	0	0
normalized size	1	1.	0.63	1.53	0.	0.	0.	0.
time (sec)	N/A	0.408	3.22	0.195	0.	0.	0.	0.

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	232	232	114	213	0	0	0	0
normalized size	1	1.	0.49	0.92	0.	0.	0.	0.
time (sec)	N/A	0.316	0.589	0.298	0.	0.	0.	0.

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	163	163	82	213	0	24677	0	0
normalized size	1	1.	0.5	1.31	0.	151.39	0.	0.
time (sec)	N/A	0.166	0.122	0.266	0.	6.75	0.	0.

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	99	99	60	173	0	9900	0	0
normalized size	1	1.	0.61	1.75	0.	100.	0.	0.
time (sec)	N/A	0.093	0.087	0.218	0.	3.48	0.	0.

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F(-2)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	83	83	49	65	0	7507	0	0
normalized size	1	1.	0.59	0.78	0.	90.45	0.	0.
time (sec)	N/A	0.1	0.065	0.086	0.	2.807	0.	0.

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	B	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	143	143	69	73	0	18669	0	0
normalized size	1	1.	0.48	0.51	0.	130.55	0.	0.
time (sec)	N/A	0.152	0.3	0.118	0.	4.733	0.	0.

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	C	F	F(-1)	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	208	208	117	73	0	0	0	0
normalized size	1	1.	0.56	0.35	0.	0.	0.	0.
time (sec)	N/A	0.225	0.424	0.122	0.	0.	0.	0.

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	333	333	252	661	0	0	0	0
normalized size	1	1.	0.76	1.98	0.	0.	0.	0.
time (sec)	N/A	0.399	3.37	0.217	0.	0.	0.	0.

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	274	274	215	799	0	0	0	0
normalized size	1	1.	0.78	2.92	0.	0.	0.	0.
time (sec)	N/A	0.28	2.601	0.192	0.	0.	0.	0.

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	251	251	190	406	0	0	0	0
normalized size	1	1.	0.76	1.62	0.	0.	0.	0.
time (sec)	N/A	0.288	1.256	0.	0.	0.	0.	0.

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	A	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	351	351	226	640	0	0	0	0
normalized size	1	1.	0.64	1.82	0.	0.	0.	0.
time (sec)	N/A	0.412	2.677	0.24	0.	0.	0.	0.

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	C	B	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	385	385	247	924	0	0	0	0
normalized size	1	1.	0.64	2.4	0.	0.	0.	0.
time (sec)	N/A	0.562	2.788	0.226	0.	0.	0.	0.

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	122	122	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.129	9.498	0.227	0.	0.	0.	0.

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	110	110	90	0	0	0	0	0
normalized size	1	1.	0.82	0.	0.	0.	0.	0.
time (sec)	N/A	0.12	0.239	0.36	0.	0.	0.	0.

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	63	63	65	0	0	0	0	0
normalized size	1	1.	1.03	0.	0.	0.	0.	0.
time (sec)	N/A	0.055	0.071	0.255	0.	0.	0.	0.

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0
normalized size	1	1.	1.	0.	0.	0.	0.	0.
time (sec)	N/A	0.063	0.06	0.279	0.	0.	0.	0.

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	F	F	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	94	94	71	0	0	0	0	0
normalized size	1	1.	0.76	0.	0.	0.	0.	0.
time (sec)	N/A	0.091	0.379	0.265	0.	0.	0.	0.

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.121	7.105	0.227	0.	0.	0.	0.

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.107	5.423	0.222	0.	0.	0.	0.

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	99	99	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	6.369	0.218	0.	0.	0.	0.

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	F	F	F	F	F(-1)	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD
size	103	103	0	0	0	0	0	0
normalized size	1	1.	0.	0.	0.	0.	0.	0.
time (sec)	N/A	0.102	6.635	0.216	0.	0.	0.	0.

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	C	F(-1)	C	F(-1)	A
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	152	152	136	132	0	2762	0	282
normalized size	1	1.	0.89	0.87	0.	18.17	0.	1.86
time (sec)	N/A	0.252	0.338	0.06	0.	11.757	0.	1.238

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	0	0	0	0
normalized size	1	1.	1.	0.75	0.	0.	0.	0.
time (sec)	N/A	0.076	0.024	0.214	0.	0.	0.	0.

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	B	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	0	5184	0	0
normalized size	1	1.	1.	0.76	0.	115.2	0.	0.
time (sec)	N/A	0.082	0.023	0.042	0.	15.551	0.	0.

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	0	390	0	0
normalized size	1	1.	1.	0.83	0.	13.45	0.	0.
time (sec)	N/A	0.087	0.024	0.033	0.	1.835	0.	0.

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac
grade	A	A	A	A	F	A	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	0	552	0	0
normalized size	1	1.	0.96	0.81	0.	11.74	0.	0.
time (sec)	N/A	0.092	0.021	0.016	0.	1.818	0.	0.

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [182] had the largest ratio of [1.]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.	21	0.143
2	A	3	2	1.	21	0.095
3	A	3	3	1.	21	0.143
4	A	2	1	1.	19	0.053
5	A	3	2	1.	12	0.167
6	A	2	2	1.	19	0.105
7	A	2	2	1.	21	0.095
8	A	2	2	1.	21	0.095
9	A	3	3	1.	21	0.143
10	A	6	6	1.	23	0.261
11	A	3	2	1.	23	0.087
12	A	2	2	1.06	23	0.087
13	A	3	2	1.	21	0.095
14	A	1	1	1.	14	0.071
15	A	4	3	1.	21	0.143
16	A	4	4	1.28	23	0.174
17	A	5	4	1.	23	0.174
18	A	4	3	1.	23	0.13
19	A	7	6	1.	23	0.261
20	A	3	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	3	2	1.	23	0.087
22	A	3	2	1.	21	0.095
23	A	2	2	1.	14	0.143
24	A	4	3	1.	21	0.143
25	A	5	5	1.	23	0.217
26	A	5	4	1.	23	0.174
27	A	5	4	1.	23	0.174
28	A	4	3	1.	23	0.13
29	A	6	6	1.	23	0.261
30	A	4	3	1.	23	0.13
31	A	5	5	1.	23	0.217
32	A	3	3	1.	23	0.13
33	A	3	3	1.	23	0.13
34	A	2	2	1.	21	0.095
35	A	2	2	1.	14	0.143
36	A	4	4	1.	21	0.19
37	A	3	3	1.	23	0.13
38	A	5	5	1.	23	0.217
39	A	4	3	1.	23	0.13
40	A	6	6	1.	23	0.261
41	A	4	3	1.	23	0.13
42	A	5	5	1.	23	0.217
43	A	3	3	1.	23	0.13
44	A	4	4	1.	23	0.174
45	A	3	3	1.	21	0.143
46	A	4	4	1.	14	0.286
47	A	5	5	1.	21	0.238
48	A	4	4	1.	23	0.174
49	A	6	6	1.	23	0.261
50	A	5	4	1.	23	0.174
51	A	4	3	1.	23	0.13
52	A	4	4	1.	23	0.174
53	A	5	4	1.	23	0.174
54	A	4	3	1.	21	0.143
55	A	5	5	1.	14	0.357
56	A	6	6	1.	21	0.286
57	A	5	5	1.	23	0.217
58	A	7	6	1.	23	0.261
59	A	6	5	1.	23	0.217
60	A	3	3	1.	8	0.375
61	A	3	2	1.	8	0.25
62	A	3	2	1.	8	0.25
63	A	2	2	1.	10	0.2
64	A	4	4	1.	10	0.4

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
65	A	5	5	1.	10	0.5
66	A	5	5	1.	25	0.2
67	A	4	4	1.	23	0.174
68	A	6	5	1.	23	0.217
69	A	4	4	1.	25	0.16
70	A	5	5	1.	25	0.2
71	A	7	7	1.	25	0.28
72	A	6	6	1.	25	0.24
73	A	2	2	1.	16	0.125
74	A	7	7	1.	25	0.28
75	A	7	7	1.	25	0.28
76	A	6	5	1.	25	0.2
77	A	5	4	1.	23	0.174
78	A	7	6	1.	23	0.261
79	A	7	6	1.	25	0.24
80	A	5	4	1.	25	0.16
81	A	6	5	1.	25	0.2
82	A	8	7	1.	25	0.28
83	A	7	6	1.	25	0.24
84	A	6	6	1.	16	0.375
85	A	6	6	1.	25	0.24
86	A	7	7	1.	25	0.28
87	A	7	7	1.	16	0.438
88	A	3	3	1.	10	0.3
89	A	3	3	1.	12	0.25
90	A	1	1	1.	12	0.083
91	A	2	2	1.	10	0.2
92	A	2	2	1.	12	0.167
93	A	4	4	1.	10	0.4
94	A	4	4	1.	12	0.333
95	A	4	4	1.	12	0.333
96	A	6	6	1.	10	0.6
97	A	6	6	1.	12	0.5
98	A	4	4	1.	25	0.16
99	A	3	3	1.	23	0.13
100	A	3	3	1.	23	0.13
101	A	4	4	1.	25	0.16
102	A	6	6	1.	25	0.24
103	A	5	5	1.	25	0.2
104	A	2	2	1.	16	0.125
105	A	5	5	1.	25	0.2
106	A	7	7	1.	25	0.28
107	A	4	4	1.	25	0.16
108	A	2	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
109	A	4	4	1.	23	0.174
110	A	6	6	1.	25	0.24
111	A	7	7	1.	25	0.28
112	A	6	6	1.	25	0.24
113	A	6	6	1.	25	0.24
114	A	4	4	1.	16	0.25
115	A	7	7	1.	25	0.28
116	A	5	5	1.	25	0.2
117	A	3	3	1.	25	0.12
118	A	3	3	1.	23	0.13
119	A	6	6	1.	23	0.261
120	A	7	7	1.	25	0.28
121	A	5	5	1.	25	0.2
122	A	7	6	1.	25	0.24
123	A	7	7	1.	16	0.438
124	A	8	8	1.	25	0.32
125	A	3	3	1.	10	0.3
126	A	1	1	1.	12	0.083
127	A	2	2	1.	10	0.2
128	A	3	3	1.	12	0.25
129	A	2	2	1.	12	0.167
130	A	3	3	1.	25	0.12
131	A	5	5	1.	23	0.217
132	A	4	4	1.	23	0.174
133	A	3	3	1.	21	0.143
134	A	3	3	1.	21	0.143
135	A	3	3	1.	23	0.13
136	A	3	3	1.	23	0.13
137	A	3	3	1.	23	0.13
138	A	3	3	1.	23	0.13
139	A	3	3	1.	23	0.13
140	A	3	3	1.	23	0.13
141	A	7	4	1.	21	0.19
142	A	8	4	1.	21	0.19
143	A	6	4	1.	21	0.19
144	A	6	4	1.	19	0.21
145	A	3	1	1.	12	0.083
146	A	5	4	1.	19	0.21
147	A	5	4	1.	21	0.19
148	A	4	3	1.	21	0.143
149	A	5	3	1.	21	0.143
150	A	10	4	1.	23	0.174
151	A	11	4	1.	23	0.174
152	A	8	5	1.	21	0.238

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
153	A	8	4	1.	14	0.286
154	A	7	5	1.	21	0.238
155	A	8	5	1.	23	0.217
156	A	6	4	1.	23	0.174
157	A	7	5	1.	23	0.217
158	A	8	6	1.	23	0.261
159	A	6	4	1.	23	0.174
160	A	9	4	1.	23	0.174
161	A	13	4	1.	23	0.174
162	A	14	5	1.	21	0.238
163	A	10	4	1.	14	0.286
164	A	12	5	1.	21	0.238
165	A	10	6	1.	23	0.261
166	A	10	6	1.	23	0.261
167	A	9	6	1.	23	0.261
168	A	11	7	1.	23	0.304
169	A	8	5	1.	23	0.217
170	A	11	6	1.	23	0.261
171	A	15	6	1.	23	0.261
172	A	15	7	1.	23	0.304
173	A	14	5	1.	23	0.217
174	A	13	5	1.	23	0.217
175	A	11	5	1.	23	0.217
176	A	11	4	1.	21	0.19
177	A	11	4	1.	14	0.286
178	A	14	6	1.	21	0.286
179	A	15	6	1.	23	0.261
180	A	15	7	1.	23	0.304
181	A	16	7	1.	23	0.304
182	A	12	8	1.	8	1.
183	A	12	8	1.	10	0.8
184	A	6	6	1.	21	0.286
185	A	3	2	1.	21	0.095
186	A	5	5	1.	21	0.238
187	A	2	1	1.	19	0.053
188	A	4	2	1.	12	0.167
189	A	4	3	1.	19	0.158
190	A	4	4	1.	21	0.19
191	A	4	4	1.	21	0.19
192	A	4	3	1.	21	0.143
193	A	4	4	1.	21	0.19
194	A	3	2	1.	21	0.095
195	A	5	5	1.	21	0.238
196	A	3	2	1.	23	0.087

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
197	A	7	6	1.	23	0.261
198	A	3	2	1.	21	0.095
199	A	6	5	1.	14	0.357
200	A	4	3	1.	21	0.143
201	A	6	5	1.	23	0.217
202	A	5	4	1.	23	0.174
203	A	6	5	1.	23	0.217
204	A	6	5	1.	23	0.217
205	A	5	4	1.	23	0.174
206	A	6	5	1.	23	0.217
207	A	3	2	1.	23	0.087
208	A	3	2	1.	23	0.087
209	A	3	2	1.	21	0.095
210	A	4	3	1.	21	0.143
211	A	5	4	1.	23	0.174
212	A	6	5	1.	23	0.217
213	A	7	5	1.	23	0.217
214	A	8	5	1.	23	0.217
215	A	8	5	1.	23	0.217
216	A	8	5	1.	23	0.217
217	A	9	6	1.	23	0.261
218	A	8	5	1.	14	0.357
219	A	8	5	1.	23	0.217
220	A	8	5	1.	23	0.217
221	A	7	5	1.	23	0.217
222	A	6	5	1.	23	0.217
223	A	5	4	1.	23	0.174
224	A	4	3	1.	23	0.13
225	A	3	2	1.	23	0.087
226	A	3	2	1.	23	0.087
227	A	3	2	1.	23	0.087
228	A	3	2	1.	23	0.087
229	A	6	5	1.	24	0.208
230	A	6	5	1.	24	0.208
231	A	4	4	1.	24	0.167
232	A	4	4	1.	22	0.182
233	A	7	6	1.	22	0.273
234	A	7	6	1.	24	0.25
235	A	7	5	1.	24	0.208
236	A	7	5	1.	24	0.208
237	A	4	3	1.	24	0.125
238	A	4	3	1.	15	0.2
239	A	6	4	1.	24	0.167
240	A	6	4	1.	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
241	A	7	6	1.	24	0.25
242	A	5	5	1.	24	0.208
243	A	5	5	1.	24	0.208
244	A	5	5	1.	24	0.208
245	A	5	5	1.	22	0.227
246	A	11	7	1.	22	0.318
247	A	14	9	1.	24	0.375
248	A	6	5	1.	24	0.208
249	A	7	6	1.	24	0.25
250	A	5	4	1.	24	0.167
251	A	5	4	1.	15	0.267
252	A	7	5	1.	24	0.208
253	A	6	6	1.	24	0.25
254	A	6	6	1.	24	0.25
255	A	6	6	1.	24	0.25
256	A	6	5	1.	24	0.208
257	A	6	6	1.	22	0.273
258	A	16	7	1.	22	0.318
259	A	9	7	1.	24	0.292
260	A	6	5	1.	24	0.208
261	A	6	5	1.	24	0.208
262	A	6	5	1.	24	0.208
263	A	6	5	1.	15	0.333
264	A	8	6	1.	24	0.25
265	A	3	3	1.	10	0.3
266	A	10	6	1.	8	0.75
267	A	17	5	1.	10	0.5
268	A	7	3	1.	10	0.3
269	A	9	3	1.	10	0.3
270	A	17	6	1.	8	0.75
271	A	8	6	1.	8	0.75
272	A	9	3	1.	8	0.375
273	A	17	6	1.	10	0.6
274	A	7	3	1.	10	0.3
275	A	10	6	1.	10	0.6
276	A	3	2	1.	15	0.133
277	A	3	3	1.	15	0.2
278	A	2	2	1.	15	0.133
279	A	2	2	1.	15	0.133
280	A	2	2	1.	13	0.154
281	A	3	3	1.	13	0.231
282	A	4	3	1.	15	0.2
283	A	5	4	1.	21	0.19
284	A	3	2	1.	21	0.095

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
285	A	4	4	1.	21	0.19
286	A	2	1	1.	19	0.053
287	A	3	3	1.	19	0.158
288	A	3	3	1.	21	0.143
289	A	3	3	1.	21	0.143
290	A	2	1	1.	21	0.048
291	A	4	4	1.	21	0.19
292	A	3	2	1.	21	0.095
293	A	6	5	1.	23	0.217
294	A	3	2	1.	23	0.087
295	A	5	5	1.	23	0.217
296	A	3	2	1.	21	0.095
297	A	4	3	1.	21	0.143
298	A	5	4	1.	23	0.174
299	A	5	4	1.	23	0.174
300	A	4	3	1.	23	0.13
301	A	4	4	1.	23	0.174
302	A	3	2	1.	23	0.087
303	A	5	5	1.	23	0.217
304	A	7	6	1.	23	0.261
305	A	3	2	1.	23	0.087
306	A	6	6	1.	23	0.261
307	A	3	2	1.	21	0.095
308	A	4	3	1.	21	0.143
309	A	6	5	1.	23	0.217
310	A	5	4	1.	23	0.174
311	A	5	4	1.	23	0.174
312	A	6	5	1.	23	0.217
313	A	4	3	1.	23	0.13
314	A	5	5	1.	23	0.217
315	A	3	2	1.	23	0.087
316	A	4	3	1.	23	0.13
317	A	6	6	1.	23	0.261
318	A	4	3	1.	23	0.13
319	A	5	5	1.	23	0.217
320	A	3	3	1.	23	0.13
321	A	4	4	1.	23	0.174
322	A	2	2	1.	21	0.095
323	A	4	4	1.	21	0.19
324	A	3	3	1.	23	0.13
325	A	5	5	1.	23	0.217
326	A	4	3	1.	23	0.13
327	A	6	6	1.	23	0.261
328	A	4	3	1.	23	0.13

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
329	A	6	6	1.	23	0.261
330	A	5	4	1.	23	0.174
331	A	5	5	1.	23	0.217
332	A	3	3	1.	23	0.13
333	A	3	3	1.	23	0.13
334	A	3	3	1.	21	0.143
335	A	5	5	1.	21	0.238
336	A	5	4	1.	23	0.174
337	A	6	6	1.	23	0.261
338	A	5	4	1.	23	0.174
339	A	6	6	1.	23	0.261
340	A	4	4	1.	23	0.174
341	A	4	3	1.	23	0.13
342	A	4	4	1.	23	0.174
343	A	4	4	1.	23	0.174
344	A	4	3	1.	21	0.143
345	A	6	6	1.	21	0.286
346	A	6	5	1.	23	0.217
347	A	7	6	1.	23	0.261
348	A	6	5	1.	23	0.217
349	A	4	3	1.	15	0.2
350	A	3	3	1.	15	0.2
351	A	5	4	1.	15	0.267
352	A	5	5	1.	25	0.2
353	A	4	4	1.	23	0.174
354	A	6	6	1.	23	0.261
355	A	4	4	1.	25	0.16
356	A	5	5	1.	25	0.2
357	A	7	7	1.	25	0.28
358	A	6	6	1.	25	0.24
359	A	2	2	1.	16	0.125
360	A	2	2	1.	25	0.08
361	A	5	5	1.	25	0.2
362	A	6	5	1.	25	0.2
363	A	5	4	1.	23	0.174
364	A	7	7	1.	23	0.304
365	A	7	7	1.	25	0.28
366	A	5	4	1.	25	0.16
367	A	6	5	1.	25	0.2
368	A	8	7	1.	25	0.28
369	A	7	7	1.	25	0.28
370	A	6	6	1.	16	0.375
371	A	6	6	1.	25	0.24
372	A	5	5	1.	25	0.2

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
373	A	4	4	1.	25	0.16
374	A	3	3	1.	23	0.13
375	A	3	3	1.	23	0.13
376	A	4	4	1.	25	0.16
377	A	6	6	1.	25	0.24
378	A	5	5	1.	25	0.2
379	A	2	2	1.	16	0.125
380	A	7	7	1.	25	0.28
381	A	5	5	1.	25	0.2
382	A	4	4	1.	25	0.16
383	A	2	2	1.	23	0.087
384	A	4	4	1.	23	0.174
385	A	6	6	1.	25	0.24
386	A	7	7	1.	25	0.28
387	A	6	6	1.	25	0.24
388	A	2	2	1.	25	0.08
389	A	4	4	1.	16	0.25
390	A	5	5	1.	25	0.2
391	A	5	5	1.	25	0.2
392	A	3	3	1.	25	0.12
393	A	3	3	1.	23	0.13
394	A	6	6	1.	23	0.261
395	A	7	7	1.	25	0.28
396	A	5	5	1.	25	0.2
397	A	5	5	1.	25	0.2
398	A	7	7	1.	16	0.438
399	A	6	6	1.	25	0.24
400	A	3	3	1.	25	0.12
401	A	5	5	1.	23	0.217
402	A	4	4	0.95	23	0.174
403	A	3	3	1.	21	0.143
404	A	3	3	1.	21	0.143
405	A	3	3	1.	23	0.13
406	A	3	3	1.	23	0.13
407	A	3	3	1.	23	0.13
408	A	3	3	1.	14	0.214
409	A	3	3	1.	23	0.13
410	A	3	3	1.	23	0.13
411	A	4	3	1.	25	0.12
412	A	4	3	1.	25	0.12
413	A	4	3	1.	23	0.13
414	A	19	13	1.	23	0.565
415	A	4	3	1.	25	0.12
416	A	4	3	1.	25	0.12

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
417	A	4	3	1.	23	0.13
418	A	19	13	1.	23	0.565
419	A	6	4	1.	23	0.174
420	A	5	4	1.	23	0.174
421	A	2	2	1.	21	0.095
422	A	6	4	1.	23	0.174
423	A	5	4	1.	23	0.174
424	A	2	2	1.	21	0.095
425	A	4	4	1.	13	0.308
426	A	5	4	1.	25	0.16
427	A	5	4	1.	25	0.16
428	A	4	4	1.	23	0.174
429	A	5	5	1.	23	0.217
430	A	7	7	1.	25	0.28
431	A	7	6	1.	25	0.24
432	A	6	6	1.	25	0.24
433	A	5	5	1.	25	0.2
434	A	5	4	1.	25	0.16
435	A	5	4	1.	25	0.16
436	A	5	4	1.	25	0.16
437	A	5	4	1.	25	0.16
438	A	5	4	1.	25	0.16
439	A	4	4	1.	23	0.174
440	A	4	4	1.	23	0.174
441	A	6	6	1.	25	0.24
442	A	5	4	1.	25	0.16
443	A	4	4	1.	25	0.16
444	A	4	4	1.	25	0.16
445	A	4	3	1.	25	0.12
446	A	5	4	1.	25	0.16
447	A	5	4	1.	25	0.16
448	A	5	4	1.	25	0.16
449	A	4	4	1.	23	0.174
450	A	5	5	1.	23	0.217
451	A	6	6	1.	25	0.24
452	A	5	5	1.	25	0.2
453	A	5	5	1.	25	0.2
454	A	4	4	1.	25	0.16
455	A	5	4	1.	25	0.16
456	A	5	4	1.	25	0.16
457	A	6	6	1.	25	0.24
458	A	5	5	1.	25	0.2
459	A	4	4	1.	23	0.174
460	A	4	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
461	A	5	5	1.	25	0.2
462	A	6	6	1.	25	0.24
463	A	7	7	1.	25	0.28
464	A	6	6	1.	25	0.24
465	A	2	2	1.	16	0.125
466	A	6	6	1.	25	0.24
467	A	7	7	1.	25	0.28
468	A	7	6	1.	25	0.24
469	A	6	5	1.	25	0.2
470	A	5	4	1.	23	0.174
471	A	5	4	1.	23	0.174
472	A	6	5	1.	25	0.2
473	A	7	6	0.98	25	0.24
474	A	8	8	1.	25	0.32
475	A	7	7	1.	25	0.28
476	A	6	6	1.	16	0.375
477	A	7	7	1.	25	0.28
478	A	8	8	1.	25	0.32
479	A	5	5	1.	25	0.2
480	A	4	4	1.	25	0.16
481	A	3	3	1.	23	0.13
482	A	3	3	1.	23	0.13
483	A	4	4	1.	25	0.16
484	A	5	5	1.	25	0.2
485	A	5	5	1.	25	0.2
486	A	6	6	1.	25	0.24
487	A	2	2	1.	16	0.125
488	A	6	6	1.	25	0.24
489	A	7	7	1.	25	0.28
490	A	6	6	1.	25	0.24
491	A	5	5	1.	25	0.2
492	A	4	4	1.	23	0.174
493	A	4	4	1.	23	0.174
494	A	5	5	1.	25	0.2
495	A	6	6	1.	25	0.24
496	A	6	6	1.	25	0.24
497	A	5	5	1.	25	0.2
498	A	4	4	1.	16	0.25
499	A	7	7	1.	25	0.28
500	A	8	7	1.	25	0.28
501	A	7	6	1.	25	0.24
502	A	6	5	1.	25	0.2
503	A	5	4	1.	23	0.174
504	A	5	4	1.	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
505	A	6	5	1.	25	0.2
506	A	7	6	1.	25	0.24
507	A	7	6	1.	25	0.24
508	A	6	6	1.	25	0.24
509	A	7	7	1.	16	0.438
510	A	8	8	1.	25	0.32
511	A	9	8	1.	25	0.32
512	A	3	3	1.	25	0.12
513	A	3	3	1.	23	0.13
514	A	2	2	1.	21	0.095
515	A	2	2	1.	21	0.095
516	A	3	3	1.	23	0.13
517	A	3	3	1.	23	0.13
518	A	3	3	1.	23	0.13
519	A	3	3	1.	23	0.13
520	A	3	3	1.	23	0.13
521	A	12	11	1.	15	0.733
522	A	4	4	1.	15	0.267
523	A	5	5	1.	15	0.333
524	A	4	4	1.	15	0.267
525	A	5	5	1.	15	0.333

Chapter 3

Listing of integrals

3.1 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$\frac{(6a-5b)\sinh^3(c+dx)\cosh(c+dx)}{24d}-\frac{(6a-5b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-5b)+\frac{b\sinh^5(c+dx)\cosh(c}{6d}$$

```
[Out] ((6*a - 5*b)*x)/16 - ((6*a - 5*b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) + ((6
*a - 5*b)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c +
 d*x]^5)/(6*d)
```

Rubi [A] time = 0.0548921, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 2635, 8}

$$\frac{(6a-5b)\sinh^3(c+dx)\cosh(c+dx)}{24d}-\frac{(6a-5b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-5b)+\frac{b\sinh^5(c+dx)\cosh(c}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((6*a - 5*b)*x)/16 - ((6*a - 5*b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) + ((6
*a - 5*b)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c +
 d*x]^5)/(6*d)
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)dx = \frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d}+\frac{1}{6}(6a-5b)\int \sinh^4(c+dx)\,dx$$
$$= \frac{(6a-5b)\cosh(c+dx)\sinh^3(c+dx)}{24d}+\frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d}+\frac{1}{8}(-6$$
$$= -\frac{(6a-5b)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(6a-5b)\cosh(c+dx)\sinh^3(c+dx}{24d}$$
$$= \frac{1}{16}(6a-5b)x-\frac{(6a-5b)\cosh(c+dx)\sinh(c+dx)}{16d}+\frac{(6a-5b)\cosh(c+dx}{24d}$$

Mathematica [A] time = 0.108146, size = 68, normalized size = 0.76

$$\frac{(45b-48a)\sinh(2(c+dx))+(6a-9b)\sinh(4(c+dx))+72ac+72adx+b\sinh(6(c+dx))-60bc-60bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (72*a*c - 60*b*c + 72*a*d*x - 60*b*d*x + (-48*a + 45*b)*Sinh[2*(c + d*x)] +
 (6*a - 9*b)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.014, size = 88, normalized size = 1.

$$\frac{1}{d}\left(b\left(\left(\frac{(\sinh(dx+c))^5}{6}-\frac{5\,(\sinh(dx+c))^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5\,dx}{16}-\frac{5\,c}{16}\right)+a\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\,\mathrm{si}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)
-5/16*d*x-5/16*c)+a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*
x+3/8*c))
```

Maxima [A] time = 1.034, size = 203, normalized size = 2.28

$$\frac{1}{64}a\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{384}b\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*
c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-
```

```
4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [A] time = 1.95221, size = 319, normalized size = 3.58

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+2\left(5\,b\cosh\left(dx+c\right)^{3}+3\,(2\,a-3\,b)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+6\,(6\,a-5\,b)dx+\cdots}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a -
 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - 5*b)*d*x + 3*(b*cosh(d*x +
c)^5 + 2*(2*a - 3*b)*cosh(d*x + c)^3 - (16*a - 15*b)*cosh(d*x + c))*sinh(d*
x + c))/d
```

Sympy [A] time = 4.7777, size = 258, normalized size = 2.9

$$\begin{cases}\frac{3ax\sinh^{4}(c+dx)}{8}-\frac{3ax\sinh^{2}(c+dx)\cosh^{2}(c+dx)}{4}+\frac{3ax\cosh^{4}(c+dx)}{8}+\frac{5a\sinh^{3}(c+dx)\cosh(c+dx)}{8d}-\frac{3a\sinh(c+dx)\cosh^{3}(c+dx)}{8d}+\frac{5bx\sinh^{6}(c\cdots}{16} \\ x\left(a+b\sinh^{2}(c)\right)\sinh^{4}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b*x*sinh(c + d*x)**6/16 - 15
*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c
+ d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d
*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x
)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c)**4, Tru
e))
```

Giac [B] time = 1.32322, size = 221, normalized size = 2.48

$$\frac{24\,(dx+c)(6\,a-5\,b)+be^{(6\,dx+6\,c)}+6\,ae^{(4\,dx+4\,c)}-9\,be^{(4\,dx+4\,c)}-48\,ae^{(2\,dx+2\,c)}+45\,be^{(2\,dx+2\,c)}-\left(132\,ae^{(6\,dx+6\,c)}-110\,be\cdots\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/384*(24*(d*x + c)*(6*a - 5*b) + b*e^(6*d*x + 6*c) + 6*a*e^(4*d*x + 4*c) -
 9*b*e^(4*d*x + 4*c) - 48*a*e^(2*d*x + 2*c) + 45*b*e^(2*d*x + 2*c) - (132*a
*e^(6*d*x + 6*c) - 110*b*e^(6*d*x + 6*c) - 48*a*e^(4*d*x + 4*c) + 45*b*e^(4
*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 9*b*e^(2*d*x + 2*c) + b)*e^(-6*d*x - 6*
c))/d
```

3.2 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=53

$$\frac{(a-2b)\cosh^3(c+dx)}{3d}-\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}$$

```
[Out] -(((a - b)*Cosh[c + d*x])/d) + ((a - 2*b)*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[
c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0557225, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3013, 373}

$$\frac{(a-2b)\cosh^3(c+dx)}{3d}-\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)*Cosh[c + d*x])/d) + ((a - 2*b)*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[
c + d*x]^5)/(5*d)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a-b+bx^2\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(a\left(1-\frac{b}{a}\right)-(a-2b)x^2-bx^4\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{(a-b)\cosh(c+dx)}{d}+\frac{(a-2b)\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0255729, size = 77, normalized size = 1.45

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) + (5*b*Cosh[c + d*x])/(8*d) + (a*Cosh[3*(c + d*x
)])/(12*d) - (5*b*Cosh[3*(c + d*x)])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d)
```

Maple [A] time = 0.013, size = 56, normalized size = 1.1

$$\frac{1}{d}\left(b\left(\frac{8}{15}+\frac{(\sinh(dx+c))^4}{5}-\frac{4(\sinh(dx+c))^2}{15}\right)\cosh(dx+c)+a\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*(-2/3+1/3*
sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.04109, size = 190, normalized size = 3.58

$$\frac{1}{480}b\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)+\frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 1
50*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*a*
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

Fricas [B] time = 1.82341, size = 273, normalized size = 5.15

$$\frac{3b\cosh(dx+c)^5+15b\cosh(dx+c)\sinh(dx+c)^4+5(4a-5b)\cosh(dx+c)^3+15\left(2b\cosh(dx+c)^3+(4a-5b\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 + 5*(4*a -
5*b)*cosh(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 + (4*a - 5*b)*cosh(d*x + c))
*sinh(d*x + c)^2 - 30*(6*a - 5*b)*cosh(d*x + c))/d
```

Sympy [A] time = 2.47998, size = 105, normalized size = 1.98

$$\begin{cases}\frac{a\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2a\cosh^3(c+dx)}{3d}+\frac{b\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4b\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{8b\cosh^5(c+dx)}{15d} & \text{for } d\neq 0\\ x\left(a+b\sinh^2(c)\right)\sinh^3(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d*x)**2*cosh(c + d*x)**
3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sin
h(c)**3, True))
```

Giac [B] time = 1.25488, size = 166, normalized size = 3.13

$$\frac{3\,be^{(5\,dx+5\,c)} + 20\,ae^{(3\,dx+3\,c)} - 25\,be^{(3\,dx+3\,c)} - 180\,ae^{(dx+c)} + 150\,be^{(dx+c)} - \left(180\,ae^{(4\,dx+4\,c)} - 150\,be^{(4\,dx+4\,c)} - 20\,ae^{(2\,dx+2\,c)} + \right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*b*e^(5*d*x + 5*c) + 20*a*e^(3*d*x + 3*c) - 25*b*e^(3*d*x + 3*c) -
180*a*e^(d*x + c) + 150*b*e^(d*x + c) - (180*a*e^(4*d*x + 4*c) - 150*b*e^(4
*d*x + 4*c) - 20*a*e^(2*d*x + 2*c) + 25*b*e^(2*d*x + 2*c) - 3*b)*e^(-5*d*x
- 5*c))/d
```

3.3 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{1}{8}x(4a-3b)+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

```
[Out] -((4*a - 3*b)*x)/8 + ((4*a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*C
osh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0428877, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 2635, 8}

$$\frac{(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}-\frac{1}{8}x(4a-3b)+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((4*a - 3*b)*x)/8 + ((4*a - 3*b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*C
osh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{4}(-4a+3b)\int \sinh^2(c+dx)\,dx\\ &= \frac{(4a-3b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d}-\frac{1}{8}(\\ &= -\frac{1}{8}(4a-3b)x+\frac{(4a-3b)\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b\cosh(c+dx)\sin}{4d}\end{aligned}$$

Mathematica [A] time = 0.0854699, size = 47, normalized size = 0.77

$$\frac{-4(4a-3b)(c+dx)+8(a-b)\sinh(2(c+dx))+b\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-4*(4*a - 3*b)*(c + d*x) + 8*(a - b)*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x
)])/(32*d)
```

Maple [A] time = 0.013, size = 66, normalized size = 1.1

$$\frac{1}{d}\left(b\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a*(1
/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.05928, size = 131, normalized size = 2.15

$$\frac{1}{64}b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c
)/d)
```

Fricas [A] time = 1.90866, size = 163, normalized size = 2.67

$$\frac{b\cosh(dx+c)\sinh(dx+c)^3-(4a-3b)dx+\left(b\cosh(dx+c)^3+4(a-b)\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 - (4*a - 3*b)*d*x + (b*cosh(d*x + c)^3
 + 4*(a - b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 1.42794, size = 158, normalized size = 2.59

$$\begin{cases}\frac{ax\sinh^2(c+dx)}{2}-\frac{ax\cosh^2(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{3bx\sinh^4(c+dx)}{8}-\frac{3bx\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3bx\cosh^4(c+dx)}{8}+\frac{5b\sinh^3(c \\ x\left(a+b\sinh^2(c)\right)\sinh^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*x)**2*c
osh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*cosh(c
+ d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a +
 b*sinh(c)**2)*sinh(c)**2, True))
```

Giac [B] time = 1.36898, size = 161, normalized size = 2.64

$$-\frac{8\,(dx+c)(4\,a-3\,b) - be^{(4\,dx+4\,c)} - 8\,ae^{(2\,dx+2\,c)} + 8\,be^{(2\,dx+2\,c)} - \left(24\,ae^{(4\,dx+4\,c)} - 18\,be^{(4\,dx+4\,c)} - 8\,ae^{(2\,dx+2\,c)} + 8\,be^{(2\,dx+}\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*(8*(d*x + c)*(4*a - 3*b) - b*e^(4*d*x + 4*c) - 8*a*e^(2*d*x + 2*c) +
8*b*e^(2*d*x + 2*c) - (24*a*e^(4*d*x + 4*c) - 18*b*e^(4*d*x + 4*c) - 8*a*e^
(2*d*x + 2*c) + 8*b*e^(2*d*x + 2*c) - b)*e^(-4*d*x - 4*c))/d
```

3.4 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

```
[Out] ((a - b)*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0289156, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3013}

$$\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - b)*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=\frac{\text{Subst}\left(\int\left(a-b+bx^2\right)dx,x,\cosh(c+dx)\right)}{d}\\&=\frac{(a-b)\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0240816, size = 53, normalized size = 1.66

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])
/(12*d) + (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.013, size = 34, normalized size = 1.1

$$\frac{1}{d}\left(b\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)+a\cosh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a*cosh(d*x+c))
```

Maxima [B] time = 1.0147, size = 90, normalized size = 2.81

$$\frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + a*cosh(d*x + c)/d
```

Fricas [A] time = 1.87512, size = 127, normalized size = 3.97

$$\frac{b\cosh\left(dx+c\right)^{3}+3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+3\,\left(4\,a-3\,b\right)\cosh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + 3*(4*a - 3*b)
*cosh(d*x + c))/d
```

Sympy [A] time = 0.768842, size = 56, normalized size = 1.75

$$\begin{cases}\frac{a\cosh\left(c+dx\right)}{d}+\frac{b\sinh^{2}\left(c+dx\right)\cosh\left(c+dx\right)}{d}-\frac{2b\cosh^{3}\left(c+dx\right)}{3d} & \text{for } d\neq 0\\ x\left(a+b\sinh^{2}\left(c\right)\right)\sinh\left(c\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*b*cos
h(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*sinh(c), True))
```

Giac [B] time = 1.26747, size = 96, normalized size = 3.

$$\frac{be^{(3\,dx+3\,c)}+12\,ae^{(dx+c)}-9\,be^{(dx+c)}+\left(12\,ae^{(2\,dx+2\,c)}-9\,be^{(2\,dx+2\,c)}+b\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(b*e^(3*d*x + 3*c) + 12*a*e^(d*x + c) - 9*b*e^(d*x + c) + (12*a*e^(2*d
*x + 2*c) - 9*b*e^(2*d*x + 2*c) + b)*e^(-3*d*x - 3*c))/d
```

3.5 $\int \left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$ax+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

```
[Out] a*x - (b*x)/2 + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0160623, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2635, 8}

$$ax+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^2,x]
```

```
[Out] a*x - (b*x)/2 + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^2(c+dx)\right)\,dx &= ax+b\int \sinh^2(c+dx)\,dx\\ &= ax+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}b\int 1\,dx\\ &= ax-\frac{bx}{2}+\frac{b\cosh(c+dx)\sinh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0316776, size = 36, normalized size = 1.2

$$ax+\frac{b(-c-dx)}{2d}+\frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^2,x]
```

```
[Out] a*x + (b*(-c - d*x))/(2*d) + (b*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.007, size = 32, normalized size = 1.1

$$ax+\frac{b}{d}\left(\frac{\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2}-\frac{dx}{2}-\frac{c}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^2,x)
```

```
[Out] a*x+b/d*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)
```

Maxima [A] time = 1.04378, size = 51, normalized size = 1.7

$$-\frac{1}{8}b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*x
```

Fricas [A] time = 1.84118, size = 74, normalized size = 2.47

$$\frac{(2a-b)dx+b\cosh\left(dx+c\right)\sinh\left(dx+c\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a - b)*d*x + b*cosh(d*x + c)*sinh(d*x + c))/d
```

Sympy [A] time = 0.408983, size = 51, normalized size = 1.7

$$ax+b\left(\begin{cases}\frac{x\sinh^2\left(c+dx\right)}{2}-\frac{x\cosh^2\left(c+dx\right)}{2}+\frac{\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{2d} & \text{for } d\neq 0\\ x\sinh^2\left(c\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((x*sinh(c + d*x)**2/2 - x*cosh(c + d*x)**2/2 + sinh(c + d
*x)*cosh(c + d*x)/(2*d), Ne(d, 0)), (x*sinh(c)**2, True))
```

Giac [B] time = 1.30807, size = 72, normalized size = 2.4

$$ax-\frac{\left(4dx-\left(2e^{(2dx+2c)}-1\right)e^{(-2dx-2c)}+4c-e^{(2dx+2c)}\right)b}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] a*x - 1/8*(4*d*x - (2*e^(2*d*x + 2*c) - 1)*e^(-2*d*x - 2*c) + 4*c - e^(2*d*
x + 2*c))*b/d
```

3.6 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=25

$$\frac{b\cosh(c+dx)}{d}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}$$

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) + (b*Cosh[c + d*x])/d
```

Rubi [A] time = 0.0350184, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3014, 3770}

$$\frac{b\cosh(c+dx)}{d}-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2),x]

[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) + (b*Cosh[c + d*x])/d
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{b\cosh(c+dx)}{d}+a\int \operatorname{csch}(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.0315701, size = 62, normalized size = 2.48

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\sinh(c)\sinh(dx)}{d}+\frac{b\cosh(c)\cosh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2),x]

[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2
+ (d*x)/2]])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.029, size = 24, normalized size = 1.

$$\frac{-2\,a\mathrm{Artanh}\left(\mathrm{e}^{dx+c}\right)+b\cosh\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*cosh(d*x+c))
```

Maxima [A] time = 1.04059, size = 58, normalized size = 2.32

$$\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 1.9737, size = 374, normalized size = 14.96

$$\frac{b\cosh\left(dx+c\right)^{2}+2\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)+b\sinh\left(dx+c\right)^{2}-2\left(a\cosh\left(dx+c\right)+a\sinh\left(dx+c\right)\right)\log\left(\cosh\left(a\right.\right.}{2\left(d\cosh\left(dx+c\right)+d\,s\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 - 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 + 1) + 2*(a*cosh(d*x + c) + a*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
+ c) - 1) + b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2144, size = 78, normalized size = 3.12

$$\frac{be^{(dx+c)}}{2\,d}+\frac{be^{(-dx-c)}}{2\,d}-\frac{a\log\left(e^{(dx+c)}+1\right)}{d}+\frac{a\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*b*e^(d*x + c)/d + 1/2*b*e^(-d*x - c)/d - a*log(e^(d*x + c) + 1)/d + a*l
og(abs(e^(d*x + c) - 1))/d
```

3.7 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$bx-\frac{a\coth(c+dx)}{d}$$

```
[Out] b*x - (a*Coth[c + d*x])/d
```

Rubi [A] time = 0.0266243, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 8}

$$bx-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x - (a*Coth[c + d*x])/d
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx&=-\frac{a\coth(c+dx)}{d}+b\int 1\,dx\\&=bx-\frac{a\coth(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.0195822, size = 16, normalized size = 1.

$$bx-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x - (a*Coth[c + d*x])/d
```

Maple [A] time = 0.027, size = 22, normalized size = 1.4

$$\frac{-\coth(dx+c)\,a+(dx+c)\,b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(-coth(d*x+c)*a+(d*x+c)*b)
```

Maxima [A] time = 1.02413, size = 31, normalized size = 1.94

$$bx+\frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*x + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.8503, size = 89, normalized size = 5.56

$$-\frac{a\cosh(dx+c)-(bdx+a)\sinh(dx+c)}{d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -(a*cosh(d*x + c) - (b*d*x + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29289, size = 41, normalized size = 2.56

$$\frac{(dx+c)b}{d}-\frac{2\,a}{d\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (d*x + c)*b/d - 2*a/(d*(e^(2*d*x + 2*c) - 1))
```

3.8 $\int \mathrm{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\mathrm{csch}(c+dx)}{2d}$$

```
[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/
(2*d)
```

Rubi [A] time = 0.04142, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 3770}

$$\frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\mathrm{csch}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - 2*b)*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/
(2*d)
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= -\frac{a\coth(c+dx)\mathrm{csch}(c+dx)}{2d}-\frac{1}{2}(a-2b)\int \mathrm{csch}(c+dx)\,dx\\ &= \frac{(a-2b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\mathrm{csch}(c+dx)}{2d}\end{aligned}$$

Mathematica [B] time = 0.0320482, size = 99, normalized size = 2.48

$$-\frac{a\,\mathrm{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\,\mathrm{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(a*Csch[(c + d*x)/2]^2)/(8*d) - (b*Log[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Si
nh[c/2 + (d*x)/2]])/d - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a*Sech[(c + d*x
)/2]^2)/(8*d)
```

Maple [A] time = 0.034, size = 40, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{Artanh}\left(e^{dx+c}\right)\right)-2\,b\operatorname{Artanh}\left(e^{dx+c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-2*b*arctanh(exp(d
*x+c)))
```

Maxima [B] time = 1.05464, size = 169, normalized size = 4.22

$$\frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c)
+ e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - b*(l
og(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Fricas [B] time = 1.9364, size = 1323, normalized size = 33.08

$$-\frac{2\,a\cosh(dx+c)^{3}+6\,a\cosh(dx+c)\sinh(dx+c)^{2}+2\,a\sinh(dx+c)^{3}+2\,a\cosh(dx+c)-\left((a-2\,b)\cosh(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d*
x + c)^3 + 2*a*cosh(d*x + c) - ((a - 2*b)*cosh(d*x + c)^4 + 4*(a - 2*b)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a - 2*b)*sinh(d*x + c)^4 - 2*(a - 2*b)*cosh(d
*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x + c)^2 + 4*(
(a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x + c) + a - 2*
b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a - 2*b)*cosh(d*x + c)^4 + 4*
(a - 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 2*b)*sinh(d*x + c)^4 - 2*(a
- 2*b)*cosh(d*x + c)^2 + 2*(3*(a - 2*b)*cosh(d*x + c)^2 - a + 2*b)*sinh(d*x
 + c)^2 + 4*((a - 2*b)*cosh(d*x + c)^3 - (a - 2*b)*cosh(d*x + c))*sinh(d*x
+ c) + a - 2*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x +
c)^2 + a)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x +
c)^3 + d*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d
)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) +
```

```
d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25302, size = 136, normalized size = 3.4

$$\frac{(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{4\,d}-\frac{(a-2\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{4\,d}-\frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(a - 2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/4*(a - 2*b)*log(e^(
d*x + c) + e^(-d*x - c) - 2)/d - a*(e^(d*x + c) + e^(-d*x - c))/(((e^(d*x +
 c) + e^(-d*x - c))^2 - 4)*d)
```

3.9 $\int \text{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=43

$$\frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}$$

```
[Out] ((2*a - 3*b)*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)
```

Rubi [A] time = 0.0394368, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3012, 3767, 8}

$$\frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((2*a - 3*b)*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \text{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= -\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}+\frac{1}{3}(-2a+3b)\int \text{csch}^2(c+dx)\,dx\\ &= -\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}+\frac{(i(2a-3b))\,\text{Subst}\left(\int 1\,dx,x,-i\coth(c+\right.}{3d}\\ &= \frac{(2a-3b)\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0314179, size = 49, normalized size = 1.14

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}-\frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (b*Coth[c + d*x])/d - (a*Coth[c + d*x]*Csch[c +
 d*x]^2)/(3*d)
```

Maple [A] time = 0.034, size = 35, normalized size = 0.8

$$\frac{1}{d}\left(a\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^{2}}{3}\right)\coth(dx+c)-b\coth(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-b*coth(d*x+c))
```

Maxima [B] time = 1.0247, size = 153, normalized size = 3.56

$$\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\frac{1}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}\right)+\frac{2b}{d\left(e^{(-2dx-2c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.80659, size = 425, normalized size = 9.88

$$\frac{4\left((a-3b)\cosh(dx+c)^{2}-2a\cosh(dx+c)\sinh(dx+c)+(a-3b)\mathrm{s}\right.}{3\left(d\cosh(dx+c)^{4}+4d\cosh(dx+c)\sinh(dx+c)^{3}+d\sinh(dx+c)^{4}-4d\cosh(dx+c)^{2}+2\left(3d\cosh(dx+c)^{2}-2d\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 4/3*((a - 3*b)*cosh(d*x + c)^2 - 2*a*cosh(d*x + c)*sinh(d*x + c) + (a - 3*b
)*sinh(d*x + c)^2 - 3*a + 3*b)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(
d*x + c)^3 + d*sinh(d*x + c)^4 - 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
^2 - 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*
x + c) + 3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.17306, size = 82, normalized size = 1.91

$$-\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-2\,a+3\,b\right)}{3\,d\left(e^{(2\,dx+2\,c)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) - 2*a
 + 3*b)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.10 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=146

$$\frac{\left(48a^2-208ab+139b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{\left(80a^2-176ab+93b^2\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(48a^2-\right.$$

```
[Out] ((48*a^2 - 80*a*b + 35*b^2)*x)/128 - ((80*a^2 - 176*a*b + 93*b^2)*Cosh[c +
d*x]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 208*a*b + 139*b^2)*Cosh[c + d*x]^3
*Sinh[c + d*x])/(192*d) + ((16*a - 13*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(
48*d) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x]^5)/(8*d)
```

Rubi [A] time = 0.1804, antiderivative size = 146, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 463, 455, 1157, 385, 206}

$$\frac{\left(48a^2-208ab+139b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{\left(80a^2-176ab+93b^2\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(48a^2-\right.$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```


```
[Out] ((48*a^2 - 80*a*b + 35*b^2)*x)/128 - ((80*a^2 - 176*a*b + 93*b^2)*Cosh[c +
d*x]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 208*a*b + 139*b^2)*Cosh[c + d*x]^3
*Sinh[c + d*x])/(192*d) + ((16*a - 13*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(
48*d) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x]^5)/(8*d)
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 463

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^2, x_Symbol] :> -Simp[((b*c - a*d)^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/
(a*b^2*e*n*(p + 1)), x] + Dist[1/(a*b^2*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)
^(p + 1)*Simp[(b*c - a*d)^2*(m + 1) + b^2*c^2*n*(p + 1) + a*b*d^2*n*(p + 1)
*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, n}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1]
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^5}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^3(c+dx)\sinh^5(c+dx)}{8d} - \frac{\text{Subst}\left(\int \frac{x^4\left(-8a^2+5b^2+8(a-b)^2x^2\right)}{\left(1-x^2\right)^4}\,dx,x,\text{ta}\right.}{8d}$$

$$= \frac{(16a-13b)b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^2\cosh^3(c+dx)\sinh^5(c+d}{8d}$$

$$= \frac{\left(48a^2-208ab+139b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{192d} + \frac{(16a-13b)b\cosh}{}$$

$$= -\frac{\left(80a^2-176ab+93b^2\right)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{\left(48a^2-208ab+1\right.}{}$$

$$= \frac{1}{128}\left(48a^2-80ab+35b^2\right)x - \frac{\left(80a^2-176ab+93b^2\right)\cosh(c+dx)\sinh(}{128d}$$

Mathematica [A] time = 0.203947, size = 133, normalized size = 0.91

$$\frac{-96\left(8a^2-15ab+7b^2\right)\sinh(2(c+dx))+24\left(4a^2-12ab+7b^2\right)\sinh(4(c+dx))+1152a^2c+1152a^2dx+32ab\sinh(6}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (1152*a^2*c - 1920*a*b*c + 840*b^2*c + 1152*a^2*d*x - 1920*a*b*d*x + 840*b^
2*d*x - 96*(8*a^2 - 15*a*b + 7*b^2)*Sinh[2*(c + d*x)] + 24*(4*a^2 - 12*a*b
+ 7*b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] - 32*b^2*Sinh[6*(c +
```

```
d*x)] + 3*b^2*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A] time = 0.048, size = 150, normalized size = 1.

$$\frac{1}{d}\left(b^2\left(\left(\frac{(\sinh(dx+c))^7}{8}-\frac{7(\sinh(dx+c))^5}{48}+\frac{35(\sinh(dx+c))^3}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh(dx+c)+\frac{35dx}{128}+\frac{35c}{128}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128
*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+2*a*b*((1/6*sinh(d*x+c)^5-5/
24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^2*((1/4*s
inh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.04378, size = 360, normalized size = 2.47

$$\frac{1}{64}a^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{6144}b^2\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d
*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d
- (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^
(-8*d*x - 8*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) -
 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [A] time = 1.97267, size = 589, normalized size = 4.03

$$\frac{3b^2\cosh(dx+c)\sinh(dx+c)^7+3\left(7b^2\cosh(dx+c)^3+8\left(ab-b^2\right)\cosh(dx+c)\right)\sinh(dx+c)^5+\left(21b^2\cosh(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 + 8*(
a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 + 80*(a
*b - b^2)*cosh(d*x + c)^3 + 12*(4*a^2 - 12*a*b + 7*b^2)*cosh(d*x + c))*sinh
(d*x + c)^3 + 3*(48*a^2 - 80*a*b + 35*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 + 8
*(a*b - b^2)*cosh(d*x + c)^5 + 4*(4*a^2 - 12*a*b + 7*b^2)*cosh(d*x + c)^3 -
 8*(8*a^2 - 15*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 15.3862, size = 490, normalized size = 3.36

$$\begin{cases} \frac{3a^2x\sinh^4(c+dx)}{8} - \frac{3a^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3a^2x\cosh^4(c+dx)}{8} + \frac{5a^2\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{3a^2\sinh(c+dx)\cosh^3(c+dx)}{8d} + \frac{5abx\,\mathrm{s}}{} \\ x\left(a+b\sinh^2(c)\right)^2\sinh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 + 5*a**2*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) - 3*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*a*b*x*sinh(c
+ d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(
c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c
 + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3
*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + 35*b**2*x*sinh(c + d*x)*
*8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**2*x*sinh(c
 + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**
6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)**7*cosh(c + d
*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**2
*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c + d*x)*cosh(c +
 d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**4, True))
```

Giac [B] time = 1.3632, size = 429, normalized size = 2.94

$$\frac{3\,b^2e^{(8\,dx+8\,c)} + 32\,abe^{(6\,dx+6\,c)} - 32\,b^2e^{(6\,dx+6\,c)} + 96\,a^2e^{(4\,dx+4\,c)} - 288\,abe^{(4\,dx+4\,c)} + 168\,b^2e^{(4\,dx+4\,c)} - 768\,a^2e^{(2\,dx+2\,c)} + 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6144*(3*b^2*e^(8*d*x + 8*c) + 32*a*b*e^(6*d*x + 6*c) - 32*b^2*e^(6*d*x +
6*c) + 96*a^2*e^(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 168*b^2*e^(4*d*x
+ 4*c) - 768*a^2*e^(2*d*x + 2*c) + 1440*a*b*e^(2*d*x + 2*c) - 672*b^2*e^(2*
d*x + 2*c) + 48*(48*a^2 - 80*a*b + 35*b^2)*(d*x + c) - (2400*a^2*e^(8*d*x +
 8*c) - 4000*a*b*e^(8*d*x + 8*c) + 1750*b^2*e^(8*d*x + 8*c) - 768*a^2*e^(6*
d*x + 6*c) + 1440*a*b*e^(6*d*x + 6*c) - 672*b^2*e^(6*d*x + 6*c) + 96*a^2*e^
(4*d*x + 4*c) - 288*a*b*e^(4*d*x + 4*c) + 168*b^2*e^(4*d*x + 4*c) + 32*a*b*
e^(2*d*x + 2*c) - 32*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-8*d*x - 8*c))/d
```

3.11 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=85

$$\frac{b(2a-3b)\cosh^5(c+dx)}{5d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}-\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}$$

```
[Out] -(((a - b)^2*Cosh[c + d*x])/d) + ((a - 3*b)*(a - b)*Cosh[c + d*x]^3)/(3*d)
+ ((2*a - 3*b)*b*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0974927, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3186, 373}

$$\frac{b(2a-3b)\cosh^5(c+dx)}{5d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}-\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -(((a - b)^2*Cosh[c + d*x])/d) + ((a - 3*b)*(a - b)*Cosh[c + d*x]^3)/(3*d)
+ ((2*a - 3*b)*b*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a-b+bx^2\right)^2\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left((a-b)^2+(a-3b)(-a+b)x^2-(2a-3b)bx^4-b^2x^6\right)dx,x,\cosh\right.}{d}\\ &= -\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{(a-3b)(a-b)\cosh^3(c+dx)}{3d}+\frac{(2a-3b)b\cosh^5(}{5d}\end{aligned}$$

Mathematica [A] time = 0.0384168, size = 154, normalized size = 1.81

$$-\frac{3a^2\cosh(c+dx)}{4d}+\frac{a^2\cosh(3(c+dx))}{12d}+\frac{5ab\cosh(c+dx)}{4d}-\frac{5ab\cosh(3(c+dx))}{24d}+\frac{ab\cosh(5(c+dx))}{40d}-\frac{35b^2\cosh(c}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-3*a^2*Cosh[c + d*x])/(4*d) + (5*a*b*Cosh[c + d*x])/(4*d) - (35*b^2*Cosh[c
 + d*x])/(64*d) + (a^2*Cosh[3*(c + d*x)])/(12*d) - (5*a*b*Cosh[3*(c + d*x)]
)/(24*d) + (7*b^2*Cosh[3*(c + d*x)])/(64*d) + (a*b*Cosh[5*(c + d*x)])/(40*d
) - (7*b^2*Cosh[5*(c + d*x)])/(320*d) + (b^2*Cosh[7*(c + d*x)])/(448*d)
```

Maple [A] time = 0.017, size = 102, normalized size = 1.2

$$\frac{1}{d}\left(b^2\left(-\frac{16}{35}+\frac{(\sinh(dx+c))^6}{7}-\frac{6(\sinh(dx+c))^4}{35}+\frac{8(\sinh(dx+c))^2}{35}\right)\cosh(dx+c)+2ab\left(\frac{8}{15}+1/5(\sinh(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*c
osh(d*x+c)+2*a*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^
2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.05969, size = 333, normalized size = 3.92

$$-\frac{1}{4480}b^2\left(\frac{\left(49e^{(-2dx-2c)}-245e^{(-4dx-4c)}+1225e^{(-6dx-6c)}-5\right)e^{(7dx+7c)}}{d}+\frac{1225e^{(-dx-c)}-245e^{(-3dx-3c)}+49e^{(-5dx-5c}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)
/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(
-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*e
^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 2.24913, size = 554, normalized size = 6.52

$$\frac{15b^2\cosh(dx+c)^7+105b^2\cosh(dx+c)\sinh(dx+c)^6+21\left(8ab-7b^2\right)\cosh(dx+c)^5+105\left(5b^2\cosh(dx+c)^3+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 + 21
*(8*a*b - 7*b^2)*cosh(d*x + c)^5 + 105*(5*b^2*cosh(d*x + c)^3 + (8*a*b - 7*
b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a^2 - 40*a*b + 21*b^2)*cosh(d*
x + c)^3 + 105*(3*b^2*cosh(d*x + c)^5 + 2*(8*a*b - 7*b^2)*cosh(d*x + c)^3 +
 (16*a^2 - 40*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48*a^2 -
```

```
80*a*b + 35*b^2)*cosh(d*x + c))/d
```

Sympy [A] time = 8.16706, size = 204, normalized size = 2.4

$$\begin{cases} \frac{a^2 \sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2\cosh^3(c+dx)}{3d} + \frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d} + \frac{16ab\cosh^5(c+dx)}{15d} + \frac{b^2\sinh^6(c+d\ldots}{\ldots} \\ x\left(a+b\sinh^2(c)\right)^2\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8*a*b*sinh(c + d*x)**2*cos
h(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*d) + b**2*sinh(c + d*x)**
6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b**2*sin
h(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(c + d*x)**7/(35*d), Ne(
d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**3, True))
```

Giac [B] time = 1.32485, size = 332, normalized size = 3.91

$$\frac{15\,b^2e^{(7\,dx+7\,c)} + 168\,abe^{(5\,dx+5\,c)} - 147\,b^2e^{(5\,dx+5\,c)} + 560\,a^2e^{(3\,dx+3\,c)} - 1400\,abe^{(3\,dx+3\,c)} + 735\,b^2e^{(3\,dx+3\,c)} - 5040\,a^2e^{(dx+c)} + \ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*b^2*e^(7*d*x + 7*c) + 168*a*b*e^(5*d*x + 5*c) - 147*b^2*e^(5*d*
x + 5*c) + 560*a^2*e^(3*d*x + 3*c) - 1400*a*b*e^(3*d*x + 3*c) + 735*b^2*e^(
3*d*x + 3*c) - 5040*a^2*e^(d*x + c) + 8400*a*b*e^(d*x + c) - 3675*b^2*e^(d*
x + c) - (5040*a^2*e^(6*d*x + 6*c) - 8400*a*b*e^(6*d*x + 6*c) + 3675*b^2*e^
(6*d*x + 6*c) - 560*a^2*e^(4*d*x + 4*c) + 1400*a*b*e^(4*d*x + 4*c) - 735*b^
2*e^(4*d*x + 4*c) - 168*a*b*e^(2*d*x + 2*c) + 147*b^2*e^(2*d*x + 2*c) - 15*
b^2)*e^(-7*d*x - 7*c))/d
```

3.12 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=110

$$\frac{\left(8a^2-20ab+11b^2\right)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}x\left(8a^2-12ab+5b^2\right)+\frac{b(4a-3b)\sinh(c+dx)\cosh^3(c+dx)}{8d}+\cdots$$

```
[Out] -((8*a^2 - 12*a*b + 5*b^2)*x)/16 + ((8*a^2 - 20*a*b + 11*b^2)*Cosh[c + d*x]
*Sinh[c + d*x])/(16*d) + ((4*a - 3*b)*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(8*d
) + (b^2*Cosh[c + d*x]^3*Sinh[c + d*x]^3)/(6*d)
```

Rubi [A] time = 0.110067, antiderivative size = 117, normalized size of antiderivative = 1.06, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3170, 3169}

$$\frac{\left(16a^2-36ab+15b^2\right)\sinh(c+dx)\cosh(c+dx)}{48d}-\frac{1}{16}x\left(8a^2-12ab+5b^2\right)+\frac{b(4a-5b)\sinh^3(c+dx)\cosh(c+dx)}{24d}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((8*a^2 - 12*a*b + 5*b^2)*x)/16 + ((16*a^2 - 36*a*b + 15*b^2)*Cosh[c + d*x
]*Sinh[c + d*x])/(48*d) + ((4*a - 5*b)*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24
*d) + (Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(6*d)
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}-\frac{1}{6}\int\left(a-(4a-5b)\,\mathrm{s}\cdots\right.$$

$$=-\frac{1}{16}\left(8a^2-12ab+5b^2\right)x+\frac{\left(16a^2-36ab+15b^2\right)\cosh(c+dx)\sinh(c+\cdots}{48d}$$

Mathematica [A] time = 0.191873, size = 99, normalized size = 0.9

$$\frac{\left(48a^2-96ab+45b^2\right)\sinh(2(c+dx))-96a^2c-96a^2dx+3b(4a-3b)\sinh(4(c+dx))+144abc+144abdx+b^2\sinh(\cdots}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-96*a^2*c + 144*a*b*c - 60*b^2*c - 96*a^2*d*x + 144*a*b*d*x - 60*b^2*d*x +
 (48*a^2 - 96*a*b + 45*b^2)*Sinh[2*(c + d*x)] + 3*(4*a - 3*b)*b*Sinh[4*(c +
 d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.017, size = 118, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(\left(\frac{(\sinh(dx+c))^5}{6}-\frac{5(\sinh(dx+c))^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+2ab\left(\left(1/4(\sinh(dx+c))^3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+
c)-5/16*d*x-5/16*c)+2*a*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+
3/8*d*x+3/8*c)+a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.04317, size = 255, normalized size = 2.32

$$\frac{1}{32}ab\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{8}a^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{384}b^2\left(\frac{(9e^{(-2dx-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x -
 2*c)/d) - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d
*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c))/d)
```

Fricas [A] time = 2.14473, size = 373, normalized size = 3.39

$$\frac{3b^2\cosh(dx+c)\sinh(dx+c)^5+2\left(5b^2\cosh(dx+c)^3+3\left(4ab-3b^2\right)\cosh(dx+c)\right)\sinh(dx+c)^3-6\left(8a^2-12ab+\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(4
*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(8*a^2 - 12*a*b + 5*b^2)*d
*x + 3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - 3*b^2)*cosh(d*x + c)^3 + (16*a^2 -
 32*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 4.87993, size = 332, normalized size = 3.02

$$\begin{cases} \frac{a^2x\sinh^2(c+dx)}{2} - \frac{a^2x\cosh^2(c+dx)}{2} + \frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{3abx\sinh^4(c+dx)}{4} - \frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2} + \frac{3abx\cosh^4(c+dx)}{4} + \\ x\left(a+b\sinh^2(c)\right)^2\sinh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(
c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c
+ d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d)
+ 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**
2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b**2*x*cosh(c + d
*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b**2*sinh(c +
 d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16
*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c)**2, True))
```

Giac [B] time = 1.35627, size = 316, normalized size = 2.87

$$\frac{b^2e^{(6\,dx+6\,c)} + 12\,abe^{(4\,dx+4\,c)} - 9\,b^2e^{(4\,dx+4\,c)} + 48\,a^2e^{(2\,dx+2\,c)} - 96\,abe^{(2\,dx+2\,c)} + 45\,b^2e^{(2\,dx+2\,c)} - 24\left(8\,a^2 - 12\,ab + 5\,b^2\right)(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/384*(b^2*e^(6*d*x + 6*c) + 12*a*b*e^(4*d*x + 4*c) - 9*b^2*e^(4*d*x + 4*c)
 + 48*a^2*e^(2*d*x + 2*c) - 96*a*b*e^(2*d*x + 2*c) + 45*b^2*e^(2*d*x + 2*c)
 - 24*(8*a^2 - 12*a*b + 5*b^2)*(d*x + c) + (176*a^2*e^(6*d*x + 6*c) - 264*a
*b*e^(6*d*x + 6*c) + 110*b^2*e^(6*d*x + 6*c) - 48*a^2*e^(4*d*x + 4*c) + 96*
a*b*e^(4*d*x + 4*c) - 45*b^2*e^(4*d*x + 4*c) - 12*a*b*e^(2*d*x + 2*c) + 9*b
^2*e^(2*d*x + 2*c) - b^2)*e^(-6*d*x - 6*c))/d
```

3.13 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{2b(a-b)\cosh^3(c+dx)}{3d}+\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}$$

```
[Out] ((a - b)^2*Cosh[c + d*x])/d + (2*(a - b)*b*Cosh[c + d*x]^3)/(3*d) + (b^2*Co
sh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0600295, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 194}

$$\frac{2b(a-b)\cosh^3(c+dx)}{3d}+\frac{(a-b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*Cosh[c + d*x])/d + (2*(a - b)*b*Cosh[c + d*x]^3)/(3*d) + (b^2*Co
sh[c + d*x]^5)/(5*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a-b+bx^2\right)^2\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^2\left(1+\frac{b(-2a+b)}{a^2}\right)+2ab\left(1-\frac{b}{a}\right)x^2+b^2x^4\right)dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{(a-b)^2\cosh(c+dx)}{d}+\frac{2(a-b)b\cosh^3(c+dx)}{3d}+\frac{b^2\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0362276, size = 111, normalized size = 1.95

$$\frac{a^2\sinh(c)\sinh(dx)}{d}+\frac{a^2\cosh(c)\cosh(dx)}{d}-\frac{3ab\cosh(c+dx)}{2d}+\frac{ab\cosh(3(c+dx))}{6d}+\frac{5b^2\cosh(c+dx)}{8d}-\frac{5b^2\cosh(3(c}{48d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Cosh[c]*Cosh[d*x])/d - (3*a*b*Cosh[c + d*x])/(2*d) + (5*b^2*Cosh[c + d
*x])/(8*d) + (a*b*Cosh[3*(c + d*x)])/(6*d) - (5*b^2*Cosh[3*(c + d*x)])/(48*
d) + (b^2*Cosh[5*(c + d*x)])/(80*d) + (a^2*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.015, size = 70, normalized size = 1.2

$$\frac{1}{d}\left(b^2\left(\frac{8}{15}+\frac{(\sinh(dx+c))^4}{5}-\frac{4(\sinh(dx+c))^2}{15}\right)\cosh(dx+c)+2ab\left(-2/3+1/3(\sinh(dx+c))^2\right)\cosh(dx+c)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(-2/
3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*cosh(d*x+c))
```

Maxima [B] time = 1.04599, size = 212, normalized size = 3.72

$$\frac{1}{480}b^2\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)+\frac{1}{12}ab\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(d}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/12*
a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3
*c)/d) + a^2*cosh(d*x + c)/d
```

Fricas [B] time = 2.01757, size = 309, normalized size = 5.42

$$\frac{3b^2\cosh(dx+c)^5+15b^2\cosh(dx+c)\sinh(dx+c)^4+5\left(8ab-5b^2\right)\cosh(dx+c)^3+15\left(2b^2\cosh(dx+c)^3+\left(8a\right.\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^2*cosh(d*x + c)^5 + 15*b^2*cosh(d*x + c)*sinh(d*x + c)^4 + 5*(8*
a*b - 5*b^2)*cosh(d*x + c)^3 + 15*(2*b^2*cosh(d*x + c)^3 + (8*a*b - 5*b^2)*
cosh(d*x + c))*sinh(d*x + c)^2 + 30*(8*a^2 - 12*a*b + 5*b^2)*cosh(d*x + c))
/d
```

Sympy [A] time = 2.35011, size = 128, normalized size = 2.25

$$\begin{cases}\frac{a^2\cosh(c+dx)}{d}+\frac{2ab\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{b^2\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4b^2\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{8b^2\cosh^5(c+}{15d}\\ x\left(a+b\sinh^2(c)\right)^2\sinh(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d -
4*a*b*cosh(c + d*x)**3/(3*d) + b**2*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*
*2*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b**2*cosh(c + d*x)**5/(15*d)
, Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*sinh(c), True))
```

Giac [B] time = 1.25033, size = 220, normalized size = 3.86

$$\frac{3\,b^2e^{(5\,dx+5\,c)}+40\,abe^{(3\,dx+3\,c)}-25\,b^2e^{(3\,dx+3\,c)}+240\,a^2e^{(dx+c)}-360\,abe^{(dx+c)}+150\,b^2e^{(dx+c)}+\left(240\,a^2e^{(4\,dx+4\,c)}-360\,abe^{(4}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^2*e^(5*d*x + 5*c) + 40*a*b*e^(3*d*x + 3*c) - 25*b^2*e^(3*d*x + 3
*c) + 240*a^2*e^(d*x + c) - 360*a*b*e^(d*x + c) + 150*b^2*e^(d*x + c) + (24
0*a^2*e^(4*d*x + 4*c) - 360*a*b*e^(4*d*x + 4*c) + 150*b^2*e^(4*d*x + 4*c) +
 40*a*b*e^(2*d*x + 2*c) - 25*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c))
/d
```

3.14 $\int \left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=72

$$\frac{1}{8}x\left(8a^2-8ab+3b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*x)/8 + ((8*a - 3*b)*b*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0217836, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3179}

$$\frac{1}{8}x\left(8a^2-8ab+3b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*x)/8 + ((8*a - 3*b)*b*Cosh[c + d*x]*Sinh[c + d*x])
/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3179

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[((8*a^2 +
 8*a*b + 3*b^2)*x)/8, x] + (-Simp[(b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f),
x] - Simp[(b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^2\,dx=\frac{1}{8}\left(8a^2-8ab+3b^2\right)x+\frac{(8a-3b)b\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^2\cosh(c+dx)\sinh^3}{4d}$$

Mathematica [A] time = 0.126727, size = 60, normalized size = 0.83

$$\frac{4\left(8a^2-8ab+3b^2\right)(c+dx)+8b(2a-b)\sinh(2(c+dx))+b^2\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (4*(8*a^2 - 8*a*b + 3*b^2)*(c + d*x) + 8*(2*a - b)*b*Sinh[2*(c + d*x)] + b^
2*Sinh[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.013, size = 79, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)+2ab\left(1/2\cosh(dx+c)\sinh(dx+c)-1/2d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+2*
a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a^2*(d*x+c))
```

Maxima [A] time = 1.00849, size = 142, normalized size = 1.97

$$\frac{1}{64}\,b^2\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)-\frac{1}{4}\,ab\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x -
 2*c)/d) + a^2*x
```

Fricas [A] time = 1.87806, size = 193, normalized size = 2.68

$$\frac{b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\left(8\,a^2-8\,ab+3\,b^2\right)dx+\left(b^2\cosh\left(dx+c\right)^3+4\left(2\,ab-b^2\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)*sinh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*d*x + (b^2
*cosh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 1.27699, size = 168, normalized size = 2.33

$$\begin{cases} a^2x+abx\sinh^2\left(c+dx\right)-abx\cosh^2\left(c+dx\right)+\frac{ab\sinh\left(c+dx\right)\cosh\left(c+dx\right)}{d}+\frac{3b^2x\sinh^4\left(c+dx\right)}{8}-\frac{3b^2x\sinh^2\left(c+dx\right)\cosh^2\left(c+dx\right)}{4}+\frac{3b^2x \\ x\left(a+b\sinh^2\left(c\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*x*sinh(c + d*x)**2 - a*b*x*cosh(c + d*x)**2 + a*b*s
inh(c + d*x)*cosh(c + d*x)/d + 3*b**2*x*sinh(c + d*x)**4/8 - 3*b**2*x*sinh(
c + d*x)**2*cosh(c + d*x)**2/4 + 3*b**2*x*cosh(c + d*x)**4/8 + 5*b**2*sinh(
c + d*x)**3*cosh(c + d*x)/(8*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*
d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2, True))
```

Giac [B] time = 1.22594, size = 204, normalized size = 2.83

$$\frac{b^2e^{(4\,dx+4\,c)} + 16\,abe^{(2\,dx+2\,c)} - 8\,b^2e^{(2\,dx+2\,c)} + 8\left(8\,a^2 - 8\,ab + 3\,b^2\right)(dx + c) - \left(48\,a^2e^{(4\,dx+4\,c)} - 48\,abe^{(4\,dx+4\,c)} + 18\,b^2e^{(4}\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/64*(b^2*e^(4*d*x + 4*c) + 16*a*b*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c)
+ 8*(8*a^2 - 8*a*b + 3*b^2)*(d*x + c) - (48*a^2*e^(4*d*x + 4*c) - 48*a*b*e^
(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) + 16*a*b*e^(2*d*x + 2*c) - 8*b^2*e^(
2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c))/d
```

3.15 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=52

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a-b)\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}$$

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) + ((2*a - b)*b*Cosh[c + d*x])/d + (b^2*Co
sh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0657536, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 390, 206}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a-b)\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) + ((2*a - b)*b*Cosh[c + d*x])/d + (b^2*Co
sh[c + d*x]^3)/(3*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx = -\frac{\text{Subst}\left(\int \frac{(a-b+bx^2)^2}{1-x^2}dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-(2a-b)b-b^2x^2+\frac{a^2}{1-x^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{(2a-b)b\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \cosh\right.}{d}$$
$$= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{(2a-b)b\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0339139, size = 104, normalized size = 2.

$$\frac{a^2\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{2ab\sinh(c)\sinh(dx)}{d}+\frac{2ab\cosh(c)\cosh(dx)}{d}-\frac{3b^2\cosh(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*a*b*Cosh[c]*Cosh[d*x])/d - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c
+ d*x)])/(12*d) - (a^2*Log[Cosh[c/2 + (d*x)/2]])/d + (a^2*Log[Sinh[c/2 + (d
*x)/2]])/d + (2*a*b*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.032, size = 50, normalized size = 1.

$$\frac{1}{d}\left(-2\,a^2\text{Artanh}\left(e^{dx+c}\right)+2\,ab\cosh\left(dx+c\right)+b^2\left(-\frac{2}{3}+\frac{\left(\sinh\left(dx+c\right)\right)^2}{3}\right)\cosh\left(dx+c\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*cosh(d*x+c)+b^2*(-2/3+1/3*sinh(d*x+c)
^2)*cosh(d*x+c))
```

Maxima [B] time = 1.04856, size = 138, normalized size = 2.65

$$\frac{1}{24}b^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+ab\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{a^2\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a^2*log(tanh(1/2*d*x +
 1/2*c))/d
```

Fricas [B] time = 1.85076, size = 1280, normalized size = 24.62

$$\frac{b^2\cosh(dx+c)^6+6\,b^2\cosh(dx+c)\sinh(dx+c)^5+b^2\sinh(dx+c)^6+3\left(8\,ab-3\,b^2\right)\cosh(dx+c)^4+3\left(5\,b^2\cosh(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(
d*x + c)^6 + 3*(8*a*b - 3*b^2)*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 +
 8*a*b - 3*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + 3*(8*a*b - 3*b
^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(8*a*b - 3*b^2)*cosh(d*x + c)^2 + 3*
(5*b^2*cosh(d*x + c)^4 + 6*(8*a*b - 3*b^2)*cosh(d*x + c)^2 + 8*a*b - 3*b^2)
*sinh(d*x + c)^2 + b^2 - 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*si
nh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) + 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh
(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(
d*x + c)^3)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(b^2*cosh(d*x + c)^5
 + 2*(8*a*b - 3*b^2)*cosh(d*x + c)^3 + (8*a*b - 3*b^2)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh
(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28008, size = 174, normalized size = 3.35

$$-\frac{a^2\log\left(e^{(dx+c)}+1\right)}{d}+\frac{a^2\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}+\frac{\left(24\,abe^{(2\,dx+2\,c)}-9\,b^2e^{(2\,dx+2\,c)}+b^2\right)e^{(-3\,dx-3\,c)}}{24\,d}+\frac{b^2d^2e^{(3\,dx+3\,c)}+24\,abd^2e^{(dx}}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -a^2*log(e^(d*x + c) + 1)/d + a^2*log(abs(e^(d*x + c) - 1))/d + 1/24*(24*a*
b*e^(2*d*x + 2*c) - 9*b^2*e^(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c)/d + 1/24*
(b^2*d^2*e^(3*d*x + 3*c) + 24*a*b*d^2*e^(d*x + c) - 9*b^2*d^2*e^(d*x + c))/
d^3
```

3.16 $\int \mathrm{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=50

$$-\frac{a^2\coth(c+dx)}{d}+\frac{1}{2}bx(4a-b)+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] ((4*a - b)*b*x)/2 - (a^2*Coth[c + d*x])/d + (b^2*Cosh[c + d*x]*Sinh[c + d*x
])/(2*d)
```

Rubi [A] time = 0.0813633, antiderivative size = 64, normalized size of antiderivative = 1.28, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 462, 385, 206}

$$\frac{\left(2a^2+b^2\right)\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d}+\frac{1}{2}bx(4a-b)$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - b)*b*x)/2 - (a^2*Cosh[c + d*x]^2*Coth[c + d*x])/d + ((2*a^2 + b^2)*
Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \text{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{x^2\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\text{Subst}\left(\int \frac{a(a+2b)+(a-b)^2x^2}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+\right.}{d}$$

$$= -\frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\left(2a^2+b^2\right)\cosh(c+dx)\sinh(c+dx)}{2d} + $$

$$= \frac{1}{2}(4a-b)bx - \frac{a^2\cosh^2(c+dx)\coth(c+dx)}{d} + \frac{\left(2a^2+b^2\right)\cosh(c+dx)\sin}{2d}$$

Mathematica [A] time = 0.162947, size = 56, normalized size = 1.12

$$-\frac{a^2\coth(c+dx)}{d} + 2abx + \frac{b^2(-c-dx)}{2d} + \frac{b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] 2*a*b*x + (b^2*(-c - d*x))/(2*d) - (a^2*Coth[c + d*x])/d + (b^2*Sinh[2*(c +
 d*x)])/(4*d)
```

Maple [A] time = 0.033, size = 52, normalized size = 1.

$$\frac{1}{d}\left(-a^2\coth(dx+c) + 2\,ab\,(dx+c) + b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*(d*x+c)+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*
x-1/2*c))
```

Maxima [A] time = 1.01429, size = 85, normalized size = 1.7

$$-\frac{1}{8}b^2\left(4\,x - \frac{e^{(2\,dx+2\,c)}}{d} + \frac{e^{(-2\,dx-2\,c)}}{d}\right) + 2\,abx + \frac{2\,a^2}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a*b*x + 2*a^2/(
d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [A] time = 1.87027, size = 215, normalized size = 4.3

$$\frac{b^2\cosh(dx+c)^3 + 3b^2\cosh(dx+c)\sinh(dx+c)^2 - (8a^2+b^2)\cosh(dx+c) + 4((4ab-b^2)dx + 2a^2)\sinh(dx+c}{8d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a^2 + b
^2)*cosh(d*x + c) + 4*((4*a*b - b^2)*d*x + 2*a^2)*sinh(d*x + c))/(d*sinh(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22766, size = 189, normalized size = 3.78

$$\frac{b^2e^{(2\,dx+2\,c)}}{8\,d} + \frac{(4\,ab-b^2)(dx+c)}{2\,d} - \frac{4\,abe^{(4\,dx+4\,c)} - b^2e^{(4\,dx+4\,c)} + 16\,a^2e^{(2\,dx+2\,c)} - 4\,abe^{(2\,dx+2\,c)} + 2\,b^2e^{(2\,dx+2\,c)} - b^2}{8\,d\left(e^{(4\,dx+4\,c)} - e^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*e^(2*d*x + 2*c)/d + 1/2*(4*a*b - b^2)*(d*x + c)/d - 1/8*(4*a*b*e^(4
*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 4*a*b*e^(2*d*x
 + 2*c) + 2*b^2*e^(2*d*x + 2*c) - b^2)/(d*(e^(4*d*x + 4*c) - e^(2*d*x + 2*c
)))
```

3.17 $\int \mathbf{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=56

$$-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\cosh(c+dx)}{d}$$

```
[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b^2*Cosh[c + d*x])/d - (a^2*C
oth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rubi [A] time = 0.0880729, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 390, 385, 206}

$$-\frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d}+\frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a*(a - 4*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + (b^2*Cosh[c + d*x])/d - (a^2*C
oth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \mathrm{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\mathrm{Subst}\left(\int \frac{\left(a-b+bx^2\right)^2}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\mathrm{Subst}\left(\int \left(b^2+\frac{a(a-2b)+2abx^2}{\left(1-x^2\right)^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh(c+dx)}{d} + \frac{\mathrm{Subst}\left(\int \frac{a(a-2b)+2abx^2}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh(c+dx)}{d} - \frac{a^2\coth(c+dx)\mathrm{csch}(c+dx)}{2d} + \frac{(a(a-4b))\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{a(a-4b)\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b^2\cosh(c+dx)}{d} - \frac{a^2\coth(c+dx)\mathrm{csc}}{2d}$$

Mathematica [B] time = 0.0639887, size = 134, normalized size = 2.39

$$-\frac{a^2\mathrm{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\mathrm{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{2ab\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{2ab\log\left(\cosh\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (a^2*Csch[(c + d*x)/2]^2)/(8*d) - (2*a*b*Log[Co
sh[c/2 + (d*x)/2]])/d + (2*a*b*Log[Sinh[c/2 + (d*x)/2]])/d - (a^2*Log[Tanh[
(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/(8*d) + (b^2*Sinh[c]*Sinh[
d*x])/d
```

Maple [A] time = 0.041, size = 53, normalized size = 1.

$$\frac{1}{d}\left(a^2\left(-\frac{\mathrm{csch}\,(dx+c)\coth\,(dx+c)}{2}+\mathrm{Artanh}\left(\mathrm{e}^{dx+c}\right)\right)-4\,ab\mathrm{Artanh}\left(\mathrm{e}^{dx+c}\right)+b^2\cosh\,(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-4*a*b*arctanh(e
xp(d*x+c))+b^2*cosh(d*x+c))
```

Maxima [B] time = 1.07468, size = 212, normalized size = 3.79

$$\frac{1}{2}\,b^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)-2\,ab\left(\frac{\log\left(e^{(\right.}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d
 - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-
2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - l
og(e^(-d*x - c) - 1)/d)
```

Fricas [B] time = 1.95247, size = 2310, normalized size = 41.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d
*x + c)^6 - (2*a^2 + b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x + c)^2 - 2*a^2
 - b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x
 + c))*sinh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x + c)^2 + (15*b^2*cosh(d*x +
 c)^4 - 6*(2*a^2 + b^2)*cosh(d*x + c)^2 - 2*a^2 - b^2)*sinh(d*x + c)^2 + b^
2 + ((a^2 - 4*a*b)*cosh(d*x + c)^5 + 5*(a^2 - 4*a*b)*cosh(d*x + c)*sinh(d*x
 + c)^4 + (a^2 - 4*a*b)*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b)*cosh(d*x + c)^3 +
 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(d*x + c)^3 + 2*(5*(
a^2 - 4*a*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x + c))*sinh(d*x + c)
^2 + (a^2 - 4*a*b)*cosh(d*x + c) + (5*(a^2 - 4*a*b)*cosh(d*x + c)^4 - 6*(a^
2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c))*log(cosh(d*x + c)
+ sinh(d*x + c) + 1) - ((a^2 - 4*a*b)*cosh(d*x + c)^5 + 5*(a^2 - 4*a*b)*cos
h(d*x + c)*sinh(d*x + c)^4 + (a^2 - 4*a*b)*sinh(d*x + c)^5 - 2*(a^2 - 4*a*b
)*cosh(d*x + c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^2 - a^2 + 4*a*b)*sinh(
d*x + c)^3 + 2*(5*(a^2 - 4*a*b)*cosh(d*x + c)^3 - 3*(a^2 - 4*a*b)*cosh(d*x
+ c))*sinh(d*x + c)^2 + (a^2 - 4*a*b)*cosh(d*x + c) + (5*(a^2 - 4*a*b)*cosh
(d*x + c)^4 - 6*(a^2 - 4*a*b)*cosh(d*x + c)^2 + a^2 - 4*a*b)*sinh(d*x + c))
*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*b^2*cosh(d*x + c)^5 - 2*(2*a
^2 + b^2)*cosh(d*x + c)^3 - (2*a^2 + b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*
cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 - 2
*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 2*(5*d*c
osh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*
d*cosh(d*x + c)^4 - 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35923, size = 180, normalized size = 3.21

$$\frac{b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{2\,d}-\frac{a^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)d}+\frac{\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{4\,d}-\frac{\left(a^2-4\,ab\right)\log\left(e^{(dx+c)}+e^{(-\cdots}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*(e^(d*x + c) + e^(-d*x - c))/d - a^2*(e^(d*x + c) + e^(-d*x - c))/(
((e^(d*x + c) + e^(-d*x - c))^2 - 4)*d) + 1/4*(a^2 - 4*a*b)*log(e^(d*x + c)
 + e^(-d*x - c) + 2)/d - 1/4*(a^2 - 4*a*b)*log(e^(d*x + c) + e^(-d*x - c) -
 2)/d
```

3.18 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=40

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a(a-2b)\coth(c+dx)}{d}+b^2x$$

```
[Out] b^2*x + (a*(a - 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0741049, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 461, 207}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a(a-2b)\coth(c+dx)}{d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] b^2*x + (a*(a - 2*b)*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{x^4\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{a^2}{x^4}-\frac{a(a-2b)}{x^2}-\frac{b^2}{-1+x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}-\frac{b^2\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh\right.}{d}\\
&= b^2x+\frac{a(a-2b)\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [B] time = 0.712014, size = 85, normalized size = 2.12

$$\frac{4\sinh^4(c+dx)\left(a\operatorname{csch}^2(c+dx)+b\right)^2\left(3b^2(c+dx)-a\coth(c+dx)\left(a\operatorname{csch}^2(c+dx)-2a+6b\right)\right)}{3d(2a+b\cosh(2(c+dx))-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (4*(b + a*Csch[c + d*x]^2)^2*(3*b^2*(c + d*x) - a*Coth[c + d*x]*(-2*a + 6*b
 + a*Csch[c + d*x]^2))*Sinh[c + d*x]^4)/(3*d*(2*a - b + b*Cosh[2*(c + d*x)]
)^2)
```

Maple [A] time = 0.04, size = 47, normalized size = 1.2

$$\frac{1}{d}\left(a^2\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)-2\,ab\coth(dx+c)+b^2(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*a*b*coth(d*x+c)+b^2*(d*x+c))
```

Maxima [B] time = 1.06279, size = 163, normalized size = 4.08

$$b^2x+\frac{4}{3}a^2\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)+\frac{4\,ab}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] b^2*x + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x -
4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*
c) + e^(-6*d*x - 6*c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.89286, size = 428, normalized size = 10.7

$$\frac{2\left(a^2-3\,ab\right)\cosh(dx+c)^3+6\left(a^2-3\,ab\right)\cosh(dx+c)\sinh(dx+c)^2+\left(3\,b^2dx-2\,a^2+6\,ab\right)\sinh(dx+c)^3-6\left(a^2\right.}{3\left(d\sinh(dx+c)^3+3\left(d\cosh(dx+c)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*(a^2 - 3*a*b)*cosh(d*x + c)^3 + 6*(a^2 - 3*a*b)*cosh(d*x + c)*sinh(d
*x + c)^2 + (3*b^2*d*x - 2*a^2 + 6*a*b)*sinh(d*x + c)^3 - 6*(a^2 - a*b)*cos
h(d*x + c) - 3*(3*b^2*d*x - (3*b^2*d*x - 2*a^2 + 6*a*b)*cosh(d*x + c)^2 - 2
*a^2 + 6*a*b)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2 - d)
```

```
*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.34544, size = 109, normalized size = 2.72

$$\frac{(dx+c)b^2}{d} - \frac{4\left(3\,abe^{(4\,dx+4\,c)} + 3\,a^2e^{(2\,dx+2\,c)} - 6\,abe^{(2\,dx+2\,c)} - a^2 + 3\,ab\right)}{3\,d\left(e^{(2\,dx+2\,c)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^2/d - 4/3*(3*a*b*e^(4*d*x + 4*c) + 3*a^2*e^(2*d*x + 2*c) - 6*a*
b*e^(2*d*x + 2*c) - a^2 + 3*a*b)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.19 $\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=261

$$\frac{\left(-272a^2b+48a^3+314ab^2-105b^3\right)\sinh(c+dx)\cosh^3(c+dx)}{640d}-\frac{\left(-1744a^2b+576a^3+1678ab^2-525b^3\right)\sinh(c+d}{1280d}$$

```
[Out] (3*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*x)/256 - ((576*a^3 - 1744*a^2*b + 1
678*a*b^2 - 525*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(1280*d) + ((48*a^3 - 272
*a^2*b + 314*a*b^2 - 105*b^3)*Cosh[c + d*x]^3*Sinh[c + d*x])/(640*d) + (3*(
2*a - 3*b)*Cosh[c + d*x]^5*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^2)
/(80*d) + (Cosh[c + d*x]^7*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^3)
/(10*d) - (b*Cosh[c + d*x]^3*Sinh[c + d*x]^3*(a*(14*a - 9*b) - (22*a - 21*b
)*(a - b)*Tanh[c + d*x]^2))/(160*d)
```

Rubi [A] time = 0.429778, antiderivative size = 261, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 467, 577, 455, 385, 206}

$$\frac{\left(-272a^2b+48a^3+314ab^2-105b^3\right)\sinh(c+dx)\cosh^3(c+dx)}{640d}-\frac{\left(-1744a^2b+576a^3+1678ab^2-525b^3\right)\sinh(c+d}{1280d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] (3*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*x)/256 - ((576*a^3 - 1744*a^2*b + 1
678*a*b^2 - 525*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(1280*d) + ((48*a^3 - 272
*a^2*b + 314*a*b^2 - 105*b^3)*Cosh[c + d*x]^3*Sinh[c + d*x])/(640*d) + (3*(
2*a - 3*b)*Cosh[c + d*x]^5*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^2)
/(80*d) + (Cosh[c + d*x]^7*Sinh[c + d*x]^3*(a - (a - b)*Tanh[c + d*x]^2)^3)
/(10*d) - (b*Cosh[c + d*x]^3*Sinh[c + d*x]^3*(a*(14*a - 9*b) - (22*a - 21*b
)*(a - b)*Tanh[c + d*x]^2))/(160*d)
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 455

```
Int[(x_)^(m_)*((a_) + (b_.)*(x_)^2)^(p_)*((c_) + (d_.)*(x_)^2), x_Symbol] :
> Simp[((-a)^(m/2 - 1)*(b*c - a*d)*x*(a + b*x^2)^(p + 1))/(2*b^(m/2 + 1)*(p
 + 1)), x] + Dist[1/(2*b^(m/2 + 1)*(p + 1)), Int[(a + b*x^2)^(p + 1)*Expand
ToSum[2*b*(p + 1)*x^2*Together[(b^(m/2)*x^(m - 2)*(c + d*x^2) - (-a)^(m/2 -
 1)*(b*c - a*d))/(a + b*x^2)] - (-a)^(m/2 - 1)*(b*c - a*d), x], x], x] /; F
reeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && IGtQ[m/2, 0] &&
 (IntegerQ[p] || EqQ[m + 2*p + 1, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{x^4\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^6}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^7(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^3}{10d} - \frac{\operatorname{Subst}\left(\int \frac{x^2(3a-}{}\right.}{}$$

$$= \frac{3(2a-3b)\cosh^5(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{80d} + \frac{\cosh}{}$$

$$= \frac{3(2a-3b)\cosh^5(c+dx)\sinh^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{80d} + \frac{\cosh}{}$$

$$= \frac{\left(48a^3-272a^2b+314ab^2-105b^3\right)\cosh^3(c+dx)\sinh(c+dx)}{640d} + \frac{3(2a-3b)}{}$$

$$= -\frac{\left(576a^3-1744a^2b+1678ab^2-525b^3\right)\cosh(c+dx)\sinh(c+dx)}{1280d} + \frac{\left(48a^3\right.}{}$$

$$= \frac{3}{256}(4a-3b)\left(8a^2-14ab+7b^2\right)x - \frac{\left(576a^3-1744a^2b+1678ab^2-525b^3\right)}{1280d}$$

Mathematica [A] time = 0.42218, size = 162, normalized size = 0.62

$$\frac{120(4a-3b)\left(8a^2-14ab+7b^2\right)(c+dx)+10b\left(16a^2-32ab+15b^2\right)\sinh(6(c+dx))-20\left(-360a^2b+128a^3+336ab^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (120*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*(c + d*x) - 20*(128*a^3 - 360*a^2
*b + 336*a*b^2 - 105*b^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 - 36*a^2*b + 42*a*b
^2 - 15*b^3)*Sinh[4*(c + d*x)] + 10*b*(16*a^2 - 32*a*b + 15*b^2)*Sinh[6*(c
+ d*x)] + 5*(6*a - 5*b)*b^2*Sinh[8*(c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(
10240*d)
```

Maple [A] time = 0.052, size = 222, normalized size = 0.9

$$\frac{1}{d}\left(b^3\left(\left(\frac{(\sinh(dx+c))^9}{10}-\frac{9(\sinh(dx+c))^7}{80}+\frac{21(\sinh(dx+c))^5}{160}-\frac{21(\sinh(dx+c))^3}{128}+\frac{63\sinh(dx+c)}{256}\right)\cosh(d\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/12
8*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+3*a*b^
2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d
*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3*a^2*b*((1/6*sinh(d*x+c)^5-5/24*si
nh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^3*((1/4*sinh(d
*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.0771, size = 547, normalized size = 2.1

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{20480}b^3\left(\frac{\left(25e^{(-2dx-2c)}-150e^{(-4dx-4c)}+600e^{(-6}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-4*
d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x +
10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c)
+ 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 1
/2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x -
6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 16
8*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) - 1/128*a
^2*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 12
0*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c
))/d)
```

Fricas [A] time = 1.80972, size = 1013, normalized size = 3.88

$$\frac{5\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^9+10\left(6\,b^3\cosh\left(dx+c\right)^3+\left(6\,ab^2-5\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^7+\left(126\,b^3\cosh\left(d\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 + (
6*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5
+ 70*(6*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - 32*a*b^2 + 15*b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - 5
*b^3)*cosh(d*x + c)^5 + 5*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d*x + c)^3 +
4*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 3
0*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*d*x + 5*(b^3*cosh(d*x + c)^9 + 2*
(6*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 3*(16*a^2*b - 32*a*b^2 + 15*b^3)*cosh(d
*x + c)^5 + 8*(8*a^3 - 36*a^2*b + 42*a*b^2 - 15*b^3)*cosh(d*x + c)^3 - 2*(1
28*a^3 - 360*a^2*b + 336*a*b^2 - 105*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 37.0623, size = 777, normalized size = 2.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 + 5*a**3*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) - 3*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 15*a**2*b*x*sin
h(c + d*x)**6/16 - 45*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 45*a*
*2*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 15*a**2*b*x*cosh(c + d*x)**6/
16 + 33*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*a**2*b*sinh(c + d*
x)**3*cosh(c + d*x)**3/(2*d) + 15*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16
*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*cos
h(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 105*
a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x)*
*8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*sin
h(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh(c
 + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 63
*b**3*x*sinh(c + d*x)**10/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**
2/256 + 315*b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(
c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x
)**8/256 - 63*b**3*x*cosh(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh
(c + d*x)/(256*d) - 237*b**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21
*b**3*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*
cosh(c + d*x)**7/(128*d) + 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d),
Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**4, True))
```

Giac [B] time = 1.39929, size = 679, normalized size = 2.6

$$\frac{2\,b^3e^{(10\,dx+10\,c)}+30\,ab^2e^{(8\,dx+8\,c)}-25\,b^3e^{(8\,dx+8\,c)}+160\,a^2be^{(6\,dx+6\,c)}-320\,ab^2e^{(6\,dx+6\,c)}+150\,b^3e^{(6\,dx+6\,c)}+320\,a^3e^{(4\,dx+4\,c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/20480*(2*b^3*e^(10*d*x + 10*c) + 30*a*b^2*e^(8*d*x + 8*c) - 25*b^3*e^(8*d
*x + 8*c) + 160*a^2*b*e^(6*d*x + 6*c) - 320*a*b^2*e^(6*d*x + 6*c) + 150*b^3
*e^(6*d*x + 6*c) + 320*a^3*e^(4*d*x + 4*c) - 1440*a^2*b*e^(4*d*x + 4*c) + 1
680*a*b^2*e^(4*d*x + 4*c) - 600*b^3*e^(4*d*x + 4*c) - 2560*a^3*e^(2*d*x + 2
*c) + 7200*a^2*b*e^(2*d*x + 2*c) - 6720*a*b^2*e^(2*d*x + 2*c) + 2100*b^3*e^
(2*d*x + 2*c) + 240*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^3)*(d*x + c) - (87
68*a^3*e^(10*d*x + 10*c) - 21920*a^2*b*e^(10*d*x + 10*c) + 19180*a*b^2*e^(1
0*d*x + 10*c) - 5754*b^3*e^(10*d*x + 10*c) - 2560*a^3*e^(8*d*x + 8*c) + 720
0*a^2*b*e^(8*d*x + 8*c) - 6720*a*b^2*e^(8*d*x + 8*c) + 2100*b^3*e^(8*d*x +
8*c) + 320*a^3*e^(6*d*x + 6*c) - 1440*a^2*b*e^(6*d*x + 6*c) + 1680*a*b^2*e^
(6*d*x + 6*c) - 600*b^3*e^(6*d*x + 6*c) + 160*a^2*b*e^(4*d*x + 4*c) - 320*a
*b^2*e^(4*d*x + 4*c) + 150*b^3*e^(4*d*x + 4*c) + 30*a*b^2*e^(2*d*x + 2*c) -
 25*b^3*e^(2*d*x + 2*c) + 2*b^3)*e^(-10*d*x - 10*c))/d
```

3.20 $\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=115

$$\frac{b^2(3a-4b)\cosh^7(c+dx)}{7d}+\frac{3b(a-2b)(a-b)\cosh^5(c+dx)}{5d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}-\frac{(a-b)^3\cosh(c+dx)}{d}+$$

```
[Out] -(((a - b)^3*Cosh[c + d*x])/d) + ((a - 4*b)*(a - b)^2*Cosh[c + d*x]^3)/(3*d
) + (3*(a - 2*b)*(a - b)*b*Cosh[c + d*x]^5)/(5*d) + ((3*a - 4*b)*b^2*Cosh[c
 + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.129173, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3186, 373}

$$\frac{b^2(3a-4b)\cosh^7(c+dx)}{7d}+\frac{3b(a-2b)(a-b)\cosh^5(c+dx)}{5d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}-\frac{(a-b)^3\cosh(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -(((a - b)^3*Cosh[c + d*x])/d) + ((a - 4*b)*(a - b)^2*Cosh[c + d*x]^3)/(3*d
) + (3*(a - 2*b)*(a - b)*b*Cosh[c + d*x]^5)/(5*d) + ((3*a - 4*b)*b^2*Cosh[c
 + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= -\frac{\operatorname{Subst}\left(\int\left(1-x^2\right)\left(a-b+bx^2\right)^3\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\operatorname{Subst}\left(\int\left((a-b)^3-(a-4b)(a-b)^2x^2+3(a-2b)b(-a+b)x^4-(3a-4b\right.\right.}{d}\\ &= -\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{(a-4b)(a-b)^2\cosh^3(c+dx)}{3d}+\frac{3(a-2b)(a-b)b}{5d}\end{aligned}$$

Mathematica [A] time = 0.796978, size = 127, normalized size = 1.1

$$\frac{-1890(4a-3b)\left(8a^2-14ab+7b^2\right)\cosh(c+dx)+420\left(-60a^2b+16a^3+63ab^2-21b^3\right)\cosh(3(c+dx))+135b^2(4a-3b}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-1890*(4*a - 3*b)*(8*a^2 - 14*a*b + 7*b^2)*Cosh[c + d*x] + 420*(16*a^3 - 6
0*a^2*b + 63*a*b^2 - 21*b^3)*Cosh[3*(c + d*x)] + 756*(4*a - 3*b)*(a - b)*b*
Cosh[5*(c + d*x)] + 135*(4*a - 3*b)*b^2*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(
c + d*x)])/(80640*d)
```

Maple [A] time = 0.049, size = 158, normalized size = 1.4

$$\frac{1}{d}\left(b^3\left(\frac{128}{315}+\frac{(\sinh(dx+c))^8}{9}-\frac{8(\sinh(dx+c))^6}{63}+\frac{16(\sinh(dx+c))^4}{105}-\frac{64(\sinh(dx+c))^2}{315}\right)\cosh(dx+c)+3ab^2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4
-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*s
inh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(8/15+1/5*sinh(d*x+c)^
4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.08331, size = 508, normalized size = 4.42

$$-\frac{1}{161280}b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)}-2268\,e^{(-4\,dx-4\,c)}+8820\,e^{(-6\,dx-6\,c)}-39690\,e^{(-8\,dx-8\,c)}-35\right)e^{(9\,dx+9\,c)}}{d}-\frac{39690\,e^{(-dx-c)}-88$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-
4*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x
 - c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
 + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c
)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) +
1/24*a^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d)
```

Fricas [B] time = 1.82041, size = 940, normalized size = 8.17

$$35\,b^3\cosh(dx+c)^9+315\,b^3\cosh(dx+c)\sinh(dx+c)^8+135\left(4\,ab^2-3\,b^3\right)\cosh(dx+c)^7+105\left(28\,b^3\cosh(dx+c\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 + 1
35*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^7 + 105*(28*b^3*cosh(d*x + c)^3 + 9*(4*a
*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 756*(4*a^2*b - 7*a*b^2 + 3*b
^3)*cosh(d*x + c)^5 + 315*(14*b^3*cosh(d*x + c)^5 + 15*(4*a*b^2 - 3*b^3)*co
sh(d*x + c)^3 + 12*(4*a^2*b - 7*a*b^2 + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)
^4 + 420*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*cosh(d*x + c)^3 + 315*(4*b
^3*cosh(d*x + c)^7 + 9*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 24*(4*a^2*b - 7*
a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 60*a^2*b + 63*a*b^2 - 21*b^3)*
cosh(d*x + c))*sinh(d*x + c)^2 - 1890*(32*a^3 - 80*a^2*b + 70*a*b^2 - 21*b^
3)*cosh(d*x + c))/d
```

Sympy [A] time = 21.724, size = 330, normalized size = 2.87

$$\begin{cases} \frac{a^3\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^3\cosh^3(c+dx)}{3d} + \frac{3a^2b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^2b\sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8a^2b\cosh^5(c+dx)}{5d} + \frac{3ab^2\sinh^6(\ldots}{} \\ x\left(a+b\sinh^2(c)\right)^3\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**3*cosh(c + d*x)**3/
(3*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*a**2*b*sinh(c + d*x)*
*2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/(5*d) + 3*a*b**2*sinh(c +
 d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 2
4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 48*a*b**2*cosh(c + d*x)*
*7/(35*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**3*sinh(c + d*x)**6
*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) -
 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**3*cosh(c + d*x)*
*9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**3, True))
```

Giac [B] time = 1.41424, size = 544, normalized size = 4.73

$$\frac{35\,b^3e^{(9\,dx+9\,c)} + 540\,ab^2e^{(7\,dx+7\,c)} - 405\,b^3e^{(7\,dx+7\,c)} + 3024\,a^2be^{(5\,dx+5\,c)} - 5292\,ab^2e^{(5\,dx+5\,c)} + 2268\,b^3e^{(5\,dx+5\,c)} + 6720\,a^3e^{(\ldots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/161280*(35*b^3*e^(9*d*x + 9*c) + 540*a*b^2*e^(7*d*x + 7*c) - 405*b^3*e^(7
*d*x + 7*c) + 3024*a^2*b*e^(5*d*x + 5*c) - 5292*a*b^2*e^(5*d*x + 5*c) + 226
8*b^3*e^(5*d*x + 5*c) + 6720*a^3*e^(3*d*x + 3*c) - 25200*a^2*b*e^(3*d*x + 3
*c) + 26460*a*b^2*e^(3*d*x + 3*c) - 8820*b^3*e^(3*d*x + 3*c) - 60480*a^3*e^
(d*x + c) + 151200*a^2*b*e^(d*x + c) - 132300*a*b^2*e^(d*x + c) + 39690*b^3
*e^(d*x + c) - (60480*a^3*e^(8*d*x + 8*c) - 151200*a^2*b*e^(8*d*x + 8*c) +
132300*a*b^2*e^(8*d*x + 8*c) - 39690*b^3*e^(8*d*x + 8*c) - 6720*a^3*e^(6*d*
x + 6*c) + 25200*a^2*b*e^(6*d*x + 6*c) - 26460*a*b^2*e^(6*d*x + 6*c) + 8820
*b^3*e^(6*d*x + 6*c) - 3024*a^2*b*e^(4*d*x + 4*c) + 5292*a*b^2*e^(4*d*x + 4
*c) - 2268*b^3*e^(4*d*x + 4*c) - 540*a*b^2*e^(2*d*x + 2*c) + 405*b^3*e^(2*d
*x + 2*c) - 35*b^3)*e^(-9*d*x - 9*c))/d
```

3.21 $\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=181

$$\frac{b\left(24a^2-64ab+35b^2\right)\sinh^3(c+dx)\cosh(c+dx)}{192d}+\frac{\left(-376a^2b+96a^3+360ab^2-105b^3\right)\sinh(c+dx)\cosh(c+dx)}{384d}$$

```
[Out] -((64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*x)/128 + ((96*a^3 - 376*a^2*b +
 360*a*b^2 - 105*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(384*d) + (b*(24*a^2 - 6
4*a*b + 35*b^2)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) + ((6*a - 7*b)*Cosh[
c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(48*d) + (Cosh[c + d*x]*S
inh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3)/(8*d)
```

Rubi [A] time = 0.192777, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3170, 3169}

$$\frac{b\left(24a^2-64ab+35b^2\right)\sinh^3(c+dx)\cosh(c+dx)}{192d}+\frac{\left(-376a^2b+96a^3+360ab^2-105b^3\right)\sinh(c+dx)\cosh(c+dx)}{384d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -((64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*x)/128 + ((96*a^3 - 376*a^2*b +
 360*a*b^2 - 105*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(384*d) + (b*(24*a^2 - 6
4*a*b + 35*b^2)*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) + ((6*a - 7*b)*Cosh[
c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)/(48*d) + (Cosh[c + d*x]*S
inh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3)/(8*d)
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3}{8d} - \frac{1}{8}\int \left(a-(6a-7b)\sinh\right.$$

$$= \frac{(6a-7b)\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{48d} + \frac{\cosh(c+dx)\,s}{}$$

$$= -\frac{1}{128}\left(64a^3-144a^2b+120ab^2-35b^3\right)x + \frac{\left(96a^3-376a^2b+360ab^2-105\right.}{384}$$

Mathematica [A] time = 0.29103, size = 130, normalized size = 0.72

$$\frac{-24\left(-144a^2b+64a^3+120ab^2-35b^3\right)(c+dx)+24b\left(12a^2-18ab+7b^2\right)\sinh(4(c+dx))+48\left(-48a^2b+16a^3+45ab^2\right.}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-24*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*(c + d*x) + 48*(16*a^3 - 48*
a^2*b + 45*a*b^2 - 14*b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 18*a*b + 7*b^
2)*Sinh[4*(c + d*x)] + 16*(3*a - 2*b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*
(c + d*x)])/(3072*d)
```

Maple [A] time = 0.014, size = 180, normalized size = 1.

$$\frac{1}{d}\left(b^3\left(\left(\frac{(\sinh(dx+c))^7}{8} - \frac{7(\sinh(dx+c))^5}{48} + \frac{35(\sinh(dx+c))^3}{192} - \frac{35\sinh(dx+c)}{128}\right)\cosh(dx+c) + \frac{35dx}{128} + \frac{35c}{128}\right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128
*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3*a*b^2*((1/6*sinh(d*x+c)^5-
5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+3*a^2*b*(
(1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^3*(1/2*cos
h(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.10491, size = 413, normalized size = 2.28

$$\frac{3}{64}a^2b\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{1}{8}a^3\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{6144}b^3\left(\frac{\left(32e^{(-2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x
 - 2*c)/d) - 1/6144*b^3*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*
e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x
```

```
- 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/
d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x +
 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c))/d)
```

Fricas [A] time = 1.81714, size = 667, normalized size = 3.69

$$\frac{3\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+3\left(7\,b^3\cosh\left(dx+c\right)^3+4\left(3\,ab^2-2\,b^3\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^5+\left(21\,b^3\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^3*cosh(d*x + c)^3 + 4*(
3*a*b^2 - 2*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^3*cosh(d*x + c)^5 +
 40*(3*a*b^2 - 2*b^3)*cosh(d*x + c)^3 + 12*(12*a^2*b - 18*a*b^2 + 7*b^3)*co
sh(d*x + c))*sinh(d*x + c)^3 - 3*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*
d*x + 3*(b^3*cosh(d*x + c)^7 + 4*(3*a*b^2 - 2*b^3)*cosh(d*x + c)^5 + 4*(12*
a^2*b - 18*a*b^2 + 7*b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 48*a^2*b + 45*a*b^2
 - 14*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 14.3627, size = 561, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x
*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a
**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c
+ d*x)**3/(8*d) + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c + d*
x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/1
6 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(c + d
*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*b**2*s
inh(c + d*x)*cosh(c + d*x)**5/(16*d) + 35*b**3*x*sinh(c + d*x)**8/128 - 35*
b**3*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b**3*x*sinh(c + d*x)**4*c
osh(c + d*x)**4/64 - 35*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b*
*3*x*cosh(c + d*x)**8/128 + 93*b**3*sinh(c + d*x)**7*cosh(c + d*x)/(128*d)
- 511*b**3*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b**3*sinh(c + d*
x)**3*cosh(c + d*x)**5/(384*d) - 35*b**3*sinh(c + d*x)*cosh(c + d*x)**7/(12
8*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c)**2, True))
```

Giac [B] time = 1.40784, size = 521, normalized size = 2.88

$$\frac{3\,b^3e^{(8\,dx+8\,c)}+48\,ab^2e^{(6\,dx+6\,c)}-32\,b^3e^{(6\,dx+6\,c)}+288\,a^2be^{(4\,dx+4\,c)}-432\,ab^2e^{(4\,dx+4\,c)}+168\,b^3e^{(4\,dx+4\,c)}+768\,a^3e^{(2\,dx+2\,c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/6144*(3*b^3*e^(8*d*x + 8*c) + 48*a*b^2*e^(6*d*x + 6*c) - 32*b^3*e^(6*d*x
+ 6*c) + 288*a^2*b*e^(4*d*x + 4*c) - 432*a*b^2*e^(4*d*x + 4*c) + 168*b^3*e^
(4*d*x + 4*c) + 768*a^3*e^(2*d*x + 2*c) - 2304*a^2*b*e^(2*d*x + 2*c) + 2160
*a*b^2*e^(2*d*x + 2*c) - 672*b^3*e^(2*d*x + 2*c) - 48*(64*a^3 - 144*a^2*b +
 120*a*b^2 - 35*b^3)*(d*x + c) + (3200*a^3*e^(8*d*x + 8*c) - 7200*a^2*b*e^(
8*d*x + 8*c) + 6000*a*b^2*e^(8*d*x + 8*c) - 1750*b^3*e^(8*d*x + 8*c) - 768*
a^3*e^(6*d*x + 6*c) + 2304*a^2*b*e^(6*d*x + 6*c) - 2160*a*b^2*e^(6*d*x + 6*
c) + 672*b^3*e^(6*d*x + 6*c) - 288*a^2*b*e^(4*d*x + 4*c) + 432*a*b^2*e^(4*d
*x + 4*c) - 168*b^3*e^(4*d*x + 4*c) - 48*a*b^2*e^(2*d*x + 2*c) + 32*b^3*e^(
2*d*x + 2*c) - 3*b^3)*e^(-8*d*x - 8*c))/d
```

3.22 $\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=79

$$\frac{3b^2(a-b)\cosh^5(c+dx)}{5d}+\frac{b(a-b)^2\cosh^3(c+dx)}{d}+\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^7(c+dx)}{7d}$$

```
[Out] ((a - b)^3*Cosh[c + d*x])/d + ((a - b)^2*b*Cosh[c + d*x]^3)/d + (3*(a - b)*
b^2*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0873647, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 194}

$$\frac{3b^2(a-b)\cosh^5(c+dx)}{5d}+\frac{b(a-b)^2\cosh^3(c+dx)}{d}+\frac{(a-b)^3\cosh(c+dx)}{d}+\frac{b^3\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^3*Cosh[c + d*x])/d + ((a - b)^2*b*Cosh[c + d*x]^3)/d + (3*(a - b)*
b^2*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/(7*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int\left(a-b+bx^2\right)^3\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^3\left(1-\frac{b(3a^2-3ab+b^2)}{a^3}\right)+3a^2b\left(1+\frac{b(-2a+b)}{a^2}\right)x^2+3ab^2\left(1-\frac{b}{a}\right)x^4\right.\right.}{d}\\ &= \frac{(a-b)^3\cosh(c+dx)}{d}+\frac{(a-b)^2b\cosh^3(c+dx)}{d}+\frac{3(a-b)b^2\cosh^5(c+d}{5d}\end{aligned}$$

Mathematica [A] time = 0.303804, size = 94, normalized size = 1.19

$$\frac{\cosh(c+dx)\left(b\left(560a^2-784ab+299b^2\right)\cosh(2(c+dx))-2800a^2b+1120a^3+6b^2(14a-9b)\cosh(4(c+dx))+249\right.}{1120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Cosh[c + d*x]*(1120*a^3 - 2800*a^2*b + 2492*a*b^2 - 762*b^3 + b*(560*a^2 -
 784*a*b + 299*b^2)*Cosh[2*(c + d*x)] + 6*(14*a - 9*b)*b^2*Cosh[4*(c + d*x)
] + 5*b^3*Cosh[6*(c + d*x)]))/(1120*d)
```

Maple [A] time = 0.016, size = 116, normalized size = 1.5

$$\frac{1}{d}\left(b^3\left(-\frac{16}{35}+\frac{(\sinh(dx+c))^6}{7}-\frac{6\,(\sinh(dx+c))^4}{35}+\frac{8\,(\sinh(dx+c))^2}{35}\right)\cosh(dx+c)+3\,ab^2\left(\frac{8}{15}+1/5\,(\sinh(dx+c))\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*c
osh(d*x+c)+3*a*b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+
3*a^2*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*cosh(d*x+c))
```

Maxima [B] time = 1.05319, size = 355, normalized size = 4.49

$$-\frac{1}{4480}\,b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/160*a*b^2*(3*e^(5*d*x + 5*
c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e
^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8*a^2*b*(e^(3*d*x + 3*c)/d -
9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^3*cosh(d*x + c
)/d
```

Fricas [B] time = 1.77414, size = 589, normalized size = 7.46

$$\frac{5\,b^3\cosh(dx+c)^7+35\,b^3\cosh(dx+c)\sinh(dx+c)^6+7\left(12\,ab^2-7\,b^3\right)\cosh(dx+c)^5+35\left(5\,b^3\cosh(dx+c)^3+\left(12\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^3*cosh(d*x + c)^7 + 35*b^3*cosh(d*x + c)*sinh(d*x + c)^6 + 7*(1
2*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^3 + (12*a*b^2 -
7*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 35*(16*a^2*b - 20*a*b^2 + 7*b^3)*co
sh(d*x + c)^3 + 35*(3*b^3*cosh(d*x + c)^5 + 2*(12*a*b^2 - 7*b^3)*cosh(d*x +
 c)^3 + 3*(16*a^2*b - 20*a*b^2 + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 35
```

```
*(64*a^3 - 144*a^2*b + 120*a*b^2 - 35*b^3)*cosh(d*x + c))/d
```

Sympy [A] time = 7.78788, size = 221, normalized size = 2.8

$$\begin{cases} \frac{a^3\cosh(c+dx)}{d} + \frac{3a^2b\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2b\cosh^3(c+dx)}{d} + \frac{3ab^2\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4ab^2\sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8ab^2\mathrm{co}}{} \\ x\left(a+b\sinh^2(c)\right)^3\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d
 - 2*a**2*b*cosh(c + d*x)**3/d + 3*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)/d
- 4*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a*b**2*cosh(c + d*x)**5/
(5*d) + b**3*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**3*sinh(c + d*x)**4*cos
h(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**3*
cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*sinh(c), True)
)
```

Giac [B] time = 1.31953, size = 386, normalized size = 4.89

$$\frac{5\,b^3e^{(7\,dx+7\,c)} + 84\,ab^2e^{(5\,dx+5\,c)} - 49\,b^3e^{(5\,dx+5\,c)} + 560\,a^2be^{(3\,dx+3\,c)} - 700\,ab^2e^{(3\,dx+3\,c)} + 245\,b^3e^{(3\,dx+3\,c)} + 2240\,a^3e^{(dx+c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*b^3*e^(7*d*x + 7*c) + 84*a*b^2*e^(5*d*x + 5*c) - 49*b^3*e^(5*d*x
+ 5*c) + 560*a^2*b*e^(3*d*x + 3*c) - 700*a*b^2*e^(3*d*x + 3*c) + 245*b^3*e^
(3*d*x + 3*c) + 2240*a^3*e^(d*x + c) - 5040*a^2*b*e^(d*x + c) + 4200*a*b^2*
e^(d*x + c) - 1225*b^3*e^(d*x + c) + (2240*a^3*e^(6*d*x + 6*c) - 5040*a^2*b
*e^(6*d*x + 6*c) + 4200*a*b^2*e^(6*d*x + 6*c) - 1225*b^3*e^(6*d*x + 6*c) +
560*a^2*b*e^(4*d*x + 4*c) - 700*a*b^2*e^(4*d*x + 4*c) + 245*b^3*e^(4*d*x +
4*c) + 84*a*b^2*e^(2*d*x + 2*c) - 49*b^3*e^(2*d*x + 2*c) + 5*b^3)*e^(-7*d*x
 - 7*c))/d
```

3.23 $\int \left(a+b\sinh^2(c+dx)\right)^3 dx$

Optimal. Leaf size=128

$$\frac{b\left(64a^2-54ab+15b^2\right)\sinh(c+dx)\cosh(c+dx)}{48d}+\frac{1}{16}x(2a-b)\left(8a^2-8ab+5b^2\right)+\frac{5b^2(2a-b)\sinh^3(c+dx)\cosh(c+$$

```
[Out] ((2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*x)/16 + (b*(64*a^2 - 54*a*b + 15*b^2)*Co
sh[c + d*x]*Sinh[c + d*x])/(48*d) + (5*(2*a - b)*b^2*Cosh[c + d*x]*Sinh[c +
 d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)
/(6*d)
```

Rubi [A] time = 0.100085, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3180, 3169}

$$\frac{b\left(64a^2-54ab+15b^2\right)\sinh(c+dx)\cosh(c+dx)}{48d}+\frac{1}{16}x(2a-b)\left(8a^2-8ab+5b^2\right)+\frac{5b^2(2a-b)\sinh^3(c+dx)\cosh(c+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*x)/16 + (b*(64*a^2 - 54*a*b + 15*b^2)*Co
sh[c + d*x]*Sinh[c + d*x])/(48*d) + (5*(2*a - b)*b^2*Cosh[c + d*x]*Sinh[c +
 d*x]^3)/(24*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2)
/(6*d)
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^3 dx=\frac{b\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2}{6d}+\frac{1}{6}\int \left(a+b\sinh^2(c+dx)\right)\left(a(6a-b\right.$$

$$=\frac{1}{16}(2a-b)\left(8a^2-8ab+5b^2\right)x+\frac{b\left(64a^2-54ab+15b^2\right)\cosh(c+dx)\sinh(c+dx)}{48d}+\frac{5(2$$

Mathematica [A] time = 0.259497, size = 95, normalized size = 0.74

$$\frac{12(2a-b)\left(8a^2-8ab+5b^2\right)(c+dx)+9b\left(16a^2-16ab+5b^2\right)\sinh(2(c+dx))+9b^2(2a-b)\sinh(4(c+dx))+b^3\sinh}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (12*(2*a - b)*(8*a^2 - 8*a*b + 5*b^2)*(c + d*x) + 9*b*(16*a^2 - 16*a*b + 5*
b^2)*Sinh[2*(c + d*x)] + 9*(2*a - b)*b^2*Sinh[4*(c + d*x)] + b^3*Sinh[6*(c
+ d*x)])/(192*d)
```

Maple [A] time = 0.013, size = 131, normalized size = 1.

$$\frac{1}{d}\left(b^3\left(\left(\frac{(\sinh(dx+c))^5}{6}-\frac{5(\sinh(dx+c))^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5dx}{16}-\frac{5c}{16}\right)+3ab^2\left(\left(1/4(\sinh(dx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+
c)-5/16*d*x-5/16*c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c
)+3/8*d*x+3/8*c)+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+a^3*(d
*x+c))
```

Maxima [A] time = 1.04539, size = 266, normalized size = 2.08

$$\frac{3}{64}ab^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{3}{8}a^2b\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+a^3x-\frac{1}{384}b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d
*x - 2*c)/d) + a^3*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c)
 - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [A] time = 1.82293, size = 398, normalized size = 3.11

$$\frac{3b^3\cosh(dx+c)\sinh(dx+c)^5+2\left(5b^3\cosh(dx+c)^3+9\left(2ab^2-b^3\right)\cosh(dx+c)\right)\sinh(dx+c)^3+6\left(16a^3-24\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^3*cosh(d*x + c)^3 + 9*(2
*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(16*a^3 - 24*a^2*b + 18*a*
b^2 - 5*b^3)*d*x + 3*(b^3*cosh(d*x + c)^5 + 6*(2*a*b^2 - b^3)*cosh(d*x + c)
^3 + 3*(16*a^2*b - 16*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 5.04416, size = 350, normalized size = 2.73

$$\begin{cases} a^3x + \frac{3a^2bx\sinh^2(c+dx)}{2} - \frac{3a^2bx\cosh^2(c+dx)}{2} + \frac{3a^2b\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{9ab^2x\sinh^4(c+dx)}{8} - \frac{9ab^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{9ab^2x\cos \\ x\left(a+b\sinh^2(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*x*sinh(c + d*x)**2/2 - 3*a**2*b*x*cosh(c + d*x
)**2/2 + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)/(2*d) + 9*a*b**2*x*sinh(c + d
*x)**4/8 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a*b**2*x*cosh
(c + d*x)**4/8 + 15*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a*b**2*
sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 5*b**3*x*sinh(c + d*x)**6/16 - 15*b*
*3*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 15*b**3*x*sinh(c + d*x)**2*cosh
(c + d*x)**4/16 - 5*b**3*x*cosh(c + d*x)**6/16 + 11*b**3*sinh(c + d*x)**5*c
osh(c + d*x)/(16*d) - 5*b**3*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*
*3*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*
*3, True))
```

Giac [B] time = 1.3218, size = 362, normalized size = 2.83

$$\frac{b^3e^{(6dx+6c)} + 18ab^2e^{(4dx+4c)} - 9b^3e^{(4dx+4c)} + 144a^2be^{(2dx+2c)} - 144ab^2e^{(2dx+2c)} + 45b^3e^{(2dx+2c)} + 24\left(16a^3 - 24a^2b + 1\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/384*(b^3*e^(6*d*x + 6*c) + 18*a*b^2*e^(4*d*x + 4*c) - 9*b^3*e^(4*d*x + 4*
c) + 144*a^2*b*e^(2*d*x + 2*c) - 144*a*b^2*e^(2*d*x + 2*c) + 45*b^3*e^(2*d*
x + 2*c) + 24*(16*a^3 - 24*a^2*b + 18*a*b^2 - 5*b^3)*(d*x + c) - (352*a^3*e
^(6*d*x + 6*c) - 528*a^2*b*e^(6*d*x + 6*c) + 396*a*b^2*e^(6*d*x + 6*c) - 11
0*b^3*e^(6*d*x + 6*c) + 144*a^2*b*e^(4*d*x + 4*c) - 144*a*b^2*e^(4*d*x + 4*
c) + 45*b^3*e^(4*d*x + 4*c) + 18*a*b^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2
*c) + b^3)*e^(-6*d*x - 6*c))/d
```

3.24 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$

Optimal. Leaf size=83

$$\frac{b\left(3a^2-3ab+b^2\right)\cosh(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2(3a-2b)\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}$$

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) + (b*(3*a^2 - 3*a*b + b^2)*Cosh[c + d*x])
/d + ((3*a - 2*b)*b^2*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0869544, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 390, 206}

$$\frac{b\left(3a^2-3ab+b^2\right)\cosh(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2(3a-2b)\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) + (b*(3*a^2 - 3*a*b + b^2)*Cosh[c + d*x])
/d + ((3*a - 2*b)*b^2*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^3}{1-x^2}dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(-b\left(3a^2-3ab+b^2\right)-(3a-2b)b^2x^2-b^3x^4+\frac{a^3}{1-x^2}\right)dx, x, \cosh(\right.}{d}$$
$$= \frac{b\left(3a^2-3ab+b^2\right)\cosh(c+dx)}{d}+\frac{(3a-2b)b^2\cosh^3(c+dx)}{3d}+\frac{b^3\cosh^5(c+}{5d}$$
$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\left(3a^2-3ab+b^2\right)\cosh(c+dx)}{d}+\frac{(3a-2b)b^2}{}$$

Mathematica [A] time = 0.214731, size = 83, normalized size = 1.

$$\frac{30b\left(24a^2-18ab+5b^2\right)\cosh(c+dx)+3\left(80a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+b^3\cosh(5(c+dx))\right)+5b^2(12a-5b)\cosh(3(c+}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (30*b*(24*a^2 - 18*a*b + 5*b^2)*Cosh[c + d*x] + 5*(12*a - 5*b)*b^2*Cosh[3*(
c + d*x)] + 3*(b^3*Cosh[5*(c + d*x)] + 80*a^3*Log[Tanh[(c + d*x)/2]]))/(240
*d)
```

Maple [A] time = 0.03, size = 86, normalized size = 1.

$$\frac{1}{d}\left(-2a^3\operatorname{Artanh}\left(e^{dx+c}\right)+3a^2b\cosh(dx+c)+3ab^2\left(-2/3+1/3\,(\sinh(dx+c))^2\right)\cosh(dx+c)+b^3\left(\frac{8}{15}+\frac{(\sinh(dx+}{5}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*cosh(d*x+c)+3*a*b^2*(-2/3+1/3*sinh(
d*x+c)^2)*cosh(d*x+c)+b^3*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(
d*x+c))
```

Maxima [B] time = 1.06502, size = 261, normalized size = 3.14

$$\frac{1}{480}b^3\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)+\frac{1}{8}ab^2\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 1/8*a
*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x -
```

```
3*c)/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + a^3*log(tanh(1/2*d*x
 + 1/2*c))/d
```

Fricas [B] time = 2.06531, size = 2889, normalized size = 34.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^10 + 30*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
3*sinh(d*x + c)^10 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 5*(27*b^3*cosh(
d*x + c)^2 + 12*a*b^2 - 5*b^3)*sinh(d*x + c)^8 + 40*(9*b^3*cosh(d*x + c)^3
+ (12*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(24*a^2*b - 18*a*b
^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(63*b^3*cosh(d*x + c)^4 + 72*a^2*b - 54*a*
b^2 + 15*b^3 + 14*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(
189*b^3*cosh(d*x + c)^5 + 70*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 45*(24*a^
2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(24*a^2*b - 18*
a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 10*(63*b^3*cosh(d*x + c)^6 + 35*(12*a*b^2
- 5*b^3)*cosh(d*x + c)^4 + 72*a^2*b - 54*a*b^2 + 15*b^3 + 45*(24*a^2*b - 18
*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(9*b^3*cosh(d*x + c)^
7 + 7*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 15*(24*a^2*b - 18*a*b^2 + 5*b^3)
*cosh(d*x + c)^3 + 3*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*b^3 + 5*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2 + 5*(27*b^3*cosh(d*x
+ c)^8 + 28*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 90*(24*a^2*b - 18*a*b^2 +
5*b^3)*cosh(d*x + c)^4 + 12*a*b^2 - 5*b^3 + 36*(24*a^2*b - 18*a*b^2 + 5*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(
d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*
cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3
*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 480*(a^3*cosh(d*
x + c)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sin
h(d*x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)
*sinh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) -
 1) + 10*(3*b^3*cosh(d*x + c)^9 + 4*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 18
*(24*a^2*b - 18*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 12*(24*a^2*b - 18*a*b^2 +
5*b^3)*cosh(d*x + c)^3 + (12*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(
d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*x + c
)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32194, size = 311, normalized size = 3.75

$$-\frac{a^3\log\left(e^{(dx+c)}+1\right)}{d}+\frac{a^3\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}+\frac{\left(720\,a^2be^{(4\,dx+4\,c)}-540\,ab^2e^{(4\,dx+4\,c)}+150\,b^3e^{(4\,dx+4\,c)}+60\,ab^2e^{(2\,dx+2\,c)}-2\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -a^3*log(e^(d*x + c) + 1)/d + a^3*log(abs(e^(d*x + c) - 1))/d + 1/480*(720*
a^2*b*e^(4*d*x + 4*c) - 540*a*b^2*e^(4*d*x + 4*c) + 150*b^3*e^(4*d*x + 4*c)
 + 60*a*b^2*e^(2*d*x + 2*c) - 25*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-5*d*x - 5
*c)/d + 1/480*(3*b^3*d^4*e^(5*d*x + 5*c) + 60*a*b^2*d^4*e^(3*d*x + 3*c) - 2
5*b^3*d^4*e^(3*d*x + 3*c) + 720*a^2*b*d^4*e^(d*x + c) - 540*a*b^2*d^4*e^(d*
x + c) + 150*b^3*d^4*e^(d*x + c))/d^5
```

3.25 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=137

$$\frac{3}{8}bx\left(8a^2-4ab+b^2\right)-\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d}+\frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d}+$$

```
[Out] (3*b*(8*a^2 - 4*a*b + b^2)*x)/8 - (a*(2*a + b)*(4*a + b)*Coth[c + d*x])/(8*
d) + (b*Cosh[c + d*x]^4*Coth[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(4*d
) + (b*Cosh[c + d*x]^2*Coth[c + d*x]*(a*(4*a + b) - (4*a - 3*b)*(a - b)*Tan
h[c + d*x]^2))/(8*d)
```

Rubi [A] time = 0.191937, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 468, 577, 453, 206}

$$\frac{3}{8}bx\left(8a^2-4ab+b^2\right)-\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d}+\frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] (3*b*(8*a^2 - 4*a*b + b^2)*x)/8 - (a*(2*a + b)*(4*a + b)*Coth[c + d*x])/(8*
d) + (b*Cosh[c + d*x]^4*Coth[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(4*d
) + (b*Cosh[c + d*x]^2*Coth[c + d*x]*(a*(4*a + b) - (4*a - 3*b)*(a - b)*Tan
h[c + d*x]^2))/(8*d)
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
```

```
*f])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{x^2\left(1-x^2\right)^3}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d} + \frac{\text{Subst}\left(\int \frac{(a(4a}{}\right.}{}$$

$$= \frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{4d} + \frac{b\cosh^2(c+dx}{}$$

$$= -\frac{a(2a+b)(4a+b)\coth(c+dx)}{8d} + \frac{b\cosh^4(c+dx)\coth(c+dx)\left(a-(a-b\right.}{4d}$$

$$= \frac{3}{8}b\left(8a^2-4ab+b^2\right)x - \frac{a(2a+b)(4a+b)\coth(c+dx)}{8d} + \frac{b\cosh^4(c+dx)\,c}{}$$

Mathematica [A] time = 1.91726, size = 113, normalized size = 0.82

$$\frac{\sinh^6(c+dx)\left(a\operatorname{csch}^2(c+dx)+b\right)^3\left(12b\left(8a^2-4ab+b^2\right)(c+dx)-32a^3\coth(c+dx)+8b^2(3a-b)\sinh(2(c+dx))+b\right.}{4d(2a+b\cosh(2(c+dx))-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(12*b*(8*a^2 - 4*a*b + b^2)*(c +
 d*x) - 32*a^3*Coth[c + d*x] + 8*(3*a - b)*b^2*Sinh[2*(c + d*x)] + b^3*Sinh
[4*(c + d*x)]))/(4*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.03, size = 94, normalized size = 0.7

$$\frac{1}{d}\left(-a^3\coth(dx+c)+3\,a^2b\,(dx+c)+3\,ab^2\,(1/2\,\cosh(dx+c)\sinh(dx+c)-1/2\,dx-c/2)+b^3\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\,\mathrm{s}}{}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-
1/2*d*x-1/2*c)+b^3*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x
+3/8*c))
```

Maxima [A] time = 1.06488, size = 176, normalized size = 1.28

$$\frac{1}{64}b^3\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{3}{8}ab^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+3a^2bx+\frac{}{d(e^{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x
 - 2*c)/d) + 3*a^2*b*x + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [A] time = 1.84056, size = 409, normalized size = 2.99

$$\frac{b^3\cosh(dx+c)^5+5b^3\cosh(dx+c)\sinh(dx+c)^4+3\left(8ab^2-3b^3\right)\cosh(dx+c)^3+\left(10b^3\cosh(dx+c)^3+9\left(8ab\right.\right.}{64ds}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)*sinh(d*x + c)^4 + 3*(8*a*b^
2 - 3*b^3)*cosh(d*x + c)^3 + (10*b^3*cosh(d*x + c)^3 + 9*(8*a*b^2 - 3*b^3)*
cosh(d*x + c))*sinh(d*x + c)^2 - 8*(8*a^3 + 3*a*b^2 - b^3)*cosh(d*x + c) +
8*(8*a^3 + 3*(8*a^2*b - 4*a*b^2 + b^3)*d*x)*sinh(d*x + c))/(d*sinh(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36316, size = 257, normalized size = 1.88

$$\frac{3\left(8a^2b-4ab^2+b^3\right)(dx+c)}{8d}-\frac{2a^3}{d\left(e^{(2dx+2c)}-1\right)}-\frac{\left(144a^2be^{(4dx+4c)}-72ab^2e^{(4dx+4c)}+18b^3e^{(4dx+4c)}+24ab^2e^{(2dx+2}\right.}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*(8*a^2*b - 4*a*b^2 + b^3)*(d*x + c)/d - 2*a^3/(d*(e^(2*d*x + 2*c) - 1))
 - 1/64*(144*a^2*b*e^(4*d*x + 4*c) - 72*a*b^2*e^(4*d*x + 4*c) + 18*b^3*e^(4
*d*x + 4*c) + 24*a*b^2*e^(2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4
*d*x - 4*c)/d + 1/64*(b^3*d*e^(4*d*x + 4*c) + 24*a*b^2*d*e^(2*d*x + 2*c) -
8*b^3*d*e^(2*d*x + 2*c))/d^2
```

3.26 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=83

$$\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b^2(3a-b)\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}$$

```
[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + ((3*a - b)*b^2*Cosh[c + d*x]
)/d + (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rubi [A] time = 0.106845, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 390, 385, 206}

$$\frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b^2(3a-b)\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^2*(a - 6*b)*ArcTanh[Cosh[c + d*x]])/(2*d) + ((3*a - b)*b^2*Cosh[c + d*x]
)/d + (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^3}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((3a-b)b^2+b^3x^2+\frac{a^2(a-3b)+3a^2bx^2}{\left(1-x^2\right)^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}+\frac{\text{Subst}\left(\int \frac{a^2(a-3b)+3a^2bx^2}{\left(1-x^2\right)^2}\,dx, x\right.}{d}$$
$$= \frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}$$
$$= \frac{a^2(a-6b)\tanh^{-1}(\cosh(c+dx))}{2d}+\frac{(3a-b)b^2\cosh(c+dx)}{d}+\frac{b^3\cosh^3(c+}{3d}$$

Mathematica [B] time = 4.70042, size = 210, normalized size = 2.53

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^3\left(-72a^2b\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+72a^2b\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+3a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+3a^3\operatorname{sec}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -((-18*(4*a - b)*b^2*Cosh[c]*Cosh[d*x] - 2*b^3*Cosh[3*c]*Cosh[3*d*x] + 3*a^
3*Csch[(c + d*x)/2]^2 - 12*a^3*Log[Cosh[(c + d*x)/2]] + 72*a^2*b*Log[Cosh[(
c + d*x)/2]] + 12*a^3*Log[Sinh[(c + d*x)/2]] - 72*a^2*b*Log[Sinh[(c + d*x)/
2]] + 3*a^3*Sech[(c + d*x)/2]^2 - 72*a*b^2*Sinh[c]*Sinh[d*x] + 18*b^3*Sinh[
c]*Sinh[d*x] - 2*b^3*Sinh[3*c]*Sinh[3*d*x])*(a + b*Sinh[c + d*x]^2)^3)/(3*d
*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.042, size = 79, normalized size = 1.

$$\frac{1}{d}\left(a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2}+\operatorname{Artanh}\left(e^{dx+c}\right)\right)-6\,a^2b\operatorname{Artanh}\left(e^{dx+c}\right)+3\,ab^2\cosh(dx+c)+b^3\left(-\frac{2}{3}+\frac{(\sinh(d}{3}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))-6*a^2*b*arctanh
(exp(d*x+c))+3*a*b^2*cosh(d*x+c)+b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.07542, size = 293, normalized size = 3.53

$$\frac{1}{24}b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{3}{2}ab^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{2}a^3\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^3*(log(e^(
-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x -
3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1))) - 3*a^2*b*(log(e^(-d
*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Fricas [B] time = 2.0839, size = 4575, normalized size = 55.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sin
h(d*x + c)^10 + (36*a*b^2 - 11*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x + c)
^2 + 36*a*b^2 - 11*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (36*a
*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(12*a^3 + 18*a*b^2 - 5*b^
3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3
 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*b^3*cosh
(d*x + c)^5 + 14*(36*a*b^2 - 11*b^3)*cosh(d*x + c)^3 - 3*(12*a^3 + 18*a*b^2
 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^3 + 18*a*b^2 - 5*b^3)*co
sh(d*x + c)^4 + 2*(105*b^3*cosh(d*x + c)^6 + 35*(36*a*b^2 - 11*b^3)*cosh(d*
x + c)^4 - 12*a^3 - 18*a*b^2 + 5*b^3 - 15*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*a*b^2 - 11*
b^3)*cosh(d*x + c)^5 - 5*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (12*
a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + b^3 + (36*a*b^2 -
11*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^8 + 28*(36*a*b^2 - 11*b^3)*
cosh(d*x + c)^6 - 30*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 36*a*b^2
 - 11*b^3 - 12*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 + 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)*cosh(d*x + c)*si
nh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3 - 6*a^2*b)*cosh(d*
x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x
 + c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(a^3 - 6*a^2*b)*cosh(d*x
 + c))*sinh(d*x + c)^4 + (a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (35*(a^3 - 6*a^2
*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + (21*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 20*(a^3 - 6*a^2*b)*c
osh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7*(a^3
 - 6*a^2*b)*cosh(d*x + c)^6 - 10*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 3*(a^3 -
 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
 + 1) - 12*((a^3 - 6*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 - 6*a^2*b)*cosh(d*x +
c)*sinh(d*x + c)^6 + (a^3 - 6*a^2*b)*sinh(d*x + c)^7 - 2*(a^3 - 6*a^2*b)*co
sh(d*x + c)^5 - (2*a^3 - 12*a^2*b - 21*(a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sin
h(d*x + c)^5 + 5*(7*(a^3 - 6*a^2*b)*cosh(d*x + c)^3 - 2*(a^3 - 6*a^2*b)*cos
h(d*x + c))*sinh(d*x + c)^4 + (a^3 - 6*a^2*b)*cosh(d*x + c)^3 + (35*(a^3 -
6*a^2*b)*cosh(d*x + c)^4 + a^3 - 6*a^2*b - 20*(a^3 - 6*a^2*b)*cosh(d*x + c)
^2)*sinh(d*x + c)^3 + (21*(a^3 - 6*a^2*b)*cosh(d*x + c)^5 - 20*(a^3 - 6*a^2
*b)*cosh(d*x + c)^3 + 3*(a^3 - 6*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (7
*(a^3 - 6*a^2*b)*cosh(d*x + c)^6 - 10*(a^3 - 6*a^2*b)*cosh(d*x + c)^4 + 3*(
a^3 - 6*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) - 1) + 2*(5*b^3*cosh(d*x + c)^9 + 4*(36*a*b^2 - 11*b^3)*cosh(d*x + c)
^7 - 6*(12*a^3 + 18*a*b^2 - 5*b^3)*cosh(d*x + c)^5 - 4*(12*a^3 + 18*a*b^2 -
 5*b^3)*cosh(d*x + c)^3 + (36*a*b^2 - 11*b^3)*cosh(d*x + c))*sinh(d*x + c))
/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7
 - 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 5*(
```

```
7*d*cosh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^
3 + (35*d*cosh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21
*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c
)^2 + (7*d*cosh(d*x + c)^6 - 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*si
nh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36298, size = 261, normalized size = 3.14

$$-\frac{a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)d}+\frac{\left(a^3-6\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{4\,d}-\frac{\left(a^3-6\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{4\,d}+\frac{b^3d^2\left(e^{(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -a^3*(e^(d*x + c) + e^(-d*x - c))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)*d)
+ 1/4*(a^3 - 6*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/4*(a^3 - 6*
a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d + 1/24*(b^3*d^2*(e^(d*x + c) +
 e^(-d*x - c))^3 + 36*a*b^2*d^2*(e^(d*x + c) + e^(-d*x - c)) - 12*b^3*d^2*(
e^(d*x + c) + e^(-d*x - c)))/d^3
```

3.27 $\int \mathrm{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=113

$$\frac{a\left(2a^2-5ab-2b^2\right)\coth(c+dx)}{2d}-\frac{a^2(2a+3b)\coth^3(c+dx)}{6d}+\frac{1}{2}b^2x(6a-b)+\frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a\right.}{2d}$$

```
[Out] ((6*a - b)*b^2*x)/2 + (a*(2*a^2 - 5*a*b - 2*b^2)*Coth[c + d*x])/(2*d) - (a^
2*(2*a + 3*b)*Coth[c + d*x]^3)/(6*d) + (b*Cosh[c + d*x]^2*Coth[c + d*x]^3*(
a - (a - b)*Tanh[c + d*x]^2)^2)/(2*d)
```

Rubi [A] time = 0.141387, antiderivative size = 113, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 468, 570, 207}

$$\frac{a\left(2a^2-5ab-2b^2\right)\coth(c+dx)}{2d}-\frac{a^2(2a+3b)\coth^3(c+dx)}{6d}+\frac{1}{2}b^2x(6a-b)+\frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((6*a - b)*b^2*x)/2 + (a*(2*a^2 - 5*a*b - 2*b^2)*Coth[c + d*x])/(2*d) - (a^
2*(2*a + 3*b)*Coth[c + d*x]^3)/(6*d) + (b*Cosh[c + d*x]^2*Coth[c + d*x]^3*(
a - (a - b)*Tanh[c + d*x]^2)^2)/(2*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int\frac{\left(a-(a-b)x^2\right)^3}{x^4\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{2d}+\frac{\text{Subst}\left(\int\frac{(a(2\cdots}{\cdots}\right.}{\cdots}$$

$$=\frac{b\cosh^2(c+dx)\coth^3(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{2d}+\frac{\text{Subst}\left(\int\left(\frac{a^2\cdots}{\cdots}\right.\right.}{\cdots}$$

$$=\frac{a\left(2a^2-5ab-2b^2\right)\coth(c+dx)}{2d}-\frac{a^2(2a+3b)\coth^3(c+dx)}{6d}+\frac{b\cosh^2(c\cdots}{\cdots}$$

$$=\frac{1}{2}(6a-b)b^2x+\frac{a\left(2a^2-5ab-2b^2\right)\coth(c+dx)}{2d}-\frac{a^2(2a+3b)\coth^3(c+a\cdots}{6d}$$

Mathematica [A] time = 2.5041, size = 107, normalized size = 0.95

$$\frac{2\sinh^6(c+dx)\left(a\operatorname{csch}^2(c+dx)+b\right)^3\left(3b^2(2(6a-b)(c+dx)+b\sinh(2(c+dx)))-4a^2\coth(c+dx)\left(a\operatorname{csch}^2(c+dx)-2\cdots\right.\right.}{3d(2a+b\cosh(2(c+dx))-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (2*(b + a*Csch[c + d*x]^2)^3*Sinh[c + d*x]^6*(-4*a^2*Coth[c + d*x]*(-2*a +
9*b + a*Csch[c + d*x]^2) + 3*b^2*(2*(6*a - b)*(c + d*x) + b*Sinh[2*(c + d*x
)])))/(3*d*(2*a - b + b*Cosh[2*(c + d*x)])^3)
```

Maple [A] time = 0.039, size = 77, normalized size = 0.7

$$\frac{1}{d}\left(a^3\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)-3a^2b\coth(dx+c)+3ab^2(dx+c)+b^3\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-3*a^2*b*coth(d*x+c)+3*a*b^2*(d
*x+c)+b^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.03046, size = 217, normalized size = 1.92

$$-\frac{1}{8}b^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+3ab^2x+\frac{4}{3}a^3\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-6c)}-1\right)}-\frac{\cdots}{d\left(3e^{(-2dx-2c)}-3e^{(-\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 3*a*b^2*x + 4/3*a
^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*
d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.7636, size = 668, normalized size = 5.91

$$\frac{3\,b^3\cosh(dx+c)^5 + 15\,b^3\cosh(dx+c)\sinh(dx+c)^4 + \left(16\,a^3 - 72\,a^2b - 9\,b^3\right)\cosh(dx+c)^3 - 4\left(4\,a^3 - 18\,a^2b - 3\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^3*cosh(d*x + c)^5 + 15*b^3*cosh(d*x + c)*sinh(d*x + c)^4 + (16*a^
3 - 72*a^2*b - 9*b^3)*cosh(d*x + c)^3 - 4*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 -
b^3)*d*x)*sinh(d*x + c)^3 + 3*(10*b^3*cosh(d*x + c)^3 + (16*a^3 - 72*a^2*b
- 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 6*(8*a^3 - 12*a^2*b - b^3)*cosh(d
*x + c) + 12*(4*a^3 - 18*a^2*b - 3*(6*a*b^2 - b^3)*d*x - (4*a^3 - 18*a^2*b
- 3*(6*a*b^2 - b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3
 + 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.36781, size = 217, normalized size = 1.92

$$\frac{b^3 e^{(2\,dx+2\,c)}}{8\,d} + \frac{\left(6\,ab^2 - b^3\right)(dx+c)}{2\,d} - \frac{\left(12\,ab^2 e^{(2\,dx+2\,c)} - 2\,b^3 e^{(2\,dx+2\,c)} + b^3\right)e^{(-2\,dx-2\,c)}}{8\,d} - \frac{2\left(9\,a^2 b e^{(4\,dx+4\,c)} + 6\,a^3 e^{(2\,dx+2\,c)}\right.}{3\,d\left(e^{(2\,dx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^3*e^(2*d*x + 2*c)/d + 1/2*(6*a*b^2 - b^3)*(d*x + c)/d - 1/8*(12*a*b^2
*e^(2*d*x + 2*c) - 2*b^3*e^(2*d*x + 2*c) + b^3)*e^(-2*d*x - 2*c)/d - 2/3*(9
*a^2*b*e^(4*d*x + 4*c) + 6*a^3*e^(2*d*x + 2*c) - 18*a^2*b*e^(2*d*x + 2*c) -
 2*a^3 + 9*a^2*b)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.28 $\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=109

$$\frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d}-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{7/2}d\sqrt{a-b}}-\frac{(a+2b)\cosh^3(c+dx)}{3b^2d}+\frac{\cosh^5(c+dx)}{5bd}$$

```
[Out] -((a^3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(7/2)*d)
) + ((a^2 + a*b + b^2)*Cosh[c + d*x])/(b^3*d) - ((a + 2*b)*Cosh[c + d*x]^3)
/(3*b^2*d) + Cosh[c + d*x]^5/(5*b*d)
```

Rubi [A] time = 0.149371, antiderivative size = 109, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 390, 205}

$$\frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d}-\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{7/2}d\sqrt{a-b}}-\frac{(a+2b)\cosh^3(c+dx)}{3b^2d}+\frac{\cosh^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((a^3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(7/2)*d)
) + ((a^2 + a*b + b^2)*Cosh[c + d*x])/(b^3*d) - ((a + 2*b)*Cosh[c + d*x]^3)
/(3*b^2*d) + Cosh[c + d*x]^5/(5*b*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{a^2+ab+b^2}{b^3}+\frac{(a+2b)x^2}{b^2}-\frac{x^4}{b}+\frac{a^3}{b^3(a-b+bx^2)}\right)dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d} - \frac{(a+2b)\cosh^3(c+dx)}{3b^2d} + \frac{\cosh^5(c+dx)}{5bd} - \frac{a^3\,\text{Subst}\left(\int \frac{}{a-b}\right.}{}$$

$$= -\frac{a^3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}b^{7/2}d} + \frac{(a^2+ab+b^2)\cosh(c+dx)}{b^3d} - \frac{(a+2b)\cosh^3(c+dx)}{3b^2d} + \cosh$$

Mathematica [C] time = 0.868052, size = 165, normalized size = 1.51

$$\frac{30\sqrt{b}\left(8a^2+6ab+5b^2\right)\cosh(c+dx) - \frac{240a^3\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}} - 5b^{3/2}(4a+5b)\cosh(3(}{240b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-240*a^3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + A
rcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] +
30*Sqrt[b]*(8*a^2 + 6*a*b + 5*b^2)*Cosh[c + d*x] - 5*b^(3/2)*(4*a + 5*b)*Co
sh[3*(c + d*x)] + 3*b^(5/2)*Cosh[5*(c + d*x)])/(240*b^(7/2)*d)
```

Maple [B] time = 0.044, size = 448, normalized size = 4.1

$$\frac{1}{5\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-5} - \frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-4} + \frac{a}{2\,db^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{3}{8\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/5/d/b/(tanh(1/2*d*x+1/2*c)+1)^5-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/b
^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+3/8/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/3/d/b^2/
(tanh(1/2*d*x+1/2*c)+1)^3*a+1/12/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/d/b^3/(tan
h(1/2*d*x+1/2*c)+1)*a^2+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a+3/8/d/b/(tanh(1
/2*d*x+1/2*c)+1)-1/5/d/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/2/d/b/(tanh(1/2*d*x+1/
2*c)-1)^4+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a+3/8/d/b/(tanh(1/2*d*x+1/2*c
)-1)^2+1/3/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3*a-1/12/d/b/(tanh(1/2*d*x+1/2*c)-
1)^3-1/d/b^3/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*
a-3/8/d/b/(tanh(1/2*d*x+1/2*c)-1)-1/d*a^3/b^3/(a*b-b^2)^(1/2)*arctan(1/4*(2
*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,b^2e^{(10\,dx+10\,c)}+3\,b^2-5\left(4\,abe^{(8\,c)}+5\,b^2e^{(8\,c)}\right)e^{(8\,dx)}+30\left(8\,a^2e^{(6\,c)}+6\,abe^{(6\,c)}+5\,b^2e^{(6\,c)}\right)e^{(6\,dx)}+30\left(8\,a^2e^{(4\,c)}+6\,abe\right.}{480\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(3*b^2*e^(10*d*x + 10*c) + 3*b^2 - 5*(4*a*b*e^(8*c) + 5*b^2*e^(8*c))*
e^(8*d*x) + 30*(8*a^2*e^(6*c) + 6*a*b*e^(6*c) + 5*b^2*e^(6*c))*e^(6*d*x) +
30*(8*a^2*e^(4*c) + 6*a*b*e^(4*c) + 5*b^2*e^(4*c))*e^(4*d*x) - 5*(4*a*b*e^(
2*c) + 5*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integrate
(256*(a^3*e^(3*d*x + 3*c) - a^3*e^(d*x + c))/(b^4*e^(4*d*x + 4*c) + b^4 + 2
*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.28412, size = 7640, normalized size = 70.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/480*(3*(a*b^3 - b^4)*cosh(d*x + c)^10 + 30*(a*b^3 - b^4)*cosh(d*x + c)*s
inh(d*x + c)^9 + 3*(a*b^3 - b^4)*sinh(d*x + c)^10 - 5*(4*a^2*b^2 + a*b^3 -
5*b^4)*cosh(d*x + c)^8 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4 - 27*(a*b^3 - b^4)*co
sh(d*x + c)^2)*sinh(d*x + c)^8 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^3 - (4*a
^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 2*a^
2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)
^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b^3 - 15*b^4 - 14*(4*a^2*b^2 + a*b^3 - 5*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*(a*b^3 - b^4)*cosh(d*x + c)^5
- 70*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^3 + 45*(8*a^3*b - 2*a^2*b^2
- a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 30*(8*a^3*b - 2*a^2*b^2 -
 a*b^3 - 5*b^4)*cosh(d*x + c)^4 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^6 - 35
*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b
^3 - 15*b^4 + 45*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 3*a*b^3 - 3*b^4 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^7 - 7*(4
*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 15*(8*a^3*b - 2*a^2*b^2 - a*b^3
 - 5*b^4)*cosh(d*x + c)^3 + 3*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*
x + c))*sinh(d*x + c)^3 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2 + 5
*(27*(a*b^3 - b^4)*cosh(d*x + c)^8 - 28*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*
x + c)^6 + 90*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 4*a^2
*b^2 - a*b^3 + 5*b^4 + 36*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^2 - 240*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*si
nh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x + c)
^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x + c
)^5)*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
 c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
 + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*c
osh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x
 + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*
x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + b)) + 10*(3*(a*b^3 - b^4)*cosh(d*x + c)^9 - 4*(
4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 18*(8*a^3*b - 2*a^2*b^2 - a*b^
3 - 5*b^4)*cosh(d*x + c)^5 + 12*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(
d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a*
b^4 - b^5)*d*cosh(d*x + c)^5 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)^4*sinh(d*x +
 c) + 10*(a*b^4 - b^5)*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a*b^4 - b^5)
*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)*sinh(d
```

```
*x + c)^4 + (a*b^4 - b^5)*d*sinh(d*x + c)^5), 1/480*(3*(a*b^3 - b^4)*cosh(d
*x + c)^10 + 30*(a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + 3*(a*b^3 - b^
4)*sinh(d*x + c)^10 - 5*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^8 - 5*(4*
a^2*b^2 + a*b^3 - 5*b^4 - 27*(a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8
 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^3 - (4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d
*x + c))*sinh(d*x + c)^7 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*
x + c)^6 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*
a*b^3 - 15*b^4 - 14*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x +
 c)^6 + 4*(189*(a*b^3 - b^4)*cosh(d*x + c)^5 - 70*(4*a^2*b^2 + a*b^3 - 5*b^
4)*cosh(d*x + c)^3 + 45*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 30*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^
4 + 10*(63*(a*b^3 - b^4)*cosh(d*x + c)^6 - 35*(4*a^2*b^2 + a*b^3 - 5*b^4)*c
osh(d*x + c)^4 + 24*a^3*b - 6*a^2*b^2 - 3*a*b^3 - 15*b^4 + 45*(8*a^3*b - 2*
a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a*b^3 - 3*b^4
 + 40*(9*(a*b^3 - b^4)*cosh(d*x + c)^7 - 7*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh
(d*x + c)^5 + 15*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^3 + 3*
(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*(4
*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^2 + 5*(27*(a*b^3 - b^4)*cosh(d*x +
c)^8 - 28*(4*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 90*(8*a^3*b - 2*a^2
*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 4*a^2*b^2 - a*b^3 + 5*b^4 + 36*(8*a
^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*(a
^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*sinh(d*x + c) + 10*a^3*cosh(d*x
+ c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*a^3*cos
h(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x + c)^5)*sqrt(a*b - b^2)*arctan(-1
/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)
^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x
 + c))/sqrt(a*b - b^2)) + 480*(a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^4*
sinh(d*x + c) + 10*a^3*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^3*cosh(d*x +
c)^2*sinh(d*x + c)^3 + 5*a^3*cosh(d*x + c)*sinh(d*x + c)^4 + a^3*sinh(d*x +
 c)^5)*sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*
x + c))/(a - b)) + 10*(3*(a*b^3 - b^4)*cosh(d*x + c)^9 - 4*(4*a^2*b^2 + a*b
^3 - 5*b^4)*cosh(d*x + c)^7 + 18*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh
(d*x + c)^5 + 12*(8*a^3*b - 2*a^2*b^2 - a*b^3 - 5*b^4)*cosh(d*x + c)^3 - (4
*a^2*b^2 + a*b^3 - 5*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a*b^4 - b^5)*d*co
sh(d*x + c)^5 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*(a*b^4
 - b^5)*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*(a*b^4 - b^5)*d*cosh(d*x + c
)^2*sinh(d*x + c)^3 + 5*(a*b^4 - b^5)*d*cosh(d*x + c)*sinh(d*x + c)^4 + (a*
b^4 - b^5)*d*sinh(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.29 $\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^3 d\sqrt{a-b}}+\frac{x\left(8a^2+4ab+3b^2\right)}{8b^3}-\frac{(4a+3b)\sinh(c+dx)\cosh(c+dx)}{8b^2 d}+\frac{\sinh^3(c+dx)\cosh(c+$$

```
[Out] ((8*a^2 + 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c
+ d*x])/Sqrt[a]])/(Sqrt[a - b]*b^3*d) - ((4*a + 3*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*b*d)
```

Rubi [A] time = 0.235035, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 470, 578, 522, 206, 208}

$$-\frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^3 d\sqrt{a-b}}+\frac{x\left(8a^2+4ab+3b^2\right)}{8b^3}-\frac{(4a+3b)\sinh(c+dx)\cosh(c+dx)}{8b^2 d}+\frac{\sinh^3(c+dx)\cosh(c+$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((8*a^2 + 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c
+ d*x])/Sqrt[a]])/(Sqrt[a - b]*b^3*d) - ((4*a + 3*b)*Cosh[c + d*x]*Sinh[c +
 d*x])/(8*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*b*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
```

```
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^3(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \frac{x^2(3a+(a+3b)x^2)}{(1-x^2)^2(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{4bd}$$

$$= -\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \frac{-a(4a+3b)+(-}{(1-x^2)(a+}\right.}{}$$

$$= -\frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh(c+dx)\sinh^3(c+dx)}{4bd} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\right.}{b^3}$$

$$= \frac{(8a^2+4ab+3b^2)x}{8b^3} - \frac{a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}b^3d} - \frac{(4a+3b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} +$$

Mathematica [A] time = 0.475127, size = 97, normalized size = 0.8

$$\frac{4(8a^2+4ab+3b^2)(c+dx) - \frac{32a^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - 8b(a+b)\sinh(2(c+dx)) + b^2\sinh(4(c+dx))}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (4*(8*a^2 + 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTanh[(Sqrt[a - b]*Tan
h[c + d*x]])/Sqrt[a]])/Sqrt[a - b] - 8*b*(a + b)*Sinh[2*(c + d*x)] + b^2*Sin
h[4*(c + d*x)])/(32*b^3*d)
```

Maple [B] time = 0.08, size = 670, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/4/d/b/(tanh(1/2*d*x+1/2*c)+1)^4+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/
b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+1/8/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b^2
/(tanh(1/2*d*x+1/2*c)+1)*a-3/8/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/d/b^3*ln(tanh(
1/2*d*x+1/2*c)+1)*a^2+1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+3/8/d/b*ln(tanh
(1/2*d*x+1/2*c)+1)+1/4/d/b/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/b/(tanh(1/2*d*x+
1/2*c)-1)^3-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1/8/d/b/(tanh(1/2*d*x+1/2
*c)-1)^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-3/8/d/b/(tanh(1/2*d*x+1/2*c)-1
)-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-
1)-3/8/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a^3/b^3/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/
2))+1/d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a^3/b^3/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.05375, size = 4313, normalized size = 35.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh
(d*x + c)^8 + 8*(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^2)
*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x + c)^
6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b + b^
2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 +
 4*a*b + 3*b^2)*d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*sinh(d*
x + c)^3 + 8*(a*b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8
*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*x + c)^4
+ 2*a*b + 2*b^2)*sinh(d*x + c)^2 + 32*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x
 + c)^3*sinh(d*x + c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^2*cosh(
```

```
d*x + c)*sinh(d*x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(a/(a - b))*log((b^2*co
sh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 +
 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*
sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b
^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b
- b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 -
3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d
*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d
*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cos
h(d*x + c))*sinh(d*x + c) + b)) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 +
 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c)^5 + 2*(a*
b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cos
h(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^
3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4), 1/64*(b^2*cosh(
d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 8*
(8*a^2 + 4*a*b + 3*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b + b^2)*cosh(d*x + c)^6
 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b - 2*b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cos
h(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cos
h(d*x + c)^4 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*x - 60*(a*b + b^2)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 + 4*a*b + 3*b^2)*
d*x*cosh(d*x + c) - 20*(a*b + b^2)*cosh(d*x + c)^3)*sinh(d*x + c)^3 + 8*(a*
b + b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8*a^2 + 4*a*b + 3
*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b + b^2)*cosh(d*x + c)^4 + 2*a*b + 2*b^2)
*sinh(d*x + c)^2 - 64*(a^2*cosh(d*x + c)^4 + 4*a^2*cosh(d*x + c)^3*sinh(d*x
 + c) + 6*a^2*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*a^2*cosh(d*x + c)*sinh(d*
x + c)^3 + a^2*sinh(d*x + c)^4)*sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-
a/(a - b))/a) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 + 4*a*b + 3*b^2)*d*
x*cosh(d*x + c)^3 - 6*(a*b + b^2)*cosh(d*x + c)^5 + 2*(a*b + b^2)*cosh(d*x
+ c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^3*d*cosh(d*x + c)*
sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20858, size = 296, normalized size = 2.45

$$-\frac{a^3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b^3d}+\frac{\left(8\,a^2+4\,ab+3\,b^2\right)(dx+c)}{8\,b^3d}-\frac{\left(48\,a^2e^{(4\,dx+4\,c)}+24\,abe^{(4\,dx+4\,c)}+18\,b^2e^{(4\,dx+4\,c)}-8\,abe^{(2\,dx+}\right.}{64\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*b^3*d) + 1/8*(8*a^2 + 4*a*b + 3*b^2)*(d*x + c)/(b^3*d) - 1/64*(48*a^
```

```
2*e^(4*d*x + 4*c) + 24*a*b*e^(4*d*x + 4*c) + 18*b^2*e^(4*d*x + 4*c) - 8*a*b
*e^(2*d*x + 2*c) - 8*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/(b^3*d) +
1/64*(b*d*e^(4*d*x + 4*c) - 8*a*d*e^(2*d*x + 2*c) - 8*b*d*e^(2*d*x + 2*c))/
(b^2*d^2)
```

3.30 $\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{5/2}d\sqrt{a-b}}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

```
[Out] (a^2*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(5/2)*d) -
 ((a + b)*Cosh[c + d*x])/(b^2*d) + Cosh[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.108028, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 390, 205}

$$\frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{5/2}d\sqrt{a-b}}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a^2*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(5/2)*d) -
 ((a + b)*Cosh[c + d*x])/(b^2*d) + Cosh[c + d*x]^3/(3*b*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a+b}{b^2}+\frac{x^2}{b}+\frac{a^2}{b^2(a-b+bx^2)}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{b^2d}$$
$$= \frac{a^2\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{5/2}d}-\frac{(a+b)\cosh(c+dx)}{b^2d}+\frac{\cosh^3(c+dx)}{3bd}$$

Mathematica [C] time = 0.434318, size = 134, normalized size = 1.7

$$\frac{\frac{12a^2\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}-3\sqrt{b}(4a+3b)\cosh(c+dx)+b^{3/2}\cosh(3(c+dx))}{12b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((12*a^2*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + Arc
Tan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b] - 3*
Sqrt[b]*(4*a + 3*b)*Cosh[c + d*x] + b^(3/2)*Cosh[3*(c + d*x)])/(12*b^(5/2)*
d)
```

Maple [B] time = 0.033, size = 227, normalized size = 2.9

$$\frac{1}{3\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{a}{db^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/d/b^2
/(tanh(1/2*d*x+1/2*c)+1)*a-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/3/d/b/(tanh(1/
2*d*x+1/2*c)-1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/d/b^2/(tanh(1/2*d*x+1
/2*c)-1)*a+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a^2/b^2/(a*b-b^2)^(1/2)*arct
an(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\left(4\,ae^{(4\,c)}+3\,be^{(4\,c)}\right)e^{(4\,dx)}+3\left(4\,ae^{(2\,c)}+3\,be^{(2\,c)}\right)e^{(2\,dx)}-be^{(6\,dx+6\,c)}-b\right)e^{(-3\,dx-3\,c)}}{24\,b^2d}+\frac{1}{32}\int \frac{64\left(a^2e^{(3\,dx+3\,c)}\right.}{b^3e^{(4\,dx+4\,c)}+b^3+2\left(2\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(4*a*e^(4*c) + 3*b*e^(4*c))*e^(4*d*x) + 3*(4*a*e^(2*c) + 3*b*e^(2*
c))*e^(2*d*x) - b*e^(6*d*x + 6*c) - b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*inte
grate(64*(a^2*e^(3*d*x + 3*c) - a^2*e^(d*x + c))/(b^3*e^(4*d*x + 4*c) + b^3
 + 2*(2*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 1.96975, size = 3993, normalized size = 50.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*((a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(a*b^2 - b^3)*cosh(d*x + c)*sinh(d
*x + c)^5 + (a*b^2 - b^3)*sinh(d*x + c)^6 - 3*(4*a^2*b - a*b^2 - 3*b^3)*cos
h(d*x + c)^4 - 3*(4*a^2*b - a*b^2 - 3*b^3 - 5*(a*b^2 - b^3)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 4*(5*(a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^2*b - a*b^2
 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - b^3 - 3*(4*a^2*b - a*b^2
 - 3*b^3)*cosh(d*x + c)^2 + 3*(5*(a*b^2 - b^3)*cosh(d*x + c)^4 - 4*a^2*b +
a*b^2 + 3*b^3 - 6*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
2 - 12*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c) + 3*a^2*c
osh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*sqrt(-a*b + b^2)*log((b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -
2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)
- 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 +
(3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b
)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) +
b)) + 6*((a*b^2 - b^3)*cosh(d*x + c)^5 - 2*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d
*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3
 - b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*cosh(d*x + c)^2*sinh(d*x + c)
 + 3*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d*sinh(d
*x + c)^3), 1/24*((a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(a*b^2 - b^3)*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a*b^2 - b^3)*sinh(d*x + c)^6 - 3*(4*a^2*b - a*b^2 -
 3*b^3)*cosh(d*x + c)^4 - 3*(4*a^2*b - a*b^2 - 3*b^3 - 5*(a*b^2 - b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^
2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - b^3 - 3*(4*a^
2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 3*(5*(a*b^2 - b^3)*cosh(d*x + c)^4 -
 4*a^2*b + a*b^2 + 3*b^3 - 6*(4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh(d*x + c)^2*sinh(d*x + c
) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(d*x + c)^3)*sqrt(a*b - b
^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*
sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a -
3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - 24*(a^2*cosh(d*x + c)^3 + 3*a^2*cosh
(d*x + c)^2*sinh(d*x + c) + 3*a^2*cosh(d*x + c)*sinh(d*x + c)^2 + a^2*sinh(
d*x + c)^3)*sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + si
nh(d*x + c))/(a - b)) + 6*((a*b^2 - b^3)*cosh(d*x + c)^5 - 2*(4*a^2*b - a*b
^2 - 3*b^3)*cosh(d*x + c)^3 - (4*a^2*b - a*b^2 - 3*b^3)*cosh(d*x + c))*sinh
(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^3 + 3*(a*b^3 - b^4)*d*cosh(d*x +
c)^2*sinh(d*x + c) + 3*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^2 + (a*b
^3 - b^4)*d*sinh(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.31 $\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=79

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a-b}}-\frac{x(2a+b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

```
[Out] -((2*a + b)*x)/(2*b^2) + (a^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a - b]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.121325, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 470, 522, 206, 208}

$$\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a-b}}-\frac{x(2a+b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((2*a + b)*x)/(2*b^2) + (a^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[
a]])/(Sqrt[a - b]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{2bd}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{b^2 d} - \frac{(2a+b)\,\text{Subs}\ldots}{\ldots}$$

$$= -\frac{(2a+b)x}{2b^2} + \frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}\,b^2 d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

Mathematica [A] time = 0.24996, size = 71, normalized size = 0.9

$$\frac{\frac{4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - 2(2a+b)(c+dx) + b\sinh(2(c+dx))}{4b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-2*(2*a + b)*(c + d*x) + (4*a^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sq
rt[a]])/Sqrt[a - b] + b*Sinh[2*(c + d*x)])/(4*b^2*d)
```

Maple [B] time = 0.043, size = 454, normalized size = 5.8

$$-\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2} + \frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{a}{db^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{1}{2\,bd}\ln\left(\tanh\left(\frac{dx}{2}+\ldots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^
2*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d/b*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b/(tanh
(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*
d*x+1/2*c)-1)+1/2/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/d*a^2/b^2/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2))-1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d
*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
```

```
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.12917, size = 2191, normalized size = 27.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a + b)*d*x - 3*b*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 - 4*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*
x + c) + a*sinh(d*x + c)^2)*sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) - 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)
*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/
(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x
 + c) + b)) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*
x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) +
b^2*d*sinh(d*x + c)^2), -1/8*(4*(2*a + b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x
+ c)^4 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a
+ b)*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*(a*cosh(d*x + c)^2 + 2*
a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*sqrt(-a/(a - b))*arctan(
1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 + 2*a - b)*sqrt(-a/(a - b))/a) + 4*(2*(2*a + b)*d*x*cosh(d*x + c) - b*cos
h(d*x + c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x
+ c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.23783, size = 181, normalized size = 2.29

$$\frac{a^2\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b^2d}-\frac{(dx+c)(2\,a+b)}{2\,b^2d}+\frac{e^{(2\,dx+2\,c)}}{8\,bd}+\frac{\left(4\,ae^{(2\,dx+2\,c)}+2\,be^{(2\,dx+2\,c)}-b\right)e^{(-2\,dx-2\,c)}}{8\,b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] a^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*b^2*d) - 1/2*(d*x + c)*(2*a + b)/(b^2*d) + 1/8*e^(2*d*x + 2*c)/(b*d)
+ 1/8*(4*a*e^(2*d*x + 2*c) + 2*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/(b^2
*d)
```

3.32 $\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=56

$$\frac{\cosh(c+dx)}{bd}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{3/2}d\sqrt{a-b}}$$

```
[Out] -((a*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(3/2)*d))
+ Cosh[c + d*x]/(b*d)
```

Rubi [A] time = 0.0851327, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 388, 205}

$$\frac{\cosh(c+dx)}{bd}-\frac{a\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{b^{3/2}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(Sqrt[a - b]*b^(3/2)*d))
+ Cosh[c + d*x]/(b*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)}{bd} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{bd}$$
$$= -\frac{a\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\,b^{3/2}d} + \frac{\cosh(c+dx)}{bd}$$

Mathematica [C] time = 0.233537, size = 107, normalized size = 1.91

$$\frac{\sqrt{b}\cosh(c+dx) - \frac{a\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{a-b}}}{b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-((a*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan
[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/Sqrt[a - b]) + Sqrt
[b]*Cosh[c + d*x])/(b^(3/2)*d)
```

Maple [B] time = 0.03, size = 98, normalized size = 1.8

$$-\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1} + \frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1} - \frac{a}{bd}\arctan\left(\frac{1}{4}\left(2\left(\tanh(1/2\,dx+c/2)\right)^2 a - 2a + 4b\right)\frac{1}{\sqrt{ab-b^2}}\right)\frac{1}{\sqrt{ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d/b/(tanh(1/2*d*x+1/2*c)-1)+1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b/(a*b-b
^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,dx+2\,c)}+1\right)e^{(-dx-c)}}{2\,bd} - \frac{1}{8}\int \frac{16\left(ae^{(3\,dx+3\,c)} - ae^{(dx+c)}\right)}{b^2e^{(4\,dx+4\,c)} + b^2 + 2\left(2\,abe^{(2\,c)} - b^2e^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*(a*e^(3*d*x
 + 3*c) - a*e^(d*x + c))/(b^2*e^(4*d*x + 4*c) + b^2 + 2*(2*a*b*e^(2*c) - b^
2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 1.91584, size = 1894, normalized size = 33.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x +
c) + (a*b - b^2)*sinh(d*x + c)^2 - sqrt(-a*b + b^2)*(a*cosh(d*x + c) + a*si
nh(d*x + c))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 -
 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 +
 sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*s
qrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c
))*sinh(d*x + c) + b)) + a*b - b^2)/((a*b^2 - b^3)*d*cosh(d*x + c) + (a*b^2
 - b^3)*d*sinh(d*x + c)), 1/2*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x + c)^2 - 2*sqrt(a*b - b^
2)*(a*cosh(d*x + c) + a*sinh(d*x + c))*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b
*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x +
 c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) + 2
*sqrt(a*b - b^2)*(a*cosh(d*x + c) + a*sinh(d*x + c))*arctan(-1/2*sqrt(a*b -
 b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + a*b - b^2)/((a*b^2 - b^3)*
d*cosh(d*x + c) + (a*b^2 - b^3)*d*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.33 $\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)} dx$

Optimal. Leaf size=50

$$\frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a-b}}$$

```
[Out] x/b - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b
*d)
```

Rubi [A] time = 0.0871054, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3171, 3181, 208}

$$\frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a - b]*b
*d)
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{b}\\ &= \frac{x}{b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx,x,\tanh(c+dx)\right)}{bd}\\ &= \frac{x}{b}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}bd}\end{aligned}$$

Mathematica [A] time = 0.131342, size = 50, normalized size = 1.

$$\frac{-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}}+c+dx}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a -
b])/(b*d)
```

Maple [B] time = 0.033, size = 312, normalized size = 6.2

$$\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{a}{bd}\mathrm{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a/b/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2))+1/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.01965, size = 1122, normalized size = 22.44

$$\left[\frac{2\,dx+\sqrt{\frac{a}{a-b}}\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+2\left(2\,ab-b^2\right)\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\,ab-b^2\right)\sinh(dx+c)^2+8}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\,c}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt(a/(a - b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c
)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
 4*((a*b - b^2)*cosh(d*x + c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)
 + (a*b - b^2)*sinh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/(
b*d), (d*x - sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b))/a))/(b*d
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.26475, size = 88, normalized size = 1.76

$$-\frac{a \arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}bd}+\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
a*b)*b*d) + (d*x + c)/(b*d)
```

3.34 $\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{b}d\sqrt{a-b}}$$

```
[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(Sqrt[a - b]*Sqrt[b]*d)
```

Rubi [A] time = 0.0468252, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{b}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(Sqrt[a - b]*Sqrt[b]*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{\sqrt{a-b}\sqrt{b}d}\end{aligned}$$

Mathematica [C] time = 0.12318, size = 91, normalized size = 2.28

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{b}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqrt
[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(Sqrt[a - b]*Sqrt[b]*d)
```

Maple [A] time = 0.016, size = 51, normalized size = 1.3

$$\frac{1}{d}\arctan\left(\frac{1}{4}\left(2\,(\tanh(1/2\,dx+c/2))^2\,a-2\,a+4\,b\right)\frac{1}{\sqrt{ab-b^2}}\right)\frac{1}{\sqrt{ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2
)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{b\sinh(dx+c)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)
```

Fricas [B] time = 2.01602, size = 1293, normalized size = 32.32

$$\left[-\frac{\sqrt{-ab+b^2}\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a-3\,b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2-2\,a+3\,b\right)\sinh(dx+c)^2+4\,(\ldots}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\,\mathrm{c}\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d
*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b))/((a*b - b^2)*d), (sqrt(a*b - b^2)*arctan(-
1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c
)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*
x + c))/sqrt(a*b - b^2)) - sqrt(a*b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cos
```

```
h(d*x + c) + sinh(d*x + c))/(a - b)))/((a*b - b^2)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.35 $\int \frac{1}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=40

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rubi [A] time = 0.0270021, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sinh^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{a-b}d}\end{aligned}$$

Mathematica [A] time = 0.0709516, size = 40, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-1),x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a - b]*d)
```

Maple [B] time = 0.042, size = 267, normalized size = 6.7

$$\frac{1}{d}\mathrm{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)\frac{1}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}-\frac{b}{d}\mathrm{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*
a)^(1/2))*b-1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.92759, size = 1068, normalized size = 26.7

$$\left[\frac{\log\left(\frac{b^2\cosh(dx+c)^4+4b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+2\left(2ab-b^2\right)\cosh(dx+c)^2+2\left(3b^2\cosh(dx+c)^2+2ab-b^2\right)\sinh(dx+c)^2+8a^2-8ab+b^2+4\left(\right.}{b\cosh(dx+c)^4+4b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2(2a-b)\cosh(dx+c)^2+2\left(3b\cosh(dx+c\right.}\right.}{2\sqrt{a^2-ab}d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*s
inh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c
)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*
b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c)
+ b))/(sqrt(a^2 - a*b)*d), -sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2
 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2
```

```
+ a*b)/(a^2 - a*b))/((a^2 - a*b)*d)]
```

Sympy [A] time = 118.484, size = 3859, normalized size = 96.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), ((-tanh(c/2 +
 d*x/2)/(2*d) - 1/(2*d*tanh(c/2 + d*x/2)))/b, Eq(a, 0)), (2*tanh(c/2 + d*x/
2)/(b*d*tanh(c/2 + d*x/2)**2 + b*d), Eq(a, b)), (x/(a + b*sinh(c)**2), Eq(d
, 0)), (-a**3*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 - 2*b/a -
 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a*
*3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 1
6*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) + a**3*sqrt(1 - 2*b/a - 2*sqrt(
-a*b + b**2)/a)*log(sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*
x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d
 - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b*
*2)) - a**3*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 - 2*b/a + 2
*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3
*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*
a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) + a**3*sqrt(1 - 2*b/a + 2*sqrt(-a
*b + b**2)/a)*log(sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/
2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d -
 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2
)) + 13*a**2*b*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 - 2*b/a
- 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a
**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) -
16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) - 13*a**2*b*sqrt(1 - 2*b/a - 2
*sqrt(-a*b + b**2)/a)*log(sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/
2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*
b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*
b + b**2)) + 5*a**2*b*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 -
 2*b/a + 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*
d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b
**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) - 5*a**2*b*sqrt(1 - 2*b
/a + 2*sqrt(-a*b + b**2)/a)*log(sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + t
anh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32
*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sq
rt(-a*b + b**2)) - 5*a**2*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b +
b**2)/a)*log(-sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/
(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*
a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) +
 5*a**2*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(sqrt(
1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3
*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b
+ b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) - 3*a**2*sqrt(-a*b +
 b**2)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 - 2*b/a + 2*sqrt
(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sq
rt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**
3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) + 3*a**2*sqrt(-a*b + b**2)*sqrt(1 - 2*
b/a + 2*sqrt(-a*b + b**2)/a)*log(sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) +
tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 3
2*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*s
qrt(-a*b + b**2)) - 28*a*b**2*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(-
```

```
sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18
*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(
-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) + 28*a*b**2*sqr
t(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**
2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b
**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*
b**2*d*sqrt(-a*b + b**2)) - 4*a*b**2*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a
)*log(-sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4
*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*
d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) + 4*a*b*
*2*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)*log(sqrt(1 - 2*b/a + 2*sqrt(-a*b
 + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a
*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d +
 16*a*b**2*d*sqrt(-a*b + b**2)) + 20*a*b*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a -
 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh
(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a*
*2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(
-a*b + b**2)) - 20*a*b*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**
2)/a)*log(sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a
**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2
*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) + 4*a
*b*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 -
2*b/a + 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d
 + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b*
*2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) - 4*a*b*sqrt(-a*b + b**2
)*sqrt(1 - 2*b/a + 2*sqrt(-a*b + b**2)/a)*log(sqrt(1 - 2*b/a + 2*sqrt(-a*b
+ b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*
b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d +
16*a*b**2*d*sqrt(-a*b + b**2)) + 16*b**3*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**
2)/a)*log(-sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*
a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24*a**
2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)) - 16
*b**3*sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a)*log(sqrt(1 - 2*b/a - 2*sqrt(-
a*b + b**2)/a) + tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt
(-a*b + b**2) + 32*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*
d + 16*a*b**2*d*sqrt(-a*b + b**2)) - 16*b**2*sqrt(-a*b + b**2)*sqrt(1 - 2*b
/a - 2*sqrt(-a*b + b**2)/a)*log(-sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) +
tanh(c/2 + d*x/2))/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 3
2*a**2*b**2*d - 24*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*s
qrt(-a*b + b**2)) + 16*b**2*sqrt(-a*b + b**2)*sqrt(1 - 2*b/a - 2*sqrt(-a*b
+ b**2)/a)*log(sqrt(1 - 2*b/a - 2*sqrt(-a*b + b**2)/a) + tanh(c/2 + d*x/2))
/(2*a**4*d - 18*a**3*b*d + 8*a**3*d*sqrt(-a*b + b**2) + 32*a**2*b**2*d - 24
*a**2*b*d*sqrt(-a*b + b**2) - 16*a*b**3*d + 16*a*b**2*d*sqrt(-a*b + b**2)),
 True))
```

Giac [A] time = 1.25799, size = 63, normalized size = 1.58

$$\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b
)*d)
```

3.36 $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*d))
- ArcTanh[Cosh[c + d*x]]/(a*d)
```

Rubi [A] time = 0.0742426, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3186, 391, 206, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a*Sqrt[a - b]*d))
- ArcTanh[Cosh[c + d*x]]/(a*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{ad} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{ad}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a\sqrt{a-b}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Mathematica [C] time = 0.209569, size = 124, normalized size = 2.07

$$\frac{\sqrt{a-b}\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right) - \sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{ad\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-(Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]) - S
qrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + Sqrt[a
 - b]*Log[Tanh[(c + d*x)/2]])/(a*Sqrt[a - b]*d)
```

Maple [A] time = 0.049, size = 74, normalized size = 1.2

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{b}{da}\arctan\left(\frac{1}{4}\left(2\left(\tanh(1/2\,dx+c/2)\right)^2 a - 2\,a + 4\,b\right)\frac{1}{\sqrt{ab-b^2}}\right)\frac{1}{\sqrt{ab-b^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c))-1/d/a*b/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/
2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad} + \frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad} - 2\int \frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{abe^{(4\,dx+4\,c)} + ab + 2\left(2\,a^2e^{(2\,c)} - abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a*b*e^(4*d*x + 4*c) + a*
b + 2*(2*a^2*e^(2*c) - a*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.01415, size = 1544, normalized size = 25.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*
cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(
d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c)
+ (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) - 2*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*log(cosh(d*x + c) + sinh(d
*x + c) - 1))/(a*d), -(sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x
 + c) + sinh(d*x + c))) - sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3
*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x
 + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b)
 + log(cosh(d*x + c) + sinh(d*x + c) + 1) - log(cosh(d*x + c) + sinh(d*x +
c) - 1))/(a*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(csch(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.37 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=57

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a-b}}-\frac{\coth(c+dx)}{ad}$$

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a - b]*d))
 - Coth[c + d*x]/(a*d)
```

Rubi [A] time = 0.0808976, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 453, 208}

$$-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a-b}}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a - b]*d))
 - Coth[c + d*x]/(a*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{x^2\left(a-(a-b)x^2\right)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth(c+dx)}{ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= -\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}d} - \frac{\coth(c+dx)}{ad}$$

Mathematica [A] time = 0.292172, size = 57, normalized size = 1.

$$\frac{-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} - \sqrt{a}\coth(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b]) - Sqrt[a]*
Coth[c + d*x])/(a^(3/2)*d)
```

Maple [B] time = 0.061, size = 319, normalized size = 5.6

$$-\frac{1}{2da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1} - \frac{b}{da}\text{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)\frac{1}{\sqrt{(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d*b/a/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2))+1/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^
2+1/d*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.21384, size = 1661, normalized size = 29.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 - b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sin
h(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3
*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 +
 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b
*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*
a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
+ c))*sinh(d*x + c) + b)) - 4*a^2 + 4*a*b)/((a^3 - a^2*b)*d*cosh(d*x + c)^2
 + 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)*d*sinh(d*x
 + c)^2 - (a^3 - a^2*b)*d), ((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*
x + c) + b*sinh(d*x + c)^2 - b)*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(
-a^2 + a*b)/(a^2 - a*b)) - 2*a^2 + 2*a*b)/((a^3 - a^2*b)*d*cosh(d*x + c)^2
+ 2*(a^3 - a^2*b)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - a^2*b)*d*sinh(d*x
+ c)^2 - (a^3 - a^2*b)*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30799, size = 100, normalized size = 1.75

$$-\frac{b\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}ad}-\frac{2}{ad\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
a*b)*a*d) - 2/(a*d*(e^(2*d*x + 2*c) - 1))
```

3.38 $\int \frac{\operatorname{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2d\sqrt{a-b}}+\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^2*Sqrt[a - b]*d) +
 ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Coth[c + d*x]*Csch[c + d*x
])/(2*a*d)
```

Rubi [A] time = 0.123493, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 414, 522, 206, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2d\sqrt{a-b}}+\frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d}-\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^2*Sqrt[a - b]*d) +
 ((a + 2*b)*ArcTanh[Cosh[c + d*x]])/(2*a^2*d) - (Coth[c + d*x]*Csch[c + d*x
])/(2*a*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\text{csch}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \frac{a+b+bx^2}{(1-x^2)(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{2ad}$$
$$= -\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{a^2d} + \frac{(a+2b)\,\text{Subst}\left(\int\right.}{}$$
$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^2\sqrt{a-b}d} + \frac{(a+2b)\tanh^{-1}(\cosh(c+dx))}{2a^2d} - \frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}$$

Mathematica [C] time = 0.693815, size = 201, normalized size = 2.28

$$-\frac{\text{csch}^4(c+dx)(2a+b\cosh(2(c+dx))-b)\left(2a\sqrt{a-b}\cosh(c+dx)-2\sinh^2(c+dx)\left(2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right.\right.}{8a^2d\sqrt{a-b}\left(a\,\text{csch}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*(2*a*Sqrt[a - b]*Cosh[c +
 d*x] - 2*(2*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a
- b]] + 2*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b
]] - Sqrt[a - b]*(a + 2*b)*Log[Tanh[(c + d*x)/2]])*Sinh[c + d*x]^2))/(8*a^2
*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

Maple [A] time = 0.062, size = 133, normalized size = 1.5

$$\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 - \frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2} - \frac{1}{2da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) - \frac{b}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{b^2}{da^2}\arctan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/8/d/a/tanh(1/2*d*x+1/2*c)^2-1/2/d/a*ln(tanh
(1/2*d*x+1/2*c))-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))+1/d/a^2*b^2/(a*b-b^2)^(1
/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{ade^{(4\,dx+4\,c)}-2\,ade^{(2\,dx+2\,c)}+ad}+\frac{(a+2\,b)\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\,a^2d}-\frac{(a+2\,b)\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\,a^2d}+8\int\frac{}{4\left(a^2be^{(4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*(a + 2*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - 1/2*(a +
2*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 8*integrate(1/4*(b^2*e^(3*d*x
+ 3*c) - b^2*e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(2*c)
 - a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.81937, size = 4867, normalized size = 55.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*sinh(d
*x + c)^3 - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*x +
c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-b/(
a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh
(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a
+ 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*
sinh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d
*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*co
sh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c
)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*c
osh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*c
osh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 2*a*cosh(d*
x + c) - ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x +
c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a +
2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c
)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) +
 sinh(d*x + c) + 1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2
 + 2*(3*(a + 2*b)*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)
*cosh(d*x + c)^3 - (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(co
sh(d*x + c) + sinh(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c
))/(a^2*d*cosh(d*x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*s
inh(d*x + c)^4 - 2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)
^2 - a^2*d)*sinh(d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c
))*sinh(d*x + c)), -1/2*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x +
 c)^2 + 2*a*sinh(d*x + c)^3 - 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh
(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x
 + c) + b)*sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh
(d*x + c))) + 2*(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*
sinh(d*x + c)^4 - 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - b)*sinh(d*
x + c)^2 + 4*(b*cosh(d*x + c)^3 - b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(
```

```
b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^
2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 +
4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b) + 2*a*cosh(d*x + c) - ((a + 2*
b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a + 2*b)*
sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)*cosh(d*x + c
)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3 - (a + 2*b)*c
osh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sinh(d*x + c) +
1) + ((a + 2*b)*cosh(d*x + c)^4 + 4*(a + 2*b)*cosh(d*x + c)*sinh(d*x + c)^3
 + (a + 2*b)*sinh(d*x + c)^4 - 2*(a + 2*b)*cosh(d*x + c)^2 + 2*(3*(a + 2*b)
*cosh(d*x + c)^2 - a - 2*b)*sinh(d*x + c)^2 + 4*((a + 2*b)*cosh(d*x + c)^3
- (a + 2*b)*cosh(d*x + c))*sinh(d*x + c) + a + 2*b)*log(cosh(d*x + c) + sin
h(d*x + c) - 1) + 2*(3*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/(a^2*d*cosh(d*
x + c)^4 + 4*a^2*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*d*sinh(d*x + c)^4 -
2*a^2*d*cosh(d*x + c)^2 + a^2*d + 2*(3*a^2*d*cosh(d*x + c)^2 - a^2*d)*sinh(
d*x + c)^2 + 4*(a^2*d*cosh(d*x + c)^3 - a^2*d*cosh(d*x + c))*sinh(d*x + c))
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.39 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=78

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a-b}}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]*d)
+ ((a + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.120393, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 461, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a-b}}+\frac{(a+b)\coth(c+dx)}{a^2d}-\frac{\coth^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]*d)
+ ((a + b)*Coth[c + d*x])/(a^2*d) - Coth[c + d*x]^3/(3*a*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2}{x^4(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4} + \frac{-a-b}{a^2x^2} + \frac{b^2}{a^2(a-(a-b)x^2)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{a^2d}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}\,d} + \frac{(a+b)\coth(c+dx)}{a^2d} - \frac{\coth^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.693455, size = 126, normalized size = 1.62

$$-\frac{\operatorname{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\sqrt{a}\sqrt{a-b}\coth(c+dx)\left(a\operatorname{csch}^2(c+dx)-2a-3b\right)-3b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right.\right.}{6a^{5/2}d\sqrt{a-b}\left(a\operatorname{csch}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(-3*b^2*ArcTanh[(Sqrt[a -
 b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(-2*a - 3*b
 + a*Csch[c + d*x]^2)))/(6*a^(5/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

Maple [B] time = 0.071, size = 401, normalized size = 5.1

$$-\frac{1}{24\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3 + \frac{3}{8\,da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right) + \frac{b}{2\,da^2}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right) - \frac{1}{24\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3} + \frac{3}{8\,da}\left(\tanh\left(\frac{dx}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)+1/2/d/a^2*tanh(
1/2*d*x+1/2*c)*b-1/24/d/a/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)
+1/2/d*b/a^2/tanh(1/2*d*x+1/2*c)+1/d*b^2/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-
1/d*b^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^2/a^2/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))-1/d*b^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.3784, size = 4710, normalized size = 60.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 + 8*a^3 + 4*a^2*b - 1
2*a*b^2 - 24*(a^3 - a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - a*b^2 - 3*(a^2*b - a
*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cos
h(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 - 3*b^2*cosh(d*x + c)^4 +
3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4
*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*c
osh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 - b^2 + 6*(b^
2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c
))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x
+ c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*c
osh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^
2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)
*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 +
b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 +
2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*
sinh(d*x + c) + b)) + 48*((a^2*b - a*b^2)*cosh(d*x + c)^3 - (a^3 - a*b^2)*c
osh(d*x + c))*sinh(d*x + c))/((a^4 - a^3*b)*d*cosh(d*x + c)^6 + 6*(a^4 - a^
3*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - a^3*b)*d*sinh(d*x + c)^6 - 3*
(a^4 - a^3*b)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^2 - (a
^4 - a^3*b)*d)*sinh(d*x + c)^4 + 3*(a^4 - a^3*b)*d*cosh(d*x + c)^2 + 4*(5*(
a^4 - a^3*b)*d*cosh(d*x + c)^3 - 3*(a^4 - a^3*b)*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^4 - 6*(a^4 - a^3*b)*d*cosh(d*x
+ c)^2 + (a^4 - a^3*b)*d)*sinh(d*x + c)^2 - (a^4 - a^3*b)*d + 6*((a^4 - a^3
*b)*d*cosh(d*x + c)^5 - 2*(a^4 - a^3*b)*d*cosh(d*x + c)^3 + (a^4 - a^3*b)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 24
*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b - a*b^2)*sinh(d*x
 + c)^4 + 4*a^3 + 2*a^2*b - 6*a*b^2 - 12*(a^3 - a*b^2)*cosh(d*x + c)^2 - 12
*(a^3 - a*b^2 - 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^2
*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^
6 - 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x + c)^4
 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*
sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 - 6*b^2*cosh(d*x + c)^2 + b^2)*s
inh(d*x + c)^2 - b^2 + 6*(b^2*cosh(d*x + c)^5 - 2*b^2*cosh(d*x + c)^3 + b^2
*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c
)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-
a^2 + a*b)/(a^2 - a*b)) + 24*((a^2*b - a*b^2)*cosh(d*x + c)^3 - (a^3 - a*b^
2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 - a^3*b)*d*cosh(d*x + c)^6 + 6*(a^4
- a^3*b)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - a^3*b)*d*sinh(d*x + c)^6
- 3*(a^4 - a^3*b)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^2
- (a^4 - a^3*b)*d)*sinh(d*x + c)^4 + 3*(a^4 - a^3*b)*d*cosh(d*x + c)^2 + 4*
(5*(a^4 - a^3*b)*d*cosh(d*x + c)^3 - 3*(a^4 - a^3*b)*d*cosh(d*x + c))*sinh(
```

```
d*x + c)^3 + 3*(5*(a^4 - a^3*b)*d*cosh(d*x + c)^4 - 6*(a^4 - a^3*b)*d*cosh(
d*x + c)^2 + (a^4 - a^3*b)*d)*sinh(d*x + c)^2 - (a^4 - a^3*b)*d + 6*((a^4 -
 a^3*b)*d*cosh(d*x + c)^5 - 2*(a^4 - a^3*b)*d*cosh(d*x + c)^3 + (a^4 - a^3*
b)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.42493, size = 159, normalized size = 2.04

$$\frac{b^2 \arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a^2d}+\frac{2\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}+2\,a+3\,b\right)}{3\,a^2d\left(e^{(2\,dx+2\,c)}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*a^2*d) + 2/3*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) - 6*b*e^(2*d*
x + 2*c) + 2*a + 3*b)/(a^2*d*(e^(2*d*x + 2*c) - 1)^3)
```

3.40 $\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=130

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3d\sqrt{a-b}}-\frac{\left(3a^2+4ab+8b^2\right)\tanh^{-1}(\cosh(c+dx))}{8a^3d}+\frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d}-\frac{\coth($$

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^3*Sqrt[a - b]*d)
) - ((3*a^2 + 4*a*b + 8*b^2)*ArcTanh[Cosh[c + d*x]])/(8*a^3*d) + ((3*a + 4*
b)*Coth[c + d*x]*Csch[c + d*x])/(8*a^2*d) - (Coth[c + d*x]*Csch[c + d*x]^3)
/(4*a*d)
```

Rubi [A] time = 0.203545, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 205}

$$-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3d\sqrt{a-b}}-\frac{\left(3a^2+4ab+8b^2\right)\tanh^{-1}(\cosh(c+dx))}{8a^3d}+\frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d}-\frac{\coth(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b^(5/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(a^3*Sqrt[a - b]*d)
) - ((3*a^2 + 4*a*b + 8*b^2)*ArcTanh[Cosh[c + d*x]])/(8*a^3*d) + ((3*a + 4*
b)*Coth[c + d*x]*Csch[c + d*x])/(8*a^2*d) - (Coth[c + d*x]*Csch[c + d*x]^3)
/(4*a*d)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{3a+b+3bx^2}{(1-x^2)^2(a-b+bx^2)}\,dx, x, \cosh(c+dx)\right)}{4ad}$$

$$= \frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{3a^2+ab+4b^2+b(3a}{(1-x^2)(a-b+b}\right.}{8}$$

$$= \frac{(3a+4b)\coth(c+dx)\operatorname{csch}(c+dx)}{8a^2d} - \frac{\coth(c+dx)\operatorname{csch}^3(c+dx)}{4ad} - \frac{b^3\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx,\right.}{a^3d}$$

$$= -\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{a^3\sqrt{a-b}d} - \frac{(3a^2+4ab+8b^2)\tanh^{-1}(\cosh(c+dx))}{8a^3d} + \frac{(3a+4b)\coth(c+}{8a^2d}$$

Mathematica [C] time = 6.28295, size = 574, normalized size = 4.42

$$\frac{(3a^2+4ab+8b^2)\operatorname{csch}^2(c+dx)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)(2a+b\cosh(2(c+dx))-b)}{16a^3d\left(a\operatorname{csch}^2(c+dx)+b\right)} - \frac{b^{5/2}\operatorname{csch}^2(c+dx)(2a+b\cosh(2(}{2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(b^(5/2)*ArcTan[(Sech[(c + d*x)/2]*(Sqrt[b]*Cosh[(c + d*x)/2] - I*Sqrt[a]*
Sinh[(c + d*x)/2]))/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d
*x]^2)/(2*a^3*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2)) - (b^(5/2)*ArcTan[(Sec
h[(c + d*x)/2]*(Sqrt[b]*Cosh[(c + d*x)/2] + I*Sqrt[a]*Sinh[(c + d*x)/2]))/S
qrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2)/(2*a^3*Sqrt[a
- b]*d*(b + a*Csch[c + d*x]^2)) + ((3*a + 4*b)*(2*a - b + b*Cosh[2*(c + d*x
)])*Csch[(c + d*x)/2]^2*Csch[c + d*x]^2)/(64*a^2*d*(b + a*Csch[c + d*x]^2))
 - ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[(c + d*x)/2]^4*Csch[c + d*x]^2)/(1
28*a*d*(b + a*Csch[c + d*x]^2)) + ((3*a^2 + 4*a*b + 8*b^2)*(2*a - b + b*Cos
```

```
h[2*(c + d*x)])*Csch[c + d*x]^2*Log[Tanh[(c + d*x)/2]])/(16*a^3*d*(b + a*Cs
ch[c + d*x]^2)) + ((3*a + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x
]^2*Sech[(c + d*x)/2]^2)/(64*a^2*d*(b + a*Csch[c + d*x]^2)) + ((2*a - b + b
*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*Sech[(c + d*x)/2]^4)/(128*a*d*(b + a*Cs
ch[c + d*x]^2))
```

Maple [A] time = 0.066, size = 232, normalized size = 1.8

$$\frac{1}{64\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}-\frac{1}{8\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{b}{8\,da^{2}}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}-\frac{1}{64\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-4}+\frac{1}{8\,da}\left(\mathrm{ta}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/64/d/a*tanh(1/2*d*x+1/2*c)^4-1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/8/d/a^2*tanh
(1/2*d*x+1/2*c)^2*b-1/64/d/a/tanh(1/2*d*x+1/2*c)^4+1/8/d/a/tanh(1/2*d*x+1/2
*c)^2+1/8/d/a^2/tanh(1/2*d*x+1/2*c)^2*b+3/8/d/a*ln(tanh(1/2*d*x+1/2*c))+1/2
/d/a^2*b*ln(tanh(1/2*d*x+1/2*c))+1/d/a^3*ln(tanh(1/2*d*x+1/2*c))*b^2-1/d/a^
3*b^3/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b
^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,ae^{(7\,c)}+4\,be^{(7\,c)}\right)e^{(7\,dx)}-\left(11\,ae^{(5\,c)}+4\,be^{(5\,c)}\right)e^{(5\,dx)}-\left(11\,ae^{(3\,c)}+4\,be^{(3\,c)}\right)e^{(3\,dx)}+\left(3\,ae^{c}+4\,be^{c}\right)e^{(dx)}}{4\left(a^{2}de^{(8\,dx+8\,c)}-4\,a^{2}de^{(6\,dx+6\,c)}+6\,a^{2}de^{(4\,dx+4\,c)}-4\,a^{2}de^{(2\,dx+2\,c)}+a^{2}d\right)}-\frac{\left(3\,a^{2}+4\,ab+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*((3*a*e^(7*c) + 4*b*e^(7*c))*e^(7*d*x) - (11*a*e^(5*c) + 4*b*e^(5*c))*e
^(5*d*x) - (11*a*e^(3*c) + 4*b*e^(3*c))*e^(3*d*x) + (3*a*e^c + 4*b*e^c)*e^(
d*x))/(a^2*d*e^(8*d*x + 8*c) - 4*a^2*d*e^(6*d*x + 6*c) + 6*a^2*d*e^(4*d*x +
 4*c) - 4*a^2*d*e^(2*d*x + 2*c) + a^2*d) - 1/8*(3*a^2 + 4*a*b + 8*b^2)*log(
(e^(d*x + c) + 1)*e^(-c))/(a^3*d) + 1/8*(3*a^2 + 4*a*b + 8*b^2)*log((e^(d*x
 + c) - 1)*e^(-c))/(a^3*d) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) - b^3*e
^(d*x + c))/(a^3*b*e^(4*d*x + 4*c) + a^3*b + 2*(2*a^4*e^(2*c) - a^3*b*e^(2*
c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.03656, size = 14357, normalized size = 110.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(3*a^2 + 4*a*b)*cosh(d*x + c)^7 + 14*(3*a^2 + 4*a*b)*cosh(d*x + c)*
sinh(d*x + c)^6 + 2*(3*a^2 + 4*a*b)*sinh(d*x + c)^7 - 2*(11*a^2 + 4*a*b)*co
sh(d*x + c)^5 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 - 11*a^2 - 4*a*b)*sin
```

```
h(d*x + c)^5 + 10*(7*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 - (11*a^2 + 4*a*b)*cos
h(d*x + c))*sinh(d*x + c)^4 - 2*(11*a^2 + 4*a*b)*cosh(d*x + c)^3 + 2*(35*(3
*a^2 + 4*a*b)*cosh(d*x + c)^4 - 10*(11*a^2 + 4*a*b)*cosh(d*x + c)^2 - 11*a^
2 - 4*a*b)*sinh(d*x + c)^3 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^5 - 10*(11
*a^2 + 4*a*b)*cosh(d*x + c)^3 - 3*(11*a^2 + 4*a*b)*cosh(d*x + c))*sinh(d*x
+ c)^2 + 4*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2
*sinh(d*x + c)^8 - 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - b^2)*
sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 - 3*b^2*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 - 30*b^2*cosh(d*
x + c)^2 + 3*b^2)*sinh(d*x + c)^4 - 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d
*x + c)^5 - 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 +
 4*(7*b^2*cosh(d*x + c)^6 - 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2
- b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 - 3*b^2*cosh(d*x + c)
^5 + 3*b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a -
 b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*
x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3
*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sin
h(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x
+ c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(
d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 2*(3*a^2 + 4*a
*b)*cosh(d*x + c) - ((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 4
*a*b + 8*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*sinh(
d*x + c)^8 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*
b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^6 + 8*(7*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*
x + c))*sinh(d*x + c)^5 + 6*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35
*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^2 + 9*a^2 + 12*a*b + 24*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^5 - 10*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b +
 8*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 - 15
*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*cosh(d
*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2
+ 8*((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*co
sh(d*x + c)^5 + 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - (3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) +
1) + ((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*c
osh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c)^8 - 4*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh
(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 6*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 9*
a^2 + 12*a*b + 24*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^5 - 10*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b
+ 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*
x + c)^2 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 - 15*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a
^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4
*a*b + 8*b^2)*cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 +
 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(3*a^
2 + 4*a*b)*cosh(d*x + c)^6 - 5*(11*a^2 + 4*a*b)*cosh(d*x + c)^4 - 3*(11*a^2
 + 4*a*b)*cosh(d*x + c)^2 + 3*a^2 + 4*a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x +
 c)^8 + 8*a^3*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*d*sinh(d*x + c)^8 - 4*a
^3*d*cosh(d*x + c)^6 + 6*a^3*d*cosh(d*x + c)^4 + 4*(7*a^3*d*cosh(d*x + c)^2
```

```
 - a^3*d)*sinh(d*x + c)^6 - 4*a^3*d*cosh(d*x + c)^2 + 8*(7*a^3*d*cosh(d*x +
 c)^3 - 3*a^3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*d*cosh(d*x + c)^
4 - 30*a^3*d*cosh(d*x + c)^2 + 3*a^3*d)*sinh(d*x + c)^4 + a^3*d + 8*(7*a^3*
d*cosh(d*x + c)^5 - 10*a^3*d*cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*a^3*d*cosh(d*x + c)^6 - 15*a^3*d*cosh(d*x + c)^4 + 9*a^3*
d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^2 + 8*(a^3*d*cosh(d*x + c)^7 - 3*a
^3*d*cosh(d*x + c)^5 + 3*a^3*d*cosh(d*x + c)^3 - a^3*d*cosh(d*x + c))*sinh(
d*x + c)), 1/8*(2*(3*a^2 + 4*a*b)*cosh(d*x + c)^7 + 14*(3*a^2 + 4*a*b)*cosh
(d*x + c)*sinh(d*x + c)^6 + 2*(3*a^2 + 4*a*b)*sinh(d*x + c)^7 - 2*(11*a^2 +
 4*a*b)*cosh(d*x + c)^5 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^2 - 11*a^2 -
4*a*b)*sinh(d*x + c)^5 + 10*(7*(3*a^2 + 4*a*b)*cosh(d*x + c)^3 - (11*a^2 +
4*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(11*a^2 + 4*a*b)*cosh(d*x + c)^3
+ 2*(35*(3*a^2 + 4*a*b)*cosh(d*x + c)^4 - 10*(11*a^2 + 4*a*b)*cosh(d*x + c)
^2 - 11*a^2 - 4*a*b)*sinh(d*x + c)^3 + 2*(21*(3*a^2 + 4*a*b)*cosh(d*x + c)^
5 - 10*(11*a^2 + 4*a*b)*cosh(d*x + c)^3 - 3*(11*a^2 + 4*a*b)*cosh(d*x + c))
*sinh(d*x + c)^2 - 8*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x +
c)^7 + b^2*sinh(d*x + c)^8 - 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)
^2 - b^2)*sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^
3 - 3*b^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 - 30*b
^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^4 - 4*b^2*cosh(d*x + c)^2 + 8*(7*
b^2*cosh(d*x + c)^5 - 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*b^2*cosh(d*x + c)^6 - 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d
*x + c)^2 - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 - 3*b^2*cos
h(d*x + c)^5 + 3*b^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*sq
rt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) +
 8*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*
x + c)^8 - 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - b^2)*sinh(d*x
 + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 - 30*b^2*cosh(d*x + c)^2
 + 3*b^2)*sinh(d*x + c)^4 - 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^
5 - 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^
2*cosh(d*x + c)^6 - 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 - b^2)*s
inh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 - 3*b^2*cosh(d*x + c)^5 + 3*b
^2*cosh(d*x + c)^3 - b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b))*arct
an(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x
+ c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh
(d*x + c))*sqrt(b/(a - b))/b) + 2*(3*a^2 + 4*a*b)*cosh(d*x + c) - ((3*a^2 +
 4*a*b + 8*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)*s
inh(d*x + c)^7 + (3*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c)^8 - 4*(3*a^2 + 4*a*b
 + 8*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 -
3*a^2 - 4*a*b - 8*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^3 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(
3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b + 8*b^2)*cosh
(d*x + c)^4 - 30*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 9*a^2 + 12*a*b +
 24*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 - 1
0*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 4*(
7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^6 - 15*(3*a^2 + 4*a*b + 8*b^2)*cosh
(d*x + c)^4 + 9*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8
*b^2)*sinh(d*x + c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4*a*b + 8*b^2)*
cosh(d*x + c)^7 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 + 4*
a*b + 8*b^2)*cosh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(
d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a^2 + 4*a*b + 8*b^2)
*cosh(d*x + c)^8 + 8*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)*sinh(d*x + c)^7
+ (3*a^2 + 4*a*b + 8*b^2)*sinh(d*x + c)^8 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(
d*x + c)^6 + 4*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b -
 8*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^3 - 3*
(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2 + 4*a*b +
 8*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 - 3
```

```
0*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 9*a^2 + 12*a*b + 24*b^2)*sinh(d
*x + c)^4 + 8*(7*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 - 10*(3*a^2 + 4*a*
b + 8*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(
d*x + c)^3 - 4*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 4*a*
b + 8*b^2)*cosh(d*x + c)^6 - 15*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^4 + 9
*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^2 - 3*a^2 - 4*a*b - 8*b^2)*sinh(d*x
+ c)^2 + 3*a^2 + 4*a*b + 8*b^2 + 8*((3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^7
 - 3*(3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 + 4*a*b + 8*b^2)*co
sh(d*x + c)^3 - (3*a^2 + 4*a*b + 8*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(c
osh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(3*a^2 + 4*a*b)*cosh(d*x + c)^6 -
5*(11*a^2 + 4*a*b)*cosh(d*x + c)^4 - 3*(11*a^2 + 4*a*b)*cosh(d*x + c)^2 + 3
*a^2 + 4*a*b)*sinh(d*x + c))/(a^3*d*cosh(d*x + c)^8 + 8*a^3*d*cosh(d*x + c)
*sinh(d*x + c)^7 + a^3*d*sinh(d*x + c)^8 - 4*a^3*d*cosh(d*x + c)^6 + 6*a^3*
d*cosh(d*x + c)^4 + 4*(7*a^3*d*cosh(d*x + c)^2 - a^3*d)*sinh(d*x + c)^6 - 4
*a^3*d*cosh(d*x + c)^2 + 8*(7*a^3*d*cosh(d*x + c)^3 - 3*a^3*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*a^3*d*cosh(d*x + c)^4 - 30*a^3*d*cosh(d*x + c)^2
+ 3*a^3*d)*sinh(d*x + c)^4 + a^3*d + 8*(7*a^3*d*cosh(d*x + c)^5 - 10*a^3*d*
cosh(d*x + c)^3 + 3*a^3*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*d*cosh(
d*x + c)^6 - 15*a^3*d*cosh(d*x + c)^4 + 9*a^3*d*cosh(d*x + c)^2 - a^3*d)*si
nh(d*x + c)^2 + 8*(a^3*d*cosh(d*x + c)^7 - 3*a^3*d*cosh(d*x + c)^5 + 3*a^3*
d*cosh(d*x + c)^3 - a^3*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.41 $\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=110

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a-b}}-\frac{\left(a^2+ab+b^2\right)\coth(c+dx)}{a^3d}+\frac{(2a+b)\coth^3(c+dx)}{3a^2d}-\frac{\coth^5(c+dx)}{5ad}$$

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a - b]*d
)) - ((a^2 + a*b + b^2)*Coth[c + d*x])/(a^3*d) + ((2*a + b)*Coth[c + d*x]^3
)/(3*a^2*d) - Coth[c + d*x]^5/(5*a*d)
```

Rubi [A] time = 0.135537, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 461, 208}

$$-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a-b}}-\frac{\left(a^2+ab+b^2\right)\coth(c+dx)}{a^3d}+\frac{(2a+b)\coth^3(c+dx)}{3a^2d}-\frac{\coth^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a - b]*d
)) - ((a^2 + a*b + b^2)*Coth[c + d*x])/(a^3*d) + ((2*a + b)*Coth[c + d*x]^3
)/(3*a^2*d) - Coth[c + d*x]^5/(5*a*d)
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6} + \frac{-2a-b}{a^2x^4} + \frac{a^2+ab+b^2}{a^3x^2} + \frac{b^3}{a^3(-a+(a-b)x^2)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{(a^2+ab+b^2)\coth(c+dx)}{a^3d} + \frac{(2a+b)\coth^3(c+dx)}{3a^2d} - \frac{\coth^5(c+dx)}{5ad} + \frac{b^3\,\text{Subst}\left(\int \frac{}{-a+(\ldots}\right.}{}$$
$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a-b}d} - \frac{(a^2+ab+b^2)\coth(c+dx)}{a^3d} + \frac{(2a+b)\coth^3(c+dx)}{3a^2d} - \text{cotl}\ldots$$

Mathematica [A] time = 1.45808, size = 155, normalized size = 1.41

$$-\frac{\operatorname{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\sqrt{a}\sqrt{a-b}\coth(c+dx)\left(3a^2\operatorname{csch}^4(c+dx)+8a^2-a(4a+5b)\operatorname{csch}^2(c+dx)\right.\right.}{30a^{7/2}d\sqrt{a-b}\left(a\operatorname{csch}^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^2*(15*b^3*ArcTanh[(Sqrt[a -
 b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a - b]*Coth[c + d*x]*(8*a^2 + 10
*a*b + 15*b^2 - a*(4*a + 5*b)*Csch[c + d*x]^2 + 3*a^2*Csch[c + d*x]^4)))/(3
0*a^(7/2)*Sqrt[a - b]*d*(b + a*Csch[c + d*x]^2))
```

Maple [B] time = 0.074, size = 519, normalized size = 4.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/160/d/a*tanh(1/2*d*x+1/2*c)^5+5/96/d/a*tanh(1/2*d*x+1/2*c)^3+1/24/d/a^2*
tanh(1/2*d*x+1/2*c)^3*b-5/16/d/a*tanh(1/2*d*x+1/2*c)-3/8/d/a^2*tanh(1/2*d*x
+1/2*c)*b-1/2/d/a^3*b^2*tanh(1/2*d*x+1/2*c)-1/160/d/a/tanh(1/2*d*x+1/2*c)^5
+5/96/d/a/tanh(1/2*d*x+1/2*c)^3+1/24/d/a^2/tanh(1/2*d*x+1/2*c)^3*b-5/16/d/a
/tanh(1/2*d*x+1/2*c)-3/8/d*b/a^2/tanh(1/2*d*x+1/2*c)-1/2/d/a^3/tanh(1/2*d*x
+1/2*c)*b^2-1/d*b^3/a^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^4/a^3/(-b*(a-b)
)^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^3/a^3/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))+1/d*b^4/a^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arcta
n(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.78102, size = 10761, normalized size = 97.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(60*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 480*(a^2*b^2 - a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^7 + 60*(a^2*b^2 - a*b^3)*sinh(d*x + c)^8 - 120*(a^3*b
 + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 120*(a^3*b + a^2*b^2 - 2*a*b^3 - 14
*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 240*(14*(a^2*b^2 - a*
b^3)*cosh(d*x + c)^3 - 3*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c))*sinh(d*
x + c)^5 + 40*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4 + 40*(1
05*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3
 - 45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*a^4
 + 8*a^3*b + 20*a^2*b^2 - 60*a*b^3 + 160*(21*(a^2*b^2 - a*b^3)*cosh(d*x + c)
^5 - 15*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^3 + (8*a^4 - a^3*b + 2*a^
2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 40*(4*a^4 + a^3*b + a^2*b
^2 - 6*a*b^3)*cosh(d*x + c)^2 + 40*(42*(a^2*b^2 - a*b^3)*cosh(d*x + c)^6 -
45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^4 - 4*a^4 - a^3*b - a^2*b^2 +
6*a*b^3 + 6*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 - 15*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
 b^3*sinh(d*x + c)^10 - 5*b^3*cosh(d*x + c)^8 + 10*b^3*cosh(d*x + c)^6 + 5*
(9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^8 + 40*(3*b^3*cosh(d*x + c)^3 -
 b^3*cosh(d*x + c))*sinh(d*x + c)^7 - 10*b^3*cosh(d*x + c)^4 + 10*(21*b^3*c
osh(d*x + c)^4 - 14*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^6 + 4*(63*b^3*
cosh(d*x + c)^5 - 70*b^3*cosh(d*x + c)^3 + 15*b^3*cosh(d*x + c))*sinh(d*x +
 c)^5 + 5*b^3*cosh(d*x + c)^2 + 10*(21*b^3*cosh(d*x + c)^6 - 35*b^3*cosh(d*
x + c)^4 + 15*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^4 + 40*(3*b^3*cosh(d
*x + c)^7 - 7*b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^3 - b^3*cosh(d*x +
c))*sinh(d*x + c)^3 - b^3 + 5*(9*b^3*cosh(d*x + c)^8 - 28*b^3*cosh(d*x + c)
^6 + 30*b^3*cosh(d*x + c)^4 - 12*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2
 + 10*(b^3*cosh(d*x + c)^9 - 4*b^3*cosh(d*x + c)^7 + 6*b^3*cosh(d*x + c)^5
- 4*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)
*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(
d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2
*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3
+ (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*c
osh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) + 80*(6*(a^2*b^2 - a*b^3)*cosh(d*x + c)^7 - 9*(a^3*b + a^2*b^2 - 2*a*b^3)
*cosh(d*x + c)^5 + 2*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3
- (4*a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 -
 a^4*b)*d*cosh(d*x + c)^10 + 10*(a^5 - a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)
^9 + (a^5 - a^4*b)*d*sinh(d*x + c)^10 - 5*(a^5 - a^4*b)*d*cosh(d*x + c)^8 +
 5*(9*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5 - a^4*b)*d)*sinh(d*x + c)^8 +
10*(a^5 - a^4*b)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - a^4*b)*d*cosh(d*x + c)^3
- (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - a^4*b)*d*c
```

```
osh(d*x + c)^4 - 14*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + (a^5 - a^4*b)*d)*sinh
(d*x + c)^6 - 10*(a^5 - a^4*b)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - a^4*b)*d*co
sh(d*x + c)^5 - 70*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + 15*(a^5 - a^4*b)*d*cos
h(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - a^4*b)*d*cosh(d*x + c)^6 - 35*(
a^5 - a^4*b)*d*cosh(d*x + c)^4 + 15*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5
- a^4*b)*d)*sinh(d*x + c)^4 + 5*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + 40*(3*(a^
5 - a^4*b)*d*cosh(d*x + c)^7 - 7*(a^5 - a^4*b)*d*cosh(d*x + c)^5 + 5*(a^5 -
 a^4*b)*d*cosh(d*x + c)^3 - (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 5*(9*(a^5 - a^4*b)*d*cosh(d*x + c)^8 - 28*(a^5 - a^4*b)*d*cosh(d*x + c)^6
 + 30*(a^5 - a^4*b)*d*cosh(d*x + c)^4 - 12*(a^5 - a^4*b)*d*cosh(d*x + c)^2
+ (a^5 - a^4*b)*d)*sinh(d*x + c)^2 - (a^5 - a^4*b)*d + 10*((a^5 - a^4*b)*d*
cosh(d*x + c)^9 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^7 + 6*(a^5 - a^4*b)*d*cos
h(d*x + c)^5 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + (a^5 - a^4*b)*d*cosh(d*x
 + c))*sinh(d*x + c)), -1/15*(30*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 240*(a
^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 30*(a^2*b^2 - a*b^3)*sinh(d
*x + c)^8 - 60*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^6 - 60*(a^3*b + a^
2*b^2 - 2*a*b^3 - 14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 1
20*(14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^3 - 3*(a^3*b + a^2*b^2 - 2*a*b^3)*co
sh(d*x + c))*sinh(d*x + c)^5 + 20*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cos
h(d*x + c)^4 + 20*(105*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - a^3*b +
2*a^2*b^2 - 9*a*b^3 - 45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 16*a^4 + 4*a^3*b + 10*a^2*b^2 - 30*a*b^3 + 80*(21*(a^2*b^2 - a
*b^3)*cosh(d*x + c)^5 - 15*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^3 + (8
*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 20*(4*
a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*cosh(d*x + c)^2 + 20*(42*(a^2*b^2 - a*b^3)
*cosh(d*x + c)^6 - 45*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh(d*x + c)^4 - 4*a^4 -
 a^3*b - a^2*b^2 + 6*a*b^3 + 6*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 - 15*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c
)*sinh(d*x + c)^9 + b^3*sinh(d*x + c)^10 - 5*b^3*cosh(d*x + c)^8 + 10*b^3*c
osh(d*x + c)^6 + 5*(9*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^8 + 40*(3*b^
3*cosh(d*x + c)^3 - b^3*cosh(d*x + c))*sinh(d*x + c)^7 - 10*b^3*cosh(d*x +
c)^4 + 10*(21*b^3*cosh(d*x + c)^4 - 14*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x
+ c)^6 + 4*(63*b^3*cosh(d*x + c)^5 - 70*b^3*cosh(d*x + c)^3 + 15*b^3*cosh(d
*x + c))*sinh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^2 + 10*(21*b^3*cosh(d*x + c)
^6 - 35*b^3*cosh(d*x + c)^4 + 15*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^4
 + 40*(3*b^3*cosh(d*x + c)^7 - 7*b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^
3 - b^3*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 + 5*(9*b^3*cosh(d*x + c)^8 - 2
8*b^3*cosh(d*x + c)^6 + 30*b^3*cosh(d*x + c)^4 - 12*b^3*cosh(d*x + c)^2 + b
^3)*sinh(d*x + c)^2 + 10*(b^3*cosh(d*x + c)^9 - 4*b^3*cosh(d*x + c)^7 + 6*b
^3*cosh(d*x + c)^5 - 4*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sin
h(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 4
0*(6*(a^2*b^2 - a*b^3)*cosh(d*x + c)^7 - 9*(a^3*b + a^2*b^2 - 2*a*b^3)*cosh
(d*x + c)^5 + 2*(8*a^4 - a^3*b + 2*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^3 - (4*
a^4 + a^3*b + a^2*b^2 - 6*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 - a^4*
b)*d*cosh(d*x + c)^10 + 10*(a^5 - a^4*b)*d*cosh(d*x + c)*sinh(d*x + c)^9 +
(a^5 - a^4*b)*d*sinh(d*x + c)^10 - 5*(a^5 - a^4*b)*d*cosh(d*x + c)^8 + 5*(9
*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5 - a^4*b)*d)*sinh(d*x + c)^8 + 10*(a
^5 - a^4*b)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - a^4*b)*d*cosh(d*x + c)^3 - (a^
5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - a^4*b)*d*cosh(d
*x + c)^4 - 14*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + (a^5 - a^4*b)*d)*sinh(d*x
+ c)^6 - 10*(a^5 - a^4*b)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - a^4*b)*d*cosh(d*
x + c)^5 - 70*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + 15*(a^5 - a^4*b)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - a^4*b)*d*cosh(d*x + c)^6 - 35*(a^5 -
 a^4*b)*d*cosh(d*x + c)^4 + 15*(a^5 - a^4*b)*d*cosh(d*x + c)^2 - (a^5 - a^4
*b)*d)*sinh(d*x + c)^4 + 5*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + 40*(3*(a^5 - a
^4*b)*d*cosh(d*x + c)^7 - 7*(a^5 - a^4*b)*d*cosh(d*x + c)^5 + 5*(a^5 - a^4*
b)*d*cosh(d*x + c)^3 - (a^5 - a^4*b)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(
9*(a^5 - a^4*b)*d*cosh(d*x + c)^8 - 28*(a^5 - a^4*b)*d*cosh(d*x + c)^6 + 30
```

```
*(a^5 - a^4*b)*d*cosh(d*x + c)^4 - 12*(a^5 - a^4*b)*d*cosh(d*x + c)^2 + (a^
5 - a^4*b)*d)*sinh(d*x + c)^2 - (a^5 - a^4*b)*d + 10*((a^5 - a^4*b)*d*cosh(
d*x + c)^9 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^7 + 6*(a^5 - a^4*b)*d*cosh(d*x
 + c)^5 - 4*(a^5 - a^4*b)*d*cosh(d*x + c)^3 + (a^5 - a^4*b)*d*cosh(d*x + c)
)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out
```

Giac [B] time = 1.49275, size = 289, normalized size = 2.63

$$-\frac{b^3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}a^3d}-\frac{2\left(15\,b^2e^{(8\,dx+8\,c)}-30\,abe^{(6\,dx+6\,c)}-60\,b^2e^{(6\,dx+6\,c)}+80\,a^2e^{(4\,dx+4\,c)}+70\,abe^{(4\,dx+4\,c)}+90\right.}{15\,a^3d\left(e^{(2\,dx+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] -b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*a^3*d) - 2/15*(15*b^2*e^(8*d*x + 8*c) - 30*a*b*e^(6*d*x + 6*c) - 60*
b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x + 4*c) + 70*a*b*e^(4*d*x + 4*c) + 90*
b^2*e^(4*d*x + 4*c) - 40*a^2*e^(2*d*x + 2*c) - 50*a*b*e^(2*d*x + 2*c) - 60*
b^2*e^(2*d*x + 2*c) + 8*a^2 + 10*a*b + 15*b^2)/(a^3*d*(e^(2*d*x + 2*c) - 1)
^5)
```

3.42 $\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=102

$$-\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^2d(a-b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

```
[Out] x/b^2 - (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/
(2*(a - b)^(3/2)*b^2*d) - (a*Tanh[c + d*x])/(2*(a - b)*b*d*(a - (a - b)*Tan
h[c + d*x]^2))
```

Rubi [A] time = 0.169097, antiderivative size = 102, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 470, 522, 206, 208}

$$-\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2b^2d(a-b)^{3/2}}-\frac{a\tanh(c+dx)}{2bd(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/
(2*(a - b)^(3/2)*b^2*d) - (a*Tanh[c + d*x])/(2*(a - b)*b*d*(a - (a - b)*Tan
h[c + d*x]^2))
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+(a-2b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)}\,dx, x, \tanh(c+d\right.}{2(a-b)bd}$$

$$= -\frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{b^2d} - \frac{(a(2a}{}$$

$$= \frac{x}{b^2} - \frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2(a-b)^{3/2}b^2d} - \frac{a\tanh(c+dx)}{2(a-b)bd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.832878, size = 99, normalized size = 0.97

$$-\frac{\frac{\sqrt{a}(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}} + \frac{ab\sinh(2(c+dx))}{(a-b)(2a+b\cosh(2(c+dx))-b)} - 2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -(-2*(c + d*x) + (Sqrt[a]*(2*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/S
qrt[a]])/(a - b)^(3/2) + (a*b*Sinh[2*(c + d*x)])/((a - b)*(2*a - b + b*Cosh
[2*(c + d*x)])))/(2*b^2*d)
```

Maple [B] time = 0.051, size = 798, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/d*a/b
/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/d*a/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/
```

```
d*a^2/b^2/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a^2/b/(a-b)/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a^2/b^2/(a-b)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2))+1/d*a^2/b/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/2/d*a
/b/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/2/d*a/(a-b)/(-b*(a-b))^(1/2)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2))-3/2/d*a/b/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/2/d
*a/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tan
h(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.55959, size = 4199, normalized size = 41.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b - b^2)*d*x*cosh(d*x + c)^4 + 16*(a*b - b^2)*d*x*cosh(d*x + c)*
sinh(d*x + c)^3 + 4*(a*b - b^2)*d*x*sinh(d*x + c)^4 + 4*(a*b - b^2)*d*x + 4
*(2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + c)^2 + 4*(6*(a*b -
b^2)*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*sinh(
d*x + c)^2 + ((2*a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(2*a*b - 3*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^3 + (2*a*b - 3*b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 8*a*b +
 3*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - 3*b^2)*cosh(d*x + c)^2 + 4*a^2 - 8*
a*b + 3*b^2)*sinh(d*x + c)^2 + 2*a*b - 3*b^2 + 4*((2*a*b - 3*b^2)*cosh(d*x
+ c)^3 + (4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a/(a -
b))*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*si
nh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2
+ 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)
^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*((a*b - b^2)*cosh(d*x +
 c)^2 + 2*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c) + (a*b - b^2)*sinh(d*x +
c)^2 + 2*a^2 - 3*a*b + b^2)*sqrt(a/(a - b)))/(b*cosh(d*x + c)^4 + 4*b*cosh(
d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2
+ 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3
+ (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*a*b + 8*(2*(a*b - b^2)*d
*x*cosh(d*x + c)^3 + (2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x +
 c))*sinh(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^4 + 4*(a*b^3 - b^4)*d*co
sh(d*x + c)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*sinh(d*x + c)^4 + 2*(2*a^2*b^
2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^2
```

```
 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 4*((a
*b^3 - b^4)*d*cosh(d*x + c)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/2*(2*(a*b - b^2)*d*x*cosh(d*x + c)^4 + 8*(a*b - b^2)*d*
x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*(a*b - b^2)*d*x*sinh(d*x + c)^4 + 2*(a*
b - b^2)*d*x + 2*(2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a*b)*cosh(d*x + c)^
2 + 2*(6*(a*b - b^2)*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - 3*a*b + b^2)*d*x + 2*
a^2 - a*b)*sinh(d*x + c)^2 - ((2*a*b - 3*b^2)*cosh(d*x + c)^4 + 4*(2*a*b -
3*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - 3*b^2)*sinh(d*x + c)^4 + 2*
(4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - 3*b^2)*cosh(d*x + c
)^2 + 4*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^2 + 2*a*b - 3*b^2 + 4*((2*a*b -
3*b^2)*cosh(d*x + c)^3 + (4*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
c))*sqrt(-a/(a - b))*arctan(1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh
(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a/(a - b))/a) + 2*a*b + 4*(2
*(a*b - b^2)*d*x*cosh(d*x + c)^3 + (2*(2*a^2 - 3*a*b + b^2)*d*x + 2*a^2 - a
*b)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 - b^4)*d*cosh(d*x + c)^4 + 4*(a*b
^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*sinh(d*x + c)^4
 + 2*(2*a^2*b^2 - 3*a*b^3 + b^4)*d*cosh(d*x + c)^2 + 2*(3*(a*b^3 - b^4)*d*c
osh(d*x + c)^2 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d)*sinh(d*x + c)^2 + (a*b^3 -
b^4)*d + 4*((a*b^3 - b^4)*d*cosh(d*x + c)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*d
*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28282, size = 228, normalized size = 2.24

$$-\frac{\left(2\,a^2-3\,ab\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{2\left(ab^2d-b^3d\right)\sqrt{-a^2+ab}}+\frac{2\,a^2e^{(2\,dx+2\,c)}-abe^{(2\,dx+2\,c)}+ab}{\left(ab^2d-b^3d\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}+\frac{dx+c}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a^2 - 3*a*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a
*b))/((a*b^2*d - b^3*d)*sqrt(-a^2 + a*b)) + (2*a^2*e^(2*d*x + 2*c) - a*b*e^
(2*d*x + 2*c) + a*b)/((a*b^2*d - b^3*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x +
 2*c) - 2*b*e^(2*d*x + 2*c) + b)) + (d*x + c)/(b^2*d)
```

3.43 $\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=90

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2b^{3/2}d(a-b)^{3/2}}-\frac{a\cosh(c+dx)}{2bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*(a - b)^(3/2)*b^
(3/2)*d) - (a*Cosh[c + d*x])/(2*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.112523, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 385, 205}

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2b^{3/2}d(a-b)^{3/2}}-\frac{a\cosh(c+dx)}{2bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*(a - b)^(3/2)*b^
(3/2)*d) - (a*Cosh[c + d*x])/(2*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)}{2(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)} + \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{2(a-b)bd}$$

$$= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}b^{3/2}d} - \frac{a\cosh(c+dx)}{2(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)}$$

Mathematica [C] time = 0.596782, size = 141, normalized size = 1.57

$$\frac{-\frac{2a\sqrt{b}\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)} + \frac{(a-2b)\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{3/2}}}{2b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (((a - 2*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(3/2)
 - (2*a*Sqrt[b]*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(
2*b^(3/2)*d)
```

Maple [B] time = 0.032, size = 341, normalized size = 3.8

$$16\,\frac{\left(\tanh\left(1/2\,dx+c/2\right)\right)^2 a}{d\left(16\,ab-16\,b^2\right)\left(\left(\tanh\left(1/2\,dx+c/2\right)\right)^4 a-2\,\left(\tanh\left(1/2\,dx+c/2\right)\right)^2 a+4\,\left(\tanh\left(1/2\,dx+c/2\right)\right)^2 b+a\right)} - 32\,\frac{}{d\left(16\,a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 16/d/(16*a*b-16*b^2)/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^2-32/d/(16*a*b-16*b^2)/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
*tanh(1/2*d*x+1/2*c)^2*b-16/d/(16*a*b-16*b^2)/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a+8/d/(16*a*b-16*b^2)/(a
*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2
))*a-16/d/(16*a*b-16*b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)
^2*a-2*a+4*b)/(a*b-b^2)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ae^{(3\,dx+3\,c)}+ae^{(dx+c)}}{ab^2d-b^3d+\left(ab^2de^{(4\,c)}-b^3de^{(4\,c)}\right)e^{(4\,dx)}+2\left(2\,a^2bde^{(2\,c)}-3\,ab^2de^{(2\,c)}+b^3de^{(2\,c)}\right)e^{(2\,dx)}} + \frac{1}{8}\int \frac{8\left(\left(ae\right.\right.}{ab^2-b^3+\left(ab^2e^{(4\,c)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(4*c) -
b^3*d*e^(4*c))*e^(4*d*x) + 2*(2*a^2*b*d*e^(2*c) - 3*a*b^2*d*e^(2*c) + b^3*d
*e^(2*c))*e^(2*d*x)) + 1/8*integrate(8*((a*e^(3*c) - 2*b*e^(3*c))*e^(3*d*x)
 - (a*e^c - 2*b*e^c)*e^(d*x))/(a*b^2 - b^3 + (a*b^2*e^(4*c) - b^3*e^(4*c))*
e^(4*d*x) + 2*(2*a^2*b*e^(2*c) - 3*a*b^2*e^(2*c) + b^3*e^(2*c))*e^(2*d*x)),
 x)
```

<u>Fricas</u> [B] time = 2.43195, size = 4536, normalized size = 50.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b - 2*b^2)*cosh(
d*x + c)^4 + 4*(a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b - 2*b^2)*
sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - 2
*b^2)*cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2)*sinh(d*x + c)^2 + a*b - 2*b^
2 + 4*((a*b - 2*b^2)*cosh(d*x + c)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 -
(2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x
+ c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 + 1)*sinh(d*x +
 c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b - a*b^2)*cosh(d*x +
 c) + 4*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/
((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*(a^2*b^3 - 2*a*b^4 + b^5)*
d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^3 - 2*a*b^4 + b^5)*d*sinh(d*x + c)
^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*cosh(d*x + c)^2 + 2*(3*(a^
2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4
 - b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 - 2*a*b^4 + b^5)*d + 4*((a^2*b^3 - 2*
a*b^4 + b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*
cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 6*
(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2*b - a*b^2)*sinh(d*x
+ c)^3 - ((a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(a*b - 2*b^2)*cosh(d*x + c)*sin
h(d*x + c)^3 + (a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 5*a*b + 2*b^2)*co
sh(d*x + c)^2 + 2*(3*(a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2 - 5*a*b + 2*b^2)
*sinh(d*x + c)^2 + a*b - 2*b^2 + 4*((a*b - 2*b^2)*cosh(d*x + c)^3 + (2*a^2
- 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(-1/2*
(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
+ (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x +
c))/sqrt(a*b - b^2)) + ((a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(a*b - 2*b^2)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 5*a
*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - 2*b^2)*cosh(d*x + c)^2 + 2*a^2 -
5*a*b + 2*b^2)*sinh(d*x + c)^2 + a*b - 2*b^2 + 4*((a*b - 2*b^2)*cosh(d*x +
c)^3 + (2*a^2 - 5*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2
)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*
(a^2*b - a*b^2)*cosh(d*x + c) + 2*(a^2*b - a*b^2 + 3*(a^2*b - a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^4 + 4*
(a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^3 - 2*a*
```

```
b^4 + b^5)*d*sinh(d*x + c)^4 + 2*(2*a^3*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d*
cosh(d*x + c)^2 + 2*(3*(a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^2 + (2*a^3
*b^2 - 5*a^2*b^3 + 4*a*b^4 - b^5)*d)*sinh(d*x + c)^2 + (a^2*b^3 - 2*a*b^4 +
 b^5)*d + 4*((a^2*b^3 - 2*a*b^4 + b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^2 - 5*a
^2*b^3 + 4*a*b^4 - b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.44 $\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=84

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a-b)^{3/2}}$$

```
[Out] -ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a - b)^(3/2)*d) +
 (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0861296, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3173, 12, 3181, 208}

$$\frac{\sinh(c+dx)\cosh(c+dx)}{2d(a-b)\left(a+b\sinh^2(c+dx)\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a - b)^(3/2)*d) +
 (Cosh[c + d*x]*Sinh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{a}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}$$
$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{2(a-b)}$$
$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2(a-b)d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}(a-b)^{3/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.444132, size = 81, normalized size = 0.96

$$\frac{\frac{\sinh(2(c+dx))}{(a-b)(2a+b\cosh(2(c+dx))-b)} - \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a - b)^(3/2))) +
Sinh[2*(c + d*x)]/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)
```

Maple [B] time = 0.041, size = 428, normalized size = 5.1

$$\frac{1}{d\left(a-b\right)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a - 2\left(\tanh\left(1/2\,dx+c/2\right)\right)^2 a + 4\left(\tanh\left(1/2\,dx+c/2\right)\right)^2 b + a\right)^{-1} + \frac{}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/2
/d/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(a-b)/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))*b+1/2/d/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/(a-
b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.39192, size = 3564, normalized size = 42.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*
(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b
+ a*b^2)*sinh(d*x + c)^2 + (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b
^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x +
 c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^
2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)
^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b
^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(
d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
 - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/((a^3
*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - 2*a^2*b^3 + a*b^
4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*sinh(d
*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 +
 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (2*a^4*b - 5*a^3*b^
2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 + (a^3*b^2 - 2*a^2*b^3 + a*b^4)*d
 + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b - 5*a^3*b^
2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b - 2*a
*b^2 + 2*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a
*b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x +
c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sin
h(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 +
 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2
)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x +
c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(
-a^2 + a*b)/(a^2 - a*b)))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4
+ 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^
2 - 2*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 5*a^3*b^2 + 4*a^2*b
^3 - a*b^4)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^2 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^2 +
(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d + 4*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^3 + (2*a^4*b - 5*a^3*b^2 + 4*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh
(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40178, size = 184, normalized size = 2.19

$$-\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{2\,\sqrt{-a^2+ab}(ad-bd)}-\frac{2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b}{\left(abd-b^2d\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2
+ a*b)*(a*d - b*d)) - (2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((a*b*d
 - b^2*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) +
b))
```

3.45 $\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=81

$$\frac{\cosh(c+dx)}{2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{b}d(a-b)^{3/2}}$$

```
[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b]*d) + C
osh[c + d*x]/(2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.0654572, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 199, 205}

$$\frac{\cosh(c+dx)}{2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2\sqrt{b}d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]]/(2*(a - b)^(3/2)*Sqrt[b]*d) + C
osh[c + d*x]/(2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\cosh(c+dx)}{2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a-b+bx^2}\,dx, x, \cosh(c+dx)\right)}{2(a-b)d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}\sqrt{b}d} + \frac{\cosh(c+dx)}{2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)}$$

Mathematica [C] time = 0.34833, size = 130, normalized size = 1.6

$$\frac{\frac{2\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)} + \frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{\sqrt{b}(a-b)^{3/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/((a - b)^(3/2)*Sqrt[b]) +
 (2*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])))/(2*d)
```

Maple [B] time = 0.027, size = 256, normalized size = 3.2

$$-\frac{1}{d(a-b)}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a - 2\left(\tanh(1/2\,dx+c/2)\right)^2 a + 4\left(\tanh(1/2\,dx+c/2)\right)^2 b + a\right)^{-1} + 2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2+2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)/a*tanh(1/2*d*x+1/2*c)^
2*b+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)/(a-b)+1/2/d/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1
/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{abd-b^2d+\left(abde^{(4\,c)}-b^2de^{(4\,c)}\right)e^{(4\,dx)}+2\left(2\,a^2de^{(2\,c)}-3\,abde^{(2\,c)}+b^2de^{(2\,c)}\right)e^{(2\,dx)}} + \frac{1}{2}\int \frac{2}{ab-b^2+\left(abe^{(4\,c)}-b^2e^{(4\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) + e^(d*x + c))/(a*b*d - b^2*d + (a*b*d*e^(4*c) - b^2*d*e^(
4*c))*e^(4*d*x) + 2*(2*a^2*d*e^(2*c) - 3*a*b*d*e^(2*c) + b^2*d*e^(2*c))*e^(
2*d*x)) + 1/2*integrate(2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a*b - b^2 + (a*b
*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^2*e^(2*c) - 3*a*b*e^(2*c) + b^2*
e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.39528, size = 3999, normalized size = 49.37

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a*b - b^2)*cosh(d*x + c)^3 + 12*(a*b - b^2)*cosh(d*x + c)*sinh(d*x
 + c)^2 + 4*(a*b - b^2)*sinh(d*x + c)^3 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x
 + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2
*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (
2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b + b^2)*log((b*cosh(d*x
 + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a -
3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh
(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d
*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh
(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*
a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(
a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b^2)*cosh(d*x + c)^2 + a*b - b^2)*si
nh(d*x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 - 2*
a*b^3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d*
sinh(d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d*cosh(d*x + c)^2
 + 2*(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*a^3*b - 5*a^2*b^2
+ 4*a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*d + 4*((a^2
*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a*b - b^2)*cosh(d*x + c)^3 +
6*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a*b - b^2)*sinh(d*x + c)^3
 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)
^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d
*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) +
 b)*sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh
(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x
 + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - (b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d
*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d
*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(a*b - b^2)*arc
tan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*(a*b
- b^2)*cosh(d*x + c) + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + a*b - b^2)*sinh(d
*x + c))/((a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^4 + 4*(a^2*b^2 - 2*a*b^
3 + b^4)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2*b^2 - 2*a*b^3 + b^4)*d*sinh
(d*x + c)^4 + 2*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*d*cosh(d*x + c)^2 + 2
*(3*(a^2*b^2 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^2 + (2*a^3*b - 5*a^2*b^2 + 4*
a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*d + 4*((a^2*b^2
 - 2*a*b^3 + b^4)*d*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)
*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.46 $\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=95

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)
^(3/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c +
d*x]^2))
```

Rubi [A] time = 0.0694698, antiderivative size = 95, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3184, 12, 3181, 208}

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}}-\frac{b\sinh(c+dx)\cosh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)
^(3/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*a*(a - b)*d*(a + b*Sinh[c +
d*x]^2))
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{-2a+b}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{(2a-b)\int \frac{1}{a+b\sinh^2(c+dx)}\,dx}{2a(a-b)}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2a(a-b)d}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{3/2}d} - \frac{b\cosh(c+dx)\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.302547, size = 96, normalized size = 1.01

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{3/2}} - \frac{b\sinh(2(c+dx))}{2ad(a-b)(2a+b\cosh(2(c+dx))-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)
^(3/2)*d) - (b*Sinh[2*(c + d*x)])/(2*a*(a - b)*d*(2*a - b + b*Cosh[2*(c + d
*x)]))
```

Maple [B] time = 0.046, size = 749, normalized size = 7.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*b/a/(a-b)*tanh(1/2*d*x+1
/2*c)+1/d/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(a-b)/(-b*(a-b))^(1/2)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2))*b-1/d/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(a
-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b-1/2/d/(a-b)*b/a/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))+1/2/d/(a-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2))*b^2+1/2/d/(a-b)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/(a-b)/a/(-b
*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.43671, size = 3767, normalized size = 39.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*(
2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b +
 a*b^2)*sinh(d*x + c)^2 + ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^2 -
4*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*a^2 -
 4*a*b + b^2)*sinh(d*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d*x + c
)^3 + (4*a^2 - 4*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*l
og((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 +
(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 +
2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x +
c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))
/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^4*b - 2*a^3*b^2 +
a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^4*b - 2*a^3*b^2 + a^2*b^3)*d*
sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)
^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + (2*a^5 - 5*a^4*
b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b - 2*a^3*b^2 + a^2*b^3)
*d + 4*((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^5 - 5*a^4*b
+ 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a^2*b - 2*a*
b^2 + 2*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*
b^2)*cosh(d*x + c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c
)^2 - ((2*a*b - b^2)*cosh(d*x + c)^4 + 4*(2*a*b - b^2)*cosh(d*x + c)*sinh(d
*x + c)^3 + (2*a*b - b^2)*sinh(d*x + c)^4 + 2*(4*a^2 - 4*a*b + b^2)*cosh(d*
x + c)^2 + 2*(3*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*a^2 - 4*a*b + b^2)*sinh(d
*x + c)^2 + 2*a*b - b^2 + 4*((2*a*b - b^2)*cosh(d*x + c)^3 + (4*a^2 - 4*a*b
 + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(
d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)
*sqrt(-a^2 + a*b)/(a^2 - a*b)))/((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x +
 c)^4 + 4*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (
a^4*b - 2*a^3*b^2 + a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^5 - 5*a^4*b + 4*a^3
*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^4*b - 2*a^3*b^2 + a^2*b^3)*d*co
sh(d*x + c)^2 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2
+ (a^4*b - 2*a^3*b^2 + a^2*b^3)*d + 4*((a^4*b - 2*a^3*b^2 + a^2*b^3)*d*cosh
(d*x + c)^3 + (2*a^5 - 5*a^4*b + 4*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh
```

```
(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21252, size = 196, normalized size = 2.06

$$\frac{(2a-b)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{2\left(a^2d-abd\right)\sqrt{-a^2+ab}}+\frac{2ae^{(2dx+2c)}-be^{(2dx+2c)}+b}{\left(a^2d-abd\right)\left(be^{(4dx+4c)}+4ae^{(2dx+2c)}-2be^{(2dx+2c)}+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((
a^2*d - a*b*d)*sqrt(-a^2 + a*b)) + (2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c)
 + b)/((a^2*d - a*b*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*
d*x + 2*c) + b))
```

3.47 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=110

$$-\frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2d(a-b)^{3/2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{b\cosh(c+dx)}{2ad(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^2*(
a - b)^(3/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^2*d) - (b*Cosh[c + d*x])/(2*a*(
a - b)*d*(a - b + b*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.149327, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3186, 414, 522, 206, 205}

$$-\frac{\sqrt{b}(3a-2b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2d(a-b)^{3/2}}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^2d}-\frac{b\cosh(c+dx)}{2ad(a-b)\left(a+b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((3*a - 2*b)*Sqrt[b]*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^2*(
a - b)^(3/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^2*d) - (b*Cosh[c + d*x])/(2*a*(
a - b)*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh(c+dx)}{2a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2a+b+bx^2}{(1-x^2)\left(a-b+bx^2\right)}\,dx, x, \cosh(c+dx)\right)}{2a(a-b)d}$$

$$= -\frac{b\cosh(c+dx)}{2a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{a^2d} - \frac{((3a-2\cdots}{\cdots}$$

$$= -\frac{(3a-2b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^2(a-b)^{3/2}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2d} - \frac{b\cosh(c+d\cdots}{2a(a-b)d\left(a-b+b\operatorname{co}\cdots\right.}$$

Mathematica [C] time = 0.644036, size = 176, normalized size = 1.6

$$\frac{-\frac{2ab\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)} + \frac{\sqrt{b}(2b-3a)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + \frac{\sqrt{b}(2b-3a)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} + 2\log\left(\tanh\left(\frac{1}{2}(c+dx\cdots\right.\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[
a - b]])/(a - b)^(3/2) + (Sqrt[b]*(-3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*
Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(3/2) - (2*a*b*Cosh[c + d*x])/((a
- b)*(2*a - b + b*Cosh[2*(c + d*x)])) + 2*Log[Tanh[(c + d*x)/2]])/(2*a^2*d)
```

Maple [B] time = 0.062, size = 350, normalized size = 3.2

$$\frac{1}{da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right) + \frac{b}{d(a-b)a}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a - 2\left(\tanh(1/2\,dx+c/2)\right)^2 a + 4\,(\tanh\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*ln(tanh(1/2*d*x+1/2*c))+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)/a*tanh(1/2*d*x+1/2*c)^2*b-2/
```

```
d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/d/a*b/(tanh(1/2*d*x+1/2*c)^4*a
-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)-3/2/d/a*b/(a-
b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)
^(1/2))+1/d/a^2*b^2/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)
^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{be^{(3\,dx+3\,c)}+be^{(dx+c)}}{a^2bd-ab^2d+\left(a^2bde^{(4\,c)}-ab^2de^{(4\,c)}\right)e^{(4\,dx)}+2\left(2\,a^3de^{(2\,c)}-3\,a^2bde^{(2\,c)}+ab^2de^{(2\,c)}\right)e^{(2\,dx)}}-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{a^2d}+\frac{\log\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
- a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a*b
^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^(
d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/2*((3*a*b*e^(3*c) - 2*b^2*e^(
3*c))*e^(3*d*x) - (3*a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b - a^2*b^2 + (a^3*
b*e^(4*c) - a^2*b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 3*a^3*b*e^(2*c)
 + a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.91061, size = 6178, normalized size = 56.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b
*sinh(d*x + c)^3 + 4*a*b*cosh(d*x + c) - ((3*a*b - 2*b^2)*cosh(d*x + c)^4 +
 4*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x
 + c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*
cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a*b - 2*b^2 +
4*((3*a*b - 2*b^2)*cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)
*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2
*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*cosh(d*x + c)^3 + 3*(a
- b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh
(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a -
 b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d
*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)
*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x
 + c) + b)) + 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c)*
sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cos
h(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh
(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b +
 b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1)
- 4*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c
)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(d*x + c)^2
```

```
 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2
+ a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b + b^2)*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*a*b*c
osh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 +
 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b - a^2*b^2)*d*
sinh(d*x + c)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*(a
^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*d)*sinh(d*x
 + c)^2 + (a^3*b - a^2*b^2)*d + 4*((a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 + (2
*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a*b*cosh
(d*x + c)^3 + 6*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 +
 2*a*b*cosh(d*x + c) + ((3*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(3*a*b - 2*b^2)
*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(6*a^2
 - 7*a*b + 2*b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*cosh(d*x + c)^2 +
6*a^2 - 7*a*b + 2*b^2)*sinh(d*x + c)^2 + 3*a*b - 2*b^2 + 4*((3*a*b - 2*b^2)
*cosh(d*x + c)^3 + (6*a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) -
 ((3*a*b - 2*b^2)*cosh(d*x + c)^4 + 4*(3*a*b - 2*b^2)*cosh(d*x + c)*sinh(d*
x + c)^3 + (3*a*b - 2*b^2)*sinh(d*x + c)^4 + 2*(6*a^2 - 7*a*b + 2*b^2)*cosh
(d*x + c)^2 + 2*(3*(3*a*b - 2*b^2)*cosh(d*x + c)^2 + 6*a^2 - 7*a*b + 2*b^2)
*sinh(d*x + c)^2 + 3*a*b - 2*b^2 + 4*((3*a*b - 2*b^2)*cosh(d*x + c)^3 + (6*
a^2 - 7*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1
/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)
^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x
 + c))*sqrt(b/(a - b))/b) + 2*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*
a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a
*b + b^2)*sinh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2
*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(
d*x + c) + 1) - 2*((a*b - b^2)*cosh(d*x + c)^4 + 4*(a*b - b^2)*cosh(d*x + c
)*sinh(d*x + c)^3 + (a*b - b^2)*sinh(d*x + c)^4 + 2*(2*a^2 - 3*a*b + b^2)*c
osh(d*x + c)^2 + 2*(3*(a*b - b^2)*cosh(d*x + c)^2 + 2*a^2 - 3*a*b + b^2)*si
nh(d*x + c)^2 + a*b - b^2 + 4*((a*b - b^2)*cosh(d*x + c)^3 + (2*a^2 - 3*a*b
 + b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1
) + 2*(3*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cos
h(d*x + c)^4 + 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b
 - a^2*b^2)*d*sinh(d*x + c)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x +
c)^2 + 2*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + (2*a^4 - 3*a^3*b + a^2*b^
2)*d)*sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 4*((a^3*b - a^2*b^2)*d*cosh(d
*x + c)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.48 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=142

$$\frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a-b)^{3/2}}-\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] -((4*a - 3*b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a
 - b)^(3/2)*d) - Coth[c + d*x]/(a*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((2*a^
2 - 4*a*b + 3*b^2)*Tanh[c + d*x])/(2*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d*
x]^2))
```

Rubi [A] time = 0.153389, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 462, 385, 208}

$$\frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a-b)^{3/2}}-\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```


```
[Out] -((4*a - 3*b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a
 - b)^(3/2)*d) - Coth[c + d*x]/(a*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((2*a^
2 - 4*a*b + 3*b^2)*Tanh[c + d*x])/(2*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d*
x]^2))
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{a-3b+ax^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{ad}$$

$$= -\frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\left(2a^2-4ab+3b^2\right)\tanh(c+dx)}{2a^2(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{((4a-3b)b)}{}$$

$$= -\frac{(4a-3b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a-b)^{3/2}d} - \frac{\coth(c+dx)}{ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\left(2a^2-4ab+\right.}{2a^2(a-b)d\left(a-\right.}$$

Mathematica [A] time = 0.811308, size = 170, normalized size = 1.2

$$-\frac{\operatorname{csch}^5(c+dx)(2a+b\cosh(2(c+dx))-b)\left(2\sqrt{a}\sqrt{a-b}\cosh(c+dx)\left(4a^2+b(2a-3b)\cosh(2(c+dx))-6ab+3b^2\right)-2\right.}{16a^{5/2}d(a-b)^{3/2}\left(a\operatorname{csch}^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*(2*Sqrt[a]*Sqrt[a - b]*Co
sh[c + d*x]*(4*a^2 - 6*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)]) - 2*b
*(-4*a + 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cos
h[2*(c + d*x)])*Sinh[c + d*x]))/(16*a^(5/2)*(a - b)^(3/2)*d*(b + a*Csch[c +
 d*x]^2)^2)
```

Maple [B] time = 0.076, size = 810, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/a^2*tanh(1/2*d*x+1/2*c)-1/2/d/a^2/tanh(1/2*d*x+1/2*c)+1/d/a^2*b^2/(t
anh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+
a)/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/d/a^2*b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)-2
/d/(a-b)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b)/a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
```

```
b))^(1/2)+a-2*b)*a)^(1/2))*b^2+2/d/(a-b)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2
/d/(a-b)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2+3/2/d/a^2*b^2/
(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/2/d/a^2*b^3/(a-b)/(-b*(a-b))^(1/2)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2))-3/2/d/a^2*b^2/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
)-3/2/d/a^2*b^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.62142, size = 6962, normalized size = 49.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 7*
a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 7*a^2*b^2 +
 3*a*b^3)*sinh(d*x + c)^4 + 8*a^3*b - 20*a^2*b^2 + 12*a*b^3 + 8*(4*a^4 - 11
*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d*x + c)^2 + 8*(4*a^4 - 11*a^3*b + 10*a
^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 - ((4*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)*cosh(
d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - 3*b^3)*sinh(d*x + c)^6 + (16*a^2*b -
24*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + (16*a^2*b - 24*a*b^2 + 9*b^3 + 15*(4*a*
b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - 3*b^3)*cosh
(d*x + c)^3 + (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
- 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2 + (15*(4*
a*b^2 - 3*b^3)*cosh(d*x + c)^4 - 16*a^2*b + 24*a*b^2 - 9*b^3 + 6*(16*a^2*b
- 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - 3*b^
3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^3 - (16*
a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log
((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
+ c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
 - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(
d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*c
osh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*
(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)
^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) +
16*((4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 11*a^3*b + 1
0*a^2*b^2 - 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 2*a^4*b^2 + a^
```

```
3*b^3)*d*cosh(d*x + c)^6 + 6*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)*
sinh(d*x + c)^5 + (a^5*b - 2*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c)^6 + (4*a^6
- 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^4 + (15*(a^5*b - 2*a^4
*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*
b^3)*d)*sinh(d*x + c)^4 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cos
h(d*x + c)^2 + 4*(5*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^3 + (4*a^
6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (
15*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 + 6*(4*a^6 - 11*a^5*b +
10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^2 - (4*a^6 - 11*a^5*b + 10*a^4*b^2
- 3*a^3*b^3)*d)*sinh(d*x + c)^2 - (a^5*b - 2*a^4*b^2 + a^3*b^3)*d + 2*(3*(a
^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 11*a^5*b + 10*a^
4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^3 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a
^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(4*a^3*b - 7*a^2*b^2 + 3*a
*b^3)*cosh(d*x + c)^4 + 8*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^3 + 2*(4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^4 + 4*a^3*b
- 10*a^2*b^2 + 6*a*b^3 + 4*(4*a^4 - 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d
*x + c)^2 + 4*(4*a^4 - 11*a^3*b + 10*a^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 7*a^2
*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((4*a*b^2 - 3*b^3)*cosh(
d*x + c)^6 + 6*(4*a*b^2 - 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 -
 3*b^3)*sinh(d*x + c)^6 + (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + (
16*a^2*b - 24*a*b^2 + 9*b^3 + 15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(5*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (16*a^2*b - 24*a*b^2 +
9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*a*b^2 + 3*b^3 - (16*a^2*b - 24*a*
b^2 + 9*b^3)*cosh(d*x + c)^2 + (15*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^4 - 16*a
^2*b + 24*a*b^2 - 9*b^3 + 6*(16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 24
*a*b^2 + 9*b^3)*cosh(d*x + c)^3 - (16*a^2*b - 24*a*b^2 + 9*b^3)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*co
sh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(
a^2 - a*b)) + 8*((4*a^3*b - 7*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (4*a^4 -
 11*a^3*b + 10*a^2*b^2 - 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 2
*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^6 + 6*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*c
osh(d*x + c)*sinh(d*x + c)^5 + (a^5*b - 2*a^4*b^2 + a^3*b^3)*d*sinh(d*x + c
)^6 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^4 + (15*(
a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^2 + (4*a^6 - 11*a^5*b + 10*a^4
*b^2 - 3*a^3*b^3)*d)*sinh(d*x + c)^4 - (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a
^3*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x +
 c)^3 + (4*a^6 - 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d
*x + c)^3 + (15*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^4 + 6*(4*a^6
- 11*a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^2 - (4*a^6 - 11*a^5*b
+ 10*a^4*b^2 - 3*a^3*b^3)*d)*sinh(d*x + c)^2 - (a^5*b - 2*a^4*b^2 + a^3*b^3
)*d + 2*(3*(a^5*b - 2*a^4*b^2 + a^3*b^3)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 11*
a^5*b + 10*a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c)^3 - (4*a^6 - 11*a^5*b + 10*
a^4*b^2 - 3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.48533, size = 312, normalized size = 2.2

$$-\frac{\left(4\,ab-3\,b^{2}\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^{2}+ab}}\right)}{2\,\left(a^{3}d-a^{2}bd\right)\sqrt{-a^{2}+ab}}-\frac{4\,abe^{(4\,dx+4\,c)}-3\,b^{2}e^{(4\,dx+4\,c)}+8\,a^{2}e^{(2\,dx+2\,c)}-14\,abe^{(2\,dx+2\,c)}+6\,b^{2}e^{(2\,dx+2\,c)}+}{\left(a^{3}d-a^{2}bd\right)\left(be^{(6\,dx+6\,c)}+4\,ae^{(4\,dx+4\,c)}-3\,be^{(4\,dx+4\,c)}-4\,ae^{(2\,dx+2\,c)}+3\,be^{(2\,dx}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(4*a*b - 3*b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a
*b))/((a^3*d - a^2*b*d)*sqrt(-a^2 + a*b)) - (4*a*b*e^(4*d*x + 4*c) - 3*b^2*
e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 14*a*b*e^(2*d*x + 2*c) + 6*b^2*e^
(2*d*x + 2*c) + 2*a*b - 3*b^2)/((a^3*d - a^2*b*d)*(b*e^(6*d*x + 6*c) + 4*a*
e^(4*d*x + 4*c) - 3*b*e^(4*d*x + 4*c) - 4*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x
+ 2*c) - b))
```

3.49 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=161

$$\frac{b^{3/2}(5a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3d(a-b)^{3/2}}-\frac{b(a-2b)\cosh(c+dx)}{2a^2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}+\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{\coth(c+}{2ad\left(a+b\right.}$$

```
[Out] ((5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^3*(a
 - b)^(3/2)*d) + ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - ((a - 2*b)*
b*Cosh[c + d*x])/(2*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)) - (Coth[c +
d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.272916, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}(5a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3d(a-b)^{3/2}}-\frac{b(a-2b)\cosh(c+dx)}{2a^2d(a-b)\left(a+b\cosh^2(c+dx)-b\right)}+\frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d}-\frac{\coth(c+}{2ad\left(a+b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```


```
[Out] ((5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(2*a^3*(a
 - b)^(3/2)*d) + ((a + 4*b)*ArcTanh[Cosh[c + d*x]])/(2*a^3*d) - ((a - 2*b)*
b*Cosh[c + d*x])/(2*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)) - (Coth[c +
d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cosh[c + d*x]^2))
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+b+3bx^2}{\left(1-x^2\right)\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{2ad}$$

$$= -\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{(a-2b)b\cosh(c+dx)}{2a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)} + \frac{((5a-4b)b}{}$$

$$= \frac{(5a-4b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{2a^3(a-b)^{3/2}d} + \frac{(a+4b)\tanh^{-1}(\cosh(c+dx))}{2a^3d} - \frac{(a-2b)}{2a^2(a-b)d\left(a-\right.}$$

Mathematica [C] time = 1.4059, size = 350, normalized size = 2.17

$$\frac{\operatorname{csch}^3(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\frac{8ab^2\coth(c+dx)}{a-b} + \frac{4b^{3/2}(5a-4b)\operatorname{csch}(c+dx)(2a+b\cosh(2(c+dx))-b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{3/2}}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^3*((8*a*b^2*Coth[c + d*x])/(
a - b) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/
2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3/
```

```
2) + (4*(5*a - 4*b)*b^(3/2)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/
Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x])/(a - b)^(3/2) -
 a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[(c + d*x)/2]^2*Csch[c + d*x] - 4*(a
 + 4*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Log[Tanh[(c + d*x)/2]
] - a*(2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]*Sech[(c + d*x)/2]^2))/(
32*a^3*d*(b + a*Csch[c + d*x]^2)^2)
```

Maple [B] time = 0.079, size = 415, normalized size = 2.6

$$\frac{1}{8da^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{8da^2}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}-\frac{1}{2da^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2\frac{\ln(\tanh(1/2\,dx+c/2))\,b}{da^3}-\frac{b^2}{da^2\,(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/a^2-1/8/d/a^2/tanh(1/2*d*x+1/2*c)^2-1/2/d/a^2*l
n(tanh(1/2*d*x+1/2*c))-2/d/a^3*ln(tanh(1/2*d*x+1/2*c))*b-1/d/a^2*b^2/(tanh(
1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(
a-b)*tanh(1/2*d*x+1/2*c)^2+2/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/d
*b^2/a^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)/(a-b)+5/2/d/a^2*b^2/(a-b)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(
1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))-2/d*b^3/a^3/(a-b)/(a*b-b^2)^(1
/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(abe^{(7c)}-2\,b^2e^{(7c)}\right)e^{(7dx)}+\left(4\,a^2e^{(5c)}-5\,abe^{(5c)}+2\,b^2e^{(5c)}\right)e^{(5dx)}+\left(4\,a^2e^{(3c)}-5\,abe^{(3c)}+2}{a^3bd-a^2b^2d+\left(a^3bde^{(8c)}-a^2b^2de^{(8c)}\right)e^{(8dx)}+4\left(a^4de^{(6c)}-2\,a^3bde^{(6c)}+a^2b^2de^{(6c)}\right)e^{(6dx)}-2\left(4\,a^4de^{(4c)}-7\,a^3bde^{(4c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -((a*b*e^(7*c) - 2*b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 5*a*b*e^(5*c)
+ 2*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 5*a*b*e^(3*c) + 2*b^2*e^(3*c)
)*e^(3*d*x) + (a*b*e^c - 2*b^2*e^c)*e^(d*x))/(a^3*b*d - a^2*b^2*d + (a^3*b*
d*e^(8*c) - a^2*b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^4*d*e^(6*c) - 2*a^3*b*d*e^(
6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x) - 2*(4*a^4*d*e^(4*c) - 7*a^3*b*d*e^(4*c
) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(2*c) +
 a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*(a + 4*b)*log((e^(d*x + c) + 1)*e^(-c)
)/(a^3*d) - 1/2*(a + 4*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) + 8*integra
te(1/8*((5*a*b^2*e^(3*c) - 4*b^3*e^(3*c))*e^(3*d*x) - (5*a*b^2*e^c - 4*b^3*
e^c)*e^(d*x))/(a^4*b - a^3*b^2 + (a^4*b*e^(4*c) - a^3*b^2*e^(4*c))*e^(4*d*x
) + 2*(2*a^5*e^(2*c) - 3*a^4*b*e^(2*c) + a^3*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.66935, size = 19038, normalized size = 118.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - 2*a*b^2)*cosh(d*x + c)^7 + 28*(a^2*b - 2*a*b^2)*cosh(d*x
+ c)*sinh(d*x + c)^6 + 4*(a^2*b - 2*a*b^2)*sinh(d*x + c)^7 + 4*(4*a^3 - 5*a
^2*b + 2*a*b^2)*cosh(d*x + c)^5 + 4*(4*a^3 - 5*a^2*b + 2*a*b^2 + 21*(a^2*b
- 2*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b - 2*a*b^2)*cosh(
d*x + c)^3 + (4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4
*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 4*(35*(a^2*b - 2*a*b^2)*cosh
(d*x + c)^4 + 4*a^3 - 5*a^2*b + 2*a*b^2 + 10*(4*a^3 - 5*a^2*b + 2*a*b^2)*co
sh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b - 2*a*b^2)*cosh(d*x + c)^5 +
10*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 - 5*a^2*b + 2*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^2 - ((5*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8
*(5*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - 4*b^3)*sinh(d
*x + c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(5*a^2*b - 9*
a*b^2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7
*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 2*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^4 +
 2*(35*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 20*a^2*b + 31*a*b^2 - 12*b^3 + 3
0*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*
b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)
^3 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^
2 - 4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 -
 4*b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 +
5*a^2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - 9*
a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x +
c)^3 + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a
 - b))*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(
d*x + c)^4 - 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a +
 3*b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*s
inh(d*x + c) + 4*((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*
x + c)^2 + (a - b)*sinh(d*x + c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cos
h(d*x + c)^2 + a - b)*sinh(d*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b - 2
*a*b^2)*cosh(d*x + c) - 2*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(a
^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b + 3*a*b^2 -
4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^
6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^3
 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(
4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4 + 2*(35*(a^2*b + 3*a*b
^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*(a^3
 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a^2*
b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)
*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c))*sin
h(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3
)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(a^
3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^2 +
4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x +
 c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(a^3 + 2*a^2*b - 7
*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cos
h(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*((a^2*b + 3*a*b^2 - 4*b^3)*c
osh(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7
+ (a^2*b + 3*a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 +
4*b^3)*cosh(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*
a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4
```

```
*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*
sinh(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4 +
 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b
^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a^2
*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^
2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cos
h(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3 +
 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d
*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7 +
 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 2
5*a*b^2 + 12*b^3)*cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(
d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(7*(a^2
*b - 2*a*b^2)*cosh(d*x + c)^6 + 5*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)
^4 + a^2*b - 2*a*b^2 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sinh(
d*x + c))/((a^4*b - a^3*b^2)*d*cosh(d*x + c)^8 + 8*(a^4*b - a^3*b^2)*d*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^4*b - a^3*b^2)*d*sinh(d*x + c)^8 + 4*(a^5 -
2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(d*x +
c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(d*x + c)^6 - 2*(4*a^5 - 7*a^4*b +
3*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^3 + 3
*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b
- a^3*b^2)*d*cosh(d*x + c)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)
^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 - 2*a^4*b +
a^3*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^5 + 10*
(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^
6 + 15*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 - 3*(4*a^5 - 7*a^4*b + 3
*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(d*x + c)^2
+ (a^4*b - a^3*b^2)*d + 8*((a^4*b - a^3*b^2)*d*cosh(d*x + c)^7 + 3*(a^5 - 2
*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d*cosh(
d*x + c)^3 + (a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)), -1/
2*(2*(a^2*b - 2*a*b^2)*cosh(d*x + c)^7 + 14*(a^2*b - 2*a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^6 + 2*(a^2*b - 2*a*b^2)*sinh(d*x + c)^7 + 2*(4*a^3 - 5*a^2*b
 + 2*a*b^2)*cosh(d*x + c)^5 + 2*(4*a^3 - 5*a^2*b + 2*a*b^2 + 21*(a^2*b - 2*
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(a^2*b - 2*a*b^2)*cosh(d*x
+ c)^3 + (4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(4*
a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 2*(35*(a^2*b - 2*a*b^2)*cosh(d*x
 + c)^4 + 4*a^3 - 5*a^2*b + 2*a*b^2 + 10*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(a^2*b - 2*a*b^2)*cosh(d*x + c)^5 + 10*(
4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^3 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*
cosh(d*x + c))*sinh(d*x + c)^2 - ((5*a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(5*
a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - 4*b^3)*sinh(d*x +
 c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(5*a^2*b - 9*a*b^
2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*
a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c
))*sinh(d*x + c)^5 - 2*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^4 + 2*(
35*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 20*a^2*b + 31*a*b^2 - 12*b^3 + 30*(5
*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2
- 4*b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^3 -
 (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^2 -
4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 - 4*b
^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 5*a^
2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - 9*a*b^
2 + 4*b^3)*cosh(d*x + c)^5 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c)^3
 + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/(a - b)
)*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x + c) + sinh(d*x + c))) + ((5*a*b^2 -
 4*b^3)*cosh(d*x + c)^8 + 8*(5*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7
```

```
 + (5*a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^6 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3 + 7*(5*a*b^2 - 4*b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b
- 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(20*a^2*b - 31*a*b^2
+ 12*b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 20*a^
2*b + 31*a*b^2 - 12*b^3 + 30*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*s
inh(d*x + c)^4 + 8*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - 9*a
*b^2 + 4*b^3)*cosh(d*x + c)^3 - (20*a^2*b - 31*a*b^2 + 12*b^3)*cosh(d*x + c
))*sinh(d*x + c)^3 + 5*a*b^2 - 4*b^3 + 4*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^2 + 4*(7*(5*a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - 9*a*b^2
+ 4*b^3)*cosh(d*x + c)^4 + 5*a^2*b - 9*a*b^2 + 4*b^3 - 3*(20*a^2*b - 31*a*b
^2 + 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - 4*b^3)*cosh(d
*x + c)^7 + 3*(5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (20*a^2*b - 31*
a*b^2 + 12*b^3)*cosh(d*x + c)^3 + (5*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)
)*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d
*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (
3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b) + 2*(a^2
*b - 2*a*b^2)*cosh(d*x + c) - ((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^8 +
8*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b + 3*a*b^
2 - 4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x +
 c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b + 3*a*b^2 - 4*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x +
c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 -
 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4 + 2*(35*(a^2*b + 3
*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a*b^2 - 12*b^3 + 30*
(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(
a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a^2*b - 7*a*b^2 + 4*
b^3)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c))
*sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4
*b^3)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15
*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3 + 2*a^2*b - 7*a*b^
2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(a^3 + 2*a^2*b
 - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)
*cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + ((a^2*b + 3*a*b^2 - 4*b^3)
*cosh(d*x + c)^8 + 8*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^
7 + (a^2*b + 3*a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(a^3 + 2*a^2*b - 7*a*b^2
+ 4*b^3)*cosh(d*x + c)^6 + 4*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3 + 7*(a^2*b +
3*a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b + 3*a*b^2 -
 4*b^3)*cosh(d*x + c)^3 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)
)*sinh(d*x + c)^5 - 2*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh(d*x + c)^4
 + 2*(35*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^4 - 4*a^3 - 9*a^2*b + 25*a
*b^2 - 12*b^3 + 30*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^4 + 8*(7*(a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(a^3 + 2*a
^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^3 - (4*a^3 + 9*a^2*b - 25*a*b^2 + 12*
b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a^2*b + 3*a*b^2 - 4*b^3 + 4*(a^3 + 2*
a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(a^2*b + 3*a*b^2 - 4*b^3)*c
osh(d*x + c)^6 + 15*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + a^3
 + 2*a^2*b - 7*a*b^2 + 4*b^3 - 3*(4*a^3 + 9*a^2*b - 25*a*b^2 + 12*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b + 3*a*b^2 - 4*b^3)*cosh(d*x + c)^7
 + 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cosh(d*x + c)^5 - (4*a^3 + 9*a^2*b -
 25*a*b^2 + 12*b^3)*cosh(d*x + c)^3 + (a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cos
h(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(a
^2*b - 2*a*b^2)*cosh(d*x + c)^6 + 5*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x +
c)^4 + a^2*b - 2*a*b^2 + 3*(4*a^3 - 5*a^2*b + 2*a*b^2)*cosh(d*x + c)^2)*sin
h(d*x + c))/((a^4*b - a^3*b^2)*d*cosh(d*x + c)^8 + 8*(a^4*b - a^3*b^2)*d*co
sh(d*x + c)*sinh(d*x + c)^7 + (a^4*b - a^3*b^2)*d*sinh(d*x + c)^8 + 4*(a^5
- 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(d*x
```

```
+ c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(d*x + c)^6 - 2*(4*a^5 - 7*a^4*b
+ 3*a^3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^3 +
 3*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*
b - a^3*b^2)*d*cosh(d*x + c)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x +
c)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d)*sinh(d*x + c)^4 + 4*(a^5 - 2*a^4*b
+ a^3*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c)^5 + 1
0*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^
2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - a^3*b^2)*d*cosh(d*x + c
)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^4 - 3*(4*a^5 - 7*a^4*b +
 3*a^3*b^2)*d*cosh(d*x + c)^2 + (a^5 - 2*a^4*b + a^3*b^2)*d)*sinh(d*x + c)^
2 + (a^4*b - a^3*b^2)*d + 8*((a^4*b - a^3*b^2)*d*cosh(d*x + c)^7 + 3*(a^5 -
 2*a^4*b + a^3*b^2)*d*cosh(d*x + c)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*d*cos
h(d*x + c)^3 + (a^5 - 2*a^4*b + a^3*b^2)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.50 $\int \frac{\mathrm{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=174

$$\frac{b^2(6a-5b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a-b)^{3/2}}+\frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3d(a-b)}-\frac{(2a-5b)\coth^3(c+dx)}{6a^2d(a-b)}-\frac{b\,\mathrm{csch}^3(c+dx}{2ad(a-b)\left(a-(a\right.}$$

```
[Out] ((6*a - 5*b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(
a - b)^(3/2)*d) + ((2*a^2 + a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(a - b)*d) -
 ((2*a - 5*b)*Coth[c + d*x]^3)/(6*a^2*(a - b)*d) - (b*Csch[c + d*x]^3*Sech[
c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.206368, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 468, 570, 208}

$$\frac{b^2(6a-5b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a-b)^{3/2}}+\frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3d(a-b)}-\frac{(2a-5b)\coth^3(c+dx)}{6a^2d(a-b)}-\frac{b\,\mathrm{csch}^3(c+dx}{2ad(a-b)\left(a-(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - 5*b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(
a - b)^(3/2)*d) + ((2*a^2 + a*b - 5*b^2)*Coth[c + d*x])/(2*a^3*(a - b)*d) -
 ((2*a - 5*b)*Coth[c + d*x]^3)/(6*a^2*(a - b)*d) - (b*Csch[c + d*x]^3*Sech[
c + d*x])/(2*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\text{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\,\text{csch}^3(c+dx)\text{sech}(c+dx)}{2a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)\left(2a-5b+(-2a+b)x^2\right)}{x^4\left(a+(-a+b)x^2\right)}\,dx, x, \tanh(c+\right.}{2a(a-b)d}$$

$$= -\frac{b\,\text{csch}^3(c+dx)\text{sech}(c+dx)}{2a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \left(\frac{2a-5b}{ax^4} + \frac{-2a^2-ab+5b^2}{a^2x^2} + \frac{(6a-5b)b^2}{a^2\left(a-(a-b)x^2\right)}\right)\right.}{2a(a-b)d}$$

$$= \frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3(a-b)d} - \frac{(2a-5b)\coth^3(c+dx)}{6a^2(a-b)d} - \frac{b\,\text{csch}^3(c+dx)\text{sech}(c+}{2a(a-b)d\left(a-(a-b)\tanh^2(\right.}$$

$$= \frac{(6a-5b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a-b)^{3/2}d} + \frac{\left(2a^2+ab-5b^2\right)\coth(c+dx)}{2a^3(a-b)d} - \frac{(2a-5b)\coth^3(c}{6a^2(a-b)d}$$

Mathematica [A] time = 1.25862, size = 210, normalized size = 1.21

$$\frac{\text{csch}^4(c+dx)(2a+b\cosh(2(c+dx))-b)\left(-2a^{3/2}\coth(c+dx)\text{csch}^2(c+dx)(2a+b\cosh(2(c+dx))-b)-\frac{3\sqrt{a}b^3\sinh(2(c+}{a-b}\right.}{24a^{7/2}d\left(a\,\text{csch}^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^4*((3*(6*a - 5*b)*b^2*ArcTan
h[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c + d*x)]))/(a
- b)^(3/2) + 4*Sqrt[a]*(a + 3*b)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d
*x] - 2*a^(3/2)*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d*x]*Csch[c + d*x]
^2 - (3*Sqrt[a]*b^3*Sinh[2*(c + d*x)])/(a - b)))/(24*a^(7/2)*d*(b + a*Csch[
c + d*x]^2)^2)
```

Maple [B] time = 0.089, size = 890, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/24/d/a^2*tanh(1/2*d*x+1/2*c)^3+3/8/d/a^2*tanh(1/2*d*x+1/2*c)+1/d/a^3*tan
h(1/2*d*x+1/2*c)*b-1/24/d/a^2/tanh(1/2*d*x+1/2*c)^3+3/8/d/a^2/tanh(1/2*d*x+
```

```
1/2*c)+1/d/a^3/tanh(1/2*d*x+1/2*c)*b-1/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/
2*c)^3-1/d*b^3/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)/(a-b)*tanh(1/2*d*x+1/2*c)+3/d/a^2*b^2/(a-b)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))-3/d/a^2*b^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))-3/d/a^2*b^2/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(
a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/d/a^2*b^3/(a-
b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*
d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/2/d*b^3/a^3/(a-b)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))+5/2/d*b^4/a^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2))+5/2/d*b^3/a^3/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*ar
ctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+5/2/d*b^4/
a^3/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.16269, size = 16662, normalized size = 95.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^8 + 96*(6*a^3*b^
2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(6*a^3*b^2 - 1
1*a^2*b^3 + 5*a*b^4)*sinh(d*x + c)^8 + 24*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^
3 - 10*a*b^4)*cosh(d*x + c)^6 + 24*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*
a*b^4 + 14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 48*(14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^3 + 3*(6*a^4*
b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 16
*a^4*b + 16*a^3*b^2 - 92*a^2*b^3 + 60*a*b^4 - 8*(24*a^5 - 14*a^4*b - 89*a^3
*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^4 - 8*(24*a^5 - 14*a^4*b - 89*
a^3*b^2 + 124*a^2*b^3 - 45*a*b^4 - 105*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*c
osh(d*x + c)^4 - 45*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 32*(21*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d
*x + c)^5 + 15*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)
^3 - (24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 8*(8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^
4)*cosh(d*x + c)^2 + 8*(42*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)
^6 + 8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4 + 45*(6*a^4*b - 2
3*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^4 - 6*(24*a^5 - 14*a^4*b -
```

89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((6*a*b^3 - 5*b^4)*cosh(d*x + c)^10 + 10*(6*a*b^3 - 5*b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (6*a*b^3 - 5*b^4)*sinh(d*x + c)^10 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4 + 45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^7 - 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^6 + 2*(105*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 36*a^2*b^2 + 60*a*b^3 - 25*b^4 + 14*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 14*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - 3*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 2*(105*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 35*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 60*a*b^3 + 25*b^4 - 15*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 6*a*b^3 + 5*b^4 + 8*(15*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^5 - 5*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^3 + (36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^2 + (45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^8 + 28*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^6 - 30*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^4 - 24*a^2*b^2 + 50*a*b^3 - 25*b^4 + 12*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^9 + 4*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^7 - 6*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^3 - (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*(6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^7 + 9*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^5 - 2*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^3 + (8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 10*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6*b - 2*a^5*b^2 + a^4*b^3)*d*sinh(d*x + c)^10 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^8 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^8 - 2*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^3 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^4 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 - (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 4*(63*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 - 3*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^6 + 35*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 - 15*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^4 - (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(15*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 7*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^5 - 5*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)

```
^8 + 28*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 - 30*
(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 12*(6*a^7 -
 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 - (4*a^7 - 13*a^6*b +
 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^2 - (a^6*b - 2*a^5*b^2 + a^4*b^3)
*d + 2*(5*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^9 + 4*(4*a^7 - 13*a
^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^7 - 6*(6*a^7 - 17*a^6*b + 16
*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^5 + 4*(6*a^7 - 17*a^6*b + 16*a^5*b^2
- 5*a^4*b^3)*d*cosh(d*x + c)^3 - (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3
)*d*cosh(d*x + c))*sinh(d*x + c)), 1/6*(6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4
)*cosh(d*x + c)^8 + 48*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)*sin
h(d*x + c)^7 + 6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*sinh(d*x + c)^8 + 12*(6
*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^6 + 12*(6*a^4*b
- 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4 + 14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(14*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b
^4)*cosh(d*x + c)^3 + 3*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh
(d*x + c))*sinh(d*x + c)^5 + 8*a^4*b + 8*a^3*b^2 - 46*a^2*b^3 + 30*a*b^4 -
4*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^4
 - 4*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4 - 105*(6*a^3*
b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^4 - 45*(6*a^4*b - 23*a^3*b^2 + 27
*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(21*(6*a^3*b^2 -
 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^5 + 15*(6*a^4*b - 23*a^3*b^2 + 27*a^2*
b^3 - 10*a*b^4)*cosh(d*x + c)^3 - (24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2
*b^3 - 45*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^5 - 2*a^4*b - 47*a
^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*cosh(d*x + c)^2 + 4*(42*(6*a^3*b^2 - 11*a^2
*b^3 + 5*a*b^4)*cosh(d*x + c)^6 + 8*a^5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3
 - 30*a*b^4 + 45*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x +
c)^4 - 6*(24*a^5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^2 - 3*((6*a*b^3 - 5*b^4)*cosh(d*x + c)^10 + 10*(6*a*
b^3 - 5*b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (6*a*b^3 - 5*b^4)*sinh(d*x + c
)^10 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 - 50*
a*b^3 + 25*b^4 + 45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*
(15*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*co
sh(d*x + c))*sinh(d*x + c)^7 - 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x
+ c)^6 + 2*(105*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^4 - 36*a^2*b^2 + 60*a*b^3 -
 25*b^4 + 14*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 4*(63*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^5 + 14*(24*a^2*b^2 - 50*a*b^3 +
 25*b^4)*cosh(d*x + c)^3 - 3*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^4 + 2*
(105*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^6 + 35*(24*a^2*b^2 - 50*a*b^3 + 25*b^4
)*cosh(d*x + c)^4 + 36*a^2*b^2 - 60*a*b^3 + 25*b^4 - 15*(36*a^2*b^2 - 60*a*
b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 6*a*b^3 + 5*b^4 + 8*(15*(6
*a*b^3 - 5*b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d
*x + c)^5 - 5*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^3 + (36*a^2*b^
2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - (24*a^2*b^2 - 50*a*
b^3 + 25*b^4)*cosh(d*x + c)^2 + (45*(6*a*b^3 - 5*b^4)*cosh(d*x + c)^8 + 28*
(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^6 - 30*(36*a^2*b^2 - 60*a*b^
3 + 25*b^4)*cosh(d*x + c)^4 - 24*a^2*b^2 + 50*a*b^3 - 25*b^4 + 12*(36*a^2*b
^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 -
5*b^4)*cosh(d*x + c)^9 + 4*(24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x + c)^7
 - 6*(36*a^2*b^2 - 60*a*b^3 + 25*b^4)*cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 60*
a*b^3 + 25*b^4)*cosh(d*x + c)^3 - (24*a^2*b^2 - 50*a*b^3 + 25*b^4)*cosh(d*x
 + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b
*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b
)/(a^2 - a*b)) + 8*(6*(6*a^3*b^2 - 11*a^2*b^3 + 5*a*b^4)*cosh(d*x + c)^7 +
9*(6*a^4*b - 23*a^3*b^2 + 27*a^2*b^3 - 10*a*b^4)*cosh(d*x + c)^5 - 2*(24*a^
5 - 14*a^4*b - 89*a^3*b^2 + 124*a^2*b^3 - 45*a*b^4)*cosh(d*x + c)^3 + (8*a^
5 - 2*a^4*b - 47*a^3*b^2 + 71*a^2*b^3 - 30*a*b^4)*cosh(d*x + c))*sinh(d*x +
 c))/((a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^10 + 10*(a^6*b - 2*a^5*
b^2 + a^4*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^6*b - 2*a^5*b^2 + a^4*b
```

```
^3)*d*sinh(d*x + c)^10 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh
(d*x + c)^8 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^2 + (4*a^7
- 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^8 - 2*(6*a^7 - 17*a^6
*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 2*a^5*b^2 +
 a^4*b^3)*d*cosh(d*x + c)^3 + (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d
*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*co
sh(d*x + c)^4 + 14*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x +
 c)^2 - (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^6 + 2*
(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 4*(63*(a^6*
b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^5 + 14*(4*a^7 - 13*a^6*b + 14*a^5*
b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 - 3*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a
^4*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6*b - 2*a^5*b^2 + a^4*
b^3)*d*cosh(d*x + c)^6 + 35*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*c
osh(d*x + c)^4 - 15*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x
+ c)^2 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^4 - (
4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 + 8*(15*(a^6*b
 - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^7 + 7*(4*a^7 - 13*a^6*b + 14*a^5*b^
2 - 5*a^4*b^3)*d*cosh(d*x + c)^5 - 5*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4
*b^3)*d*cosh(d*x + c)^3 + (6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (45*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x
+ c)^8 + 28*(4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^6 -
 30*(6*a^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^4 + 12*(6*a
^7 - 17*a^6*b + 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^2 - (4*a^7 - 13*a^6
*b + 14*a^5*b^2 - 5*a^4*b^3)*d)*sinh(d*x + c)^2 - (a^6*b - 2*a^5*b^2 + a^4*
b^3)*d + 2*(5*(a^6*b - 2*a^5*b^2 + a^4*b^3)*d*cosh(d*x + c)^9 + 4*(4*a^7 -
13*a^6*b + 14*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^7 - 6*(6*a^7 - 17*a^6*b
+ 16*a^5*b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^5 + 4*(6*a^7 - 17*a^6*b + 16*a^5*
b^2 - 5*a^4*b^3)*d*cosh(d*x + c)^3 - (4*a^7 - 13*a^6*b + 14*a^5*b^2 - 5*a^4
*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.50775, size = 301, normalized size = 1.73

$$\frac{\left(6\,ab^2-5\,b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{2\left(a^4d-a^3bd\right)\sqrt{-a^2+ab}}+\frac{2\,ab^2e^{(2\,dx+2\,c)}-b^3e^{(2\,dx+2\,c)}+b^3}{\left(a^4d-a^3bd\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)}+\frac{4\left(3\,be^{(4\,dx+4\,c)}-3\,ae\right.}{3\,a^3c}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(6*a*b^2 - 5*b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 +
a*b))/((a^4*d - a^3*b*d)*sqrt(-a^2 + a*b)) + (2*a*b^2*e^(2*d*x + 2*c) - b^3
*e^(2*d*x + 2*c) + b^3)/((a^4*d - a^3*b*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*
x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)) + 4/3*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*
d*x + 2*c) - 6*b*e^(2*d*x + 2*c) + a + 3*b)/(a^3*d*(e^(2*d*x + 2*c) - 1)^3)
```

3.51 $\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=124

$$\frac{\tanh^3(c+dx)}{4d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{3\tanh(c+dx)}{8d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}d(a-b)^{5/2}}$$

```
[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^(5/2)*d
) + Tanh[c + d*x]^3/(4*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - (3*Tanh
[c + d*x])/(8*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.123065, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3187, 288, 208}

$$\frac{\tanh^3(c+dx)}{4d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}-\frac{3\tanh(c+dx)}{8d(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}d(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a - b)^(5/2)*d
) + Tanh[c + d*x]^3/(4*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2)^2) - (3*Tanh
[c + d*x])/(8*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{3\,\text{Subst}\left(\int \frac{x^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4(a-b)d}$$

$$= \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{3\tanh(c+dx)}{8(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{3\,\text{Subst}}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}(a-b)^{5/2}d} + \frac{\tanh^3(c+dx)}{4(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{3\tanh(}{8(a-b)^2d\left(a-(a-\right.}$$

Mathematica [A] time = 1.3691, size = 104, normalized size = 0.84

$$\frac{\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}} + \frac{\sinh(2(c+dx))((2a-5b)\cosh(2(c+dx))-8a+5b)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)) +
 ((-8*a + 5*b + (2*a - 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)^
2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Maple [B] time = 0.052, size = 768, normalized size = 6.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+11/4/d/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a-5/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^5*b+11/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*a-5/d/(tanh(1/
2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(
1/2*d*x+1/2*c)+3/8/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d/(a
^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b-3/8/d/(a^2-2
*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
```

```
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.94201, size = 11786, normalized size = 95.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^6 + 2
4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)*sinh(d*x + c)
^5 + 4*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*sinh(d*x + c)^6 + 8*a^
3*b^2 - 28*a^2*b^3 + 20*a*b^4 + 4*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2
*b^3 + 15*a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*
a^2*b^3 + 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(
d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*
a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15
*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^4*b - 40*a^3*b^2 + 47*a^2*b
^3 - 15*a*b^4)*cosh(d*x + c)^2 + 4*(8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*
a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^4 +
6*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 - 3*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x +
 c)^7 + b^4*sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(7*b^4*
cosh(d*x + c)^2 + 2*a*b^3 - b^4)*sinh(d*x + c)^6 + 8*(7*b^4*cosh(d*x + c)^3
 + 3*(2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^
3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*b^4*cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^
3 + 3*b^4 + 30*(2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + b^4 + 8*(
7*b^4*cosh(d*x + c)^5 + 10*(2*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 8
*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*cosh(d*x
 + c)^2 + 4*(7*b^4*cosh(d*x + c)^6 + 15*(2*a*b^3 - b^4)*cosh(d*x + c)^4 + 2
*a*b^3 - b^4 + 3*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^2*
b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a*b^3 - b^4)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*
sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2
*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^
2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4
*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 +
 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
+ c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d
*x + c))*sinh(d*x + c) + b)) + 8*(3*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*
```

```
a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 +
15*a*b^4)*cosh(d*x + c)^3 + (8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)*
cosh(d*x + c))*sinh(d*x + c))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^8 + 8*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x +
c)*sinh(d*x + c)^7 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*sinh(d*x +
 c)^8 + 4*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*
x + c)^6 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^2
 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d)*sinh(d*x + c)
^6 + 2*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a
*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6
 - a*b^7)*d*cosh(d*x + c)^4 + 30*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*
b^6 + a*b^7)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*
a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 7*a^4*b
^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - 3*a
^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^5 + 10*(2*a^5*b^3 - 7*a^4*b^4 +
 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 32*a^5*b^3
 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^6
 + 15*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x +
c)^4 + 3*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3
*a*b^7)*d*cosh(d*x + c)^2 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6
+ a*b^7)*d)*sinh(d*x + c)^2 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d +
 8*((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^7 + 3*(2*a^5*
b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^5 + (8*a^6
*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(
d*x + c)^3 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh
(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a
*b^4)*cosh(d*x + c)^6 + 12*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a*b^
4)*sinh(d*x + c)^6 + 4*a^3*b^2 - 14*a^2*b^3 + 10*a*b^4 + 2*(16*a^5 - 72*a^4
*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(d*x + c)^4 + 2*(16*a^5 - 72*
a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21
*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(8*a^4*b - 24*a
^3*b^2 + 21*a^2*b^3 - 5*a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 72*a^4*b + 102*a
^3*b^2 - 61*a^2*b^3 + 15*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^4*b
 - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)*cosh(d*x + c)^2 + 2*(8*a^4*b - 40*a^
3*b^2 + 47*a^2*b^3 - 15*a*b^4 + 15*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*a
*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 + 1
5*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^4*cosh(d*x + c)^8 + 8*b^4*
cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*cos
h(d*x + c)^6 + 4*(7*b^4*cosh(d*x + c)^2 + 2*a*b^3 - b^4)*sinh(d*x + c)^6 +
8*(7*b^4*cosh(d*x + c)^3 + 3*(2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5
 + 2*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*b^4*cosh(d*x + c
)^4 + 8*a^2*b^2 - 8*a*b^3 + 3*b^4 + 30*(2*a*b^3 - b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*(2*a*b^3 - b^4)*cosh(d*x
 + c)^3 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(2*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*b^4*cosh(d*x + c)^6 + 15*(2*a*b^3 -
b^4)*cosh(d*x + c)^4 + 2*a*b^3 - b^4 + 3*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*c
osh(d*x + c)^5 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a*b^3 -
 b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*s
qrt(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(8*a^4*b - 24*a^3*b^2 + 21*a^2*b^3 - 5*
a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 72*a^4*b + 102*a^3*b^2 - 61*a^2*b^3 +
15*a*b^4)*cosh(d*x + c)^3 + (8*a^4*b - 40*a^3*b^2 + 47*a^2*b^3 - 15*a*b^4)*
cosh(d*x + c))*sinh(d*x + c))/((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^8 + 8*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x +
```

```
c)*sinh(d*x + c)^7 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*sinh(d*x +
 c)^8 + 4*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*
x + c)^6 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^2
 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d)*sinh(d*x + c)
^6 + 2*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a
*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*
cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6
- a*b^7)*d*cosh(d*x + c)^4 + 30*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*
b^6 + a*b^7)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*
a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 7*a^4*b
^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - 3*a
^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^5 + 10*(2*a^5*b^3 - 7*a^4*b^4 +
 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 32*a^5*b^3
 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(d*x + c))*sinh(d*
x + c)^3 + 4*(7*(a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^6
 + 15*(2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x +
c)^4 + 3*(8*a^6*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3
*a*b^7)*d*cosh(d*x + c)^2 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6
+ a*b^7)*d)*sinh(d*x + c)^2 + (a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d +
 8*((a^4*b^4 - 3*a^3*b^5 + 3*a^2*b^6 - a*b^7)*d*cosh(d*x + c)^7 + 3*(2*a^5*
b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh(d*x + c)^5 + (8*a^6
*b^2 - 32*a^5*b^3 + 51*a^4*b^4 - 41*a^3*b^5 + 17*a^2*b^6 - 3*a*b^7)*d*cosh(
d*x + c)^3 + (2*a^5*b^3 - 7*a^4*b^4 + 9*a^3*b^5 - 5*a^2*b^6 + a*b^7)*d*cosh
(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.56499, size = 386, normalized size = 3.11

$$\frac{3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{8\left(a^2d-2\,abd+b^2d\right)\sqrt{-a^2+ab}}-\frac{8\,a^2be^{(6\,dx+6\,c)}-16\,ab^2e^{(6\,dx+6\,c)}+5\,b^3e^{(6\,dx+6\,c)}+16\,a^3e^{(4\,dx+4\,c)}-56\,a^2be^{(4\,dx+4\,c)}+\cdots}{4\left(a^2b^2d-2\,ab^3d+b^4d\right)\left(be^{(\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^2*d - 2*
a*b*d + b^2*d)*sqrt(-a^2 + a*b)) - 1/4*(8*a^2*b*e^(6*d*x + 6*c) - 16*a*b^2*
e^(6*d*x + 6*c) + 5*b^3*e^(6*d*x + 6*c) + 16*a^3*e^(4*d*x + 4*c) - 56*a^2*b
*e^(4*d*x + 4*c) + 46*a*b^2*e^(4*d*x + 4*c) - 15*b^3*e^(4*d*x + 4*c) + 8*a^
2*b*e^(2*d*x + 2*c) - 32*a*b^2*e^(2*d*x + 2*c) + 15*b^3*e^(2*d*x + 2*c) + 2
*a*b^2 - 5*b^3)/((a^2*b^2*d - 2*a*b^3*d + b^4*d)*(b*e^(4*d*x + 4*c) + 4*a*e
^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2)
```

3.52 $\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=135

$$\frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8b^{3/2}d(a-b)^{5/2}}+\frac{(a-4b)\cosh(c+dx)}{8bd(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{a\cosh(c+dx)}{4bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}$$

```
[Out] ((a - 4*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*b^
(3/2)*d) - (a*Cosh[c + d*x])/(4*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2)^2)
+ ((a - 4*b)*Cosh[c + d*x])/(8*(a - b)^2*b*d*(a - b + b*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.141105, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 385, 199, 205}

$$\frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8b^{3/2}d(a-b)^{5/2}}+\frac{(a-4b)\cosh(c+dx)}{8bd(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{a\cosh(c+dx)}{4bd(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 4*b)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*b^
(3/2)*d) - (a*Cosh[c + d*x])/(4*(a - b)*b*d*(a - b + b*Cosh[c + d*x]^2)^2)
+ ((a - 4*b)*Cosh[c + d*x])/(8*(a - b)^2*b*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+bx^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{(a-4b)\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+ \ldots\right.}{4(a-b)bd}$$

$$= -\frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{(a-4b)\cosh(c+dx)}{8(a-b)^2bd\left(a-b+b\cosh^2(c+dx)\right)} + \frac{(a- \ldots}{\ldots}$$

$$= \frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}b^{3/2}d} - \frac{a\cosh(c+dx)}{4(a-b)bd\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{(a-4 \ldots}{8(a-b)^2bd\,(a \ldots}$$

Mathematica [C] time = 1.22242, size = 170, normalized size = 1.26

$$\frac{\frac{2\sqrt{b}\cosh(c+dx)\left(-2a^2+b(a-4b)\cosh(2(c+dx))-5ab+4b^2\right)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2}+\frac{(a-4b)\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{(a-b)^{5/2}}}{8b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (((a - 4*b)*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] +
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/(a - b)^(5/2)
 + (2*Sqrt[b]*Cosh[c + d*x]*(-2*a^2 - 5*a*b + 4*b^2 + (a - 4*b)*b*Cosh[2*(c
 + d*x)]))/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*b^(3/2)*d)
```

Maple [B] time = 0.042, size = 961, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/d/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^4+1/2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+2/d/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*
```

```
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^4+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
2*a^2+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*a-4/d/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*
d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a^2/b/(a^2-2*a*b+b^2)-1/2/d/(
tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b
+a)^2*a/(a^2-2*a*b+b^2)+1/8/d/b/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*
(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))*a-1/2/d/(a^2-2*a*b+b^2
)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^
(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(abe^{(7\,c)} - 4\,b^2e^{(7\,c)}\right)e^{(7\,dx)} - \left(4\,a^2e^{(5\,c)} + 9\,abe^{(5\,c)} -\right.}{4\left(a^2b^3d - 2\,ab^4d + b^5d + \left(a^2b^3de^{(8\,c)} - 2\,ab^4de^{(8\,c)} + b^5de^{(8\,c)}\right)e^{(8\,dx)} + 4\left(2\,a^3b^2de^{(6\,c)} - 5\,a^2b^3de^{(6\,c)} + 4\,ab^4de^{(6\,c)} - b^5de^{(6\,c}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a*b*e^(7*c) - 4*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) + 9*a*b*e^(5*
c) - 4*b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) + 9*a*b*e^(3*c) - 4*b^2*e^(3
*c))*e^(3*d*x) + (a*b*e^c - 4*b^2*e^c)*e^(d*x))/(a^2*b^3*d - 2*a*b^4*d + b^
5*d + (a^2*b^3*d*e^(8*c) - 2*a*b^4*d*e^(8*c) + b^5*d*e^(8*c))*e^(8*d*x) + 4
*(2*a^3*b^2*d*e^(6*c) - 5*a^2*b^3*d*e^(6*c) + 4*a*b^4*d*e^(6*c) - b^5*d*e^(
6*c))*e^(6*d*x) + 2*(8*a^4*b*d*e^(4*c) - 24*a^3*b^2*d*e^(4*c) + 27*a^2*b^3*
d*e^(4*c) - 14*a*b^4*d*e^(4*c) + 3*b^5*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b^2*
d*e^(2*c) - 5*a^2*b^3*d*e^(2*c) + 4*a*b^4*d*e^(2*c) - b^5*d*e^(2*c))*e^(2*d
*x)) + 1/8*integrate(2*((a*e^(3*c) - 4*b*e^(3*c))*e^(3*d*x) - (a*e^c - 4*b*
e^c)*e^(d*x))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^(4*c) - 2*a*b^3*e^(4*c)
 + b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^3*b*e^(2*c) - 5*a^2*b^2*e^(2*c) + 4*a*b^
3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.96323, size = 14074, normalized size = 104.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^7 + 28*(a^2*b^2 - 5*a*b^
3 + 4*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(a^2*b^2 - 5*a*b^3 + 4*b^4)*si
nh(d*x + c)^7 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^5
- 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4 - 21*(a^2*b^2 - 5*a*b^3 + 4*b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh
(d*x + c)^3 - (4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(
d*x + c)^4 - 4*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + 4
*(35*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 13
*a*b^3 - 4*b^4 - 10*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 4*(21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 10
```

```
*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 - 3*(4*a^3*b + 5*
a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 - 4*b^
3)*cosh(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b
^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6
 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b - 9*a*b^2 + 4
*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*
b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c)^4 + 8*a^3 - 40*a
^2*b + 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - 9*a*
b^2 + 4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(
d*x + c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*c
osh(d*x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b - 9*a*b
^2 + 4*b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 + 3*(8*a^3 - 40*a^2
*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 - 4*b^
3)*cosh(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (8*a^3
 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2*b - 9*a*b^2 + 4*b
^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b + b^2)*log((b*cosh(d*x + c)^4 +
 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + 4*(cosh(d*x + c)
^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2
 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) + b)/(b*cosh(d*x + c)
^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*co
sh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*co
sh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(a^2*b^2 -
 5*a*b^3 + 4*b^4)*cosh(d*x + c) + 4*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x
 + c)^6 - 5*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^4 + a^2*
b^2 - 5*a*b^3 + 4*b^4 - 3*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x +
 c)^8 + 8*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)*sinh(d*x +
c)^7 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*sinh(d*x + c)^8 + 4*(2*a^4*b
^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b
^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4
+ 9*a^2*b^5 - 5*a*b^6 + b^7)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 32*a^4*b^3
 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a^
3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - 7*a^3
*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*
(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 - 7
*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (8*a^5*b^2 - 32*a
^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d)*sinh(d*x + c)^4 + 4
*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + 8*
(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3
- 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^3 + (8*a^5*b^2 - 3
2*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 4*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^
6 + 15*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^
4 + 3*(8*a^5*b^2 - 32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)
*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d)
*sinh(d*x + c)^2 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d + 8*((a^3*b^4 -
3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - 7*a^3*b^4 + 9
*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 32*a^4*b^3 + 51*
a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^3 + (2*a^4*b^3 - 7
*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*
(2*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^7 + 14*(a^2*b^2 - 5*a*b^3 + 4*
b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b^2 - 5*a*b^3 + 4*b^4)*sinh(d*x
 + c)^7 - 2*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 2*(4
*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4 - 21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x +
```

```
c)^3 - (4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x +
c)^4 - 2*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 + 2*(35*(
a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^4 - 4*a^3*b - 5*a^2*b^2 + 13*a*b^3
 - 4*b^4 - 10*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^3 + 2*(21*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*x + c)^5 - 10*(4*a^
3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^3 - 3*(4*a^3*b + 5*a^2*b^
2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 - 4*b^3)*cos
h(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - 4
*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^6 + 4*(
2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*c
osh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c)^4 + 8*a^3 - 40*a^2*b +
 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b - 9*a*b^2 +
4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x +
c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*
x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b - 9*a*b^2 + 4
*b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 + 3*(8*a^3 - 40*a^2*b + 3
5*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 - 4*b^3)*cos
h(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^5 + (8*a^3 - 40*
a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2*b - 9*a*b^2 + 4*b^3)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3
+ 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(
d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)
) - ((a*b^2 - 4*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 - 4*b^3)*cosh(d*x + c)*sinh
(d*x + c)^7 + (a*b^2 - 4*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b - 9*a*b^2 + 4*b^
3)*cosh(d*x + c)^6 + 4*(2*a^2*b - 9*a*b^2 + 4*b^3 + 7*(a*b^2 - 4*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^3 + 3*(2*a
^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 - 40*a^2*
b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 - 4*b^3)*cosh(d*x + c
)^4 + 8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3 + 30*(2*a^2*b - 9*a*b^2 + 4*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^5 +
10*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^3 + (8*a^3 - 40*a^2*b + 35*a*b
^2 - 12*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 - 4*b^3 + 4*(2*a^2*b -
9*a*b^2 + 4*b^3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 - 4*b^3)*cosh(d*x + c)^6 + 1
5*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c)^4 + 2*a^2*b - 9*a*b^2 + 4*b^3 +
 3*(8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 8*((a*b^2 - 4*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b - 9*a*b^2 + 4*b^3)*cosh(d
*x + c)^5 + (8*a^3 - 40*a^2*b + 35*a*b^2 - 12*b^3)*cosh(d*x + c)^3 + (2*a^2
*b - 9*a*b^2 + 4*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b - b^2)*arctan(
-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b)) + 2*(a^2*b^2
- 5*a*b^3 + 4*b^4)*cosh(d*x + c) + 2*(7*(a^2*b^2 - 5*a*b^3 + 4*b^4)*cosh(d*
x + c)^6 - 5*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*x + c)^4 + a^2
*b^2 - 5*a*b^3 + 4*b^4 - 3*(4*a^3*b + 5*a^2*b^2 - 13*a*b^3 + 4*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c))/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x
+ c)^8 + 8*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)*sinh(d*x +
 c)^7 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*sinh(d*x + c)^8 + 4*(2*a^4*
b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^6 + 4*(7*(a^3*
b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4
 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 32*a^4*b^
3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^4 + 8*(7*(a
^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - 7*a^
3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35
*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 -
7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + (8*a^5*b^2 - 32*
a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d)*sinh(d*x + c)^4 +
4*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^2 + 8
*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3
```

```
 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^3 + (8*a^5*b^2 -
32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c))*s
inh(d*x + c)^3 + 4*(7*(a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)
^6 + 15*(2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)
^4 + 3*(8*a^5*b^2 - 32*a^4*b^3 + 51*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7
)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - 7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d
)*sinh(d*x + c)^2 + (a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d + 8*((a^3*b^4 -
 3*a^2*b^5 + 3*a*b^6 - b^7)*d*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - 7*a^3*b^4 +
9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 32*a^4*b^3 + 51
*a^3*b^4 - 41*a^2*b^5 + 17*a*b^6 - 3*b^7)*d*cosh(d*x + c)^3 + (2*a^4*b^3 -
7*a^3*b^4 + 9*a^2*b^5 - 5*a*b^6 + b^7)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.53 $\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=139

$$-\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a-b)^{5/2}}+\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{8ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] -((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a - b
)^(5/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(4*(a - b)*d*(a + b*Sinh[c + d*x
]^2)^2) + ((2*a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a*(a - b)^2*d*(a + b*S
inh[c + d*x]^2))
```

Rubi [A] time = 0.156531, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3173, 12, 3181, 208}

$$-\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a-b)^{5/2}}+\frac{(2a+b)\sinh(c+dx)\cosh(c+dx)}{8ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)\cosh(c+dx)}{4d(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] -((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a - b
)^(5/2)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(4*(a - b)*d*(a + b*Sinh[c + d*x
]^2)^2) + ((2*a + b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*a*(a - b)^2*d*(a + b*S
inh[c + d*x]^2))
```


Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{\int \frac{a-2a\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx}{4a(a-b)}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{a(4a-b)}{a+b\sinh^2(c+\ldots}}{8a^2(a-b\ldots}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{(4a-b)\int \frac{\ldots}{a+\ldots}}{8a(a\ldots}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)\cosh(c+dx)\sinh(c+dx)}{8a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{(4a-b)\,\mathrm{Subs}\ldots}{\ldots}$$

$$= -\frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a-b)^{5/2}d} + \frac{\cosh(c+dx)\sinh(c+dx)}{4(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(2a+b)\cosh\ldots}{8a(a-b)^2d\left(\ldots\right.}$$

Mathematica [A] time = 1.34558, size = 121, normalized size = 0.87

$$\frac{\frac{\sinh(2(c+dx))\left(8a^2+b(2a+b)\cosh(2(c+dx))-4ab-b^2\right)}{a(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} - \frac{(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-(((4*a - b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a - b
)^(5/2))) + ((8*a^2 - 4*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*(
c + d*x)])/(a*(a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Maple [B] time = 0.049, size = 1408, normalized size = 10.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1/4/d/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^7*b-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
^5*a+9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*
x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b+1/d/(tanh(1/2*d*x
+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b^2-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3*a+9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
```

```
nh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b+1/d/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2+1/d/(tanh(1/2*d*x+1/2*c)^4*a
-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)-1/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b-1/2
/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(a^2-2*a*b+b^2)/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b+1/2/d/(a^2-2*a*b+b^2)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))+1/2/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2))*b+1/8/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8
/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-1/8
/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/8/d/(a^2-2*a*b+b^2)/a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.00344, size = 12358, normalized size = 88.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 24*(4*a^3*b^2 -
 5*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(4*a^3*b^2 - 5*a^2*b^
3 + a*b^4)*sinh(d*x + c)^6 + 8*a^3*b^2 - 4*a^2*b^3 - 4*a*b^4 + 4*(16*a^5 -
24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 2
4*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 15*(4*a^3*b^2 - 5*a^2*b^3 + a*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4
)*cosh(d*x + c)^3 + (16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^
4)*cosh(d*x + c)^2 + 4*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^4 + 15*(4*a
^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 24*a^4*b + 6*a^3*
b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a*b^3 - b
^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*
a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^
6 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4 + 7*(4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^6 + 8*(7*(4*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 6*a*b^3
```

```
+ b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(32*a^3*b - 40*a^2*b^2 + 20*a*b^3
 - 3*b^4)*cosh(d*x + c)^4 + 2*(35*(4*a*b^3 - b^4)*cosh(d*x + c)^4 + 32*a^3*
b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4 + 30*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 4*a*b^3 - b^4 + 8*(7*(4*a*b^3 - b^4)*cosh(d*x +
c)^5 + 10*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (32*a^3*b - 40*a^2*
b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 6*a
*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - b^4)*cosh(d*x + c)^6 + 15*(8*
a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 6*a*b^3 + b^4 + 3*(3
2*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((4*a*b^3 - b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*
x + c)^5 + (32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (8*
a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log(
(b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
 c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
- b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*
a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*co
sh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(
2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^
2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 8
*(3*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 24*a^4*b
+ 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (16*a^4*b - 20*a^3*b^2
 + a^2*b^3 + 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 3*a^4*b^4 +
 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^
5 - a^2*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*d*sinh(d*x + c)^8 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 -
5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*
b^5 - a^2*b^6)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a
^3*b^5 + a^2*b^6)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3
 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 -
 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 7*a^5*
b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(35*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^4 + 30*(2*
a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 +
(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d
)*sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d
*cosh(d*x + c)^5 + 10*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*
b^6)*d*cosh(d*x + c)^3 + (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 +
17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 -
3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^6 + 15*(2*a^6*b^2 - 7*a^5*
b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 3*(8*a^7*b - 32*
a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)
^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d)*sinh(d*x
+ c)^2 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d + 8*((a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^7 + 3*(2*a^6*b^2 - 7*a^5*b^3 +
 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + (8*a^7*b - 32*a^6*b^2
 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^3 + (2
*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*si
nh(d*x + c)), -1/8*(2*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 12*
(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(4*a^3*b^
2 - 5*a^2*b^3 + a*b^4)*sinh(d*x + c)^6 + 4*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 +
2*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 2
*(16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4 + 15*(4*a^3*b^2 - 5*a
^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(4*a^3*b^2 - 5*a^2*
b^3 + a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 24*a^4*b + 6*a^3*b^2 + 5*a^2*b^3 -
 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(16*a^4*b - 20*a^3*b^2 + a^2*b
^3 + 3*a*b^4)*cosh(d*x + c)^2 + 2*(16*a^4*b - 20*a^3*b^2 + a^2*b^3 + 3*a*b^
4 + 15*(4*a^3*b^2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 24*a^4
```

```
*b + 6*a^3*b^2 + 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((
4*a*b^3 - b^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x +
 c)^7 + (4*a*b^3 - b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4)*cos
h(d*x + c)^6 + 4*(8*a^2*b^2 - 6*a*b^3 + b^4 + 7*(4*a*b^3 - b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(8*a^2*b^2
 - 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(32*a^3*b - 40*a^2*b^2
 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 2*(35*(4*a*b^3 - b^4)*cosh(d*x + c)^
4 + 32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4 + 30*(8*a^2*b^2 - 6*a*b^3 + b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*a*b^3 - b^4 + 8*(7*(4*a*b^3 - b^4)*
cosh(d*x + c)^5 + 10*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^3 + (32*a^3*
b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^
2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - b^4)*cosh(d*x + c)
^6 + 15*(8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 6*a*b^3 +
 b^4 + 3*(32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d
*x + c)^2 + 8*((4*a*b^3 - b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 6*a*b^3 + b
^4)*cosh(d*x + c)^5 + (32*a^3*b - 40*a^2*b^2 + 20*a*b^3 - 3*b^4)*cosh(d*x +
 c)^3 + (8*a^2*b^2 - 6*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2
 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) +
b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(4*a^3*b^
2 - 5*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 + 2*(16*a^5 - 24*a^4*b + 6*a^3*b^2 +
 5*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (16*a^4*b - 20*a^3*b^2 + a^2*b^3 +
3*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 -
a^2*b^6)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*
d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^
6)*d*sinh(d*x + c)^8 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a
^2*b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6
)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*
b^6)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4
 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - 3*a^4*b^4 +
3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b
^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3
 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 7*a
^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^2 + (8*a^7*b - 32
*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d)*sinh(d*x +
c)^4 + 4*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)^2 + 8*(7*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c
)^5 + 10*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d
*x + c)^3 + (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 -
3*a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 - 3*a^4*b^4 + 3
*a^3*b^5 - a^2*b^6)*d*cosh(d*x + c)^6 + 15*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b
^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^4 + 3*(8*a^7*b - 32*a^6*b^2 + 51*
a^5*b^3 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^2 + (2*a^6*b
^2 - 7*a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d)*sinh(d*x + c)^2 + (a^5
*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d + 8*((a^5*b^3 - 3*a^4*b^4 + 3*a^3
*b^5 - a^2*b^6)*d*cosh(d*x + c)^7 + 3*(2*a^6*b^2 - 7*a^5*b^3 + 9*a^4*b^4 -
5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c)^5 + (8*a^7*b - 32*a^6*b^2 + 51*a^5*b^3
 - 41*a^4*b^4 + 17*a^3*b^5 - 3*a^2*b^6)*d*cosh(d*x + c)^3 + (2*a^6*b^2 - 7*
a^5*b^3 + 9*a^4*b^4 - 5*a^3*b^5 + a^2*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.52648, size = 378, normalized size = 2.72

$$-\frac{(4a-b)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{8\left(a^3d-2a^2bd+ab^2d\right)\sqrt{-a^2+ab}} - \frac{4ab^2e^{(6dx+6c)} - b^3e^{(6dx+6c)} + 16a^3e^{(4dx+4c)} - 8a^2be^{(4dx+4c)} - 2ab^2e^{(4dx+4c)} + 3}{4\left(a^3bd-2a^2b^2d+ab^3d\right)\left(be^{(4dx+4c)} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(4*a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(
(a^3*d - 2*a^2*b*d + a*b^2*d)*sqrt(-a^2 + a*b)) - 1/4*(4*a*b^2*e^(6*d*x + 6
*c) - b^3*e^(6*d*x + 6*c) + 16*a^3*e^(4*d*x + 4*c) - 8*a^2*b*e^(4*d*x + 4*c
) - 2*a*b^2*e^(4*d*x + 4*c) + 3*b^3*e^(4*d*x + 4*c) + 16*a^2*b*e^(2*d*x + 2
*c) - 4*a*b^2*e^(2*d*x + 2*c) - 3*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + b^3)/((a^
3*b*d - 2*a^2*b^2*d + a*b^3*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2
*b*e^(2*d*x + 2*c) + b)^2)
```

3.54 $\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=118

$$\frac{3\cosh(c+dx)}{8d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\cosh(c+dx)}{4d(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8\sqrt{b}d(a-b)^{5/2}}$$

```
[Out] (3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*Sqrt[b]*d)
 + Cosh[c + d*x]/(4*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) + (3*Cosh[c +
d*x])/(8*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.0831678, antiderivative size = 118, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 199, 205}

$$\frac{3\cosh(c+dx)}{8d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}+\frac{\cosh(c+dx)}{4d(a-b)\left(a+b\cosh^2(c+dx)-b\right)^2}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8\sqrt{b}d(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a - b]])/(8*(a - b)^(5/2)*Sqrt[b]*d)
 + Cosh[c + d*x]/(4*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) + (3*Cosh[c +
d*x])/(8*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{4(a-b)d}$$

$$= \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{3\cosh(c+dx)}{8(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)} + \frac{3\,\text{Subst}}{}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}\sqrt{b}d} + \frac{\cosh(c+dx)}{4(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{3\cosh(c+}{8(a-b)^2d\left(a-b+b\,c\right.}$$

Mathematica [C] time = 0.746678, size = 149, normalized size = 1.26

$$\frac{\frac{2\cosh(c+dx)(10a+3b\cosh(2(c+dx))-7b)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} + \frac{3\left(\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)\right)}{\sqrt{b}(a-b)^{5/2}}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*(ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]] + ArcTan[(
Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]))/((a - b)^(5/2)*Sqrt[b
]) + (2*Cosh[c + d*x]*(10*a - 7*b + 3*b*Cosh[2*(c + d*x)]))/((a - b)^2*(2*a
 - b + b*Cosh[2*(c + d*x)])^2))/(8*d)
```

Maple [B] time = 0.034, size = 964, normalized size = 8.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+4/d/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^6*b-2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1
/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)^6*b^2+15/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-23/2/d/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+14/d/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
4*b^3-15/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*a+8/d/(tanh(1/2*d
```

```
*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2
*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^2*b^2+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2*a/(a^2-2*a*b+b^2)-1/2/d/(tanh(1/2*d*x+1/2*c)^4
*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)
*b+3/8/d/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^
2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(20\,ae^{(5\,c)} - 11\,be^{(5\,c)}\right)e^{(5\,dx)} + }{4\left(a^2b^2d - 2\,ab^3d + b^4d + \left(a^2b^2de^{(8\,c)} - 2\,ab^3de^{(8\,c)} + b^4de^{(8\,c)}\right)e^{(8\,dx)} + 4\left(2\,a^3bde^{(6\,c)} - 5\,a^2b^2de^{(6\,c)} + 4\,ab^3de^{(6\,c)} - b^4de^{(6\,c)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((20*a*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) + (20*a*e^(3*c) - 11*b*e^(3*c)
)*e^(3*d*x) + 3*b*e^(7*d*x + 7*c) + 3*b*e^(d*x + c))/(a^2*b^2*d - 2*a*b^3*d
 + b^4*d + (a^2*b^2*d*e^(8*c) - 2*a*b^3*d*e^(8*c) + b^4*d*e^(8*c))*e^(8*d*x
) + 4*(2*a^3*b*d*e^(6*c) - 5*a^2*b^2*d*e^(6*c) + 4*a*b^3*d*e^(6*c) - b^4*d*
e^(6*c))*e^(6*d*x) + 2*(8*a^4*d*e^(4*c) - 24*a^3*b*d*e^(4*c) + 27*a^2*b^2*d
*e^(4*c) - 14*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^3*b*d*e
^(2*c) - 5*a^2*b^2*d*e^(2*c) + 4*a*b^3*d*e^(2*c) - b^4*d*e^(2*c))*e^(2*d*x)
) + 1/2*integrate(3/2*(e^(3*d*x + 3*c) - e^(d*x + c))/(a^2*b - 2*a*b^2 + b^
3 + (a^2*b*e^(4*c) - 2*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^3*e^
(2*c) - 5*a^2*b*e^(2*c) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.86219, size = 12081, normalized size = 102.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*(a*b^2 - b^3)*cosh(d*x + c)^7 + 84*(a*b^2 - b^3)*cosh(d*x + c)*si
nh(d*x + c)^6 + 12*(a*b^2 - b^3)*sinh(d*x + c)^7 + 4*(20*a^2*b - 31*a*b^2 +
 11*b^3)*cosh(d*x + c)^5 + 4*(20*a^2*b - 31*a*b^2 + 11*b^3 + 63*(a*b^2 - b^
3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(21*(a*b^2 - b^3)*cosh(d*x + c)^3
+ (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(20*a^2
*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 4*(105*(a*b^2 - b^3)*cosh(d*x + c
)^4 + 20*a^2*b - 31*a*b^2 + 11*b^3 + 10*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^3 + 4*(63*(a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(20
*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 3*(20*a^2*b - 31*a*b^2 + 11*b
^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*
x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c
)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*co
sh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 -
 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b
^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cos
h(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*
```

```
cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b
^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x
+ c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x
 + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b + b^2)*log(
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
- 2*(2*a - 3*b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c
) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3
+ (3*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + cosh(d*x + c))*sqrt(-a*b + b^2) +
 b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c
)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(
d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c)
+ b)) + 12*(a*b^2 - b^3)*cosh(d*x + c) + 4*(21*(a*b^2 - b^3)*cosh(d*x + c)^
6 + 5*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 3*a*b^2 - 3*b^3 + 3*
(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^3*b^3 -
3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^8 + 8*(a^3*b^3 - 3*a^2*b^4 + 3*a
*b^5 - b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^
5 - b^6)*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5
 + b^6)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*co
sh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh
(d*x + c)^6 + 2*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5
- 3*b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*c
osh(d*x + c)^3 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*
d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*
d*cosh(d*x + c)^2 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*
b^5 - 3*b^6)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*
a*b^5 + b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)
*d*cosh(d*x + c)^5 + 10*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)
*d*cosh(d*x + c)^3 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a
*b^5 - 3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b^3 - 3*a^2*b^4
+ 3*a*b^5 - b^6)*d*cosh(d*x + c)^6 + 15*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4
- 5*a*b^5 + b^6)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 -
 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3
+ 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 - 3*a^2*b^4 + 3*
a*b^5 - b^6)*d + 8*((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^7
 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d*x + c)^5
+ (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cos
h(d*x + c)^3 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d
*x + c))*sinh(d*x + c)), 1/8*(6*(a*b^2 - b^3)*cosh(d*x + c)^7 + 42*(a*b^2 -
 b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 6*(a*b^2 - b^3)*sinh(d*x + c)^7 + 2*(
20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^5 + 2*(20*a^2*b - 31*a*b^2 + 11
*b^3 + 63*(a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(21*(a*b^2 -
b^3)*cosh(d*x + c)^3 + (20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d
*x + c)^4 + 2*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 2*(105*(a*b^
2 - b^3)*cosh(d*x + c)^4 + 20*a^2*b - 31*a*b^2 + 11*b^3 + 10*(20*a^2*b - 31
*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*(a*b^2 - b^3)*cos
h(d*x + c)^5 + 10*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 3*(20*a^
2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x +
 c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*
b - b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x
 + c)^6 + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x +
 c)^4 + 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x
+ c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a
^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d
*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3
*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b
^2 + 8*(b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*
```

```
a*b + 3*b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(a*b - b^2)*arctan(-1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x
 + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c) + (3*b*cosh(d*x + c
)^2 + 4*a - 3*b)*sinh(d*x + c))/sqrt(a*b - b^2)) - 3*(b^2*cosh(d*x + c)^8 +
 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2
)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6
 + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 +
 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4
+ 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*
a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)
^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2
 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*
(b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3
*b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*
b - b^2)*arctan(-1/2*sqrt(a*b - b^2)*(cosh(d*x + c) + sinh(d*x + c))/(a - b
)) + 6*(a*b^2 - b^3)*cosh(d*x + c) + 2*(21*(a*b^2 - b^3)*cosh(d*x + c)^6 +
5*(20*a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 3*a*b^2 - 3*b^3 + 3*(20*
a^2*b - 31*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^3*b^3 - 3*a^
2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^8 + 8*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5
 - b^6)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 -
b^6)*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b
^6)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d
*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x
 + c)^6 + 2*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3*
b^6)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(
d*x + c)^3 + 3*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*co
sh(d*x + c)^4 + 30*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*co
sh(d*x + c)^2 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5
- 3*b^6)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^
5 + b^6)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*c
osh(d*x + c)^5 + 10*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*c
osh(d*x + c)^3 + (8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5
 - 3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b^3 - 3*a^2*b^4 + 3*
a*b^5 - b^6)*d*cosh(d*x + c)^6 + 15*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*
a*b^5 + b^6)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*
a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - 7*a^3*b^3 + 9*
a^2*b^4 - 5*a*b^5 + b^6)*d)*sinh(d*x + c)^2 + (a^3*b^3 - 3*a^2*b^4 + 3*a*b^
5 - b^6)*d + 8*((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cosh(d*x + c)^7 + 3
*(2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d*x + c)^5 + (8
*a^5*b - 32*a^4*b^2 + 51*a^3*b^3 - 41*a^2*b^4 + 17*a*b^5 - 3*b^6)*d*cosh(d*
x + c)^3 + (2*a^4*b^2 - 7*a^3*b^3 + 9*a^2*b^4 - 5*a*b^5 + b^6)*d*cosh(d*x +
 c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.55 $\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=154

$$\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{5/2}}-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a
^(5/2)*(a - b)^(5/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(4*a*(a - b)*d*(a
 + b*Sinh[c + d*x]^2)^2) - (3*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*a
^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.154646, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3184, 3173, 12, 3181, 208}

$$\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{5/2}}-\frac{3b(2a-b)\sinh(c+dx)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}-\frac{b\sinh(c+dx)\cosh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(-3),x]
```

```
[Out] ((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a
^(5/2)*(a - b)^(5/2)*d) - (b*Cosh[c + d*x]*Sinh[c + d*x])/(4*a*(a - b)*d*(a
 + b*Sinh[c + d*x]^2)^2) - (3*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*a
^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{\int \frac{-4a+3b+2b\sinh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx}{4a(a-b)}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{\int \frac{-8a^2+8\ldots}{a+b\sinh\ldots}}{8a^2(\ldots}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\left(8a^2-8\ldots\right.}{\ldots}$$

$$= -\frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a-b)b\cosh(c+dx)\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\left(8a^2-8\ldots\right.}{\ldots}$$

$$= \frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{5/2}d} - \frac{b\cosh(c+dx)\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3(2a\ldots}{8a^2\ldots}$$

Mathematica [A] time = 1.21876, size = 132, normalized size = 0.86

$$\frac{\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{a}b\sinh(2(c+dx))\left(-16a^2+3b(b-2a)\cosh(2(c+dx))+16ab-3b^2\right)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(-3),x]
```

```
[Out] (((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a
- b)^(5/2) + (Sqrt[a]*b*(-16*a^2 + 16*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(
c + d*x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2)
)/(8*a^(5/2)*d)
```

Maple [B] time = 0.053, size = 1768, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b+5/4/d/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^5*b-29/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*b^2+3/d/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+2/d/(tanh(1/2*d*x+1/2*c)^
4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^3*b-29/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^3*b^2+3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-2/d/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2/a/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)+1/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(
a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*
d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d/a^2/(a^2-2*a*b+b^2)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-1/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2))*b+1/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2))*b^2-3/8/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))*b^3-1/d/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
)+1/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tan
h(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/a^2/(a^2-2*a*b
+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2-1/d/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2))*b+1/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))*b^2-3/8/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2))*b^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.96866, size = 13509, normalized size = 87.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")

[Out] [1/16*(4*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 24
*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^
5 + 4*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 24*a^
3*b^2 - 36*a^2*b^3 + 12*a*b^4 + 12*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2
*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 12*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a
^2*b^3 + 3*a*b^4 + 5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 16*(5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b
^4)*cosh(d*x + c)^3 + 3*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*
b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3
 - 9*a*b^4)*cosh(d*x + c)^2 + 4*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b
^4 + 15*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 18*
(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + ((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^2*b
^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^2*b^2 - 8*a*b^3
+ 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos
h(d*x + c)^6 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 -
 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 - 8*a*
b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*
a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d
*x + c)^4 + 64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4 + 30*(16*a^
3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a
^2*b^2 - 8*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)
^5 + 10*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (64*a^
4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(
8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b - 24*a^2*b^2 +
14*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^
4 + 3*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^2
)*sinh(d*x + c)^2 + 8*((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(1
6*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 128*a^
3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (16*a^3*b - 24*a^2*
b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((
b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x +
c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b -
 b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a
*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*
x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2
*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 8*
(3*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 6*(16*a^
5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (40*a^4
*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6
*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 - 3*
a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 -
 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^7*b - 7*a^6*b^
2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 - 3*
a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^7*b - 7*a^6*b^2 + 9
*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^8 - 32*a^7*b +
51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*
(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^7*b
- 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4
+ 30*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c
)^2 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)
*d)*sinh(d*x + c)^4 + 4*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*
```

```
b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d
*cosh(d*x + c)^5 + 10*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^
5)*d*cosh(d*x + c)^3 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4
*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^6*b^2 - 3*a^5*
b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^7*b - 7*a^6*b^2 + 9*
a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^8 - 32*a^7*b + 51
*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^7*
b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^2 + (a^6*
b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d + 8*((a^6*b^2 - 3*a^5*b^3 + 3*a^4*
b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a
^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a
^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^7*b - 7*a^6*b^2 +
 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(
8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 + 12*(8*a^4*b
- 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(8*a
^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^6 + 12*a^3*b^2 - 18
*a^2*b^3 + 6*a*b^4 + 6*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b
^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*
b^4 + 5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^4 + 8*(5*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x +
 c)^3 + 3*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 2*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4)*co
sh(d*x + c)^2 + 2*(40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4 + 15*(8*a^4
*b - 16*a^3*b^2 + 11*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 + 18*(16*a^5 - 40*a
^4*b + 38*a^3*b^2 - 17*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2
- ((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(8*a^2*b^2 - 8*a*b^3 +
 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*sinh(
d*x + c)^8 + 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^6 +
 4*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 - 8*a*b^3 + 3*b
^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*c
osh(d*x + c)^3 + 3*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(64*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)
*cosh(d*x + c)^4 + 2*(35*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64
*a^4 - 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4 + 30*(16*a^3*b - 24*a^2*b
^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b
^3 + 3*b^4 + 8*(7*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^
3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (64*a^4 - 128*a^3*b
+ 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^
3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(8*a^2*b^2 - 8*
a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b
^4)*cosh(d*x + c)^4 + 16*a^3*b - 24*a^2*b^2 + 14*a*b^3 - 3*b^4 + 3*(64*a^4
- 128*a^3*b + 112*a^2*b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 + 8*((8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b - 24*a
^2*b^2 + 14*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 128*a^3*b + 112*a^2*
b^2 - 48*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (16*a^3*b - 24*a^2*b^2 + 14*a*b^3
 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cos
h(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a -
b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(8*a^4*b - 16*a^3*b^2 + 11*a^2*b^3
- 3*a*b^4)*cosh(d*x + c)^5 + 6*(16*a^5 - 40*a^4*b + 38*a^3*b^2 - 17*a^2*b^3
 + 3*a*b^4)*cosh(d*x + c)^3 + (40*a^4*b - 80*a^3*b^2 + 49*a^2*b^3 - 9*a*b^4
)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5
)*d*cosh(d*x + c)^8 + 8*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*si
nh(d*x + c)^8 + 4*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^6 + 4*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(
d*x + c)^2 + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d)*sin
h(d*x + c)^6 + 2*(8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 -
 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3
*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 +
```

```
a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^6*b^2 - 3*a^5*b^3 + 3*
a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 30*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3
- 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^8 - 32*a^7*b + 51*a^6*b^2 -
 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^7*b - 7*a
^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^6*b^2
 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + 10*(2*a^7*b - 7*a^6
*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^8 - 32*a^7
*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(7*(a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x
+ c)^6 + 15*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(
d*x + c)^4 + 3*(8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3
*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4
+ a^3*b^5)*d)*sinh(d*x + c)^2 + (a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)
*d + 8*((a^6*b^2 - 3*a^5*b^3 + 3*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(
2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 +
(8*a^8 - 32*a^7*b + 51*a^6*b^2 - 41*a^5*b^3 + 17*a^4*b^4 - 3*a^3*b^5)*d*cos
h(d*x + c)^3 + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d*co
sh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4622, size = 413, normalized size = 2.68

$$\frac{\left(8\,a^2-8\,ab+3\,b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{8\left(a^4d-2\,a^3bd+a^2b^2d\right)\sqrt{-a^2+ab}}+\frac{8\,a^2be^{(6\,dx+6\,c)}-8\,ab^2e^{(6\,dx+6\,c)}+3\,b^3e^{(6\,dx+6\,c)}+48\,a^3e^{(4\,dx+4\,c)}-72\,a^2be^{(}}{4\left(a^4d-2\,a^3bd+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 - 8*a*b + 3*b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-
a^2 + a*b))/((a^4*d - 2*a^3*b*d + a^2*b^2*d)*sqrt(-a^2 + a*b)) + 1/4*(8*a^2
*b*e^(6*d*x + 6*c) - 8*a*b^2*e^(6*d*x + 6*c) + 3*b^3*e^(6*d*x + 6*c) + 48*a
^3*e^(4*d*x + 4*c) - 72*a^2*b*e^(4*d*x + 4*c) + 42*a*b^2*e^(4*d*x + 4*c) -
9*b^3*e^(4*d*x + 4*c) + 40*a^2*b*e^(2*d*x + 2*c) - 40*a*b^2*e^(2*d*x + 2*c)
 + 9*b^3*e^(2*d*x + 2*c) + 6*a*b^2 - 3*b^3)/((a^4*d - 2*a^3*b*d + a^2*b^2*d
)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2)
```

3.56 $\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=166

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3d(a-b)^{5/2}}-\frac{b(7a-4b)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}-\frac{}{4ad(a}$$

```
[Out] -(Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a -
 b]])/(8*a^3*(a - b)^(5/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^3*d) - (b*Cosh[c
+ d*x])/(4*a*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) - ((7*a - 4*b)*b*Cosh
[c + d*x])/(8*a^2*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))
```

Rubi [A] time = 0.266536, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3186, 414, 527, 522, 206, 205}

$$-\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3d(a-b)^{5/2}}-\frac{b(7a-4b)\cosh(c+dx)}{8a^2d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{\tanh^{-1}(\cosh(c+dx))}{a^3d}-\frac{}{4ad(a}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] -(Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a -
 b]])/(8*a^3*(a - b)^(5/2)*d) - ArcTanh[Cosh[c + d*x]]/(a^3*d) - (b*Cosh[c
+ d*x])/(4*a*(a - b)*d*(a - b + b*Cosh[c + d*x]^2)^2) - ((7*a - 4*b)*b*Cosh
[c + d*x])/(8*a^2*(a - b)^2*d*(a - b + b*Cosh[c + d*x]^2))
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+bx^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{-4a+b+3bx^2}{(1-x^2)\left(a-b+bx^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{4a(a-b)d}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(7a-4b)b\cosh(c+dx)}{8a^2(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\text{Su}}{}$$

$$= -\frac{b\cosh(c+dx)}{4a(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(7a-4b)b\cosh(c+dx)}{8a^2(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{\text{Su}}{}$$

$$= -\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^3(a-b)^{5/2}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3d} - \frac{b}{4a(a-b)d\left(\right.}$$

Mathematica [C] time = 3.42551, size = 237, normalized size = 1.43

$$-\frac{\frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{\sqrt{b}\left(15a^2-20ab+8b^2\right)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a-b}}\right)}{(a-b)^{5/2}} + \frac{8a^2b\cosh(c+dx)}{(a-b)(2a+b\cosh(2(c+dx))-b)^2} + \frac{2ab(}{(a-b)^2(2}}{8a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -((Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tanh[(c +
d*x)/2])/Sqrt[a - b]])/(a - b)^(5/2) + (Sqrt[b]*(15*a^2 - 20*a*b + 8*b^2)*A
rcTan[(Sqrt[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]])/(a - b)^(5/2) +
 (8*a^2*b*Cosh[c + d*x])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2) + (2*a
```

```
*(7*a - 4*b)*b*Cosh[c + d*x])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])) -
 8*Log[Tanh[(c + d*x)/2]])/(8*a^3*d)
```

Maple [B] time = 0.069, size = 1145, normalized size = 6.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d/a^3*ln(tanh(1/2*d*x+1/2*c))+9/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^6*b-7/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*b^2+4/d/a^2*b^
3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-27/4/d/(tanh(1/2*d*x+1/2*c)^
4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^4+45/2/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^4-30/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4*b^3+12/d/a^3
*b^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+27/4/d/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^2-17/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1
/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)^2*b^2+8/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-9/4/d/(t
anh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+
a)^2/(a^2-2*a*b+b^2)*b+3/2/d/a*b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+
1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)-15/8/d/a*b/(a^2-2
*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a
*b-b^2)^(1/2))+5/2/d/a^2*b^2/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*
tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))-1/d/a^3*b^3/(a^2-2*a*b+b^
2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((7*a*b^2*e^(7*c) - 4*b^3*e^(7*c))*e^(7*d*x) + (36*a^2*b*e^(5*c) - 31*
a*b^2*e^(5*c) + 4*b^3*e^(5*c))*e^(5*d*x) + (36*a^2*b*e^(3*c) - 31*a*b^2*e^(
3*c) + 4*b^3*e^(3*c))*e^(3*d*x) + (7*a*b^2*e^c - 4*b^3*e^c)*e^(d*x))/(a^4*b
^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) - 2*a^3*b^3*d*e^(8*c) +
 a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(2*a^5*b*d*e^(6*c) - 5*a^4*b^2*d*e^(6*c)
+ 4*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^6*d*e^(4*c) -
 24*a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) - 14*a^3*b^3*d*e^(4*c) + 3*a^2*b
^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^5*b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c) + 4*a^
```

```
3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(
-c))/(a^3*d) + log((e^(d*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/8*((15
*a^2*b*e^(3*c) - 20*a*b^2*e^(3*c) + 8*b^3*e^(3*c))*e^(3*d*x) - (15*a^2*b*e^
c - 20*a*b^2*e^c + 8*b^3*e^c)*e^(d*x))/(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*
b*e^(4*c) - 2*a^4*b^2*e^(4*c) + a^3*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^6*e^(2*
c) - 5*a^5*b*e^(2*c) + 4*a^4*b^2*e^(2*c) - a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 4.05571, size = 22814, normalized size = 137.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^7 + 28*(7*a^2*b^2 - 4*a*b^3)*
cosh(d*x + c)*sinh(d*x + c)^6 + 4*(7*a^2*b^2 - 4*a*b^3)*sinh(d*x + c)^7 + 4
*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 4*(36*a^3*b - 31*a^2*b
^2 + 4*a*b^3 + 21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 +
20*(7*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^3 + (36*a^3*b - 31*a^2*b^2 + 4*a*
b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*c
osh(d*x + c)^3 + 4*(35*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 + 36*a^3*b - 3
1*a^2*b^2 + 4*a*b^3 + 10*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 4*(21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 10*(36*a^3
*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a
*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cos
h(d*x + c)^8 + 8*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c
)^7 + (15*a^2*b^2 - 20*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^
2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a
*b^3 - 8*b^4 + 7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 8*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b
 - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a
^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*
(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269
*a^2*b^2 - 124*a*b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*
(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2
 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 -
 124*a*b^3 + 24*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*
b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b
^4)*cosh(d*x + c)^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*
x + c)^4 + 30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*
b + 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*
((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2
 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 -
 124*a*b^3 + 24*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 -
8*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/(a - b))*log((b*cosh(d*x + c)^
4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a - 3*b)*c
osh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a + 3*b)*sinh(d*x + c)^2 + 4*(b
*cosh(d*x + c)^3 - (2*a - 3*b)*cosh(d*x + c))*sinh(d*x + c) - 4*((a - b)*co
sh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x
+ c)^3 + (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 + a - b)*sinh(d
*x + c))*sqrt(-b/(a - b)) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b)) + 4*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c) +
16*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)
```

*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(d*x + c)^8
+ 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(2*a^3*b - 5*
a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b -
5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2
*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 +
3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x
+ c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^4
- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^2*b^2 -
2*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b
+ 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^
2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(d*x + c)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*
cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh
(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 16*((a^2*b^2 - 2*a*b^3
+ b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x
+ c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4
+ 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2
*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14
*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x
+ c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5
*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^2*b^2 - 2*a*
b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(2*a^3*b -
5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2
- 14*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*
x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a^3
*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^
3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*
cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^5 +
(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a^
3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x
+ c) + sinh(d*x + c) - 1) + 4*(7*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^6 + 5
*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^4 + 7*a^2*b^2 - 4*a*b^3 +
3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5*b
^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3*
b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*si
nh(d*x + c)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x +
c)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^2 + (2*a^6*b -
5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(8*a^7 - 24*a^6*b +
27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 2
*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^3 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3
- a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 +
a^3*b^4)*d*cosh(d*x + c)^4 + 30*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)
*d*cosh(d*x + c)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^
4)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cos
h(d*x + c)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^5 + 10*
(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3 + (8*a^7 - 24
*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c
)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^6 + 15*(2*a^6*b
- 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4 + 3*(8*a^7 - 24*a^6*b
+ 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^2 + (2*a^6*b - 5*a^5
*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3

```
*b^4)*d + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^7 + 3*(2*a^6*b
 - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (8*a^7 - 24*a^6*b +
 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (2*a^6*b - 5*a^5*
b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(7*a^2*
b^2 - 4*a*b^3)*cosh(d*x + c)^7 + 14*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)*sin
h(d*x + c)^6 + 2*(7*a^2*b^2 - 4*a*b^3)*sinh(d*x + c)^7 + 2*(36*a^3*b - 31*a
^2*b^2 + 4*a*b^3)*cosh(d*x + c)^5 + 2*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3 + 21
*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(7*a^2*b^2
- 4*a*b^3)*cosh(d*x + c)^3 + (36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c
))*sinh(d*x + c)^4 + 2*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^3 +
2*(35*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^4 + 36*a^3*b - 31*a^2*b^2 + 4*a*b
^3 + 10*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3
+ 2*(21*(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c)^5 + 10*(36*a^3*b - 31*a^2*b^2 +
 4*a*b^3)*cosh(d*x + c)^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh(d*x +
c))*sinh(d*x + c)^2 + ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8*
(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b^2
 - 20*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3
- 8*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 7*
(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15
*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b - 55*a^2*b^2 + 3
6*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a^4 - 280*a^3*b +
269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 20*
a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*
b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*(15*a^2*b^2 - 20*
a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b
^4)*cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b
^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 -
8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)
^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 30*a^3
*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*b + 269*a^2*b^2 -
 124*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 20
*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b
^4)*cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*b
^4)*cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x +
 c))*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1/2*sqrt(b/(a - b))*(cosh(d*x +
c) + sinh(d*x + c))) - ((15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^8 + 8
*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b^
2 - 20*a*b^3 + 8*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3
 - 8*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 7
*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(1
5*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b - 55*a^2*b^2 +
36*a*b^3 - 8*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a^4 - 280*a^3*b +
 269*a^2*b^2 - 124*a*b^3 + 24*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 20
*a*b^3 + 8*b^4)*cosh(d*x + c)^4 + 120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a
*b^3 + 24*b^4 + 30*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 20*a*b^3 + 8*b^4 + 8*(7*(15*a^2*b^2 - 20
*a*b^3 + 8*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*
b^4)*cosh(d*x + c)^3 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*
b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 -
 8*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 20*a*b^3 + 8*b^4)*cosh(d*x + c
)^6 + 15*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x + c)^4 + 30*a^
3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4 + 3*(120*a^4 - 280*a^3*b + 269*a^2*b^2
- 124*a*b^3 + 24*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 2
0*a*b^3 + 8*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*
b^4)*cosh(d*x + c)^5 + (120*a^4 - 280*a^3*b + 269*a^2*b^2 - 124*a*b^3 + 24*
b^4)*cosh(d*x + c)^3 + (30*a^3*b - 55*a^2*b^2 + 36*a*b^3 - 8*b^4)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt(b/(a - b))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*co
sh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - 3*b)*cosh(d*x + c)
```

```
 + (3*b*cosh(d*x + c)^2 + 4*a - 3*b)*sinh(d*x + c))*sqrt(b/(a - b))/b) + 2*
(7*a^2*b^2 - 4*a*b^3)*cosh(d*x + c) + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x
 + c)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*
b^2 - 2*a*b^3 + b^4)*sinh(d*x + c)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b
^4)*cosh(d*x + c)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 -
 2*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3
+ b^4)*cosh(d*x + c)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)
*cosh(d*x + c)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^4
- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*
b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(
7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a
*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3
*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b
^4)*cosh(d*x + c)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(
2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2
+ 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^7
 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^4 - 24*a^
3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2
 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*
x + c) + 1) - 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^8 + 8*(a^2*b^2 - 2
*a*b^3 + b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sin
h(d*x + c)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*
(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x
 + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^3 +
 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 2*(35
*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2
- 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)
^2)*sinh(d*x + c)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b
^4)*cosh(d*x + c)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c
)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c))*sin
h(d*x + c)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*
(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*
a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a
^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(d*x + c)^7 + 3*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*
b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(d
*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*(7*a^
2*b^2 - 4*a*b^3)*cosh(d*x + c)^6 + 5*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cosh
(d*x + c)^4 + 7*a^2*b^2 - 4*a*b^3 + 3*(36*a^3*b - 31*a^2*b^2 + 4*a*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x +
c)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 +
(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*sinh(d*x + c)^8 + 4*(2*a^6*b - 5*a^5*b^2
+ 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*
b^4)*d*cosh(d*x + c)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*sin
h(d*x + c)^6 + 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d
*cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^3 +
 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x +
c)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cosh(d*x + c)^4 + 30*(2*a^6*
b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + (8*a^7 - 24*a^6*b
+ 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b - 5*
a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^
3 + a^3*b^4)*d*cosh(d*x + c)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*
b^4)*d*cosh(d*x + c)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^
3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b
^4)*d*cosh(d*x + c)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*co
```

```
sh(d*x + c)^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*
d*cosh(d*x + c)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d)*sinh(d*x
 + c)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d + 8*((a^5*b^2 - 2*a^4*b^3 + a^3
*b^4)*d*cosh(d*x + c)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*c
osh(d*x + c)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*d
*cosh(d*x + c)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*d*cosh(d*x +
 c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.57 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=215

$$-\frac{3b\left(8a^2-12ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a-b)^{5/2}}-\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3d(a-b)^2}-\frac{b\coth(c+dx)\left(-(4a-b)\tanh^2(c+\right.}{8a^2d(a-b)^2\left(a-(a-b)\tanh\right.}$$

```
[Out] (-3*b*(8*a^2 - 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(8*a^(7/2)*(a - b)^(5/2)*d) - ((4*a - 5*b)*(2*a - 3*b)*Coth[c + d*x])/(8*
a^3*(a - b)^2*d) - (b*Csch[c + d*x]*Sech[c + d*x]^3)/(4*a*(a - b)*d*(a - (a
 - b)*Tanh[c + d*x]^2)^2) - (b*Coth[c + d*x]*(4*a - 5*b - (4*a - b)*Tanh[c
+ d*x]^2))/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.287169, antiderivative size = 215, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 468, 577, 453, 208}

$$-\frac{3b\left(8a^2-12ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a-b)^{5/2}}-\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3d(a-b)^2}-\frac{b\coth(c+dx)\left(-(4a-b)\tanh^2(c+\right.}{8a^2d(a-b)^2\left(a-(a-b)\tanh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 - 12*a*b + 5*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(8*a^(7/2)*(a - b)^(5/2)*d) - ((4*a - 5*b)*(2*a - 3*b)*Coth[c + d*x])/(8*
a^3*(a - b)^2*d) - (b*Csch[c + d*x]*Sech[c + d*x]^3)/(4*a*(a - b)*d*(a - (a
 - b)*Tanh[c + d*x]^2)^2) - (b*Coth[c + d*x]*(4*a - 5*b - (4*a - b)*Tanh[c
+ d*x]^2))/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^2\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)\left(4a-5b+(-4a+b)x^2\right)}{x^2\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh\right.}{4a(a-b)d}$$

$$= -\frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{b\coth(c+dx)\left(4a-5b-(4a-b)\tanh^2(c\right.}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+dx\right.}$$

$$= -\frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3(a-b)^2d} - \frac{b\operatorname{csch}(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{b\coth(}{8a^2}$$

$$= -\frac{3b\left(8a^2-12ab+5b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a-b)^{5/2}d} - \frac{(4a-5b)(2a-3b)\coth(c+dx)}{8a^3(a-b)^2d} -$$

Mathematica [A] time = 1.81292, size = 225, normalized size = 1.05

$$\frac{\operatorname{csch}^6(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\frac{4a^{3/2}b^2\sinh(2(c+dx))}{a-b} - \frac{3b\left(8a^2-12ab+5b^2\right)(2a+b\cosh(2(c+dx))-b)^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} -\right.}{64a^{7/2}d\left(a\operatorname{csch}^2(c+dx)+b\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^6*((-3*b*(8*a^2 - 12*a*b + 5
*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]*(2*a - b + b*Cosh[2*(c +
```

```
d*x)])^2)/(a - b)^(5/2) - 8*Sqrt[a]*(2*a - b + b*Cosh[2*(c + d*x)])^2*Coth
[c + d*x] + (4*a^(3/2)*b^2*Sinh[2*(c + d*x)])/(a - b) + (Sqrt[a]*(10*a - 7*
b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a - b)^2))/(64*a
^(7/2)*d*(b + a*Csch[c + d*x]^2)^3)
```

Maple [B] time = 0.089, size = 1850, normalized size = 8.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/a^3*tanh(1/2*d*x+1/2*c)-1/2/d/a^3/tanh(1/2*d*x+1/2*c)+3/d/(tanh(1/2*
d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*b^2
/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-9/4/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)
^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^7-3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)
^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5
*b^2+49/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d
*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-7/d/a^3*b^
4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-3/d/(tanh(1/2*d*x+1/2*c)^4*a
-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^3*b^2+49/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1
/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3-7/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+
4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+3/d/(t
anh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+
a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-9/4/d/a^2*b^3/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)-3/d/(a^2-2*a*b+b^2)*b/a/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))+9/2/d/a^2/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-15/8/d/
a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d/(a^2-2*a*b+b^2)/a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-9/2/d/a^2/(a^2-2*a*b+b^
2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^3+15/8/d/a^3*b^4/(a^2-2
*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d/(a^2-2*a*b+b^2
)*b/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-9/2/d/a^2/(a^2-2*a*b+b^2)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2))*b^2+15/8/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))+3/d/(a^2-2*a*b+b^2)/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2
-9/2/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^
3+15/8/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 3.58571, size = 21173, normalized size = 98.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(12*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^8
+ 96*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)*sinh(d*x
 + c)^7 + 12*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*sinh(d*x + c)^
8 + 24*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*
x + c)^6 + 24*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5 +
 14*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^2)*sinh(d
*x + c)^6 + 32*a^4*b^2 - 136*a^3*b^3 + 164*a^2*b^4 - 60*a*b^5 + 48*(14*(8*a
^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b -
 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 8*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 4
5*a*b^5)*cosh(d*x + c)^4 + 8*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^
3 + 238*a^2*b^4 - 45*a*b^5 + 105*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a
*b^5)*cosh(d*x + c)^4 + 45*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4
 + 10*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(8*a^4*b^2 - 20*a^3*
b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 76*a^4*b^2 + 9
1*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 296*a^5*b +
548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c))*sinh(d*x
 + c)^3 + 8*(32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5)
*cosh(d*x + c)^2 + 8*(42*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*co
sh(d*x + c)^6 + 32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b
^5 + 45*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d
*x + c)^4 + 6*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4
 - 45*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b^3 - 12*a*b^4 +
5*b^5)*cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)*s
inh(d*x + c)^9 + (8*a^2*b^3 - 12*a*b^4 + 5*b^5)*sinh(d*x + c)^10 + (64*a^3*
b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 - 136
*a^2*b^3 + 100*a*b^4 - 25*b^5 + 45*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c
)^3 + (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c))*sinh(d
*x + c)^7 + 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*c
osh(d*x + c)^6 + 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b
^5 + 105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 -
136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*
(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^5 + 14*(64*a^3*b^2 - 136*a^2*b
^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^3 + 3*(64*a^4*b - 192*a^3*b^2 + 224*
a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*a^2*b^3 +
12*a*b^4 - 5*b^5 - 2*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25
*b^5)*cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)
^6 - 64*a^4*b + 192*a^3*b^2 - 224*a^2*b^3 + 120*a*b^4 - 25*b^5 + 35*(64*a^3
```

*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^4 + 15*(64*a^4*b - 1
92*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2
 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 192*a^
3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^3 - (64*a^4*b - 192
*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^3
 - (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^2 + (45*(8
*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 136*a^2*b^3
 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^6 - 64*a^3*b^2 + 136*a^2*b^3 - 100*a*b
^4 + 25*b^5 + 30*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5
)*cosh(d*x + c)^4 - 12*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 +
25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(8*a^2*b^3 - 12*a*b^4 + 5*b
^5)*cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cos
h(d*x + c)^7 + 6*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5
)*cosh(d*x + c)^5 - 4*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 2
5*b^5)*cosh(d*x + c)^3 - (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) + 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*(6*(8*a^4*b^2 - 20*a^3*b^
3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^7 + 9*(24*a^5*b - 76*a^4*b^2 + 91*a
^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^5 + 2*(64*a^6 - 296*a^5*b + 5
48*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c)^3 + (32*a^
5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5)*cosh(d*x + c))*si
nh(d*x + c))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^1
0 + 10*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)*sinh(d*x
 + c)^9 + (a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*sinh(d*x + c)^10 +
(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c
)^8 + (45*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^2 + (
8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d)*sinh(d*x + c
)^8 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^
5)*d*cosh(d*x + c)^6 + 8*(15*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*
cosh(d*x + c)^3 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*
b^4 - a^4*b^5)*d*cosh(d*x + c)^4 + 14*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 -
23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2
 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d)*sinh(d*x + c)^6 - 2*(8*a^9 - 36*a
^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^4
 + 4*(63*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^5 + 14*
(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c
)^3 + 3*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^
5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b
^4 - a^4*b^5)*d*cosh(d*x + c)^6 + 35*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 2
3*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^4 + 15*(8*a^9 - 36*a^8*b + 65*a^7*b^
2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^2 - (8*a^9 - 36*a^
8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d)*sinh(d*x + c)^4
 - (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x +
 c)^2 + 8*(15*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^7
 + 7*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*
x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a
^4*b^5)*d*cosh(d*x + c)^3 - (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 2
7*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^7*b^2 - 3*
a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^8 + 28*(8*a^8*b - 29*a^7*b^2
 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^6 + 30*(8*a^9 - 36*

a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^4
 - 12*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)
*d*cosh(d*x + c)^2 - (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^
4*b^5)*d)*sinh(d*x + c)^2 - (a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d +
 2*(5*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^9 + 4*(8*
a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^7
 + 6*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*
d*cosh(d*x + c)^5 - 4*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*
b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^3 - (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 2
3*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(6*(8*a^4*b^2
- 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^8 + 48*(8*a^4*b^2 - 20*a
^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)*sinh(d*x + c)^7 + 6*(8*a^4*b^2
 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*sinh(d*x + c)^8 + 12*(24*a^5*b - 76*a
^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^6 + 12*(24*a^5*b
 - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5 + 14*(8*a^4*b^2 - 20*a^3
*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*a^4*b^2
- 68*a^3*b^3 + 82*a^2*b^4 - 30*a*b^5 + 24*(14*(8*a^4*b^2 - 20*a^3*b^3 + 17*
a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3
- 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(64*a^6 - 296*a
^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c)^4
+ 4*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^
5 + 105*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^4 + 4
5*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 + 16*(21*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^
5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 +
10*a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3
 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(32*a^5*b - 1
44*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5)*cosh(d*x + c)^2 + 4*(42*
(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^6 + 32*a^5*b
- 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30*a*b^5 + 45*(24*a^5*b - 76*a^
4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^4 + 6*(64*a^6 - 2
96*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)*cosh(d*x + c
)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^10 +
 10*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)*sinh(d*x + c)^9 + (8*a^2*b
^3 - 12*a*b^4 + 5*b^5)*sinh(d*x + c)^10 + (64*a^3*b^2 - 136*a^2*b^3 + 100*a
*b^4 - 25*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25
b^5 + 45(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^8 +
 8*(15*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^3 + (64*a^3*b^2 - 136*a
^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(64*a^4*b -
 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^6 + 2*(64*a^
4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5 + 105*(8*a^2*b^3 - 12*
a*b^4 + 5*b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 -
 25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(8*a^2*b^3 - 12*a*b^4 + 5
*b^5)*cosh(d*x + c)^5 + 14*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*
cosh(d*x + c)^3 + 3*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*
b^5)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*a^2*b^3 + 12*a*b^4 - 5*b^5 - 2*(64*
a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^4 + 2
(105(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^6 - 64*a^4*b + 192*a^3*b
^2 - 224*a^2*b^3 + 120*a*b^4 - 25*b^5 + 35*(64*a^3*b^2 - 136*a^2*b^3 + 100*
a*b^4 - 25*b^5)*cosh(d*x + c)^4 + 15*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3
- 120*a*b^4 + 25*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(15*(8*a^2*b^3 -
 12*a*b^4 + 5*b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^
4 - 25*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120
*a*b^4 + 25*b^5)*cosh(d*x + c)^3 - (64*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 -
120*a*b^4 + 25*b^5)*cosh(d*x + c))*sinh(d*x + c)^3 - (64*a^3*b^2 - 136*a^2*
b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 12*a*b^4 + 5*b
^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*co
sh(d*x + c)^6 - 64*a^3*b^2 + 136*a^2*b^3 - 100*a*b^4 + 25*b^5 + 30*(64*a^4*
b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^4 - 12*(6

```
4*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 2*(5*(8*a^2*b^3 - 12*a*b^4 + 5*b^5)*cosh(d*x + c)^9 + 4*(
64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c)^7 + 6*(64*a^4*
b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^5 - 4*(64
*a^4*b - 192*a^3*b^2 + 224*a^2*b^3 - 120*a*b^4 + 25*b^5)*cosh(d*x + c)^3 -
(64*a^3*b^2 - 136*a^2*b^3 + 100*a*b^4 - 25*b^5)*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh
(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 8*
(6*(8*a^4*b^2 - 20*a^3*b^3 + 17*a^2*b^4 - 5*a*b^5)*cosh(d*x + c)^7 + 9*(24*
a^5*b - 76*a^4*b^2 + 91*a^3*b^3 - 49*a^2*b^4 + 10*a*b^5)*cosh(d*x + c)^5 +
2*(64*a^6 - 296*a^5*b + 548*a^4*b^2 - 509*a^3*b^3 + 238*a^2*b^4 - 45*a*b^5)
*cosh(d*x + c)^3 + (32*a^5*b - 144*a^4*b^2 + 219*a^3*b^3 - 137*a^2*b^4 + 30
*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a
^4*b^5)*d*cosh(d*x + c)^10 + 10*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)
*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b
^5)*d*sinh(d*x + c)^10 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 +
5*a^4*b^5)*d*cosh(d*x + c)^8 + (45*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b
^5)*d*cosh(d*x + c)^2 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5
*a^4*b^5)*d)*sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^
3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^7*b^2 - 3*a^6*b^3
+ 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^3 + (8*a^8*b - 29*a^7*b^2 + 39*a^6*b
^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7
*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^4 + 14*(8*a^8*b - 2
9*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^2 + (8*a^9
 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d)*sinh(d*x
 + c)^6 - 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^
4*b^5)*d*cosh(d*x + c)^4 + 4*(63*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5
)*d*cosh(d*x + c)^5 + 14*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 +
5*a^4*b^5)*d*cosh(d*x + c)^3 + 3*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^
3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7*
b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^6 + 35*(8*a^8*b - 29
*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c)^4 + 15*(8*a
^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*
x + c)^2 - (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4
*b^5)*d)*sinh(d*x + c)^4 - (8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4
+ 5*a^4*b^5)*d*cosh(d*x + c)^2 + 8*(15*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a
^4*b^5)*d*cosh(d*x + c)^7 + 7*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b
^4 + 5*a^4*b^5)*d*cosh(d*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a
^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^3 - (8*a^9 - 36*a^8*b + 65
*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + (45*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*d*cosh(d*x + c)^8 +
 28*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x
 + c)^6 + 30*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a
^4*b^5)*d*cosh(d*x + c)^4 - 12*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3
+ 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^2 - (8*a^8*b - 29*a^7*b^2 + 39*a^
6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d)*sinh(d*x + c)^2 - (a^7*b^2 - 3*a^6*b^3 +
 3*a^5*b^4 - a^4*b^5)*d + 2*(5*(a^7*b^2 - 3*a^6*b^3 + 3*a^5*b^4 - a^4*b^5)*
d*cosh(d*x + c)^9 + 4*(8*a^8*b - 29*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a
^4*b^5)*d*cosh(d*x + c)^7 + 6*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 59*a^6*b^3 +
 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^5 - 4*(8*a^9 - 36*a^8*b + 65*a^7*b
^2 - 59*a^6*b^3 + 27*a^5*b^4 - 5*a^4*b^5)*d*cosh(d*x + c)^3 - (8*a^8*b - 29
*a^7*b^2 + 39*a^6*b^3 - 23*a^5*b^4 + 5*a^4*b^5)*d*cosh(d*x + c))*sinh(d*x +
 c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.5999, size = 455, normalized size = 2.12

$$-\frac{3\left(8a^{2}b-12ab^{2}+5b^{3}\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^{2}+ab}}\right)}{8\left(a^{5}d-2a^{4}bd+a^{3}b^{2}d\right)\sqrt{-a^{2}+ab}}-\frac{16a^{2}b^{2}e^{(6dx+6c)}-20ab^{3}e^{(6dx+6c)}+7b^{4}e^{(6dx+6c)}+80a^{3}be^{(4dx+4c}}{4\left(a^{5}d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -3/8*(8*a^2*b - 12*a*b^2 + 5*b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/
sqrt(-a^2 + a*b))/((a^5*d - 2*a^4*b*d + a^3*b^2*d)*sqrt(-a^2 + a*b)) - 1/4*
(16*a^2*b^2*e^(6*d*x + 6*c) - 20*a*b^3*e^(6*d*x + 6*c) + 7*b^4*e^(6*d*x + 6
*c) + 80*a^3*b*e^(4*d*x + 4*c) - 136*a^2*b^2*e^(4*d*x + 4*c) + 86*a*b^3*e^(
4*d*x + 4*c) - 21*b^4*e^(4*d*x + 4*c) + 64*a^2*b^2*e^(2*d*x + 2*c) - 76*a*b
^3*e^(2*d*x + 2*c) + 21*b^4*e^(2*d*x + 2*c) + 10*a*b^3 - 7*b^4)/((a^5*d - 2
*a^4*b*d + a^3*b^2*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d
*x + 2*c) + b)^2) - 2/(a^3*d*(e^(2*d*x + 2*c) - 1))
```

3.58 $\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=224

$$\frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4d(a-b)^{5/2}}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{8a^3d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{b(2a-3b)\cosh(c+dx}{4a^2d(a-b)\left(a+b\cosh^2(c+d\right.}$$

```
[Out] (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a -
 b]])/(8*a^4*(a - b)^(5/2)*d) + ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d
) - ((2*a - 3*b)*b*Cosh[c + d*x])/(4*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]
^2)^2) - ((a - 4*b)*(4*a - 3*b)*b*Cosh[c + d*x])/(8*a^3*(a - b)^2*d*(a - b
+ b*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cos
h[c + d*x]^2)^2)
```

Rubi [A] time = 0.43874, antiderivative size = 224, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4d(a-b)^{5/2}}-\frac{b(a-4b)(4a-3b)\cosh(c+dx)}{8a^3d(a-b)^2\left(a+b\cosh^2(c+dx)-b\right)}-\frac{b(2a-3b)\cosh(c+dx}{4a^2d(a-b)\left(a+b\cosh^2(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b]*Cosh[c + d*x])/Sqrt[a -
 b]])/(8*a^4*(a - b)^(5/2)*d) + ((a + 6*b)*ArcTanh[Cosh[c + d*x]])/(2*a^4*d
) - ((2*a - 3*b)*b*Cosh[c + d*x])/(4*a^2*(a - b)*d*(a - b + b*Cosh[c + d*x]
^2)^2) - ((a - 4*b)*(4*a - 3*b)*b*Cosh[c + d*x])/(8*a^3*(a - b)^2*d*(a - b
+ b*Cosh[c + d*x]^2)) - (Coth[c + d*x]*Csch[c + d*x])/(2*a*d*(a - b + b*Cos
h[c + d*x]^2)^2)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-b+bx^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+b+5bx^2}{\left(1-x^2\right)\left(a-b+bx^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{2ad}$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{\coth(c+dx)\operatorname{csch}(c+dx)}{2ad\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(a-4b)(4a-3b)b\cosh(c+dx)}{8a^3(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{}{2a}$$

$$= -\frac{(2a-3b)b\cosh(c+dx)}{4a^2(a-b)d\left(a-b+b\cosh^2(c+dx)\right)^2} - \frac{(a-4b)(4a-3b)b\cosh(c+dx)}{8a^3(a-b)^2d\left(a-b+b\cosh^2(c+dx)\right)} - \frac{}{2a}$$

$$= \frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\cosh(c+dx)}{\sqrt{a-b}}\right)}{8a^4(a-b)^{5/2}d} + \frac{(a+6b)\tanh^{-1}(\cosh(c+dx))}{2a^4d} - \frac{}{4a}$$

Mathematica [C] time = 2.94302, size = 419, normalized size = 1.87

$$\operatorname{csch}^5(c+dx)(2a+b\cosh(2(c+dx))-b)\left(\frac{8a^2b^2\coth(c+dx)}{a-b} + \frac{b^{3/2}\left(35a^2-56ab+24b^2\right)\operatorname{csch}(c+dx)(2a+b\cosh(2(c+dx))-b)^2\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\,\text{ta}}{}\right.}{(a-b)^{5/2}}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((2*a - b + b*Cosh[2*(c + d*x)])*Csch[c + d*x]^5*((8*a^2*b^2*Coth[c + d*x])
/(a - b) + (2*a*(11*a - 8*b)*b^2*(2*a - b + b*Cosh[2*(c + d*x)])*Coth[c + d
*x])/(a - b)^2 + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqrt[b] - I*Sq
rt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d*x)])^2*Csc
h[c + d*x])/(a - b)^(5/2) + (b^(3/2)*(35*a^2 - 56*a*b + 24*b^2)*ArcTan[(Sqr
t[b] + I*Sqrt[a]*Tanh[(c + d*x)/2])/Sqrt[a - b]]*(2*a - b + b*Cosh[2*(c + d
*x)])^2*Csch[c + d*x])/(a - b)^(5/2) - a*(2*a - b + b*Cosh[2*(c + d*x)])^2*
Csch[(c + d*x)/2]^2*Csch[c + d*x] - 4*(a + 6*b)*(2*a - b + b*Cosh[2*(c + d*
x)])^2*Csch[c + d*x]*Log[Tanh[(c + d*x)/2]] - a*(2*a - b + b*Cosh[2*(c + d*
x)])^2*Csch[c + d*x]*Sech[(c + d*x)/2]^2))/(64*a^4*d*(b + a*Csch[c + d*x]^2
)^3)
```

Maple [B] time = 0.091, size = 1225, normalized size = 5.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/8/d*tanh(1/2*d*x+1/2*c)^2/a^3-1/8/d/a^3/tanh(1/2*d*x+1/2*c)^2-1/2/d/a^3*l
n(tanh(1/2*d*x+1/2*c))-3/d/a^4*ln(tanh(1/2*d*x+1/2*c))*b-13/4/d/(tanh(1/2*d
*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*b^2+10/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4
*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^6-6/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+
1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^6+39/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d
*x+1/2*c)^2*b+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-67/2/d/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4*b^3+46/d/a^3*b^4/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-20/d*b^5/a^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2
*d*x+1/2*c)^4-39/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*t
anh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*b^2+26/
d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-14/d*b^4/a^3/(tanh(1
/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+13/4/d/a*b^2/(tanh(1/2*d*x+1/2*c)^4*a
-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)-5
/2/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)+35/8/d/a^2*b^2/(a^2-2*a*b+b^2)/(a*b-b^2
)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))-7/d
/a^3*b^3/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^
2*a-2*a+4*b)/(a*b-b^2)^(1/2))+3/d*b^4/a^4/(a^2-2*a*b+b^2)/(a*b-b^2)^(1/2)*a
rctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a-2*a+4*b)/(a*b-b^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((4*a^2*b^2*e^(11*c) - 19*a*b^3*e^(11*c) + 12*b^4*e^(11*c))*e^(11*d*x)
 + (32*a^3*b*e^(9*c) - 128*a^2*b^2*e^(9*c) + 129*a*b^3*e^(9*c) - 36*b^4*e^(
9*c))*e^(9*d*x) + 2*(32*a^4*e^(7*c) - 80*a^3*b*e^(7*c) + 94*a^2*b^2*e^(7*c)
 - 55*a*b^3*e^(7*c) + 12*b^4*e^(7*c))*e^(7*d*x) + 2*(32*a^4*e^(5*c) - 80*a^
3*b*e^(5*c) + 94*a^2*b^2*e^(5*c) - 55*a*b^3*e^(5*c) + 12*b^4*e^(5*c))*e^(5*
d*x) + (32*a^3*b*e^(3*c) - 128*a^2*b^2*e^(3*c) + 129*a*b^3*e^(3*c) - 36*b^4
*e^(3*c))*e^(3*d*x) + (4*a^2*b^2*e^c - 19*a*b^3*e^c + 12*b^4*e^c)*e^(d*x))/
(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d + (a^5*b^2*d*e^(12*c) - 2*a^4*b^3*d*e^
(12*c) + a^3*b^4*d*e^(12*c))*e^(12*d*x) + 2*(4*a^6*b*d*e^(10*c) - 11*a^5*b^
2*d*e^(10*c) + 10*a^4*b^3*d*e^(10*c) - 3*a^3*b^4*d*e^(10*c))*e^(10*d*x) + (
16*a^7*d*e^(8*c) - 64*a^6*b*d*e^(8*c) + 95*a^5*b^2*d*e^(8*c) - 62*a^4*b^3*d
*e^(8*c) + 15*a^3*b^4*d*e^(8*c))*e^(8*d*x) - 4*(8*a^7*d*e^(6*c) - 28*a^6*b*
d*e^(6*c) + 37*a^5*b^2*d*e^(6*c) - 22*a^4*b^3*d*e^(6*c) + 5*a^3*b^4*d*e^(6*
c))*e^(6*d*x) + (16*a^7*d*e^(4*c) - 64*a^6*b*d*e^(4*c) + 95*a^5*b^2*d*e^(4*
c) - 62*a^4*b^3*d*e^(4*c) + 15*a^3*b^4*d*e^(4*c))*e^(4*d*x) + 2*(4*a^6*b*d*
e^(2*c) - 11*a^5*b^2*d*e^(2*c) + 10*a^4*b^3*d*e^(2*c) - 3*a^3*b^4*d*e^(2*c)
)*e^(2*d*x)) + 1/2*(a + 6*b)*log((e^(d*x + c) + 1)*e^(-c))/(a^4*d) - 1/2*(a
 + 6*b)*log((e^(d*x + c) - 1)*e^(-c))/(a^4*d) + 8*integrate(1/32*((35*a^2*b
^2*e^(3*c) - 56*a*b^3*e^(3*c) + 24*b^4*e^(3*c))*e^(3*d*x) - (35*a^2*b^2*e^c
 - 56*a*b^3*e^c + 24*b^4*e^c)*e^(d*x))/(a^6*b - 2*a^5*b^2 + a^4*b^3 + (a^6*
b*e^(4*c) - 2*a^5*b^2*e^(4*c) + a^4*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^7*e^(2*
c) - 5*a^6*b*e^(2*c) + 4*a^5*b^2*e^(2*c) - a^4*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.59 $\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=259

$$\frac{b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a-b)^{5/2}}-\frac{\left(8a^2-52ab+35b^2\right)\coth^3(c+dx)}{24a^3d(a-b)^2}+\frac{\left(-4a^2b+8a^3-45ab^2+35b\cdots\right.}{8a^4d(a-b)^2}$$

```
[Out] (b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]
])/(8*a^(9/2)*(a - b)^(5/2)*d) + ((8*a^3 - 4*a^2*b - 45*a*b^2 + 35*b^3)*Cot
h[c + d*x])/(8*a^4*(a - b)^2*d) - ((8*a^2 - 52*a*b + 35*b^2)*Coth[c + d*x]^
3)/(24*a^3*(a - b)^2*d) - (b*Csch[c + d*x]^3*Sech[c + d*x]^3)/(4*a*(a - b)*
d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((10*a - 7*b)*b*Csch[c + d*x]^3*Sech[c
 + d*x])/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.337981, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 468, 577, 570, 208}

$$\frac{b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}d(a-b)^{5/2}}-\frac{\left(8a^2-52ab+35b^2\right)\coth^3(c+dx)}{24a^3d(a-b)^2}+\frac{\left(-4a^2b+8a^3-45ab^2+35b\cdots\right.}{8a^4d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] (b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]
])/(8*a^(9/2)*(a - b)^(5/2)*d) + ((8*a^3 - 4*a^2*b - 45*a*b^2 + 35*b^3)*Cot
h[c + d*x])/(8*a^4*(a - b)^2*d) - ((8*a^2 - 52*a*b + 35*b^2)*Coth[c + d*x]^
3)/(24*a^3*(a - b)^2*d) - (b*Csch[c + d*x]^3*Sech[c + d*x]^3)/(4*a*(a - b)*
d*(a - (a - b)*Tanh[c + d*x]^2)^2) - ((10*a - 7*b)*b*Csch[c + d*x]^3*Sech[c
 + d*x])/(8*a^2*(a - b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```


Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^4\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2\left(4a-7b+(-4a+b)x^2\right)}{x^4\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c\right.}{4a(a-b)d}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(10a-7b)b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Su}}{}$$

$$= -\frac{b\operatorname{csch}^3(c+dx)\operatorname{sech}^3(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(10a-7b)b\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)}{8a^2(a-b)^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Su}}{}$$

$$= \frac{\left(8a^3-4a^2b-45ab^2+35b^3\right)\coth(c+dx)}{8a^4(a-b)^2d} - \frac{\left(8a^2-52ab+35b^2\right)\coth^3(c+dx)}{24a^3(a-b)^2d} - \frac{b}{4a(a-}$$

$$= \frac{b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{9/2}(a-b)^{5/2}d} + \frac{\left(8a^3-4a^2b-45ab^2+35b^3\right)\coth(c+}{8a^4(a-b)^2d}$$

Mathematica [A] time = 2.76237, size = 167, normalized size = 0.64

$$\frac{\frac{3b^2\left(48a^2-80ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{5/2}} + \sqrt{a}\left(\frac{3b^3\sinh(2(c+dx))\left(-32a^2+b(11b-14a)\cosh(2(c+dx))+40ab-11b^2\right)}{(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} - 8\coth(c+dx)\left(a\operatorname{csch}^2\right.\right.}{24a^{9/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*b^2*(48*a^2 - 80*a*b + 35*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt
[a]])/(a - b)^(5/2) + Sqrt[a]*(-8*Coth[c + d*x]*(-2*a - 9*b + a*Csch[c + d*
x]^2) + (3*b^3*(-32*a^2 + 40*a*b - 11*b^2 + b*(-14*a + 11*b)*Cosh[2*(c + d*
x)])*Sinh[2*(c + d*x)])/((a - b)^2*(2*a - b + b*Cosh[2*(c + d*x)])^2)))/(24
*a^(9/2)*d)
```

Maple [B] time = 0.111, size = 1930, normalized size = 7.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 13/4/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+35/8/d*b^4/a^4/(a
^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-35/8/d*b^4/a^4/(a^2-2*a*b+b^2)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2))+13/4/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2
*d*x+1/2*c)^7+11/d*b^5/a^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+11/
d*b^5/a^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-69/4/d/a^3*b^4/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-69/4/d/a^3*b^4/(tanh(1/2*d*x+1/2*c)
^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^3-4/d/a^2*b^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)-10/d/a^3*b^3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arct
anh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+10/d/a^3*b^
3/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-4/d/a^2*b^3/(tanh(1/2*d*x+1/
2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^7+4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*
x+1/2*c)^5+4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/
2*d*x+1/2*c)^2*b+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+6/d/a^2
/(a^2-2*a*b+b^2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^2-6/d/a^2/(a^2-2*a*b+b^2)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2-35/8/d*b^5/a^4/(a^2-2*a*b+b^2)/(-b*(a-b))^
(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-35/8/d*b^5/a^4/(a^2-2*a*b+b^2)/(-b*(a-b)
)^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+10/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+10/d/a^3*b^4/(a^2-2*a*b+b^2)/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-6/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b^3-6/d/a^2/(a^2-2*a*b+b^2)/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^3+3/2/d/a^4*tanh(1/2*d*x+1/2*c)*b+3/2/d
/a^4/tanh(1/2*d*x+1/2*c)*b+3/8/d/a^3*tanh(1/2*d*x+1/2*c)+3/8/d/a^3/tanh(1/2
*d*x+1/2*c)-1/24/d/a^3*tanh(1/2*d*x+1/2*c)^3-1/24/d/a^3/tanh(1/2*d*x+1/2*c)
```

```
^3
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.60946, size = 518, normalized size = 2.

$$\frac{\left(48\,a^2b^2 - 80\,ab^3 + 35\,b^4\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{8\left(a^6d - 2\,a^5bd + a^4b^2d\right)\sqrt{-a^2+ab}} + \frac{24\,a^2b^3e^{(6\,dx+6\,c)} - 32\,ab^4e^{(6\,dx+6\,c)} + 11\,b^5e^{(6\,dx+6\,c)} + 112\,a^3b^2e^{(4\,dx+4\,c}}{4\left(a^6d\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(48*a^2*b^2 - 80*a*b^3 + 35*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a -
b)/sqrt(-a^2 + a*b))/((a^6*d - 2*a^5*b*d + a^4*b^2*d)*sqrt(-a^2 + a*b)) + 1
/4*(24*a^2*b^3*e^(6*d*x + 6*c) - 32*a*b^4*e^(6*d*x + 6*c) + 11*b^5*e^(6*d*x
 + 6*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 200*a^2*b^3*e^(4*d*x + 4*c) + 130*a
*b^4*e^(4*d*x + 4*c) - 33*b^5*e^(4*d*x + 4*c) + 88*a^2*b^3*e^(2*d*x + 2*c)
- 112*a*b^4*e^(2*d*x + 2*c) + 33*b^5*e^(2*d*x + 2*c) + 14*a*b^4 - 11*b^5)/(
(a^6*d - 2*a^5*b*d + a^4*b^2*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) -
2*b*e^(2*d*x + 2*c) + b)^2) + 2/3*(9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c
) - 18*b*e^(2*d*x + 2*c) + 2*a + 9*b)/(a^4*d*(e^(2*d*x + 2*c) - 1)^3)
```

3.60 $\int \frac{1}{1+\sinh^2(x)} \, dx$

Optimal. Leaf size=2

$$\tanh(x)$$

[Out] Tanh[x]

Rubi [A] time = 0.0163758, antiderivative size = 2, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3175, 3767, 8}

$$\tanh(x)$$

Antiderivative was successfully verified.

[In] Int[(1 + Sinh[x]^2)^(-1),x]

[Out] Tanh[x]

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned} \int \frac{1}{1+\sinh^2(x)} \, dx &= \int \operatorname{sech}^2(x) \, dx \\ &= i \operatorname{Subst}\left(\int 1 \, dx, x, -i \tanh(x)\right) \\ &= \tanh(x) \end{aligned}$$

Mathematica [A] time = 0.0036714, size = 2, normalized size = 1.

$$\tanh(x)$$

Antiderivative was successfully verified.

[In] Integrate[(1 + Sinh[x]^2)^(-1),x]

```
[Out] Tanh[x]
```

Maple [B] time = 0.016, size = 17, normalized size = 8.5

$$2\frac{\tanh(x/2)}{(\tanh(x/2))^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2),x)
```

```
[Out] 2*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

Maxima [B] time = 1.02647, size = 14, normalized size = 7.

$$\frac{2}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 2/(e^(-2*x) + 1)
```

Fricas [B] time = 1.75827, size = 70, normalized size = 35.

$$-\frac{2}{\cosh(x)^2+2\cosh(x)\sinh(x)+\sinh(x)^2+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -2/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [B] time = 0.894588, size = 14, normalized size = 7.

$$\frac{2\tanh\left(\frac{x}{2}\right)}{\tanh^2\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2),x)
```

```
[Out] 2*tanh(x/2)/(tanh(x/2)**2 + 1)
```

<u>Giac</u> [B] time = 1.29848, size = 14, normalized size = 7.

$$-\frac{2}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2),x, algorithm="giac")
```

```
[Out] -2/(e^(2*x) + 1)
```

3.61 $\int \frac{1}{\left(1+\sinh^2(x)\right)^2} \, dx$

Optimal. Leaf size=11

$$\tanh(x)-\frac{\tanh^3(x)}{3}$$

```
[Out] Tanh[x] - Tanh[x]^3/3
```

Rubi [A] time = 0.0190848, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3175, 3767}

$$\tanh(x)-\frac{\tanh^3(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(-2),x]
```

```
[Out] Tanh[x] - Tanh[x]^3/3
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1+\sinh^2(x)\right)^2} \, dx &= \int \text{sech}^4(x) \, dx\\ &= i \,\text{Subst}\left(\int \left(1+x^2\right) \, dx, x, -i \tanh(x)\right)\\ &= \tanh(x)-\frac{\tanh^3(x)}{3}\end{aligned}$$

Mathematica [A] time = 0.0027172, size = 17, normalized size = 1.55

$$\frac{2 \tanh(x)}{3}+\frac{1}{3} \tanh(x) \text{sech}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(-2),x]
```

```
[Out] (2*Tanh[x])/3 + (Sech[x]^2*Tanh[x])/3
```

Maple [B] time = 0.015, size = 36, normalized size = 3.3

$$-2\frac{-(\tanh(x/2))^5 - 2/3\,(\tanh(x/2))^3 - \tanh(x/2)}{\left((\tanh(x/2))^2+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^2,x)
```

```
[Out] -2*(-tanh(1/2*x)^5-2/3*tanh(1/2*x)^3-tanh(1/2*x))/(tanh(1/2*x)^2+1)^3
```

Maxima [B] time = 1.03861, size = 66, normalized size = 6.

$$\frac{4\,e^{(-2\,x)}}{3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1}+\frac{4}{3\left(3\,e^{(-2\,x)}+3\,e^{(-4\,x)}+e^{(-6\,x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 4*e^(-2*x)/(3*e^(-2*x) + 3*e^(-4*x) + e^(-6*x) + 1) + 4/3/(3*e^(-2*x) + 3*e
^(-4*x) + e^(-6*x) + 1)
```

Fricas [B] time = 1.79279, size = 286, normalized size = 26.

$$-\frac{8\,(2\,\cosh(x)+\sinh(x))}{3\left(\cosh(x)^5+5\,\cosh(x)\sinh(x)^4+\sinh(x)^5+\left(10\,\cosh(x)^2+3\right)\sinh(x)^3+3\,\cosh(x)^3+\left(10\,\cosh(x)^3+9\,\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -8/3*(2*cosh(x) + sinh(x))/(cosh(x)^5 + 5*cosh(x)*sinh(x)^4 + sinh(x)^5 + (
10*cosh(x)^2 + 3)*sinh(x)^3 + 3*cosh(x)^3 + (10*cosh(x)^3 + 9*cosh(x))*sinh
(x)^2 + (5*cosh(x)^4 + 9*cosh(x)^2 + 2)*sinh(x) + 4*cosh(x))
```

Sympy [B] time = 4.90601, size = 104, normalized size = 9.45

$$\frac{6\tanh^5\left(\frac{x}{2}\right)}{3\tanh^6\left(\frac{x}{2}\right)+9\tanh^4\left(\frac{x}{2}\right)+9\tanh^2\left(\frac{x}{2}\right)+3}+\frac{4\tanh^3\left(\frac{x}{2}\right)}{3\tanh^6\left(\frac{x}{2}\right)+9\tanh^4\left(\frac{x}{2}\right)+9\tanh^2\left(\frac{x}{2}\right)+3}+\frac{6\,\mathrm{ta}}{3\tanh^6\left(\frac{x}{2}\right)+9\,\mathrm{tanh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**2,x)
```

```
[Out] 6*tanh(x/2)**5/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3) + 4*t
anh(x/2)**3/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3) + 6*tanh
(x/2)/(3*tanh(x/2)**6 + 9*tanh(x/2)**4 + 9*tanh(x/2)**2 + 3)
```

Giac [A] time = 1.27509, size = 24, normalized size = 2.18

$$-\frac{4\left(3\,e^{(2\,x)}+1\right)}{3\left(e^{(2\,x)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^2,x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*x) + 1)/(e^(2*x) + 1)^3
```

3.62 $\int \frac{1}{\left(1+\sinh^2(x)\right)^3} \, dx$

Optimal. Leaf size=19

$$\frac{\tanh^5(x)}{5}-\frac{2\tanh^3(x)}{3}+\tanh(x)$$

```
[Out] Tanh[x] - (2*Tanh[x]^3)/3 + Tanh[x]^5/5
```

Rubi [A] time = 0.0201132, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3175, 3767}

$$\frac{\tanh^5(x)}{5}-\frac{2\tanh^3(x)}{3}+\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(-3),x]
```

```
[Out] Tanh[x] - (2*Tanh[x]^3)/3 + Tanh[x]^5/5
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(1+\sinh^2(x)\right)^3} \, dx &= \int \operatorname{sech}^6(x) \, dx \\
&= i \operatorname{Subst}\left(\int \left(1+2x^2+x^4\right) \, dx, x, -i \tanh(x)\right) \\
&= \tanh(x)-\frac{2\tanh^3(x)}{3}+\frac{\tanh^5(x)}{5}
\end{aligned}$$

Mathematica [A] time = 0.0029111, size = 27, normalized size = 1.42

$$\frac{8\tanh(x)}{15}+\frac{1}{5}\tanh(x)\operatorname{sech}^4(x)+\frac{4}{15}\tanh(x)\operatorname{sech}^2(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(-3),x]
```

```
[Out] (8*Tanh[x])/15 + (4*Sech[x]^2*Tanh[x])/15 + (Sech[x]^4*Tanh[x])/5
```

Maple [B] time = 0.019, size = 52, normalized size = 2.7

$$-2\frac{1}{\left((\tanh(x/2))^2+1\right)^5}\left(-(\tanh(x/2))^9-4/3\,(\tanh(x/2))^7-\frac{58\,(\tanh(x/2))^5}{15}-4/3\,(\tanh(x/2))^3-\tanh(x/2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^3,x)
```

```
[Out] -2*(-tanh(1/2*x)^9-4/3*tanh(1/2*x)^7-58/15*tanh(1/2*x)^5-4/3*tanh(1/2*x)^3-
tanh(1/2*x))/(tanh(1/2*x)^2+1)^5
```

Maxima [B] time = 1.04492, size = 150, normalized size = 7.89

$$\frac{16\,e^{(-2\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}+\frac{32\,e^{(-4\,x)}}{3\left(5\,e^{(-2\,x)}+10\,e^{(-4\,x)}+10\,e^{(-6\,x)}+5\,e^{(-8\,x)}+e^{(-10\,x)}+1\right)}+\frac{}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 16/3*e^(-2*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*x) + e^(-10
*x) + 1) + 32/3*e^(-4*x)/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(-8*
x) + e^(-10*x) + 1) + 16/15/(5*e^(-2*x) + 10*e^(-4*x) + 10*e^(-6*x) + 5*e^(
-8*x) + e^(-10*x) + 1)
```

Fricas [B] time = 1.78008, size = 628, normalized size = 33.05

$$-\frac{}{15\left(\cosh(x)^8+8\,\cosh(x)\sinh(x)^7+\sinh(x)^8+\left(28\,\cosh(x)^2+5\right)\sinh(x)^6+5\,\cosh(x)^6+2\left(28\,\cosh(x)^3+15\,\cos\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -16/15*(11*cosh(x)^2 + 18*cosh(x)*sinh(x) + 11*sinh(x)^2 + 5)/(cosh(x)^8 +
8*cosh(x)*sinh(x)^7 + sinh(x)^8 + (28*cosh(x)^2 + 5)*sinh(x)^6 + 5*cosh(x)^
6 + 2*(28*cosh(x)^3 + 15*cosh(x))*sinh(x)^5 + 5*(14*cosh(x)^4 + 15*cosh(x)^
2 + 2)*sinh(x)^4 + 10*cosh(x)^4 + 4*(14*cosh(x)^5 + 25*cosh(x)^3 + 10*cosh(
x))*sinh(x)^3 + (28*cosh(x)^6 + 75*cosh(x)^4 + 60*cosh(x)^2 + 11)*sinh(x)^2
 + 11*cosh(x)^2 + 2*(4*cosh(x)^7 + 15*cosh(x)^5 + 20*cosh(x)^3 + 9*cosh(x))
*sinh(x) + 5)
```

Sympy [B] time = 18.1921, size = 260, normalized size = 13.68

$$\frac{30\tanh^9\left(\frac{x}{2}\right)}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^8\left(\frac{x}{2}\right)+150\tanh^6\left(\frac{x}{2}\right)+150\tanh^4\left(\frac{x}{2}\right)+75\tanh^2\left(\frac{x}{2}\right)+15}+\frac{}{15\tanh^{10}\left(\frac{x}{2}\right)+75\tanh^8\left(\frac{x}{2}\right)+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**3,x)
```

```
[Out] 30*tanh(x/2)**9/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 15
0*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15) + 40*tanh(x/2)**7/(15*tanh(x/2)**10
+ 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 +
 15) + 116*tanh(x/2)**5/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)
**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15) + 40*tanh(x/2)**3/(15*tanh(x
/2)**10 + 75*tanh(x/2)**8 + 150*tanh(x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x
/2)**2 + 15) + 30*tanh(x/2)/(15*tanh(x/2)**10 + 75*tanh(x/2)**8 + 150*tanh(
x/2)**6 + 150*tanh(x/2)**4 + 75*tanh(x/2)**2 + 15)
```

Giac [A] time = 1.27453, size = 32, normalized size = 1.68

$$-\frac{16\left(10\,e^{(4\,x)}+5\,e^{(2\,x)}+1\right)}{15\left(e^{(2\,x)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -16/15*(10*e^(4*x) + 5*e^(2*x) + 1)/(e^(2*x) + 1)^5
```

3.63 $\int \frac{1}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=15

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}$$

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2]

Rubi [A] time = 0.0127272, antiderivative size = 15, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3181, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}$$

Antiderivative was successfully verified.

[In] Int[(1 - Sinh[x]^2)^(-1),x]

[Out] ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2]

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}\end{aligned}$$

Mathematica [F] time = 0.0215643, size = 0, normalized size = 0.

$$\int \frac{1}{1-\sinh^2(x)}\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(1 - Sinh[x]^2)^(-1),x]

[Out] Integrate[(1 - Sinh[x]^2)^(-1), x]

Maple [B] time = 0.016, size = 40, normalized size = 2.7

$$\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)-2)\right)+\frac{\sqrt{2}}{2}\operatorname{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2),x)
```

```
[Out] 1/2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+1/2*2^(1/2)*arctanh(1/4*
(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [B] time = 1.53046, size = 82, normalized size = 5.47

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{4}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/4*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1))
```

Fricas [B] time = 1.84293, size = 215, normalized size = 14.33

$$\frac{1}{4}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^{2}-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^{2}-2\sqrt{2}+3}{\cosh(x)^{2}+\sinh(x)^{2}-3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3))
```

Sympy [B] time = 3.56741, size = 209, normalized size = 13.93

$$\frac{12\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{48+34\sqrt{2}}+\frac{17\log\left(\tanh\left(\frac{x}{2}\right)-1+\sqrt{2}\right)}{48+34\sqrt{2}}+\frac{12\sqrt{2}\log\left(\tanh\left(\frac{x}{2}\right)+1+\sqrt{2}\right)}{48+34\sqrt{2}}+\frac{17\log\left(\tanh\left(\frac{x}{2}\right)+\right.}{48+34\sqrt{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2),x)
```

```
[Out] 12*sqrt(2)*log(tanh(x/2) - 1 + sqrt(2))/(48 + 34*sqrt(2)) + 17*log(tanh(x/2
) - 1 + sqrt(2))/(48 + 34*sqrt(2)) + 12*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2)
```

```
)/(48 + 34*sqrt(2)) + 17*log(tanh(x/2) + 1 + sqrt(2))/(48 + 34*sqrt(2)) - 1
7*log(tanh(x/2) - sqrt(2) - 1)/(48 + 34*sqrt(2)) - 12*sqrt(2)*log(tanh(x/2)
 - sqrt(2) - 1)/(48 + 34*sqrt(2)) - 17*log(tanh(x/2) - sqrt(2) + 1)/(48 + 3
4*sqrt(2)) - 12*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)/(48 + 34*sqrt(2))
```

Giac [B] time = 1.26601, size = 50, normalized size = 3.33

$$-\frac{1}{4}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/4*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6))
```

3.64 $\int \frac{1}{\left(1-\sinh^2(x)\right)^2} dx$

Optimal. Leaf size=37

$$\frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\sinh(x)\cosh(x)}{4\left(1-\sinh^2(x)\right)}$$

```
[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + (Cosh[x]*Sinh[x])/(4*(1 - Sinh[x
]^2))
```

Rubi [A] time = 0.0268138, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3184, 12, 3181, 206}

$$\frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\sinh(x)\cosh(x)}{4\left(1-\sinh^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(-2),x]
```

```
[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) + (Cosh[x]*Sinh[x])/(4*(1 - Sinh[x
]^2))
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{\left(1-\sinh^2(x)\right)^2}\,dx = \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)} - \frac{1}{4}\int -\frac{3}{1-\sinh^2(x)}\,dx$$
$$= \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)} + \frac{3}{4}\int \frac{1}{1-\sinh^2(x)}\,dx$$
$$= \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)} + \frac{3}{4}\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)$$
$$= \frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}} + \frac{\cosh(x)\sinh(x)}{4\left(1-\sinh^2(x)\right)}$$

Mathematica [A] time = 0.128136, size = 35, normalized size = 0.95

$$\frac{3\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}} - \frac{\sinh(2x)}{4(\cosh(2x)-3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(-2),x]
```

```
[Out] (3*ArcTanh[Sqrt[2]*Tanh[x]])/(4*Sqrt[2]) - Sinh[2*x]/(4*(-3 + Cosh[2*x]))
```

Maple [B] time = 0.02, size = 92, normalized size = 2.5

$$-\left(-\frac{1}{4}\tanh\left(\frac{x}{2}\right)-\frac{1}{4}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\tanh(x/2)-1\right)^{-1}+\frac{3\sqrt{2}}{8}\text{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)-2)\right)-\left(-\frac{1}{4}\tanh\left(\frac{x}{2}\right)+\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^2,x)
```

```
[Out] -(-1/4*tanh(1/2*x)-1/4)/(tanh(1/2*x)^2-2*tanh(1/2*x)-1)+3/8*2^(1/2)*arctanh
(1/4*(2*tanh(1/2*x)-2)*2^(1/2))-(-1/4*tanh(1/2*x)+1/4)/(tanh(1/2*x)^2+2*tan
h(1/2*x)-1)+3/8*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [B] time = 1.54868, size = 117, normalized size = 3.16

$$\frac{3}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{3}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-\frac{3e^{(-2x)}-1}{2\left(6e^{(-2x)}-e^{(-4x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 3/16*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 3/16*sqr
t(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 1/2*(3*e^(-2*x)
- 1)/(6*e^(-2*x) - e^(-4*x) - 1)
```

Fricas [B] time = 1.8539, size = 729, normalized size = 19.7

$$-\frac{24 \cosh(x)^2 - 3\left(\sqrt{2}\cosh(x)^4 + 4\sqrt{2}\cosh(x)\sinh(x)^3 + \sqrt{2}\sinh(x)^4 + 6\left(\sqrt{2}\cosh(x)^2 - \sqrt{2}\right)\sinh(x)^2 - 6\sqrt{2}\cosh}{16\left(\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(24*cosh(x)^2 - 3*(sqrt(2)*cosh(x)^4 + 4*sqrt(2)*cosh(x)*sinh(x)^3 +
sqrt(2)*sinh(x)^4 + 6*(sqrt(2)*cosh(x)^2 - sqrt(2))*sinh(x)^2 - 6*sqrt(2)*c
osh(x)^2 + 4*(sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*log
(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(2*s
qrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 48*co
sh(x)*sinh(x) + 24*sinh(x)^2 - 8)/(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x
)^4 + 6*(cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 4*(cosh(x)^3 - 3*cosh(x))
*sinh(x) + 1)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{(\sinh(x) - 1)^2 (\sinh(x) + 1)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**2,x)
```

```
[Out] Integral(1/((sinh(x) - 1)**2*(sinh(x) + 1)**2), x)
```

Giac [B] time = 1.2771, size = 84, normalized size = 2.27

$$-\frac{3}{16}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2} + 2e^{(2x)} - 6\right|}{\left|4\sqrt{2} + 2e^{(2x)} - 6\right|}\right) - \frac{3e^{(2x)} - 1}{2\left(e^{(4x)} - 6e^{(2x)} + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^2,x, algorithm="giac")
```

```
[Out] -3/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - 1/2*(3*e^(2*x) - 1)/(e^(4*x) - 6*e^(2*x) + 1)
```

3.65 $\int \frac{1}{\left(1-\sinh^2(x)\right)^3} dx$

Optimal. Leaf size=55

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{9\sinh(x)\cosh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{\sinh(x)\cosh(x)}{8\left(1-\sinh^2(x)\right)^2}$$

```
[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + (Cosh[x]*Sinh[x])/(8*(1 - Sinh
[x]^2)^2) + (9*Cosh[x]*Sinh[x])/(32*(1 - Sinh[x]^2))
```

Rubi [A] time = 0.0581055, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3184, 3173, 12, 3181, 206}

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{9\sinh(x)\cosh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{\sinh(x)\cosh(x)}{8\left(1-\sinh^2(x)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(-3),x]
```

```
[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + (Cosh[x]*Sinh[x])/(8*(1 - Sinh
[x]^2)^2) + (9*Cosh[x]*Sinh[x])/(32*(1 - Sinh[x]^2))
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(1-\sinh^2(x)\right)^3}\,dx &= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}-\frac{1}{8}\int \frac{-7-2\sinh^2(x)}{\left(1-\sinh^2(x)\right)^2}\,dx\\ &= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}-\frac{1}{32}\int -\frac{19}{1-\sinh^2(x)}\,dx\\ &= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{19}{32}\int \frac{1}{1-\sinh^2(x)}\,dx\\ &= \frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}+\frac{19}{32}\mathrm{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)\\ &= \frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}+\frac{\cosh(x)\sinh(x)}{8\left(1-\sinh^2(x)\right)^2}+\frac{9\cosh(x)\sinh(x)}{32\left(1-\sinh^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.177785, size = 51, normalized size = 0.93

$$\frac{19\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{32\sqrt{2}}-\frac{9\sinh(2x)}{32(\cosh(2x)-3)}+\frac{\sinh(2x)}{4(\cosh(2x)-3)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(-3),x]
```

```
[Out] (19*ArcTanh[Sqrt[2]*Tanh[x]])/(32*Sqrt[2]) + Sinh[2*x]/(4*(-3 + Cosh[2*x])^
2) - (9*Sinh[2*x])/(32*(-3 + Cosh[2*x]))
```

Maple [B] time = 0.023, size = 124, normalized size = 2.3

$$-\frac{1}{4}\left(-\frac{13}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^3+\frac{11}{8}\left(\tanh\left(\frac{x}{2}\right)\right)^2+\frac{31}{8}\tanh\left(\frac{x}{2}\right)+\frac{11}{8}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\tanh(x/2)-1\right)^{-2}+\frac{19\sqrt{2}}{64}\mathrm{Artanh}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^3,x)
```

```
[Out] -1/4*(-13/8*tanh(1/2*x)^3+11/8*tanh(1/2*x)^2+31/8*tanh(1/2*x)+11/8)/(tanh(1
/2*x)^2-2*tanh(1/2*x)-1)^2+19/64*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1
/2))-1/4*(-13/8*tanh(1/2*x)^3-11/8*tanh(1/2*x)^2+31/8*tanh(1/2*x)-11/8)/(ta
nh(1/2*x)^2+2*tanh(1/2*x)-1)^2+19/64*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*
2^(1/2))
```

Maxima [B] time = 1.56568, size = 150, normalized size = 2.73

$$\frac{19}{128}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{19}{128}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-\frac{89\,e^{(-2x)}-171\,e^{(-4x)}+19\,e^{(-6x)}-9}{16\left(12\,e^{(-2x)}-38\,e^{(-4x)}+12\,e^{(-6x)}-e^{(-8x)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 19/128*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 19/128
*sqrt(2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 1/16*(89*e^(
-2*x) - 171*e^(-4*x) + 19*e^(-6*x) - 9)/(12*e^(-2*x) - 38*e^(-4*x) + 12*e^(
-6*x) - e^(-8*x) - 1)
```

Fricas [B] time = 2.06877, size = 1936, normalized size = 35.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(152*cosh(x)^6 + 912*cosh(x)*sinh(x)^5 + 152*sinh(x)^6 + 456*(5*cosh
(x)^2 - 3)*sinh(x)^4 - 1368*cosh(x)^4 + 608*(5*cosh(x)^3 - 9*cosh(x))*sinh(
x)^3 + 8*(285*cosh(x)^4 - 1026*cosh(x)^2 + 89)*sinh(x)^2 + 712*cosh(x)^2 -
19*(sqrt(2)*cosh(x)^8 + 8*sqrt(2)*cosh(x)*sinh(x)^7 + sqrt(2)*sinh(x)^8 + 4
*(7*sqrt(2)*cosh(x)^2 - 3*sqrt(2))*sinh(x)^6 - 12*sqrt(2)*cosh(x)^6 + 8*(7*
sqrt(2)*cosh(x)^3 - 9*sqrt(2)*cosh(x))*sinh(x)^5 + 2*(35*sqrt(2)*cosh(x)^4
- 90*sqrt(2)*cosh(x)^2 + 19*sqrt(2))*sinh(x)^4 + 38*sqrt(2)*cosh(x)^4 + 8*(
7*sqrt(2)*cosh(x)^5 - 30*sqrt(2)*cosh(x)^3 + 19*sqrt(2)*cosh(x))*sinh(x)^3
+ 4*(7*sqrt(2)*cosh(x)^6 - 45*sqrt(2)*cosh(x)^4 + 57*sqrt(2)*cosh(x)^2 - 3*
sqrt(2))*sinh(x)^2 - 12*sqrt(2)*cosh(x)^2 + 8*(sqrt(2)*cosh(x)^7 - 9*sqrt(2
)*cosh(x)^5 + 19*sqrt(2)*cosh(x)^3 - 3*sqrt(2)*cosh(x))*sinh(x) + sqrt(2))*
log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(x) + 3*(
2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 - 3)) + 16
*(57*cosh(x)^5 - 342*cosh(x)^3 + 89*cosh(x))*sinh(x) - 72)/(cosh(x)^8 + 8*c
osh(x)*sinh(x)^7 + sinh(x)^8 + 4*(7*cosh(x)^2 - 3)*sinh(x)^6 - 12*cosh(x)^6
 + 8*(7*cosh(x)^3 - 9*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 - 90*cosh(x)^2 +
 19)*sinh(x)^4 + 38*cosh(x)^4 + 8*(7*cosh(x)^5 - 30*cosh(x)^3 + 19*cosh(x))
*sinh(x)^3 + 4*(7*cosh(x)^6 - 45*cosh(x)^4 + 57*cosh(x)^2 - 3)*sinh(x)^2 -
12*cosh(x)^2 + 8*(cosh(x)^7 - 9*cosh(x)^5 + 19*cosh(x)^3 - 3*cosh(x))*sinh(
x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27688, size = 100, normalized size = 1.82

$$-\frac{19}{128}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{19e^{(6x)}-171e^{(4x)}+89e^{(2x)}-9}{16\left(e^{(4x)}-6e^{(2x)}+1\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^3,x, algorithm="giac")
```

```
[Out] -19/128*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*
x) - 6)) - 1/16*(19*e^(6*x) - 171*e^(4*x) + 89*e^(2*x) - 9)/(e^(4*x) - 6*e^
(2*x) + 1)^2
```

3.66 $\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=130

$$-\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4bf}-\frac{(a+3b)\cosh(e+fx)\sqrt{a+b\cdots}}{8bf}$$

```
[Out] -((a - b)*(a + 3*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
 f*x]^2]])/(8*b^(3/2)*f) - ((a + 3*b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e +
 f*x]^2])/(8*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*b*
f)
```

Rubi [A] time = 0.148348, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 388, 195, 217, 206}

$$-\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4bf}-\frac{(a+3b)\cosh(e+fx)\sqrt{a+b\cdots}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((a - b)*(a + 3*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
 f*x]^2]])/(8*b^(3/2)*f) - ((a + 3*b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e +
 f*x]^2])/(8*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*b*
f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)\sqrt{a-b+bx^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a+3b)\,\text{Subst}\left(\int \sqrt{a-b+b\right.}{4bf}$$

$$= -\frac{(a+3b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8bf} + \frac{\cosh(e+fx)\left(a-b+\right.}{4b}$$

$$= -\frac{(a+3b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8bf} + \frac{\cosh(e+fx)\left(a-b+\right.}{4b}$$

$$= -\frac{(a-b)(a+3b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a+3b)\cosh(e+fx)\sqrt{a-\right.}{8bf}$$

Mathematica [A] time = 0.651416, size = 114, normalized size = 0.88

$$\frac{\frac{(b-a)(a+3b)\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{b^{3/2}} + \frac{\cosh(e+fx)\sqrt{4a+2b\cosh(2(e+fx))-2b}(a+b\cosh(2(e+fx))-4b)}{2b}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((Cosh[e + f*x]*(a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh[2
*(e + f*x)]])/(2*b) + ((-a + b)*(a + 3*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x]
 + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])/b^(3/2))/(8*f)
```

Maple [B] time = 0.114, size = 339, normalized size = 2.6

$$\frac{1}{16f\cosh(fx+e)}\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+e)\right)^2}\left(4\sqrt{b\left(\cosh(fx+e)\right)^4+(a-b)\left(\cosh(fx+e)\right)^2}b^{5/2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*(b*cosh(f*x+e)^4+(a-b)*co
sh(f*x+e)^2)^(1/2)*b^(5/2)*cosh(f*x+e)^2-10*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x
```

```
+e)^2)^(1/2)*b^(5/2)+2*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(3/2
)-ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b
^(1/2)+a-b)/b^(1/2))*a^2*b-2*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4
+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*b^2+3*b^3*ln(1/2*(2*b*cos
h(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/
2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \sinh\left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^3, x)
```

Fricas [B] time = 3.89635, size = 7794, normalized size = 59.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(2*((a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*
cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sin
h(f*x + e)^2 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2
 + 2*a*b - 3*b^2)*sinh(f*x + e)^4)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a
^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b
)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e
)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x
 + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x +
e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x
 + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 +
(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2
 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*co
sh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh
(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f
*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*
x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x
+ e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3
*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))
*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) +
4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e
))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*
sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*s
inh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*((
a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e
)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2
 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b - 3
```

```
*b^2)*sinh(f*x + e)^4)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*
sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*co
sh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))
*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2)) - sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x
 + e)^5 + b^2*sinh(f*x + e)^6 + (2*a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*c
osh(f*x + e)^2 + 2*a*b - 7*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3
+ (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh(f*x
 + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(2*a*b - 7*b^2)*cosh(f*x + e)^2 + 2*a
*b - 7*b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(2*a*b - 7
*b^2)*cosh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2
*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^2
+ 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*f*sinh(f*x + e)^4), 1/64*(4*(
(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x +
e)^3*sinh(f*x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^
2 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b -
3*b^2)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*co
sh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e
)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (
6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(
f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 4*((a^2 + 2*a*b
 - 3*b^2)*cosh(f*x + e)^4 + 4*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^3*sinh(f*
x + e) + 6*(a^2 + 2*a*b - 3*b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 +
 2*a*b - 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 + 2*a*b - 3*b^2)*sinh(
f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*
sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*
a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*si
nh(f*x + e) + b)) + sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh
(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (2*a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b
^2*cosh(f*x + e)^2 + 2*a*b - 7*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e
)^3 + (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh
(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(2*a*b - 7*b^2)*cosh(f*x + e)^2 +
 2*a*b - 7*b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(2*a*b
 - 7*b^2)*cosh(f*x + e)^3 + (2*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^4 + 4
*b^2*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e
)^2 + 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + b^2*f*sinh(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\sinh\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^3, x)
```

3.67 $\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}f}$$

```
[Out] ((a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*Sqrt[b]*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0702846, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 195, 217, 206}

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*Sqrt[b]*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a-b+bx^2}\,dx,x,\cosh(e+fx)\right)}{f}$$
$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx,x,\cos$$
$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\cosh}{\sqrt{a-b+b}}$$
$$= \frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f}$$

Mathematica [A] time = 0.161826, size = 97, normalized size = 1.18

$$\frac{\cosh(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{2\sqrt{2}f} + \frac{(a-b)\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*f) + ((a - b
)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]])
/(2*Sqrt[b]*f)
```

Maple [B] time = 0.066, size = 200, normalized size = 2.4

$$\frac{1}{4f\cosh(fx+e)}\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+e)\right)^2}\left(2\sqrt{b\left(\cosh(fx+e)\right)^4+(a-b)\left(\cosh(fx+e)\right)^2}\sqrt{b}+a\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)*b^(1/2)+a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a
-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-b*ln(1/2*(2*b*cosh(f*x+e)^2+
2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(1/2
)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(fx+e)^2+a}\sinh(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e), x)
```

Fricas [B] time = 3.46369, size = 5562, normalized size = 67.83

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (
a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f
*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x
+ e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14
*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (
9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^
2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4
*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b -
4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cos
h(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)
^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x
+ e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
 a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sin
h(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18
*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f
*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*
cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*
x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e
)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a - b)*cosh(f
*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2
)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b
)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(
2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2
 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^
2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2), -1/8*(2*((a -
 b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(
f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*
sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x +
 e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f
*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 +
 (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a - b)*cosh(f*x + e)^2
+ 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(-b)
*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
```

```
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*c
osh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*
(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)
^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) -
sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x +
 e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x
+ e)^2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \sinh\left(fx+e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e), x)
```

3.68 $\int \mathbf{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x
]^2]])/f
```

Rubi [A] time = 0.105076, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 402, 217, 206, 377}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/f) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x
]^2]])/f
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \operatorname{csch}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{1-x^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f} + \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \right.}{f}$$

$$= -\frac{a\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{b\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cosh(}{\sqrt{a-b+b\cosh}}\right.}{f}$$

$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.123564, size = 97, normalized size = 1.15

$$\frac{\sqrt{b}\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]]) + Sqrt[b]*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b +
b*Cosh[2*(e + f*x)]]])/f
```

Maple [B] time = 0.098, size = 174, normalized size = 2.1

$$\frac{1}{2f\cosh(fx+e)}\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+e)\right)^2}\left(\sqrt{b}\ln\left(\frac{1}{2}\left(2b\left(\cosh(fx+e)\right)^2+2\sqrt{b\left(\cosh(fx+e)\right)^4+}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(b^(1/2)*ln(1/2*(2*b*cosh(f*x
+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-a
^(1/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)
^2)^(1/2)+a-b)/sinh(f*x+e)^2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,\mathrm{csch}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e), x)
```

Fricas [B] time = 3.26454, size = 11636, normalized size = 138.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x +
 e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2
*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x +
e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^
3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*
x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(
14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3
 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(
a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x +
e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e
)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)
^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*
cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 +
3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3
 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(
f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x
 + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x +
 e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*sqrt(a)*log(-((a + b)*cosh(f*x
+ e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4
+ 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sin
h(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) +
 a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^
4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh
(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(b)*log(-(b*cosh(f*x
 + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2
)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*s
qrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e
)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f
```

```
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/f, 1/4*(4*sqrt(-a)*arctan(sqrt(2
)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*s
qrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4
 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh
(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh
(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(b)*log((a
^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f
*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 +
 a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*
b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)
^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)
*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 +
 a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
 + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x +
e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*
b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6
+ 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*
x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(
f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x
+ e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*si
nh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4
*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh
(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*c
osh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x
+ e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e
)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5
+ sinh(f*x + e)^6)) + sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*
sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/f, -1/2*(sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cos
h(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)
*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6
*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f
*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + sqrt(-b)*arctan(
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
- 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(
f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x
+ e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b
)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(
b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(a)*
log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (
a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*
x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*
x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)))/f,
 1/2*(2*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
```

```
 + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh
(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f
*x + e) + b)) - sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x +
 e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f
*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*co
sh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)
^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) - sqrt(-b)*arctan(sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sq
rt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4
+ 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(
f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\operatorname{csch}\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*csch(e + f*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.69 $\int \mathrm{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=88

$$\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}f}-\frac{\coth(e+fx)\mathrm{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}$$

```
[Out] ((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*Sqrt[a]*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x
])/(2*f)
```

Rubi [A] time = 0.118046, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 378, 377, 206}

$$\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2\sqrt{a}f}-\frac{\coth(e+fx)\mathrm{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*Sqrt[a]*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x
])/(2*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \mathrm{csch}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\mathrm{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{\left(1-x^2\right)^2}dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\mathrm{csch}(e+fx)}{2f} + \frac{(a-b)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\mathrm{csch}(e+fx)}{2f} + \frac{(a-b)\,\mathrm{Subst}\left(\int\right.}{}$$

$$= \frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+}{2f}$$

Mathematica [A] time = 0.293015, size = 104, normalized size = 1.18

$$\frac{2(a-b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right) - \sqrt{2}\sqrt{a}\coth(e+fx)\mathrm{csch}(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{4\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x])/(4*Sqrt[a]*f)
```

Maple [B] time = 0.09, size = 230, normalized size = 2.6

$$\frac{1}{4\left(\sinh\left(fx+e\right)\right)^2\cosh\left(fx+e\right)f}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)\left(\cosh\left(fx+e\right)\right)^2}\left(a\ln\left(\frac{1}{\left(\sinh\left(fx+e\right)\right)^2}\left((a+b)\left(\cosh\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(a*ln(((a+b)*cosh(f*x+e)^2+2*
a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sin
h(f*x+e)^2-b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(
f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-2*a^(1/2)*((a+b*sinh(f*x+
e)^2)*cosh(f*x+e)^2)^(1/2))/a^(1/2)/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,\mathrm{csch}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^3, x)
```

Fricas [B] time = 2.27941, size = 3421, normalized size = 38.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*
x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 - (a - b)*co
sh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 +
 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a
 - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x +
e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/
(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3
*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2
*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^4 + 4*a*f*cosh(f*x + e)*s
inh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 + 2*(3*a*f*cos
h(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a*f*cosh(f*x + e)^3 - a*f*co
sh(f*x + e))*sinh(f*x + e)), -1/2*(((a - b)*cosh(f*x + e)^4 + 4*(a - b)*cos
h(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4 - 2*(a - b)*cosh(f*x +
 e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 - a + b)*sinh(f*x + e)^2 + 4*((a - b)*
cosh(f*x + e)^3 - (a - b)*cosh(f*x + e))*sinh(f*x + e) + a - b)*sqrt(-a)*ar
ctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*
a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2
+ 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqr
t(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)
^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e
)^4 + 4*a*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*f*sinh(f*x + e)^4 - 2*a*f*cos
h(f*x + e)^2 + 2*(3*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 4*(a
*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37309, size = 938, normalized size = 10.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(a - b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*
a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))/(sqrt(-a)
*f) + 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a + (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b + 5*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) - 3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2) +
 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
- 2*b*e^(2*f*x + 2*e) + b))*a^2 - 9*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b + 3*(sqrt(
b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(
2*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 3*a*b^(3/2) - b^(5/2))/(((sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))^2 - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) - 4*a + b)^2*f)
```

3.70 $\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=144

$$-\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4af}+\frac{(3a+b)\coth(e}{}$$

```
[Out] -((a - b)*(3*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
 f*x]^2]])/(8*a^(3/2)*f) + ((3*a + b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[
e + f*x]*Csch[e + f*x])/(8*a*f) - ((a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e
 + f*x]*Csch[e + f*x]^3)/(4*a*f)
```

Rubi [A] time = 0.150017, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 382, 378, 377, 206}

$$-\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4af}+\frac{(3a+b)\coth(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((a - b)*(3*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
 f*x]^2]])/(8*a^(3/2)*f) + ((3*a + b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[
e + f*x]*Csch[e + f*x])/(8*a*f) - ((a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e
 + f*x]*Csch[e + f*x]^3)/(4*a*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a-b+bx^2}}{(1-x^2)^3}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{4af} - \frac{(3a+b)\operatorname{Subs}}{}$$

$$= \frac{(3a+b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{8af} - \frac{\left(a-b+b\right.}{}$$

$$= \frac{(3a+b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{8af} - \frac{\left(a-b+b\right.}{}$$

$$= -\frac{(a-b)(3a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8a^{3/2}f} + \frac{(3a+b)\sqrt{a-b+b\cosh^2(}}{}$$

Mathematica [A] time = 0.538088, size = 129, normalized size = 0.9

$$\frac{\left(-6a^2+4ab+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right) - \sqrt{2}\sqrt{a}\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}\left(2ac\right.}{16a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-6*a^2 + 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a
- b + b*Cosh[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e +
f*x)]]*Coth[e + f*x]*Csch[e + f*x]*(-3*a + b + 2*a*Csch[e + f*x]^2))/(16*a^
(3/2)*f)
```

Maple [B] time = 0.099, size = 381, normalized size = 2.7

$$\frac{1}{16\left(\sinh\left(fx+e\right)\right)^4\cosh\left(fx+e\right)f}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)\left(\cosh\left(fx+e\right)\right)^2}\left(6\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)\left(\cosh\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(6*((a+b*sinh(f*x+e)^2)*cosh
(f*x+e)^2)^(1/2)*sinh(f*x+e)^2*a^(5/2)-3*a^3*ln(((a+b)*cosh(f*x+e)^2+2*a^(1
/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*
x+e)^4+2*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*
x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4*a^2+ln(((a+b)*cosh(f*x+e)^2
+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*
b^2*sinh(f*x+e)^4*a-2*b*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+
e)^2*a^(3/2)-4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*a^(5/2))/sinh(f*x+
e)^4/a^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\operatorname{csch}\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^5, x)
```

Fricas [B] time = 4.09698, size = 8303, normalized size = 57.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^8 + 8*(3*a^2 - 2*a*b - b^2)*co
sh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 2*a*b - b^2)*sinh(f*x + e)^8 - 4*(3*
a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x +
e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e)^6 + 8*(7*(3*a^2 - 2*a*b - b^2)*co
sh(f*x + e)^3 - 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*
(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(3*a^2 - 2*a*b - b^2)*cosh(f*
x + e)^4 - 30*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 + 9*a^2 - 6*a*b - 3*b^2
)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^5 - 10*(3*a^2
- 2*a*b - b^2)*cosh(f*x + e)^3 + 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sin
h(f*x + e)^3 - 4*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*(3*a^2 - 2*a*
b - b^2)*cosh(f*x + e)^6 - 15*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 + 9*(3*
a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e)^2 +
 3*a^2 - 2*a*b - b^2 + 8*((3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^7 - 3*(3*a^2
- 2*a*b - b^2)*cosh(f*x + e)^5 + 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 -
(3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*c
osh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x
 + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a
- b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*
x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f
*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 +
```

```
 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((3*a^
2 - a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - a*b)*cosh(f*x + e)*sinh(f*x + e)^5 +
(3*a^2 - a*b)*sinh(f*x + e)^6 - (11*a^2 - a*b)*cosh(f*x + e)^4 + (15*(3*a^2
 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*sinh(f*x + e)^4 + 4*(5*(3*a^2 - a*b
)*cosh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2
 - a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - a*b)*cosh(f*x + e)^4 - 6*(11*a^2 - a
*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*sinh(f*x + e)^2 + 3*a^2 - a*b + 2*(3*(3
*a^2 - a*b)*cosh(f*x + e)^5 - 2*(11*a^2 - a*b)*cosh(f*x + e)^3 - (11*a^2 -
a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e
)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)))/(a^2*f*cosh(f*x + e)^8 + 8*a^2*f*cosh(f*x + e)*sinh(f*x + e)^7 +
a^2*f*sinh(f*x + e)^8 - 4*a^2*f*cosh(f*x + e)^6 + 6*a^2*f*cosh(f*x + e)^4 +
 4*(7*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^6 + 8*(7*a^2*f*cosh(f*x
+ e)^3 - 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^2 +
 2*(35*a^2*f*cosh(f*x + e)^4 - 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*sinh(f*x
 + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 - 10*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*c
osh(f*x + e))*sinh(f*x + e)^3 + a^2*f + 4*(7*a^2*f*cosh(f*x + e)^6 - 15*a^2
*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 8*(
a^2*f*cosh(f*x + e)^7 - 3*a^2*f*cosh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^3 -
 a^2*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((3*a^2 - 2*a*b - b^2)*cosh(f*x
+ e)^8 + 8*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 2
*a*b - b^2)*sinh(f*x + e)^8 - 4*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 + 4*(
7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a^2 + 2*a*b + b^2)*sinh(f*x + e
)^6 + 8*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 - 3*(3*a^2 - 2*a*b - b^2)*
cosh(f*x + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 +
2*(35*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^4 - 30*(3*a^2 - 2*a*b - b^2)*cosh
(f*x + e)^2 + 9*a^2 - 6*a*b - 3*b^2)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 2*a*b
- b^2)*cosh(f*x + e)^5 - 10*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^3 + 3*(3*a^
2 - 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(3*a^2 - 2*a*b - b^2)*c
osh(f*x + e)^2 + 4*(7*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^6 - 15*(3*a^2 - 2
*a*b - b^2)*cosh(f*x + e)^4 + 9*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^2 - 3*a
^2 + 2*a*b + b^2)*sinh(f*x + e)^2 + 3*a^2 - 2*a*b - b^2 + 8*((3*a^2 - 2*a*b
 - b^2)*cosh(f*x + e)^7 - 3*(3*a^2 - 2*a*b - b^2)*cosh(f*x + e)^5 + 3*(3*a^
2 - 2*a*b - b^2)*cosh(f*x + e)^3 - (3*a^2 - 2*a*b - b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*
sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*
a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*si
nh(f*x + e) + b)) + sqrt(2)*((3*a^2 - a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - a*b
)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b)*sinh(f*x + e)^6 - (11*a^2 -
 a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*s
inh(f*x + e)^4 + 4*(5*(3*a^2 - a*b)*cosh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f
*x + e))*sinh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - a*
b)*cosh(f*x + e)^4 - 6*(11*a^2 - a*b)*cosh(f*x + e)^2 - 11*a^2 + a*b)*sinh(
f*x + e)^2 + 3*a^2 - a*b + 2*(3*(3*a^2 - a*b)*cosh(f*x + e)^5 - 2*(11*a^2 -
 a*b)*cosh(f*x + e)^3 - (11*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^8 + 8*a^2*
f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*f*sinh(f*x + e)^8 - 4*a^2*f*cosh(f*x
+ e)^6 + 6*a^2*f*cosh(f*x + e)^4 + 4*(7*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh
(f*x + e)^6 + 8*(7*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*sinh(f*x
+ e)^5 - 4*a^2*f*cosh(f*x + e)^2 + 2*(35*a^2*f*cosh(f*x + e)^4 - 30*a^2*f*c
osh(f*x + e)^2 + 3*a^2*f)*sinh(f*x + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 - 10
*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*f + 4
*(7*a^2*f*cosh(f*x + e)^6 - 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e
)^2 - a^2*f)*sinh(f*x + e)^2 + 8*(a^2*f*cosh(f*x + e)^7 - 3*a^2*f*cosh(f*x
+ e)^5 + 3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.57355, size = 3086, normalized size = 21.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*a^2 - 2*a*b - b^2)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(
4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sq
rt(-a))/(sqrt(-a)*a*f) - 1/2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2 - 2*(sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))^7*a*b - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*b^2 - 21*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b))^6*a^2*sqrt(b) - 18*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x +
4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2) + 7*(sqr
t(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e
^(2*f*x + 2*e) + b))^6*b^(5/2) - 44*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3 - 121*(s
qrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b
*e^(2*f*x + 2*e) + b))^5*a^2*b + 122*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2 - 21*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^5*b^3 - 292*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4
*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^3*sqrt(b)
 + 559*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2
*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2) - 270*(sqrt(b)*e^(2*f*x + 2*e
) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)
)^4*a*b^(5/2) + 35*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(7/2) - 176*(sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))^3*a^4 + 872*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4
*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^3*b - 823*(sqrt(b)*e^(2*
f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
2*e) + b))^3*a^2*b^2 + 290*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b^3 - 35*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^3*b^4 + 528*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^4*sqrt(b) - 936*(sq
rt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*
e^(2*f*x + 2*e) + b))^2*a^3*b^(3/2) + 577*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b
```

```
*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^2*b^
(5/2) - 158*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(7/2) + 21*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))^2*b^(9/2) + 192*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a
*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^5 - 656*(sqrt(b)*e^(2*f*x +
2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
 b))*a^4*b + 580*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^
(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^3*b^2 - 211*(sqrt(b)*e^(2*f*x +
 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
+ b))*a^2*b^3 + 38*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*
e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^4 - 7*(sqrt(b)*e^(2*f*x + 2
*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b))*b^5 - 192*a^5*sqrt(b) + 304*a^4*b^(3/2) - 180*a^3*b^(5/2) + 37*a^2*b^(7
/2) - 2*a*b^(9/2) + b^(11/2))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 - 2*(sqrt(b)*e^(
2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
+ 2*e) + b))*sqrt(b) - 4*a + b)^4*a*f)
```

3.71 $\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=300

$$-\frac{(a-4b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\mathrm{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{15bf\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\left(2a^2+3ab-8b^2\right)\tanh(e+fx)\sqrt{a}}{15b^2f}$$

```
[Out] ((a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f
) + (Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) + ((2
*a^2 + 3*a*b - 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*
x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) - ((a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a +
 b*Sinh[e + f*x]^2))/a]) - ((2*a^2 + 3*a*b - 8*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rubi [A] time = 0.327036, antiderivative size = 300, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 478, 582, 531, 418, 492, 411}

$$-\frac{\left(2a^2+3ab-8b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f}+\frac{\left(2a^2+3ab-8b^2\right)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\right.}{15b^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f
) + (Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) + ((2
*a^2 + 3*a*b - 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*
x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) - ((a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a +
 b*Sinh[e + f*x]^2))/a]) - ((2*a^2 + 3*a*b - 8*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
```

```
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sinh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{1+x^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e\right.}{}$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+fx)\sin}{}$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+fx)\sin}{}$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+fx)\sin}{}$$

$$= \frac{(a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+fx)\sin}{}$$

Mathematica [C] time = 1.3688, size = 210, normalized size = 0.7

$$\frac{-32ia\left(a^2+ab-2b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \sqrt{2}b\sinh(2(e+fx))\left(8a^2+4b(4a-7b)\cosh(2(e+f\right.}{240b^2f\sqrt{2a+b\cosh(2(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((16*I)*a*(2*a^2 + 3*a*b - 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 + a*b - 2*b^2)*Sqrt[(2*a - b + b
*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 - 48*
a*b + 25*b^2 + 4*(4*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x)])
*Sinh[2*(e + f*x)])/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.086, size = 512, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^7+4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5
-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-4*(-1/a*
b)^(1/2)*b^2*sinh(f*x+e)^3+a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+7*a*((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
```

```
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*((a+b*sinh(f*x+e)^2)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))*a^2-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh
(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+(-
1/a*b)^(1/2)*a^2*sinh(f*x+e)-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/b/(-1/a*b)^(
1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^2 + a} \sinh \left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b \sinh \left(fx+e\right)^2 + a} \sinh \left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^2 + a} \sinh \left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4, x)
```

3.72 $\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=177

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\operatorname{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{3bf\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{i(a-2b)\sqrt{a+b\sinh^2(e+fx)}\ldots}{3bf\sqrt{\ldots}}$$

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((I/3)*(a
 - 2*b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*
Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.213308, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}F\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i(a-2b)\sqrt{a+b\sinh^2(e+\ldots}}{3bf\sqrt{\frac{b\sinh^2(e+\ldots}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((I/3)*(a
 - 2*b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b/a]*
Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{1}{3}\int \frac{a-(a-2b)\sinh^2}{\sqrt{a+b\sinh^2(e}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(a-2b)\int\sqrt{a+b\sinh}}{3b}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left((a-2b)\sqrt{a+b\sinh^2}\right.}{3b\sqrt{}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{i(a-2b)E\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf\sqrt{1+}}$$

Mathematica [A] time = 0.84108, size = 170, normalized size = 0.96

$$\frac{2i\sqrt{2}a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-2i\sqrt{2}a(a-}{6bf\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.065, size = 343, normalized size = 1.9

$$-\frac{1}{3f\cosh(fx+e)}\left(-\sqrt{-\frac{b}{a}}b\sinh(fx+e)(\cosh(fx+e))^4+\left(-\sqrt{-\frac{b}{a}}a+\sqrt{-\frac{b}{a}}b\right)(\cosh(fx+e))^2\sinh(fx+e)+2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(-(-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^4+(-(-1/a*b)^(1/2)*a+(-1/a*
b)^(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+2*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+2*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^{2}+a} \sinh \left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh \left(fx+e\right)^{2}+a} \sinh \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^{2}+a} \sinh \left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2, x)
```

3.73 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\left|\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rubi [A] time = 0.0363792, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\left|\frac{b}{a}\right.\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\\ &= -\frac{iE\left(ie+ifx\left|\frac{b}{a}\right.\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0960923, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.065, size = 140, normalized size = 2.3

$$\frac{1}{f\cosh\left(fx+e\right)}\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\left(a\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\mathrm{EllipticF}\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.74 $\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=199

$$\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\text{co}}{}$$

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (EllipticE[ArcTan[Sinh[e
 + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (b*EllipticF[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e +
 f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e
+ f*x])/f
```

Rubi [A] time = 0.181818, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 475, 21, 422, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan\right.}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (EllipticE[ArcTan[Sinh[e
 + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (b*EllipticF[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech[e +
 f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e
+ f*x])/f
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1+x^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subs}}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Sub}}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Sub}}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{bF\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b}{a}}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\,\mathrm{si}}{a}}}$$

Mathematica [C] time = 0.593719, size = 151, normalized size = 0.76

$$\frac{2i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)-2ia\sqrt{\frac{2a+b\cosh(}{}}}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*(a - b)*
Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*f*S
qrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.072, size = 160, normalized size = 0.8

$$\frac{1}{\cosh\left(fx+e\right)\sinh\left(fx+e\right)f}\left(b\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^{2}}{a}+\frac{a-b}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\sinh\left(fx+e\right)\text{EllipticE}\left(\sinh\left(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] (b*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)*Elli
pticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(-1/a*b)^(1/2)*b*cosh(f*x+e)^
4+(-(-1/a*b)^(1/2)*a+(-1/a*b)^(1/2)*b)*cosh(f*x+e)^2)/sinh(f*x+e)/(-1/a*b)^
(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\operatorname{csch}\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\operatorname{csch}\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \operatorname{csch}\left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2, x)
```

3.75 $\int \operatorname{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=276

$$-\frac{b\,\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+}}{3af}$$

```
[Out] ((2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e + f
*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + ((2*a - b)*Ellipti
cE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a - b)*Sqr
t[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rubi [A] time = 0.281847, antiderivative size = 276, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 475, 583, 531, 418, 492, 411}

$$-\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}+\frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth(e+fx)\text{csch}^2(e+}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e + f
*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + ((2*a - b)*Ellipti
cE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*EllipticF
[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a - b)*Sqr
t[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{1+x^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\text{se}\right.}{}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\text{csch}^2(e+fx)}{3f}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\text{csch}^2(e+fx)}{3f}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\text{csch}^2(e+fx)}{3f}$$

$$= \frac{(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth(e+fx)\text{csch}^2(e+fx)}{3f}$$

Mathematica [C] time = 3.02826, size = 208, normalized size = 0.75

$$\frac{-4ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \frac{\coth(e+fx)\text{csch}^2(e+fx)\left(4\left(2a^2-4ab+b^2\right)\cosh(2(e+fx))-(2a-b)(8a-b\cosh(4(e+\right.}{2\sqrt{2}}}{6af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (((4*(2*a^2 - 4*a*b + b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(8*a - 3*b - b*Cos
h[4*(e + f*x)]))*Coth[e + f*x]*Csch[e + f*x]^2)/(2*Sqrt[2]) + (2*I)*a*(2*a
- b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] -
(4*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*
x), b/a])/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.121, size = 436, normalized size = 1.6

$$\frac{1}{3a\left(\sinh\left(fx+e\right)\right)^3\cosh\left(fx+e\right)f}\left(2\sqrt{-\frac{b}{a}}ab\left(\sinh\left(fx+e\right)\right)^6 - \sqrt{-\frac{b}{a}}b^2\left(\sinh\left(fx+e\right)\right)^6 + b\sqrt{\frac{a+b\left(\sinh\left(fx+e\right.\right.}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+b*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*s
```

```
inh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+((a+b*sinh(f*x+
e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-(-1/a*b)^(
1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2-2*(-1/a*b)^(1/2)*a*
b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x
+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,\mathrm{csch}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^2+a}\,\mathrm{csch}\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,\mathrm{csch}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^4, x)
```

3.76 $\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=177

$$-\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6bf}-\frac{(a+5b)\cosh(e+fx)\left(a\ -\right.}{24b}$$

```
[Out] -((a - b)^2*(a + 5*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e
 + f*x]^2]])/(16*b^(3/2)*f) - ((a - b)*(a + 5*b)*Cosh[e + f*x]*Sqrt[a - b +
 b*Cosh[e + f*x]^2])/(16*b*f) - ((a + 5*b)*Cosh[e + f*x]*(a - b + b*Cosh[e
+ f*x]^2)^(3/2))/(24*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(5/2
))/(6*b*f)
```

Rubi [A] time = 0.181481, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 388, 195, 217, 206}

$$-\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16b^{3/2}f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6bf}-\frac{(a+5b)\cosh(e+fx)\left(a\ -\right.}{24b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - b)^2*(a + 5*b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e
 + f*x]^2]])/(16*b^(3/2)*f) - ((a - b)*(a + 5*b)*Cosh[e + f*x]*Sqrt[a - b +
 b*Cosh[e + f*x]^2])/(16*b*f) - ((a + 5*b)*Cosh[e + f*x]*(a - b + b*Cosh[e
+ f*x]^2)^(3/2))/(24*b*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(5/2
))/(6*b*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a-b+bx^2\right)^{3/2}dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{5/2}}{6bf} - \frac{(a+5b)\,\text{Subst}\left(\int\left(a-b+bx\right.\right.}{6bf}$$

$$= -\frac{(a+5b)\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{24bf} + \frac{\cosh(e+fx)\left(a-b\right.}{}$$

$$= -\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} - \frac{(a+5b)\cosh(e}{}$$

$$= -\frac{(a-b)(a+5b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{16bf} - \frac{(a+5b)\cosh(e}{}$$

$$= -\frac{(a-b)^2(a+5b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{(a-b)(a+5b)\cosh(e+f}{1}$$

Mathematica [A] time = 0.582482, size = 151, normalized size = 0.85

$$\frac{\sqrt{2}\sqrt{b}\sqrt{2a+b\cosh(2(e+fx))-b}\left(\left(6a^2-51ab+37b^2\right)\cosh(e+fx)+b((7a-8b)\cosh(3(e+fx))+b\cosh(5(e+fx))\right.}{192b^{3/2}f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*Sqrt[b]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*((6*a^2 - 51*a*b + 37*
b^2)*Cosh[e + f*x] + b*((7*a - 8*b)*Cosh[3*(e + f*x)] + b*Cosh[5*(e + f*x)]
)) - 12*(a - b)^2*(a + 5*b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]]])/(192*b^(3/2)*f)
```

Maple [B] time = 0.091, size = 483, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/96*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(16*b^(7/2)*(b*cosh(f*x+e)^4
+(a-b)*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)^4+4*b^(5/2)*(b*cosh(f*x+e)^4+(a-b)*
cosh(f*x+e)^2)^(1/2)*(-13*b+7*a)*cosh(f*x+e)^2+66*b^(7/2)*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)-72*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)
*b^(5/2)+6*a^2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(3/2)-3*ln(1/2
*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a
-b)/b^(1/2))*a^3*b-9*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cos
h(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))*a^2*b^2+27*b^3*a*ln(1/2*(2*b*cosh(f
*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))
-15*b^4*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(
1/2)*b^(1/2)+a-b)/b^(1/2)))/b^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^3, x)
```

Fricas [B] time = 3.80683, size = 11580, normalized size = 65.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(6*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a
^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 + 3*a^2*b -
 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a
*b^2 + 5*b^3)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2
 + 5*b^3)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh
(f*x + e)^6)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sin
h(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*
(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh
(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b
^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*
cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b
^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^
2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^
2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 - sq
rt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh
(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(
f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^
3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f
*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 +
 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x +
```

e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x
 + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 +
6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*c
osh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cos
h(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^3 + 3*a^2*b - 9*a*b^
2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x +
 e)^5*sinh(f*x + e) + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4*
sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^3*sinh
(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^2*sinh(f*x
 + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)*sinh(f*x + e)^5
 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh(f*x + e)^6)*sqrt(b)*log(-(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a -
 b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 - sqr
t(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1
)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x
+ e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2)*(b^3*cosh(f*x + e)^1
0 + 10*b^3*cosh(f*x + e)*sinh(f*x + e)^9 + b^3*sinh(f*x + e)^10 + (7*a*b^2
- 8*b^3)*cosh(f*x + e)^8 + (45*b^3*cosh(f*x + e)^2 + 7*a*b^2 - 8*b^3)*sinh(
f*x + e)^8 + 8*(15*b^3*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e))*s
inh(f*x + e)^7 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^6 + (210*b^3*c
osh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b^3 + 28*(7*a*b^2 - 8*b^3)*cosh(f*
x + e)^2)*sinh(f*x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 - 8*b^
3)*cosh(f*x + e)^3 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*
x + e)^5 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*
x + e)^6 + 70*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b
^3 + 15*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*
(30*b^3*cosh(f*x + e)^7 + 14*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b
 - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(
f*x + e))*sinh(f*x + e)^3 + b^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)^2 + (45*b
^3*cosh(f*x + e)^8 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b - 5
1*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + 7*a*b^2 - 8*b^3 + 6*(6*a^2*b - 51*a*b^2
 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*
(7*a*b^2 - 8*b^3)*cosh(f*x + e)^7 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*
x + e)^5 + 2*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (7*a*b^2 - 8*b
^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2)))/(b^2*f*cosh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 1
5*b^2*f*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x
 + e)^3 + 15*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*
sinh(f*x + e)^5 + b^2*f*sinh(f*x + e)^6), 1/384*(12*((a^3 + 3*a^2*b - 9*a*b
^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x
+ e)^5*sinh(f*x + e) + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4
*sinh(f*x + e)^2 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^3*sin
h(f*x + e)^3 + 15*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^2*sinh(f*
x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)*sinh(f*x + e)^
5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh(f*x + e)^6)*sqrt(-b)*arctan(sqrt
(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^
2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cos
h(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(
f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3
*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*
x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*si
nh(f*x + e))) + 12*((a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^6 + 6*(
a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 +
3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 + 3*a^
2*b - 9*a*b^2 + 5*b^3)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 + 3*a^2*b
- 9*a*b^2 + 5*b^3)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 + 3*a^2*b - 9*a

```
*b^2 + 5*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*sinh(f*x + e)^6)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
 e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
 e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*
x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b^3*cosh(f*x + e)^10 + 10*b^3*cosh(f
*x + e)*sinh(f*x + e)^9 + b^3*sinh(f*x + e)^10 + (7*a*b^2 - 8*b^3)*cosh(f*x
 + e)^8 + (45*b^3*cosh(f*x + e)^2 + 7*a*b^2 - 8*b^3)*sinh(f*x + e)^8 + 8*(1
5*b^3*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 +
(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^6 + (210*b^3*cosh(f*x + e)^4 +
6*a^2*b - 51*a*b^2 + 37*b^3 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^
3 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (6*a^2
*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*x + e)^6 + 70*(7*
a*b^2 - 8*b^3)*cosh(f*x + e)^4 + 6*a^2*b - 51*a*b^2 + 37*b^3 + 15*(6*a^2*b
- 51*a*b^2 + 37*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(30*b^3*cosh(f*x
+ e)^7 + 14*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b - 51*a*b^2 + 37*
b^3)*cosh(f*x + e)^3 + (6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e))*sinh(f*
x + e)^3 + b^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)^2 + (45*b^3*cosh(f*x + e)^
8 + 28*(7*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b - 51*a*b^2 + 37*b^3)
*cosh(f*x + e)^4 + 7*a*b^2 - 8*b^3 + 6*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f
*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*a*b^2 - 8*b^3)
*cosh(f*x + e)^7 + 3*(6*a^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^5 + 2*(6*a
^2*b - 51*a*b^2 + 37*b^3)*cosh(f*x + e)^3 + (7*a*b^2 - 8*b^3)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(co
sh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*c
osh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*cosh(f*x
+ e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*b^2*
f*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x + e)^5 +
 b^2*f*sinh(f*x + e)^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^3, x)
```

3.77 $\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$\frac{3(a-b)\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{8f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{b}\cos}{\sqrt{a+b\cos}}\right.}{8\sqrt{b}f}$$

```
[Out] (3*(a - b)^2*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2
]])/(8*Sqrt[b]*f) + (3*(a - b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2
])/(8*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.096654, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 195, 217, 206}

$$\frac{3(a-b)\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{8f}+\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{b}\cos}{\sqrt{a+b\cos}}\right.}{8\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] (3*(a - b)^2*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2
]])/(8*Sqrt[b]*f) + (3*(a - b)*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2
])/(8*f) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(3/2))/(4*f)
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a-b+bx^2\right)^{3/2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}{4f} + \frac{(3(a-b))\,\text{Subst}\left(\int \sqrt{a-b}\right.}{4}$$

$$= \frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8f} + \frac{\cosh(e+fx)\left(a-b+\right.}{4}$$

$$= \frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{8f} + \frac{\cosh(e+fx)\left(a-b+\right.}{4}$$

$$= \frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3(a-b)\cosh(e+fx)\sqrt{a-b+b}}{8f}$$

Mathematica [A] time = 0.312135, size = 111, normalized size = 0.92

$$\frac{\frac{1}{2}\cosh(e+fx)(5a+b\cosh(2(e+fx))-4b)\sqrt{4a+2b\cosh(2(e+fx))-2b}+\frac{3(a-b)^2\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e\right.}{\sqrt{b}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((Cosh[e + f*x]*(5*a - 4*b + b*Cosh[2*(e + f*x)])*Sqrt[4*a - 2*b + 2*b*Cosh
[2*(e + f*x)]])/2 + (3*(a - b)^2*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2
*a - b + b*Cosh[2*(e + f*x)]]])/Sqrt[b])/(8*f)
```

Maple [B] time = 0.071, size = 336, normalized size = 2.8

$$\frac{1}{16f\cosh\left(fx+e\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)\left(\cosh\left(fx+e\right)\right)^2}\left(4b^{3/2}\sqrt{b\left(\cosh\left(fx+e\right)\right)^4+(a-b)\left(\cosh\left(fx+e\right)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(4*b^(3/2)*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2-10*b^(3/2)*(b*cosh(f*x+e)^4+(a-b)*
cosh(f*x+e)^2)^(1/2)+10*a*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/
2)+3*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2
)*b^(1/2)+a-b)/b^(1/2))*a^2-6*b*a*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e
)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))+3*b^2*ln(1/2*(2*b*cosh
(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2
)))/b^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e), x)
```

Fricas [B] time = 2.92628, size = 7737, normalized size = 63.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(6*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x +
 e)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b
+ b^2)*sinh(f*x + e)^4)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f
*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x
+ e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14
*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (
9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^
2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4
*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b -
4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cos
h(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)
^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x
+ e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
 a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sin
h(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18
*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f
*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*
cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*
x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e
)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^2 - 2*a*b
 + b^2)*cosh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x +
e) + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b
 + b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4
)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*s
inh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b
)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(
2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*
x + e)^6 + (10*a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*
```

```
a*b - 7*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 7*b^2)*
cosh(f*x + e))*sinh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e)^2 + (15*b^2
*cosh(f*x + e)^4 + 6*(10*a*b - 7*b^2)*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*sin
h(f*x + e)^2 + b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 7*b^2)*cosh(f*x
 + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^
3*sinh(f*x + e) + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x +
e)*sinh(f*x + e)^3 + b*f*sinh(f*x + e)^4), -1/64*(12*((a^2 - 2*a*b + b^2)*c
osh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*c
osh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4)*sqrt(-b
)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*s
inh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*
x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b -
 b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh
(f*x + e))*sinh(f*x + e))) + 12*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 4*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 2*a*b + b^2)*cos
h(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x
 + e)^3 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cos
h(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
 e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
 e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b^2*cosh(f*
x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (10*
a*b - 7*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*si
nh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e))*
sinh(f*x + e)^3 + (10*a*b - 7*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^
4 + 6*(10*a*b - 7*b^2)*cosh(f*x + e)^2 + 10*a*b - 7*b^2)*sinh(f*x + e)^2 +
b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 7*b^2)*cosh(f*x + e)^3 + (10*a
*b - 7*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^3*sinh(f*x + e)
 + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x + e)*sinh(f*x + e
)^3 + b*f*sinh(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e), x)
```

3.78 $\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{b\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{\sqrt{b}(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}$$

```
[Out] -((a^(3/2)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Co
sh[e + f*x]^2]])/(2*f) + (b*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/
(2*f)
```

Rubi [A] time = 0.16497, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 416, 523, 217, 206, 377}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{b\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2f}+\frac{\sqrt{b}(3a-b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/f) + ((3*a - b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Co
sh[e + f*x]^2]])/(2*f) + (b*Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/
(2*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{1-x^2}dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} + \frac{\text{Subst}\left(\int \frac{-(a-b)(2a-b)-(3a-b)bx^2}{\left(1-x^2\right)\sqrt{a-b+bx^2}} d\right.}{2f}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a-b+bx^2}}dx,\right.}{f}$$

$$= \frac{b\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2f} - \frac{a^2\,\text{Subst}\left(\int \frac{1}{1-ax^2}dx, x, \frac{\cos}{\sqrt{a-b+b}}\right.}{f}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f} + \frac{(3a-b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2f}$$

Mathematica [A] time = 0.553553, size = 136, normalized size = 1.07

$$\frac{-4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)+b\cosh(e+fx)\sqrt{4a+2b\cosh(2(e+fx))-2b}-2\sqrt{b}(b-3a)\log\left(\sqrt{2a+b\cosh(2(\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-4*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]] + b*Cosh[e + f*x]*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]] - 2
*Sqrt[b]*(-3*a + b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Co
sh[2*(e + f*x)]]])/(4*f)
```

Maple [B] time = 0.086, size = 268, normalized size = 2.1

$$-\frac{1}{4f\cosh(fx+e)}\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+e)\right)^2}\left(2a^{3/2}\ln\left(\frac{(a+b)\left(\cosh(fx+e)\right)^2+2\sqrt{a}\sqrt{b\left(\cosh(fx-\right.}}{\left(\cosh(fx+e\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*a^(3/2)*ln(((a+b)*cosh(f*
x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/(cosh(f*x
+e)^2-1))+b^(3/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f
*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-3*b^(1/2)*a*ln(1/2*(2*b*cosh(f*x+e)^2+
2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-2*b*(b*
cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e), x)
```

Fricas [B] time = 3.68536, size = 14642, normalized size = 115.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e)
 + (3*a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*
cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cos
h(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 +
 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)
^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4
+ 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)
^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^
2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^
3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*
x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x
+ e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x +
e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e
))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^
4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*
cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh
(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*
a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^
2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(c
osh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(
f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f
```

```
*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) - 4*(a*cosh
(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(a)*
log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (
a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*
x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*
x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + (
(3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e) + (3*a
- b)*sinh(f*x + e)^2)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 + a - b)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*
sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e)
 + b*sinh(f*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x + e)^2),
 1/8*(8*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x +
 e)^2)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(
f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b
)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*
x + e) + b)) - ((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(
f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 +
8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^
2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x
+ e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(
f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x
 + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(
f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3
 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*
b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)
*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e
)^2)*sinh(f*x + e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*s
inh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cos
h(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh
(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f
*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2
*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x +
e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
 + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b
- 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x
+ e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)
^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^
2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) - (
(3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e) + (3*a
- b)*sinh(f*x + e)^2)*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 + a - b)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^
```

```
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*
sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e)
 + b*sinh(f*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x + e)^2),
 -1/8*(2*((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x +
e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2
+ 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh
(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x +
e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a
*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((3*a
 - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - b)
*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^
3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^
2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x +
e))*sinh(f*x + e) + b)) - 4*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x
 + e) + a*sinh(f*x + e)^2)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b
)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cos
h(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*
sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
+ 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*
cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x
 + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
 + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cos
h(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x
 + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) +
f*sinh(f*x + e)^2), 1/8*(8*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x
+ e) + a*sinh(f*x + e)^2)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f
*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f
*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cos
h(f*x + e))*sinh(f*x + e) + b)) - 2*((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b
)*cosh(f*x + e)*sinh(f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(
sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x +
 e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*c
osh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4
+ (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sin
h(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e)
)*sinh(f*x + e))) - 2*((3*a - b)*cosh(f*x + e)^2 + 2*(3*a - b)*cosh(f*x + e
)*sinh(f*x + e) + (3*a - b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(
f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b*cosh(f*x +
```

```
e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 + b)*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x +
e)*sinh(f*x + e) + f*sinh(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e), x)
```

3.79 $\int \mathbf{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=130

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{\sqrt{a}(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}-\frac{a\coth(e+fx)\mathrm{csch}(e+fx)\sqrt{a+b\cosh^2(}}{2f}$$

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*C
osh[e + f*x]^2]])/f - (a*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch
[e + f*x])/(2*f)
```

Rubi [A] time = 0.168235, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3186, 413, 523, 217, 206, 377}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{f}+\frac{\sqrt{a}(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2f}-\frac{a\coth(e+fx)\mathrm{csch}(e+fx)\sqrt{a+b\cosh^2(}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e +
f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*C
osh[e + f*x]^2]])/f - (a*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch
[e + f*x])/(2*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rubi steps

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^2}dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2f} - \frac{\text{Subst}\left(\int \frac{-(a-2b}{(1-x}\right.}{}$$

$$= -\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2f} + \frac{(a(a-3b))\,\text{Sub}}{}$$

$$= -\frac{a\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2f} + \frac{(a(a-3b))\,\text{Sub}}{}$$

$$= \frac{\sqrt{a}(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2f} + \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.71128, size = 143, normalized size = 1.1

$$\frac{4b^{3/2}\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)+2\sqrt{a}(a-3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)-a\coth(e+}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[a]*(a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b +
 b*Cosh[2*(e + f*x)]]] - a*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x] + 4*b^(3/2)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*
a - b + b*Cosh[2*(e + f*x)]]])/(4*f)
```

Maple [B] time = 0.097, size = 297, normalized size = 2.3

$$\frac{1}{4\left(\sinh\left(fx+e\right)\right)^2\cosh\left(fx+e\right)f}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)\left(\cosh\left(fx+e\right)\right)^2}\left(2b^{3/2}\ln\left(1/2\,\frac{2b\left(\cosh\left(fx+e\right)\right)^2+2\sqrt{b}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*b^(3/2)*ln(1/2*(2*b*cosh(f
*x+e)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))
*sinh(f*x+e)^2+a^(3/2)*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(
a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-3*a^(1/2)*b*ln(
((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+
a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^2-2*a*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(
1/2))/sinh(f*x+e)^2/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^3, x)
```

Fricas [B] time = 4.75341, size = 17554, normalized size = 135.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
 e)^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 +
 4*(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log((a^
2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*
x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 +
a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b
)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^
4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 +
a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
+ e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e
)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b
^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 +
 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x
 + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f
*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x +
 e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sin
h(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(
f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*co
sh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x +
```

```
e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)
^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 +
 sinh(f*x + e)^6)) - ((a - 3*b)*cosh(f*x + e)^4 + 4*(a - 3*b)*cosh(f*x + e)
*sinh(f*x + e)^3 + (a - 3*b)*sinh(f*x + e)^4 - 2*(a - 3*b)*cosh(f*x + e)^2
+ 2*(3*(a - 3*b)*cosh(f*x + e)^2 - a + 3*b)*sinh(f*x + e)^2 + 4*((a - 3*b)*
cosh(f*x + e)^3 - (a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(a)
*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 +
(a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f
*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f
*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
 + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*c
osh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) +
(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
- 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*
cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log(-(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a -
 b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqr
t(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1
)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x
+ e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 2*sqrt(2)*(a*cosh(f*x + e)^2
 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*s
inh(f*x + e)^3 + f*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x
+ e)^2 - f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 - f*cosh(f*x + e))*sinh(
f*x + e) + f), -1/4*(2*((a - 3*b)*cosh(f*x + e)^4 + 4*(a - 3*b)*cosh(f*x +
e)*sinh(f*x + e)^3 + (a - 3*b)*sinh(f*x + e)^4 - 2*(a - 3*b)*cosh(f*x + e)^
2 + 2*(3*(a - 3*b)*cosh(f*x + e)^2 - a + 3*b)*sinh(f*x + e)^2 + 4*((a - 3*b
)*cosh(f*x + e)^3 - (a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(
-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x +
 e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))
 - (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)
^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*
(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log((a^2*b
*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x +
 e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3
 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*c
osh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 +
 (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cos
h(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2
*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e
)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6
 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*
a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x +
e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x
+ e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)
^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f
*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b
 - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*s
```

```
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x
 + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(
f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e)
 + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3
+ 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + si
nh(f*x + e)^6)) - (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
b*sinh(f*x + e)^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(
f*x + e)^2 + 4*(b*cosh(f*x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqr
t(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sin
h(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(
f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 2*sqrt(2)*
(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 +
a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4
*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^2
+ 2*(3*f*cosh(f*x + e)^2 - f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 - f*co
sh(f*x + e))*sinh(f*x + e) + f), -1/4*(2*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x
+ e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*cosh(f*x + e)^2 + 2*(3*b*cos
h(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 - b*cosh(f*x + e))
*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f
*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*si
nh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*
b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x
+ e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*(b*cosh(f*x + e)^
4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 -
 b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)
^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x
+ e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2
*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + ((a - 3*b)*cosh(f*x + e)^4 + 4
*(a - 3*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 3*b)*sinh(f*x + e)^4 - 2*(a
 - 3*b)*cosh(f*x + e)^2 + 2*(3*(a - 3*b)*cosh(f*x + e)^2 - a + 3*b)*sinh(f*
x + e)^2 + 4*((a - 3*b)*cosh(f*x + e)^3 - (a - 3*b)*cosh(f*x + e))*sinh(f*x
 + e) + a - 3*b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x
 + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)
^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(c
osh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(
a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x +
 e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 +
4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 -
1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e)
)*sinh(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*si
nh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh
(f*x + e)^4 - 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x + e)^2 - f)*sinh(f*x +
e)^2 + 4*(f*cosh(f*x + e)^3 - f*cosh(f*x + e))*sinh(f*x + e) + f), -1/2*(((
a - 3*b)*cosh(f*x + e)^4 + 4*(a - 3*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a -
```

```
 3*b)*sinh(f*x + e)^4 - 2*(a - 3*b)*cosh(f*x + e)^2 + 2*(3*(a - 3*b)*cosh(f
*x + e)^2 - a + 3*b)*sinh(f*x + e)^2 + 4*((a - 3*b)*cosh(f*x + e)^3 - (a -
3*b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt(-a)*arctan(sqrt(2)*(cosh(
f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + (b*cosh(f*x + e)^4 + 4
*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*cosh(f*x + e)^2
+ 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 - b*co
sh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2
+ 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh
(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x +
e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a
*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + (b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 - 2*b*c
osh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*
x + e)^3 - b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(cos
h(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
 e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
 e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(a*cosh(f*x
+ e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x
+ e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 - 2*f*cosh(f*x + e)^2 + 2*(3*f*cos
h(f*x + e)^2 - f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 - f*cosh(f*x + e))
*sinh(f*x + e) + f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^3, x)
```

3.80 $\int \mathrm{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=135

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8\sqrt{a}f}-\frac{\coth(e+fx)\mathrm{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)\coth(e+\ldots}{}$$

```
[Out] (-3*(a - b)^2*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^
2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*
x]*Csch[e + f*x])/(8*f) - ((a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e + f*x]*
Csch[e + f*x]^3)/(4*f)
```

Rubi [A] time = 0.140604, antiderivative size = 135, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 378, 377, 206}

$$-\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{8\sqrt{a}f}-\frac{\coth(e+fx)\mathrm{csch}^3(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}{4f}+\frac{3(a-b)\coth(e+\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a - b)^2*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^
2]])/(8*Sqrt[a]*f) + (3*(a - b)*Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*
x]*Csch[e + f*x])/(8*f) - ((a - b + b*Cosh[e + f*x]^2)^(3/2)*Coth[e + f*x]*
Csch[e + f*x]^3)/(4*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^3}dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{4f} - \frac{(3(a-b))\operatorname{Sul}}{}$$

$$= \frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{8f} - \frac{\left(a-b+b\right.}{}$$

$$= \frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{8f} - \frac{\left(a-b+b\right.}{}$$

$$= -\frac{3(a-b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{8\sqrt{a}f} + \frac{3(a-b)\sqrt{a-b+b\cosh^2(e+fx}}{8f}$$

Mathematica [A] time = 0.630421, size = 123, normalized size = 0.91

$$\frac{\sqrt{2}\sqrt{a}\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}\left(-2a\operatorname{csch}^2(e+fx)+3a-5b\right)-6(a-b)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\,c}{\sqrt{2a+b\cos}}\right.}{16\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-6*(a - b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh
[2*(e + f*x)]]] + Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[
e + f*x]*Csch[e + f*x]*(3*a - 5*b - 2*a*Csch[e + f*x]^2))/(16*Sqrt[a]*f)
```

Maple [B] time = 0.095, size = 379, normalized size = 2.8

$$-\frac{1}{16\left(\sinh\left(fx+e\right)\right)^4\cosh\left(fx+e\right)f}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)\left(\cosh\left(fx+e\right)\right)^2}\left(3a^2\ln\left(\frac{(a+b)\left(\cosh\left(fx+e\right)\right)^2+2\sqrt{}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/16*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(3*a^2*ln(((a+b)*cosh(f*x+e
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)*sinh(f*x+e)^4-6*a*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(
a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^4+3*b^2*ln(((a+b)
*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/s
inh(f*x+e)^2)*sinh(f*x+e)^4-6*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(
```

```
1/2)*sinh(f*x+e)^2+10*b*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*a^(1/2)*s
inh(f*x+e)^2+4*a^(3/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/a^(1/2)/s
inh(f*x+e)^4/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^5, x)
```

Fricas [B] time = 4.11192, size = 8128, normalized size = 60.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(3*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)*sinh(f*x + e)^7 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 - 4*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^
2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3
 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(a^2 - 2*a*b +
b^2)*cosh(f*x + e)^4 + 2*(35*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 30*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^2 + 3*a^2 - 6*a*b + 3*b^2)*sinh(f*x + e)^4 + 8
*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 10*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^2 + 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 - 15*(
a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2
- a^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + a^2 - 2*a*b + b^2 + 8*((a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^7 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 3*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^3 - (a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f
*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a
 + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)
^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a
 - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x +
 e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x
+ e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x +
 e) + 1)) + 2*sqrt(2)*((3*a^2 - 5*a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - 5*a*b)*
cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b)*sinh(f*x + e)^6 - (11*a^2 -
 5*a*b)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*
a*b)*sinh(f*x + e)^4 + 4*(5*(3*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a
*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e)^2 + (15
*(3*a^2 - 5*a*b)*cosh(f*x + e)^4 - 6*(11*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*
a^2 + 5*a*b)*sinh(f*x + e)^2 + 3*a^2 - 5*a*b + 2*(3*(3*a^2 - 5*a*b)*cosh(f*
x + e)^5 - 2*(11*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x +
 e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
```

```
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*
cosh(f*x + e)^8 + 8*a*f*cosh(f*x + e)*sinh(f*x + e)^7 + a*f*sinh(f*x + e)^8
 - 4*a*f*cosh(f*x + e)^6 + 4*(7*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^6
+ 6*a*f*cosh(f*x + e)^4 + 8*(7*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*s
inh(f*x + e)^5 + 2*(35*a*f*cosh(f*x + e)^4 - 30*a*f*cosh(f*x + e)^2 + 3*a*f
)*sinh(f*x + e)^4 - 4*a*f*cosh(f*x + e)^2 + 8*(7*a*f*cosh(f*x + e)^5 - 10*a
*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f
*x + e)^6 - 15*a*f*cosh(f*x + e)^4 + 9*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x
+ e)^2 + a*f + 8*(a*f*cosh(f*x + e)^7 - 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(
f*x + e)^3 - a*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(3*((a^2 - 2*a*b + b^2)
*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (a
^2 - 2*a*b + b^2)*sinh(f*x + e)^8 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 +
 4*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e
)^6 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e)^5 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 2*(35*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 30*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^
2 + 3*a^2 - 6*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*(a^2 - 2*a*b + b^2)*cosh(
f*x + e)^5 - 10*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)
*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 4
*(7*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 - 15*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^4 + 9*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x
 + e)^2 + a^2 - 2*a*b + b^2 + 8*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^7 - 3*(a
^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 -
 (a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(sqrt(2)*
(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqr
t(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 +
 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f
*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f
*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2
- 5*a*b)*cosh(f*x + e)^6 + 6*(3*a^2 - 5*a*b)*cosh(f*x + e)*sinh(f*x + e)^5
+ (3*a^2 - 5*a*b)*sinh(f*x + e)^6 - (11*a^2 - 5*a*b)*cosh(f*x + e)^4 + (15*
(3*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^4 + 4*(5*(3
*a^2 - 5*a*b)*cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x +
e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e)^2 + (15*(3*a^2 - 5*a*b)*cosh(f*x + e)
^4 - 6*(11*a^2 - 5*a*b)*cosh(f*x + e)^2 - 11*a^2 + 5*a*b)*sinh(f*x + e)^2 +
 3*a^2 - 5*a*b + 2*(3*(3*a^2 - 5*a*b)*cosh(f*x + e)^5 - 2*(11*a^2 - 5*a*b)*
cosh(f*x + e)^3 - (11*a^2 - 5*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^8 + 8*a*f*cosh(f
*x + e)*sinh(f*x + e)^7 + a*f*sinh(f*x + e)^8 - 4*a*f*cosh(f*x + e)^6 + 4*(
7*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 8*(7
*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a*f*cos
h(f*x + e)^4 - 30*a*f*cosh(f*x + e)^2 + 3*a*f)*sinh(f*x + e)^4 - 4*a*f*cosh
(f*x + e)^2 + 8*(7*a*f*cosh(f*x + e)^5 - 10*a*f*cosh(f*x + e)^3 + 3*a*f*cos
h(f*x + e))*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f*x + e)^6 - 15*a*f*cosh(f*x +
e)^4 + 9*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^2 + a*f + 8*(a*f*cosh(f*x
 + e)^7 - 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e)
)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^5, x)
```

3.81 $\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=199

$$\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}^5(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6af}+\frac{(5a+b)\coth(e}{}$$

```
[Out] ((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e
+ f*x]^2]])/(16*a^(3/2)*f) - ((a - b)*(5*a + b)*Sqrt[a - b + b*Cosh[e + f*x
]^2]*Coth[e + f*x]*Csch[e + f*x])/(16*a*f) + ((5*a + b)*(a - b + b*Cosh[e +
 f*x]^2)^(3/2)*Coth[e + f*x]*Csch[e + f*x]^3)/(24*a*f) - ((a - b + b*Cosh[e
 + f*x]^2)^(5/2)*Coth[e + f*x]*Csch[e + f*x]^5)/(6*a*f)
```

Rubi [A] time = 0.199041, antiderivative size = 199, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 382, 378, 377, 206}

$$\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{16a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}^5(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{5/2}}{6af}+\frac{(5a+b)\coth(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e
+ f*x]^2]])/(16*a^(3/2)*f) - ((a - b)*(5*a + b)*Sqrt[a - b + b*Cosh[e + f*x
]^2]*Coth[e + f*x]*Csch[e + f*x])/(16*a*f) + ((5*a + b)*(a - b + b*Cosh[e +
 f*x]^2)^(3/2)*Coth[e + f*x]*Csch[e + f*x]^3)/(24*a*f) - ((a - b + b*Cosh[e
 + f*x]^2)^(5/2)*Coth[e + f*x]*Csch[e + f*x]^5)/(6*a*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a-b+bx^2\right)^{3/2}}{\left(1-x^2\right)^4}dx,x,\cosh(e+fx)\right)}{f}$$

$$=-\frac{\left(a-b+b\cosh^2(e+fx)\right)^{5/2}\coth(e+fx)\operatorname{csch}^5(e+fx)}{6af}+\frac{(5a+b)\,\mathrm{S}}{}$$

$$=\frac{(5a+b)\left(a-b+b\cosh^2(e+fx)\right)^{3/2}\coth(e+fx)\operatorname{csch}^3(e+fx)}{24af}-\frac{(a}{}$$

$$=-\frac{(a-b)(5a+b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{16af}+$$

$$=-\frac{(a-b)(5a+b)\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{16af}+$$

$$=\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{16a^{3/2}f}-\frac{(a-b)(5a+b)\sqrt{a-b+}}{}$$

Mathematica [A] time = 1.01245, size = 174, normalized size = 0.87

$$\frac{\frac{(a-b)^2(5a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{3/2}}-\frac{\coth(e+fx)\operatorname{csch}^5(e+fx)\sqrt{a+\frac{1}{2}b\cosh(2(e+fx))-\frac{b}{2}}\left(-4\left(25a^2-36ab+3b^2\right)\cosh(2(e+fx))+\left(15a^2-22ab+3b^2\right)\right.}{24a}}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((a - b)^2*(5*a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b
+ b*Cosh[2*(e + f*x)]]])/a^(3/2) - (Sqrt[a - b/2 + (b*Cosh[2*(e + f*x)])/2]
*(149*a^2 - 122*a*b + 9*b^2 - 4*(25*a^2 - 36*a*b + 3*b^2)*Cosh[2*(e + f*x)]
 + (15*a^2 - 22*a*b + 3*b^2)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]
^5)/(24*a))/(16*f)
```

Maple [B] time = 0.122, size = 569, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/96*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(30*a^(7/2)*((a+b*sinh(f*x+
e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^4-44*b*((a+b*sinh(f*x+e)^2)*cosh(f*x
+e)^2)^(1/2)*sinh(f*x+e)^4*a^(5/2)-15*a^4*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)
*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e
)^6+27*a^3*b*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(
f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^6-9*b^2*ln(((a+b)*cosh(f*x+
e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)
^2)*sinh(f*x+e)^6*a^2-3*b^3*ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e
)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(f*x+e)^6*a-20*a^(7/
2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^2+6*b^2*((a+b*sinh
(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^4*a^(3/2)+28*b*((a+b*sinh(f*x+e
)^2)*cosh(f*x+e)^2)^(1/2)*sinh(f*x+e)^2*a^(5/2)+16*a^(7/2)*((a+b*sinh(f*x+e
)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^6/a^(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+
e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^7, x)
```

Fricas [B] time = 10.1742, size = 18294, normalized size = 91.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/96*(3*((5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^12 + 12*(5*a^3 -
9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (5*a^3 - 9*a^2*b
+ 3*a*b^2 + b^3)*sinh(f*x + e)^12 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cos
h(f*x + e)^10 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 11*(5*a^3 - 9*a^2*b +
3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 9*a^2*b
+ 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh
(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x
+ e)^8 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 -
 9*a^2*b + 3*a*b^2 + b^3 - 18*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x +
e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x +
 e)^5 - 30*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 5*(5*a^3 - 9
*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^7 - 20*(5*a^3 - 9*a^2*
b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 4*(231*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^
3)*cosh(f*x + e)^6 - 315*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4
- 25*a^3 + 45*a^2*b - 15*a*b^2 - 5*b^3 + 105*(5*a^3 - 9*a^2*b + 3*a*b^2 + b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 +
```

b^3)*cosh(f*x + e)^7 - 63*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5
 + 35*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 5*(5*a^3 - 9*a^2*
b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 9*a^2*b + 3
*a*b^2 + b^3)*cosh(f*x + e)^4 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*co
sh(f*x + e)^8 - 84*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 70*(
5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*a*b^
2 + b^3 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x +
e)^4 + 20*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 - 36*(5*a^3
 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 + 42*(5*a^3 - 9*a^2*b + 3*a*b^2
 + b^3)*cosh(f*x + e)^5 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e
)^3 + 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 +
5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(
f*x + e)^2 + 6*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10 - 45*
(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 70*(5*a^3 - 9*a^2*b + 3
*a*b^2 + b^3)*cosh(f*x + e)^6 - 50*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f
*x + e)^4 - 5*a^3 + 9*a^2*b - 3*a*b^2 - b^3 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2
 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 9*a^2*b + 3*a*b^2 +
 b^3)*cosh(f*x + e)^11 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^
9 + 10*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 - 10*(5*a^3 - 9*a^
2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*
cosh(f*x + e)^3 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh
(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*
(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x +
e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3
*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*
x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x
 + e) + 1)) - 2*sqrt(2)*((15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^10 + 1
0*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*a^3 - 2
2*a^2*b + 3*a*b^2)*sinh(f*x + e)^10 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f
*x + e)^8 - (85*a^3 - 122*a^2*b + 9*a*b^2 - 45*(15*a^3 - 22*a^2*b + 3*a*b^2
)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*co
sh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e))*sinh(f*x + e)
^7 + 2*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 22*
a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 - 14*(85*a^3
 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(15*a^3 -
22*a^2*b + 3*a*b^2)*cosh(f*x + e)^5 - 14*(85*a^3 - 122*a^2*b + 9*a*b^2)*cos
h(f*x + e)^3 + 3*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)
^5 + 2*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 22*
a^2*b + 3*a*b^2)*cosh(f*x + e)^6 - 35*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f
*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 + 15*(99*a^3 - 50*a^2*b + 3*a*b^2)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh
(f*x + e)^7 - 7*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^5 + 5*(99*a^3
- 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 + (99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 + 15*a^3 - 22*a^2*b + 3*a*b^2 - (85*a^3 - 122*a^2
*b + 9*a*b^2)*cosh(f*x + e)^2 + (45*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x
+ e)^8 - 28*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^6 + 30*(99*a^3 - 5
0*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 - 85*a^3 + 122*a^2*b - 9*a*b^2 + 12*(99*
a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 -
 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^9 - 4*(85*a^3 - 122*a^2*b + 9*a*b^2)*cos
h(f*x + e)^7 + 6*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^5 + 4*(99*a^3
- 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))
/(a^2*f*cosh(f*x + e)^12 + 12*a^2*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^2*f*
sinh(f*x + e)^12 - 6*a^2*f*cosh(f*x + e)^10 + 15*a^2*f*cosh(f*x + e)^8 + 6*

(11*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^10 + 20*(11*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 - 20*a^2*f*cosh(f*x + e)^6 + 15*(33*a^2*f*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)^8 + 24*(33*a^2*f*cosh(f*x + e)^5 - 30*a^2*f*cosh(f*x + e)^3 + 5*a^2*f*cosh(f*x + e))*sinh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^4 + 4*(231*a^2*f*cosh(f*x + e)^6 - 315*a^2*f*cosh(f*x + e)^4 + 105*a^2*f*cosh(f*x + e)^2 - 5*a^2*f)*sinh(f*x + e)^6 + 24*(33*a^2*f*cosh(f*x + e)^7 - 63*a^2*f*cosh(f*x + e)^5 + 35*a^2*f*cosh(f*x + e)^3 - 5*a^2*f*cosh(f*x + e))*sinh(f*x + e)^5 - 6*a^2*f*cosh(f*x + e)^2 + 15*(33*a^2*f*cosh(f*x + e)^8 - 84*a^2*f*cosh(f*x + e)^6 + 70*a^2*f*cosh(f*x + e)^4 - 20*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)^4 + 20*(11*a^2*f*cosh(f*x + e)^9 - 36*a^2*f*cosh(f*x + e)^7 + 42*a^2*f*cosh(f*x + e)^5 - 20*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*f + 6*(11*a^2*f*cosh(f*x + e)^10 - 45*a^2*f*cosh(f*x + e)^8 + 70*a^2*f*cosh(f*x + e)^6 - 50*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 12*(a^2*f*cosh(f*x + e)^11 - 5*a^2*f*cosh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^7 - 10*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), -1/48*(3*((5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^12 + 12*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*sinh(f*x + e)^12 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 18*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 - 30*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^7 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 4*(231*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 - 315*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 - 25*a^3 + 45*a^2*b - 15*a*b^2 - 5*b^3 + 105*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 24*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 - 63*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 + 35*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 15*(33*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 - 84*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 + 70*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 + 5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 20*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 - 36*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 + 42*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 - 20*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 5*a^3 - 9*a^2*b + 3*a*b^2 + b^3 - 6*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2 + 6*(11*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^10 - 45*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^8 + 70*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^6 - 50*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^4 - 5*a^3 + 9*a^2*b - 3*a*b^2 - b^3 + 15*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^11 - 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^9 + 10*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^7 - 10*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e)^3 - (5*a^3 - 9*a^2*b + 3*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x +

```
e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))
+ sqrt(2)*((15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^10 + 10*(15*a^3 - 22
*a^2*b + 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*a^3 - 22*a^2*b + 3*a*
b^2)*sinh(f*x + e)^10 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^8 - (8
5*a^3 - 122*a^2*b + 9*a*b^2 - 45*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e
)^2)*sinh(f*x + e)^8 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^3
- (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(99*a^3
 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 22*a^2*b + 3*a*b^
2)*cosh(f*x + e)^4 + 99*a^3 - 50*a^2*b + 3*a*b^2 - 14*(85*a^3 - 122*a^2*b +
 9*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(15*a^3 - 22*a^2*b + 3*a
*b^2)*cosh(f*x + e)^5 - 14*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^3 +
 3*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(99*a^3
 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 22*a^2*b + 3*a*b^
2)*cosh(f*x + e)^6 - 35*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^4 + 99
*a^3 - 50*a^2*b + 3*a*b^2 + 15*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^
2)*sinh(f*x + e)^4 + 8*(15*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^7 -
7*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^5 + 5*(99*a^3 - 50*a^2*b + 3
*a*b^2)*cosh(f*x + e)^3 + (99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh
(f*x + e)^3 + 15*a^3 - 22*a^2*b + 3*a*b^2 - (85*a^3 - 122*a^2*b + 9*a*b^2)*
cosh(f*x + e)^2 + (45*(15*a^3 - 22*a^2*b + 3*a*b^2)*cosh(f*x + e)^8 - 28*(8
5*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^6 + 30*(99*a^3 - 50*a^2*b + 3*a*
b^2)*cosh(f*x + e)^4 - 85*a^3 + 122*a^2*b - 9*a*b^2 + 12*(99*a^3 - 50*a^2*b
 + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 - 22*a^2*b + 3*
a*b^2)*cosh(f*x + e)^9 - 4*(85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e)^7 +
 6*(99*a^3 - 50*a^2*b + 3*a*b^2)*cosh(f*x + e)^5 + 4*(99*a^3 - 50*a^2*b + 3
*a*b^2)*cosh(f*x + e)^3 - (85*a^3 - 122*a^2*b + 9*a*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f
*x + e)^12 + 12*a^2*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^2*f*sinh(f*x + e)^
12 - 6*a^2*f*cosh(f*x + e)^10 + 15*a^2*f*cosh(f*x + e)^8 + 6*(11*a^2*f*cosh
(f*x + e)^2 - a^2*f)*sinh(f*x + e)^10 + 20*(11*a^2*f*cosh(f*x + e)^3 - 3*a^
2*f*cosh(f*x + e))*sinh(f*x + e)^9 - 20*a^2*f*cosh(f*x + e)^6 + 15*(33*a^2*
f*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)^8 + 24*
(33*a^2*f*cosh(f*x + e)^5 - 30*a^2*f*cosh(f*x + e)^3 + 5*a^2*f*cosh(f*x + e
))*sinh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^4 + 4*(231*a^2*f*cosh(f*x + e)^
6 - 315*a^2*f*cosh(f*x + e)^4 + 105*a^2*f*cosh(f*x + e)^2 - 5*a^2*f)*sinh(f
*x + e)^6 + 24*(33*a^2*f*cosh(f*x + e)^7 - 63*a^2*f*cosh(f*x + e)^5 + 35*a^
2*f*cosh(f*x + e)^3 - 5*a^2*f*cosh(f*x + e))*sinh(f*x + e)^5 - 6*a^2*f*cosh
(f*x + e)^2 + 15*(33*a^2*f*cosh(f*x + e)^8 - 84*a^2*f*cosh(f*x + e)^6 + 70*
a^2*f*cosh(f*x + e)^4 - 20*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)^4 +
 20*(11*a^2*f*cosh(f*x + e)^9 - 36*a^2*f*cosh(f*x + e)^7 + 42*a^2*f*cosh(f*
x + e)^5 - 20*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*sinh(f*x + e)^
3 + a^2*f + 6*(11*a^2*f*cosh(f*x + e)^10 - 45*a^2*f*cosh(f*x + e)^8 + 70*a^
2*f*cosh(f*x + e)^6 - 50*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 -
 a^2*f)*sinh(f*x + e)^2 + 12*(a^2*f*cosh(f*x + e)^11 - 5*a^2*f*cosh(f*x + e
)^9 + 10*a^2*f*cosh(f*x + e)^7 - 10*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*
x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^{7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^7, x)
```

3.82 $\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=367

$$-\frac{\left(a^2-11ab+8b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{35bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a-2b)\left(a^2+4ab-\right.}{}$$

```
[Out] ((a^2 - 11*a*b + 8*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x
]^2])/(35*b*f) + (2*(4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Si
nh[e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]*Sinh[e + f*x]^5*Sqrt[a + b*Sinh[e
 + f*x]^2])/(7*f) + (2*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 11*a*b + 8*b^2
)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2
*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(35*b^2*f)
```

Rubi [A] time = 0.470834, antiderivative size = 367, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 477, 582, 531, 418, 492, 411}

$$-\frac{2(a-2b)\left(a^2+4ab-4b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f}+\frac{\left(a^2-11ab+8b^2\right)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b}}{35bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a^2 - 11*a*b + 8*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x
]^2])/(35*b*f) + (2*(4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]^3*Sqrt[a + b*Si
nh[e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]*Sinh[e + f*x]^5*Sqrt[a + b*Sinh[e
 + f*x]^2])/(7*f) + (2*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 11*a*b + 8*b^2
)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2
*(a - 2*b)*(a^2 + 4*a*b - 4*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(35*b^2*f)
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
```

```
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{1+x^2}}dx, x, \sinh(e+fx\right.}{f}$$

$$= \frac{b\cosh(e+fx)\sinh^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f} + \frac{\left(\sqrt{\cosh^2(e+fx\right.}{}$$

$$= \frac{2(4a-3b)\cosh(e+fx)\sinh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35f} + \frac{b\cosh(}{}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2}{}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2}{}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2}{}$$

$$= \frac{\left(a^2-11ab+8b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2}{}$$

Mathematica [C] time = 2.87267, size = 262, normalized size = 0.71

$$\frac{-64ia\left(3a^2b+2a^3-13ab^2+8b^3\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(b\left(144a^2-480a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((128*I)*a*(a^3 + 2*a^2*b - 12*a*b^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e +
 f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 + 3*a^2*b - 13*a*b
^2 + 8*b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x),
b/a] + Sqrt[2]*b*(32*a^3 - 496*a^2*b + 684*a*b^2 - 250*b^3 + b*(144*a^2 - 4
80*a*b + 299*b^2)*Cosh[2*(e + f*x)] + 2*(26*a - 27*b)*b^2*Cosh[4*(e + f*x)]
 + 5*b^3*Cosh[6*(e + f*x)])*Sinh[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a - b + b
*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.074, size = 743, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/35*(5*(-1/a*b)^(1/2)*b^3*sinh(f*x+e)*cosh(f*x+e)^8+(13*(-1/a*b)^(1/2)*a*b
^2-21*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-1/a*b)^(1/2)*a^2*b
```

```
-43*(-1/a*b)^(1/2)*a*b^2+35*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^4*sinh(f*x+e)+(
(-1/a*b)^(1/2)*a^3-20*(-1/a*b)^(1/2)*a^2*b+38*(-1/a*b)^(1/2)*a*b^2-19*(-1/a
*b)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3
+15*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-32*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b^2+16*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(
sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)
)*a^3-4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+24*(b/a*cosh(f*x+e)^2+(a-b)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b^2-16*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/b/(-1/a*b)^(1/2)/cosh(f*x+
e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^6+a\sinh\left(fx+e\right)^4\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^6 + a*sinh(f*x + e)^4)*sqrt(b*sinh(f*x + e)^2 + a
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

<u>Giac</u> [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^4, x)
```

3.83 $\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$\frac{ia(3a-4b)(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{15bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\left(3a^2-13ab+8b^2\right)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}+$$

```
[Out] ((3*a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f
) + (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*f) - ((I
/15)*(3*a^2 - 13*a*b + 8*b^2)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/15)*a*(3*a - 4*b)*(
a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sq
rt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.325431, antiderivative size = 236, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$-\frac{i\left(3a^2-13ab+8b^2\right)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}+\frac{\sinh(e+fx)\cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f}+\frac{(3a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((3*a - 4*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f
) + (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*f) - ((I
/15)*(3*a^2 - 13*a*b + 8*b^2)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/15)*a*(3*a - 4*b)*(
a - b)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(b*f*Sq
rt[a + b*Sinh[e + f*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule steps

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5f} - \frac{1}{5}\int \left(a-(3a-4\right.$$

$$= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\cosh(e+}{}$$

$$= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\cosh(e+}{}$$

$$= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\cosh(e+}{}$$

$$= \frac{(3a-4b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{\cosh(e+}{}$$

Mathematica [A] time = 1.36271, size = 213, normalized size = 0.9

$$\frac{16ia\left(3a^2-7ab+4b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(48a^2+4b(9a-7b)\cosh(2\right.}{240bf\sqrt{2a+b\cosh(2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-16*I)*a*(3*a^2 - 13*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 7*a*b + 4*b^2)*Sqrt[(2*a -
 b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^
2 - 68*a*b + 25*b^2 + 4*(9*a - 7*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e +
```

```
f*x)])*Sinh[2*(e + f*x)])/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.071, size = 535, normalized size = 2.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15*(-3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+(-9*(-1/a*b)^(1/2)*a
*b+10*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(-6*(-1/a*b)^(1/2)*a^2+
13*(-1/a*b)^(1/2)*a*b-7*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+9*a^2
*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-17*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+8*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/(-1/a
*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^{4}+a\sinh\left(fx+e\right)^{2}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^4 + a*sinh(f*x + e)^2)*sqrt(b*sinh(f*x + e)^2 + a
), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sinh\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sinh(f*x + e)^2, x)
```

3.84 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=174

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)\sqrt{a+b}}{3f}$$

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.193697, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(}}{3f\sqrt{\frac{b\sinh^2(}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a-b)+2(2a-b)b\sinh}{\sqrt{a+b\sinh^2(e+f}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}(a(a-b))\int \frac{1}{\sqrt{a+b\sinh^2(e+}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{\left(2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\right.}{3\sqrt{1+\frac{b\sinh^2}{a}}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b}}{3f\sqrt{1+\frac{b\sinh^2(e+fx}{a}}}$$

Mathematica [A] time = 0.679463, size = 169, normalized size = 0.97

$$\frac{2i\sqrt{2}a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-4i\sqrt{2}a(2a}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.069, size = 416, normalized size = 2.4

$$\frac{1}{3f\cosh(fx+e)}\left(\sqrt{-\frac{b}{a}}b^2\sinh(fx+e)\left(\cosh(fx+e)\right)^4+\left(\sqrt{-\frac{b}{a}}ab-\sqrt{-\frac{b}{a}}b^2\right)\left(\cosh(fx+e)\right)^2\sinh(fx+e)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a*b-(-1/a
*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^
2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(fx + e\right)^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.85 $\int \mathrm{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=204

$$\frac{2b\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\mathrm{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

```
[Out] -((a*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - ((a + b)*EllipticE[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*b*EllipticF[ArcTan[
Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*Sqrt[a + b*Sinh[e
 + f*x]^2]*Tanh[e + f*x])/f
```

Rubi [A] time = 0.211493, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3188, 474, 531, 418, 492, 411}

$$\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{2b\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - ((a + b)*EllipticE[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*b*EllipticF[ArcTan[
Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*Sqrt[a + b*Sinh[e
 + f*x]^2]*Tanh[e + f*x])/f
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^2\sqrt{1+x^2}}\,dx, x, \sinh(e+fx)\right.}{f}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx\right.}{}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(2ab\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+\right.}{}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{2bF\left(\tan^{-1}(\sinh(e+fx))\right|1-}{f\sqrt{\frac{\operatorname{sech}^2(}{}}}$$

$$= -\frac{a\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{(a+b)E\left(\tan^{-1}(\sinh(e+fx)\right.}{f\sqrt{\frac{\operatorname{sech}}{}}}$$

Mathematica [C] time = 1.04665, size = 155, normalized size = 0.76

$$-\frac{a\left(-2i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}\coth(e+fx)(2a+b\cosh(2(e+fx))-b)+2i(a+b)\right.}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(a*(Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Coth[e + f*x] + (2*I)*(a + b)*
Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)
*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a
]))/(2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.079, size = 243, normalized size = 1.2

$$\frac{1}{\cosh(fx+e)\sinh(fx+e)f}\left(\sinh(fx+e)\sqrt{(\cosh(fx+e))^2}\sqrt{\frac{b(\cosh(fx+e))^2}{a}+\frac{a-b}{a}}b\left(a\mathrm{EllipticF}\left(\sinh(fx+\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] (sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*E
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))
*a+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))-(-1/a*b)^(1/2)*a*b*
cosh(f*x+e)^4+(-(-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2)/sinh(
f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}\operatorname{csch}(fx+e)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\operatorname{csch}(fx+e)^2\sinh(fx+e)^2+a\operatorname{csch}(fx+e)^2\right)\sqrt{b\sinh(fx+e)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*csch(f*x + e)^2*sinh(f*x + e)^2 + a*csch(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \operatorname{csch}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^2, x)
```

3.86 $\int \mathbf{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=267

$$-\frac{b(a-3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sin}}{3f}$$

```
[Out] (2*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (a*Coth[e +
 f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (2*(a - 2*b)*Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)
*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2
*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rubi [A] time = 0.305941, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 474, 583, 531, 418, 492, 411}

$$-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{a\coth(e+fx)\text{csch}^2(e+}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (a*Coth[e +
 f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) + (2*(a - 2*b)*Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a - 3*b)
*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2
*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{x^4\sqrt{1+x^2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= -\frac{a\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{s}\right.}{}$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)\operatorname{csch}^2(e+f}{3f}$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)\operatorname{csch}^2(e+f}{3f}$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)\operatorname{csch}^2(e+f}{3f}$$

$$= \frac{2(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{a\coth(e+fx)\operatorname{csch}^2(e+f}{3f}$$

Mathematica [C] time = 4.06128, size = 213, normalized size = 0.8

$$\frac{-2i\left(2a^2-5ab+3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\left(2\left(2a^2-7ab+4b^2\right)\cosh(2(e+fx))-8a^2+b(a-2}{\sqrt{2}}}{6f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((-8*a^2 + 13*a*b - 6*b^2 + 2*(2*a^2 - 7*a*b + 4*b^2)*Cosh[2*(e + f*x)] +
(a - 2*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*
I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x
), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)]
)/a]*EllipticF[I*(e + f*x), b/a])/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.087, size = 454, normalized size = 1.7

$$\frac{1}{3\left(\sinh\left(fx+e\right)\right)^3\cosh\left(fx+e\right)f}\left(2\sqrt{-\frac{b}{a}}ab\left(\sinh\left(fx+e\right)\right)^6-4\sqrt{-\frac{b}{a}}b^2\left(\sinh\left(fx+e\right)\right)^6+b\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^2}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+
b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
```

```
*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+4*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4-3*(-1/
a*b)^(1/2)*a*b*sinh(f*x+e)^4-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^(1
/2)*a^2*sinh(f*x+e)^2-5*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2
)/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\operatorname{csch}\left(fx+e\right)^{4}\sinh\left(fx+e\right)^{2}+a\operatorname{csch}\left(fx+e\right)^{4}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*csch(f*x + e)^4*sinh(f*x + e)^2 + a*csch(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{csch}\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*csch(f*x + e)^4, x)
```

3.87 $\int \left(a+b\sinh^2(c+dx)\right)^{5/2} dx$

Optimal. Leaf size=232

$$\frac{4ia(a-b)(2a-b)\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}\text{EllipticF}\left(ic+idx,\frac{b}{a}\right)}{15d\sqrt{a+b\sinh^2(c+dx)}}-\frac{i\left(23a^2-23ab+8b^2\right)\sqrt{a+b\sinh^2(c+dx)}E\left(ic+idx\middle|\frac{b}{a}\right)}{15d\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}}+$$

```
[Out] (4*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x]*Sqrt[a + b*Sinh[c + d*x]^2])/(15
*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^(3/2))/(5*d) -
 ((I/15)*(23*a^2 - 23*a*b + 8*b^2)*EllipticE[I*c + I*d*x, b/a]*Sqrt[a + b*S
inh[c + d*x]^2])/(d*Sqrt[1 + (b*Sinh[c + d*x]^2)/a]) + (((4*I)/15)*a*(a - b
)*(2*a - b)*EllipticF[I*c + I*d*x, b/a]*Sqrt[1 + (b*Sinh[c + d*x]^2)/a])/(d
*Sqrt[a + b*Sinh[c + d*x]^2])
```

Rubi [A] time = 0.303784, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3180, 3170, 3172, 3178, 3177, 3183, 3182}

$$-\frac{i\left(23a^2-23ab+8b^2\right)\sqrt{a+b\sinh^2(c+dx)}E\left(ic+idx\middle|\frac{b}{a}\right)}{15d\sqrt{\frac{b\sinh^2(c+dx)}{a}+1}}+\frac{b\sinh(c+dx)\cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^{3/2}}{5d}+\frac{4b(}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^(5/2),x]
```


```
[Out] (4*(2*a - b)*b*Cosh[c + d*x]*Sinh[c + d*x]*Sqrt[a + b*Sinh[c + d*x]^2])/(15
*d) + (b*Cosh[c + d*x]*Sinh[c + d*x]*(a + b*Sinh[c + d*x]^2)^(3/2))/(5*d) -
 ((I/15)*(23*a^2 - 23*a*b + 8*b^2)*EllipticE[I*c + I*d*x, b/a]*Sqrt[a + b*S
inh[c + d*x]^2])/(d*Sqrt[1 + (b*Sinh[c + d*x]^2)/a]) + (((4*I)/15)*a*(a - b
)*(2*a - b)*EllipticF[I*c + I*d*x, b/a]*Sqrt[1 + (b*Sinh[c + d*x]^2)/a])/(d
*Sqrt[a + b*Sinh[c + d*x]^2])
```


Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^{5/2} dx = \frac{b\cosh(c+dx)\sinh(c+dx)\left(a+b\sinh^2(c+dx)\right)^{3/2}}{5d}+\frac{1}{5}\int \sqrt{a+b\sinh^2(c+dx)}\,\big(a(\ldots$$

$$= \frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+dx)\sinh(c+\ldots}{\ldots}$$

$$= \frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+dx)\sinh(c+\ldots}{\ldots}$$

$$= \frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+dx)\sinh(c+\ldots}{\ldots}$$

$$= \frac{4(2a-b)b\cosh(c+dx)\sinh(c+dx)\sqrt{a+b\sinh^2(c+dx)}}{15d}+\frac{b\cosh(c+dx)\sinh(c+\ldots}{\ldots}$$

Mathematica [A] time = 1.39443, size = 208, normalized size = 0.9

$$\frac{64ia\left(2a^2-3ab+b^2\right)\sqrt{\frac{2a+b\cosh(2(c+dx))-b}{a}}\,\text{EllipticF}\left(i(c+dx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(c+dx))\left(88a^2+28b(2a-b)\cosh(2(c\ldots\right.}{240d\sqrt{2a+b\cosh(2(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^(5/2),x]
```

```
[Out] ((-16*I)*a*(23*a^2 - 23*a*b + 8*b^2)*Sqrt[(2*a - b + b*Cosh[2*(c + d*x)])/a
]*EllipticE[I*(c + d*x), b/a] + (64*I)*a*(2*a^2 - 3*a*b + b^2)*Sqrt[(2*a -
b + b*Cosh[2*(c + d*x)])/a]*EllipticF[I*(c + d*x), b/a] + Sqrt[2]*b*(88*a^2
 - 88*a*b + 25*b^2 + 28*(2*a - b)*b*Cosh[2*(c + d*x)] + 3*b^2*Cosh[4*(c + d
*x)])*Sinh[2*(c + d*x)])/(240*d*Sqrt[2*a - b + b*Cosh[2*(c + d*x)]])
```

Maple [B] time = 0.085, size = 609, normalized size = 2.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^(5/2),x)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^3*sinh(d*x+c)*cosh(d*x+c)^6+(14*(-1/a*b)^(1/2)*a*b
^2-10*(-1/a*b)^(1/2)*b^3)*cosh(d*x+c)^4*sinh(d*x+c)+(11*(-1/a*b)^(1/2)*a^2*
b-18*(-1/a*b)^(1/2)*a*b^2+7*(-1/a*b)^(1/2)*b^3)*cosh(d*x+c)^2*sinh(d*x+c)+1
5*a^3*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sin
h(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))-34*a^2*b*(b/a*cosh(d*x+c)^2+(a-b)/a)^(
1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2)
)+27*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh
(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-8*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/
2)*(cosh(d*x+c)^2)^(1/2)*EllipticF(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*
b^3+23*a^2*b*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*Ellipt
icE(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))-23*(b/a*cosh(d*x+c)^2+(a-b)/a)^
(1/2)*(cosh(d*x+c)^2)^(1/2)*EllipticE(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2
))*a*b^2+8*(b/a*cosh(d*x+c)^2+(a-b)/a)^(1/2)*(cosh(d*x+c)^2)^(1/2)*Elliptic
E(sinh(d*x+c)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/(-1/a*b)^(1/2)/cosh(d*x+c)/(
a+b*sinh(d*x+c)^2)^(1/2)/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(dx+c\right)^2+a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b^2\sinh\left(dx+c\right)^4+2\,ab\sinh\left(dx+c\right)^2+a^2\right)\sqrt{b\sinh\left(dx+c\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*sinh(d*x + c)^4 + 2*a*b*sinh(d*x + c)^2 + a^2)*sqrt(b*sinh(d*
x + c)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(dx+c\right)^{2}+a\right)^{\frac{5}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^(5/2), x)
```

3.88 $\int \sqrt{1+\sinh^2(x)}\,dx$

Optimal. Leaf size=11

$$\sqrt{\cosh^2(x)}\tanh(x)$$

[Out] Sqrt[Cosh[x]^2]*Tanh[x]

Rubi [A] time = 0.0239073, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3176, 3207, 2637}

$$\sqrt{\cosh^2(x)}\tanh(x)$$

Antiderivative was successfully verified.

[In] Int[Sqrt[1 + Sinh[x]^2],x]

[Out] Sqrt[Cosh[x]^2]*Tanh[x]

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{1+\sinh^2(x)}\,dx &= \int \sqrt{\cosh^2(x)}\,dx\\ &= \left(\sqrt{\cosh^2(x)}\,\mathrm{sech}(x)\right)\int \cosh(x)\,dx\\ &= \sqrt{\cosh^2(x)}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0065058, size = 11, normalized size = 1.

$$\sqrt{\cosh^2(x)}\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 + Sinh[x]^2],x]
```

```
[Out] Sqrt[Cosh[x]^2]*Tanh[x]
```

Maple [A] time = 0.049, size = 14, normalized size = 1.3

$$\frac{\sinh(x)}{\cosh(x)}\sqrt{(\cosh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)^(1/2),x)
```

```
[Out] (cosh(x)^2)^(1/2)*sinh(x)/cosh(x)
```

Maxima [A] time = 1.55792, size = 15, normalized size = 1.36

$$-\frac{1}{2}e^{(-x)} + \frac{1}{2}e^x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

Fricas [A] time = 1.87428, size = 12, normalized size = 1.09

$$\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] sinh(x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sinh^2(x)+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x)**2 + 1), x)
```

Giac [A] time = 1.25695, size = 15, normalized size = 1.36

$$-\frac{1}{2}e^{(-x)}+\frac{1}{2}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*e^(-x) + 1/2*e^x
```

3.89 $\int \sqrt{-1-\sinh^2(x)}\,dx$

Optimal. Leaf size=13

$$\sqrt{-\cosh^2(x)}\tanh(x)$$

```
[Out] Sqrt[-Cosh[x]^2]*Tanh[x]
```

Rubi [A] time = 0.0256116, antiderivative size = 13, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3176, 3207, 2637}

$$\sqrt{-\cosh^2(x)}\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] Sqrt[-Cosh[x]^2]*Tanh[x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1-\sinh^2(x)}\,dx &= \int \sqrt{-\cosh^2(x)}\,dx\\ &= \left(\sqrt{-\cosh^2(x)}\,\mathbf{sech}(x)\right)\int \cosh(x)\,dx\\ &= \sqrt{-\cosh^2(x)}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0053552, size = 13, normalized size = 1.

$$\sqrt{-\cosh^2(x)}\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] Sqrt[-Cosh[x]^2]*Tanh[x]
```

Maple [A] time = 0.033, size = 15, normalized size = 1.2

$$-\cosh(x)\sinh(x)\frac{1}{\sqrt{-(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-sinh(x)^2)^(1/2),x)
```

```
[Out] -cosh(x)*sinh(x)/(-cosh(x)^2)^(1/2)
```

Maxima [B] time = 1.55258, size = 34, normalized size = 2.62

$$-\frac{e^{(-2x)}}{2\sqrt{-e^{(-2x)}}}+\frac{1}{2\sqrt{-e^{(-2x)}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-2*x)/sqrt(-e^(-2*x)) + 1/2/sqrt(-e^(-2*x))
```

Fricas [C] time = 1.7685, size = 38, normalized size = 2.92

$$\frac{1}{2}\left(ie^{(2x)}-i\right)e^{(-x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(I*e^(2*x) - I)*e^(-x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{-\sinh^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-sinh(x)**2 - 1), x)
```

Giac [C] time = 1.30061, size = 15, normalized size = 1.15

$$-\frac{1}{2}ie^{(-x)} + \frac{1}{2}ie^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*I*e^(-x) + 1/2*I*e^x
```

3.90 $\int \sqrt{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=11

$$-iE(ix|-1)$$

```
[Out] (-I)*EllipticE[I*x, -1]
```

Rubi [A] time = 0.0102307, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3177}

$$-iE(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[1 - Sinh[x]^2],x]

[Out] (-I)*EllipticE[I*x, -1]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \sqrt{1-\sinh^2(x)}\,dx=-iE(ix|-1)$$

Mathematica [A] time = 0.024372, size = 11, normalized size = 1.

$$-iE(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[1 - Sinh[x]^2],x]

[Out] (-I)*EllipticE[I*x, -1]
```

Maple [B] time = 0.07, size = 51, normalized size = 4.6

$$\frac{2\,\mathrm{EllipticF}\left(\sinh(x),i\right)-\mathrm{EllipticE}\left(\sinh(x),i\right)}{\cosh(x)}\sqrt{-\left(-1+(\sinh(x))^2\right)(\cosh(x))^2}\sqrt{(\cosh(x))^2}\frac{1}{\sqrt{1-(\sinh(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sinh(x)^2)^(1/2),x)
```

```
[Out] (-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(2*EllipticF(sinh(x),I)
-EllipticE(sinh(x),I))/(1-sinh(x)^4)^(1/2)/cosh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sinh(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(-sinh(x)^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{-\sinh(x)^2+1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-sinh(x)^2 + 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{1-\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(1 - sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{-\sinh(x)^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(-sinh(x)^2 + 1), x)
```

3.91 $\int \sqrt{-1+\sinh^2(x)}\,dx$

Optimal. Leaf size=33

$$-\frac{i\sqrt{\sinh^2(x)-1}E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

```
[Out] ((-I)*EllipticE[I*x, -1]*Sqrt[-1 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]
```

Rubi [A] time = 0.0202611, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3178, 3177}

$$-\frac{i\sqrt{\sinh^2(x)-1}E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[-1 + Sinh[x]^2],x]

[Out] ((-I)*EllipticE[I*x, -1]*Sqrt[-1 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{-1+\sinh^2(x)}\,dx &= \frac{\sqrt{-1+\sinh^2(x)}\int \sqrt{1-\sinh^2(x)}\,dx}{\sqrt{1-\sinh^2(x)}}\\ &= -\frac{iE(ix|-1)\sqrt{-1+\sinh^2(x)}}{\sqrt{1-\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0329341, size = 33, normalized size = 1.

$$\frac{i\sqrt{3-\cosh(2x)}E(ix|-1)}{\sqrt{\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] (I*Sqrt[3 - Cosh[2*x]]*EllipticE[I*x, -1])/Sqrt[-3 + Cosh[2*x]]
```

Maple [A] time = 0.064, size = 61, normalized size = 1.9

$$\frac{i\text{EllipticE}\left(i\sinh\left(x\right),i\right)}{\cosh\left(x\right)}\sqrt{\left(-1+\left(\sinh\left(x\right)\right)^{2}\right)\left(\cosh\left(x\right)\right)^{2}}\sqrt{\left(\cosh\left(x\right)\right)^{2}}\sqrt{1-\left(\sinh\left(x\right)\right)^{2}}\frac{1}{\sqrt{\left(\sinh\left(x\right)\right)^{4}-1}}\frac{1}{\sqrt{-1+(\sinh(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sinh(x)^2)^(1/2),x)
```

```
[Out] I*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(1-sinh(x)^2)^(1/2)*El
lipticE(I*sinh(x),I)/(sinh(x)^4-1)^(1/2)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sinh\left(x\right)^{2}-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(sinh(x)^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{\sinh\left(x\right)^{2}-1},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(sinh(x)^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{\sinh^{2}\left(x\right)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sinh(x)**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{\sinh(x)^2 - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(sinh(x)^2 - 1), x)
```

3.92 $\int \sqrt{a+b\sinh^2(x)}\,dx$

Optimal. Leaf size=42

$$-\frac{i\sqrt{a+b\sinh^2(x)}E\left(ix\left|\frac{b}{a}\right.\right)}{\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

```
[Out] ((-I)*EllipticE[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a]
```

Rubi [A] time = 0.0317896, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\sinh^2(x)}E\left(ix\left|\frac{b}{a}\right.\right)}{\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^2(x)}\,dx &= \frac{\sqrt{a+b\sinh^2(x)}\int \sqrt{1+\frac{b\sinh^2(x)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}\\ &= -\frac{iE\left(ix\left|\frac{b}{a}\right.\right)\sqrt{a+b\sinh^2(x)}}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0439838, size = 54, normalized size = 1.29

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2x)-b}{a}}E\left(ix\left|\frac{b}{a}\right.\right)}{\sqrt{2a+b\cosh(2x)-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticE[I*x, b/a])/Sqrt[2*a - b +
 b*Cosh[2*x]]
```

Maple [B] time = 0.067, size = 109, normalized size = 2.6

$$\frac{1}{\cosh(x)}\sqrt{\frac{a+b(\sinh(x))^2}{a}}\sqrt{(\cosh(x))^2}\left(a\mathrm{EllipticF}\left(\sinh(x)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\mathrm{EllipticF}\left(\sinh(x)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)+b\mathrm{Ellip}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(x)^2)/a)^(1/2)*(cosh(x)^2)^(1/2)*(a*EllipticF(sinh(x)*(-1/a*b)^(
1/2),(a/b)^(1/2))-b*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))+b*Ellipti
cE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/cosh(x)/(a+b*sinh(x)
^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh(x)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh(x)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(x)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sinh^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(x)^2 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^2 + a), x)
```

3.93 $\int \left(1+\sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=29

$$\frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{2}{3}\sqrt{\cosh^2(x)}\tanh(x)$$

```
[Out] (2*Sqrt[Cosh[x]^2]*Tanh[x])/3 + ((Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rubi [A] time = 0.026434, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.4, Rules used = {3176, 3203, 3207, 2637}

$$\frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)+\frac{2}{3}\sqrt{\cosh^2(x)}\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[Cosh[x]^2]*Tanh[x])/3 + ((Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \left(1+\sinh^2(x)\right)^{3/2} dx &= \int \cosh^2(x)^{3/2}\, dx\\ &= \frac{1}{3}\cosh^2(x)^{3/2}\tanh(x) + \frac{2}{3}\int \sqrt{\cosh^2(x)}\, dx\\ &= \frac{1}{3}\cosh^2(x)^{3/2}\tanh(x) + \frac{1}{3}\left(2\sqrt{\cosh^2(x)}\text{sech}(x)\right)\int \cosh(x)\, dx\\ &= \frac{2}{3}\sqrt{\cosh^2(x)}\tanh(x) + \frac{1}{3}\cosh^2(x)^{3/2}\tanh(x)\end{aligned}$$

Mathematica [A] time = 0.0203646, size = 23, normalized size = 0.79

$$\frac{1}{12}(9\sinh(x)+\sinh(3x))\sqrt{\cosh^2(x)}\text{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] (Sqrt[Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12
```

Maple [A] time = 0.041, size = 21, normalized size = 0.7

$$\frac{\sinh(x)\left((\sinh(x))^2+3\right)}{3\cosh(x)}\sqrt{(\cosh(x))^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1+sinh(x)^2)^(3/2),x)
```

```
[Out] 1/3*(cosh(x)^2)^(1/2)*sinh(x)*(sinh(x)^2+3)/cosh(x)
```

Maxima [A] time = 1.5582, size = 31, normalized size = 1.07

$$\frac{1}{24}e^{(3x)} - \frac{3}{8}e^{(-x)} - \frac{1}{24}e^{(-3x)} + \frac{3}{8}e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/24*e^(3*x) - 3/8*e^(-x) - 1/24*e^(-3*x) + 3/8*e^x
```

Fricas [A] time = 1.78809, size = 62, normalized size = 2.14

$$\frac{1}{12}\sinh(x)^3 + \frac{1}{4}\left(\cosh(x)^2+3\right)\sinh(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/12*sinh(x)^3 + 1/4*(cosh(x)^2 + 3)*sinh(x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.22696, size = 34, normalized size = 1.17

$$-\frac{1}{24}\left(9 e^{(2x)}+1\right)e^{(-3x)}+\frac{1}{24} e^{(3x)}+\frac{3}{8} e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1+sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/24*(9*e^(2*x) + 1)*e^(-3*x) + 1/24*e^(3*x) + 3/8*e^x
```

3.94 $\int \left(-1-\sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=33

$$\frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{2}{3}\sqrt{-\cosh^2(x)}\tanh(x)$$

```
[Out] (-2*Sqrt[-Cosh[x]^2]*Tanh[x])/3 + ((-Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rubi [A] time = 0.0293963, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3176, 3203, 3207, 2637}

$$\frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)-\frac{2}{3}\sqrt{-\cosh^2(x)}\tanh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(-1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[-Cosh[x]^2]*Tanh[x])/3 + ((-Cosh[x]^2)^(3/2)*Tanh[x])/3
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\int \left(-1-\sinh^2(x)\right)^{3/2} dx = \int \left(-\cosh^2(x)\right)^{3/2} dx$$
$$= \frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x) - \frac{2}{3}\int \sqrt{-\cosh^2(x)}\,dx$$
$$= \frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x) - \frac{1}{3}\left(2\sqrt{-\cosh^2(x)}\operatorname{sech}(x)\right)\int \cosh(x)\,dx$$
$$= -\frac{2}{3}\sqrt{-\cosh^2(x)}\tanh(x) + \frac{1}{3}\left(-\cosh^2(x)\right)^{3/2}\tanh(x)$$

Mathematica [A] time = 0.0063351, size = 25, normalized size = 0.76

$$-\frac{1}{12}(9\sinh(x)+\sinh(3x))\sqrt{-\cosh^2(x)}\operatorname{sech}(x)$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[-Cosh[x]^2]*Sech[x]*(9*Sinh[x] + Sinh[3*x]))/12
```

Maple [A] time = 0.043, size = 21, normalized size = 0.6

$$\frac{\cosh(x)\sinh(x)\left((\cosh(x))^2+2\right)}{3}\frac{1}{\sqrt{-(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1-sinh(x)^2)^(3/2),x)
```

```
[Out] 1/3*cosh(x)*sinh(x)*(cosh(x)^2+2)/(-cosh(x)^2)^(1/2)
```

Maxima [B] time = 1.54951, size = 72, normalized size = 2.18

$$\frac{3\,e^{(-2\,x)}}{8\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}} - \frac{3\,e^{(-4\,x)}}{8\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}} - \frac{e^{(-6\,x)}}{24\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}} + \frac{1}{24\left(-e^{(-2\,x)}\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 3/8*e^(-2*x)/(-e^(-2*x))^(3/2) - 3/8*e^(-4*x)/(-e^(-2*x))^(3/2) - 1/24*e^(-
6*x)/(-e^(-2*x))^(3/2) + 1/24/(-e^(-2*x))^(3/2)
```

Fricas [C] time = 1.80619, size = 81, normalized size = 2.45

$$\frac{1}{24}\left(-i\,e^{(6\,x)} - 9i\,e^{(4\,x)} + 9i\,e^{(2\,x)} + i\right)e^{(-3\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/24*(-I*e^(6*x) - 9*I*e^(4*x) + 9*I*e^(2*x) + I)*e^(-3*x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.19878, size = 34, normalized size = 1.03

$$\frac{1}{24} i \left(9 e^{(2x)} + 1\right) e^{(-3x)} - \frac{1}{24} i e^{(3x)} - \frac{3}{8} i e^{x}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1-sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/24*I*(9*e^(2*x) + 1)*e^(-3*x) - 1/24*I*e^(3*x) - 3/8*I*e^x
```

3.95 $\int \left(1-\sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=45

$$\frac{2}{3}i\text{EllipticF}(ix,-1)-2iE(ix|-1)-\frac{1}{3}\sinh(x)\sqrt{1-\sinh^2(x)}\cosh(x)$$

```
[Out] (-2*I)*EllipticE[I*x, -1] + ((2*I)/3)*EllipticF[I*x, -1] - (Cosh[x]*Sinh[x]
*Sqrt[1 - Sinh[x]^2])/3
```

Rubi [A] time = 0.0635546, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3180, 3172, 3177, 3182}

$$\frac{2}{3}iF(ix|-1)-2iE(ix|-1)-\frac{1}{3}\sinh(x)\sqrt{1-\sinh^2(x)}\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] (-2*I)*EllipticE[I*x, -1] + ((2*I)/3)*EllipticF[I*x, -1] - (Cosh[x]*Sinh[x]
*Sqrt[1 - Sinh[x]^2])/3
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(1-\sinh^2(x)\right)^{3/2} dx = -\frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)} + \frac{1}{3}\int \frac{4-6\sinh^2(x)}{\sqrt{1-\sinh^2(x)}}dx$$
$$= -\frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)} - \frac{2}{3}\int \frac{1}{\sqrt{1-\sinh^2(x)}}dx + 2\int \sqrt{1-\sinh^2(x)}\,dx$$
$$= -2iE(ix|-1) + \frac{2}{3}iF(ix|-1) - \frac{1}{3}\cosh(x)\sinh(x)\sqrt{1-\sinh^2(x)}$$

Mathematica [A] time = 0.0688293, size = 45, normalized size = 1.

$$\frac{1}{12}\left(8i\text{EllipticF}(ix,-1) - 24iE(ix|-1) - \sinh(2x)\sqrt{6-2\cosh(2x)}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^2)^(3/2),x]
```

```
[Out] ((-24*I)*EllipticE[I*x, -1] + (8*I)*EllipticF[I*x, -1] - Sqrt[6 - 2*Cosh[2*
x]]*Sinh[2*x])/12
```

Maple [A] time = 0.077, size = 103, normalized size = 2.3

$$\frac{1}{3\cosh(x)}\sqrt{-\left(-1+(\sinh(x))^2\right)(\cosh(x))^2}\left(\sinh(x)(\cosh(x))^4 + 10\sqrt{-(\cosh(x))^2+2}\sqrt{(\cosh(x))^2}\text{EllipticF}(\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((1-sinh(x)^2)^(3/2),x)
```

```
[Out] 1/3*(-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(sinh(x)*cosh(x)^4+10*(-cosh(x)^2+2)^
(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x),I)-6*(-cosh(x)^2+2)^(1/2)*(cosh(x
)^2)^(1/2)*EllipticE(sinh(x),I)-2*cosh(x)^2*sinh(x))/(1-sinh(x)^4)^(1/2)/co
sh(x)/(1-sinh(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sinh(x)^2+1\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((-sinh(x)^2 + 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(-\sinh(x)^2+1\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-sinh(x)^2 + 1)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(-\sinh\left(x\right)^{2}+1\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((1-sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((-sinh(x)^2 + 1)^(3/2), x)
```

3.96 $\int \left(-1+\sinh^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=87

$$\frac{2i\sqrt{1-\sinh^2(x)}\,\text{EllipticF}(ix,-1)}{3\sqrt{\sinh^2(x)-1}}+\frac{1}{3}\sinh(x)\sqrt{\sinh^2(x)-1}\cosh(x)+\frac{2i\sqrt{\sinh^2(x)-1}\,E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

```
[Out] (((2*I)/3)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2] + (
Cosh[x]*Sinh[x]*Sqrt[-1 + Sinh[x]^2])/3 + ((2*I)*EllipticE[I*x, -1]*Sqrt[-1
 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]
```

Rubi [A] time = 0.0855633, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{1}{3}\sinh(x)\sqrt{\sinh^2(x)-1}\cosh(x)+\frac{2i\sqrt{1-\sinh^2(x)}F(ix|-1)}{3\sqrt{\sinh^2(x)-1}}+\frac{2i\sqrt{\sinh^2(x)-1}\,E(ix|-1)}{\sqrt{1-\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(-1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] (((2*I)/3)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2] + (
Cosh[x]*Sinh[x]*Sqrt[-1 + Sinh[x]^2])/3 + ((2*I)*EllipticE[I*x, -1]*Sqrt[-1
 + Sinh[x]^2])/Sqrt[1 - Sinh[x]^2]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \left(-1+\sinh^2(x)\right)^{3/2}dx &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}+\frac{1}{3}\int \frac{4-6\sinh^2(x)}{\sqrt{-1+\sinh^2(x)}}dx\\ &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}-\frac{2}{3}\int \frac{1}{\sqrt{-1+\sinh^2(x)}}dx-2\int \sqrt{-1+\sinh^2(x)}\,dx\\ &= \frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}-\frac{\left(2\sqrt{1-\sinh^2(x)}\right)\int \frac{1}{\sqrt{1-\sinh^2(x)}}dx}{3\sqrt{-1+\sinh^2(x)}}-\frac{\left(2\sqrt{-1+\sinh^2(x)}\right.}{\sqrt{1-s}}\\ &= \frac{2iF(ix|-1)\sqrt{1-\sinh^2(x)}}{3\sqrt{-1+\sinh^2(x)}}+\frac{1}{3}\cosh(x)\sinh(x)\sqrt{-1+\sinh^2(x)}+\frac{2iE(ix|-1)\sqrt{-1+\sinh^2(x}}{\sqrt{1-\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.12193, size = 78, normalized size = 0.9

$$\frac{8i\sqrt{3-\cosh(2x)}\text{EllipticF}(ix,-1)+\frac{\sinh(4x)-6\sinh(2x)}{\sqrt{2}}-24i\sqrt{3-\cosh(2x)}E(ix|-1)}{12\sqrt{\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(-1 + Sinh[x]^2)^(3/2),x]
```

```
[Out] ((-24*I)*Sqrt[3 - Cosh[2*x]]*EllipticE[I*x, -1] + (8*I)*Sqrt[3 - Cosh[2*x]]
*EllipticF[I*x, -1] + (-6*Sinh[2*x] + Sinh[4*x])/Sqrt[2])/(12*Sqrt[-3 + Cos
h[2*x]])
```

Maple [A] time = 0.079, size = 106, normalized size = 1.2

$$\frac{1}{3\cosh(x)}\sqrt{\left(-1+(\sinh(x))^2\right)(\cosh(x))^2}\left(\sinh(x)(\cosh(x))^4+2i\sqrt{(\cosh(x))^2}\sqrt{-(\cosh(x))^2+2}\text{EllipticF}(i\sinh(x)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((-1+sinh(x)^2)^(3/2),x)
```

```
[Out] 1/3*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(sinh(x)*cosh(x)^4+2*I*(cosh(x)^2)^(1/
2)*(-cosh(x)^2+2)^(1/2)*EllipticF(I*sinh(x),I)-6*I*(cosh(x)^2)^(1/2)*(-cosh
(x)^2+2)^(1/2)*EllipticE(I*sinh(x),I)-2*cosh(x)^2*sinh(x))/(sinh(x)^4-1)^(1
```

```
/2)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sinh(x)^2-1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((sinh(x)^2 - 1)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(\sinh(x)^2-1\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((sinh(x)^2 - 1)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(\sinh(x)^2-1\right)^{\frac{3}{2}}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((-1+sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((sinh(x)^2 - 1)^(3/2), x)
```

3.97 $\int \left(a+b\sinh^2(x)\right)^{3/2} dx$

Optimal. Leaf size=123

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(x)}{a}+1}\text{EllipticF}\left(ix,\frac{b}{a}\right)}{3\sqrt{a+b\sinh^2(x)}}+\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\sinh^2(x)}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(x)}E\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

```
[Out] (b*Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2])/3 - (((2*I)/3)*(2*a - b)*Elliptic
E[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a] + ((I/3)*a*(a
- b)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]
```

Rubi [A] time = 0.162822, antiderivative size = 123, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{1}{3}b\sinh(x)\cosh(x)\sqrt{a+b\sinh^2(x)}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(x)}{a}+1}F\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{a+b\sinh^2(x)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(x)}E\left(ix\middle|\frac{b}{a}\right)}{3\sqrt{\frac{b\sinh^2(x)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[x]*Sinh[x]*Sqrt[a + b*Sinh[x]^2])/3 - (((2*I)/3)*(2*a - b)*Elliptic
E[I*x, b/a]*Sqrt[a + b*Sinh[x]^2])/Sqrt[1 + (b*Sinh[x]^2)/a] + ((I/3)*a*(a
- b)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(x)\right)^{3/2}\,dx = \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}+\frac{1}{3}\int \frac{a(3a-b)+2(2a-b)b\sinh^2(x)}{\sqrt{a+b\sinh^2(x)}}\,dx$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}-\frac{1}{3}(a(a-b))\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx+\frac{1}{3}(2(2a-b))\int$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}+\frac{\left(2(2a-b)\sqrt{a+b\sinh^2(x)}\right)\int \sqrt{1+\frac{b\sinh^2(x)}{a}}\,dx}{3\sqrt{1+\frac{b\sinh^2(x)}{a}}}-$$

$$= \frac{1}{3}b\cosh(x)\sinh(x)\sqrt{a+b\sinh^2(x)}-\frac{2i(2a-b)E\left(ix\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(x)}}{3\sqrt{1+\frac{b\sinh^2(x)}{a}}}+\frac{ia(a-b)F\left(ix\right.}{3\sqrt{a}}$$

Mathematica [A] time = 0.378756, size = 132, normalized size = 1.07

$$\frac{4ia(a-b)\sqrt{\frac{2a+b\cosh(2x)-b}{a}}\text{EllipticF}\left(ix,\frac{b}{a}\right)+\sqrt{2}b\sinh(2x)(2a+b\cosh(2x)-b)-8ia(2a-b)\sqrt{\frac{2a+b\cosh(2x)-b}{a}}E\left(ix\middle|\frac{b}{a}\right)}{12\sqrt{2a+b\cosh(2x)-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^2)^(3/2),x]
```

```
[Out] ((-8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticE[I*x, b/a] + (
4*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticF[I*x, b/a] + Sqrt[2
]*b*(2*a - b + b*Cosh[2*x])*Sinh[2*x])/(12*Sqrt[2*a - b + b*Cosh[2*x]])
```

Maple [B] time = 0.07, size = 329, normalized size = 2.7

$$\frac{1}{3\cosh(x)}\left(\sqrt{-\frac{b}{a}}b^2\sinh(x)(\cosh(x))^4+\left(\sqrt{-\frac{b}{a}}ab-\sqrt{-\frac{b}{a}}b^2\right)(\cosh(x))^2\sinh(x)+3a^2\sqrt{\frac{b(\cosh(x))^2}{a}+\frac{a-b}{a}}\sqrt{(c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(x)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(x)*cosh(x)^4+((-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2
)*b^2)*cosh(x)^2*sinh(x)+3*a^2*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1
/2)*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))-5*a*b*(b/a*cosh(x)^2+(a-b
)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))+
2*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh(x)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2+4*a*b*(b/a*cosh(x)^2+(a-b)/a)^(1/2)*(cosh(x)^2)^(1
/2)*EllipticE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*(b/a*cosh(x)^2+(a-b)/a)
^(1/2)*(cosh(x)^2)^(1/2)*EllipticE(sinh(x)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)
/(-1/a*b)^(1/2)/cosh(x)/(a+b*sinh(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(x\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(x)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh\left(x\right)^{2}+a\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(x)^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(x\right)^{2}+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(x)^2 + a)^(3/2), x)
```

3.98 $\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2bf}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2b^{3/2}f}$$

```
[Out] -((a + b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])
/(2*b^(3/2)*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*b*f)
```

Rubi [A] time = 0.0992391, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 388, 217, 206}

$$\frac{\cosh(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2bf}-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((a + b)*ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])
/(2*b^(3/2)*f) + (Cosh[e + f*x]*Sqrt[a - b + b*Cosh[e + f*x]^2])/(2*b*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{2bf}$$

$$= \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2bf}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\cosh(e+fx)\sqrt{a-b+b\cosh^2(e+fx)}}{2bf}$$

Mathematica [A] time = 0.269282, size = 98, normalized size = 1.18

$$\frac{\cosh(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{2\sqrt{2}bf} - \frac{(a+b)\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])/(2*Sqrt[2]*b*f) - ((a +
 b)*Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
])/(2*b^(3/2)*f)
```

Maple [B] time = 0.109, size = 204, normalized size = 2.5

$$\frac{1}{4f\cosh(fx+e)}\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+e)\right)^2}\left(2b^{3/2}\sqrt{b\left(\cosh(fx+e)\right)^4+(a-b)\left(\cosh(fx+e)\right)^2}-b^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(2*b^(3/2)*(b*cosh(f*x+e)^4+(
a-b)*cosh(f*x+e)^2)^(1/2)-b^2*ln(1/2*(2*b*cosh(f*x+e)^2+2*(b*cosh(f*x+e)^4+
(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))-b*a*ln(1/2*(2*b*cosh(f*x+e
)^2+2*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2)))/b^
(5/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx+e)^3}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sinh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.5622, size = 5576, normalized size = 67.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [1/8*(((a + b)*cosh(f*x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a
 + b)*sinh(f*x + e)^2)*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*
x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x +
 e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*
a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9
*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2
*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*
(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4
*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh
(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^
4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x +
 e)^2 - sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5
+ a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 +
a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh
(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*
a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*
x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*c
osh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3
)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x
 + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e
)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)
^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a + b)*cosh(f*
x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*sinh(f*x + e)^2)
*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*si
nh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)
*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(c
osh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2
)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2
+ b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)
^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2), 1/8*(2*(
(a + b)*cosh(f*x + e)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*s
inh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x +
 e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f
*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*b*co
sh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x + e)
^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a + b)*cosh(f*x + e
)^2 + 2*(a + b)*cosh(f*x + e)*sinh(f*x + e) + (a + b)*sinh(f*x + e)^2)*sqrt
```

```
(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/
(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
+ 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
+ e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)
) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f
*x + e)^2 + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cos
h(f*x + e)^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2)
]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.99 $\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{b}f}$$

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[b]*f
)
```

Rubi [A] time = 0.0505043, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[b]*f
)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{b}f}$$

Mathematica [A] time = 0.10851, size = 49, normalized size = 1.2

$$\frac{\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/(S
qrt[b]*f)
```

Maple [B] time = 0.054, size = 108, normalized size = 2.6

$$\frac{1}{2f\cosh\left(fx+e\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\ln\left(\frac{1}{2}\left(2b\left(\cosh\left(fx+e\right)\right)^{2}+2\sqrt{b\left(\cosh\left(fx+e\right)\right)^{4}+(a-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*ln(1/2*(2*b*cosh(f*x+e)^2+2*(
b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)*b^(1/2)+a-b)/b^(1/2))/b^(1/2)/co
sh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.36186, size = 4286, normalized size = 104.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)*sinh(f*x +
 e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6 + 2*(14*a^2
*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*cosh(f*x +
e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 4*a*b^2
+ b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4*a*b^2 + b^
3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^2*b*cosh(f*
x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2 + b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(
14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*a*b^2 - b^3
 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*(
a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*sinh(f*x +
e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e
)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 + (4*
a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*cosh(f*x + e)
^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)^5 + 6*a^2*
cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*x + e)^7 +
3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^3
 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(
f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x
 + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x +
 e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + sqrt(b)*log(-(b*cosh(f*x + e)^4 +
 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x
 + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sq
rt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a
- b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2)))/(b*f), -1/2*(sqrt(-b)*arctan(sqrt(2)*(a*c
osh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*s
qrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)
^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b
^2)*cosh(f*x + e)^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2
 + b^2 + 2*(2*a*b*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))) + sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh
(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f
*x + e) + b)))/(b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.100 $\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[a]
*f))
```

Rubi [A] time = 0.0802426, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(Sqrt[a]
*f))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$
$$= -\frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{f}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{\sqrt{a}f}$$

Mathematica [A] time = 0.17785, size = 49, normalized size = 1.17

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]]/(Sqrt[a]*f))
```

Maple [B] time = 0.069, size = 113, normalized size = 2.7

$$-\frac{1}{2f\cosh(fx+e)}\sqrt{\left(a+b(\sinh(fx+e))^2\right)(\cosh(fx+e))^2}\ln\left(\frac{1}{(\sinh(fx+e))^2}\left((a+b)(\cosh(fx+e))^2+2\sqrt{a}\sqrt{b}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/a^(1/2)*ln(((a+b)*cosh(f*x+e
)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^
2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.07714, size = 1561, normalized size = 37.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^
3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*co
sh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*co
sh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 -
 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)
)/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
 b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))
*sinh(f*x + e) + b))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(e + fx\right)}{\sqrt{a + b\sinh^{2}\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [B] time = 1.4009, size = 143, normalized size = 3.4

$$\frac{2\arctan\left(-\frac{\sqrt{b}e^{\left(-2fx-2e\right)}-\sqrt{4ae^{\left(-2fx-2e\right)}-2be^{\left(-2fx-2e\right)}+be^{\left(-4fx-4e\right)}+b}-\sqrt{b}}{2\sqrt{-a}}\right)}{\sqrt{-a}f}-\frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{-a}}{\sqrt{b}}\right)}{af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(b)*e^(-2*f*x - 2*e) - sqrt(4*a*e^(-2*f*x - 2*e) - 2*b*e
^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b) - sqrt(b))/sqrt(-a))/(sqrt(-a)*f)
 - 2*sqrt(-a)*arctan(sqrt(-a)/sqrt(b))/(a*f)
```

3.101 $\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2af}$$

```
[Out] ((a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*a^(3/2)*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x
])/(2*a*f)
```

Rubi [A] time = 0.114886, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 382, 377, 206}

$$\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{3/2}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{a+b\cosh^2(e+fx)-b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a + b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]])/
(2*a^(3/2)*f) - (Sqrt[a - b + b*Cosh[e + f*x]^2]*Coth[e + f*x]*Csch[e + f*x
])/(2*a*f)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2af} + \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx}}\right.}{2af}$$

$$= -\frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+fx)}{2af} + \frac{(a+b)\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, -\right.}{2af}$$

$$= \frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\sqrt{a-b+b\cosh^2(e+fx)}\coth(e+fx)\operatorname{csch}(e+f}{2af}$$

Mathematica [A] time = 0.318659, size = 102, normalized size = 1.15

$$\frac{2(a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right) - \sqrt{2}\sqrt{a}\coth(e+fx)\operatorname{csch}(e+fx)\sqrt{2a+b\cosh(2(e+fx))-b}}{4a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*
(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*Coth[e +
 f*x]*Csch[e + f*x])/(4*a^(3/2)*f)
```

Maple [B] time = 0.091, size = 232, normalized size = 2.6

$$\frac{1}{4\left(\sinh\left(fx+e\right)\right)^2\cosh\left(fx+e\right)f}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)\left(\cosh\left(fx+e\right)\right)^2}\left(\ln\left(\frac{1}{\left(\sinh\left(fx+e\right)\right)^2}\left((a+b)\left(\cosh\left(\right.\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(ln(((a+b)*cosh(f*x+e)^2+2*a^
(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*sinh(
f*x+e)^2*a^2+ln(((a+b)*cosh(f*x+e)^2+2*a^(1/2)*(b*cosh(f*x+e)^4+(a-b)*cosh(
f*x+e)^2)^(1/2)+a-b)/sinh(f*x+e)^2)*b*sinh(f*x+e)^2*a-2*a^(3/2)*((a+b*sinh(
f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2/a^(5/2)/cosh(f*x+e)/(a+b*sinh
(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{3}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.56671, size = 3468, normalized size = 38.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 +
(a + b)*sinh(f*x + e)^4 - 2*(a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x
 + e)^2 - a - b)*sinh(f*x + e)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*cos
h(f*x + e))*sinh(f*x + e) + a + b)*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 +
4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a
- b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e
)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
(a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(
cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*
cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)
^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*cosh(f*x + e)^2 + 2*
a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e
)*sinh(f*x + e)^3 + a^2*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f
 + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x
+ e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a + b)*cosh(f*x + e)^
4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 - 2*(
a + b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 - a - b)*sinh(f*x + e
)^2 + 4*((a + b)*cosh(f*x + e)^3 - (a + b)*cosh(f*x + e))*sinh(f*x + e) + a
 + b)*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)
*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x
 + e) + b)) + sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e)
+ a*sinh(f*x + e)^2 + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh
(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2
- a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*
sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

3.102 $\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=229

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx}}{3b^2f}$$

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a +
 b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
 (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*
(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f)
```

Rubi [A] time = 0.217968, antiderivative size = 229, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3188, 479, 531, 418, 492, 411}

$$-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f}+\frac{2(a+b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a +
 b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
 (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*
(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f)
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Sub}\ldots}{3\ldots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(a\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Su}\ldots}{\ldots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(\ldots}{3bf\sqrt{\frac{\text{sech}^2(e+fx)(a+\ldots}{a\ldots}}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a+b)E\left(\tan^{-1}(\sinh(e+fx))|1-\ldots\right.}{3b^2f\sqrt{\frac{\text{sech}^2(e+\ldots}{\ldots}}}$$

Mathematica [C] time = 0.951974, size = 168, normalized size = 0.73

$$\frac{-2i\sqrt{2}a(2a+b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)+4i\sqrt{2}a(\ldots}{6b^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[
I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e +
 f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*
Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.068, size = 344, normalized size = 1.5

$$\frac{1}{3\,b\cosh\left(fx+e\right)f}\left(\sqrt{-\frac{b}{a}}b\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^{4}+\left(\sqrt{-\frac{b}{a}}a-\sqrt{-\frac{b}{a}}b\right)\left(\cosh\left(fx+e\right)\right)^{2}\sinh\left(fx+e\right)+a\sqrt{\frac{b(c}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^
(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*(b/a*
cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a-2*(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1
/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sinh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx+e)^4}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.103 $\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=128

$$\frac{ia\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1
+ (b*Sinh[e + f*x]^2)/a]) + (I*a*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.135813, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3172, 3178, 3177, 3183, 3182}

$$\frac{ia\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}\,E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(b*f*Sqrt[1
+ (b*Sinh[e + f*x]^2)/a]) + (I*a*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```


Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{b\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} - \frac{\left(a\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\right)\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,d}{b\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} + \frac{iaF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{bf\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.256913, size = 89, normalized size = 0.7

$$-\frac{i\sqrt{2a+b\cosh(2(e+fx))-b}\left(E\left(i(e+fx)\middle|\frac{b}{a}\right)-\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]*(EllipticE[I*(e + f*x), b/a] - El
lipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])
```

Maple [A] time = 0.066, size = 113, normalized size = 0.9

$$-\frac{1}{f\cosh(fx+e)}\sqrt{\frac{a+b\left(\sinh(fx+e)\right)^2}{a}}\sqrt{\left(\cosh(fx+e)\right)^2}\left(\text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-\text{EllipticE}\left(\sin\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2)))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sinh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.104 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0362947, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}\\
&= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0811603, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.047, size = 86, normalized size = 1.4

$$\frac{1}{f\cosh\left(fx+e\right)}\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\text{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}\frac{1}{\sqrt{a+b\left(\sinh\left(fx\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.105 $\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=134

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(s\right.}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx\right.}{a}}}$$

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/(a*f)
```

Rubi [A] time = 0.148971, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3188, 480, 12, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(s\right.}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/(a*f)
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] && !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{b}{\sqrt{1+x^2}\ldots}\right.}{af}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{}{\sqrt{1+\ldots}}\right.}{af}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{af} - \frac{\left(\sqrt{\cosh^2\ldots}\right.}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a\ldots}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 0.514988, size = 150, normalized size = 1.12

$$\frac{2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx\ldots}{a}}}{2af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (2*I)*a*Sqrt[(
2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(2*a*f*Sqrt[
2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.079, size = 189, normalized size = 1.4

$$-\frac{1}{a\sinh\left(fx+e\right)\cosh\left(fx+e\right)f}\left(\sinh\left(fx+e\right)\sqrt{\left(\cosh\left(fx+e\right)\right)^2}\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^2}{a}+\frac{a-b}{a}}b\left(\operatorname{EllipticF}\left(\sinh\left(\ldots\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2)))+(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-
1/a*b)^(1/2)*b)*cosh(f*x+e)^2)/a/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+
b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{csch}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}^2\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csch(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csch(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.106 $\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=267

$$-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+f}}{3a^2f}$$

```
[Out] (2*(a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[e +
 f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) + (2*(a + b)*Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*Ell
ipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a
+ b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)
```

Rubi [A] time = 0.283546, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 480, 583, 531, 418, 492, 411}

$$-\frac{2(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}+\frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh}}{3a^2f\sqrt{\operatorname{sec}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] (2*(a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[e +
 f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) + (2*(a + b)*Ell
ipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (b*Ell
ipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*
x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a
+ b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^4\sqrt{1+x^2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Sub}}{3}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+}}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+}}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+}}{3af}$$

$$= \frac{2(a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+}}{3af}$$

Mathematica [C] time = 3.79533, size = 201, normalized size = 0.75

$$\frac{-2ia(2a+b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) + \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\left(\left(4a^2-2ab-4b^2\right)\cosh(2(e+fx))-8a^2+b(a+b)\cosh(4(e+fx}{\sqrt{2}}}{6a^2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (((-8*a^2 + a*b + 3*b^2 + (4*a^2 - 2*a*b - 4*b^2)*Cosh[2*(e + f*x)] + b*(a
+ b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2] + (4*I)*a*(a
 + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] -
 (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e +
 f*x), b/a])/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.098, size = 456, normalized size = 1.7

$$\frac{1}{3a^2\left(\sinh\left(fx+e\right)\right)^3\cosh\left(fx+e\right)f}\left(2\sqrt{-\frac{b}{a}}ab\left(\sinh\left(fx+e\right)\right)^6 + 2\sqrt{-\frac{b}{a}}b^2\left(\sinh\left(fx+e\right)\right)^6 + b\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right.}{a}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6+
b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2
```

```
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b
^2*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-2*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-
1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+(-1/a*b)^
(1/2)*a^2*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^2
)/a^2/sinh(f*x+e)^3/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{csch}\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csch(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.107 $\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{3/2}f}-\frac{a\cosh(e+fx)}{bf(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(3/2)*f
) - (a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.112454, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3186, 385, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{3/2}f}-\frac{a\cosh(e+fx)}{bf(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(3/2)*f
) - (a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a-b+bx^2}}\,dx, x, \cosh(e+fx)\right)}{bf}$$

$$= -\frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{bf}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{3/2}f} - \frac{a\cosh(e+fx)}{(a-b)bf\sqrt{a-b+b\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.422344, size = 98, normalized size = 1.18

$$\frac{\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{b^{3/2}f} - \frac{\sqrt{2}a\cosh(e+fx)}{bf(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Sqrt[2]*a*Cosh[e + f*x])/((a - b)*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]
])) + Log[Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]]/(b^(3/2)*f)
```

Maple [A] time = 0.109, size = 146, normalized size = 1.8

$$\frac{1}{f\cosh\left(fx+e\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)\left(\cosh\left(fx+e\right)\right)^2}\left(\frac{1}{2}\ln\left(\left(\frac{a}{2}+\frac{b}{2}+b\left(\sinh\left(fx+e\right)\right)^2\right)\frac{1}{\sqrt{b}}+\sqrt{\left(a+b\left(\sinh\left(f\right.\right.\right.}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*ln((1/2*a+1/2*b+b*si
nh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))-a/b*cosh(f*
x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.16763, size = 7602, normalized size = 91.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^3 + (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)
^2 + 2*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^
2 + a*b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh
(f*x + e)*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*
x + e)^6 + 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(
14*a^2*b*cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 +
 (9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*
a^2*b - 4*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
 4*(14*a^2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b
- 4*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*c
osh(f*x + e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x +
e)^4 + 3*a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)
^5 + a^2*sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2
 + a^2)*sinh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*s
inh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 +
18*a^2*cosh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh
(f*x + e)^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*
b*cosh(f*x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 +
b^3)*cosh(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(
f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x
+ e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
 e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a*b - b^2)*
cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)
*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^
2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*(
(a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x
 + e))*sqrt(b)*log(-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
+ b*sinh(f*x + e)^4 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
a - b)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) +
 b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) -
4*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e) + a*b*si
nh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a*
b^3 - b^4)*f*cosh(f*x + e)^4 + 4*(a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e
)^3 + (a*b^3 - b^4)*f*sinh(f*x + e)^4 + 2*(2*a^2*b^2 - 3*a*b^3 + b^4)*f*cos
```

```
h(f*x + e)^2 + 2*(3*(a*b^3 - b^4)*f*cosh(f*x + e)^2 + (2*a^2*b^2 - 3*a*b^3
+ b^4)*f)*sinh(f*x + e)^2 + (a*b^3 - b^4)*f + 4*((a*b^3 - b^4)*f*cosh(f*x +
 e)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(
((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3
+ (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2
*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*
b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x +
 e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 + 2*a*cosh(f
*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*si
nh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)^2 + (6*a*
b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b*cosh(f*x
+ e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + ((a*b - b^2)*cosh(f
*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(
f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*cos
h(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b -
 b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e)
 + b)) + 2*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e)
 + a*b*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)))/((a*b^3 - b^4)*f*cosh(f*x + e)^4 + 4*(a*b^3 - b^4)*f*cosh(f*x + e)*sin
h(f*x + e)^3 + (a*b^3 - b^4)*f*sinh(f*x + e)^4 + 2*(2*a^2*b^2 - 3*a*b^3 + b
^4)*f*cosh(f*x + e)^2 + 2*(3*(a*b^3 - b^4)*f*cosh(f*x + e)^2 + (2*a^2*b^2 -
 3*a*b^3 + b^4)*f)*sinh(f*x + e)^2 + (a*b^3 - b^4)*f + 4*((a*b^3 - b^4)*f*c
osh(f*x + e)^3 + (2*a^2*b^2 - 3*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)
)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.108 $\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=36

$$\frac{\cosh(e+fx)}{f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] Cosh[e + f*x]/((a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.0526382, antiderivative size = 36, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3186, 191}

$$\frac{\cosh(e+fx)}{f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Cosh[e + f*x]/((a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}\\ &= \frac{\cosh(e+fx)}{(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.15519, size = 43, normalized size = 1.19

$$\frac{\sqrt{2}\cosh(e+fx)}{f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*Cosh[e + f*x])/((a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.051, size = 32, normalized size = 0.9

$$\frac{\cosh\left(fx+e\right)}{(a-b)f}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] cosh(f*x+e)/(a-b)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.60599, size = 319, normalized size = 8.86

$$\frac{b^{2}e^{(-6fx-6e)}+2ab-b^{2}+\left(8a^{2}-8ab+3b^{2}\right)e^{(-2fx-2e)}+3\left(2ab-b^{2}\right)e^{(-4fx-4e)}}{2\left(a^{2}-ab\right)\left(2(2a-b)e^{(-2fx-2e)}+be^{(-4fx-4e)}+b\right)^{\frac{3}{2}}f}+\frac{b^{2}+3\left(2ab-b^{2}\right)e^{(-2fx-2e)}+\left(8a\right.}{2\left(a^{2}-ab\right)\left(2(2a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*e^(-6*f*x - 6*e) + 2*a*b - b^2 + (8*a^2 - 8*a*b + 3*b^2)*e^(-2*f*x
 - 2*e) + 3*(2*a*b - b^2)*e^(-4*f*x - 4*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(3/2)*f) + 1/2*(b^2 + 3*(2*a*b - b^2)
*e^(-2*f*x - 2*e) + (8*a^2 - 8*a*b + 3*b^2)*e^(-4*f*x - 4*e) + (2*a*b - b^2
)*e^(-6*f*x - 6*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*
x - 4*e) + b)^(3/2)*f)
```

Fricas [B] time = 2.17215, size = 738, normalized size = 20.5

$$\frac{\sqrt{2}\left(\cosh\left(fx+e\right)^{2}+2\,\mathrm{c}\right.}{\left(ab-b^{2}\right)f\cosh\left(fx+e\right)^{4}+4\left(ab-b^{2}\right)f\cosh\left(fx+e\right)\sinh\left(fx+e\right)^{3}+\left(ab-b^{2}\right)f\sinh\left(fx+e\right)^{4}+2\left(2a^{2}-3ab+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
+ 1)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*f*cosh(f
*x + e)^4 + 4*(a*b - b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*f*s
inh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a*b - b^
2)*f*cosh(f*x + e)^2 + (2*a^2 - 3*a*b + b^2)*f)*sinh(f*x + e)^2 + (a*b - b^
2)*f + 4*((a*b - b^2)*f*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*f*cosh(f*x
```

```
+ e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22891, size = 207, normalized size = 5.75

$$-\frac{\sqrt{b}f}{256\left(ab^4-b^5\right)}+\frac{\frac{\left(a^3f-a^2bf\right)e^{\left(2\,fx+2\,e\right)}}{a^4b^3-2\,a^3b^4+a^2b^5}+\frac{a^3f-a^2bf}{a^4b^3-2\,a^3b^4+a^2b^5}}{256\,\sqrt{be^{\left(4\,fx+4\,e\right)}+4\,ae^{\left(2\,fx+2\,e\right)}-2\,be^{\left(2\,fx+2\,e\right)}+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/256*sqrt(b)*f/(a*b^4 - b^5) + 1/256*((a^3*f - a^2*b*f)*e^(2*f*x + 2*e)/(
a^4*b^3 - 2*a^3*b^4 + a^2*b^5) + (a^3*f - a^2*b*f)/(a^4*b^3 - 2*a^3*b^4 + a
^2*b^5))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e)
 + b)
```

3.109 $\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=84

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\cosh(e+fx)}{af(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(3/2)
*f)) - (b*Cosh[e + f*x])/(a*(a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.10844, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 382, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{3/2}f}-\frac{b\cosh(e+fx)}{af(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(3/2)
*f)) - (b*Cosh[e + f*x])/(a*(a - b)*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=-\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+bx^2\right)^{3/2}}\,dx,x,\cosh(e+fx)\right)}{f}$$

$$=-\frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}-\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a-b+bx^2}}\,dx,x,\cosh(e+fx)\right)}{af}$$

$$=-\frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}-\frac{\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx,x,\frac{\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{af}$$

$$=-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{3/2}f}-\frac{b\cosh(e+fx)}{a(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.400944, size = 98, normalized size = 1.17

$$\frac{-\frac{\sqrt{2}\sqrt{a}b\cosh(e+fx)}{(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}-\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)
]]] - (Sqrt[2]*Sqrt[a]*b*Cosh[e + f*x])/((a - b)*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]]))/(a^(3/2)*f)
```

Maple [B] time = 0.123, size = 154, normalized size = 1.8

$$\frac{1}{f\cosh(fx+e)}\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+e)\right)^2}\left(-\frac{b\left(\cosh(fx+e)\right)^2}{a(a-b)}\frac{1}{\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/a*b*cosh(f*x+e)^2/(a-b)/((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)-1/2/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e)^
2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2))/cosh
(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.58476, size = 4082, normalized size = 48.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*(((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^3 + (a*b - b^2)*sinh(f*x + e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)
^2 + 2*(3*(a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^
2 + a*b - b^2 + 4*((a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)
*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh
(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*s
qrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 +
 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*c
osh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x
+ e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
+ e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh
(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cos
h(f*x + e)*sinh(f*x + e) + a*b*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)
^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f
*x + e) + sinh(f*x + e)^2)))/((a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 4*(a^3*
b - a^2*b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^3*b - a^2*b^2)*f*sinh(f*x
 + e)^4 + 2*(2*a^4 - 3*a^3*b + a^2*b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - a
^2*b^2)*f*cosh(f*x + e)^2 + (2*a^4 - 3*a^3*b + a^2*b^2)*f)*sinh(f*x + e)^2
+ (a^3*b - a^2*b^2)*f + 4*((a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^4 - 3
*a^3*b + a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), (((a*b - b^2)*cosh(f*x +
 e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x
+ e)^4 + 2*(2*a^2 - 3*a*b + b^2)*cosh(f*x + e)^2 + 2*(3*(a*b - b^2)*cosh(f*
x + e)^2 + 2*a^2 - 3*a*b + b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*((a*b - b^2
)*cosh(f*x + e)^3 + (2*a^2 - 3*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e))*sqr
t(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sin
h(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
 + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)) - sqrt(2)*(a*b*cosh(f*x + e)^2 + 2*a*b*cosh(f*x + e)*sinh(f*x + e) + a*b
*sinh(f*x + e)^2 + a*b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(
(a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 4*(a^3*b - a^2*b^2)*f*cosh(f*x + e)*s
```

```
inh(f*x + e)^3 + (a^3*b - a^2*b^2)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 3*a^3*b +
 a^2*b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (2
*a^4 - 3*a^3*b + a^2*b^2)*f)*sinh(f*x + e)^2 + (a^3*b - a^2*b^2)*f + 4*((a^
3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^4 - 3*a^3*b + a^2*b^2)*f*cosh(f*x +
 e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4239, size = 331, normalized size = 3.94

$$\frac{a\sqrt{b}f}{8\left(ab^3-b^4\right)}-\frac{\frac{\left(a^4bf-a^3b^2f\right)e^{(2fx+2e)}}{a^4b^3-2a^3b^4+a^2b^5}+\frac{a^4bf-a^3b^2f}{a^4b^3-2a^3b^4+a^2b^5}}{8\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}}+\frac{2\arctan\left(-\frac{\sqrt{b}e^{(2fx+2e)}-\sqrt{be^{(4fx+4e)}+4ae^{(2fx+2e)}-2be^{(2fx+2e)}+b}-}{2\sqrt{-a}}\right.}{\sqrt{-a}af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*a*sqrt(b)*f/(a*b^3 - b^4) - 1/8*((a^4*b*f - a^3*b^2*f)*e^(2*f*x + 2*e)/
(a^4*b^3 - 2*a^3*b^4 + a^2*b^5) + (a^4*b*f - a^3*b^2*f)/(a^4*b^3 - 2*a^3*b^
4 + a^2*b^5))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x +
 2*e) + b) + 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e
) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/sqrt(-a))/(sq
rt(-a)*a*f)
```

3.110 $\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=139

$$-\frac{b(a-3b)\cosh(e+fx)}{2a^2f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] ((a + 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/(2*a^(5/2)*f) - ((a - 3*b)*b*Cosh[e + f*x])/(2*a^2*(a - b)*f*Sqrt[a - b +
 b*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x])/(2*a*f*Sqrt[a - b + b*
Cosh[e + f*x]^2])
```

Rubi [A] time = 0.181834, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3186, 414, 527, 12, 377, 206}

$$-\frac{b(a-3b)\cosh(e+fx)}{2a^2f(a-b)\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{2a^{5/2}f}-\frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 3*b)*ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]
)/(2*a^(5/2)*f) - ((a - 3*b)*b*Cosh[e + f*x])/(2*a^2*(a - b)*f*Sqrt[a - b +
 b*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x])/(2*a*f*Sqrt[a - b + b*
Cosh[e + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{a+b+2bx^2}{\left(1-x^2\right)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{2af}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{(}{(1-}\right.}{}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{(a+3b)\operatorname{Subs}}{}$$

$$= -\frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)}{2af\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{(a+3b)\operatorname{Subs}}{}$$

$$= \frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{(a-3b)b\cosh(e+fx)}{2a^2(a-b)f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\coth(e}{2af\sqrt{a-}}$$

Mathematica [A] time = 0.71249, size = 134, normalized size = 0.96

$$\frac{\frac{(a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{5/2}} - \frac{\coth(e+fx)\operatorname{csch}(e+fx)\left(2a^2+b(a-3b)\cosh(2(e+fx))-3ab+3b^2\right)}{a^2(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (((a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]])/a^(5/2) - ((2*a^2 - 3*a*b + 3*b^2 + (a - 3*b)*b*Cosh[2*(e +
f*x)])*Coth[e + f*x]*Csch[e + f*x])/(a^2*(a - b)*Sqrt[4*a - 2*b + 2*b*Cosh[
2*(e + f*x)]]))/(2*f)
```

Maple [B] time = 0.132, size = 251, normalized size = 1.8

$$\frac{1}{f\cosh\left(fx+e\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\left(-\frac{1}{2\,a^{2}\left(\sinh\left(fx+e\right)\right)^{2}}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/2/a^2/sinh(f*x+e)^2*((a+b*sin
h(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)+1/4/a^(3/2)*ln((2*a+(a+b)*sinh(f*x+e)^2+2*
a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2)+3/4/a^(5/
2)*b*ln((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)
^2)^(1/2))/sinh(f*x+e)^2)+b^2/a^2*cosh(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*
cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.79229, size = 10539, normalized size = 75.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 8*(a^2*b + 2*a*b^2 - 3*b
^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b + 2*a*b^2 - 3*b^3)*sinh(f*x + e)
^8 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 4*(a^3 + a^2*b - 5
*a*b^2 + 3*b^3 + 7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)
^6 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(a^3 + a^2*b - 5*a*
b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(4*a^3 + 5*a^2*b - 18*a*b^2
```

```
 + 9*b^3)*cosh(f*x + e)^4 + 2*(35*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4
 - 4*a^3 - 5*a^2*b + 18*a*b^2 - 9*b^3 + 30*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*
cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x
+ e)^5 + 10*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^3 - (4*a^3 + 5*a^
2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b + 2*a*b^2 -
3*b^3 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2 + 4*(7*(a^2*b + 2
*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 15*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f
*x + e)^4 + a^3 + a^2*b - 5*a*b^2 + 3*b^3 - 3*(4*a^3 + 5*a^2*b - 18*a*b^2 +
 9*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b + 2*a*b^2 - 3*b^3)*cos
h(f*x + e)^7 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^5 - (4*a^3 +
 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b
^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4
*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a -
 b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)
^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/
(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((
a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(c
osh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*c
osh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^
3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a^2*b - 3*a*b^2)*cosh(
f*x + e)^6 + 6*(a^2*b - 3*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b - 3
*a*b^2)*sinh(f*x + e)^6 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + (4*
a^3 - 5*a^2*b + 3*a*b^2 + 15*(a^2*b - 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x +
e)^4 + 4*(5*(a^2*b - 3*a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)
*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 3*a*b^2 + (4*a^3 - 5*a^2*b + 3*a*
b^2)*cosh(f*x + e)^2 + (15*(a^2*b - 3*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 5*a^
2*b + 3*a*b^2 + 6*(4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e
)^2 + 2*(3*(a^2*b - 3*a*b^2)*cosh(f*x + e)^5 + 2*(4*a^3 - 5*a^2*b + 3*a*b^2
)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e
))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - a^3*b^2)*f*c
osh(f*x + e)^8 + 8*(a^4*b - a^3*b^2)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4
*b - a^3*b^2)*f*sinh(f*x + e)^8 + 4*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x +
e)^6 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)
*f)*sinh(f*x + e)^6 - 2*(4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8
*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^3 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cos
h(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 3
0*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^
2)*f)*sinh(f*x + e)^4 + 4*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^2 + 8*(
7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^5 + 10*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh
(f*x + e)^3 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^
3 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)
*f*cosh(f*x + e)^4 - 3*(4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (a
^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x + e)^2 + (a^4*b - a^3*b^2)*f + 8*((a^4*
b - a^3*b^2)*f*cosh(f*x + e)^7 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e
)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^3 + (a^5 - 2*a^4*b + a^
3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((a^2*b + 2*a*b^2 - 3*b^3)*co
sh(f*x + e)^8 + 8*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^7 +
 (a^2*b + 2*a*b^2 - 3*b^3)*sinh(f*x + e)^8 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b
^3)*cosh(f*x + e)^6 + 4*(a^3 + a^2*b - 5*a*b^2 + 3*b^3 + 7*(a^2*b + 2*a*b^2
 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b + 2*a*b^2 - 3*b^3)
*cosh(f*x + e)^3 + 3*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*
x + e)^5 - 2*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^4 + 2*(35*(
a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 - 4*a^3 - 5*a^2*b + 18*a*b^2 - 9*b
^3 + 30*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 +
8*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 10*(a^3 + a^2*b - 5*a*b^2
+ 3*b^3)*cosh(f*x + e)^3 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x +
e))*sinh(f*x + e)^3 + a^2*b + 2*a*b^2 - 3*b^3 + 4*(a^3 + a^2*b - 5*a*b^2 +
```

```
3*b^3)*cosh(f*x + e)^2 + 4*(7*(a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 1
5*(a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + a^3 + a^2*b - 5*a*b^2 +
 3*b^3 - 3*(4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f*x + e)^2)*sinh(f*x +
 e)^2 + 8*((a^2*b + 2*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 3*(a^3 + a^2*b - 5*a
*b^2 + 3*b^3)*cosh(f*x + e)^5 - (4*a^3 + 5*a^2*b - 18*a*b^2 + 9*b^3)*cosh(f
*x + e)^3 + (a^3 + a^2*b - 5*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e))*s
qrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)
^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f
*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) +
 b)) + sqrt(2)*((a^2*b - 3*a*b^2)*cosh(f*x + e)^6 + 6*(a^2*b - 3*a*b^2)*cos
h(f*x + e)*sinh(f*x + e)^5 + (a^2*b - 3*a*b^2)*sinh(f*x + e)^6 + (4*a^3 - 5
*a^2*b + 3*a*b^2)*cosh(f*x + e)^4 + (4*a^3 - 5*a^2*b + 3*a*b^2 + 15*(a^2*b
- 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a^2*b - 3*a*b^2)*cosh(f
*x + e)^3 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + a^
2*b - 3*a*b^2 + (4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^2 + (15*(a^2*b -
3*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 5*a^2*b + 3*a*b^2 + 6*(4*a^3 - 5*a^2*b +
 3*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a^2*b - 3*a*b^2)*cosh(f*
x + e)^5 + 2*(4*a^3 - 5*a^2*b + 3*a*b^2)*cosh(f*x + e)^3 + (4*a^3 - 5*a^2*b
 + 3*a*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2)))/((a^4*b - a^3*b^2)*f*cosh(f*x + e)^8 + 8*(a^4*b - a^3*b^2)
*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4*b - a^3*b^2)*f*sinh(f*x + e)^8 + 4*
(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^6 + 4*(7*(a^4*b - a^3*b^2)*f*cosh
(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x + e)^6 - 2*(4*a^5 - 7*a
^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e
)^3 + 3*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*
(a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 30*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f
*x + e)^2 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e)^4 + 4*(a^5 - 2*a
^4*b + a^3*b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^4*b - a^3*b^2)*f*cosh(f*x + e)^
5 + 10*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^3 - (4*a^5 - 7*a^4*b + 3*a
^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^4*b - a^3*b^2)*f*cosh(f*
x + e)^6 + 15*(a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^4 - 3*(4*a^5 - 7*a^
4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (a^5 - 2*a^4*b + a^3*b^2)*f)*sinh(f*x
+ e)^2 + (a^4*b - a^3*b^2)*f + 8*((a^4*b - a^3*b^2)*f*cosh(f*x + e)^7 + 3*(
a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e)^5 - (4*a^5 - 7*a^4*b + 3*a^3*b^2)*
f*cosh(f*x + e)^3 + (a^5 - 2*a^4*b + a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e
))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csch(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.111 $\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}} \, dx$

Optimal. Leaf size=341

$$-\frac{(4a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3b^2f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\left(8a^2-3ab-2b^2\right)\tanh(e+fx)}{3b^3f(a-b)}$$

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
3*(a - b)*b^2*f) + ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*b^3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((4*a - b)*EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a
 - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*b^3*f)
```

Rubi [A] time = 0.340662, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 470, 582, 531, 418, 492, 411}

$$-\frac{\left(8a^2-3ab-2b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)}+\frac{\left(8a^2-3ab-2b^2\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}\right.}{3b^3f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x]^3)/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
3*(a - b)*b^2*f) + ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]]
, 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*b^3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((4*a - b)*EllipticF[ArcTa
n[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a
 - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*(a - b)*b^3*f)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
```

```
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1+x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^2(3a+(4}{\sqrt{1+x^2}}\right.}{(a-b)bf}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(}}{3(a-b)b^2f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(}}{3(a-b)b^2f}$$

Mathematica [C] time = 1.25201, size = 211, normalized size = 0.62

$$\frac{-2i\sqrt{2}a\left(8a^2-7ab-b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)-b\sinh(2(e+fx))\left(-8a^2+b(b-a)\cosh(2(e+\right.}{6b^3f(a-b)\sqrt{4a+2b\cosh(2(e+fx))-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*I)*Sqrt[2]*a*(8*a^2 - 3*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)
])/a]*EllipticE[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*a*(8*a^2 - 7*a*b - b^2)*S
qrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] - b*(-8*
a^2 + 3*a*b - b^2 + b*(-a + b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*(a
- b)*b^3*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.102, size = 500, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+4*(-
1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+4*a^2*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))-2*a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
```

```
2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*((a+b*sinh(f*x+e)^
2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b
)^(1/2))*b^2-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elliptic
E(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+3*((a+b*sinh(f*x+e)^2)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)-(-1/a
*b)^(1/2)*a*b*sinh(f*x+e))/b^2/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^{6}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\sinh\left(fx+e\right)^{6}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^{6}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.112 $\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=256

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{bf(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b^2f(a-b)}$$

```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/((a - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
 + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*
Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(
(a - b)*b^2*f)
```

Rubi [A] time = 0.235018, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3188, 470, 531, 418, 492, 411}

$$\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{b^2f(a-b)}-\frac{(2a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{b^2f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```



```
[Out] -((a*Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/((a - b)*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
 + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*
Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(
(a - b)*b^2*f)
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{a+(2a-}{\sqrt{1+x^2}\sqrt{}}\right.}{(a-b)bf}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(a\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{}{\sqrt{1+x^2}}\right.}{(a-b)bf}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e+fx)\sqrt{a+}}{(a-b)bf\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx\right.}{a}}}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{(a-b)bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e+f}{(a-b)b^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2\right.}{a}}}$$

Mathematica [C] time = 1.05016, size = 156, normalized size = 0.61

$$\frac{a\left(4i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)-2i(2a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)-\sqrt{2}b\sinh(2(e+\right.}{2b^2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*((-2*I)*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (4*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellip
ticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]))/(2*(a - b)*b^2*f*Sqr
t[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.093, size = 313, normalized size = 1.2

$$-\frac{1}{b(a-b)\cosh(fx+e)f}\left(\sqrt{-\frac{b}{a}}a\sinh(fx+e)(\cosh(fx+e))^2+a\sqrt{\frac{b(\cosh(fx+e))^2}{a}+\frac{a-b}{a}}\sqrt{(\cosh(fx+e))^2}\text{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/b*((-1/a*b)^(1/2)*a*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)
/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sin
h(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh(fx+e)^4}{\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh(fx+e)^2+a}\sinh(fx+e)^4}{b^2\sinh(fx+e)^4+2ab\sinh(fx+e)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.113 $\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=173

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (I*
EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) - (I*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.212076, antiderivative size = 173, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{\sinh(e+fx)\cosh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}F\left(ie+ifx\middle|\frac{b}{a}\right)}{bf\sqrt{a+b\sinh^2(e+fx)}}+\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{bf(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/((a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (I*
EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/((a - b)*b*f*Sqrt[
1 + (b*Sinh[e + f*x]^2)/a]) - (I*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Si
nh[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```


Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{a+a\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{b} - \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{(a-b)b}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{(a-b)b\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} + \frac{\sqrt{1}}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)bf\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}} - \frac{iF\left(ie+ifx\right.}{bf\sqrt{a}}$$

Mathematica [A] time = 0.481106, size = 151, normalized size = 0.87

$$\frac{-i\sqrt{2}(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)+b\sinh(2(e+fx)}{bf(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x),
 b/a] - I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF
[I*(e + f*x), b/a] + b*Sinh[2*(e + f*x)])/((a - b)*b*f*Sqrt[4*a - 2*b + 2*b
*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.084, size = 127, normalized size = 0.7

$$-\frac{1}{(a-b)\cosh\left(fx+e\right)f}\left(-\sqrt{-\frac{b}{a}}\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^{2}+\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^{2}}{a}+\frac{a-b}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\text{Ellip}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(-(-1/a*b)^(1/2)*sinh(f*x+e)*cosh(f*x+e)^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))
/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\sinh\left(fx+e\right)^{2}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.114 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rubi [A] time = 0.0616497, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3184, 21, 3178, 3177}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.159245, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}b\sinh(2(e+fx)) - 2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{2af(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

Maple [A] time = 0.078, size = 252, normalized size = 2.2

$$\frac{1}{a(a-b)\cosh(fx+e)f}\left(-\sqrt{-\frac{b}{a}}b\sinh(fx+e)\left(\cosh(fx+e)\right)^2 + a\sqrt{\frac{b\left(\cosh(fx+e)\right)^2}{a}+\frac{a-b}{a}}\sqrt{\left(\cosh(fx+e)\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x
+ e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

3.115 $\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=290

$$-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{a^2f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+}{a^2f(a-b)}$$

```
[Out] -((b*Coth[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - ((a - 2*b)
*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)*f) - ((a - 2*b)*El
lipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
 (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2*(a - b)*f
)
```

Rubi [A] time = 0.310008, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 472, 583, 531, 418, 492, 411}

$$\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f(a-b)}-\frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f(a-b)}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sin}}{a^2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] -((b*Coth[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])) - ((a - 2*b)
*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)*f) - ((a - 2*b)*El
lipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
 (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(a^2*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2*(a - b)*f
)
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
```

```
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{a-2b}{x^2\sqrt{1+x^2}}\right.}{a(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f} - \frac{\left(\sqrt{\right.}}{}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f} - \frac{(b}{}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f} - \frac{bF}{}$$

$$= -\frac{b\coth(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2(a-b)f} - \frac{(a}{}$$

Mathematica [C] time = 1.26166, size = 185, normalized size = 0.64

$$\frac{i\sqrt{2}a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\coth(e+fx)\left(-\left(2a^2+b(a-2b)\cosh(2(e+fx))-3ab+2b\right.\right.}{a^2f(a-b)\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((2*a^2 - 3*a*b + 2*b^2 + (a - 2*b)*b*Cosh[2*(e + f*x)])*Coth[e + f*x]) -
 I*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*
(e + f*x), b/a] + I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/
a]*EllipticF[I*(e + f*x), b/a])/(a^2*(a - b)*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*
(e + f*x)]])
```

Maple [A] time = 0.104, size = 284, normalized size = 1.

$$-\frac{1}{a^2\sinh\left(fx+e\right)(a-b)\cosh\left(fx+e\right)f}\left(\sinh\left(fx+e\right)\sqrt{\left(\cosh\left(fx+e\right)\right)^2}\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^2}{a}+\frac{a-b}{a}}b\left(2\,a\text{Ellipti}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(2*
a*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*b*EllipticF(sinh(f*x+
```

```
e)*(-1/a*b)^(1/2),(a/b)^(1/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a+2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))+((-1/a*b)^(1
/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a^2-2*(-1/a*b)^
(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2)/a^2/sinh(f*x+e)/(-1/a*b)^(1/
2)/(a-b)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\operatorname{csch}\left(fx+e\right)^{2}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.116 $\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$-\frac{a(3a-5b)\cosh(e+fx)}{3b^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{5/2}f}-\frac{a\sinh^2(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(5/2)*f
) - (a*(3*a - 5*b)*Cosh[e + f*x])/(3*(a - b)^2*b^2*f*Sqrt[a - b + b*Cosh[e
+ f*x]^2]) - (a*Cosh[e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*b*f*(a - b + b*Co
sh[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.166095, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3186, 413, 385, 217, 206}

$$-\frac{a(3a-5b)\cosh(e+fx)}{3b^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{b^{5/2}f}-\frac{a\sinh^2(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(b^(5/2)*f
) - (a*(3*a - 5*b)*Cosh[e + f*x])/(3*(a - b)^2*b^2*f*Sqrt[a - b + b*Cosh[e
+ f*x]^2]) - (a*Cosh[e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*b*f*(a - b + b*Co
sh[e + f*x]^2)^(3/2))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+bx^2\right)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)bf\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a+3b+3(a-b)x^2}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+f\right.}{3(a-b)bf}$$

$$= -\frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)bf\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} +$$

$$= -\frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)bf\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} +$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{a(3a-5b)\cosh(e+fx)}{3(a-b)^2b^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{a\cosh(e}{3(a-b)bf\left(a\right.}$$

Mathematica [A] time = 0.871048, size = 130, normalized size = 0.91

$$\frac{\frac{\log\left(\sqrt{2a+b\cosh(2(e+fx))-b}+\sqrt{2}\sqrt{b}\cosh(e+fx)\right)}{b^{5/2}} - \frac{2\sqrt{2}a\cosh(e+fx)\left(3a^2+b(2a-3b)\cosh(2(e+fx))-7ab+3b^2\right)}{3b^2(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}}{f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*Sqrt[2]*a*Cosh[e + f*x]*(3*a^2 - 7*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*
(e + f*x)]))/(3*(a - b)^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)) + Log[
Sqrt[2]*Sqrt[b]*Cosh[e + f*x] + Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]]/b^(5/2
))/f
```

Maple [A] time = 0.16, size = 230, normalized size = 1.6

$$\frac{1}{f\cosh\left(fx+e\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\left(\frac{1}{2}\ln\left(\left(\frac{a}{2}+\frac{b}{2}+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\frac{1}{\sqrt{b}}+\sqrt{\left(a+b\left(\sinh\left(f\right.\right.}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*ln((1/2*a+1/2*b+b*si
nh(f*x+e)^2)/b^(1/2)+((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))+1/3*a^2/b^2
*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2
)^(1/2)/(a+b*sinh(f*x+e)^2)/(a^2-2*a*b+b^2)-2*a/b^2*cosh(f*x+e)^2/(a-b)/((a
+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^{5}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 6.88764, size = 18590, normalized size = 130.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3
+ b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x +
 e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*
b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2
)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a
^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4
 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b
^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*
b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sin
h(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cos
h(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (
8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x +
 e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2
*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
 b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24
```

```
*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 +
8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a
*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3
*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)
)*sinh(f*x + e))*sqrt(b)*log((a^2*b*cosh(f*x + e)^8 + 8*a^2*b*cosh(f*x + e)
*sinh(f*x + e)^7 + a^2*b*sinh(f*x + e)^8 + 2*(a^3 + a^2*b)*cosh(f*x + e)^6
+ 2*(14*a^2*b*cosh(f*x + e)^2 + a^3 + a^2*b)*sinh(f*x + e)^6 + 4*(14*a^2*b*
cosh(f*x + e)^3 + 3*(a^3 + a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b
 - 4*a*b^2 + b^3)*cosh(f*x + e)^4 + (70*a^2*b*cosh(f*x + e)^4 + 9*a^2*b - 4
*a*b^2 + b^3 + 30*(a^3 + a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*a^
2*b*cosh(f*x + e)^5 + 10*(a^3 + a^2*b)*cosh(f*x + e)^3 + (9*a^2*b - 4*a*b^2
 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - b^3)*cosh(f*x +
 e)^2 + 2*(14*a^2*b*cosh(f*x + e)^6 + 15*(a^3 + a^2*b)*cosh(f*x + e)^4 + 3*
a*b^2 - b^3 + 3*(9*a^2*b - 4*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2
+ sqrt(2)*(a^2*cosh(f*x + e)^6 + 6*a^2*cosh(f*x + e)*sinh(f*x + e)^5 + a^2*
sinh(f*x + e)^6 + 3*a^2*cosh(f*x + e)^4 + 3*(5*a^2*cosh(f*x + e)^2 + a^2)*s
inh(f*x + e)^4 + 4*(5*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x +
 e)^3 + (4*a*b - b^2)*cosh(f*x + e)^2 + (15*a^2*cosh(f*x + e)^4 + 18*a^2*co
sh(f*x + e)^2 + 4*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(3*a^2*cosh(f*x + e)
^5 + 6*a^2*cosh(f*x + e)^3 + (4*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sq
rt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*a^2*b*cosh(f*
x + e)^7 + 3*(a^3 + a^2*b)*cosh(f*x + e)^5 + (9*a^2*b - 4*a*b^2 + b^3)*cosh
(f*x + e)^3 + (3*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^
6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 +
20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6
*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 3*((a^2*b^2 - 2*a*b^3
+ b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x
 + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4
 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2
*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14
*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x
+ e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5
*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*
b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b -
5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2
 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*
x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3
*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^
3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*
cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 +
 (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^
3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log(
-(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a - b)*sinh(f*x + e
)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2 - 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*
cosh(f*x + e)^3 + (a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 8*sqrt(2)*((2*a^2*b
^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)*sinh(
f*x + e)^5 + (2*a^2*b^2 - 3*a*b^3)*sinh(f*x + e)^6 + 3*(2*a^3*b - 4*a^2*b^2
 + a*b^3)*cosh(f*x + e)^4 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3 + 5*(2*a^2*b^2 -
 3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*a^2*b^2 - 3*a*b^3 + 4*(5*(2*
a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f
*x + e))*sinh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^2
```

```
+ 3*(5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 + 2*a^3*b - 4*a^2*b^2 + a*b^3
+ 6*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*
a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^5 + 2*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f
*x + e)^3 + (2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqr
t((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*co
sh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2*b^5 - 2*a*b^6 + b^7)*f
*cosh(f*x + e)^8 + 8*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)*sinh(f*x + e
)^7 + (a^2*b^5 - 2*a*b^6 + b^7)*f*sinh(f*x + e)^8 + 4*(2*a^3*b^4 - 5*a^2*b^
5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cos
h(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f)*sinh(f*x + e)^6 +
 2*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)
^4 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^3 + 3*(2*a^3*b^4 - 5*a^
2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^5 -
2*a*b^6 + b^7)*f*cosh(f*x + e)^4 + 30*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^
7)*f*cosh(f*x + e)^2 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*
b^7)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(
f*x + e)^2 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^5 + 10*(2*a^3*b
^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^3 + (8*a^4*b^3 - 24*a^3*b^4
 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(
a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^6 + 15*(2*a^3*b^4 - 5*a^2*b^5 + 4*
a*b^6 - b^7)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 1
4*a*b^6 + 3*b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7
)*f)*sinh(f*x + e)^2 + (a^2*b^5 - 2*a*b^6 + b^7)*f + 8*((a^2*b^5 - 2*a*b^6
+ b^7)*f*cosh(f*x + e)^7 + 3*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh
(f*x + e)^5 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*co
sh(f*x + e)^3 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e))*si
nh(f*x + e)), -1/6*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b
^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^
4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^
6 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*co
sh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x +
e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)
^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 +
 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^
2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*
x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b
^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f
*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e
))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^
2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 +
3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(
f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*
a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 -
 14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(a*cosh(f*x + e)^2 +
2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f
*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + (3*a*b - b^2)*cosh(f*x + e)
^2 + (6*a*b*cosh(f*x + e)^2 + 3*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 2*(2*a*b
*cosh(f*x + e)^3 + (3*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))) + 3*((a^2*b
^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x
+ e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3
*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b - 5*a^2*b^2 +
4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^
6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2
 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27
*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b
```

```
^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30
*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a
^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 1
0*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b
 + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3
*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 +
 b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x +
 e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*
b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*
a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh
(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x +
 e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2 - 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e)
 + b)) + 4*sqrt(2)*((2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6*(2*a^2*b^2 -
3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b^2 - 3*a*b^3)*sinh(f*x + e
)^6 + 3*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3*(2*a^3*b - 4*a^2*
b^2 + a*b^3 + 5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 2*
a^2*b^2 - 3*a*b^3 + 4*(5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + 3*(2*a^3*b
 - 4*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(2*a^3*b - 4*a^2*b
^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 +
2*a^3*b - 4*a^2*b^2 + a*b^3 + 6*(2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)
^2)*sinh(f*x + e)^2 + 6*((2*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^5 + 2*(2*a^3*b
 - 4*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (2*a^3*b - 4*a^2*b^2 + a*b^3)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
((a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^8 + 8*(a^2*b^5 - 2*a*b^6 + b^7)*
f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^5 - 2*a*b^6 + b^7)*f*sinh(f*x + e)
^8 + 4*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^6 + 4*(7*(a^
2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6
 - b^7)*f)*sinh(f*x + e)^6 + 2*(8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*
b^6 + 3*b^7)*f*cosh(f*x + e)^4 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x
+ e)^3 + 3*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e))*sinh(f*
x + e)^5 + 2*(35*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^4 + 30*(2*a^3*b^
4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^2 + (8*a^4*b^3 - 24*a^3*b^4
+ 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b^4 - 5*a^2*
b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^2 + 8*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*c
osh(f*x + e)^5 + 10*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)
^3 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x +
e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^6 + 15
*(2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 -
 24*a^3*b^4 + 27*a^2*b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)^2 + (2*a^3*b^4
 - 5*a^2*b^5 + 4*a*b^6 - b^7)*f)*sinh(f*x + e)^2 + (a^2*b^5 - 2*a*b^6 + b^7
)*f + 8*((a^2*b^5 - 2*a*b^6 + b^7)*f*cosh(f*x + e)^7 + 3*(2*a^3*b^4 - 5*a^2
*b^5 + 4*a*b^6 - b^7)*f*cosh(f*x + e)^5 + (8*a^4*b^3 - 24*a^3*b^4 + 27*a^2*
b^5 - 14*a*b^6 + 3*b^7)*f*cosh(f*x + e)^3 + (2*a^3*b^4 - 5*a^2*b^5 + 4*a*b^
6 - b^7)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.117 $\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=87

$$\frac{\sinh^2(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}$$

```
[Out] (-2*Cosh[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2]) + (Cosh[
e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.10722, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3186, 378, 191}

$$\frac{\sinh^2(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}-\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```



```
[Out] (-2*Cosh[e + f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2]) + (Cosh[
e + f*x]*Sinh[e + f*x]^2)/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sinh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+bx^2\right)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{3(a-b)f}$$

$$= -\frac{2\cosh(e+fx)}{3(a-b)^2 f\sqrt{a-b+b\cosh^2(e+fx)}} + \frac{\cosh(e+fx)\sinh^2(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.328655, size = 67, normalized size = 0.77

$$\frac{\sqrt{2}\cosh(e+fx)((a-3b)\cosh(2(e+fx))-5a+3b)}{3f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[2]*Cosh[e + f*x]*(-5*a + 3*b + (a - 3*b)*Cosh[2*(e + f*x)]))/(3*(a -
b)^2*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [A] time = 0.077, size = 58, normalized size = 0.7

$$\frac{\left(a\left(\sinh\left(fx+e\right)\right)^2-3\,b\left(\sinh\left(fx+e\right)\right)^2-2\,a\right)\cosh\left(fx+e\right)}{3\,\left(a-b\right)^2f}\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(a*sinh(f*x+e)^2-3*b*sinh(f*x+e)^2-2*a)*cosh(f*x+e)/(a-b)^2/(a+b*sinh(f
*x+e)^2)^(3/2)/f
```

Maxima [B] time = 1.77411, size = 1251, normalized size = 14.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^4*e^(-10*f*x - 10*e) - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a^
3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-2*f*x - 2*e) + 10*(2*a^3*b - 3*a^2*
b^2 + 3*a*b^3 - b^4)*e^(-4*f*x - 4*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-
6*f*x - 6*e) + 5*(2*a*b^3 - b^4)*e^(-8*f*x - 8*e))/((a^4 - 2*a^3*b + a^2*b^
2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/4*(
2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-2*f
```

```
*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-4*f*x
 - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) + 5*(4*
a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x - 10
*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x
 - 4*e) + b)^(5/2)*f) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*
e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x - 4*e
) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x - 6*e)
+ 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^2 - 2
*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e
^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) - 1/12*(b^4 + 5*(2*a*b^3
 - b^4)*e^(-2*f*x - 2*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-4*f*x - 4*e)
+ 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^(-6*f*x - 6*e) - (16*a^4 - 32*
a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-8*f*x - 8*e) - (4*a^3*b - 6*a^2*b
^2 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Fricas [B] time = 3.41105, size = 2873, normalized size = 33.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*((a - 3*b)*cosh(f*x + e)^6 + 6*(a - 3*b)*cosh(f*x + e)*sinh(f*x
 + e)^5 + (a - 3*b)*sinh(f*x + e)^6 - 3*(3*a - b)*cosh(f*x + e)^4 + 3*(5*(a
 - 3*b)*cosh(f*x + e)^2 - 3*a + b)*sinh(f*x + e)^4 + 4*(5*(a - 3*b)*cosh(f*
x + e)^3 - 3*(3*a - b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - b)*cosh(f*
x + e)^2 + 3*(5*(a - 3*b)*cosh(f*x + e)^4 - 6*(3*a - b)*cosh(f*x + e)^2 - 3
*a + b)*sinh(f*x + e)^2 + 6*((a - 3*b)*cosh(f*x + e)^5 - 2*(3*a - b)*cosh(f
*x + e)^3 - (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a - 3*b)*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x +
 e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*
b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 -
 b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^2
+ (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 24*
a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^2*b^2 -
2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*
f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f
*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + (8
*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*
a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^2*b^2 - 2*a*
b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*c
osh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(
f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^
6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^4 + 3*(8*a^4 -
 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^2 + (2*a^3*b - 5
*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^2 + (a^2*b^2 - 2*a*b^3 + b^4)*f
+ 8*((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 +
 4*a*b^3 - b^4)*f*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b
^3 + 3*b^4)*f*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cos
h(f*x + e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.33895, size = 699, normalized size = 8.03

$$-\frac{a-3\,b}{3\left(a^{2}b^{\frac{3}{2}}f-2\,ab^{\frac{5}{2}}f+b^{\frac{7}{2}}f\right)}+\frac{\left(\left(\frac{\left(a^{7}b^{2}f^{3}-5\,a^{6}b^{3}f^{3}+7\,a^{5}b^{4}f^{3}-3\,a^{4}b^{5}f^{3}\right)e^{\left(2\,fx+2\,e\right)}}{a^{8}b^{2}f^{4}-4\,a^{7}b^{3}f^{4}+6\,a^{6}b^{4}f^{4}-4\,a^{5}b^{5}f^{4}+a^{4}b^{6}f^{4}}-\frac{3\left(3\,a^{7}b^{2}f^{3}-7\,a^{6}b^{3}f^{3}+5\,a^{5}b^{4}f^{3}-a^{4}b^{5}f^{3}\right)}{a^{8}b^{2}f^{4}-4\,a^{7}b^{3}f^{4}+6\,a^{6}b^{4}f^{4}-4\,a^{5}b^{5}f^{4}+a^{4}b^{6}f^{4}}\right)e^{\left(2\,fx+2\,e\right)}-\cdots}{3\left(be^{\left(4\,fx+4\,e\right)}+4\,ae^{\left(2\,fx+2\,e\right)}\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(a - 3*b)/(a^2*b^(3/2)*f - 2*a*b^(5/2)*f + b^(7/2)*f) + 1/3*((((a^7*b^
2*f^3 - 5*a^6*b^3*f^3 + 7*a^5*b^4*f^3 - 3*a^4*b^5*f^3)*e^(2*f*x + 2*e)/(a^8
*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4) - 3
*(3*a^7*b^2*f^3 - 7*a^6*b^3*f^3 + 5*a^5*b^4*f^3 - a^4*b^5*f^3)/(a^8*b^2*f^4
 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4))*e^(2*f*x +
 2*e) - 3*(3*a^7*b^2*f^3 - 7*a^6*b^3*f^3 + 5*a^5*b^4*f^3 - a^4*b^5*f^3)/(a^
8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4))*e
^(2*f*x + 2*e) + (a^7*b^2*f^3 - 5*a^6*b^3*f^3 + 7*a^5*b^4*f^3 - 3*a^4*b^5*f
^3)/(a^8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*
f^4))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(
3/2)
```

3.118 $\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=79

$$\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

```
[Out] Cosh[e + f*x]/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2)) + (2*Cosh[e +
 f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.0675662, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 192, 191}

$$\frac{2\cosh(e+fx)}{3f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}+\frac{\cosh(e+fx)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Cosh[e + f*x]/(3*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2)) + (2*Cosh[e +
 f*x])/(3*(a - b)^2*f*Sqrt[a - b + b*Cosh[e + f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\sinh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{2\operatorname{Subst}\left(\int \frac{1}{\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx)\right)}{3(a-b)f}$$

$$= \frac{\cosh(e+fx)}{3(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{2\cosh(e+fx)}{3(a-b)^2f\sqrt{a-b+b\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.179514, size = 63, normalized size = 0.8

$$\frac{2\sqrt{2}\cosh(e+fx)(3a+b\cosh(2(e+fx))-2b)}{3f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*Sqrt[2]*Cosh[e + f*x]*(3*a - 2*b + b*Cosh[2*(e + f*x)]))/(3*(a - b)^2*f*
(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [A] time = 0.066, size = 57, normalized size = 0.7

$$\frac{\left(2\,b\left(\sinh\left(fx+e\right)\right)^2+3\,a-b\right)\cosh\left(fx+e\right)}{\left(3\,a^2-6\,ab+3\,b^2\right)f}\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(2*b*sinh(f*x+e)^2+3*a-b)*cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2)/(a^2-2*
a*b+b^2)/f
```

Maxima [B] time = 1.67485, size = 655, normalized size = 8.29

$$\frac{2\,a^2b^2-2\,ab^3+b^4+5\left(4\,a^3b-6\,a^2b^2+4\,ab^3-b^4\right)e^{(-2\,fx-2\,e)}+2\left(24\,a^4-48\,a^3b+49\,a^2b^2-25\,ab^3+5\,b^4\right)e^{(-4\,fx-4\,e)}+1}{3\left(a^4-2\,a^3b+a^2b^2\right)\left(2\,(2\,a-b)e^{(-2\,fx-2\,e)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^
(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) +
5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x
```

```
 - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-
4*f*x - 4*e) + b)^(5/2)*f) + 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 +
b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x
- 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x -
6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^
2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a -
 b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Fricas [B] time = 3.32509, size = 2789, normalized size = 35.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(2)*(b*cosh(f*x + e)^6 + 6*b*cosh(f*x + e)*sinh(f*x + e)^5 + b*sinh
(f*x + e)^6 + 3*(2*a - b)*cosh(f*x + e)^4 + 3*(5*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^4 + 4*(5*b*cosh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e))*si
nh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e)^2 + 3*(5*b*cosh(f*x + e)^4 + 6*(2
*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 6*(b*cosh(f*x + e)^5 +
 2*(2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a^2*b^2 - 2*a*b^3 + b^4)
*f*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)*sinh(f*x +
 e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*f*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^
2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cos
h(f*x + e)^2 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^6 + 2
*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^4 + 8*(
7*(a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*
a*b^3 - b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2*b^2 - 2*a*b^3 +
b^4)*f*cosh(f*x + e)^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*
x + e)^2 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f)*sinh(f*x +
 e)^4 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^2 + 8*(7*(a
^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b
^3 - b^4)*f*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3
*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*f*c
osh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^4
 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^2 +
 (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*f)*sinh(f*x + e)^2 + (a^2*b^2 - 2*a*
b^3 + b^4)*f + 8*((a^2*b^2 - 2*a*b^3 + b^4)*f*cosh(f*x + e)^7 + 3*(2*a^3*b
- 5*a^2*b^2 + 4*a*b^3 - b^4)*f*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2
*b^2 - 14*a*b^3 + 3*b^4)*f*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3
 - b^4)*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37829, size = 622, normalized size = 7.87

$$\frac{2\left(\left(\left(\frac{\left(a^6b^3f-2\,a^5b^4f+a^4b^5f\right)e^{\left(2\,fx+2\,e\right)}}{a^8b^2f^2-4\,a^7b^3f^2+6\,a^6b^4f^2-4\,a^5b^5f^2+a^4b^6f^2}+\frac{3\left(2\,a^7b^2f-5\,a^6b^3f+4\,a^5b^4f-a^4b^5f\right)}{a^8b^2f^2-4\,a^7b^3f^2+6\,a^6b^4f^2-4\,a^5b^5f^2+a^4b^6f^2}\right)e^{\left(2\,fx+2\,e\right)}+\frac{3\left(2\,a^7b^2f-5\,a^6b^3f+4\,a^5b^4f-a^4b^5f\right.}{a^8b^2f^2-4\,a^7b^3f^2+6\,a^6b^4f^2-4\,a^5b^5f^2+a^{\cdots}}\cdots\right.\right.\right.}{3\left(be^{\left(4\,fx+4\,e\right)}+4\,ae^{\left(2\,fx+2\,e\right)}-2\,be^{\left(2\,fx+2\,e\right)}+b\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((((a^6*b^3*f - 2*a^5*b^4*f + a^4*b^5*f)*e^(2*f*x + 2*e)/(a^8*b^2*f^2 -
 4*a^7*b^3*f^2 + 6*a^6*b^4*f^2 - 4*a^5*b^5*f^2 + a^4*b^6*f^2) + 3*(2*a^7*b^
2*f - 5*a^6*b^3*f + 4*a^5*b^4*f - a^4*b^5*f)/(a^8*b^2*f^2 - 4*a^7*b^3*f^2 +
 6*a^6*b^4*f^2 - 4*a^5*b^5*f^2 + a^4*b^6*f^2))*e^(2*f*x + 2*e) + 3*(2*a^7*b
^2*f - 5*a^6*b^3*f + 4*a^5*b^4*f - a^4*b^5*f)/(a^8*b^2*f^2 - 4*a^7*b^3*f^2
+ 6*a^6*b^4*f^2 - 4*a^5*b^5*f^2 + a^4*b^6*f^2))*e^(2*f*x + 2*e) + (a^6*b^3*
f - 2*a^5*b^4*f + a^4*b^5*f)/(a^8*b^2*f^2 - 4*a^7*b^3*f^2 + 6*a^6*b^4*f^2 -
 4*a^5*b^5*f^2 + a^4*b^6*f^2))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b)^(3/2) - 2/3/(a^2*sqrt(b)*f - 2*a*b^(3/2)*f + b^(5/2
)*f)
```

3.119 $\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=136

$$-\frac{b(5a-3b)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b\cosh(e+fx)}{3af(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(5/2)
*f)) - (b*Cosh[e + f*x])/(3*a*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
- ((5*a - 3*b)*b*Cosh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Cosh[e +
f*x]^2])
```

Rubi [A] time = 0.171386, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 12, 377, 206}

$$-\frac{b(5a-3b)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\cosh^2(e+fx)-b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a+b\cosh^2(e+fx)-b}}\right)}{a^{5/2}f}-\frac{b\cosh(e+fx)}{3af(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cosh[e + f*x])/Sqrt[a - b + b*Cosh[e + f*x]^2]]/(a^(5/2)
*f)) - (b*Cosh[e + f*x])/(3*a*(a - b)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
- ((5*a - 3*b)*b*Cosh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a - b + b*Cosh[e +
f*x]^2])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+bx^2\right)^{5/2}}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3a+b+2bx^2}{(1-x^2)\left(a-b+bx^2\right)^{3/2}}\,dx, x, \cosh(e+fx\right.}{3a(a-b)f}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\text{Su}}{}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\text{Su}}{}$$

$$= -\frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3b)b\cosh(e+fx)}{3a^2(a-b)^2f\sqrt{a-b+b\cosh^2(e+fx)}} - \frac{\text{Su}}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cosh(e+fx)}{\sqrt{a-b+b\cosh^2(e+fx)}}\right)}{a^{5/2}f} - \frac{b\cosh(e+fx)}{3a(a-b)f\left(a-b+b\cosh^2(e+fx)\right)^{3/2}} - \frac{(5a-3}{3a^2(a-b)^2f\sqrt{}}$$

Mathematica [A] time = 0.797738, size = 130, normalized size = 0.96

$$\frac{\frac{\sqrt{2}b\cosh(e+fx)\left(-12a^2+b(3b-5a)\cosh(2(e+fx))+13ab-3b^2\right)}{3a^2(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cosh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{a^{5/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cosh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x
)]]]/a^(5/2)) + (Sqrt[2]*b*Cosh[e + f*x]*(-12*a^2 + 13*a*b - 3*b^2 + b*(-5*
a + 3*b)*Cosh[2*(e + f*x)]))/(3*a^2*(a - b)^2*(2*a - b + b*Cosh[2*(e + f*x)
])^(3/2)))/f
```

Maple [A] time = 0.176, size = 236, normalized size = 1.7

$$\frac{1}{f\cosh\left(fx+e\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\left(-\frac{b\left(2b\left(\sinh\left(fx+e\right)\right)^{2}+3a-b\right)\left(\cosh\left(fx+e\right)\right)^{2}}{3a\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(a^{2}-2ab+b^{2}\right)\sqrt{(\ldots}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a*b*(2*b*sinh(f*x+e)^2+3*a-
b)*cosh(f*x+e)^2/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(a+b*sinh(f*x+e)
^2)/(a^2-2*a*b+b^2)-1/a^2*b*cosh(f*x+e)^2/(a-b)/((a+b*sinh(f*x+e)^2)*cosh(f
*x+e)^2)^(1/2)-1/2/a^(5/2)*ln((2*a+(a+b)*sinh(f*x+e)^2+2*a^(1/2)*((a+b*sinh
(f*x+e)^2)*cosh(f*x+e)^2)^(1/2))/sinh(f*x+e)^2))/cosh(f*x+e)/(a+b*sinh(f*x+
e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 5.34341, size = 12269, normalized size = 90.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 +
 b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x +
e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2*a^3*b
 - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)
*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^
3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4
```

- 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log(-((a + b)*cosh(f*x + e)^4 + 4*(a + b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + b)*sinh(f*x + e)^4 + 2*(3*a - b)*cosh(f*x + e)^2 + 2*(3*(a + b)*cosh(f*x + e)^2 + 3*a - b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a + b)*cosh(f*x + e)^3 + (3*a - b)*cosh(f*x + e))*sinh(f*x + e) + a + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 3*a*b^3)*sinh(f*x + e)^6 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3 + 5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 5*a^2*b^2 - 3*a*b^3 + 4*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 + 8*a^3*b - 7*a^2*b^2 + a*b^3 + 6*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^5 + 2*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^6 + 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)), 1/3*(3*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^8 + 8*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b^2 - 2*a*b^3 + b^4)*sinh(f*x + e)^8 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^6 + 4*(2* a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^4 + 2*(35*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^4 + 8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4 + 30*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 - 2*a*b^3 + b^4 + 8*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^5 + 10*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^3 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^2 + 4*(7*(a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^6 + 15*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^4 + 2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4 + 3*(8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b^2 - 2*a*b^3 + b^4)*cosh(f*x + e)^7 + 3*(2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e)^5 + (8*a^4 - 24*a^3*b + 27*a^2*b^2 - 14*a*b^3 + 3*b^4)*cosh(f*x + e)^3 + (2*a^3*b - 5*a^2*b^2 + 4*a*b^3 - b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^6 + 6*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 3*a*b^3)*sinh(f*x + e)^6 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^4 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3 + 5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 5*a^2*b^2 - 3*a*b^3 + 4*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 3*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^4 + 8*a^3*b - 7*a^2*b^2 + a*b^3 + 6*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 3*a*b^3)*cosh(f*x + e)^5 + 2*(8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e)^3 + (8*a^3*b - 7*a^2*b^2 + a*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^8 + 8*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^6 + 2*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f)*sinh(f*x + e)^4 + 4*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^2 + 8*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^5 + 10*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^3 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^6 + 15*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^4 + 3*(8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^2 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f)*sinh(f*x + e)^2 + (a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f + 8*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*f*cosh(f*x + e)^7 + 3*(2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e)^5 + (8*a^7 - 24*a^6*b + 27*a^5*b^2 - 14*a^4*b^3 + 3*a^3*b^4)*f*cosh(f*x + e)^3 + (2*a^6*b - 5*a^5*b^2 + 4*a^4*b^3 - a^3*b^4)*f*cosh(f*x + e))*sinh(f*x + e))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.51809, size = 857, normalized size = 6.3

$$\frac{5\,ab-3\,b^2}{3\left(a^4\sqrt{b}f-2\,a^3b^{\frac{3}{2}}f+a^2b^{\frac{5}{2}}f\right)}-\frac{\left(\left(\left(\frac{\left(5\,a^{13}b^4f^3-13\,a^{12}b^5f^3+11\,a^{11}b^6f^3-3\,a^{10}b^7f^3\right)e^{(2\,fx+2\,e)}}{a^{16}b^2f^4-4\,a^{15}b^3f^4+6\,a^{14}b^4f^4-4\,a^{13}b^5f^4+a^{12}b^6f^4}+\frac{3\left(8\,a^{14}b^3f^3-23\,a^{13}b^4f^3+23\,a^{12}b^5f^3-9\,a^{11}b^6f^3+a^{10}\cdots\right.}{a^{16}b^2f^4-4\,a^{15}b^3f^4+6\,a^{14}b^4f^4-4\,a^{13}b^5f^4+a^{12}b^6\cdots}\right.\right.\right.}{3\left(be^{(4\,fx+4\cdots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(5*a*b - 3*b^2)/(a^4*sqrt(b)*f - 2*a^3*b^(3/2)*f + a^2*b^(5/2)*f) - 1/3
*((((5*a^13*b^4*f^3 - 13*a^12*b^5*f^3 + 11*a^11*b^6*f^3 - 3*a^10*b^7*f^3)*e
^(2*f*x + 2*e)/(a^16*b^2*f^4 - 4*a^15*b^3*f^4 + 6*a^14*b^4*f^4 - 4*a^13*b^5
*f^4 + a^12*b^6*f^4) + 3*(8*a^14*b^3*f^3 - 23*a^13*b^4*f^3 + 23*a^12*b^5*f^
3 - 9*a^11*b^6*f^3 + a^10*b^7*f^3)/(a^16*b^2*f^4 - 4*a^15*b^3*f^4 + 6*a^14*
b^4*f^4 - 4*a^13*b^5*f^4 + a^12*b^6*f^4))*e^(2*f*x + 2*e) + 3*(8*a^14*b^3*f
^3 - 23*a^13*b^4*f^3 + 23*a^12*b^5*f^3 - 9*a^11*b^6*f^3 + a^10*b^7*f^3)/(a^
16*b^2*f^4 - 4*a^15*b^3*f^4 + 6*a^14*b^4*f^4 - 4*a^13*b^5*f^4 + a^12*b^6*f^
4))*e^(2*f*x + 2*e) + (5*a^13*b^4*f^3 - 13*a^12*b^5*f^3 + 11*a^11*b^6*f^3 -
 3*a^10*b^7*f^3)/(a^16*b^2*f^4 - 4*a^15*b^3*f^4 + 6*a^14*b^4*f^4 - 4*a^13*b
^5*f^4 + a^12*b^6*f^4))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b)^(3/2) + 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^
(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) - sqrt(b))/s
qrt(-a))/(sqrt(-a)*a^2*f)
```

3.120 $\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=344

$$\frac{2(2a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3b^2f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\left(8a^2-13ab+3b^2\right)\tanh(e+f}{3b^3f(a-}$$

```
[Out] -(a*Cosh[e + f*x]*Sinh[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*a*(2*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*b^2*f*Sq
rt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)
^2*b^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*(2*a - 3*b
)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*(a - b)^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
))/a]) + ((8*a^2 - 13*a*b + 3*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x
])/(3*(a - b)^2*b^3*f)
```

Rubi [A] time = 0.366433, antiderivative size = 344, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3188, 470, 578, 531, 418, 492, 411}

$$\frac{\left(8a^2-13ab+3b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^3f(a-b)^2}-\frac{\left(8a^2-13ab+3b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan\right.}{3b^3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] -(a*Cosh[e + f*x]*Sinh[e + f*x]^3)/(3*(a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*a*(2*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*b^2*f*Sq
rt[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)
^2*b^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*(2*a - 3*b
)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*(a - b)^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
))/a]) + ((8*a^2 - 13*a*b + 3*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x
])/(3*(a - b)^2*b^3*f)
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sinh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1+x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^2(\cdots}{\sqrt{1\cdots}}\right.}{3(a-b)bf}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} - \cdots$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \cdots$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{2(2\cdots}{\cdots}$$

$$= -\frac{a\cosh(e+fx)\sinh^3(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2a(2a-3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2b^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a\cdots}{\cdots}$$

Mathematica [C] time = 2.03492, size = 207, normalized size = 0.6

$$\frac{a\left(2ia\left(8a^2-17ab+9b^2\right)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(-8a^2+b(7b-5a)\cos\cdots\right.\right.}{6b^3f(a-b)^2(2a+b\cosh(2(e+f\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*((-2*I)*a*(8*a^2 - 13*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(
3/2)*EllipticE[I*(e + f*x), b/a] + (2*I)*a*(8*a^2 - 17*a*b + 9*b^2)*((2*a -
 b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*
(-8*a^2 + 17*a*b - 7*b^2 + b*(-5*a + 7*b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*
x)]))/(6*(a - b)^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.13, size = 868, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((5*(-1/a*b)^(1/2)*a^2*b-7*(-1/a*b)^(1/2)*a*b^2)*sinh(f*x+e)*cosh(f*x+
e)^4+(4*(-1/a*b)^(1/2)*a^3-11*(-1/a*b)^(1/2)*a^2*b+7*(-1/a*b)^(1/2)*a*b^2)*
cosh(f*x+e)^2*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)
```

```
^(1/2)*b*(4*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-7*Ellipti
cF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+3*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^2+13*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-3*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2+4*(b/a*cosh(f
*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*a^3-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+10*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b^2-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-8*(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*a^3+21*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-16*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/(-1/
a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/b^2/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(fx+e)^6}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh(fx+e)^2+a}\,\sinh(fx+e)^6}{b^3\sinh(fx+e)^6+3\,ab^2\sinh(fx+e)^4+3\,a^2b\sinh(fx+e)^2+a^3}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^6/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^6}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.121 $\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=244

$$-\frac{(a-3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3abf(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2\sqrt{a}(a-2b)\cosh(e+fx)E\left(\tan^{-1}\right.}{3b^{3/2}f(a-b)^2\sqrt{a+b\sinh^2(e+\right.}}$$

```
[Out] -(a*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) + (2*Sqrt[a]*(a - 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^2*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/
(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - ((a - 3*b)*Elliptic
F[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]
)/(3*a*(a - b)^2*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rubi [A] time = 0.249229, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3188, 470, 525, 418, 411}

$$\frac{2\sqrt{a}(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3b^{3/2}f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{a\sinh(e+fx)\cosh(e+fx)}{3bf(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(a-3b)\text{sech}(e+fx)\sqrt{a}}{3abf(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] -(a*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*b*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) + (2*Sqrt[a]*(a - 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^2*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/
(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - ((a - 3*b)*Elliptic
F[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]
)/(3*a*(a - b)^2*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```


Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\sinh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4}{\sqrt{1+x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{a}{\sqrt{1}}\right.}{3(a-b)bf}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left((a-3b)\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subs}}{3(a-b)^2b}$$

$$= -\frac{a\cosh(e+fx)\sinh(e+fx)}{3(a-b)bf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\sqrt{a}(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+}{\sqrt{a}}\right.\right.}{3(a-b)^2b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2}}$$

Mathematica [C] time = 1.60918, size = 198, normalized size = 0.81

$$\frac{-ia\left(2a^2-5ab+3b^2\right)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right)-\sqrt{2}b\sinh(2(e+fx))\left(-a^2-b(a-2b)\cosh(2(e\right.}{3b^2f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*
(e + f*x), b/a] - I*a*(2*a^2 - 5*a*b + 3*b^2)*((2*a - b + b*Cosh[2*(e + f*x
)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-a^2 + 4*a*b - 2*b^2
- (a - 2*b)*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*(a - b)^2*b^2*f*(2*a
 - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.107, size = 659, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2+4*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-(-1/a*b)
^(1/2)*a*b*sinh(f*x+e)^3-4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+a^2*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*((a+b*sinh(f*x+e)^2)/a)^(1
/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))
*a^2+4*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+(-1/a*b)^(1/2)*a^2*sinh(f*x+e)-3*(-1/
a*b)^(1/2)*a*b*sinh(f*x+e))/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^
2/b/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\sinh\left(fx+e\right)^4}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.122 $\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=241

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{3bf(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{3af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{i(}{}$$

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) +
 ((a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Sinh[e +
 f*x]^2]) + ((I/3)*(a + b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e +
f*x]^2])/(a*(a - b)^2*b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) - ((I/3)*Ellipti
cF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/((a - b)*b*f*Sqrt[a +
 b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.325695, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{(a+b)\sinh(e+fx)\cosh(e+fx)}{3af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}F\left(ie+ifx\middle|\frac{b}{a}\right)}{3bf(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{i(a+b)\sqrt{a}}{3abf}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) +
 ((a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*Sqrt[a + b*Sinh[e +
 f*x]^2]) + ((I/3)*(a + b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e +
f*x]^2])/(a*(a - b)^2*b*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]) - ((I/3)*Ellipti
cF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/((a - b)*b*f*Sqrt[a +
 b*Sinh[e + f*x]^2])
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{a-a\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{2a^2+a(a+b)}{\sqrt{a+b\sin}}}{3a^2(}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \frac{1}{\sqrt{a+b\sinh}}}{3(a-}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left((a+b)\sqrt{a}\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} + \frac{i(a+b)E\left(\right.}{3a(}$$

Mathematica [A] time = 1.40341, size = 187, normalized size = 0.78

$$\frac{-2ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(4a^2+b(a+b)\cosh(2(e+fx))-a\right.}{6abf(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e
 + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)
*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(4*a^2 - a*b - b^2 + b*(a + b)*Cos
h[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*a*(a - b)^2*b*f*(2*a - b + b*Cosh[2*(
e + f*x)])^(3/2))
```

Maple [B] time = 0.114, size = 598, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((-(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(-
2*(-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*s
inh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*Ell
ipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))+EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a
+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2+a^2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))-2*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^
(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-(b/a*cosh(f*x+e
)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/a/cos
h(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\sinh\left(fx+e\right)^2}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sinh(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sinh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.123 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{3af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie\right.}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+}}$$

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.279692, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+fx}{3af(a-b)\left(a+b\sinh^2(e+fx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \cdots$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \cdots$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \cdots$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{2i(2a\cdots}{\cdots}$$

Mathematica [A] time = 1.31988, size = 190, normalized size = 0.76

$$\frac{ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(-5a^2+b(b-2a)\cosh(2(e+fx))+5\cdots\right.}{3a^2f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

Maple [A] time = 0.198, size = 406, normalized size = 1.6

$$\frac{1}{f\cosh\left(fx+e\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\left(-\frac{\sinh\left(fx+e\right)}{3\,ab\left(a-b\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/
2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a
*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*
x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

3.124 $\int \frac{\text{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=385

$$-\frac{2b(3a-2b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^3f(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\left(3a^2-13ab+8b^2\right)\tanh(e+f}{3a^3f(a-b}$$

```
[Out] -(b*Coth[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(3*a
- 2*b)*b*Coth[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (
(3*a^2 - 13*a*b + 8*b^2)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*
(a - b)^2*f) - ((3*a^2 - 13*a*b + 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f*Sqrt[
(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(3*a - 2*b)*b*EllipticF[
ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*a^3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3
*a^2 - 13*a*b + 8*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^3*(a
 - b)^2*f)
```

Rubi [A] time = 0.456218, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3188, 472, 579, 583, 531, 418, 492, 411}

$$\frac{\left(3a^2-13ab+8b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)^2}-\frac{\left(3a^2-13ab+8b^2\right)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)^2}-\frac{\left(3a^2-\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(b*Coth[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - (2*(3*a
- 2*b)*b*Coth[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (
(3*a^2 - 13*a*b + 8*b^2)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*
(a - b)^2*f) - ((3*a^2 - 13*a*b + 8*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)^2*f*Sqrt[
(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(3*a - 2*b)*b*EllipticF[
ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*a^3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3
*a^2 - 13*a*b + 8*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^3*(a
 - b)^2*f)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1+x^2}(a+bx^2)^{5/2}}dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{3a-}{x^2\sqrt{1+x}\cdots}\right.}{3a(a-b)f}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e\cdots}\right.}{\cdots}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(3a^2-13ab\cdots\right.}{\cdots}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(3a^2-13ab\cdots\right.}{\cdots}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(3a^2-13ab\cdots\right.}{\cdots}$$

$$= -\frac{b\coth(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(3a-2b)b\coth(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(3a^2-13ab\cdots\right.}{\cdots}$$

Mathematica [C] time = 2.3105, size = 234, normalized size = 0.61

$$\frac{i\left(4a^2\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left(\left(3a^2-7ab+4b^2\right)\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\left(-3a^2+13ab-8b^2\right)E\left(i(e+fx)\middle|\frac{b}{a}\right)\right)+2i\sqrt{2}\left(\cdots\right.\right.}{12a^3f(a-\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((I/12)*(4*a^2*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*((-3*a^2 + 13*a*b
- 8*b^2)*EllipticE[I*(e + f*x), b/a] + (3*a^2 - 7*a*b + 4*b^2)*EllipticF[I*
(e + f*x), b/a]) + (2*I)*Sqrt[2]*(3*(a - b)^2*(2*a - b + b*Cosh[2*(e + f*x)
])^2*Coth[e + f*x] - 2*a*(a - b)*b^2*Sinh[2*(e + f*x)] - (7*a - 5*b)*b^2*(2
*a - b + b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])))/(a^3*(a - b)^2*f*(2*a -
b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [A] time = 0.128, size = 747, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b^2*(9*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-17*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))*b^2-3*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*EllipticE(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*b^2)*sinh(f*x+e)*cosh(f*x+e)^2+(b/a*cosh(f*x+e)^2
+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(9*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))*a^3-26*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a^2*b+25*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-8*Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-3*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a^3+16*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^2*b-21*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+8*Ell
ipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)*sinh(f*x+e)+(3*(-1/a*b)
^(1/2)*a^2*b^2-13*(-1/a*b)^(1/2)*a*b^3+8*(-1/a*b)^(1/2)*b^4)*cosh(f*x+e)^6+
(6*(-1/a*b)^(1/2)*a^3*b-26*(-1/a*b)^(1/2)*a^2*b^2+38*(-1/a*b)^(1/2)*a*b^3-1
6*(-1/a*b)^(1/2)*b^4)*cosh(f*x+e)^4+(3*(-1/a*b)^(1/2)*a^4-12*(-1/a*b)^(1/2)
*a^3*b+26*(-1/a*b)^(1/2)*a^2*b^2-25*(-1/a*b)^(1/2)*a*b^3+8*(-1/a*b)^(1/2)*b
^4)*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^2/(-1/a*b)^(1/2)/sinh(f*
x+e)/a^3/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\operatorname{csch}\left(fx+e\right)^{2}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*csch(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csch(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.125 $\int \frac{1}{\sqrt{1+\sinh^2(x)}}\,dx$

Optimal. Leaf size=14

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}$$

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[Cosh[x]^2]
```

Rubi [A] time = 0.0240236, antiderivative size = 14, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3176, 3207, 3770}

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 + Sinh[x]^2],x]

[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[Cosh[x]^2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{1+\sinh^2(x)}}\,dx &= \int \frac{1}{\sqrt{\cosh^2(x)}}\,dx\\ &= \frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{\sqrt{\cosh^2(x)}}\\ &= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{\cosh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0108901, size = 19, normalized size = 1.36

$$\frac{2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 + Sinh[x]^2],x]
```

```
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[Cosh[x]^2]
```

Maple [A] time = 0.05, size = 15, normalized size = 1.1

$$\frac{\arctan\left(\sinh\left(x\right)\right)}{\cosh\left(x\right)}\sqrt{\left(\cosh\left(x\right)\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^2)^(1/2),x)
```

```
[Out] (cosh(x)^2)^(1/2)*arctan(sinh(x))/cosh(x)
```

Maxima [A] time = 1.65678, size = 7, normalized size = 0.5

$$2\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^x)
```

Fricas [A] time = 2.03552, size = 39, normalized size = 2.79

$$2\arctan\left(\cosh\left(x\right)+\sinh\left(x\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{\sinh^{2}\left(x\right)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sinh(x)**2 + 1), x)
```

Giac [A] time = 1.14593, size = 7, normalized size = 0.5

$$2\arctan\left(e^{x}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)
```

3.126 $\int \frac{1}{\sqrt{1-\sinh^2(x)}} \, dx$

Optimal. Leaf size=11

$$-i\text{EllipticF}(ix,-1)$$

```
[Out] (-I)*EllipticF[I*x, -1]
```

Rubi [A] time = 0.0108332, antiderivative size = 11, normalized size of antiderivative = 1., number of steps used = 1, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3182}

$$-iF(ix|-1)$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticF[I*x, -1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{1-\sinh^2(x)}} \, dx = -iF(ix|-1)$$

Mathematica [A] time = 0.0398722, size = 11, normalized size = 1.

$$-i\text{EllipticF}(ix,-1)$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[1 - Sinh[x]^2],x]
```

```
[Out] (-I)*EllipticF[I*x, -1]
```

Maple [A] time = 0.147, size = 41, normalized size = 3.7

$$\frac{\text{EllipticF}(\sinh(x), i)}{\cosh(x)} \sqrt{-\left(-1+(\sinh(x))^2\right)(\cosh(x))^2} \sqrt{(\cosh(x))^2} \frac{1}{\sqrt{1-(\sinh(x))^4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^2)^(1/2),x)
```

```
[Out] (-(-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)/(1-sinh(x)^4)^(1/2)*Ell
ipticF(sinh(x),I)/cosh(x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sinh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(-sinh(x)^2 + 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(-\frac{\sqrt{-\sinh(x)^2+1}}{\sinh(x)^2-1}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-sinh(x)^2 + 1)/(sinh(x)^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(1 - sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{-\sinh(x)^2+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-sinh(x)^2 + 1), x)
```

3.127 $\int \frac{1}{\sqrt{-1+\sinh^2(x)}}\,dx$

Optimal. Leaf size=33

$$-\frac{i\sqrt{1-\sinh^2(x)}\,\text{EllipticF}(ix,-1)}{\sqrt{\sinh^2(x)-1}}$$

```
[Out] ((-I)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2]
```

Rubi [A] time = 0.021624, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3183, 3182}

$$-\frac{i\sqrt{1-\sinh^2(x)}F(ix|-1)}{\sqrt{\sinh^2(x)-1}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*x, -1]*Sqrt[1 - Sinh[x]^2])/Sqrt[-1 + Sinh[x]^2]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{-1+\sinh^2(x)}}\,dx &= \frac{\sqrt{1-\sinh^2(x)}\int \frac{1}{\sqrt{1-\sinh^2(x)}}\,dx}{\sqrt{-1+\sinh^2(x)}}\\ &= -\frac{iF(ix|-1)\sqrt{1-\sinh^2(x)}}{\sqrt{-1+\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0414932, size = 33, normalized size = 1.

$$-\frac{i\sqrt{3-\cosh(2x)}\,\text{EllipticF}(ix,-1)}{\sqrt{\cosh(2x)-3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 + Sinh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[3 - Cosh[2*x]]*EllipticF[I*x, -1])/Sqrt[-3 + Cosh[2*x]]
```

Maple [A] time = 0.068, size = 61, normalized size = 1.9

$$\frac{-i\text{EllipticF}\left(i\sinh\left(x\right),i\right)}{\cosh\left(x\right)}\sqrt{\left(-1+\left(\sinh\left(x\right)\right)^{2}\right)\left(\cosh\left(x\right)\right)^{2}}\sqrt{\left(\cosh\left(x\right)\right)^{2}}\sqrt{1-\left(\sinh\left(x\right)\right)^{2}}\frac{1}{\sqrt{\left(\sinh\left(x\right)\right)^{4}-1}}\frac{1}{\sqrt{-1+(\sinh}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1+sinh(x)^2)^(1/2),x)
```

```
[Out] -I*((-1+sinh(x)^2)*cosh(x)^2)^(1/2)*(cosh(x)^2)^(1/2)*(1-sinh(x)^2)^(1/2)/(
sinh(x)^4-1)^(1/2)*EllipticF(I*sinh(x),I)/cosh(x)/(-1+sinh(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh\left(x\right)^{2}-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(sinh(x)^2 - 1), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{\sinh\left(x\right)^{2}-1}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(sinh(x)^2 - 1), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh^{2}\left(x\right)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(sinh(x)**2 - 1), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{\sinh{(x)}^2 - 1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1+sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(sinh(x)^2 - 1), x)
```

3.128 $\int \frac{1}{\sqrt{-1-\sinh^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{-\cosh^2(x)}}$$

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[-Cosh[x]^2]
```

Rubi [A] time = 0.0237084, antiderivative size = 16, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3176, 3207, 3770}

$$\frac{\cosh(x)\tan^{-1}(\sinh(x))}{\sqrt{-\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] (ArcTan[Sinh[x]]*Cosh[x])/Sqrt[-Cosh[x]^2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{-1-\sinh^2(x)}}\,dx &= \int \frac{1}{\sqrt{-\cosh^2(x)}}\,dx \\
&= \frac{\cosh(x)\int \operatorname{sech}(x)\,dx}{\sqrt{-\cosh^2(x)}} \\
&= \frac{\tan^{-1}(\sinh(x))\cosh(x)}{\sqrt{-\cosh^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.0079731, size = 21, normalized size = 1.31

$$\frac{2\cosh(x)\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{\sqrt{-\cosh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[-1 - Sinh[x]^2],x]
```

```
[Out] (2*ArcTan[Tanh[x/2]]*Cosh[x])/Sqrt[-Cosh[x]^2]
```

Maple [B] time = 0.04, size = 34, normalized size = 2.1

$$-\frac{\cosh(x)}{\sinh(x)}\sqrt{-(\sinh(x))^2}\operatorname{Artanh}\left(\frac{1}{\sqrt{-(\sinh(x))^2}}\right)\frac{1}{\sqrt{-(\cosh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(-1-sinh(x)^2)^(1/2),x)
```

```
[Out] -cosh(x)*(-sinh(x)^2)^(1/2)*arctanh(1/(-sinh(x)^2)^(1/2))/sinh(x)/(-cosh(x)
^2)^(1/2)
```

Maxima [C] time = 1.92463, size = 7, normalized size = 0.44

$$-2i\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*I*arctan(e^x)
```

Fricas [C] time = 1.77536, size = 39, normalized size = 2.44

$$\log(e^x + i) - \log(e^x - i)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] log(e^x + I) - log(e^x - I)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{-\sinh^2(x)-1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-sinh(x)**2 - 1), x)
```

Giac [C] time = 1.15006, size = 7, normalized size = 0.44

$$-2i\arctan(e^x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(-1-sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*I*arctan(e^x)
```

3.129 $\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx$

Optimal. Leaf size=42

$$-\frac{i\sqrt{\frac{b\sinh^2(x)}{a}+1}\text{EllipticF}\left(ix,\frac{b}{a}\right)}{\sqrt{a+b\sinh^2(x)}}$$

```
[Out] ((-I)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]
```

Rubi [A] time = 0.0329238, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\sinh^2(x)}{a}+1}F\left(ix\left|\frac{b}{a}\right.\right)}{\sqrt{a+b\sinh^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*x, b/a]*Sqrt[1 + (b*Sinh[x]^2)/a])/Sqrt[a + b*Sinh[x]^2]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sinh^2(x)}}\,dx &= \frac{\sqrt{1+\frac{b\sinh^2(x)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(x)}{a}}}\,dx}{\sqrt{a+b\sinh^2(x)}}\\ &= -\frac{iF\left(ix\left|\frac{b}{a}\right.\right)\sqrt{1+\frac{b\sinh^2(x)}{a}}}{\sqrt{a+b\sinh^2(x)}}\end{aligned}$$

Mathematica [A] time = 0.0507961, size = 53, normalized size = 1.26

$$-\frac{i\sqrt{\frac{2a+b\cosh(2x)-b}{a}}\text{EllipticF}\left(ix,\frac{b}{a}\right)}{\sqrt{2a+b\cosh(2x)-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*x])/a]*EllipticF[I*x, b/a])/Sqrt[2*a - b + b
*Cosh[2*x]]
```

Maple [A] time = 0.059, size = 63, normalized size = 1.5

$$\frac{1}{\cosh(x)}\sqrt{\frac{a+b(\sinh(x))^2}{a}}\sqrt{(\cosh(x))^2}\mathrm{EllipticF}\left(\sinh(x)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}\frac{1}{\sqrt{a+b(\sinh(x))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(x)^2)/a)^(1/2)*(cosh(x)^2)^(1/2)*EllipticF(sinh
(x)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(x)/(a+b*sinh(x)^2)^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(x)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{1}{\sqrt{b\sinh(x)^2 + a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(x)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sinh^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(x)^2 + a), x)
```

3.130 $\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=128

$$\frac{d\cosh(e+fx)\left(-\sinh^2(e+fx)\right)^{\frac{1-m}{2}}(d\sinh(e+fx))^{m-1}\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p\right.}{f}$$

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)
/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*(d*Sinh[e + f*x])^(-
1 + m)*(-Sinh[e + f*x]^2)^((1 - m)/2))/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))
^p)
```

Rubi [A] time = 0.116103, antiderivative size = 128, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3189, 430, 429}

$$\frac{d\cosh(e+fx)\left(-\sinh^2(e+fx)\right)^{\frac{1-m}{2}}(d\sinh(e+fx))^{m-1}\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```


```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)
/(a - b))]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*(d*Sinh[e + f*x])^(-
1 + m)*(-Sinh[e + f*x]^2)^((1 - m)/2))/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))
^p)
```


Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\, dx = \frac{\left(d(d\sinh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\left(-\sinh^2(e+fx)\right)^{\frac{1}{2}-\frac{m}{2}}\right)\operatorname{Subst}\left(\int\left(1-x^2\right)^{\frac{1}{2}(-\cdots}\right.}{f}$$

$$= \frac{\left(d\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}(d\sinh(e+fx))^{2\left(-\frac{1}{2}+\cdots\right.}\right.}{\cdots}$$

$$= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+\cdots\right.}{\cdots}$$

Mathematica [F] time = 8.65666, size = 0, normalized size = 0.

$$\int (d\sinh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p\, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]

[Out] Integrate[(d*Sinh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.641, size = 0, normalized size = 0.

$$\int \left(d\sinh\left(fx+e\right)\right)^m \left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)

[Out] int((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\sinh\left(fx+e\right)\right)^m\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\sinh\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\left(d\sinh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sinh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*sinh(f*x + e))^m, x)
```

3.131 $\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=226

$$\frac{\left(3a^2+4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2}{a-}\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] -(((3*a + 2*b*(2 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(
b^2*f*(3 + 2*p)*(5 + 2*p))) + ((3*a^2 + 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^
2))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Co
sh[e + f*x]^2)/(a - b))^p) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1
+ p)*Sinh[e + f*x]^2)/(b*f*(5 + 2*p))
```

Rubi [A] time = 0.25908, antiderivative size = 226, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 416, 388, 246, 245}

$$\frac{\left(3a^2+4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2}{a-}\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```


```
[Out] -(((3*a + 2*b*(2 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(
b^2*f*(3 + 2*p)*(5 + 2*p))) + ((3*a^2 + 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^
2))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p,
3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Co
sh[e + f*x]^2)/(a - b))^p) + (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1
+ p)*Sinh[e + f*x]^2)/(b*f*(5 + 2*p))
```


Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a-b+bx^2\right)^p\,dx,x,\cosh(e+fx)\right)}{f}$$

$$=\frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}\sinh^2(e+fx)}{bf(5+2p)}+\frac{\text{Subst}\left(\int\left(a\right.\right.}{}$$

$$=-\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{\cosh(e+}{}$$

$$=-\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{\cosh(e+}{}$$

$$=-\frac{(3a+2b(2+p))\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}+\frac{\left(3a^2+4\right.}{}$$

Mathematica [F] time = 11.4338, size = 0, normalized size = 0.

$$\int \sinh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sinh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.629, size = 0, normalized size = 0.

$$\int\left(\sinh\left(fx+e\right)\right)^5\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^5, x)
```

3.132 $\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=137

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{bf(2p+3)}-\frac{(a+2b(p+1))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right.}{bf(2p+3)}$$

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a +
 2*b*(1 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1
[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b*f*(3 + 2*p)*(1 + (b*Cosh
[e + f*x]^2)/(a - b))^p)
```

Rubi [A] time = 0.122933, antiderivative size = 137, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 388, 246, 245}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^{p+1}}{bf(2p+3)}-\frac{(a+2b(p+1))\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a +
 2*b*(1 + p))*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1
[1/2, -p, 3/2, -((b*Cosh[e + f*x]^2)/(a - b))])/(b*f*(3 + 2*p)*(1 + (b*Cosh
[e + f*x]^2)/(a - b))^p)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

GtQ[a, 0])

Rubi steps

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = -\frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\left(a-b+bx^2\right)^p\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a+2b(1+p))\operatorname{Subst}\left(\int \left(a-\right.\right.}{bf(3}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{\left((a+2b(1+p))\left(a-b+b\cos\right.\right.}{}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^{1+p}}{bf(3+2p)} - \frac{(a+2b(1+p))\cosh(e+fx)\left(\right.}{}$$

Mathematica [F] time = 15.0967, size = 0, normalized size = 0.

$$\int \sinh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sinh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.436, size = 0, normalized size = 0.

$$\int \left(\sinh\left(fx+e\right)\right)^3\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^3, x)
```

3.133 $\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)
```

Rubi [A] time = 0.057892, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 246, 245}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \frac{\text{Subst}\left(\int \left(a-b+bx^2\right)^p dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\cosh^2(e+fx)\right)^p \left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\right)\text{Subst}\left(\int \left(1+\frac{bx^2}{a-b}\right)^p dx\right.}{f}$$

$$= \frac{\cosh(e+fx)\left(a-b+b\cosh^2(e+fx)\right)^p \left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\right.}{f}$$

Mathematica [A] time = 0.207017, size = 78, normalized size = 1.

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p \left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p} {}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, -((b*Cosh[e + f*x]^2)/(a - b))])/(f*(1 + (b*Cosh[e + f*x]^2)/(a - b))^p)
```

Maple [F] time = 0.331, size = 0, normalized size = 0.

$$\int \sinh\left(fx+e\right)\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \sinh\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p \sinh\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(fx+e\right)^2+a\right)^p \sinh\left(fx+e\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e), x)
```

3.134 $\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b)
)]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p))
```

Rubi [A] time = 0.0878811, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 430, 429}

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b)
)]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^p}{1-x^2}\,dx,x,\cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{1-x^2}\,dx,x,c\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh^2\right.}{f}$$

Mathematica [F] time = 4.1886, size = 0, normalized size = 0.

$$\int \operatorname{csch}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csch[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.264, size = 0, normalized size = 0.

$$\int \operatorname{csch}(fx+e)\left(a+b\left(\sinh(fx+e)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(fx+e)^2+a\right)^p\operatorname{csch}(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh(fx+e)^2+a\right)^p\operatorname{csch}(fx+e),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e), x)
```

3.135 $\int \mathbf{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=87

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] (AppellF1[1/2, 2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]
*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(
a - b))^p)
```

Rubi [A] time = 0.0968984, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 430, 429}

$$\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b))]
*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)/(
a - b))^p)
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^p}{\left(1-x^2\right)^2}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{\left(1-x^2\right)^2}\,dx, x\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cos\right.}{f}$$

Mathematica [F] time = 7.89471, size = 0, normalized size = 0.

$$\int \operatorname{csch}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csch[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.294, size = 0, normalized size = 0.

$$\int \left(\operatorname{csch}\left(fx+e\right)\right)^3\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^3, x)
```

3.136 $\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=88

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

```
[Out] -((AppellF1[1/2, 3, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b)
)]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p))
```

Rubi [A] time = 0.0970843, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3186, 430, 429}

$$-\frac{\cosh(e+fx)\left(a+b\cosh^2(e+fx)-b\right)^p\left(\frac{b\cosh^2(e+fx)}{a-b}+1\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 3, -p, 3/2, Cosh[e + f*x]^2, -((b*Cosh[e + f*x]^2)/(a - b)
)]*Cosh[e + f*x]*(a - b + b*Cosh[e + f*x]^2)^p)/(f*(1 + (b*Cosh[e + f*x]^2)
/(a - b))^p))
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a-b+bx^2\right)^p}{\left(1-x^2\right)^3}\,dx, x, \cosh(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a-b+b\cosh^2(e+fx)\right)^p\left(1+\frac{b\cosh^2(e+fx)}{a-b}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a-b}\right)^p}{\left(1-x^2\right)^3}\,dx, x,\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cosh^2(e+fx),-\frac{b\cosh^2(e+fx)}{a-b}\right)\cosh(e+fx)\left(a-b+b\cosh\right.}{f}$$

Mathematica [F] time = 15.0046, size = 0, normalized size = 0.

$$\int \operatorname{csch}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csch[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.273, size = 0, normalized size = 0.

$$\int \left(\operatorname{csch}\left(fx+e\right)\right)^5\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{p} \operatorname{csch}\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^5, x)
```

3.137 $\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$\frac{\sinh^4(e+fx)\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(e+fx),-\frac{b}{}\right.}{5f}$$

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]
)/(5*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.109966, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3188, 511, 510}

$$\frac{\sinh^4(e+fx)\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(e+fx),-\frac{b}{}\right.}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]
)/(5*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4(a+bx^2)^p}{\sqrt{1+x^2}}dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-}}{f}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\sinh^4(e+}{5f}$$

Mathematica [F] time = 10.2633, size = 0, normalized size = 0.

$$\int \sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sinh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.497, size = 0, normalized size = 0.

$$\int \left(\sinh\left(fx+e\right)\right)^4\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \sinh\left(fx+e\right)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{p} \sinh \left(fx+e\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^4, x)
```

3.138 $\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{\tanh^3(e+fx)\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(1-\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b)\tan\ldots}{\ldots}\right.}{3f}$$

```
[Out] (AppellF1[3/2, 2 + p, -p, 5/2, Tanh[e + f*x]^2, ((a - b)*Tanh[e + f*x]^2)/a
]*(Sech[e + f*x]^2)^p*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]^3)/(3*f*(1 -
((a - b)*Tanh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.192478, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3174, 511, 510}

$$\frac{\tanh^3(e+fx)\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(1-\frac{(a-b)\tanh^2(e+fx)}{a}\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b)\tan\ldots}{\ldots}\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2 + p, -p, 5/2, Tanh[e + f*x]^2, ((a - b)*Tanh[e + f*x]^2)/a
]*(Sech[e + f*x]^2)^p*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x]^3)/(3*f*(1 -
((a - b)*Tanh[e + f*x]^2)/a)^p)
```

Rule 3174

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[(ff*(a + b*Sin[e + f*x]^2)^p*(Sec[e + f*x]^2)^p)/(f*(a + (a + b)*Tan[e
+ f*x]^2)^p), Subst[Int[((a + (a + b)*ff^2*x^2)^p*(A + (A + B)*ff^2*x^2))/(
1 + ff^2*x^2)^(p + 2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, A
, B}, x] && !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(a-(a-b)\tanh^2(e+fx)\right)^{-p}\right)\text{Subs}}{f}$$

$$= \frac{\left(\operatorname{sech}^2(e+fx)^p\left(a+b\sinh^2(e+fx)\right)^p\left(a-(a-b)\tanh^2(e+fx)\right)^{-p}\left(a+(\right.\right.}{}$$

$$= \frac{F_1\left(\frac{3}{2};2+p,-p;\frac{5}{2};\tanh^2(e+fx),\frac{(a-b)\tanh^2(e+fx)}{a}\right)\operatorname{sech}^2(e+fx)^p\left(a+b\sin\right.}{3f}$$

Mathematica [B] time = 0.744882, size = 250, normalized size = 2.48

$$\frac{2^{-p-2}\operatorname{csch}(2(e+fx))\sqrt{-\frac{b\sinh^2(e+fx)}{a}}\sqrt{\frac{b\cosh^2(e+fx)}{b-a}}(2a+b\cosh(2(e+fx))-b)^{p+1}\left((p+1)(2a+b\cosh(2(e+fx))-b)F_1\right.}{b^2f(p}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (2^(-2 - p)*Sqrt[(b*Cosh[e + f*x]^2)/(-a + b)]*(2*a - b + b*Cosh[2*(e + f*x
)])^(1 + p)*(-2*a*(2 + p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a - b + b*Cos
h[2*(e + f*x)])/(2*a), (2*a - b + b*Cosh[2*(e + f*x)])/(2*(a - b))] + (1 +
p)*AppellF1[2 + p, 1/2, 1/2, 3 + p, (2*a - b + b*Cosh[2*(e + f*x)])/(2*a),
(2*a - b + b*Cosh[2*(e + f*x)])/(2*(a - b))]*(2*a - b + b*Cosh[2*(e + f*x)]
))*Csch[2*(e + f*x)]*Sqrt[-((b*Sinh[e + f*x]^2)/a)])/(b^2*f*(1 + p)*(2 + p)
)
```

Maple [F] time = 0.356, size = 0, normalized size = 0.

$$\int \left(\sinh\left(fx+e\right)\right)^2\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\sinh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\sinh\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sinh(f*x + e)^2, x)
```

3.139 $\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh^2(e+fx),-\right.}{f}$$

```
[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]
*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^
p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p))
```

Rubi [A] time = 0.104391, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3188, 511, 510}

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh^2(e+fx),-\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]
*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^
p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p))
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{x^2\sqrt{1+x^2}}dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-}}{f}$$

$$= -\frac{F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\operatorname{csch}(}{f}$$

Mathematica [F] time = 5.55103, size = 0, normalized size = 0.

$$\int \operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.26, size = 0, normalized size = 0.

$$\int \left(\operatorname{csch}\left(fx+e\right)\right)^2\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^2 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^p \operatorname{csch}\left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^2, x)
```

3.140 $\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2(e+\right.}{3f}$$

```
[Out] -(AppellF1[-3/2, 1/2, -p, -1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]
*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2
)^p)/(3*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.104743, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3188, 511, 510}

$$-\frac{\sqrt{\cosh^2(e+fx)}\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2(e+\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, 1/2, -p, -1/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]
*Sqrt[Cosh[e + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x]*(a + b*Sinh[e + f*x]^2
)^p)/(3*f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{x^4\sqrt{1+x^2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\operatorname{csch}^3(e}{3f}$$

Mathematica [F] time = 10.5505, size = 0, normalized size = 0.

$$\int \operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Csch[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.249, size = 0, normalized size = 0.

$$\int \left(\operatorname{csch}\left(fx+e\right)\right)^4\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{csch}\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\operatorname{csch}\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*csch(f*x + e)^4, x)
```

3.141 $\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)dx$

Optimal. Leaf size=106

$$\frac{a\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3a\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh^3(c+}{d}$$

```
[Out] (3*a*x)/8 - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/d - (3*b*Cosh[c + d*x
]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d) - (3*a*Cosh[c + d*x]*Sinh[c + d*x])/
(8*d) + (a*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0943742, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3220, 2635, 8, 2633}

$$\frac{a\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3a\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3ax}{8}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh^3(c+}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*a*x)/8 - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/d - (3*b*Cosh[c + d*x
]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d) - (3*a*Cosh[c + d*x]*Sinh[c + d*x])/
(8*d) + (a*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^3(c+dx)\right)dx = \int \left(a\sinh^4(c+dx)+b\sinh^7(c+dx)\right)dx$$
$$= a\int \sinh^4(c+dx)\,dx + b\int \sinh^7(c+dx)\,dx$$
$$= \frac{a\cosh(c+dx)\sinh^3(c+dx)}{4d} - \frac{1}{4}(3a)\int \sinh^2(c+dx)\,dx - \frac{b\,\text{Subst}\left(\int \left(\right.\right.}{}$$
$$= -\frac{b\cosh(c+dx)}{d} + \frac{b\cosh^3(c+dx)}{d} - \frac{3b\cosh^5(c+dx)}{5d} + \frac{b\cosh^7(c+dx)}{7d}$$
$$= \frac{3ax}{8} - \frac{b\cosh(c+dx)}{d} + \frac{b\cosh^3(c+dx)}{d} - \frac{3b\cosh^5(c+dx)}{5d} + \frac{b\cosh^7(c}{7d}$$

Mathematica [A] time = 0.102702, size = 81, normalized size = 0.76

$$\frac{-560a\sinh(2(c+dx))+70a\sinh(4(c+dx))+840ac+840adx-1225b\cosh(c+dx)+245b\cosh(3(c+dx))-49b\,\mathrm{c}}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (840*a*c + 840*a*d*x - 1225*b*Cosh[c + d*x] + 245*b*Cosh[3*(c + d*x)] - 49*
b*Cosh[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] - 560*a*Sinh[2*(c + d*x)] + 70*
a*Sinh[4*(c + d*x)])/(2240*d)
```

Maple [A] time = 0.016, size = 82, normalized size = 0.8

$$\frac{1}{d}\left(b\left(-\frac{16}{35}+\frac{(\sinh(dx+c))^6}{7}-\frac{6(\sinh(dx+c))^4}{35}+\frac{8(\sinh(dx+c))^2}{35}\right)\cosh(dx+c)+a\left(\left(\frac{(\sinh(dx+c))^3}{4}-3\sinh\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(b*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cos
h(d*x+c)+a*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.19747, size = 221, normalized size = 2.08

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{4480}b\left(\frac{\left(49e^{(-2dx-2c)}-245e^{(-4dx-4c)}+1225e^{(-6dx}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/4480*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x -
 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) -
 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
```

Fricas [A] time = 1.93103, size = 539, normalized size = 5.08

$$\frac{5\,b\cosh\left(dx+c\right)^{7}+35\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{6}-49\,b\cosh\left(dx+c\right)^{5}+280\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{3}+35\left(5\,b\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b*cosh(d*x + c)^7 + 35*b*cosh(d*x + c)*sinh(d*x + c)^6 - 49*b*cos
h(d*x + c)^5 + 280*a*cosh(d*x + c)*sinh(d*x + c)^3 + 35*(5*b*cosh(d*x + c)^
3 - 7*b*cosh(d*x + c))*sinh(d*x + c)^4 + 245*b*cosh(d*x + c)^3 + 840*a*d*x
+ 35*(3*b*cosh(d*x + c)^5 - 14*b*cosh(d*x + c)^3 + 21*b*cosh(d*x + c))*sinh
(d*x + c)^2 - 1225*b*cosh(d*x + c) + 280*(a*cosh(d*x + c)^3 - 4*a*cosh(d*x
+ c))*sinh(d*x + c))/d
```

Sympy [A] time = 8.10677, size = 192, normalized size = 1.81

$$\begin{cases} \frac{3ax\sinh^{4}(c+dx)}{8}-\frac{3ax\sinh^{2}(c+dx)\cosh^{2}(c+dx)}{4}+\frac{3ax\cosh^{4}(c+dx)}{8}+\frac{5a\sinh^{3}(c+dx)\cosh(c+dx)}{8d}-\frac{3a\sinh(c+dx)\cosh^{3}(c+dx)}{8d}+\frac{b\sinh^{6}(c+dx)\cdots}{d} \\ x\left(a+b\sinh^{3}(c)\right)\sinh^{4}(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*sinh(c + d*x)**6*cosh(c + d*
x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**3/d + 8*b*sinh(c + d*x)**2*cosh(
c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh
(c)**3)*sinh(c)**4, True))
```

Giac [A] time = 1.21197, size = 213, normalized size = 2.01

$$\frac{1680\,(dx+c)a+5\,be^{(7\,dx+7\,c)}-49\,be^{(5\,dx+5\,c)}+70\,ae^{(4\,dx+4\,c)}+245\,be^{(3\,dx+3\,c)}-560\,ae^{(2\,dx+2\,c)}-1225\,be^{(dx+c)}-\left(1225\,be^{(6\,d\cdots}\right.}{4480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/4480*(1680*(d*x + c)*a + 5*b*e^(7*d*x + 7*c) - 49*b*e^(5*d*x + 5*c) + 70*
a*e^(4*d*x + 4*c) + 245*b*e^(3*d*x + 3*c) - 560*a*e^(2*d*x + 2*c) - 1225*b*
e^(d*x + c) - (1225*b*e^(6*d*x + 6*c) - 560*a*e^(5*d*x + 5*c) - 245*b*e^(4*
d*x + 4*c) + 70*a*e^(3*d*x + 3*c) + 49*b*e^(2*d*x + 2*c) - 5*b)*e^(-7*d*x -
 7*c))/d
```

3.142 $\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=99

$$\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b\sinh(c+dx)\cdots}{16d}$$

```
[Out] (-5*b*x)/16 - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^3)/(3*d) + (5*b*Cosh[c
 + d*x]*Sinh[c + d*x])/(16*d) - (5*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d)
+ (b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.108472, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3220, 2633, 2635, 8}

$$\frac{a\cosh^3(c+dx)}{3d}-\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b\sinh(c+dx)\cdots}{16d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-5*b*x)/16 - (a*Cosh[c + d*x])/d + (a*Cosh[c + d*x]^3)/(3*d) + (5*b*Cosh[c
 + d*x]*Sinh[c + d*x])/(16*d) - (5*b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(24*d)
+ (b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)dx = i\int \left(-ia\sinh^3(c+dx)-ib\sinh^6(c+dx)\right)dx$$
$$= a\int \sinh^3(c+dx)\,dx + b\int \sinh^6(c+dx)\,dx$$
$$= \frac{b\cosh(c+dx)\sinh^5(c+dx)}{6d} - \frac{1}{6}(5b)\int \sinh^4(c+dx)\,dx - \frac{a\,\mathrm{Subst}\left(\int (1-\right.}{}$$
$$= -\frac{a\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} - \frac{5b\cosh(c+dx)\sinh^3(c+dx)}{24d} + \frac{b\cos}{}$$
$$= -\frac{a\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} + \frac{5b\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{5b\cos}{}$$
$$= -\frac{5bx}{16} - \frac{a\cosh(c+dx)}{d} + \frac{a\cosh^3(c+dx)}{3d} + \frac{5b\cosh(c+dx)\sinh(c+dx)}{16d} -$$

Mathematica [A] time = 0.129329, size = 66, normalized size = 0.67

$$\frac{-144a\cosh(c+dx)+16a\cosh(3(c+dx))+b(45\sinh(2(c+dx))-9\sinh(4(c+dx))+\sinh(6(c+dx))-60c-60dx)}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-144*a*Cosh[c + d*x] + 16*a*Cosh[3*(c + d*x)] + b*(-60*c - 60*d*x + 45*Sin
h[2*(c + d*x)] - 9*Sinh[4*(c + d*x)] + Sinh[6*(c + d*x)]))/(192*d)
```

Maple [A] time = 0.015, size = 72, normalized size = 0.7

$$\frac{1}{d}\left(b\left(\left(\frac{(\sinh(dx+c))^5}{6} - \frac{5(\sinh(dx+c))^3}{24} + \frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c) - \frac{5dx}{16} - \frac{5c}{16}\right) + a\left(-\frac{2}{3} + \frac{(\sinh(dx+c))^2}{3}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)
-5/16*d*x-5/16*c)+a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [A] time = 1.11576, size = 193, normalized size = 1.95

$$-\frac{1}{384}b\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d} + \frac{120(dx+c)}{d} + \frac{45e^{(-2dx-2c)}-9e^{(-4dx-4c)}+e^{(-6dx-6c)}}{d}\right) + \frac{1}{24}a\left(\frac{e^{(3dx}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/384*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d
+ 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x -
 6*c))/d) + 1/24*a*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d
+ e^(-3*d*x - 3*c)/d)
```

Fricas [A] time = 1.88429, size = 375, normalized size = 3.79

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+8\,a\cosh\left(dx+c\right)^{3}+24\,a\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2}+2\left(5\,b\cosh\left(dx+c\right)^{3}-9\,b\cosh\ldots\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*a*cosh(d*x + c)^3 + 24*a*cosh(d
*x + c)*sinh(d*x + c)^2 + 2*(5*b*cosh(d*x + c)^3 - 9*b*cosh(d*x + c))*sinh(
d*x + c)^3 - 30*b*d*x - 72*a*cosh(d*x + c) + 3*(b*cosh(d*x + c)^5 - 6*b*cos
h(d*x + c)^3 + 15*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 4.72493, size = 194, normalized size = 1.96

$$\begin{cases}\frac{a\sinh^{2}(c+dx)\cosh(c+dx)}{d}-\frac{2a\cosh^{3}(c+dx)}{3d}+\frac{5bx\sinh^{6}(c+dx)}{16}-\frac{15bx\sinh^{4}(c+dx)\cosh^{2}(c+dx)}{16}+\frac{15bx\sinh^{2}(c+dx)\cosh^{4}(c+dx)}{16}-\frac{5bx\cosh^{6}\ldots}{16}\ldots \\ x\left(a+b\sinh^{3}(c)\right)\sinh^{3}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 +
 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*cosh(c + d*x)**6/16 +
11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh(c + d*x)**3*cosh(c +
d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a
 + b*sinh(c)**3)*sinh(c)**3, True))
```

Giac [A] time = 1.15322, size = 177, normalized size = 1.79

$$-\frac{120\,(dx+c)b-be^{(6\,dx+6\,c)}+9\,be^{(4\,dx+4\,c)}-16\,ae^{(3\,dx+3\,c)}-45\,be^{(2\,dx+2\,c)}+144\,ae^{(dx+c)}+\left(144\,ae^{(5\,dx+5\,c)}+45\,be^{(4\,dx+4\,c}\ldots\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/384*(120*(d*x + c)*b - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c) - 16*a*e^
(3*d*x + 3*c) - 45*b*e^(2*d*x + 2*c) + 144*a*e^(d*x + c) + (144*a*e^(5*d*x
+ 5*c) + 45*b*e^(4*d*x + 4*c) - 16*a*e^(3*d*x + 3*c) - 9*b*e^(2*d*x + 2*c)
+ b)*e^(-6*d*x - 6*c))/d
```

3.143 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)dx$

Optimal. Leaf size=70

$$\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{ax}{2}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh(c+dx)}{d}$$

```
[Out] -(a*x)/2 + (b*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c +
d*x]^5)/(5*d) + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0708939, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3220, 2635, 8, 2633}

$$\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{ax}{2}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -(a*x)/2 + (b*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c +
d*x]^5)/(5*d) + (a*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)dx = -\int \left(-a\sinh^2(c+dx)-b\sinh^5(c+dx)\right)dx$$
$$= a\int \sinh^2(c+dx)\,dx + b\int \sinh^5(c+dx)\,dx$$
$$= \frac{a\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{1}{2}a\int 1\,dx + \frac{b\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx\right.}{d}$$
$$= -\frac{ax}{2} + \frac{b\cosh(c+dx)}{d} - \frac{2b\cosh^3(c+dx)}{3d} + \frac{b\cosh^5(c+dx)}{5d} + \frac{a\cosh(c}{}$$

Mathematica [A] time = 0.0954682, size = 79, normalized size = 1.13

$$\frac{a(-c-dx)}{2d} + \frac{a\sinh(2(c+dx))}{4d} + \frac{5b\cosh(c+dx)}{8d} - \frac{5b\cosh(3(c+dx))}{48d} + \frac{b\cosh(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (a*(-c - d*x))/(2*d) + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)])/
(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.013, size = 60, normalized size = 0.9

$$\frac{1}{d}\left(b\left(\frac{8}{15}+\frac{(\sinh(dx+c))^4}{5}-\frac{4\,(\sinh(dx+c))^2}{15}\right)\cosh(dx+c)+a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*(1/2*cosh(
d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.11831, size = 162, normalized size = 2.31

$$-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{1}{480}b\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b*(3*e^(5*d*x
 + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d -
 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Fricas [A] time = 2.0538, size = 302, normalized size = 4.31

$$\frac{3b\cosh(dx+c)^5+15b\cosh(dx+c)\sinh(dx+c)^4-25b\cosh(dx+c)^3-120adx+120a\cosh(dx+c)\sinh(dx+}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cosh
(d*x + c)^3 - 120*a*d*x + 120*a*cosh(d*x + c)*sinh(d*x + c) + 15*(2*b*cosh(
d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2 + 150*b*cosh(d*x + c))/d
```

Sympy [A] time = 2.30679, size = 117, normalized size = 1.67

$$\begin{cases} \frac{ax\sinh^2(c+dx)}{2} - \frac{ax\cosh^2(c+dx)}{2} + \frac{a\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4b\sinh^2(c+dx)\cosh^3(c+dx)}{3d} + \frac{8b\cosh^5(c+dx)}{15d} & \text{fo} \\ x\left(a+b\sinh^3(c)\right)\sinh^2(c) & \text{ot} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sinh(c + d
*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0)), (x*
(a + b*sinh(c)**3)*sinh(c)**2, True))
```

Giac [A] time = 1.16479, size = 149, normalized size = 2.13

$$-\frac{240\,(dx+c)a - 3\,be^{(5\,dx+5\,c)} + 25\,be^{(3\,dx+3\,c)} - 60\,ae^{(2\,dx+2\,c)} - 150\,be^{(dx+c)} - \left(150\,be^{(4\,dx+4\,c)} - 60\,ae^{(3\,dx+3\,c)} - 25\,be^{(2\,dx+2\,c)}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/480*(240*(d*x + c)*a - 3*b*e^(5*d*x + 5*c) + 25*b*e^(3*d*x + 3*c) - 60*a
*e^(2*d*x + 2*c) - 150*b*e^(d*x + c) - (150*b*e^(4*d*x + 4*c) - 60*a*e^(3*d
*x + 3*c) - 25*b*e^(2*d*x + 2*c) + 3*b)*e^(-5*d*x - 5*c))/d
```

3.144 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=60

$$\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

```
[Out] (3*b*x)/8 + (a*Cosh[c + d*x])/d - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) +
 (b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0684057, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3220, 2638, 2635, 8}

$$\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*b*x)/8 + (a*Cosh[c + d*x])/d - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) +
 (b*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)dx = -\left(i\int\left(ia\sinh(c+dx)+ib\sinh^4(c+dx)\right)dx\right)$$
$$= a\int\sinh(c+dx)\,dx + b\int\sinh^4(c+dx)\,dx$$
$$= \frac{a\cosh(c+dx)}{d} + \frac{b\cosh(c+dx)\sinh^3(c+dx)}{4d} - \frac{1}{4}(3b)\int\sinh^2(c+dx)\,dx$$
$$= \frac{a\cosh(c+dx)}{d} - \frac{3b\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh(c+dx)\sinh^3(c+dx}{4d}$$
$$= \frac{3bx}{8} + \frac{a\cosh(c+dx)}{d} - \frac{3b\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh(c+dx)\sinh^3}{4d}$$

Mathematica [A] time = 0.125911, size = 45, normalized size = 0.75

$$\frac{32a\cosh(c+dx)+b(12(c+dx)-8\sinh(2(c+dx))+\sinh(4(c+dx)))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (32*a*Cosh[c + d*x] + b*(12*(c + d*x) - 8*Sinh[2*(c + d*x)] + Sinh[4*(c + d
*x)]))/(32*d)
```

Maple [A] time = 0.014, size = 50, normalized size = 0.8

$$\frac{1}{d}\left(b\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3\,dx}{8}+\frac{3\,c}{8}\right)+a\cosh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a*co
sh(d*x+c))
```

Maxima [A] time = 1.15149, size = 100, normalized size = 1.67

$$\frac{1}{64}b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+\frac{a\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + a*cosh(d*x + c)/d
```

Fricas [A] time = 1.80175, size = 171, normalized size = 2.85

$$\frac{b\cosh(dx+c)\sinh(dx+c)^3+3\,bdx+8\,a\cosh(dx+c)+\left(b\cosh(dx+c)^3-4\,b\cosh(dx+c)\right)\sinh(dx+c)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + 3*b*d*x + 8*a*cosh(d*x + c) + (b*cos
h(d*x + c)^3 - 4*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 1.24827, size = 121, normalized size = 2.02

$$\begin{cases} \frac{a\cosh(c+dx)}{d} + \frac{3bx\sinh^4(c+dx)}{8} - \frac{3bx\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3bx\cosh^4(c+dx)}{8} + \frac{5b\sinh^3(c+dx)\cosh(c+dx)}{8d} - \frac{3b\sinh(c+dx)\cosh^3(c+d\ldots}{8d} \\ x\left(a + b\sinh^3(c)\right)\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + 3*b*x*sinh(c + d*x)**4/8 - 3*b*x*sinh(c + d*
x)**2*cosh(c + d*x)**2/4 + 3*b*x*cosh(c + d*x)**4/8 + 5*b*sinh(c + d*x)**3*
cosh(c + d*x)/(8*d) - 3*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)),
(x*(a + b*sinh(c)**3)*sinh(c), True))
```

Giac [A] time = 1.17231, size = 113, normalized size = 1.88

$$\frac{24\,(dx+c)b + be^{(4\,dx+4\,c)} - 8\,be^{(2\,dx+2\,c)} + 32\,ae^{(dx+c)} + \left(32\,ae^{(3\,dx+3\,c)} + 8\,be^{(2\,dx+2\,c)} - b\right)e^{(-4\,dx-4\,c)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/64*(24*(d*x + c)*b + b*e^(4*d*x + 4*c) - 8*b*e^(2*d*x + 2*c) + 32*a*e^(d*
x + c) + (32*a*e^(3*d*x + 3*c) + 8*b*e^(2*d*x + 2*c) - b)*e^(-4*d*x - 4*c))
/d
```

3.145 $\int \left(a+b \sinh^3(c+dx)\right) \, dx$

Optimal. Leaf size=32

$$ax+\frac{b \cosh^3(c+dx)}{3d}-\frac{b \cosh(c+dx)}{d}$$

```
[Out] a*x - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0211492, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 1, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {2633}

$$ax+\frac{b \cosh^3(c+dx)}{3d}-\frac{b \cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^3,x]
```

```
[Out] a*x - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)/(3*d)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \left(a+b \sinh^3(c+dx)\right) \, dx &= ax+b \int \sinh^3(c+dx) \, dx\\
&= ax-\frac{b \operatorname{Subst}\left(\int \left(1-x^2\right) \, dx,x,\cosh(c+dx)\right)}{d}\\
&= ax-\frac{b \cosh(c+dx)}{d}+\frac{b \cosh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0105271, size = 34, normalized size = 1.06

$$ax-\frac{3b \cosh(c+dx)}{4d}+\frac{b \cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^3,x]
```

```
[Out] a*x - (3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d)
```

Maple [A] time = 0.005, size = 28, normalized size = 0.9

$$ax+\frac{b \cosh(dx+c)}{d}\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^3,x)
```

```
[Out] a*x+b/d*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)
```

Maxima [A] time = 1.15456, size = 80, normalized size = 2.5

$$ax + \frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] a*x + 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-
3*d*x - 3*c)/d)
```

Fricas [A] time = 1.90505, size = 128, normalized size = 4.

$$\frac{b\cosh\left(dx+c\right)^{3} + 3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{2} + 12\,adx - 9\,b\cosh\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] 1/12*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + 12*a*d*x - 9*
b*cosh(d*x + c))/d
```

Sympy [A] time = 0.549777, size = 41, normalized size = 1.28

$$ax + b\left(\begin{cases} \frac{\sinh^{2}(c+dx)\cosh(c+dx)}{d} - \frac{2\cosh^{3}(c+dx)}{3d} & \text{for}\ d \neq 0 \\ x\sinh^{3}(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**3,x)
```

```
[Out] a*x + b*Piecewise((sinh(c + d*x)**2*cosh(c + d*x)/d - 2*cosh(c + d*x)**3/(3
*d), Ne(d, 0)), (x*sinh(c)**3, True))
```

Giac [A] time = 1.12415, size = 72, normalized size = 2.25

$$ax - \frac{\left(\left(9\,e^{(2\,dx+2\,c)} - 1\right)e^{(-3\,dx-3\,c)} - e^{(3\,dx+3\,c)} + 9\,e^{(dx+c)}\right)b}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] a*x - 1/24*((9*e^(2*d*x + 2*c) - 1)*e^(-3*d*x - 3*c) - e^(3*d*x + 3*c) + 9*
e^(d*x + c))*b/d
```

3.146 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)dx$

Optimal. Leaf size=40

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

```
[Out] -(b*x)/2 - (a*ArcTanh[Cosh[c + d*x]])/d + (b*Cosh[c + d*x]*Sinh[c + d*x])/(
2*d)
```

Rubi [A] time = 0.0559113, antiderivative size = 40, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.21, Rules used = {3220, 3770, 2635, 8}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -(b*x)/2 - (a*ArcTanh[Cosh[c + d*x]])/d + (b*Cosh[c + d*x]*Sinh[c + d*x])/(
2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \text{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)dx = i\int\left(-ia\text{csch}(c+dx)-ib\sinh^2(c+dx)\right)dx$$
$$= a\int \text{csch}(c+dx)\,dx + b\int \sinh^2(c+dx)\,dx$$
$$= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d} + \frac{b\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{1}{2}b\int 1\,dx$$
$$= -\frac{bx}{2} - \frac{a\tanh^{-1}(\cosh(c+dx))}{d} + \frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A] time = 0.0542062, size = 72, normalized size = 1.8

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} - \frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d} + \frac{b(-c-dx)}{2d} + \frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (b*(-c - d*x))/(2*d) - (a*Log[Cosh[c/2 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (
d*x)/2]])/d + (b*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.036, size = 40, normalized size = 1.

$$\frac{1}{d}\left(-2\,a\text{Artanh}\left(e^{dx+c}\right) + b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)
)
```

Maxima [A] time = 1.1445, size = 68, normalized size = 1.7

$$-\frac{1}{8}b\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + \frac{a\log\left(\tanh\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + a*log(tanh(1/2*d*x
+ 1/2*c))/d
```

Fricas [B] time = 1.99806, size = 721, normalized size = 18.02

$$-\frac{4\,bdx\cosh(dx+c)^2 - b\cosh(dx+c)^4 - 4\,b\cosh(dx+c)\sinh(dx+c)^3 - b\sinh(dx+c)^4 + 2\left(2\,bdx - 3\,b\cosh(dx+c\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/8*(4*b*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*b*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x
 + c)^2 + 8*(a*cosh(d*x + c)^2 + 2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d
*x + c)^2)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 8*(a*cosh(d*x + c)^2 +
2*a*cosh(d*x + c)*sinh(d*x + c) + a*sinh(d*x + c)^2)*log(cosh(d*x + c) + si
nh(d*x + c) - 1) + 4*(2*b*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x +
 c) + b)/(d*cosh(d*x + c)^2 + 2*d*cosh(d*x + c)*sinh(d*x + c) + d*sinh(d*x
+ c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.13918, size = 97, normalized size = 2.42

$$-\frac{(dx+c)b}{2\,d}+\frac{be^{(2\,dx+2\,c)}}{8\,d}-\frac{be^{(-2\,dx-2\,c)}}{8\,d}-\frac{a\log\left(e^{(dx+c)}+1\right)}{d}+\frac{a\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*b/d + 1/8*b*e^(2*d*x + 2*c)/d - 1/8*b*e^(-2*d*x - 2*c)/d - a
*log(e^(d*x + c) + 1)/d + a*log(abs(e^(d*x + c) - 1))/d
```

3.147 $\int \mathrm{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=24

$$\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

```
[Out] (b*Cosh[c + d*x])/d - (a*Coth[c + d*x])/d
```

Rubi [A] time = 0.0500271, antiderivative size = 24, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3220, 3767, 8, 2638}

$$\frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (b*Cosh[c + d*x])/d - (a*Coth[c + d*x])/d
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \mathrm{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx &= -\int \left(-a\,\mathrm{csch}^2(c+dx)-b\sinh(c+dx)\right)\,dx\\
&= a\int \mathrm{csch}^2(c+dx)\,dx+b\int \sinh(c+dx)\,dx\\
&= \frac{b\cosh(c+dx)}{d}-\frac{(ia)\,\mathrm{Subst}\left(\int 1\,dx,x,-i\coth(c+dx)\right)}{d}\\
&= \frac{b\cosh(c+dx)}{d}-\frac{a\coth(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0291625, size = 35, normalized size = 1.46

$$-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(c)\sinh(dx)}{d}+\frac{b\cosh(c)\cosh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Coth[c + d*x])/d + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.036, size = 23, normalized size = 1.

$$\frac{-\coth(dx+c)\,a+b\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(-coth(d*x+c)*a+b*cosh(d*x+c))
```

Maxima [A] time = 1.13484, size = 63, normalized size = 2.62

$$\frac{1}{2}\,b\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [A] time = 1.95052, size = 103, normalized size = 4.29

$$-\frac{a\cosh(dx+c)-(b\cosh(dx+c)+a)\sinh(dx+c)}{d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -(a*cosh(d*x + c) - (b*cosh(d*x + c) + a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.20409, size = 84, normalized size = 3.5

$$\frac{be^{(dx+c)}}{2\,d} + \frac{be^{(2\,dx+2\,c)} - 4\,ae^{(dx+c)} - b}{2\,d\left(e^{(3\,dx+3\,c)} - e^{(dx+c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/2*b*e^(d*x + c)/d + 1/2*(b*e^(2*d*x + 2*c) - 4*a*e^(d*x + c) - b)/(d*(e^(
3*d*x + 3*c) - e^(d*x + c)))
```

3.148 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)dx$

Optimal. Leaf size=39

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+bx$$

```
[Out] b*x + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/(2
*d)
```

Rubi [A] time = 0.0626266, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3220, 3768, 3770}

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] b*x + (a*ArcTanh[Cosh[c + d*x]])/(2*d) - (a*Coth[c + d*x]*Csch[c + d*x])/(2
*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)dx &= -\left(i\int\left(ib+ia\operatorname{csch}^3(c+dx)\right)dx\right)\\
&= bx+a\int \operatorname{csch}^3(c+dx)\,dx\\
&= bx-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}-\frac{1}{2}a\int \operatorname{csch}(c+dx)\,dx\\
&= bx+\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}
\end{aligned}$$

Mathematica [A] time = 0.0129128, size = 63, normalized size = 1.62

$$-\frac{a\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d}-\frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] b*x - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c + d*x)/2]])/(2*d) - (a
*Sech[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.04, size = 37, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{\text{csch}\,(dx+c)\coth\,(dx+c)}{2}+\text{Artanh}\left(e^{dx+c}\right)\right)+(dx+c)\,b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+(d*x+c)*b)
```

Maxima [B] time = 1.09883, size = 123, normalized size = 3.15

$$bx+\frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] b*x + 1/2*a*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x
 - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.12639, size = 1413, normalized size = 36.23

$$2\,bdx\cosh\,(dx+c)^4+2\,bdx\sinh\,(dx+c)^4-4\,bdx\cosh\,(dx+c)^2-2\,a\cosh\,(dx+c)^3+2\,(4\,bdx\cosh\,(dx+c)-a)\sinh\,($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/2*(2*b*d*x*cosh(d*x + c)^4 + 2*b*d*x*sinh(d*x + c)^4 - 4*b*d*x*cosh(d*x +
 c)^2 - 2*a*cosh(d*x + c)^3 + 2*(4*b*d*x*cosh(d*x + c) - a)*sinh(d*x + c)^3
 + 2*b*d*x + 2*(6*b*d*x*cosh(d*x + c)^2 - 2*b*d*x - 3*a*cosh(d*x + c))*sinh
(d*x + c)^2 - 2*a*cosh(d*x + c) + (a*cosh(d*x + c)^4 + 4*a*cosh(d*x + c)*si
nh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2 + 2*(3*a*cosh(d*x +
 c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*cosh(d*x + c))*sinh(d
```

```
*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d*x + c)^4 +
4*a*cosh(d*x + c)*sinh(d*x + c)^3 + a*sinh(d*x + c)^4 - 2*a*cosh(d*x + c)^2
 + 2*(3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^2 + 4*(a*cosh(d*x + c)^3 - a*c
osh(d*x + c))*sinh(d*x + c) + a)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2
*(4*b*d*x*cosh(d*x + c)^3 - 4*b*d*x*cosh(d*x + c) - 3*a*cosh(d*x + c)^2 - a
)*sinh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d
*sinh(d*x + c)^4 - 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 - d)*sinh(d
*x + c)^2 + 4*(d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16116, size = 108, normalized size = 2.77

$$\frac{(dx+c)b}{d}+\frac{a\log\left(e^{(dx+c)}+1\right)}{2\,d}-\frac{a\log\left(\left|e^{(dx+c)}-1\right|\right)}{2\,d}-\frac{ae^{(3\,dx+3\,c)}+ae^{(dx+c)}}{d\left(e^{(2\,dx+2\,c)}-1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] (d*x + c)*b/d + 1/2*a*log(e^(d*x + c) + 1)/d - 1/2*a*log(abs(e^(d*x + c) -
1))/d - (a*e^(3*d*x + 3*c) + a*e^(d*x + c))/(d*(e^(2*d*x + 2*c) - 1)^2)
```

3.149 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)\,dx$

Optimal. Leaf size=41

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

```
[Out] -((b*ArcTanh[Cosh[c + d*x]])/d) + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)
/(3*d)
```

Rubi [A] time = 0.0571616, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3220, 3770, 3767}

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}-\frac{b\tanh^{-1}(\cosh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -((b*ArcTanh[Cosh[c + d*x]])/d) + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)
/(3*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)dx &= \int \left(b\operatorname{csch}(c+dx)+a\operatorname{csch}^4(c+dx)\right)dx\\
&= a\int \operatorname{csch}^4(c+dx)\,dx+b\int \operatorname{csch}(c+dx)\,dx\\
&= -\frac{b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{(ia)\operatorname{Subst}\left(\int\left(1+x^2\right)dx,x,-i\coth(c+dx)\right)}{d}\\
&= -\frac{b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.0302817, size = 76, normalized size = 1.85

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\text{csch}^2(c+dx)}{3d}+\frac{b\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{b\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d) - (b*Lo
g[Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d
```

Maple [A] time = 0.043, size = 36, normalized size = 0.9

$$\frac{1}{d}\left(a\left(\frac{2}{3}-\frac{(\text{csch}\,(dx+c))^2}{3}\right)\coth\,(dx+c)-2\,b\text{Artanh}\left(e^{dx+c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*b*arctanh(exp(d*x+c)))
```

Maxima [B] time = 1.16159, size = 177, normalized size = 4.32

$$-b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+\frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a*(3*e^(-2*d*x
 - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)
) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.02867, size = 1773, normalized size = 43.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/3*(12*a*cosh(d*x + c)^2 + 24*a*cosh(d*x + c)*sinh(d*x + c) + 12*a*sinh(d
*x + c)^2 + 3*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*si
nh(d*x + c)^6 - 3*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x
+ c)^4 + 4*(5*b*cosh(d*x + c)^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*
cosh(d*x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*
x + c)^2 + 6*(b*cosh(d*x + c)^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*si
```

```
nh(d*x + c) - b)*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*(b*cosh(d*x + c
)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c)^6 - 3*b*cosh(d*x
+ c)^4 + 3*(5*b*cosh(d*x + c)^2 - b)*sinh(d*x + c)^4 + 4*(5*b*cosh(d*x + c)
^3 - 3*b*cosh(d*x + c))*sinh(d*x + c)^3 + 3*b*cosh(d*x + c)^2 + 3*(5*b*cosh
(d*x + c)^4 - 6*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 6*(b*cosh(d*x + c)
^5 - 2*b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) - b)*log(cosh(d*x
 + c) + sinh(d*x + c) - 1) - 4*a)/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*si
nh(d*x + c)^5 + d*sinh(d*x + c)^6 - 3*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x +
 c)^2 - d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*si
nh(d*x + c)^3 + 3*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x + c)^4 - 6*d*cosh(d*x
 + c)^2 + d)*sinh(d*x + c)^2 + 6*(d*cosh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 +
 d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.1937, size = 88, normalized size = 2.15

$$-\frac{b\log\left(e^{(dx+c)}+1\right)}{d}+\frac{b\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}-\frac{4\left(3\,ae^{(2\,dx+2\,c)}-a\right)}{3\,d\left(e^{(2\,dx+2\,c)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -b*log(e^(d*x + c) + 1)/d + b*log(abs(e^(d*x + c) - 1))/d - 4/3*(3*a*e^(2*d
*x + 2*c) - a)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.150 $\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=192

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^5(c+dx)\cosh(c+dx)}{3d}-\frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{12d}+\frac{5ab\sinh(c-}{}$$

```
[Out] (-5*a*b*x)/8 - (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x])/d + (a^2*Cosh[c
+ d*x]^3)/(3*d) - (4*b^2*Cosh[c + d*x]^3)/(3*d) + (6*b^2*Cosh[c + d*x]^5)/(
5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[c + d*x]^9)/(9*d) + (5*a*b
*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) - (5*a*b*Cosh[c + d*x]*Sinh[c + d*x]^3)
/(12*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(3*d)
```

Rubi [A] time = 0.179161, antiderivative size = 192, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 2633, 2635, 8}

$$\frac{a^2\cosh^3(c+dx)}{3d}-\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^5(c+dx)\cosh(c+dx)}{3d}-\frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{12d}+\frac{5ab\sinh(c-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-5*a*b*x)/8 - (a^2*Cosh[c + d*x])/d + (b^2*Cosh[c + d*x])/d + (a^2*Cosh[c
+ d*x]^3)/(3*d) - (4*b^2*Cosh[c + d*x]^3)/(3*d) + (6*b^2*Cosh[c + d*x]^5)/(
5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[c + d*x]^9)/(9*d) + (5*a*b
*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) - (5*a*b*Cosh[c + d*x]*Sinh[c + d*x]^3)
/(12*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x]^5)/(3*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = i\int \left(-ia^2\sinh^3(c+dx)-2iab\sinh^6(c+dx)-ib^2\sinh^9(c+dx)\right) dx$$
$$= a^2\int \sinh^3(c+dx)\,dx+(2ab)\int \sinh^6(c+dx)\,dx+b^2\int \sinh^9(c+dx)\,dx$$
$$= \frac{ab\cosh(c+dx)\sinh^5(c+dx)}{3d}-\frac{1}{3}(5ab)\int \sinh^4(c+dx)\,dx-\frac{a^2\,\mathrm{Subst}\left(\int\right.}{}$$
$$= -\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}-\frac{4b^2\cosh^3(c+dx}{3d}$$
$$= -\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}-\frac{4b^2\cosh^3(c+dx}{3d}$$
$$= -\frac{5}{8}abx-\frac{a^2\cosh(c+dx)}{d}+\frac{b^2\cosh(c+dx)}{d}+\frac{a^2\cosh^3(c+dx)}{3d}-\frac{4b^2\cosh}{3}$$

Mathematica [A] time = 0.683032, size = 125, normalized size = 0.65

$$\frac{-1890\left(32a^2-21b^2\right)\cosh(c+dx)+420\left(16a^2-21b^2\right)\cosh(3(c+dx))+b(-840a(-45\sinh(2(c+dx))+9\sinh(4(c+dx}{80640d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-1890*(32*a^2 - 21*b^2)*Cosh[c + d*x] + 420*(16*a^2 - 21*b^2)*Cosh[3*(c +
d*x)] + b*(2268*b*Cosh[5*(c + d*x)] - 405*b*Cosh[7*(c + d*x)] + 35*b*Cosh[9
*(c + d*x)] - 840*a*(60*c + 60*d*x - 45*Sinh[2*(c + d*x)] + 9*Sinh[4*(c + d
*x)] - Sinh[6*(c + d*x)])))/(80640*d)
```

Maple [A] time = 0.02, size = 128, normalized size = 0.7

$$\frac{1}{d}\left(b^2\left(\frac{128}{315}+\frac{(\sinh(dx+c))^8}{9}-\frac{8(\sinh(dx+c))^6}{63}+\frac{16(\sinh(dx+c))^4}{105}-\frac{64(\sinh(dx+c))^2}{315}\right)\cosh(dx+c)+2ab\left(\left(1/\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4
-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+
c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^2*(-2/3+1/3*sinh(d*x+
c)^2)*cosh(d*x+c))
```

Maxima [A] time = 1.18855, size = 367, normalized size = 1.91

$$-\frac{1}{161280}b^2\left(\frac{\left(405e^{(-2dx-2c)}-2268e^{(-4dx-4c)}+8820e^{(-6dx-6c)}-39690e^{(-8dx-8c)}-35\right)e^{(9dx+9c)}}{d}-\frac{39690e^{(-dx-c)}-8820}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x
 - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9
*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d - 9*
e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.91839, size = 973, normalized size = 5.07

$$\frac{35\,b^2\cosh(dx+c)^9 + 315\,b^2\cosh(dx+c)\sinh(dx+c)^8 - 405\,b^2\cosh(dx+c)^7 + 5040\,ab\cosh(dx+c)\sinh(dx+c\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 - 4
05*b^2*cosh(d*x + c)^7 + 5040*a*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2268*b^2*
cosh(d*x + c)^5 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*sinh(
d*x + c)^6 + 315*(14*b^2*cosh(d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 36*b^2*
cosh(d*x + c))*sinh(d*x + c)^4 - 50400*a*b*d*x + 420*(16*a^2 - 21*b^2)*cosh
(d*x + c)^3 + 3360*(5*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*sinh(d*x +
 c)^3 + 315*(4*b^2*cosh(d*x + c)^7 - 27*b^2*cosh(d*x + c)^5 + 72*b^2*cosh(d
*x + c)^3 + 4*(16*a^2 - 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 1890*(32*a
^2 - 21*b^2)*cosh(d*x + c) + 5040*(a*b*cosh(d*x + c)^5 - 6*a*b*cosh(d*x + c
)^3 + 15*a*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 31.0145, size = 325, normalized size = 1.69

$$\begin{cases} \frac{a^2\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2\cosh^3(c+dx)}{3d} + \frac{5abx\sinh^6(c+dx)}{8} - \frac{15abx\sinh^4(c+dx)\cosh^2(c+dx)}{8} + \frac{15abx\sinh^2(c+dx)\cosh^4(c+dx)}{8} - \frac{5ab\cdots}{} \\ x\left(a+b\sinh^3(c)\right)^2\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh(c + d*x)**4*cosh(c + d*x
)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/8 - 5*a*b*x*cosh(c + d*
x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8*d) - 5*a*b*sinh(c + d*x)
**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*cosh(c + d*x)**5/(8*d) + b
**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(c + d*x
)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b**2*sinh
(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(315*d), N
e(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c)**3, True))
```

Giac [A] time = 1.26917, size = 369, normalized size = 1.92

$$-\frac{100800\,(dx+c)ab - 35\,b^2e^{(9\,dx+9\,c)} + 405\,b^2e^{(7\,dx+7\,c)} - 840\,abe^{(6\,dx+6\,c)} - 2268\,b^2e^{(5\,dx+5\,c)} + 7560\,abe^{(4\,dx+4\,c)} - 6720\,a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/161280*(100800*(d*x + c)*a*b - 35*b^2*e^(9*d*x + 9*c) + 405*b^2*e^(7*d*x
 + 7*c) - 840*a*b*e^(6*d*x + 6*c) - 2268*b^2*e^(5*d*x + 5*c) + 7560*a*b*e^(
4*d*x + 4*c) - 6720*a^2*e^(3*d*x + 3*c) + 8820*b^2*e^(3*d*x + 3*c) - 37800*
a*b*e^(2*d*x + 2*c) + 60480*a^2*e^(d*x + c) - 39690*b^2*e^(d*x + c) + (3780
0*a*b*e^(7*d*x + 7*c) - 7560*a*b*e^(5*d*x + 5*c) - 2268*b^2*e^(4*d*x + 4*c)
 + 840*a*b*e^(3*d*x + 3*c) + 405*b^2*e^(2*d*x + 2*c) - 35*b^2 + 1890*(32*a^
2 - 21*b^2)*e^(8*d*x + 8*c) - 420*(16*a^2 - 21*b^2)*e^(6*d*x + 6*c))*e^(-9*
d*x - 9*c))/d
```

3.151 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=180

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2x}{2}+\frac{2ab\cosh^5(c+dx)}{5d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^7(c+dx)}{8d}$$

```
[Out] -(a^2*x)/2 + (35*b^2*x)/128 + (2*a*b*Cosh[c + d*x])/d - (4*a*b*Cosh[c + d*x
]^3)/(3*d) + (2*a*b*Cosh[c + d*x]^5)/(5*d) + (a^2*Cosh[c + d*x]*Sinh[c + d*
x])/(2*d) - (35*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^2*Cosh[c +
 d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*
d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)
```

Rubi [A] time = 0.154268, antiderivative size = 180, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 2635, 8, 2633}

$$\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^2x}{2}+\frac{2ab\cosh^5(c+dx)}{5d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^7(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```


```
[Out] -(a^2*x)/2 + (35*b^2*x)/128 + (2*a*b*Cosh[c + d*x])/d - (4*a*b*Cosh[c + d*x
]^3)/(3*d) + (2*a*b*Cosh[c + d*x]^5)/(5*d) + (a^2*Cosh[c + d*x]*Sinh[c + d*
x])/(2*d) - (35*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (35*b^2*Cosh[c +
 d*x]*Sinh[c + d*x]^3)/(192*d) - (7*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*
d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^7)/(8*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = -\int \left(-a^2\sinh^2(c+dx) - 2ab\sinh^5(c+dx) - b^2\sinh^8(c+dx)\right) dx$$
$$= a^2\int \sinh^2(c+dx)\,dx + (2ab)\int \sinh^5(c+dx)\,dx + b^2\int \sinh^8(c+dx)\,dx$$
$$= \frac{a^2\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{b^2\cosh(c+dx)\sinh^7(c+dx)}{8d} - \frac{1}{2}a^2\int 1\,dx$$
$$= -\frac{a^2x}{2} + \frac{2ab\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + \frac{2ab\cosh^5(c+dx)}{5d} + \frac{a^2\cosh}{}$$
$$= -\frac{a^2x}{2} + \frac{2ab\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + \frac{2ab\cosh^5(c+dx)}{5d} + \frac{a^2\cosh}{}$$
$$= -\frac{a^2x}{2} + \frac{2ab\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + \frac{2ab\cosh^5(c+dx)}{5d} + \frac{a^2\cosh}{}$$
$$= -\frac{a^2x}{2} + \frac{35b^2x}{128} + \frac{2ab\cosh(c+dx)}{d} - \frac{4ab\cosh^3(c+dx)}{3d} + \frac{2ab\cosh^5(c+dx)}{5d}$$

Mathematica [A] time = 0.129859, size = 133, normalized size = 0.74

$$\frac{3840a^2\sinh(2(c+dx)) - 7680a^2c - 7680a^2dx + 19200ab\cosh(c+dx) - 3200ab\cosh(3(c+dx)) + 384ab\cosh(5(c+dx)}{153}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-7680*a^2*c + 4200*b^2*c - 7680*a^2*d*x + 4200*b^2*d*x + 19200*a*b*Cosh[c
+ d*x] - 3200*a*b*Cosh[3*(c + d*x)] + 384*a*b*Cosh[5*(c + d*x)] + 3840*a^2*
Sinh[2*(c + d*x)] - 3360*b^2*Sinh[2*(c + d*x)] + 840*b^2*Sinh[4*(c + d*x)]
- 160*b^2*Sinh[6*(c + d*x)] + 15*b^2*Sinh[8*(c + d*x)])/(15360*d)
```

Maple [A] time = 0.021, size = 122, normalized size = 0.7

$$\frac{1}{d}\left(b^2\left(\left(\frac{(\sinh(dx+c))^7}{8} - \frac{7(\sinh(dx+c))^5}{48} + \frac{35(\sinh(dx+c))^3}{192} - \frac{35\sinh(dx+c)}{128}\right)\cosh(dx+c) + \frac{35dx}{128} + \frac{35c}{128}\right) +$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128
*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+2*a*b*(8/15+1/5*sinh(d*x+c)^
4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-
1/2*c))
```

Maxima [A] time = 1.15235, size = 320, normalized size = 1.78

$$-\frac{1}{8}a^2\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{6144}b^2\left(\frac{\left(32e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 672e^{(-6dx-6c)} - 3\right)e^{(8dx+8c)}}{d} - \frac{1680(dx+}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^2*((32*e
^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x
 + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c)
 + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5
*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*
e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Fricas [A] time = 1.90907, size = 734, normalized size = 4.08

$$15\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+48\,ab\cosh\left(dx+c\right)^5+240\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4+15\left(7\,b^2\cosh\left(dx+c\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/1920*(15*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 48*a*b*cosh(d*x + c)^5 + 240
*a*b*cosh(d*x + c)*sinh(d*x + c)^4 + 15*(7*b^2*cosh(d*x + c)^3 - 8*b^2*cosh
(d*x + c))*sinh(d*x + c)^5 - 400*a*b*cosh(d*x + c)^3 + 5*(21*b^2*cosh(d*x +
 c)^5 - 80*b^2*cosh(d*x + c)^3 + 84*b^2*cosh(d*x + c))*sinh(d*x + c)^3 - 15
*(64*a^2 - 35*b^2)*d*x + 2400*a*b*cosh(d*x + c) + 240*(2*a*b*cosh(d*x + c)^
3 - 5*a*b*cosh(d*x + c))*sinh(d*x + c)^2 + 15*(b^2*cosh(d*x + c)^7 - 8*b^2*
cosh(d*x + c)^5 + 28*b^2*cosh(d*x + c)^3 + 8*(8*a^2 - 7*b^2)*cosh(d*x + c))
*sinh(d*x + c))/d
```

Sympy [A] time = 15.3381, size = 340, normalized size = 1.89

$$\begin{cases} \frac{a^2x\sinh^2(c+dx)}{2}-\frac{a^2x\cosh^2(c+dx)}{2}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{16ab\cosh^5(}{15d} \\ x\left(a+b\sinh^3(c)\right)^2\sinh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d - 8
*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15*
d) + 35*b**2*x*sinh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d
*x)**2/32 + 105*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sin
h(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**
2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c
 + d*x)**3/(384*d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 3
5*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)
**3)**2*sinh(c)**2, True))
```

Giac [A] time = 1.25071, size = 317, normalized size = 1.76

$$15\,b^2e^{(8\,dx+8\,c)}-160\,b^2e^{(6\,dx+6\,c)}+384\,abe^{(5\,dx+5\,c)}+840\,b^2e^{(4\,dx+4\,c)}-3200\,abe^{(3\,dx+3\,c)}+3840\,a^2e^{(2\,dx+2\,c)}-3360\,b^2e^{(2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/30720*(15*b^2*e^(8*d*x + 8*c) - 160*b^2*e^(6*d*x + 6*c) + 384*a*b*e^(5*d*
x + 5*c) + 840*b^2*e^(4*d*x + 4*c) - 3200*a*b*e^(3*d*x + 3*c) + 3840*a^2*e^
(2*d*x + 2*c) - 3360*b^2*e^(2*d*x + 2*c) + 19200*a*b*e^(d*x + c) - 240*(64*
a^2 - 35*b^2)*(d*x + c) + (19200*a*b*e^(7*d*x + 7*c) - 3200*a*b*e^(5*d*x +
5*c) - 840*b^2*e^(4*d*x + 4*c) + 384*a*b*e^(3*d*x + 3*c) + 160*b^2*e^(2*d*x
 + 2*c) - 15*b^2 - 480*(8*a^2 - 7*b^2)*e^(6*d*x + 6*c))*e^(-8*d*x - 8*c))/d
```

3.152 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=130

$$\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^3(c+dx)\cosh(c+dx)}{2d}-\frac{3ab\sinh(c+dx)\cosh(c+dx)}{4d}+\frac{3abx}{4}+\frac{b^2\cosh^7(c+dx)}{7d}-3b^2\,c$$

```
[Out] (3*a*b*x)/4 + (a^2*Cosh[c + d*x])/d - (b^2*Cosh[c + d*x])/d + (b^2*Cosh[c +
 d*x]^3)/d - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d) -
(3*a*b*Cosh[c + d*x]*Sinh[c + d*x])/(4*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x
]^3)/(2*d)
```

Rubi [A] time = 0.118934, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3220, 2638, 2635, 8, 2633}

$$\frac{a^2\cosh(c+dx)}{d}+\frac{ab\sinh^3(c+dx)\cosh(c+dx)}{2d}-\frac{3ab\sinh(c+dx)\cosh(c+dx)}{4d}+\frac{3abx}{4}+\frac{b^2\cosh^7(c+dx)}{7d}-3b^2\,c$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*a*b*x)/4 + (a^2*Cosh[c + d*x])/d - (b^2*Cosh[c + d*x])/d + (b^2*Cosh[c +
 d*x]^3)/d - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]^7)/(7*d) -
(3*a*b*Cosh[c + d*x]*Sinh[c + d*x])/(4*d) + (a*b*Cosh[c + d*x]*Sinh[c + d*x
]^3)/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = -\left(i\int \left(ia^2\sinh(c+dx)+2iab\sinh^4(c+dx)+ib^2\sinh^7(c+dx)\right)dx\right)$$
$$= a^2\int \sinh(c+dx)\,dx+(2ab)\int \sinh^4(c+dx)\,dx+b^2\int \sinh^7(c+dx)\,dx$$
$$= \frac{a^2\cosh(c+dx)}{d}+\frac{ab\cosh(c+dx)\sinh^3(c+dx)}{2d}-\frac{1}{2}(3ab)\int \sinh^2(c+dx)$$
$$= \frac{a^2\cosh(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{d}-\frac{3b^2\cosh^5(c+dx)}{5d}$$
$$= \frac{3abx}{4}+\frac{a^2\cosh(c+dx)}{d}-\frac{b^2\cosh(c+dx)}{d}+\frac{b^2\cosh^3(c+dx)}{d}-\frac{3b^2\cosh^5(}{5d}$$

Mathematica [A] time = 0.40963, size = 92, normalized size = 0.71

$$\frac{35\left(64a^2-35b^2\right)\cosh(c+dx)+b(140a(12(c+dx)-8\sinh(2(c+dx))+\sinh(4(c+dx)))+245b\cosh(3(c+dx))-49b}{2240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (35*(64*a^2 - 35*b^2)*Cosh[c + d*x] + b*(245*b*Cosh[3*(c + d*x)] - 49*b*Cos
h[5*(c + d*x)] + 5*b*Cosh[7*(c + d*x)] + 140*a*(12*(c + d*x) - 8*Sinh[2*(c
+ d*x)] + Sinh[4*(c + d*x)])))/(2240*d)
```

Maple [A] time = 0.019, size = 96, normalized size = 0.7

$$\frac{1}{d}\left(b^2\left(-\frac{16}{35}+\frac{(\sinh(dx+c))^6}{7}-\frac{6(\sinh(dx+c))^4}{35}+\frac{8(\sinh(dx+c))^2}{35}\right)\cosh(dx+c)+2ab\left(\left(1/4(\sinh(dx+c))^3-3/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*c
osh(d*x+c)+2*a*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3
/8*c)+a^2*cosh(d*x+c))
```

Maxima [A] time = 1.07251, size = 243, normalized size = 1.87

$$\frac{1}{32}ab\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{4480}b^2\left(\frac{\left(49e^{(-2dx-2c)}-245e^{(-4dx-4c)}+1225e^{(-6dx-}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d
*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x -
```

```
c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) +
a^2*cosh(d*x + c)/d
```

Fricas [A] time = 1.90338, size = 594, normalized size = 4.57

$$5\,b^2\cosh\left(dx+c\right)^7+35\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-49\,b^2\cosh\left(dx+c\right)^5+560\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)^3+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^2*cosh(d*x + c)^7 + 35*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 49*b
^2*cosh(d*x + c)^5 + 560*a*b*cosh(d*x + c)*sinh(d*x + c)^3 + 245*b^2*cosh(d
*x + c)^3 + 35*(5*b^2*cosh(d*x + c)^3 - 7*b^2*cosh(d*x + c))*sinh(d*x + c)^
4 + 1680*a*b*d*x + 35*(3*b^2*cosh(d*x + c)^5 - 14*b^2*cosh(d*x + c)^3 + 21*
b^2*cosh(d*x + c))*sinh(d*x + c)^2 + 35*(64*a^2 - 35*b^2)*cosh(d*x + c) + 5
60*(a*b*cosh(d*x + c)^3 - 4*a*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 8.4291, size = 219, normalized size = 1.68

$$\begin{cases}\frac{a^2\cosh\left(c+dx\right)}{d}+\frac{3abx\sinh^4\left(c+dx\right)}{4}-\frac{3abx\sinh^2\left(c+dx\right)\cosh^2\left(c+dx\right)}{2}+\frac{3abx\cosh^4\left(c+dx\right)}{4}+\frac{5ab\sinh^3\left(c+dx\right)\cosh\left(c+dx\right)}{4d}-\frac{3ab\sinh\left(c+dx\right)\cosh\ldots}{4d} \\ x\left(a+b\sinh^3\left(c\right)\right)^2\sinh\left(c\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*cosh(c + d*x)/d + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh
(c + d*x)**2*cosh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c
 + d*x)**3*cosh(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d)
 + b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b**2*sinh(c + d*x)**4*cosh(c +
 d*x)**3/d + 8*b**2*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b**2*cosh(
c + d*x)**7/(35*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2*sinh(c), True))
```

Giac [A] time = 1.20251, size = 266, normalized size = 2.05

$$3360\left(dx+c\right)ab+5\,b^2e^{\left(7\,dx+7\,c\right)}-49\,b^2e^{\left(5\,dx+5\,c\right)}+140\,abe^{\left(4\,dx+4\,c\right)}+245\,b^2e^{\left(3\,dx+3\,c\right)}-1120\,abe^{\left(2\,dx+2\,c\right)}+2240\,a^2e^{\left(dx+c\right)}\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/4480*(3360*(d*x + c)*a*b + 5*b^2*e^(7*d*x + 7*c) - 49*b^2*e^(5*d*x + 5*c)
 + 140*a*b*e^(4*d*x + 4*c) + 245*b^2*e^(3*d*x + 3*c) - 1120*a*b*e^(2*d*x +
2*c) + 2240*a^2*e^(d*x + c) - 1225*b^2*e^(d*x + c) + (1120*a*b*e^(5*d*x + 5
*c) + 245*b^2*e^(4*d*x + 4*c) - 140*a*b*e^(3*d*x + 3*c) - 49*b^2*e^(2*d*x +
 2*c) + 5*b^2 + 35*(64*a^2 - 35*b^2)*e^(6*d*x + 6*c))*e^(-7*d*x - 7*c))/d
```

3.153 $\int \left(a+b\sinh^3(c+dx)\right)^2 \, dx$

Optimal. Leaf size=114

$$a^2x+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b^2\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b^2\sin}{}$$

```
[Out] a^2*x - (5*b^2*x)/16 - (2*a*b*Cosh[c + d*x])/d + (2*a*b*Cosh[c + d*x]^3)/(3
*d) + (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^2*Cosh[c + d*x]*Sin
h[c + d*x]^3)/(24*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.0832559, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2633, 2635, 8}

$$a^2x+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^5(c+dx)\cosh(c+dx)}{6d}-\frac{5b^2\sinh^3(c+dx)\cosh(c+dx)}{24d}+\frac{5b^2\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] a^2*x - (5*b^2*x)/16 - (2*a*b*Cosh[c + d*x])/d + (2*a*b*Cosh[c + d*x]^3)/(3
*d) + (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^2*Cosh[c + d*x]*Sin
h[c + d*x]^3)/(24*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\sinh^3(c+dx)\right)^2 dx = \int \left(a^2+2ab\sinh^3(c+dx)+b^2\sinh^6(c+dx)\right) dx$$
$$= a^2x+(2ab)\int \sinh^3(c+dx)\,dx+b^2\int \sinh^6(c+dx)\,dx$$
$$= a^2x+\frac{b^2\cosh(c+dx)\sinh^5(c+dx)}{6d}-\frac{1}{6}\left(5b^2\right)\int \sinh^4(c+dx)\,dx-\frac{(2ab)\,\mathrm{Subst}\left(\int\left(\right.\right.}{}$$
$$= a^2x-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}-\frac{5b^2\cosh(c+dx)\sinh^3(c+dx)}{24d}+\frac{b^2\,\mathrm{co}}{}$$
$$= a^2x-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}+\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{16d}-\frac{5b^2\,\mathrm{co}}{}$$
$$= a^2x-\frac{5b^2x}{16}-\frac{2ab\cosh(c+dx)}{d}+\frac{2ab\cosh^3(c+dx)}{3d}+\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{16d}$$

Mathematica [A] time = 0.146479, size = 94, normalized size = 0.82

$$\frac{192a^2c+192a^2dx-288ab\cosh(c+dx)+32ab\cosh(3(c+dx))+45b^2\sinh(2(c+dx))-9b^2\sinh(4(c+dx))+b^2\,\mathrm{sin}}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (192*a^2*c - 60*b^2*c + 192*a^2*d*x - 60*b^2*d*x - 288*a*b*Cosh[c + d*x] +
32*a*b*Cosh[3*(c + d*x)] + 45*b^2*Sinh[2*(c + d*x)] - 9*b^2*Sinh[4*(c + d*x
)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.017, size = 85, normalized size = 0.8

$$\frac{1}{d}\left(b^2\left(\left(\frac{(\sinh(dx+c))^5}{6}-\frac{5\,(\sinh(dx+c))^3}{24}+\frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c)-\frac{5\,dx}{16}-\frac{5\,c}{16}\right)+2\,ab\left(-2/3+1/3\,(\sinh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+
c)-5/16*d*x-5/16*c)+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*(d*x+c))
```

Maxima [A] time = 1.14157, size = 204, normalized size = 1.79

$$a^2x-\frac{1}{384}b^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}-9\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}}{d}\right)+\frac{1}{12}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] a^2*x - 1/384*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x
+ 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-
```

```
d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [A] time = 1.98662, size = 421, normalized size = 3.69

$$\frac{3\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^5+16\,ab\cosh\left(dx+c\right)^3+48\,ab\cosh\left(dx+c\right)\sinh\left(dx+c\right)^2+2\left(5\,b^2\cosh\left(dx+c\right)^3-9\,b^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 16*a*b*cosh(d*x + c)^3 + 48*a*b
*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(5*b^2*cosh(d*x + c)^3 - 9*b^2*cosh(d*x
+ c))*sinh(d*x + c)^3 + 6*(16*a^2 - 5*b^2)*d*x - 144*a*b*cosh(d*x + c) + 3*
(b^2*cosh(d*x + c)^5 - 6*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c))*sinh(d
*x + c))/d
```

Sympy [A] time = 5.48219, size = 212, normalized size = 1.86

$$\begin{cases} a^2x+\frac{2ab\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{4ab\cosh^3(c+dx)}{3d}+\frac{5b^2x\sinh^6(c+dx)}{16}-\frac{15b^2x\sinh^4(c+dx)\cosh^2(c+dx)}{16}+\frac{15b^2x\sinh^2(c+dx)\cosh^4(c+dx)}{16}- \\ x\left(a+b\sinh^3(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Piecewise((a**2*x + 2*a*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 4*a*b*cosh(c +
 d*x)**3/(3*d) + 5*b**2*x*sinh(c + d*x)**6/16 - 15*b**2*x*sinh(c + d*x)**4*
cosh(c + d*x)**2/16 + 15*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*
*2*x*cosh(c + d*x)**6/16 + 11*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) -
5*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b**2*sinh(c + d*x)*cosh(
c + d*x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**2, True))
```

Giac [A] time = 1.12821, size = 212, normalized size = 1.86

$$\frac{b^2e^{(6\,dx+6\,c)}-9\,b^2e^{(4\,dx+4\,c)}+32\,abe^{(3\,dx+3\,c)}+45\,b^2e^{(2\,dx+2\,c)}-288\,abe^{(dx+c)}+24\left(16\,a^2-5\,b^2\right)(dx+c)-\left(288\,abe^{(5\,dx+5\,c)}+\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/384*(b^2*e^(6*d*x + 6*c) - 9*b^2*e^(4*d*x + 4*c) + 32*a*b*e^(3*d*x + 3*c)
 + 45*b^2*e^(2*d*x + 2*c) - 288*a*b*e^(d*x + c) + 24*(16*a^2 - 5*b^2)*(d*x
+ c) - (288*a*b*e^(5*d*x + 5*c) + 45*b^2*e^(4*d*x + 4*c) - 32*a*b*e^(3*d*x
+ 3*c) - 9*b^2*e^(2*d*x + 2*c) + b^2)*e^(-6*d*x - 6*c))/d
```

3.154 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx$

Optimal. Leaf size=88

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\sinh(c+dx)\cosh(c+dx)}{d}-abx+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh(c$$

```
[Out] -(a*b*x) - (a^2*ArcTanh[Cosh[c + d*x]])/d + (b^2*Cosh[c + d*x])/d - (2*b^2*
Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) + (a*b*Cosh[c + d*x]*S
inh[c + d*x])/d
```

Rubi [A] time = 0.101747, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3220, 3770, 2635, 8, 2633}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\sinh(c+dx)\cosh(c+dx)}{d}-abx+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c+dx)}{3d}+\frac{b^2\cosh(c$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] -(a*b*x) - (a^2*ArcTanh[Cosh[c + d*x]])/d + (b^2*Cosh[c + d*x])/d - (2*b^2*
Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) + (a*b*Cosh[c + d*x]*S
inh[c + d*x])/d
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = i\int \left(-ia^2\operatorname{csch}(c+dx)-2iab\sinh^2(c+dx)-ib^2\sinh^5(c+dx)\right)dx$$
$$= a^2\int \operatorname{csch}(c+dx)\,dx+(2ab)\int \sinh^2(c+dx)\,dx+b^2\int \sinh^5(c+dx)\,dx$$
$$= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{ab\cosh(c+dx)\sinh(c+dx)}{d}-(ab)\int 1\,dx+$$
$$= -abx-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\cosh(c+dx)}{d}-\frac{2b^2\cosh^3(c+dx)}{3d}+$$

Mathematica [A] time = 0.301818, size = 96, normalized size = 1.09

$$\frac{120a\left(b\sinh(2(c+dx))-2\left(-a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+bc+bdx\right)\right)+150b^2\cosh(c+dx)-2}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (150*b^2*Cosh[c + d*x] - 25*b^2*Cosh[3*(c + d*x)] + 3*b^2*Cosh[5*(c + d*x)]
 + 120*a*(-2*(b*c + b*d*x + a*Log[Cosh[(c + d*x)/2]] - a*Log[Sinh[(c + d*x)
/2]]) + b*Sinh[2*(c + d*x)]))/(240*d)
```

Maple [A] time = 0.056, size = 76, normalized size = 0.9

$$\frac{1}{d}\left(-2a^2\operatorname{Artanh}\left(e^{dx+c}\right)+2ab\left(1/2\cosh(dx+c)\sinh(dx+c)-1/2dx-c/2\right)+b^2\left(\frac{8}{15}+\frac{(\sinh(dx+c))^4}{5}-\frac{4(\sinh(dx}{15}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-
1/2*c)+b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [A] time = 1.17573, size = 189, normalized size = 2.15

$$-\frac{1}{4}ab\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{1}{480}b^2\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b^2*(3*e^(5
*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)
/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^2*log(tanh(1/2*d*x +
 1/2*c))/d
```

Fricas [B] time = 2.16028, size = 2851, normalized size = 32.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^2*cosh(d*x + c)^10 + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
2*sinh(d*x + c)^10 - 25*b^2*cosh(d*x + c)^8 - 480*a*b*d*x*cosh(d*x + c)^5 +
 120*a*b*cosh(d*x + c)^7 + 5*(27*b^2*cosh(d*x + c)^2 - 5*b^2)*sinh(d*x + c)
^8 + 150*b^2*cosh(d*x + c)^6 + 40*(9*b^2*cosh(d*x + c)^3 - 5*b^2*cosh(d*x +
 c) + 3*a*b)*sinh(d*x + c)^7 + 10*(63*b^2*cosh(d*x + c)^4 - 70*b^2*cosh(d*x
 + c)^2 + 84*a*b*cosh(d*x + c) + 15*b^2)*sinh(d*x + c)^6 + 150*b^2*cosh(d*x
 + c)^4 + 4*(189*b^2*cosh(d*x + c)^5 - 350*b^2*cosh(d*x + c)^3 - 120*a*b*d*
x + 630*a*b*cosh(d*x + c)^2 + 225*b^2*cosh(d*x + c))*sinh(d*x + c)^5 - 120*
a*b*cosh(d*x + c)^3 + 10*(63*b^2*cosh(d*x + c)^6 - 175*b^2*cosh(d*x + c)^4
- 240*a*b*d*x*cosh(d*x + c) + 420*a*b*cosh(d*x + c)^3 + 225*b^2*cosh(d*x +
c)^2 + 15*b^2)*sinh(d*x + c)^4 - 25*b^2*cosh(d*x + c)^2 + 40*(9*b^2*cosh(d*
x + c)^7 - 35*b^2*cosh(d*x + c)^5 - 120*a*b*d*x*cosh(d*x + c)^2 + 105*a*b*c
osh(d*x + c)^4 + 75*b^2*cosh(d*x + c)^3 + 15*b^2*cosh(d*x + c) - 3*a*b)*sin
h(d*x + c)^3 + 5*(27*b^2*cosh(d*x + c)^8 - 140*b^2*cosh(d*x + c)^6 - 960*a*
b*d*x*cosh(d*x + c)^3 + 504*a*b*cosh(d*x + c)^5 + 450*b^2*cosh(d*x + c)^4 +
 180*b^2*cosh(d*x + c)^2 - 72*a*b*cosh(d*x + c) - 5*b^2)*sinh(d*x + c)^2 +
3*b^2 - 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^4*sinh(d*x + c) + 10
*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*x + c)^2*sinh(d*x + c)
^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d*x + c)^5)*log(cosh(d*
x + c) + sinh(d*x + c) + 1) + 480*(a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c
)^4*sinh(d*x + c) + 10*a^2*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a^2*cosh(d*
x + c)^2*sinh(d*x + c)^3 + 5*a^2*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*sinh(d
*x + c)^5)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(3*b^2*cosh(d*x + c)
^9 - 20*b^2*cosh(d*x + c)^7 - 240*a*b*d*x*cosh(d*x + c)^4 + 84*a*b*cosh(d*x
 + c)^6 + 90*b^2*cosh(d*x + c)^5 + 60*b^2*cosh(d*x + c)^3 - 36*a*b*cosh(d*x
 + c)^2 - 5*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh
(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*d*cos
h(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d
*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25366, size = 240, normalized size = 2.73

$$-\frac{(dx+c)ab}{d}-\frac{a^2\log\left(e^{(dx+c)}+1\right)}{d}+\frac{a^2\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}+\frac{\left(150\,b^2e^{(4\,dx+4\,c)}-120\,abe^{(3\,dx+3\,c)}-25\,b^2e^{(2\,dx+2\,c)}+3\,b^2\right)e^{(}}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -(d*x + c)*a*b/d - a^2*log(e^(d*x + c) + 1)/d + a^2*log(abs(e^(d*x + c) - 1
))/d + 1/480*(150*b^2*e^(4*d*x + 4*c) - 120*a*b*e^(3*d*x + 3*c) - 25*b^2*e^
(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c)/d + 1/480*(3*b^2*d^4*e^(5*d*x + 5*c
) - 25*b^2*d^4*e^(3*d*x + 3*c) + 120*a*b*d^4*e^(2*d*x + 2*c) + 150*b^2*d^4*
e^(d*x + c))/d^5
```

3.155 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=82

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3b^2x}{8}$$

```
[Out] (3*b^2*x)/8 + (2*a*b*Cosh[c + d*x])/d - (a^2*Coth[c + d*x])/d - (3*b^2*Cosh
[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0979725, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3767, 8, 2638, 2635}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{2ab\cosh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{3b^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{3b^2x}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*b^2*x)/8 + (2*a*b*Cosh[c + d*x])/d - (a^2*Coth[c + d*x])/d - (3*b^2*Cosh
[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = -\int \left(-a^2\operatorname{csch}^2(c+dx)-2ab\sinh(c+dx)-b^2\sinh^4(c+dx)\right) dx$$
$$= a^2\int \operatorname{csch}^2(c+dx)\,dx + (2ab)\int \sinh(c+dx)\,dx + b^2\int \sinh^4(c+dx)\,dx$$
$$= \frac{2ab\cosh(c+dx)}{d} + \frac{b^2\cosh(c+dx)\sinh^3(c+dx)}{4d} - \frac{1}{4}\left(3b^2\right)\int \sinh^2(c+d$$
$$= \frac{2ab\cosh(c+dx)}{d} - \frac{a^2\coth(c+dx)}{d} - \frac{3b^2\cosh(c+dx)\sinh(c+dx)}{8d} + b^2 \,c$$
$$= \frac{3b^2x}{8} + \frac{2ab\cosh(c+dx)}{d} - \frac{a^2\coth(c+dx)}{d} - \frac{3b^2\cosh(c+dx)\sinh(c+dx}{8d}$$

Mathematica [A] time = 0.284753, size = 92, normalized size = 1.12

$$-\frac{a^2\coth(c+dx)}{d} + \frac{2ab\sinh(c)\sinh(dx)}{d} + \frac{2ab\cosh(c)\cosh(dx)}{d} + \frac{3b^2(c+dx)}{8d} - \frac{b^2\sinh(2(c+dx))}{4d} + \frac{b^2\sinh(4(c+dx}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (3*b^2*(c + d*x))/(8*d) + (2*a*b*Cosh[c]*Cosh[d*x])/d - (a^2*Coth[c + d*x])
/d + (2*a*b*Sinh[c]*Sinh[d*x])/d - (b^2*Sinh[2*(c + d*x)])/(4*d) + (b^2*Sin
h[4*(c + d*x)])/(32*d)
```

Maple [A] time = 0.046, size = 65, normalized size = 0.8

$$\frac{1}{d}\left(-a^2\coth(dx+c) + 2ab\cosh(dx+c) + b^2\left(\left(\frac{(\sinh(dx+c))^3}{4} - \frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3dx}{8} + \frac{3c}{8}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*cosh(d*x+c)+b^2*((1/4*sinh(d*x+c)^3-3/8*sinh(d*
x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.07606, size = 153, normalized size = 1.87

$$\frac{1}{64}b^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + ab\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{2a^2}{d\left(e^{(-2dx-2c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^2/(
d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [A] time = 1.91237, size = 360, normalized size = 4.39

$$\frac{b^2\cosh(dx+c)^5+5\,b^2\cosh(dx+c)\sinh(dx+c)^4-9\,b^2\cosh(dx+c)^3+\left(10\,b^2\cosh(dx+c)^3-27\,b^2\cosh(dx+c)\right.}{64\,d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/64*(b^2*cosh(d*x + c)^5 + 5*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 9*b^2*cos
h(d*x + c)^3 + (10*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c))*sinh(d*x + c
)^2 - 8*(8*a^2 - b^2)*cosh(d*x + c) + 8*(3*b^2*d*x + 16*a*b*cosh(d*x + c) +
 8*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22934, size = 224, normalized size = 2.73

$$\frac{3\,(dx+c)b^2}{8\,d}+\frac{\left(64\,abe^{(5\,dx+5\,c)}-64\,abe^{(3\,dx+3\,c)}-9\,b^2e^{(2\,dx+2\,c)}+b^2-8\left(16\,a^2-b^2\right)e^{(4\,dx+4\,c)}\right)e^{(-4\,dx-4\,c)}}{64\,d\left(e^{(dx+c)}+1\right)\left(e^{(dx+c)}-1\right)}+\frac{b^2d^3e^{(4\,dx+4\,c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 3/8*(d*x + c)*b^2/d + 1/64*(64*a*b*e^(5*d*x + 5*c) - 64*a*b*e^(3*d*x + 3*c)
 - 9*b^2*e^(2*d*x + 2*c) + b^2 - 8*(16*a^2 - b^2)*e^(4*d*x + 4*c))*e^(-4*d*
x - 4*c)/(d*(e^(d*x + c) + 1)*(e^(d*x + c) - 1)) + 1/64*(b^2*d^3*e^(4*d*x +
 4*c) - 8*b^2*d^3*e^(2*d*x + 2*c) + 64*a*b*d^3*e^(d*x + c))/d^4
```

3.156 $\int \mathrm{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=77

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\mathrm{csch}(c+dx)}{2d}+2abx+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{b^2\cosh(c+dx)}{d}$$

```
[Out] 2*a*b*x + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^2*Cosh[c + d*x])/d + (b^2
*Cosh[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rubi [A] time = 0.105334, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 3768, 3770, 2633}

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\mathrm{csch}(c+dx)}{2d}+2abx+\frac{b^2\cosh^3(c+dx)}{3d}-\frac{b^2\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] 2*a*b*x + (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^2*Cosh[c + d*x])/d + (b^2
*Cosh[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \text{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = -\left(i\int \left(2iab+ia^2\text{csch}^3(c+dx)+ib^2\sinh^3(c+dx)\right) dx\right)$$
$$= 2abx + a^2\int \text{csch}^3(c+dx)\,dx + b^2\int \sinh^3(c+dx)\,dx$$
$$= 2abx - \frac{a^2\coth(c+dx)\text{csch}(c+dx)}{2d} - \frac{1}{2}a^2\int \text{csch}(c+dx)\,dx - \frac{b^2\,\text{Sub}}{}$$
$$= 2abx + \frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cosh^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0305111, size = 105, normalized size = 1.36

$$-\frac{a^2\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + 2abx - \frac{3b^2\cosh(c+dx)}{4d} + \frac{b^2\cosh(3(c+}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] 2*a*b*x - (3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) - (a
^2*Csch[(c + d*x)/2]^2)/(8*d) - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*S
ech[(c + d*x)/2]^2)/(8*d)
```

Maple [A] time = 0.059, size = 63, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(-\frac{\text{csch}\,(dx+c)\coth\,(dx+c)}{2}+\text{Artanh}\left(e^{dx+c}\right)\right)+2\,ab\,(dx+c)+b^2\left(-\frac{2}{3}+\frac{(\sinh\,(dx+c))^2}{3}\right)\cosh\,(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*(d*x+c)+b
^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.22024, size = 205, normalized size = 2.66

$$2\,abx + \frac{1}{24}\,b^2\left(\frac{e^{(3\,dx+3\,c)}}{d} - \frac{9\,e^{(dx+c)}}{d} - \frac{9\,e^{(-dx-c)}}{d} + \frac{e^{(-3\,dx-3\,c)}}{d}\right) + \frac{1}{2}\,a^2\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-}\right.}{d\left(2\,e^{(-2\,d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*x + 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d
+ e^(-3*d*x - 3*c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c)
 - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-
4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.20708, size = 4246, normalized size = 55.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^10 + 10*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + b^2*sin
h(d*x + c)^10 + 48*a*b*d*x*cosh(d*x + c)^7 - 11*b^2*cosh(d*x + c)^8 - 96*a*
b*d*x*cosh(d*x + c)^5 + (45*b^2*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x + c)^8 +
 8*(15*b^2*cosh(d*x + c)^3 + 6*a*b*d*x - 11*b^2*cosh(d*x + c))*sinh(d*x + c
)^7 + 48*a*b*d*x*cosh(d*x + c)^3 - 2*(12*a^2 - 5*b^2)*cosh(d*x + c)^6 + 2*(
105*b^2*cosh(d*x + c)^4 + 168*a*b*d*x*cosh(d*x + c) - 154*b^2*cosh(d*x + c)
^2 - 12*a^2 + 5*b^2)*sinh(d*x + c)^6 + 4*(63*b^2*cosh(d*x + c)^5 + 252*a*b*
d*x*cosh(d*x + c)^2 - 154*b^2*cosh(d*x + c)^3 - 24*a*b*d*x - 3*(12*a^2 - 5*
b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(12*a^2 - 5*b^2)*cosh(d*x + c)^4 +
2*(105*b^2*cosh(d*x + c)^6 + 840*a*b*d*x*cosh(d*x + c)^3 - 385*b^2*cosh(d*x
 + c)^4 - 240*a*b*d*x*cosh(d*x + c) - 15*(12*a^2 - 5*b^2)*cosh(d*x + c)^2 -
 12*a^2 + 5*b^2)*sinh(d*x + c)^4 - 11*b^2*cosh(d*x + c)^2 + 8*(15*b^2*cosh(
d*x + c)^7 + 210*a*b*d*x*cosh(d*x + c)^4 - 77*b^2*cosh(d*x + c)^5 - 120*a*b
*d*x*cosh(d*x + c)^2 + 6*a*b*d*x - 5*(12*a^2 - 5*b^2)*cosh(d*x + c)^3 - (12
*a^2 - 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (45*b^2*cosh(d*x + c)^8 + 10
08*a*b*d*x*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^6 - 960*a*b*d*x*cosh(d*x
 + c)^3 + 144*a*b*d*x*cosh(d*x + c) - 30*(12*a^2 - 5*b^2)*cosh(d*x + c)^4 -
 12*(12*a^2 - 5*b^2)*cosh(d*x + c)^2 - 11*b^2)*sinh(d*x + c)^2 + b^2 + 12*(
a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6 + a^2*sinh(d*x +
c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)^2 - 2*a^2)*sinh(d*x +
c)^5 + a^2*cosh(d*x + c)^3 + 5*(7*a^2*cosh(d*x + c)^3 - 2*a^2*cosh(d*x + c)
)*sinh(d*x + c)^4 + (35*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(d*x + c)^2 + a^2)
*sinh(d*x + c)^3 + (21*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d*x + c)^3 + 3*a^2
*cosh(d*x + c))*sinh(d*x + c)^2 + (7*a^2*cosh(d*x + c)^6 - 10*a^2*cosh(d*x
+ c)^4 + 3*a^2*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x
 + c) + 1) - 12*(a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6
+ a^2*sinh(d*x + c)^7 - 2*a^2*cosh(d*x + c)^5 + (21*a^2*cosh(d*x + c)^2 - 2
*a^2)*sinh(d*x + c)^5 + a^2*cosh(d*x + c)^3 + 5*(7*a^2*cosh(d*x + c)^3 - 2*
a^2*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^2*cosh(d*x + c)^4 - 20*a^2*cosh(
d*x + c)^2 + a^2)*sinh(d*x + c)^3 + (21*a^2*cosh(d*x + c)^5 - 20*a^2*cosh(d
*x + c)^3 + 3*a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (7*a^2*cosh(d*x + c)^6 -
 10*a^2*cosh(d*x + c)^4 + 3*a^2*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*
x + c) + sinh(d*x + c) - 1) + 2*(5*b^2*cosh(d*x + c)^9 + 168*a*b*d*x*cosh(d
*x + c)^6 - 44*b^2*cosh(d*x + c)^7 - 240*a*b*d*x*cosh(d*x + c)^4 + 72*a*b*d
*x*cosh(d*x + c)^2 - 6*(12*a^2 - 5*b^2)*cosh(d*x + c)^5 - 4*(12*a^2 - 5*b^2
)*cosh(d*x + c)^3 - 11*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7
 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 - 2*d*cosh(d*x + c
)^5 + (21*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3
 - 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (35*d*cosh(d*x
+ c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5
- 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (7*d*cosh(d*x
 + c)^6 - 10*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2288, size = 244, normalized size = 3.17

$$\frac{2\,(dx+c)ab}{d}+\frac{a^2\log\left(e^{(dx+c)}+1\right)}{2\,d}-\frac{a^2\log\left(\left|e^{(dx+c)}-1\right|\right)}{2\,d}+\frac{b^2d^2e^{(3\,dx+3\,c)}-9\,b^2d^2e^{(dx+c)}}{24\,d^3}-\frac{\left(11\,b^2e^{(2\,dx+2\,c)}-b^2+3\left(8\,a\right.\right.}{24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 2*(d*x + c)*a*b/d + 1/2*a^2*log(e^(d*x + c) + 1)/d - 1/2*a^2*log(abs(e^(d*x
 + c) - 1))/d + 1/24*(b^2*d^2*e^(3*d*x + 3*c) - 9*b^2*d^2*e^(d*x + c))/d^3
- 1/24*(11*b^2*e^(2*d*x + 2*c) - b^2 + 3*(8*a^2 + 3*b^2)*e^(6*d*x + 6*c) +
(24*a^2 - 19*b^2)*e^(4*d*x + 4*c))*e^(-3*d*x - 3*c)/(d*(e^(d*x + c) + 1)^2*
(e^(d*x + c) - 1)^2)
```

3.157 $\int \mathrm{\mathbf{csch}}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=76

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

```
[Out] -(b^2*x)/2 - (2*a*b*ArcTanh[Cosh[c + d*x]])/d + (a^2*Coth[c + d*x])/d - (a^
2*Coth[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0910175, antiderivative size = 76, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3770, 3767, 2635, 8}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] -(b^2*x)/2 - (2*a*b*ArcTanh[Cosh[c + d*x]])/d + (a^2*Coth[c + d*x])/d - (a^
2*Coth[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = \int \left(2ab\operatorname{csch}(c+dx)+a^2\operatorname{csch}^4(c+dx)+b^2\sinh^2(c+dx)\right) dx$$
$$= a^2\int \operatorname{csch}^4(c+dx)\,dx+(2ab)\int \operatorname{csch}(c+dx)\,dx+b^2\int \sinh^2(c+dx)$$
$$= -\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}b^2\int 1$$
$$= -\frac{b^2x}{2}-\frac{2ab\tanh^{-1}(\cosh(c+dx))}{d}+\frac{a^2\coth(c+dx)}{d}-\frac{a^2\coth^3(c+dx)}{3d}$$

Mathematica [A] time = 0.398911, size = 81, normalized size = 1.07

$$\frac{3b\left(b\sinh(2(c+dx))-2\left(-4a\log\left(\sinh\left(\frac{1}{2}(c+dx)\right)\right)+4a\log\left(\cosh\left(\frac{1}{2}(c+dx)\right)\right)+bc+bdx\right)\right)-4a^2\coth(c+dx)\left(c\right.}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-4*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(-2*(b*c + b*d*x + 4*a*L
og[Cosh[(c + d*x)/2]] - 4*a*Log[Sinh[(c + d*x)/2]]) + b*Sinh[2*(c + d*x)]))
/(12*d)
```

Maple [A] time = 0.069, size = 65, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)-4ab\operatorname{Artanh}\left(e^{dx+c}\right)+b^2\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2}-\frac{dx}{2}-\frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-4*a*b*arctanh(exp(d*x+c))+b^2*
(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [B] time = 1.25967, size = 230, normalized size = 3.03

$$-\frac{1}{8}b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-2ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}\right)+\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 2*a*b*(log(e^(-d*
x - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(
3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*
e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.08082, size = 4551, normalized size = 59.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^2*cosh(d*x + c)^10 + 30*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^2
*sinh(d*x + c)^10 - 3*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^8 - 3*(4*b^2*d*x -
45*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^8 + 24*(15*b^2*cosh(d*x + c)^
3 - (4*b^2*d*x + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 6*(6*b^2*d*x + b^2
)*cosh(d*x + c)^6 + 6*(105*b^2*cosh(d*x + c)^4 + 6*b^2*d*x - 14*(4*b^2*d*x
+ 3*b^2)*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 12*(63*b^2*cosh(d*x + c)^
5 - 14*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^3 + 3*(6*b^2*d*x + b^2)*cosh(d*x +
 c))*sinh(d*x + c)^5 - 6*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c)^4 + 6*(10
5*b^2*cosh(d*x + c)^6 - 35*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^4 - 6*b^2*d*x
+ 15*(6*b^2*d*x + b^2)*cosh(d*x + c)^2 - 16*a^2 + b^2)*sinh(d*x + c)^4 + 24
*(15*b^2*cosh(d*x + c)^7 - 7*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^5 + 5*(6*b^2
*d*x + b^2)*cosh(d*x + c)^3 - (6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c))*sin
h(d*x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c)^2 + (135*b^2*cos
h(d*x + c)^8 - 84*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^6 + 90*(6*b^2*d*x + b^2
)*cosh(d*x + c)^4 + 12*b^2*d*x - 36*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*x + c
)^2 + 32*a^2 - 9*b^2)*sinh(d*x + c)^2 + 3*b^2 - 48*(a*b*cosh(d*x + c)^8 + 8
*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 - 3*a*b*cosh(d*x +
 c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6 + 3*a*b*cosh(d*x +
 c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*sinh(d*x + c)^5 +
(70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3*a*b)*sinh(d*x + c)^4 -
 a*b*cosh(d*x + c)^2 + 4*(14*a*b*cosh(d*x + c)^5 - 15*a*b*cosh(d*x + c)^3 +
 3*a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (28*a*b*cosh(d*x + c)^6 - 45*a*b*co
sh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^2 + 2*(4*a*b*co
sh(d*x + c)^7 - 9*a*b*cosh(d*x + c)^5 + 6*a*b*cosh(d*x + c)^3 - a*b*cosh(d*
x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 48*(a*b*cos
h(d*x + c)^8 + 8*a*b*cosh(d*x + c)*sinh(d*x + c)^7 + a*b*sinh(d*x + c)^8 -
3*a*b*cosh(d*x + c)^6 + (28*a*b*cosh(d*x + c)^2 - 3*a*b)*sinh(d*x + c)^6 +
3*a*b*cosh(d*x + c)^4 + 2*(28*a*b*cosh(d*x + c)^3 - 9*a*b*cosh(d*x + c))*si
nh(d*x + c)^5 + (70*a*b*cosh(d*x + c)^4 - 45*a*b*cosh(d*x + c)^2 + 3*a*b)*s
inh(d*x + c)^4 - a*b*cosh(d*x + c)^2 + 4*(14*a*b*cosh(d*x + c)^5 - 15*a*b*c
osh(d*x + c)^3 + 3*a*b*cosh(d*x + c))*sinh(d*x + c)^3 + (28*a*b*cosh(d*x +
c)^6 - 45*a*b*cosh(d*x + c)^4 + 18*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)
^2 + 2*(4*a*b*cosh(d*x + c)^7 - 9*a*b*cosh(d*x + c)^5 + 6*a*b*cosh(d*x + c)
^3 - a*b*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) -
1) + 2*(15*b^2*cosh(d*x + c)^9 - 12*(4*b^2*d*x + 3*b^2)*cosh(d*x + c)^7 + 1
8*(6*b^2*d*x + b^2)*cosh(d*x + c)^5 - 12*(6*b^2*d*x + 16*a^2 - b^2)*cosh(d*
x + c)^3 + (12*b^2*d*x + 32*a^2 - 9*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*c
osh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 3*
d*cosh(d*x + c)^6 + (28*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^6 + 2*(28*d*
cosh(d*x + c)^3 - 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 3*d*cosh(d*x + c)^4
+ (70*d*cosh(d*x + c)^4 - 45*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 4*(
14*d*cosh(d*x + c)^5 - 15*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x +
 c)^3 - d*cosh(d*x + c)^2 + (28*d*cosh(d*x + c)^6 - 45*d*cosh(d*x + c)^4 +
18*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 2*(4*d*cosh(d*x + c)^7 - 9*d*co
sh(d*x + c)^5 + 6*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.25275, size = 219, normalized size = 2.88

$$-\frac{(dx+c)b^2}{2d}+\frac{b^2e^{(2dx+2c)}}{8d}-\frac{2ab\log\left(e^{(dx+c)}+1\right)}{d}+\frac{2ab\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}-\frac{\left(3b^2e^{(6dx+6c)}-3b^2+3\left(32a^2-3b^2\right)e^{(4c}}{24d\left(e^{(dx+c)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*b^2/d + 1/8*b^2*e^(2*d*x + 2*c)/d - 2*a*b*log(e^(d*x + c) +
1)/d + 2*a*b*log(abs(e^(d*x + c) - 1))/d - 1/24*(3*b^2*e^(6*d*x + 6*c) - 3*
b^2 + 3*(32*a^2 - 3*b^2)*e^(4*d*x + 4*c) - (32*a^2 - 9*b^2)*e^(2*d*x + 2*c)
)*e^(-2*d*x - 2*c)/(d*(e^(d*x + c) + 1)^3*(e^(d*x + c) - 1)^3)
```

3.158 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=90

$$-\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{2ab\coth(c+dx)}{d}+\frac{b^2\,\mathrm{co}}{}$$

```
[Out] (-3*a^2*ArcTanh[Cosh[c + d*x]])/(8*d) + (b^2*Cosh[c + d*x])/d - (2*a*b*Coth
[c + d*x])/d + (3*a^2*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (a^2*Coth[c + d*
x]*Csch[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.132429, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3767, 8, 3768, 3770, 2638}

$$-\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{2ab\coth(c+dx)}{d}+\frac{b^2\,\mathrm{co}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-3*a^2*ArcTanh[Cosh[c + d*x]])/(8*d) + (b^2*Cosh[c + d*x])/d - (2*a*b*Coth
[c + d*x])/d + (3*a^2*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (a^2*Coth[c + d*
x]*Csch[c + d*x]^3)/(4*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = i\int \left(-2iab\operatorname{csch}^2(c+dx) - ia^2\operatorname{csch}^5(c+dx) - ib^2\sinh(c+dx)\right) dx$$
$$= a^2\int \operatorname{csch}^5(c+dx)\,dx + (2ab)\int \operatorname{csch}^2(c+dx)\,dx + b^2\int \sinh(c+dx)$$
$$= \frac{b^2\cosh(c+dx)}{d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{1}{4}\left(3a^2\right)\int \operatorname{csch}^3(c+$$
$$= \frac{b^2\cosh(c+dx)}{d} - \frac{2ab\coth(c+dx)}{d} + \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} -$$
$$= -\frac{3a^2\tanh^{-1}(\cosh(c+dx))}{8d} + \frac{b^2\cosh(c+dx)}{d} - \frac{2ab\coth(c+dx)}{d} + 3a$$

Mathematica [A] time = 0.0380619, size = 149, normalized size = 1.66

$$-\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a^2\log\left(\tanh\left(\frac{1}{2}(c\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (2*a*b*Coth[c + d*x])/d + (3*a^2*Csch[(c + d*x)
/2]^2)/(32*d) - (a^2*Csch[(c + d*x)/2]^4)/(64*d) + (3*a^2*Log[Tanh[(c + d*x
)/2]])/(8*d) + (3*a^2*Sech[(c + d*x)/2]^2)/(32*d) + (a^2*Sech[(c + d*x)/2]^
4)/(64*d) + (b^2*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.109, size = 66, normalized size = 0.7

$$\frac{1}{d}\left(a^2\left(\left(-\frac{(\operatorname{csch}(dx+c))^3}{4} + \frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c) - \frac{3\operatorname{Artanh}\left(e^{dx+c}\right)}{4}\right) - 2ab\coth(dx+c) + b^2\cosh(dx+c$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(
d*x+c)))-2*a*b*coth(d*x+c)+b^2*cosh(d*x+c))
```

Maxima [B] time = 1.12204, size = 254, normalized size = 2.82

$$\frac{1}{2}b^2\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) - \frac{1}{8}a^2\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(3e^{(-dx-c)} - 11e^{(-3dx-3c)} - 11e^{(-5dx-5c)}\right.}{d\left(4e^{(-2dx-2c)} - 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1)
/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) + 4*a*b/(d*(e^(-2
*d*x - 2*c) - 1))
```

Fricas [B] time = 2.26968, size = 5495, normalized size = 61.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(4*b^2*cosh(d*x + c)^10 + 40*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 4*b^2*
sinh(d*x + c)^10 - 32*a*b*cosh(d*x + c)^7 + 6*(a^2 - 2*b^2)*cosh(d*x + c)^8
 + 6*(30*b^2*cosh(d*x + c)^2 + a^2 - 2*b^2)*sinh(d*x + c)^8 + 16*(30*b^2*co
sh(d*x + c)^3 - 2*a*b + 3*(a^2 - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 96
*a*b*cosh(d*x + c)^5 - 2*(11*a^2 - 4*b^2)*cosh(d*x + c)^6 + 2*(420*b^2*cosh
(d*x + c)^4 - 112*a*b*cosh(d*x + c) + 84*(a^2 - 2*b^2)*cosh(d*x + c)^2 - 11
*a^2 + 4*b^2)*sinh(d*x + c)^6 + 12*(84*b^2*cosh(d*x + c)^5 - 56*a*b*cosh(d*
x + c)^2 + 28*(a^2 - 2*b^2)*cosh(d*x + c)^3 + 8*a*b - (11*a^2 - 4*b^2)*cosh
(d*x + c))*sinh(d*x + c)^5 - 96*a*b*cosh(d*x + c)^3 - 2*(11*a^2 - 4*b^2)*co
sh(d*x + c)^4 + 2*(420*b^2*cosh(d*x + c)^6 - 560*a*b*cosh(d*x + c)^3 + 210*
(a^2 - 2*b^2)*cosh(d*x + c)^4 + 240*a*b*cosh(d*x + c) - 15*(11*a^2 - 4*b^2)
*cosh(d*x + c)^2 - 11*a^2 + 4*b^2)*sinh(d*x + c)^4 + 8*(60*b^2*cosh(d*x + c
)^7 - 140*a*b*cosh(d*x + c)^4 + 42*(a^2 - 2*b^2)*cosh(d*x + c)^5 + 120*a*b*
cosh(d*x + c)^2 - 5*(11*a^2 - 4*b^2)*cosh(d*x + c)^3 - 12*a*b - (11*a^2 - 4
*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 32*a*b*cosh(d*x + c) + 6*(a^2 - 2*b^
2)*cosh(d*x + c)^2 + 6*(30*b^2*cosh(d*x + c)^8 - 112*a*b*cosh(d*x + c)^5 +
28*(a^2 - 2*b^2)*cosh(d*x + c)^6 + 160*a*b*cosh(d*x + c)^3 - 5*(11*a^2 - 4*
b^2)*cosh(d*x + c)^4 - 48*a*b*cosh(d*x + c) - 2*(11*a^2 - 4*b^2)*cosh(d*x +
 c)^2 + a^2 - 2*b^2)*sinh(d*x + c)^2 + 4*b^2 - 3*(a^2*cosh(d*x + c)^9 + 9*a
^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 - 4*a^2*cosh(d*x + c
)^7 + 4*(9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 + 6*a^2*cosh(d*x + c)
^5 + 28*(3*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^6 + 6*(21
*a^2*cosh(d*x + c)^4 - 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x + c)^5 - 4*a^
2*cosh(d*x + c)^3 + 2*(63*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x + c)^3 + 15
*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^2*cosh(d*x + c)^6 - 35*a^2*co
sh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^3 + a^2*cosh(d*
x + c) + 12*(3*a^2*cosh(d*x + c)^7 - 7*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x
 + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^2*cosh(d*x + c)^8 - 28*
a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 - 12*a^2*cosh(d*x + c)^2 + a^2
)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*(a^2*cosh(d*x +
 c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 - 4*a^2*c
osh(d*x + c)^7 + 4*(9*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 + 6*a^2*co
sh(d*x + c)^5 + 28*(3*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c
)^6 + 6*(21*a^2*cosh(d*x + c)^4 - 14*a^2*cosh(d*x + c)^2 + a^2)*sinh(d*x +
c)^5 - 4*a^2*cosh(d*x + c)^3 + 2*(63*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x
+ c)^3 + 15*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*a^2*cosh(d*x + c)^6
- 35*a^2*cosh(d*x + c)^4 + 15*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^3 +
a^2*cosh(d*x + c) + 12*(3*a^2*cosh(d*x + c)^7 - 7*a^2*cosh(d*x + c)^5 + 5*a
^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d*x + c)^2 + (9*a^2*cosh(d*x +
 c)^8 - 28*a^2*cosh(d*x + c)^6 + 30*a^2*cosh(d*x + c)^4 - 12*a^2*cosh(d*x +
 c)^2 + a^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(10*
b^2*cosh(d*x + c)^9 - 56*a*b*cosh(d*x + c)^6 + 12*(a^2 - 2*b^2)*cosh(d*x +
c)^7 + 120*a*b*cosh(d*x + c)^4 - 3*(11*a^2 - 4*b^2)*cosh(d*x + c)^5 - 72*a*
```

```
b*cosh(d*x + c)^2 - 2*(11*a^2 - 4*b^2)*cosh(d*x + c)^3 + 8*a*b + 3*(a^2 - 2
*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*
sinh(d*x + c)^8 + d*sinh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 4*(9*d*cosh(d*x
 + c)^2 - d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*s
inh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x + c)^4 - 14*d*cosh(
d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^5 - 70*d*cosh(d*x +
 c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 - 4*d*cosh(d*x + c)^3 + 4*(21*d
*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*
x + c)^3 + 12*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c
)^3 - d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (9*d*cosh(d*x +
c)^8 - 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 +
 d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26709, size = 246, normalized size = 2.73

$$\frac{b^2e^{(dx+c)}}{2\,d} + \frac{b^2e^{(-dx-c)}}{2\,d} - \frac{3\,a^2\log\left(e^{(dx+c)}+1\right)}{8\,d} + \frac{3\,a^2\log\left(\left|e^{(dx+c)}-1\right|\right)}{8\,d} + \frac{3\,a^2e^{(7\,dx+7\,c)} - 16\,abe^{(6\,dx+6\,c)} - 11\,a^2e^{(5\,dx+5\,c)} + \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*e^(d*x + c)/d + 1/2*b^2*e^(-d*x - c)/d - 3/8*a^2*log(e^(d*x + c) +
1)/d + 3/8*a^2*log(abs(e^(d*x + c) - 1))/d + 1/4*(3*a^2*e^(7*d*x + 7*c) - 1
6*a*b*e^(6*d*x + 6*c) - 11*a^2*e^(5*d*x + 5*c) + 48*a*b*e^(4*d*x + 4*c) - 1
1*a^2*e^(3*d*x + 3*c) - 48*a*b*e^(2*d*x + 2*c) + 3*a^2*e^(d*x + c) + 16*a*b
)/(d*(e^(2*d*x + 2*c) - 1)^4)
```

3.159 $\int \mathrm{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=88

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{ab\coth(c+dx)\mathrm{csch}(c+dx)}{d}+\ldots$$

```
[Out] b^2*x + (a*b*ArcTanh[Cosh[c + d*x]])/d - (a^2*Coth[c + d*x])/d + (2*a^2*Cot
h[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]^5)/(5*d) - (a*b*Coth[c + d*x]*Csch
[c + d*x])/d
```

Rubi [A] time = 0.102543, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 3768, 3770, 3767}

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a^2\coth(c+dx)}{d}+\frac{ab\tanh^{-1}(\cosh(c+dx))}{d}-\frac{ab\coth(c+dx)\mathrm{csch}(c+dx)}{d}+\ldots$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```



```
[Out] b^2*x + (a*b*ArcTanh[Cosh[c + d*x]])/d - (a^2*Coth[c + d*x])/d + (2*a^2*Cot
h[c + d*x]^3)/(3*d) - (a^2*Coth[c + d*x]^5)/(5*d) - (a*b*Coth[c + d*x]*Csch
[c + d*x])/d
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \text{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx = -\int \left(-b^2-2ab\text{csch}^3(c+dx)-a^2\text{csch}^6(c+dx)\right)\,dx$$
$$= b^2x + a^2\int \text{csch}^6(c+dx)\,dx + (2ab)\int \text{csch}^3(c+dx)\,dx$$
$$= b^2x - \frac{ab\coth(c+dx)\text{csch}(c+dx)}{d} - (ab)\int \text{csch}(c+dx)\,dx - \frac{\left(ia^2\right)\text{Su}}{}$$
$$= b^2x + \frac{ab\tanh^{-1}(\cosh(c+dx))}{d} - \frac{a^2\coth(c+dx)}{d} + \frac{2a^2\coth^3(c+dx)}{3d}$$

Mathematica [B] time = 0.941217, size = 197, normalized size = 2.24

$$\frac{16\left(-16a^2\tanh\left(\frac{1}{2}(c+dx)\right)-12a^2\sinh^6\left(\frac{1}{2}(c+dx)\right)\text{csch}^5(c+dx)-19a^2\sinh^4\left(\frac{1}{2}(c+dx)\right)\text{csch}^3(c+dx)-15ab\text{sec}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (-256*a^2*Coth[(c + d*x)/2] - 240*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c
+ d*x)/2]^4*Sinh[c + d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 16*(6
0*b^2*c + 60*b^2*d*x - 60*a*b*Log[Tanh[(c + d*x)/2]] - 15*a*b*Sech[(c + d*x
)/2]^2 - 19*a^2*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 12*a^2*Csch[c + d*x]^
5*Sinh[(c + d*x)/2]^6 - 16*a^2*Tanh[(c + d*x)/2]))/(960*d)
```

Maple [A] time = 0.108, size = 73, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(-\frac{8}{15}-\frac{(\text{csch}\,(dx+c))^4}{5}+\frac{4\,(\text{csch}\,(dx+c))^2}{15}\right)\coth\,(dx+c)+2\,ab\left(-1/2\,\text{csch}\,(dx+c)\coth\,(dx+c)+\text{Artanh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(-1
/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b^2*(d*x+c))
```

Maxima [B] time = 1.1531, size = 409, normalized size = 4.65

$$b^2x + ab\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right) - \frac{16}{15}\,a^2\left(\frac{}{d\left(5\,e^{(-2\,dx-2\,c)}-10\,e^{(-4\,dx-4\,c}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] b^2*x + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x
 - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
- 16/15*a^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1
0*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*
```

```
d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*
d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c)
 + e^(-10*d*x - 10*c) - 1)))
```

Fricas [B] time = 2.31189, size = 6268, normalized size = 71.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(15*b^2*d*x*cosh(d*x + c)^10 + 15*b^2*d*x*sinh(d*x + c)^10 - 75*b^2*d*
x*cosh(d*x + c)^8 - 30*a*b*cosh(d*x + c)^9 + 150*b^2*d*x*cosh(d*x + c)^6 +
30*(5*b^2*d*x*cosh(d*x + c) - a*b)*sinh(d*x + c)^9 + 60*a*b*cosh(d*x + c)^7
 + 15*(45*b^2*d*x*cosh(d*x + c)^2 - 5*b^2*d*x - 18*a*b*cosh(d*x + c))*sinh(
d*x + c)^8 + 60*(30*b^2*d*x*cosh(d*x + c)^3 - 10*b^2*d*x*cosh(d*x + c) - 18
*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^7 + 30*(105*b^2*d*x*cosh(d*x + c)
^4 - 70*b^2*d*x*cosh(d*x + c)^2 - 84*a*b*cosh(d*x + c)^3 + 5*b^2*d*x + 14*a
*b*cosh(d*x + c))*sinh(d*x + c)^6 + 60*(63*b^2*d*x*cosh(d*x + c)^5 - 70*b^2
*d*x*cosh(d*x + c)^3 - 63*a*b*cosh(d*x + c)^4 + 15*b^2*d*x*cosh(d*x + c) +
21*a*b*cosh(d*x + c)^2)*sinh(d*x + c)^5 - 60*a*b*cosh(d*x + c)^3 - 10*(15*b
^2*d*x + 16*a^2)*cosh(d*x + c)^4 + 10*(315*b^2*d*x*cosh(d*x + c)^6 - 525*b^
2*d*x*cosh(d*x + c)^4 - 378*a*b*cosh(d*x + c)^5 + 225*b^2*d*x*cosh(d*x + c)
^2 + 210*a*b*cosh(d*x + c)^3 - 15*b^2*d*x - 16*a^2)*sinh(d*x + c)^4 - 15*b^
2*d*x + 20*(90*b^2*d*x*cosh(d*x + c)^7 - 210*b^2*d*x*cosh(d*x + c)^5 - 126*
a*b*cosh(d*x + c)^6 + 150*b^2*d*x*cosh(d*x + c)^3 + 105*a*b*cosh(d*x + c)^4
 - 3*a*b - 2*(15*b^2*d*x + 16*a^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*a*b*
cosh(d*x + c) + 5*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 + 5*(135*b^2*d*x*co
sh(d*x + c)^8 - 420*b^2*d*x*cosh(d*x + c)^6 - 216*a*b*cosh(d*x + c)^7 + 450
*b^2*d*x*cosh(d*x + c)^4 + 252*a*b*cosh(d*x + c)^5 + 15*b^2*d*x - 36*a*b*co
sh(d*x + c) - 12*(15*b^2*d*x + 16*a^2)*cosh(d*x + c)^2 + 16*a^2)*sinh(d*x +
 c)^2 - 16*a^2 + 15*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x + c)*sinh(d*x +
 c)^9 + a*b*sinh(d*x + c)^10 - 5*a*b*cosh(d*x + c)^8 + 5*(9*a*b*cosh(d*x +
c)^2 - a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(3*a*b*cosh(d*x +
 c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cosh(d*x + c)^4 - 1
4*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 - 10*a*b*cosh(d*x + c)^4 + 4*(
63*a*b*cosh(d*x + c)^5 - 70*a*b*cosh(d*x + c)^3 + 15*a*b*cosh(d*x + c))*sin
h(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 - 35*a*b*cosh(d*x + c)^4 + 15*a*b
*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x + c)^2 + 40*(3*a*b
*cosh(d*x + c)^7 - 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x + c)^3 - a*b*cosh
(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 - 28*a*b*cosh(d*x + c
)^6 + 30*a*b*cosh(d*x + c)^4 - 12*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^
2 - a*b + 10*(a*b*cosh(d*x + c)^9 - 4*a*b*cosh(d*x + c)^7 + 6*a*b*cosh(d*x
+ c)^5 - 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*x + c))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) - 15*(a*b*cosh(d*x + c)^10 + 10*a*b*cosh(d*x
 + c)*sinh(d*x + c)^9 + a*b*sinh(d*x + c)^10 - 5*a*b*cosh(d*x + c)^8 + 5*(9
*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^8 + 10*a*b*cosh(d*x + c)^6 + 40*(
3*a*b*cosh(d*x + c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*a*b*cos
h(d*x + c)^4 - 14*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c)^6 - 10*a*b*cosh(
d*x + c)^4 + 4*(63*a*b*cosh(d*x + c)^5 - 70*a*b*cosh(d*x + c)^3 + 15*a*b*co
sh(d*x + c))*sinh(d*x + c)^5 + 10*(21*a*b*cosh(d*x + c)^6 - 35*a*b*cosh(d*x
 + c)^4 + 15*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x + c)^4 + 5*a*b*cosh(d*x +
c)^2 + 40*(3*a*b*cosh(d*x + c)^7 - 7*a*b*cosh(d*x + c)^5 + 5*a*b*cosh(d*x +
 c)^3 - a*b*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*a*b*cosh(d*x + c)^8 - 28*
a*b*cosh(d*x + c)^6 + 30*a*b*cosh(d*x + c)^4 - 12*a*b*cosh(d*x + c)^2 + a*b
)*sinh(d*x + c)^2 - a*b + 10*(a*b*cosh(d*x + c)^9 - 4*a*b*cosh(d*x + c)^7 +
```

```
 6*a*b*cosh(d*x + c)^5 - 4*a*b*cosh(d*x + c)^3 + a*b*cosh(d*x + c))*sinh(d*
x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 10*(15*b^2*d*x*cosh(d*x +
c)^9 - 60*b^2*d*x*cosh(d*x + c)^7 - 27*a*b*cosh(d*x + c)^8 + 90*b^2*d*x*cos
h(d*x + c)^5 + 42*a*b*cosh(d*x + c)^6 - 18*a*b*cosh(d*x + c)^2 - 4*(15*b^2*
d*x + 16*a^2)*cosh(d*x + c)^3 + 3*a*b + (15*b^2*d*x + 16*a^2)*cosh(d*x + c)
)*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 d*sinh(d*x + c)^10 - 5*d*cosh(d*x + c)^8 + 5*(9*d*cosh(d*x + c)^2 - d)*sin
h(d*x + c)^8 + 40*(3*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^7 +
 10*d*cosh(d*x + c)^6 + 10*(21*d*cosh(d*x + c)^4 - 14*d*cosh(d*x + c)^2 + d
)*sinh(d*x + c)^6 + 4*(63*d*cosh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 15*d*c
osh(d*x + c))*sinh(d*x + c)^5 - 10*d*cosh(d*x + c)^4 + 10*(21*d*cosh(d*x +
c)^6 - 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^4 + 4
0*(3*d*cosh(d*x + c)^7 - 7*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 - d*cosh
(d*x + c))*sinh(d*x + c)^3 + 5*d*cosh(d*x + c)^2 + 5*(9*d*cosh(d*x + c)^8 -
 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 - 12*d*cosh(d*x + c)^2 + d)*si
nh(d*x + c)^2 + 10*(d*cosh(d*x + c)^9 - 4*d*cosh(d*x + c)^7 + 6*d*cosh(d*x
+ c)^5 - 4*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) - d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.24754, size = 197, normalized size = 2.24

$$\frac{(dx+c)b^2}{d} + \frac{ab\log\left(e^{(dx+c)}+1\right)}{d} - \frac{ab\log\left(\left|e^{(dx+c)}-1\right|\right)}{d} - \frac{2\left(15\,abe^{(9\,dx+9\,c)} - 30\,abe^{(7\,dx+7\,c)} + 80\,a^2e^{(4\,dx+4\,c)} + 30\,abe^{(3}\right.}{15\,d\left(e^{(2\,dx+2\,c)}-1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^2/d + a*b*log(e^(d*x + c) + 1)/d - a*b*log(abs(e^(d*x + c) - 1)
)/d - 2/15*(15*a*b*e^(9*d*x + 9*c) - 30*a*b*e^(7*d*x + 7*c) + 80*a^2*e^(4*d
*x + 4*c) + 30*a*b*e^(3*d*x + 3*c) - 40*a^2*e^(2*d*x + 2*c) - 15*a*b*e^(d*x
 + c) + 8*a^2)/(d*(e^(2*d*x + 2*c) - 1)^5)
```

3.160 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2\,dx$

Optimal. Leaf size=133

$$\frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}-\frac{5a^2\coth(c+dx)\operatorname{csch}(c+$$

```
[Out] (5*a^2*ArcTanh[Cosh[c + d*x]])/(16*d) - (b^2*ArcTanh[Cosh[c + d*x]])/d + (2
*a*b*Coth[c + d*x])/d - (2*a*b*Coth[c + d*x]^3)/(3*d) - (5*a^2*Coth[c + d*x
]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a
^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.174964, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 3770, 3767, 3768}

$$\frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}-\frac{5a^2\coth(c+dx)\operatorname{csch}(c+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (5*a^2*ArcTanh[Cosh[c + d*x]])/(16*d) - (b^2*ArcTanh[Cosh[c + d*x]])/d + (2
*a*b*Coth[c + d*x])/d - (2*a*b*Coth[c + d*x]^3)/(3*d) - (5*a^2*Coth[c + d*x
]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a
^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rubi steps

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^2 dx = -\left(i\int \left(ib^2\operatorname{csch}(c+dx)+2iab\operatorname{csch}^4(c+dx)+ia^2\operatorname{csch}^7(c+dx)\right)dx\right)$$
$$= a^2\int \operatorname{csch}^7(c+dx)\,dx+(2ab)\int \operatorname{csch}^4(c+dx)\,dx+b^2\int \operatorname{csch}(c+dx)$$
$$= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}-\frac{1}{6}\left(5a^2\right)\int$$
$$= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth(c+dx)}{d}-\frac{2ab\coth^3(c+dx)}{3d}+5$$
$$= -\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth(c+dx)}{d}-\frac{2ab\coth^3(c+dx)}{3d}-5$$
$$= \frac{5a^2\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{b^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{2ab\coth(c+d}{d}$$

Mathematica [A] time = 0.0525052, size = 235, normalized size = 1.77

$$-\frac{a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{5a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^2,x]
```

```
[Out] (4*a*b*Coth[c + d*x])/(3*d) - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) + (a^2*Csc
h[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c + d*x)/2]^6)/(384*d) - (2*a*b*Coth[
c + d*x]*Csch[c + d*x]^2)/(3*d) - (b^2*Log[Cosh[c/2 + (d*x)/2]])/d + (b^2*L
og[Sinh[c/2 + (d*x)/2]])/d - (5*a^2*Log[Tanh[(c + d*x)/2]])/(16*d) - (5*a^2
*Sech[(c + d*x)/2]^2)/(64*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech
[(c + d*x)/2]^6)/(384*d)
```

Maple [A] time = 0.099, size = 90, normalized size = 0.7

$$\frac{1}{d}\left(a^2\left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6}+\frac{5\,(\operatorname{csch}(dx+c))^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{Artanh}\left(e^{dx+c}\right)}{8}\right)+2\,ab\left(2/3-1/\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x)
```

```
[Out] 1/d*(a^2*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x
+c)+5/8*arctanh(exp(d*x+c)))+2*a*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-2*b^
2*arctanh(exp(d*x+c)))
```

Maxima [B] time = 1.19719, size = 427, normalized size = 3.21

$$\frac{1}{48}a^2\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}+198e^{(-5dx-5c)}+198e^{(-7dx-7c)}-85\right.}{d\left(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/48*a^2*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e
^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x -
7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c)
- 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-1
0*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) - b^2*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d) + 8/3*a*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*
c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c)
 - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.28431, size = 9393, normalized size = 70.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/48*(30*a^2*cosh(d*x + c)^11 + 330*a^2*cosh(d*x + c)*sinh(d*x + c)^10 + 3
0*a^2*sinh(d*x + c)^11 - 170*a^2*cosh(d*x + c)^9 + 384*a*b*cosh(d*x + c)^8
+ 10*(165*a^2*cosh(d*x + c)^2 - 17*a^2)*sinh(d*x + c)^9 + 396*a^2*cosh(d*x
+ c)^7 + 6*(825*a^2*cosh(d*x + c)^3 - 255*a^2*cosh(d*x + c) + 64*a*b)*sinh(
d*x + c)^8 - 1280*a*b*cosh(d*x + c)^6 + 12*(825*a^2*cosh(d*x + c)^4 - 510*a
^2*cosh(d*x + c)^2 + 256*a*b*cosh(d*x + c) + 33*a^2)*sinh(d*x + c)^7 + 396*
a^2*cosh(d*x + c)^5 + 4*(3465*a^2*cosh(d*x + c)^5 - 3570*a^2*cosh(d*x + c)^
3 + 2688*a*b*cosh(d*x + c)^2 + 693*a^2*cosh(d*x + c) - 320*a*b)*sinh(d*x +
c)^6 + 1536*a*b*cosh(d*x + c)^4 + 12*(1155*a^2*cosh(d*x + c)^6 - 1785*a^2*c
osh(d*x + c)^4 + 1792*a*b*cosh(d*x + c)^3 + 693*a^2*cosh(d*x + c)^2 - 640*a
*b*cosh(d*x + c) + 33*a^2)*sinh(d*x + c)^5 - 170*a^2*cosh(d*x + c)^3 + 12*(
825*a^2*cosh(d*x + c)^7 - 1785*a^2*cosh(d*x + c)^5 + 2240*a*b*cosh(d*x + c)
^4 + 1155*a^2*cosh(d*x + c)^3 - 1600*a*b*cosh(d*x + c)^2 + 165*a^2*cosh(d*x
 + c) + 128*a*b)*sinh(d*x + c)^4 - 768*a*b*cosh(d*x + c)^2 + 2*(2475*a^2*co
sh(d*x + c)^8 - 7140*a^2*cosh(d*x + c)^6 + 10752*a*b*cosh(d*x + c)^5 + 6930
*a^2*cosh(d*x + c)^4 - 12800*a*b*cosh(d*x + c)^3 + 1980*a^2*cosh(d*x + c)^2
 + 3072*a*b*cosh(d*x + c) - 85*a^2)*sinh(d*x + c)^3 + 30*a^2*cosh(d*x + c)
+ 6*(275*a^2*cosh(d*x + c)^9 - 1020*a^2*cosh(d*x + c)^7 + 1792*a*b*cosh(d*x
 + c)^6 + 1386*a^2*cosh(d*x + c)^5 - 3200*a*b*cosh(d*x + c)^4 + 660*a^2*cos
h(d*x + c)^3 + 1536*a*b*cosh(d*x + c)^2 - 85*a^2*cosh(d*x + c) - 128*a*b)*s
inh(d*x + c)^2 + 128*a*b - 3*((5*a^2 - 16*b^2)*cosh(d*x + c)^12 + 12*(5*a^2
 - 16*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^2 - 16*b^2)*sinh(d*x + c)^
12 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^10 + 6*(11*(5*a^2 - 16*b^2)*cosh(d*x
+ c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^10 + 20*(11*(5*a^2 - 16*b^2)*cosh(d*
x + c)^3 - 3*(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^2 -
16*b^2)*cosh(d*x + c)^8 + 15*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 18*(5*a
^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^8 + 24*(33*(5*
a^2 - 16*b^2)*cosh(d*x + c)^5 - 30*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 5*(5*
a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a^2 - 16*b^2)*cosh(d*x
 + c)^6 + 4*(231*(5*a^2 - 16*b^2)*cosh(d*x + c)^6 - 315*(5*a^2 - 16*b^2)*co
sh(d*x + c)^4 + 105*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 - 25*a^2 + 80*b^2)*sin
h(d*x + c)^6 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 - 63*(5*a^2 - 16*b^2
)*cosh(d*x + c)^5 + 35*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - 5*(5*a^2 - 16*b^2
)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 15
*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 - 84*(5*a^2 - 16*b^2)*cosh(d*x + c)^6
 + 70*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^
2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^4 + 20*(11*(5*a^2 - 16*b^2)*cosh(d*x + c)
^9 - 36*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 + 42*(5*a^2 - 16*b^2)*cosh(d*x + c
)^5 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 3*(5*a^2 - 16*b^2)*cosh(d*x + c
))*sinh(d*x + c)^3 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 + 6*(11*(5*a^2 - 16
```

```
*b^2)*cosh(d*x + c)^10 - 45*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 + 70*(5*a^2 -
16*b^2)*cosh(d*x + c)^6 - 50*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 15*(5*a^2 -
 16*b^2)*cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^2 + 5*a^2 - 16*b^2
 + 12*((5*a^2 - 16*b^2)*cosh(d*x + c)^11 - 5*(5*a^2 - 16*b^2)*cosh(d*x + c)
^9 + 10*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 - 10*(5*a^2 - 16*b^2)*cosh(d*x + c
)^5 + 5*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - (5*a^2 - 16*b^2)*cosh(d*x + c))*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((5*a^2 - 16*b^2)
*cosh(d*x + c)^12 + 12*(5*a^2 - 16*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (5
*a^2 - 16*b^2)*sinh(d*x + c)^12 - 6*(5*a^2 - 16*b^2)*cosh(d*x + c)^10 + 6*(
11*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*sinh(d*x + c)^10 + 20
*(11*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 - 16*b^2)*cosh(d*x + c))*s
inh(d*x + c)^9 + 15*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 + 15*(33*(5*a^2 - 16*b
^2)*cosh(d*x + c)^4 - 18*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)
*sinh(d*x + c)^8 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^5 - 30*(5*a^2 - 16
*b^2)*cosh(d*x + c)^3 + 5*(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 -
 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^6 + 4*(231*(5*a^2 - 16*b^2)*cosh(d*x + c
)^6 - 315*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 + 105*(5*a^2 - 16*b^2)*cosh(d*x
+ c)^2 - 25*a^2 + 80*b^2)*sinh(d*x + c)^6 + 24*(33*(5*a^2 - 16*b^2)*cosh(d*
x + c)^7 - 63*(5*a^2 - 16*b^2)*cosh(d*x + c)^5 + 35*(5*a^2 - 16*b^2)*cosh(d
*x + c)^3 - 5*(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 -
 16*b^2)*cosh(d*x + c)^4 + 15*(33*(5*a^2 - 16*b^2)*cosh(d*x + c)^8 - 84*(5*
a^2 - 16*b^2)*cosh(d*x + c)^6 + 70*(5*a^2 - 16*b^2)*cosh(d*x + c)^4 - 20*(5
*a^2 - 16*b^2)*cosh(d*x + c)^2 + 5*a^2 - 16*b^2)*sinh(d*x + c)^4 + 20*(11*(
5*a^2 - 16*b^2)*cosh(d*x + c)^9 - 36*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 + 42*
(5*a^2 - 16*b^2)*cosh(d*x + c)^5 - 20*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 + 3*
(5*a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(5*a^2 - 16*b^2)*cosh(d
*x + c)^2 + 6*(11*(5*a^2 - 16*b^2)*cosh(d*x + c)^10 - 45*(5*a^2 - 16*b^2)*c
osh(d*x + c)^8 + 70*(5*a^2 - 16*b^2)*cosh(d*x + c)^6 - 50*(5*a^2 - 16*b^2)*
cosh(d*x + c)^4 + 15*(5*a^2 - 16*b^2)*cosh(d*x + c)^2 - 5*a^2 + 16*b^2)*sin
h(d*x + c)^2 + 5*a^2 - 16*b^2 + 12*((5*a^2 - 16*b^2)*cosh(d*x + c)^11 - 5*(
5*a^2 - 16*b^2)*cosh(d*x + c)^9 + 10*(5*a^2 - 16*b^2)*cosh(d*x + c)^7 - 10*
(5*a^2 - 16*b^2)*cosh(d*x + c)^5 + 5*(5*a^2 - 16*b^2)*cosh(d*x + c)^3 - (5*
a^2 - 16*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x +
c) - 1) + 6*(55*a^2*cosh(d*x + c)^10 - 255*a^2*cosh(d*x + c)^8 + 512*a*b*co
sh(d*x + c)^7 + 462*a^2*cosh(d*x + c)^6 - 1280*a*b*cosh(d*x + c)^5 + 330*a^
2*cosh(d*x + c)^4 + 1024*a*b*cosh(d*x + c)^3 - 85*a^2*cosh(d*x + c)^2 - 256
*a*b*cosh(d*x + c) + 5*a^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(
d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(
11*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d
*cosh(d*x + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x
+ c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)
^5 - 30*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(
d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(
d*x + c)^2 - 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*
x + c)^5 + 35*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d
*cosh(d*x + c)^4 + 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*c
osh(d*x + c)^4 - 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(
d*x + c)^9 - 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x +
c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*c
osh(d*x + c)^10 - 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d
*x + c)^4 + 15*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)
^11 - 5*d*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5
*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.21541, size = 281, normalized size = 2.11

$$\frac{\left(5\,a^2-16\,b^2\right)\log\left(e^{(dx+c)}+1\right)}{16\,d}-\frac{\left(5\,a^2-16\,b^2\right)\log\left(\left|e^{(dx+c)}-1\right|\right)}{16\,d}-\frac{15\,a^2e^{(11\,dx+11\,c)}-85\,a^2e^{(9\,dx+9\,c)}+192\,abe^{(8\,dx+8\,c)}+19}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/16*(5*a^2 - 16*b^2)*log(e^(d*x + c) + 1)/d - 1/16*(5*a^2 - 16*b^2)*log(ab
s(e^(d*x + c) - 1))/d - 1/24*(15*a^2*e^(11*d*x + 11*c) - 85*a^2*e^(9*d*x +
9*c) + 192*a*b*e^(8*d*x + 8*c) + 198*a^2*e^(7*d*x + 7*c) - 640*a*b*e^(6*d*x
 + 6*c) + 198*a^2*e^(5*d*x + 5*c) + 768*a*b*e^(4*d*x + 4*c) - 85*a^2*e^(3*d
*x + 3*c) - 384*a*b*e^(2*d*x + 2*c) + 15*a^2*e^(d*x + c) + 64*a*b)/(d*(e^(2
*d*x + 2*c) - 1)^6)
```

3.161 $\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=291

$$\frac{3a^2b\cosh^5(c+dx)}{5d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)}{d}+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^3x}{2}+\frac{3ab^2\sinh^7(c\cdots}{}$$

```
[Out] -(a^3*x)/2 + (105*a*b^2*x)/128 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c +
d*x])/d - (2*a^2*b*Cosh[c + d*x]^3)/d + (5*b^3*Cosh[c + d*x]^3)/(3*d) + (3*
a^2*b*Cosh[c + d*x]^5)/(5*d) - (2*b^3*Cosh[c + d*x]^5)/d + (10*b^3*Cosh[c +
 d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11
*d) + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (105*a*b^2*Cosh[c + d*x]*Si
nh[c + d*x])/(128*d) + (35*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(64*d) - (7
*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(16*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[
c + d*x]^7)/(8*d)
```

Rubi [A] time = 0.208646, antiderivative size = 291, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3220, 2635, 8, 2633}

$$\frac{3a^2b\cosh^5(c+dx)}{5d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)}{d}+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{a^3x}{2}+\frac{3ab^2\sinh^7(c\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```


```
[Out] -(a^3*x)/2 + (105*a*b^2*x)/128 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c +
d*x])/d - (2*a^2*b*Cosh[c + d*x]^3)/d + (5*b^3*Cosh[c + d*x]^3)/(3*d) + (3*
a^2*b*Cosh[c + d*x]^5)/(5*d) - (2*b^3*Cosh[c + d*x]^5)/d + (10*b^3*Cosh[c +
 d*x]^7)/(7*d) - (5*b^3*Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11
*d) + (a^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (105*a*b^2*Cosh[c + d*x]*Si
nh[c + d*x])/(128*d) + (35*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(64*d) - (7
*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(16*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[
c + d*x]^7)/(8*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\int\left(-a^3\sinh^2(c+dx)-3a^2b\sinh^5(c+dx)-3ab^2\sinh^8(c+dx)-b^3\sinh\right.$$
$$= a^3\int\sinh^2(c+dx)\,dx+\left(3a^2b\right)\int\sinh^5(c+dx)\,dx+\left(3ab^2\right)\int\sinh^8(c+d$$
$$= \frac{a^3\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{3ab^2\cosh(c+dx)\sinh^7(c+dx)}{8d}-\frac{1}{2}a^3\int 1$$
$$= -\frac{a^3x}{2}+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{5b^3\,c}{}$$
$$= -\frac{a^3x}{2}+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{5b^3\,c}{}$$
$$= -\frac{a^3x}{2}+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{5b^3\,c}{}$$
$$= -\frac{a^3x}{2}+\frac{105}{128}ab^2x+\frac{3a^2b\cosh(c+dx)}{d}-\frac{b^3\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+}{d}$$

Mathematica [A] time = 0.476869, size = 194, normalized size = 0.67

$$\frac{-27720a\left(64a^2-105b^2\right)(c+dx)+110880a\left(8a^2-21b^2\right)\sinh(2(c+dx))-20790b\left(77b^2-320a^2\right)\cosh(c+dx)+34650}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-27720*a*(64*a^2 - 105*b^2)*(c + d*x) - 20790*b*(-320*a^2 + 77*b^2)*Cosh[c
 + d*x] + 34650*b*(-32*a^2 + 11*b^2)*Cosh[3*(c + d*x)] - 2079*b*(-64*a^2 +
55*b^2)*Cosh[5*(c + d*x)] + 27225*b^3*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[9*(
c + d*x)] + 315*b^3*Cosh[11*(c + d*x)] + 110880*a*(8*a^2 - 21*b^2)*Sinh[2*(
c + d*x)] + 582120*a*b^2*Sinh[4*(c + d*x)] - 110880*a*b^2*Sinh[6*(c + d*x)]
 + 10395*a*b^2*Sinh[8*(c + d*x)])/(3548160*d)
```

Maple [A] time = 0.06, size = 188, normalized size = 0.7

$$\frac{1}{d}\left(b^3\left(-\frac{256}{693}+\frac{(\sinh(dx+c))^{10}}{11}-\frac{10\,(\sinh(dx+c))^8}{99}+\frac{80\,(\sinh(dx+c))^6}{693}-\frac{32\,(\sinh(dx+c))^4}{231}+\frac{128\,(\sinh(dx+c)}{693}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(b^3*(-256/693+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/693*sinh(d*x+
c)^6-32/231*sinh(d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*((1/8*
sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*c
osh(d*x+c)+35/128*d*x+35/128*c)+3*a^2*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d
*x+c)^2)*cosh(d*x+c)+a^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.09023, size = 522, normalized size = 1.79

$$-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{1419264}b^3\left(\frac{\left(847e^{(-2dx-2c)}-5445e^{(-4dx-4c)}+22869e^{(-6dx-6c)}-76230e^{(-8dx-}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/1419264*b^3*((8
47*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6*d*x - 6*c) - 7623
0*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(11*d*x + 11*c)/d +
(320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(-5*d*x - 5*c) - 54
45*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*x - 11*c))/d) - 1/
2048*a*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6
*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168
*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/160*a^
2*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e
^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Fricas [B] time = 1.93065, size = 1546, normalized size = 5.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/3548160*(315*b^3*cosh(d*x + c)^11 + 3465*b^3*cosh(d*x + c)*sinh(d*x + c)^
10 - 4235*b^3*cosh(d*x + c)^9 + 83160*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7 +
 27225*b^3*cosh(d*x + c)^7 + 3465*(15*b^3*cosh(d*x + c)^3 - 11*b^3*cosh(d*x
 + c))*sinh(d*x + c)^8 + 1155*(126*b^3*cosh(d*x + c)^5 - 308*b^3*cosh(d*x +
 c)^3 + 165*b^3*cosh(d*x + c))*sinh(d*x + c)^6 + 2079*(64*a^2*b - 55*b^3)*c
osh(d*x + c)^5 + 83160*(7*a*b^2*cosh(d*x + c)^3 - 8*a*b^2*cosh(d*x + c))*si
nh(d*x + c)^5 + 3465*(30*b^3*cosh(d*x + c)^7 - 154*b^3*cosh(d*x + c)^5 + 27
5*b^3*cosh(d*x + c)^3 + 3*(64*a^2*b - 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^
4 - 34650*(32*a^2*b - 11*b^3)*cosh(d*x + c)^3 + 27720*(21*a*b^2*cosh(d*x +
c)^5 - 80*a*b^2*cosh(d*x + c)^3 + 84*a*b^2*cosh(d*x + c))*sinh(d*x + c)^3 -
 27720*(64*a^3 - 105*a*b^2)*d*x + 3465*(5*b^3*cosh(d*x + c)^9 - 44*b^3*cosh
(d*x + c)^7 + 165*b^3*cosh(d*x + c)^5 + 6*(64*a^2*b - 55*b^3)*cosh(d*x + c)
^3 - 30*(32*a^2*b - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 20790*(320*a^2
*b - 77*b^3)*cosh(d*x + c) + 27720*(3*a*b^2*cosh(d*x + c)^7 - 24*a*b^2*cosh
(d*x + c)^5 + 84*a*b^2*cosh(d*x + c)^3 + 8*(8*a^3 - 21*a*b^2)*cosh(d*x + c)
)*sinh(d*x + c))/d
```

Sympy [A] time = 83.4585, size = 498, normalized size = 1.71

$$\begin{cases}\frac{a^3x\sinh^2(c+dx)}{2}-\frac{a^3x\cosh^2(c+dx)}{2}+\frac{a^3\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{3a^2b\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4a^2b\sinh^2(c+dx)\cosh^3(c+dx)}{d}+\frac{8a^2b\cosh}{5a} \\ x\left(a+b\sinh^3(c)\right)^3\sinh^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*x*sinh(c + d*x)**2/2 - a**3*x*cosh(c + d*x)**2/2 + a**3*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d
- 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5/
(5*d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*c
osh(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 10
5*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x
)**8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*s
inh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh
(c + d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) +
b**3*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**3*sinh(c + d*x)**8*cosh(c +
d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)**6*cosh(c + d*x)**5/(3*d) - 32*b**3*s
inh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)**2*cosh(c +
 d*x)**9/(63*d) - 256*b**3*cosh(c + d*x)**11/(693*d), Ne(d, 0)), (x*(a + b*
sinh(c)**3)**3*sinh(c)**2, True))
```

Giac [A] time = 1.44249, size = 545, normalized size = 1.87

$$\frac{315\,b^3e^{(11\,dx+11\,c)} - 4235\,b^3e^{(9\,dx+9\,c)} + 10395\,ab^2e^{(8\,dx+8\,c)} + 27225\,b^3e^{(7\,dx+7\,c)} - 110880\,ab^2e^{(6\,dx+6\,c)} + 133056\,a^2be^{(5\,dx+5\,c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/7096320*(315*b^3*e^(11*d*x + 11*c) - 4235*b^3*e^(9*d*x + 9*c) + 10395*a*b
^2*e^(8*d*x + 8*c) + 27225*b^3*e^(7*d*x + 7*c) - 110880*a*b^2*e^(6*d*x + 6*
c) + 133056*a^2*b*e^(5*d*x + 5*c) - 114345*b^3*e^(5*d*x + 5*c) + 582120*a*b
^2*e^(4*d*x + 4*c) - 1108800*a^2*b*e^(3*d*x + 3*c) + 381150*b^3*e^(3*d*x +
3*c) + 887040*a^3*e^(2*d*x + 2*c) - 2328480*a*b^2*e^(2*d*x + 2*c) + 6652800
*a^2*b*e^(d*x + c) - 1600830*b^3*e^(d*x + c) - 55440*(64*a^3 - 105*a*b^2)*(
d*x + c) - (582120*a*b^2*e^(7*d*x + 7*c) - 110880*a*b^2*e^(5*d*x + 5*c) - 2
7225*b^3*e^(4*d*x + 4*c) + 10395*a*b^2*e^(3*d*x + 3*c) + 4235*b^3*e^(2*d*x
+ 2*c) - 315*b^3 - 20790*(320*a^2*b - 77*b^3)*e^(10*d*x + 10*c) + 110880*(8
*a^3 - 21*a*b^2)*e^(9*d*x + 9*c) + 34650*(32*a^2*b - 11*b^3)*e^(8*d*x + 8*c
) - 2079*(64*a^2*b - 55*b^3)*e^(6*d*x + 6*c))*e^(-11*d*x - 11*c))/d
```

3.162 $\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=267

$$\frac{3a^2b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9a^2b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh^7(c+dx)}{7d}$$

```
[Out] (9*a^2*b*x)/8 - (63*b^3*x)/256 + (a^3*Cosh[c + d*x])/d - (3*a*b^2*Cosh[c +
d*x])/d + (3*a*b^2*Cosh[c + d*x]^3)/d - (9*a*b^2*Cosh[c + d*x]^5)/(5*d) + (
3*a*b^2*Cosh[c + d*x]^7)/(7*d) - (9*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d
) + (63*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + (3*a^2*b*Cosh[c + d*x]*S
inh[c + d*x]^3)/(4*d) - (21*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(128*d) + (2
1*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(160*d) - (9*b^3*Cosh[c + d*x]*Sinh[c
+ d*x]^7)/(80*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^9)/(10*d)
```

Rubi [A] time = 0.222053, antiderivative size = 267, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3220, 2638, 2635, 8, 2633}

$$\frac{3a^2b\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9a^2b\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}+\frac{3ab^2\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (9*a^2*b*x)/8 - (63*b^3*x)/256 + (a^3*Cosh[c + d*x])/d - (3*a*b^2*Cosh[c +
d*x])/d + (3*a*b^2*Cosh[c + d*x]^3)/d - (9*a*b^2*Cosh[c + d*x]^5)/(5*d) + (
3*a*b^2*Cosh[c + d*x]^7)/(7*d) - (9*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d
) + (63*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + (3*a^2*b*Cosh[c + d*x]*S
inh[c + d*x]^3)/(4*d) - (21*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(128*d) + (2
1*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(160*d) - (9*b^3*Cosh[c + d*x]*Sinh[c
+ d*x]^7)/(80*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^9)/(10*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx = -\left(i\int \left(ia^3\sinh(c+dx)+3ia^2b\sinh^4(c+dx)+3iab^2\sinh^7(c+dx)+ib^3\sin\right.\right.$$
$$= a^3\int \sinh(c+dx)\,dx+\left(3a^2b\right)\int \sinh^4(c+dx)\,dx+\left(3ab^2\right)\int \sinh^7(c+dx)$$
$$= \frac{a^3\cosh(c+dx)}{d}+\frac{3a^2b\cosh(c+dx)\sinh^3(c+dx)}{4d}+\frac{b^3\cosh(c+dx)\sinh^9(}{10d}$$
$$= \frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}-\frac{9ab^2\cosh^5(c-}{5d}$$
$$= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}-\frac{9ab^2}{}$$
$$= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}-\frac{9ab^2}{}$$
$$= \frac{9}{8}a^2bx+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx)}{d}-\frac{9ab^2}{}$$
$$= \frac{9}{8}a^2bx-\frac{63b^3x}{256}+\frac{a^3\cosh(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{3ab^2\cosh^3(c+dx}{d}$$

Mathematica [A] time = 0.459003, size = 184, normalized size = 0.69

$$\frac{1120a\left(64a^2-105b^2\right)\cosh(c+dx)+b\left(-53760a^2\sinh(2(c+dx))+6720a^2\sinh(4(c+dx))+80640a^2c+80640a^2dx+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (1120*a*(64*a^2 - 105*b^2)*Cosh[c + d*x] + b*(80640*a^2*c - 17640*b^2*c + 8
0640*a^2*d*x - 17640*b^2*d*x + 23520*a*b*Cosh[3*(c + d*x)] - 4704*a*b*Cosh[
5*(c + d*x)] + 480*a*b*Cosh[7*(c + d*x)] - 53760*a^2*Sinh[2*(c + d*x)] + 14
700*b^2*Sinh[2*(c + d*x)] + 6720*a^2*Sinh[4*(c + d*x)] - 4200*b^2*Sinh[4*(c
 + d*x)] + 1050*b^2*Sinh[6*(c + d*x)] - 175*b^2*Sinh[8*(c + d*x)] + 14*b^2*
Sinh[10*(c + d*x)]))/(71680*d)
```

Maple [A] time = 0.023, size = 168, normalized size = 0.6

$$\frac{1}{d}\left(b^3\left(\left(\frac{(\sinh(dx+c))^9}{10}-\frac{9(\sinh(dx+c))^7}{80}+\frac{21(\sinh(dx+c))^5}{160}-\frac{21(\sinh(dx+c))^3}{128}+\frac{63\sinh(dx+c)}{256}\right)\cosh(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/12
8*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+3*a*b^
2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x
```

```
+c)+3*a^2*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)
+a^3*cosh(d*x+c))
```

Maxima [A] time = 1.14679, size = 429, normalized size = 1.61

$$\frac{3}{64}a^2b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{20480}b^3\left(\frac{\left(25e^{(-2dx-2c)}-150e^{(-4dx-4c)}+600e^{(-}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((25*e^(-2*d*x - 2*c) - 150*e^(-
4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x
+ 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c
) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) -
 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x
 - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c)
+ 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + a^3*cosh(d*x + c)/d
```

Fricas [A] time = 2.00611, size = 1193, normalized size = 4.47

$$\frac{35\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^9+120\,ab^2\cosh\left(dx+c\right)^7+840\,ab^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-1176\,ab^2\cosh\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/17920*(35*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 120*a*b^2*cosh(d*x + c)^7 +
 840*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 1176*a*b^2*cosh(d*x + c)^5 + 70*
(6*b^3*cosh(d*x + c)^3 - 5*b^3*cosh(d*x + c))*sinh(d*x + c)^7 + 5880*a*b^2*
cosh(d*x + c)^3 + 7*(126*b^3*cosh(d*x + c)^5 - 350*b^3*cosh(d*x + c)^3 + 22
5*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 840*(5*a*b^2*cosh(d*x + c)^3 - 7*a*b
^2*cosh(d*x + c))*sinh(d*x + c)^4 + 70*(6*b^3*cosh(d*x + c)^7 - 35*b^3*cosh
(d*x + c)^5 + 75*b^3*cosh(d*x + c)^3 + 12*(8*a^2*b - 5*b^3)*cosh(d*x + c))*
sinh(d*x + c)^3 + 630*(32*a^2*b - 7*b^3)*d*x + 840*(3*a*b^2*cosh(d*x + c)^5
 - 14*a*b^2*cosh(d*x + c)^3 + 21*a*b^2*cosh(d*x + c))*sinh(d*x + c)^2 + 280
*(64*a^3 - 105*a*b^2)*cosh(d*x + c) + 35*(b^3*cosh(d*x + c)^9 - 10*b^3*cosh
(d*x + c)^7 + 45*b^3*cosh(d*x + c)^5 + 24*(8*a^2*b - 5*b^3)*cosh(d*x + c)^3
 - 6*(128*a^2*b - 35*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 97.2029, size = 496, normalized size = 1.86

$$\begin{cases}\frac{a^3\cosh(c+dx)}{d}+\frac{9a^2bx\sinh^4(c+dx)}{8}-\frac{9a^2bx\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{9a^2bx\cosh^4(c+dx)}{8}+\frac{15a^2b\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{9a^2b\sinh(c+d}{8} \\ x\left(a+b\sinh^3(c)\right)^3\sinh(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*
x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*
a**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c
 + d*x)**3/(8*d) + 3*a*b**2*sinh(c + d*x)**6*cosh(c + d*x)/d - 6*a*b**2*sin
h(c + d*x)**4*cosh(c + d*x)**3/d + 24*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)
**5/(5*d) - 48*a*b**2*cosh(c + d*x)**7/(35*d) + 63*b**3*x*sinh(c + d*x)**10
/256 - 315*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 315*b**3*x*sinh(c
 + d*x)**6*cosh(c + d*x)**4/128 - 315*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)
**6/128 + 315*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**3*x*cosh
(c + d*x)**10/256 + 193*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b
**3*sinh(c + d*x)**7*cosh(c + d*x)**3/(128*d) + 21*b**3*sinh(c + d*x)**5*co
sh(c + d*x)**5/(10*d) - 147*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d)
+ 63*b**3*sinh(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh
(c)**3)**3*sinh(c), True))
```

Giac [A] time = 1.47622, size = 474, normalized size = 1.78

$$\frac{14\,b^3e^{(10\,dx+10\,c)} - 175\,b^3e^{(8\,dx+8\,c)} + 480\,ab^2e^{(7\,dx+7\,c)} + 1050\,b^3e^{(6\,dx+6\,c)} - 4704\,ab^2e^{(5\,dx+5\,c)} + 6720\,a^2be^{(4\,dx+4\,c)} - 4200\,b^3e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/143360*(14*b^3*e^(10*d*x + 10*c) - 175*b^3*e^(8*d*x + 8*c) + 480*a*b^2*e^
(7*d*x + 7*c) + 1050*b^3*e^(6*d*x + 6*c) - 4704*a*b^2*e^(5*d*x + 5*c) + 672
0*a^2*b*e^(4*d*x + 4*c) - 4200*b^3*e^(4*d*x + 4*c) + 23520*a*b^2*e^(3*d*x +
 3*c) - 53760*a^2*b*e^(2*d*x + 2*c) + 14700*b^3*e^(2*d*x + 2*c) + 71680*a^3
*e^(d*x + c) - 117600*a*b^2*e^(d*x + c) + 5040*(32*a^2*b - 7*b^3)*(d*x + c)
 + (23520*a*b^2*e^(7*d*x + 7*c) - 4704*a*b^2*e^(5*d*x + 5*c) - 1050*b^3*e^(
4*d*x + 4*c) + 480*a*b^2*e^(3*d*x + 3*c) + 175*b^3*e^(2*d*x + 2*c) - 14*b^3
 + 1120*(64*a^3 - 105*a*b^2)*e^(9*d*x + 9*c) + 420*(128*a^2*b - 35*b^3)*e^(
8*d*x + 8*c) - 840*(8*a^2*b - 5*b^3)*e^(6*d*x + 6*c))*e^(-10*d*x - 10*c))/d
```

3.163 $\int \left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=204

$$\frac{a^2b\cosh^3(c+dx)}{d}-\frac{3a^2b\cosh(c+dx)}{d}+a^3x+\frac{ab^2\sinh^5(c+dx)\cosh(c+dx)}{2d}-\frac{5ab^2\sinh^3(c+dx)\cosh(c+dx)}{8d}+$$

```
[Out] a^3*x - (15*a*b^2*x)/16 - (3*a^2*b*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x])/d
 + (a^2*b*Cosh[c + d*x]^3)/d - (4*b^3*Cosh[c + d*x]^3)/(3*d) + (6*b^3*Cosh[
c + d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(
9*d) + (15*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*a*b^2*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(8*d) + (a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(2*d)
```

Rubi [A] time = 0.127754, antiderivative size = 204, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2633, 2635, 8}

$$\frac{a^2b\cosh^3(c+dx)}{d}-\frac{3a^2b\cosh(c+dx)}{d}+a^3x+\frac{ab^2\sinh^5(c+dx)\cosh(c+dx)}{2d}-\frac{5ab^2\sinh^3(c+dx)\cosh(c+dx)}{8d}+$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] a^3*x - (15*a*b^2*x)/16 - (3*a^2*b*Cosh[c + d*x])/d + (b^3*Cosh[c + d*x])/d
 + (a^2*b*Cosh[c + d*x]^3)/d - (4*b^3*Cosh[c + d*x]^3)/(3*d) + (6*b^3*Cosh[
c + d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(
9*d) + (15*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*a*b^2*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(8*d) + (a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^5)/(2*d)
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \left(a+b\sinh^3(c+dx)\right)^3 dx = \int \left(a^3+3a^2b\sinh^3(c+dx)+3ab^2\sinh^6(c+dx)+b^3\sinh^9(c+dx)\right) dx$$
$$= a^3x+\left(3a^2b\right)\int \sinh^3(c+dx)\,dx+\left(3ab^2\right)\int \sinh^6(c+dx)\,dx+b^3\int \sinh^9(c+dx)\,dx$$
$$= a^3x+\frac{ab^2\cosh(c+dx)\sinh^5(c+dx)}{2d}-\frac{1}{2}\left(5ab^2\right)\int \sinh^4(c+dx)\,dx-\frac{\left(3a^2b\right)\text{Subst}\left(\int\left(\right.\right.}{}$$
$$= a^3x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3(c+dx)}{3d}+\frac{6}{}$$
$$= a^3x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3(c+dx)}{3d}+\frac{6}{}$$
$$= a^3x-\frac{15}{16}ab^2x-\frac{3a^2b\cosh(c+dx)}{d}+\frac{b^3\cosh(c+dx)}{d}+\frac{a^2b\cosh^3(c+dx)}{d}-\frac{4b^3\cosh^3(c}{3d}$$

Mathematica [A] time = 0.251226, size = 159, normalized size = 0.78

$$\frac{5670b\left(7b^2-32a^2\right)\cosh(c+dx)+1260\left(16a^2b-7b^3\right)\cosh(3(c+dx))+80640a^3c+80640a^3dx+56700ab^2\sinh(2(c+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (80640*a^3*c - 75600*a*b^2*c + 80640*a^3*d*x - 75600*a*b^2*d*x + 5670*b*(-3
2*a^2 + 7*b^2)*Cosh[c + d*x] + 1260*(16*a^2*b - 7*b^3)*Cosh[3*(c + d*x)] +
2268*b^3*Cosh[5*(c + d*x)] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c +
 d*x)] + 56700*a*b^2*Sinh[2*(c + d*x)] - 11340*a*b^2*Sinh[4*(c + d*x)] + 12
60*a*b^2*Sinh[6*(c + d*x)])/(80640*d)
```

Maple [A] time = 0.02, size = 141, normalized size = 0.7

$$\frac{1}{d}\left(b^3\left(\frac{128}{315}+\frac{(\sinh(dx+c))^8}{9}-\frac{8(\sinh(dx+c))^6}{63}+\frac{16(\sinh(dx+c))^4}{105}-\frac{64(\sinh(dx+c))^2}{315}\right)\cosh(dx+c)+3ab^2\left(\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(b^3*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4
-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*
x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+3*a^2*b*(-2/3+1/3*sin
h(d*x+c)^2)*cosh(d*x+c)+a^3*(d*x+c))
```

Maxima [A] time = 1.17724, size = 378, normalized size = 1.85

$$a^3x-\frac{1}{161280}b^3\left(\frac{\left(405e^{(-2dx-2c)}-2268e^{(-4dx-4c)}+8820e^{(-6dx-6c)}-39690e^{(-8dx-8c)}-35\right)e^{(9dx+9c)}}{d}-\frac{39690e^{(-dx-c)}-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] a^3*x - 1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*
e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*
e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*
x - 7*c) + 35*e^(-9*d*x - 9*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*
e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x -
 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 1/8*a^2*b*(e^(3*d*x + 3
*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.81282, size = 1017, normalized size = 4.99

$$\frac{35\,b^3\cosh\left(dx+c\right)^9+315\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^8-405\,b^3\cosh\left(dx+c\right)^7+7560\,ab^2\cosh\left(dx+c\right)\sinh\left(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*cosh(d*x + c)^9 + 315*b^3*cosh(d*x + c)*sinh(d*x + c)^8 - 4
05*b^3*cosh(d*x + c)^7 + 7560*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2268*b^
3*cosh(d*x + c)^5 + 105*(28*b^3*cosh(d*x + c)^3 - 27*b^3*cosh(d*x + c))*sin
h(d*x + c)^6 + 315*(14*b^3*cosh(d*x + c)^5 - 45*b^3*cosh(d*x + c)^3 + 36*b^
3*cosh(d*x + c))*sinh(d*x + c)^4 + 1260*(16*a^2*b - 7*b^3)*cosh(d*x + c)^3
+ 5040*(5*a*b^2*cosh(d*x + c)^3 - 9*a*b^2*cosh(d*x + c))*sinh(d*x + c)^3 +
5040*(16*a^3 - 15*a*b^2)*d*x + 315*(4*b^3*cosh(d*x + c)^7 - 27*b^3*cosh(d*x
 + c)^5 + 72*b^3*cosh(d*x + c)^3 + 12*(16*a^2*b - 7*b^3)*cosh(d*x + c))*sin
h(d*x + c)^2 - 5670*(32*a^2*b - 7*b^3)*cosh(d*x + c) + 7560*(a*b^2*cosh(d*x
 + c)^5 - 6*a*b^2*cosh(d*x + c)^3 + 15*a*b^2*cosh(d*x + c))*sinh(d*x + c))/
d
```

Sympy [A] time = 33.4784, size = 340, normalized size = 1.67

$$\begin{cases} a^3x+\frac{3a^2b\sinh^2(c+dx)\cosh(c+dx)}{d}-\frac{2a^2b\cosh^3(c+dx)}{d}+\frac{15ab^2x\sinh^6(c+dx)}{16}-\frac{45ab^2x\sinh^4(c+dx)\cosh^2(c+dx)}{16}+\frac{45ab^2x\sinh^2(c+dx)\cosh}{16} \\ x\left(a+b\sinh^3(c)\right)^3 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Piecewise((a**3*x + 3*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*b*co
sh(c + d*x)**3/d + 15*a*b**2*x*sinh(c + d*x)**6/16 - 45*a*b**2*x*sinh(c + d
*x)**4*cosh(c + d*x)**2/16 + 45*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/
16 - 15*a*b**2*x*cosh(c + d*x)**6/16 + 33*a*b**2*sinh(c + d*x)**5*cosh(c +
d*x)/(16*d) - 5*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) + 15*a*b**2*
sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + b**3*sinh(c + d*x)**8*cosh(c + d*x)
/d - 8*b**3*sinh(c + d*x)**6*cosh(c + d*x)**3/(3*d) + 16*b**3*sinh(c + d*x)
**4*cosh(c + d*x)**5/(5*d) - 64*b**3*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*
d) + 128*b**3*cosh(c + d*x)**9/(315*d), Ne(d, 0)), (x*(a + b*sinh(c)**3)**3
, True))
```

Giac [A] time = 1.16286, size = 405, normalized size = 1.99

$$\frac{35\,b^3e^{(9\,dx+9\,c)}-405\,b^3e^{(7\,dx+7\,c)}+1260\,ab^2e^{(6\,dx+6\,c)}+2268\,b^3e^{(5\,dx+5\,c)}-11340\,ab^2e^{(4\,dx+4\,c)}+20160\,a^2be^{(3\,dx+3\,c)}-882}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/161280*(35*b^3*e^(9*d*x + 9*c) - 405*b^3*e^(7*d*x + 7*c) + 1260*a*b^2*e^(
6*d*x + 6*c) + 2268*b^3*e^(5*d*x + 5*c) - 11340*a*b^2*e^(4*d*x + 4*c) + 201
60*a^2*b*e^(3*d*x + 3*c) - 8820*b^3*e^(3*d*x + 3*c) + 56700*a*b^2*e^(2*d*x
+ 2*c) - 181440*a^2*b*e^(d*x + c) + 39690*b^3*e^(d*x + c) + 10080*(16*a^3 -
 15*a*b^2)*(d*x + c) - (56700*a*b^2*e^(7*d*x + 7*c) - 11340*a*b^2*e^(5*d*x
+ 5*c) - 2268*b^3*e^(4*d*x + 4*c) + 1260*a*b^2*e^(3*d*x + 3*c) + 405*b^3*e^
(2*d*x + 2*c) - 35*b^3 + 5670*(32*a^2*b - 7*b^3)*e^(8*d*x + 8*c) - 1260*(16
*a^2*b - 7*b^3)*e^(6*d*x + 6*c))*e^(-9*d*x - 9*c))/d
```

3.164 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=201

$$\frac{3a^2b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh^5(c+dx)}{5d}-\frac{2ab^2\cosh^3(c+dx)}{d}+$$

```
[Out] (-3*a^2*b*x)/2 + (35*b^3*x)/128 - (a^3*ArcTanh[Cosh[c + d*x]])/d + (3*a*b^2
*Cosh[c + d*x])/d - (2*a*b^2*Cosh[c + d*x]^3)/d + (3*a*b^2*Cosh[c + d*x]^5)
/(5*d) + (3*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (35*b^3*Cosh[c + d*x
]*Sinh[c + d*x])/(128*d) + (35*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) -
 (7*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*d) + (b^3*Cosh[c + d*x]*Sinh[c +
 d*x]^7)/(8*d)
```

Rubi [A] time = 0.188718, antiderivative size = 201, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3220, 3770, 2635, 8, 2633}

$$\frac{3a^2b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh^5(c+dx)}{5d}-\frac{2ab^2\cosh^3(c+dx)}{d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```


```
[Out] (-3*a^2*b*x)/2 + (35*b^3*x)/128 - (a^3*ArcTanh[Cosh[c + d*x]])/d + (3*a*b^2
*Cosh[c + d*x])/d - (2*a*b^2*Cosh[c + d*x]^3)/d + (3*a*b^2*Cosh[c + d*x]^5)
/(5*d) + (3*a^2*b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) - (35*b^3*Cosh[c + d*x
]*Sinh[c + d*x])/(128*d) + (35*b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(192*d) -
 (7*b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(48*d) + (b^3*Cosh[c + d*x]*Sinh[c +
 d*x]^7)/(8*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = i\int\left(-ia^3\operatorname{csch}(c+dx)-3ia^2b\sinh^2(c+dx)-3iab^2\sinh^5(c+dx)-ib^3\sinh\right.$$
$$= a^3\int \operatorname{csch}(c+dx)\,dx+\left(3a^2b\right)\int\sinh^2(c+dx)\,dx+\left(3ab^2\right)\int\sinh^5(c+dx$$
$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{b^3\cosh(c+}{}$$
$$= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab^2\cosh^3(c+d}{d}$$
$$= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab^2\cosh^3(c+d}{d}$$
$$= -\frac{3}{2}a^2bx-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab^2\cosh^3(c+d}{d}$$
$$= -\frac{3}{2}a^2bx+\frac{35b^3x}{128}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)}{d}-\frac{2ab^2\operatorname{co}}{}$$

Mathematica [A] time = 0.21892, size = 158, normalized size = 0.79

$$\frac{11520a^2b\sinh(2(c+dx))-23040a^2bc-23040a^2bdx+15360a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+28800ab^2\cosh(c+dx)-4800}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-23040*a^2*b*c + 4200*b^3*c - 23040*a^2*b*d*x + 4200*b^3*d*x + 28800*a*b^2
*Cosh[c + d*x] - 4800*a*b^2*Cosh[3*(c + d*x)] + 576*a*b^2*Cosh[5*(c + d*x)]
 + 15360*a^3*Log[Tanh[(c + d*x)/2]] + 11520*a^2*b*Sinh[2*(c + d*x)] - 3360*
b^3*Sinh[2*(c + d*x)] + 840*b^3*Sinh[4*(c + d*x)] - 160*b^3*Sinh[6*(c + d*x
)] + 15*b^3*Sinh[8*(c + d*x)])/(15360*d)
```

Maple [A] time = 0.072, size = 138, normalized size = 0.7

$$\frac{1}{d}\left(-2\,a^3\operatorname{Artanh}\left(e^{dx+c}\right)+3\,a^2b\,(1/2\,\cosh(dx+c)\sinh(dx+c)-1/2\,dx-c/2)+3\,ab^2\left(\frac{8}{15}+1/5\,(\sinh(dx+c))^4-\frac{4\,(\mathrm{si}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*
x-1/2*c)+3*a*b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+b^
3*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d
*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c))
```

Maxima [A] time = 1.18444, size = 347, normalized size = 1.73

$$-\frac{3}{8}\,a^2b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{6144}\,b^3\left(\frac{\left(32\,e^{(-2\,dx-2\,c)}-168\,e^{(-4\,dx-4\,c)}+672\,e^{(-6\,dx-6\,c)}-3\right)e^{(8\,dx+8\,c)}}{d}-\frac{1680\,(c}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/6144*b^3*((32
*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d
*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*
c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 1/160*a*b^2*(3*e^(5*d*x
 + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d -
 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*log(tanh(1/2*d*x + 1/2
*c))/d
```

Fricas [B] time = 2.07054, size = 7191, normalized size = 35.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/30720*(15*b^3*cosh(d*x + c)^16 + 240*b^3*cosh(d*x + c)*sinh(d*x + c)^15 +
 15*b^3*sinh(d*x + c)^16 - 160*b^3*cosh(d*x + c)^14 + 576*a*b^2*cosh(d*x +
c)^13 + 840*b^3*cosh(d*x + c)^12 + 40*(45*b^3*cosh(d*x + c)^2 - 4*b^3)*sinh
(d*x + c)^14 - 4800*a*b^2*cosh(d*x + c)^11 + 16*(525*b^3*cosh(d*x + c)^3 -
140*b^3*cosh(d*x + c) + 36*a*b^2)*sinh(d*x + c)^13 + 4*(6825*b^3*cosh(d*x +
 c)^4 - 3640*b^3*cosh(d*x + c)^2 + 1872*a*b^2*cosh(d*x + c) + 210*b^3)*sinh
(d*x + c)^12 + 28800*a*b^2*cosh(d*x + c)^9 + 16*(4095*b^3*cosh(d*x + c)^5 -
 3640*b^3*cosh(d*x + c)^3 + 2808*a*b^2*cosh(d*x + c)^2 + 630*b^3*cosh(d*x +
 c) - 300*a*b^2)*sinh(d*x + c)^11 - 240*(192*a^2*b - 35*b^3)*d*x*cosh(d*x +
 c)^8 + 480*(24*a^2*b - 7*b^3)*cosh(d*x + c)^10 + 8*(15015*b^3*cosh(d*x + c
)^6 - 20020*b^3*cosh(d*x + c)^4 + 20592*a*b^2*cosh(d*x + c)^3 + 6930*b^3*co
sh(d*x + c)^2 - 6600*a*b^2*cosh(d*x + c) + 1440*a^2*b - 420*b^3)*sinh(d*x +
 c)^10 + 28800*a*b^2*cosh(d*x + c)^7 + 80*(2145*b^3*cosh(d*x + c)^7 - 4004*
b^3*cosh(d*x + c)^5 + 5148*a*b^2*cosh(d*x + c)^4 + 2310*b^3*cosh(d*x + c)^3
 - 3300*a*b^2*cosh(d*x + c)^2 + 360*a*b^2 + 60*(24*a^2*b - 7*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^9 + 6*(32175*b^3*cosh(d*x + c)^8 - 80080*b^3*cosh(d*x +
 c)^6 + 123552*a*b^2*cosh(d*x + c)^5 + 69300*b^3*cosh(d*x + c)^4 - 132000*a
*b^2*cosh(d*x + c)^3 + 43200*a*b^2*cosh(d*x + c) - 40*(192*a^2*b - 35*b^3)*
d*x + 3600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 - 4800*a*b^2
*cosh(d*x + c)^5 + 48*(3575*b^3*cosh(d*x + c)^9 - 11440*b^3*cosh(d*x + c)^7
 + 20592*a*b^2*cosh(d*x + c)^6 + 13860*b^3*cosh(d*x + c)^5 - 33000*a*b^2*co
sh(d*x + c)^4 + 21600*a*b^2*cosh(d*x + c)^2 - 40*(192*a^2*b - 35*b^3)*d*x*c
osh(d*x + c) + 1200*(24*a^2*b - 7*b^3)*cosh(d*x + c)^3 + 600*a*b^2)*sinh(d*
x + c)^7 - 840*b^3*cosh(d*x + c)^4 - 480*(24*a^2*b - 7*b^3)*cosh(d*x + c)^6
 + 24*(5005*b^3*cosh(d*x + c)^10 - 20020*b^3*cosh(d*x + c)^8 + 41184*a*b^2*
cosh(d*x + c)^7 + 32340*b^3*cosh(d*x + c)^6 - 92400*a*b^2*cosh(d*x + c)^5 +
 100800*a*b^2*cosh(d*x + c)^3 - 280*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^
2 + 4200*(24*a^2*b - 7*b^3)*cosh(d*x + c)^4 + 8400*a*b^2*cosh(d*x + c) - 48
0*a^2*b + 140*b^3)*sinh(d*x + c)^6 + 576*a*b^2*cosh(d*x + c)^3 + 16*(4095*b
^3*cosh(d*x + c)^11 - 20020*b^3*cosh(d*x + c)^9 + 46332*a*b^2*cosh(d*x + c)
^8 + 41580*b^3*cosh(d*x + c)^7 - 138600*a*b^2*cosh(d*x + c)^6 + 226800*a*b^
```

```
2*cosh(d*x + c)^4 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^3 + 7560*(24
*a^2*b - 7*b^3)*cosh(d*x + c)^5 + 37800*a*b^2*cosh(d*x + c)^2 - 300*a*b^2 -
 180*(24*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 160*b^3*cosh(d*x +
 c)^2 + 20*(1365*b^3*cosh(d*x + c)^12 - 8008*b^3*cosh(d*x + c)^10 + 20592*a
*b^2*cosh(d*x + c)^9 + 20790*b^3*cosh(d*x + c)^8 - 79200*a*b^2*cosh(d*x + c
)^7 + 181440*a*b^2*cosh(d*x + c)^5 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x
+ c)^4 + 5040*(24*a^2*b - 7*b^3)*cosh(d*x + c)^6 + 50400*a*b^2*cosh(d*x + c
)^3 - 1200*a*b^2*cosh(d*x + c) - 42*b^3 - 360*(24*a^2*b - 7*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^4 + 16*(525*b^3*cosh(d*x + c)^13 - 3640*b^3*cosh(d*x +
 c)^11 + 10296*a*b^2*cosh(d*x + c)^10 + 11550*b^3*cosh(d*x + c)^9 - 49500*a
*b^2*cosh(d*x + c)^8 + 151200*a*b^2*cosh(d*x + c)^6 - 840*(192*a^2*b - 35*b
^3)*d*x*cosh(d*x + c)^5 + 3600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^7 + 63000*a
*b^2*cosh(d*x + c)^4 - 3000*a*b^2*cosh(d*x + c)^2 - 210*b^3*cosh(d*x + c) -
 600*(24*a^2*b - 7*b^3)*cosh(d*x + c)^3 + 36*a*b^2)*sinh(d*x + c)^3 - 15*b^
3 + 8*(225*b^3*cosh(d*x + c)^14 - 1820*b^3*cosh(d*x + c)^12 + 5616*a*b^2*co
sh(d*x + c)^11 + 6930*b^3*cosh(d*x + c)^10 - 33000*a*b^2*cosh(d*x + c)^9 +
129600*a*b^2*cosh(d*x + c)^7 - 840*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^6
 + 2700*(24*a^2*b - 7*b^3)*cosh(d*x + c)^8 + 75600*a*b^2*cosh(d*x + c)^5 -
6000*a*b^2*cosh(d*x + c)^3 - 630*b^3*cosh(d*x + c)^2 - 900*(24*a^2*b - 7*b^
3)*cosh(d*x + c)^4 + 216*a*b^2*cosh(d*x + c) + 20*b^3)*sinh(d*x + c)^2 - 30
720*(a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)^7*sinh(d*x + c) + 28*a^3*cos
h(d*x + c)^6*sinh(d*x + c)^2 + 56*a^3*cosh(d*x + c)^5*sinh(d*x + c)^3 + 70*
a^3*cosh(d*x + c)^4*sinh(d*x + c)^4 + 56*a^3*cosh(d*x + c)^3*sinh(d*x + c)^
5 + 28*a^3*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*a^3*cosh(d*x + c)*sinh(d*x +
 c)^7 + a^3*sinh(d*x + c)^8)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 30720
*(a^3*cosh(d*x + c)^8 + 8*a^3*cosh(d*x + c)^7*sinh(d*x + c) + 28*a^3*cosh(d
*x + c)^6*sinh(d*x + c)^2 + 56*a^3*cosh(d*x + c)^5*sinh(d*x + c)^3 + 70*a^3
*cosh(d*x + c)^4*sinh(d*x + c)^4 + 56*a^3*cosh(d*x + c)^3*sinh(d*x + c)^5 +
 28*a^3*cosh(d*x + c)^2*sinh(d*x + c)^6 + 8*a^3*cosh(d*x + c)*sinh(d*x + c)
^7 + a^3*sinh(d*x + c)^8)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 16*(15*b
^3*cosh(d*x + c)^15 - 140*b^3*cosh(d*x + c)^13 + 468*a*b^2*cosh(d*x + c)^12
 + 630*b^3*cosh(d*x + c)^11 - 3300*a*b^2*cosh(d*x + c)^10 + 16200*a*b^2*cos
h(d*x + c)^8 - 120*(192*a^2*b - 35*b^3)*d*x*cosh(d*x + c)^7 + 300*(24*a^2*b
 - 7*b^3)*cosh(d*x + c)^9 + 12600*a*b^2*cosh(d*x + c)^6 - 1500*a*b^2*cosh(d
*x + c)^4 - 210*b^3*cosh(d*x + c)^3 - 180*(24*a^2*b - 7*b^3)*cosh(d*x + c)^
5 + 108*a*b^2*cosh(d*x + c)^2 + 20*b^3*cosh(d*x + c))*sinh(d*x + c))/(d*cos
h(d*x + c)^8 + 8*d*cosh(d*x + c)^7*sinh(d*x + c) + 28*d*cosh(d*x + c)^6*sin
h(d*x + c)^2 + 56*d*cosh(d*x + c)^5*sinh(d*x + c)^3 + 70*d*cosh(d*x + c)^4*
sinh(d*x + c)^4 + 56*d*cosh(d*x + c)^3*sinh(d*x + c)^5 + 28*d*cosh(d*x + c)
^2*sinh(d*x + c)^6 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40701, size = 427, normalized size = 2.12

$$-\frac{a^3 \log\left(e^{(dx+c)}+1\right)}{d}+\frac{a^3 \log\left(\left|e^{(dx+c)}-1\right|\right)}{d}-\frac{\left(192\,a^2b-35\,b^3\right)(dx+c)}{128\,d}+\frac{\left(28800\,ab^2e^{(7\,dx+7\,c)}-4800\,ab^2e^{(5\,dx+5\,c)}-840\,b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -a^3*log(e^(d*x + c) + 1)/d + a^3*log(abs(e^(d*x + c) - 1))/d - 1/128*(192*
a^2*b - 35*b^3)*(d*x + c)/d + 1/30720*(28800*a*b^2*e^(7*d*x + 7*c) - 4800*a
*b^2*e^(5*d*x + 5*c) - 840*b^3*e^(4*d*x + 4*c) + 576*a*b^2*e^(3*d*x + 3*c)
+ 160*b^3*e^(2*d*x + 2*c) - 15*b^3 - 480*(24*a^2*b - 7*b^3)*e^(6*d*x + 6*c)
)*e^(-8*d*x - 8*c)/d + 1/30720*(15*b^3*d^7*e^(8*d*x + 8*c) - 160*b^3*d^7*e^
(6*d*x + 6*c) + 576*a*b^2*d^7*e^(5*d*x + 5*c) + 840*b^3*d^7*e^(4*d*x + 4*c)
 - 4800*a*b^2*d^7*e^(3*d*x + 3*c) + 11520*a^2*b*d^7*e^(2*d*x + 2*c) - 3360*
b^3*d^7*e^(2*d*x + 2*c) + 28800*a*b^2*d^7*e^(d*x + c))/d^8
```

3.165 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=152

$$\frac{3a^2b\cosh(c+dx)}{d}-\frac{a^3\coth(c+dx)}{d}+\frac{3ab^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9ab^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}ab^2x+\frac{b^3}{}$$

```
[Out] (9*a*b^2*x)/8 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c + d*x])/d + (b^3*Co
sh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/(7
*d) - (a^3*Coth[c + d*x])/d - (9*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) +
 (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.138335, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3767, 8, 2638, 2635, 2633}

$$\frac{3a^2b\cosh(c+dx)}{d}-\frac{a^3\coth(c+dx)}{d}+\frac{3ab^2\sinh^3(c+dx)\cosh(c+dx)}{4d}-\frac{9ab^2\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{9}{8}ab^2x+\frac{b^3}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (9*a*b^2*x)/8 + (3*a^2*b*Cosh[c + d*x])/d - (b^3*Cosh[c + d*x])/d + (b^3*Co
sh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) + (b^3*Cosh[c + d*x]^7)/(7
*d) - (a^3*Coth[c + d*x])/d - (9*a*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) +
 (3*a*b^2*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx = -\int \left(-a^3\operatorname{csch}^2(c+dx) - 3a^2b\sinh(c+dx) - 3ab^2\sinh^4(c+dx) - b^3 \mathrm{si}\right.$$
$$= a^3\int \operatorname{csch}^2(c+dx)\,dx + \left(3a^2b\right)\int \sinh(c+dx)\,dx + \left(3ab^2\right)\int \sinh^4(c$$
$$= \frac{3a^2b\cosh(c+dx)}{d} + \frac{3ab^2\cosh(c+dx)\sinh^3(c+dx)}{4d} - \frac{1}{4}\left(9ab^2\right)\int \mathrm{sin}$$
$$= \frac{3a^2b\cosh(c+dx)}{d} - \frac{b^3\cosh(c+dx)}{d} + \frac{b^3\cosh^3(c+dx)}{d} - \frac{3b^3\cosh^5(c}{5d}$$
$$= \frac{9}{8}ab^2x + \frac{3a^2b\cosh(c+dx)}{d} - \frac{b^3\cosh(c+dx)}{d} + \frac{b^3\cosh^3(c+dx)}{d} - \frac{3b^3}{}$$

Mathematica [A] time = 1.14364, size = 140, normalized size = 0.92

$$\frac{35b\left(192a^2-35b^2\right)\cosh(c+dx) - 1120a^3\tanh\left(\frac{1}{2}(c+dx)\right) - 1120a^3\coth\left(\frac{1}{2}(c+dx)\right) - 1680ab^2\sinh(2(c+dx)) + 2}{224}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (2520*a*b^2*c + 2520*a*b^2*d*x + 35*b*(192*a^2 - 35*b^2)*Cosh[c + d*x] + 24
5*b^3*Cosh[3*(c + d*x)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)
] - 1120*a^3*Coth[(c + d*x)/2] - 1680*a*b^2*Sinh[2*(c + d*x)] + 210*a*b^2*S
inh[4*(c + d*x)] - 1120*a^3*Tanh[(c + d*x)/2])/(2240*d)
```

Maple [A] time = 0.06, size = 111, normalized size = 0.7

$$\frac{1}{d}\left(-a^3\coth(dx+c) + 3a^2b\cosh(dx+c) + 3ab^2\left(\left(1/4\,(\sinh(dx+c))^3 - 3/8\sinh(dx+c)\right)\cosh(dx+c) + 3/8\,dx +\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*cosh(d*x+c)+3*a*b^2*((1/4*sinh(d*x+c)^3-3/8*s
inh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*s
inh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [A] time = 1.20085, size = 297, normalized size = 1.95

$$\frac{3}{64}ab^2\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{1}{4480}b^3\left(\frac{\left(49e^{(-2dx-2c)} - 245e^{(-4dx-4c)} + 1225e^{(-}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) - 1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4
*d*x - 4*c) + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x
- c) - 245*e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d)
+ 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 2*a^3/(d*(e^(-2*d*x - 2*c) -
 1))
```

Fricas [B] time = 1.87147, size = 809, normalized size = 5.32

$$\frac{20\,b^3\cosh\left(dx+c\right)\sinh\left(dx+c\right)^7+105\,ab^2\cosh\left(dx+c\right)^5+525\,ab^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^4-945\,ab^2\cosh\left(dx+c\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(20*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 105*a*b^2*cosh(d*x + c)^5 +
525*a*b^2*cosh(d*x + c)*sinh(d*x + c)^4 - 945*a*b^2*cosh(d*x + c)^3 + 2*(70
*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(35*b^3*co
sh(d*x + c)^5 - 135*b^3*cosh(d*x + c)^3 + 147*b^3*cosh(d*x + c))*sinh(d*x +
 c)^3 + 105*(10*a*b^2*cosh(d*x + c)^3 - 27*a*b^2*cosh(d*x + c))*sinh(d*x +
c)^2 - 280*(8*a^3 - 3*a*b^2)*cosh(d*x + c) + 2*(10*b^3*cosh(d*x + c)^7 - 81
*b^3*cosh(d*x + c)^5 + 294*b^3*cosh(d*x + c)^3 + 1260*a*b^2*d*x + 1120*a^3
+ 105*(32*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44803, size = 410, normalized size = 2.7

$$\frac{9\,(dx+c)ab^2}{8\,d}-\frac{\left(1890\,ab^2e^{(5\,dx+5\,c)}+294\,b^3e^{(4\,dx+4\,c)}-210\,ab^2e^{(3\,dx+3\,c)}-54\,b^3e^{(2\,dx+2\,c)}+5\,b^3-35\left(192\,a^2b-35\,b^3\right)e^{(8\,dx}}{4480\,d\left(e^{(dx+c)}+1\right)\left(e^{(dx+c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 9/8*(d*x + c)*a*b^2/d - 1/4480*(1890*a*b^2*e^(5*d*x + 5*c) + 294*b^3*e^(4*d
*x + 4*c) - 210*a*b^2*e^(3*d*x + 3*c) - 54*b^3*e^(2*d*x + 2*c) + 5*b^3 - 35
*(192*a^2*b - 35*b^3)*e^(8*d*x + 8*c) + 560*(16*a^3 - 3*a*b^2)*e^(7*d*x + 7
```

```
*c) + 210*(32*a^2*b - 7*b^3)*e^(6*d*x + 6*c))*e^(-7*d*x - 7*c)/(d*(e^(d*x +
 c) + 1)*(e^(d*x + c) - 1)) + 1/4480*(5*b^3*d^6*e^(7*d*x + 7*c) - 49*b^3*d^
6*e^(5*d*x + 5*c) + 210*a*b^2*d^6*e^(4*d*x + 4*c) + 245*b^3*d^6*e^(3*d*x +
3*c) - 1680*a*b^2*d^6*e^(2*d*x + 2*c) + 6720*a^2*b*d^6*e^(d*x + c) - 1225*b
^3*d^6*e^(d*x + c))/d^7
```

3.166 $\int \mathbf{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=156

$$3a^2bx+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\mathbf{csch}(c+dx)}{2d}+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{b^3\sinh^5(c}{}$$

```
[Out] 3*a^2*b*x - (5*b^3*x)/16 + (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) - (3*a*b^2*Co
sh[c + d*x])/d + (a*b^2*Cosh[c + d*x]^3)/d - (a^3*Coth[c + d*x]*Csch[c + d*
x])/(2*d) + (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^3*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(24*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.168601, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3768, 3770, 2633, 2635, 8}

$$3a^2bx+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\mathbf{csch}(c+dx)}{2d}+\frac{ab^2\cosh^3(c+dx)}{d}-\frac{3ab^2\cosh(c+dx)}{d}+\frac{b^3\sinh^5(c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] 3*a^2*b*x - (5*b^3*x)/16 + (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) - (3*a*b^2*Co
sh[c + d*x])/d + (a*b^2*Cosh[c + d*x]^3)/d - (a^3*Coth[c + d*x]*Csch[c + d*
x])/(2*d) + (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (5*b^3*Cosh[c + d*
x]*Sinh[c + d*x]^3)/(24*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^5)/(6*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\left(i\int\left(3ia^2b+ia^3\operatorname{csch}^3(c+dx)+3iab^2\sinh^3(c+dx)+ib^3\sinh^6(c+\right.\right.$$

$$= 3a^2bx + a^3\int \operatorname{csch}^3(c+dx)\,dx + \left(3ab^2\right)\int \sinh^3(c+dx)\,dx + b^3\int \sin$$

$$= 3a^2bx - \frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d} + \frac{b^3\cosh(c+dx)\sinh^5(c+dx)}{6d} -$$

$$= 3a^2bx + \frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{3ab^2\cosh(c+dx)}{d} + \frac{ab^2\cosh^3(c+}{d}$$

$$= 3a^2bx + \frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{3ab^2\cosh(c+dx)}{d} + \frac{ab^2\cosh^3(c+}{d}$$

$$= 3a^2bx - \frac{5b^3x}{16} + \frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{3ab^2\cosh(c+dx)}{d} + \frac{ab^2\operatorname{co}}{}$$

Mathematica [A] time = 3.70225, size = 150, normalized size = 0.96

$$\frac{576a^2bc + 576a^2bdx - 24a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) - 24a^3\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right) - 96a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 432ab^2\cosh(}{192}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (576*a^2*b*c - 60*b^3*c + 576*a^2*b*d*x - 60*b^3*d*x - 432*a*b^2*Cosh[c + d
*x] + 48*a*b^2*Cosh[3*(c + d*x)] - 24*a^3*Csch[(c + d*x)/2]^2 - 96*a^3*Log[
Tanh[(c + d*x)/2]] - 24*a^3*Sech[(c + d*x)/2]^2 + 45*b^3*Sinh[2*(c + d*x)]
- 9*b^3*Sinh[4*(c + d*x)] + b^3*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.066, size = 115, normalized size = 0.7

$$\frac{1}{d}\left(a^3\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2} + \operatorname{Artanh}\left(e^{dx+c}\right)\right) + 3a^2b(dx+c) + 3ab^2\left(-2/3+1/3(\sinh(dx+c))^2\right)\cosh(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*(d*x+c)
+3*a*b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*((1/6*sinh(d*x+c)^5-5/24*
sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c))
```

Maxima [A] time = 1.17251, size = 329, normalized size = 2.11

$$3\,a^2bx - \frac{1}{384}\,b^3\left(\frac{\left(9\,e^{(-2\,dx-2\,c)} - 45\,e^{(-4\,dx-4\,c)} - 1\right)e^{(6\,dx+6\,c)}}{d} + \frac{120\,(dx+c)}{d} + \frac{45\,e^{(-2\,dx-2\,c)} - 9\,e^{(-4\,dx-4\,c)} + e^{(-6\,dx-6\,c)}}{d}\right) + \frac{1}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*x - 1/384*b^3*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*
d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c)
+ e^(-6*d*x - 6*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9
*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + 1/2*a^3*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.61754, size = 9546, normalized size = 61.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(b^3*cosh(d*x + c)^16 + 16*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + b^3*s
inh(d*x + c)^16 - 11*b^3*cosh(d*x + c)^14 + 48*a*b^2*cosh(d*x + c)^13 + 64*
b^3*cosh(d*x + c)^12 + (120*b^3*cosh(d*x + c)^2 - 11*b^3)*sinh(d*x + c)^14
- 528*a*b^2*cosh(d*x + c)^11 + 2*(280*b^3*cosh(d*x + c)^3 - 77*b^3*cosh(d*x
 + c) + 24*a*b^2)*sinh(d*x + c)^13 + (1820*b^3*cosh(d*x + c)^4 - 1001*b^3*c
osh(d*x + c)^2 + 624*a*b^2*cosh(d*x + c) + 64*b^3)*sinh(d*x + c)^12 + 4*(10
92*b^3*cosh(d*x + c)^5 - 1001*b^3*cosh(d*x + c)^3 + 936*a*b^2*cosh(d*x + c)
^2 + 192*b^3*cosh(d*x + c) - 132*a*b^2)*sinh(d*x + c)^11 - 48*(48*a^2*b - 5
*b^3)*d*x*cosh(d*x + c)^8 - 3*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x
+ c)^10 + (8008*b^3*cosh(d*x + c)^6 - 11011*b^3*cosh(d*x + c)^4 + 13728*a*b
^2*cosh(d*x + c)^3 + 4224*b^3*cosh(d*x + c)^2 - 5808*a*b^2*cosh(d*x + c) -
99*b^3 + 24*(48*a^2*b - 5*b^3)*d*x)*sinh(d*x + c)^10 - 96*(4*a^3 - 5*a*b^2)
*cosh(d*x + c)^9 + 2*(5720*b^3*cosh(d*x + c)^7 - 11011*b^3*cosh(d*x + c)^5
+ 17160*a*b^2*cosh(d*x + c)^4 + 7040*b^3*cosh(d*x + c)^3 - 14520*a*b^2*cosh
(d*x + c)^2 - 192*a^3 + 240*a*b^2 - 15*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*
cosh(d*x + c))*sinh(d*x + c)^9 + 3*(4290*b^3*cosh(d*x + c)^8 - 11011*b^3*co
sh(d*x + c)^6 + 20592*a*b^2*cosh(d*x + c)^5 + 10560*b^3*cosh(d*x + c)^4 - 2
9040*a*b^2*cosh(d*x + c)^3 - 16*(48*a^2*b - 5*b^3)*d*x - 45*(33*b^3 - 8*(48
*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^2 - 288*(4*a^3 - 5*a*b^2)*cosh(d*x + c))
*sinh(d*x + c)^8 - 528*a*b^2*cosh(d*x + c)^5 - 96*(4*a^3 - 5*a*b^2)*cosh(d*
x + c)^7 + 8*(1430*b^3*cosh(d*x + c)^9 - 4719*b^3*cosh(d*x + c)^7 + 10296*a
*b^2*cosh(d*x + c)^6 + 6336*b^3*cosh(d*x + c)^5 - 21780*a*b^2*cosh(d*x + c)
^4 - 48*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c) - 45*(33*b^3 - 8*(48*a^2*b - 5
*b^3)*d*x)*cosh(d*x + c)^3 - 48*a^3 + 60*a*b^2 - 432*(4*a^3 - 5*a*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^7 - 64*b^3*cosh(d*x + c)^4 + 3*(33*b^3 + 8*(48*a
^2*b - 5*b^3)*d*x)*cosh(d*x + c)^6 + (8008*b^3*cosh(d*x + c)^10 - 33033*b^3
*cosh(d*x + c)^8 + 82368*a*b^2*cosh(d*x + c)^7 + 59136*b^3*cosh(d*x + c)^6
- 243936*a*b^2*cosh(d*x + c)^5 - 1344*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^
2 - 630*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^4 - 8064*(4*a^3 -
 5*a*b^2)*cosh(d*x + c)^3 + 99*b^3 + 24*(48*a^2*b - 5*b^3)*d*x - 672*(4*a^3
 - 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 48*a*b^2*cosh(d*x + c)^3 + 2*(
2184*b^3*cosh(d*x + c)^11 - 11011*b^3*cosh(d*x + c)^9 + 30888*a*b^2*cosh(d*
```

```
x + c)^8 + 25344*b^3*cosh(d*x + c)^7 - 121968*a*b^2*cosh(d*x + c)^6 - 1344*
(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^3 - 378*(33*b^3 - 8*(48*a^2*b - 5*b^3)
*d*x)*cosh(d*x + c)^5 - 6048*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^4 - 264*a*b^2
- 1008*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^2 + 9*(33*b^3 + 8*(48*a^2*b - 5*b^3)
*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 11*b^3*cosh(d*x + c)^2 + (1820*b^3*c
osh(d*x + c)^12 - 11011*b^3*cosh(d*x + c)^10 + 34320*a*b^2*cosh(d*x + c)^9
+ 31680*b^3*cosh(d*x + c)^8 - 174240*a*b^2*cosh(d*x + c)^7 - 3360*(48*a^2*b
 - 5*b^3)*d*x*cosh(d*x + c)^4 - 630*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cos
h(d*x + c)^6 - 12096*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^5 - 2640*a*b^2*cosh(d*
x + c) - 3360*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^3 - 64*b^3 + 45*(33*b^3 + 8*(
48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(140*b^3*cosh(d
*x + c)^13 - 1001*b^3*cosh(d*x + c)^11 + 3432*a*b^2*cosh(d*x + c)^10 + 3520
*b^3*cosh(d*x + c)^9 - 21780*a*b^2*cosh(d*x + c)^8 - 672*(48*a^2*b - 5*b^3)
*d*x*cosh(d*x + c)^5 - 90*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)
^7 - 2016*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^6 - 1320*a*b^2*cosh(d*x + c)^2 -
840*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^4 - 64*b^3*cosh(d*x + c) + 15*(33*b^3 +
 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^3 + 12*a*b^2)*sinh(d*x + c)^3 - b^
3 + (120*b^3*cosh(d*x + c)^14 - 1001*b^3*cosh(d*x + c)^12 + 3744*a*b^2*cosh
(d*x + c)^11 + 4224*b^3*cosh(d*x + c)^10 - 29040*a*b^2*cosh(d*x + c)^9 - 13
44*(48*a^2*b - 5*b^3)*d*x*cosh(d*x + c)^6 - 135*(33*b^3 - 8*(48*a^2*b - 5*b
^3)*d*x)*cosh(d*x + c)^8 - 3456*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^7 - 5280*a*
b^2*cosh(d*x + c)^3 - 2016*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^5 - 384*b^3*cosh
(d*x + c)^2 + 45*(33*b^3 + 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^4 + 144*
a*b^2*cosh(d*x + c) + 11*b^3)*sinh(d*x + c)^2 + 192*(a^3*cosh(d*x + c)^10 +
 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x + c)^10 - 2*a^3*cosh(d
*x + c)^8 + a^3*cosh(d*x + c)^6 + (45*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x
 + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^7
+ (210*a^3*cosh(d*x + c)^4 - 56*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^6
+ 2*(126*a^3*cosh(d*x + c)^5 - 56*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c)
)*sinh(d*x + c)^5 + 5*(42*a^3*cosh(d*x + c)^6 - 28*a^3*cosh(d*x + c)^4 + 3*
a^3*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(30*a^3*cosh(d*x + c)^7 - 28*a^3*c
osh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3)*sinh(d*x + c)^3 + (45*a^3*cosh(d*x
+ c)^8 - 56*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4)*sinh(d*x + c)^2 +
 2*(5*a^3*cosh(d*x + c)^9 - 8*a^3*cosh(d*x + c)^7 + 3*a^3*cosh(d*x + c)^5)*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 192*(a^3*cosh(d*x +
 c)^10 + 10*a^3*cosh(d*x + c)*sinh(d*x + c)^9 + a^3*sinh(d*x + c)^10 - 2*a^
3*cosh(d*x + c)^8 + a^3*cosh(d*x + c)^6 + (45*a^3*cosh(d*x + c)^2 - 2*a^3)*
sinh(d*x + c)^8 + 8*(15*a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x
 + c)^7 + (210*a^3*cosh(d*x + c)^4 - 56*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x
 + c)^6 + 2*(126*a^3*cosh(d*x + c)^5 - 56*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(
d*x + c))*sinh(d*x + c)^5 + 5*(42*a^3*cosh(d*x + c)^6 - 28*a^3*cosh(d*x + c
)^4 + 3*a^3*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(30*a^3*cosh(d*x + c)^7 -
28*a^3*cosh(d*x + c)^5 + 5*a^3*cosh(d*x + c)^3)*sinh(d*x + c)^3 + (45*a^3*c
osh(d*x + c)^8 - 56*a^3*cosh(d*x + c)^6 + 15*a^3*cosh(d*x + c)^4)*sinh(d*x
+ c)^2 + 2*(5*a^3*cosh(d*x + c)^9 - 8*a^3*cosh(d*x + c)^7 + 3*a^3*cosh(d*x
+ c)^5)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(8*b^3*co
sh(d*x + c)^15 - 77*b^3*cosh(d*x + c)^13 + 312*a*b^2*cosh(d*x + c)^12 + 384
*b^3*cosh(d*x + c)^11 - 2904*a*b^2*cosh(d*x + c)^10 - 192*(48*a^2*b - 5*b^3
)*d*x*cosh(d*x + c)^7 - 15*(33*b^3 - 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c
)^9 - 432*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^8 - 1320*a*b^2*cosh(d*x + c)^4 -
336*(4*a^3 - 5*a*b^2)*cosh(d*x + c)^6 - 128*b^3*cosh(d*x + c)^3 + 9*(33*b^3
 + 8*(48*a^2*b - 5*b^3)*d*x)*cosh(d*x + c)^5 + 72*a*b^2*cosh(d*x + c)^2 + 1
1*b^3*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^10 + 10*d*cosh(d*x + c
)*sinh(d*x + c)^9 + d*sinh(d*x + c)^10 - 2*d*cosh(d*x + c)^8 + (45*d*cosh(d
*x + c)^2 - 2*d)*sinh(d*x + c)^8 + 8*(15*d*cosh(d*x + c)^3 - 2*d*cosh(d*x +
 c))*sinh(d*x + c)^7 + d*cosh(d*x + c)^6 + (210*d*cosh(d*x + c)^4 - 56*d*co
sh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 2*(126*d*cosh(d*x + c)^5 - 56*d*cosh(d
*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 5*(42*d*cosh(d*x + c)^6 -
28*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(30*d*cosh(
```

```
d*x + c)^7 - 28*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3)*sinh(d*x + c)^3 +
(45*d*cosh(d*x + c)^8 - 56*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)^4)*sinh(d
*x + c)^2 + 2*(5*d*cosh(d*x + c)^9 - 8*d*cosh(d*x + c)^7 + 3*d*cosh(d*x + c
)^5)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.45249, size = 428, normalized size = 2.74

$$\frac{a^3 \log\left(e^{(dx+c)}+1\right)}{2d} - \frac{a^3 \log\left(\left|e^{(dx+c)}-1\right|\right)}{2d} + \frac{\left(48a^2b-5b^3\right)(dx+c)}{16d} - \frac{\left(45b^3e^{(8dx+8c)} - 99b^3e^{(6dx+6c)} + 528ab^2e^{(5dx+5c)} \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 1/2*a^3*log(e^(d*x + c) + 1)/d - 1/2*a^3*log(abs(e^(d*x + c) - 1))/d + 1/16
*(48*a^2*b - 5*b^3)*(d*x + c)/d - 1/384*(45*b^3*e^(8*d*x + 8*c) - 99*b^3*e^
(6*d*x + 6*c) + 528*a*b^2*e^(5*d*x + 5*c) + 64*b^3*e^(4*d*x + 4*c) - 48*a*b
^2*e^(3*d*x + 3*c) - 11*b^3*e^(2*d*x + 2*c) + b^3 + 48*(8*a^3 + 9*a*b^2)*e^
(9*d*x + 9*c) + 48*(8*a^3 - 19*a*b^2)*e^(7*d*x + 7*c))*e^(-6*d*x - 6*c)/(d*
(e^(d*x + c) + 1)^2*(e^(d*x + c) - 1)^2) + 1/384*(b^3*d^5*e^(6*d*x + 6*c) -
 9*b^3*d^5*e^(4*d*x + 4*c) + 48*a*b^2*d^5*e^(3*d*x + 3*c) + 45*b^3*d^5*e^(2
*d*x + 2*c) - 432*a*b^2*d^5*e^(d*x + c))/d^6
```

3.167 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=129

$$-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}ab^2x+\frac{b^3 c}{}$$

```
[Out] (-3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cosh[c + d*x]])/d + (b^3*Cosh[c + d*x])/d
 - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) + (a^3*Coth[
c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rubi [A] time = 0.120472, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3770, 3767, 2635, 8, 2633}

$$-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}+\frac{3ab^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{3}{2}ab^2x+\frac{b^3 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-3*a*b^2*x)/2 - (3*a^2*b*ArcTanh[Cosh[c + d*x]])/d + (b^3*Cosh[c + d*x])/d
 - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) + (a^3*Coth[
c + d*x])/d - (a^3*Coth[c + d*x]^3)/(3*d) + (3*a*b^2*Cosh[c + d*x]*Sinh[c +
 d*x])/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx = \int \left(3a^2b\operatorname{csch}(c+dx)+a^3\operatorname{csch}^4(c+dx)+3ab^2\sinh^2(c+dx)+b^3\sinh^5(c\right.$$
$$= a^3\int \operatorname{csch}^4(c+dx)\,dx+\left(3a^2b\right)\int \operatorname{csch}(c+dx)\,dx+\left(3ab^2\right)\int \sinh^2(c+d$$
$$= -\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3ab^2\cosh(c+dx)\sinh(c+dx)}{2d}-\frac{1}{2}\left(3ab^2\right)$$
$$= -\frac{3}{2}ab^2x-\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^3\cosh(c+dx)}{d}-\frac{2b^3\cosh^3(c+d}{3d}$$

Mathematica [A] time = 0.444961, size = 169, normalized size = 1.31

$$\frac{720a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+80a^3\tanh\left(\frac{1}{2}(c+dx)\right)+80a^3\coth\left(\frac{1}{2}(c+dx)\right)+80a^3\sinh^4\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^3(c+dx)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-360*a*b^2*c - 360*a*b^2*d*x + 150*b^3*Cosh[c + d*x] - 25*b^3*Cosh[3*(c +
d*x)] + 3*b^3*Cosh[5*(c + d*x)] + 80*a^3*Coth[(c + d*x)/2] + 720*a^2*b*Log[
Tanh[(c + d*x)/2]] + 80*a^3*Csch[c + d*x]^3*Sinh[(c + d*x)/2]^4 - 5*a^3*Csc
h[(c + d*x)/2]^4*Sinh[c + d*x] + 180*a*b^2*Sinh[2*(c + d*x)] + 80*a^3*Tanh[
(c + d*x)/2])/(240*d)
```

Maple [A] time = 0.077, size = 101, normalized size = 0.8

$$\frac{1}{d}\left(a^3\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)-6\,a^2b\operatorname{Artanh}\left(e^{dx+c}\right)+3\,ab^2\left(1/2\,\cosh(dx+c)\sinh(dx+c)-1/2\,dx-c/2\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-6*a^2*b*arctanh(exp(d*x+c))+3*
a*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*(8/15+1/5*sinh(d*x+c)
^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.20844, size = 351, normalized size = 2.72

$$-\frac{3}{8}ab^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{1}{480}b^3\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/480*b^3*(3*e^
(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x -
c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) - 3*a^2*b*(log(e^(-d*x
 - c) + 1)/d - log(e^(-d*x - c) - 1)/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3
*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e
^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.43673, size = 10166, normalized size = 78.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^16 + 48*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 3*b
^3*sinh(d*x + c)^16 - 34*b^3*cosh(d*x + c)^14 + 180*a*b^2*cosh(d*x + c)^13
+ 234*b^3*cosh(d*x + c)^12 + 2*(180*b^3*cosh(d*x + c)^2 - 17*b^3)*sinh(d*x
+ c)^14 + 4*(420*b^3*cosh(d*x + c)^3 - 119*b^3*cosh(d*x + c) + 45*a*b^2)*si
nh(d*x + c)^13 - 378*b^3*cosh(d*x + c)^10 + 26*(210*b^3*cosh(d*x + c)^4 - 1
19*b^3*cosh(d*x + c)^2 + 90*a*b^2*cosh(d*x + c) + 9*b^3)*sinh(d*x + c)^12 -
 180*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^11 + 4*(3276*b^3*cosh(d*x + c)^5
 - 3094*b^3*cosh(d*x + c)^3 - 180*a*b^2*d*x + 3510*a*b^2*cosh(d*x + c)^2 +
702*b^3*cosh(d*x + c) - 135*a*b^2)*sinh(d*x + c)^11 + 2*(12012*b^3*cosh(d*x
 + c)^6 - 17017*b^3*cosh(d*x + c)^4 + 25740*a*b^2*cosh(d*x + c)^3 + 7722*b^
3*cosh(d*x + c)^2 - 189*b^3 - 990*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c))*si
nh(d*x + c)^10 + 360*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^9 + 4*(8580*b^3*co
sh(d*x + c)^7 - 17017*b^3*cosh(d*x + c)^5 + 32175*a*b^2*cosh(d*x + c)^4 + 1
2870*b^3*cosh(d*x + c)^3 + 540*a*b^2*d*x - 945*b^3*cosh(d*x + c) + 90*a*b^2
 - 2475*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 378*b^3*
cosh(d*x + c)^6 + 6*(6435*b^3*cosh(d*x + c)^8 - 17017*b^3*cosh(d*x + c)^6 +
 38610*a*b^2*cosh(d*x + c)^5 + 19305*b^3*cosh(d*x + c)^4 - 2835*b^3*cosh(d*
x + c)^2 - 4950*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^3 + 540*(6*a*b^2*d*x
+ a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 - 120*(18*a*b^2*d*x + 16*a^3 - 3*a*
b^2)*cosh(d*x + c)^7 + 24*(1430*b^3*cosh(d*x + c)^9 - 4862*b^3*cosh(d*x + c
)^7 + 12870*a*b^2*cosh(d*x + c)^6 + 7722*b^3*cosh(d*x + c)^5 - 1890*b^3*cos
h(d*x + c)^3 - 90*a*b^2*d*x - 2475*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^4
- 80*a^3 + 15*a*b^2 + 540*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^7 - 234*b^3*cosh(d*x + c)^4 + 6*(4004*b^3*cosh(d*x + c)^10 - 17017*b^3*
cosh(d*x + c)^8 + 51480*a*b^2*cosh(d*x + c)^7 + 36036*b^3*cosh(d*x + c)^6 -
 13230*b^3*cosh(d*x + c)^4 - 13860*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^5
+ 5040*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^3 + 63*b^3 - 140*(18*a*b^2*d*x +
 16*a^3 - 3*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 180*a*b^2*cosh(d*x + c)
^3 + 20*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)^5 + 4*(3276*b^3*co
sh(d*x + c)^11 - 17017*b^3*cosh(d*x + c)^9 + 57915*a*b^2*cosh(d*x + c)^8 +
46332*b^3*cosh(d*x + c)^7 - 23814*b^3*cosh(d*x + c)^5 - 20790*(4*a*b^2*d*x
+ 3*a*b^2)*cosh(d*x + c)^6 + 180*a*b^2*d*x + 11340*(6*a*b^2*d*x + a*b^2)*co
sh(d*x + c)^4 + 567*b^3*cosh(d*x + c) + 160*a^3 - 135*a*b^2 - 630*(18*a*b^2
*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 34*b^3*cosh(d*x
 + c)^2 + 2*(2730*b^3*cosh(d*x + c)^12 - 17017*b^3*cosh(d*x + c)^10 + 64350
*a*b^2*cosh(d*x + c)^9 + 57915*b^3*cosh(d*x + c)^8 - 39690*b^3*cosh(d*x + c
)^6 - 29700*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^7 + 22680*(6*a*b^2*d*x +
a*b^2)*cosh(d*x + c)^5 + 2835*b^3*cosh(d*x + c)^2 - 2100*(18*a*b^2*d*x + 16
*a^3 - 3*a*b^2)*cosh(d*x + c)^3 - 117*b^3 + 50*(36*a*b^2*d*x + 32*a^3 - 27*
a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(420*b^3*cosh(d*x + c)^13 - 3094*
```

b^3*cosh(d*x + c)^11 + 12870*a*b^2*cosh(d*x + c)^10 + 12870*b^3*cosh(d*x + c)^9 - 11340*b^3*cosh(d*x + c)^7 - 7425*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^8 + 7560*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^6 + 1890*b^3*cosh(d*x + c)^3 - 1050*(18*a*b^2*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^4 - 234*b^3*cosh(d*x + c) + 45*a*b^2 + 50*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 - 3*b^3 + 2*(180*b^3*cosh(d*x + c)^14 - 1547*b^3*cosh(d*x + c)^12 + 7020*a*b^2*cosh(d*x + c)^11 + 7722*b^3*cosh(d*x + c)^10 - 8505*b^3*cosh(d*x + c)^8 - 4950*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^9 + 6480*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^7 + 2835*b^3*cosh(d*x + c)^4 - 1260*(18*a*b^2*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^5 - 702*b^3*cosh(d*x + c)^2 + 270*a*b^2*cosh(d*x + c) + 100*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)^3 + 17*b^3)*sinh(d*x + c)^2 - 1440*(a^2*b*cosh(d*x + c)^11 + 11*a^2*b*cosh(d*x + c)*sinh(d*x + c)^10 + a^2*b*sinh(d*x + c)^11 - 3*a^2*b*cosh(d*x + c)^9 + 3*a^2*b*cosh(d*x + c)^7 + (55*a^2*b*cosh(d*x + c)^2 - 3*a^2*b)*sinh(d*x + c)^9 + 3*(55*a^2*b*cosh(d*x + c)^3 - 9*a^2*b*cosh(d*x + c))*sinh(d*x + c)^8 - a^2*b*cosh(d*x + c)^5 + 3*(110*a^2*b*cosh(d*x + c)^4 - 36*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^7 + 21*(22*a^2*b*cosh(d*x + c)^5 - 12*a^2*b*cosh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c)^6 + (462*a^2*b*cosh(d*x + c)^6 - 378*a^2*b*cosh(d*x + c)^4 + 63*a^2*b*cosh(d*x + c)^2 - a^2*b)*sinh(d*x + c)^5 + (330*a^2*b*cosh(d*x + c)^7 - 378*a^2*b*cosh(d*x + c)^5 + 105*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^4 + (165*a^2*b*cosh(d*x + c)^8 - 252*a^2*b*cosh(d*x + c)^6 + 105*a^2*b*cosh(d*x + c)^4 - 10*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*a^2*b*cosh(d*x + c)^9 - 108*a^2*b*cosh(d*x + c)^7 + 63*a^2*b*cosh(d*x + c)^5 - 10*a^2*b*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (11*a^2*b*cosh(d*x + c)^10 - 27*a^2*b*cosh(d*x + c)^8 + 21*a^2*b*cosh(d*x + c)^6 - 5*a^2*b*cosh(d*x + c)^4)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 1440*(a^2*b*cosh(d*x + c)^11 + 11*a^2*b*cosh(d*x + c)*sinh(d*x + c)^10 + a^2*b*sinh(d*x + c)^11 - 3*a^2*b*cosh(d*x + c)^9 + 3*a^2*b*cosh(d*x + c)^7 + (55*a^2*b*cosh(d*x + c)^2 - 3*a^2*b)*sinh(d*x + c)^9 + 3*(55*a^2*b*cosh(d*x + c)^3 - 9*a^2*b*cosh(d*x + c))*sinh(d*x + c)^8 - a^2*b*cosh(d*x + c)^5 + 3*(110*a^2*b*cosh(d*x + c)^4 - 36*a^2*b*cosh(d*x + c)^2 + a^2*b)*sinh(d*x + c)^7 + 21*(22*a^2*b*cosh(d*x + c)^5 - 12*a^2*b*cosh(d*x + c)^3 + a^2*b*cosh(d*x + c))*sinh(d*x + c)^6 + (462*a^2*b*cosh(d*x + c)^6 - 378*a^2*b*cosh(d*x + c)^4 + 63*a^2*b*cosh(d*x + c)^2 - a^2*b)*sinh(d*x + c)^5 + (330*a^2*b*cosh(d*x + c)^7 - 378*a^2*b*cosh(d*x + c)^5 + 105*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^4 + (165*a^2*b*cosh(d*x + c)^8 - 252*a^2*b*cosh(d*x + c)^6 + 105*a^2*b*cosh(d*x + c)^4 - 10*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*a^2*b*cosh(d*x + c)^9 - 108*a^2*b*cosh(d*x + c)^7 + 63*a^2*b*cosh(d*x + c)^5 - 10*a^2*b*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (11*a^2*b*cosh(d*x + c)^10 - 27*a^2*b*cosh(d*x + c)^8 + 21*a^2*b*cosh(d*x + c)^6 - 5*a^2*b*cosh(d*x + c)^4)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(12*b^3*cosh(d*x + c)^15 - 119*b^3*cosh(d*x + c)^13 + 585*a*b^2*cosh(d*x + c)^12 + 702*b^3*cosh(d*x + c)^11 - 945*b^3*cosh(d*x + c)^9 - 495*(4*a*b^2*d*x + 3*a*b^2)*cosh(d*x + c)^10 + 810*(6*a*b^2*d*x + a*b^2)*cosh(d*x + c)^8 + 567*b^3*cosh(d*x + c)^5 - 210*(18*a*b^2*d*x + 16*a^3 - 3*a*b^2)*cosh(d*x + c)^6 - 234*b^3*cosh(d*x + c)^3 + 135*a*b^2*cosh(d*x + c)^2 + 25*(36*a*b^2*d*x + 32*a^3 - 27*a*b^2)*cosh(d*x + c)^4 + 17*b^3*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 - 3*d*cosh(d*x + c)^9 + (55*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 - 9*d*cosh(d*x + c))*sinh(d*x + c)^8 + 3*d*cosh(d*x + c)^7 + 3*(110*d*cosh(d*x + c)^4 - 36*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 21*(22*d*cosh(d*x + c)^5 - 12*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^6 - d*cosh(d*x + c)^5 + (462*d*cosh(d*x + c)^6 - 378*d*cosh(d*x + c)^4 + 63*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^5 + (330*d*cosh(d*x + c)^7 - 378*d*cosh(d*x + c)^5 + 105*d*cosh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^4 + (165*d*cosh(d*x + c)^8 - 252*d*cosh(d*x + c)^6 + 105*d*cosh(d*x + c)^4 - 10*d*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (55*d*cosh(d*x + c)^9 - 108*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 - 10*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (11

```
*d*cosh(d*x + c)^10 - 27*d*cosh(d*x + c)^8 + 21*d*cosh(d*x + c)^6 - 5*d*cos
h(d*x + c)^4)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43691, size = 419, normalized size = 3.25

$$-\frac{3\,(dx+c)ab^2}{2\,d}-\frac{3\,a^2b\log\left(e^{(dx+c)}+1\right)}{d}+\frac{3\,a^2b\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}+\frac{3\,b^3d^4e^{(5\,dx+5\,c)}-25\,b^3d^4e^{(3\,dx+3\,c)}+180\,ab^2d^4e^{(2\,dx}}{480\,d^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -3/2*(d*x + c)*a*b^2/d - 3*a^2*b*log(e^(d*x + c) + 1)/d + 3*a^2*b*log(abs(e
^(d*x + c) - 1))/d + 1/480*(3*b^3*d^4*e^(5*d*x + 5*c) - 25*b^3*d^4*e^(3*d*x
 + 3*c) + 180*a*b^2*d^4*e^(2*d*x + 2*c) + 150*b^3*d^4*e^(d*x + c))/d^5 + 1/
480*(150*b^3*e^(10*d*x + 10*c) - 180*a*b^2*e^(9*d*x + 9*c) - 475*b^3*e^(8*d
*x + 8*c) + 528*b^3*e^(6*d*x + 6*c) - 234*b^3*e^(4*d*x + 4*c) + 180*a*b^2*e
^(3*d*x + 3*c) + 34*b^3*e^(2*d*x + 2*c) - 3*b^3 - 60*(32*a^3 - 9*a*b^2)*e^(
7*d*x + 7*c) + 20*(32*a^3 - 27*a*b^2)*e^(5*d*x + 5*c))*e^(-5*d*x - 5*c)/(d*
(e^(d*x + c) + 1)^3*(e^(d*x + c) - 1)^3)
```

3.168 $\int \mathrm{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=148

$$-\frac{3a^2b\coth(c+dx)}{d}-\frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^3\coth(c+dx)\mathrm{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\mathrm{csch}(c+dx)}{8d}+\frac{3ab}{}$$

```
[Out] (3*b^3*x)/8 - (3*a^3*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*b^2*Cosh[c + d*x]
)/d - (3*a^2*b*Coth[c + d*x])/d + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d)
 - (a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d) - (3*b^3*Cosh[c + d*x]*Sinh[c
+ d*x])/(8*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.172692, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 3767, 8, 3768, 3770, 2638, 2635}

$$-\frac{3a^2b\coth(c+dx)}{d}-\frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^3\coth(c+dx)\mathrm{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\mathrm{csch}(c+dx)}{8d}+\frac{3ab}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (3*b^3*x)/8 - (3*a^3*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*b^2*Cosh[c + d*x]
)/d - (3*a^2*b*Coth[c + d*x])/d + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d)
 - (a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d) - (3*b^3*Cosh[c + d*x]*Sinh[c
+ d*x])/(8*d) + (b^3*Cosh[c + d*x]*Sinh[c + d*x]^3)/(4*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\int \mathrm{csch}^5(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = i\int\left(-3ia^2b\,\mathrm{csch}^2(c+dx) - ia^3\mathrm{csch}^5(c+dx) - 3iab^2\sinh(c+dx) - ib^3$$

$$= a^3\int \mathrm{csch}^5(c+dx)\,dx + \left(3a^2b\right)\int \mathrm{csch}^2(c+dx)\,dx + \left(3ab^2\right)\int \sinh(c$$

$$= \frac{3ab^2\cosh(c+dx)}{d} - \frac{a^3\coth(c+dx)\mathrm{csch}^3(c+dx)}{4d} + \frac{b^3\cosh(c+dx)\sinh}{4d}$$

$$= \frac{3ab^2\cosh(c+dx)}{d} - \frac{3a^2b\coth(c+dx)}{d} + \frac{3a^3\coth(c+dx)\mathrm{csch}(c+dx)}{8d}$$

$$= \frac{3b^3x}{8} - \frac{3a^3\tanh^{-1}(\cosh(c+dx))}{8d} + \frac{3ab^2\cosh(c+dx)}{d} - \frac{3a^2b\coth(c+}{d}$$

Mathematica [A] time = 6.13235, size = 218, normalized size = 1.47

$$-\frac{3a^2b\tanh\left(\frac{1}{2}(c+dx)\right)}{2d} - \frac{3a^2b\coth\left(\frac{1}{2}(c+dx)\right)}{2d} - \frac{a^3\mathrm{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^3\mathrm{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a^3\mathrm{sech}^4\left(\frac{1}{2}(c+dx\right.}{64d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (3*b^3*(c + d*x))/(8*d) + (3*a*b^2*Cosh[c + d*x])/d - (3*a^2*b*Coth[(c + d*
x)/2])/(2*d) + (3*a^3*Csch[(c + d*x)/2]^2)/(32*d) - (a^3*Csch[(c + d*x)/2]^
4)/(64*d) + (3*a^3*Log[Tanh[(c + d*x)/2]])/(8*d) + (3*a^3*Sech[(c + d*x)/2]
^2)/(32*d) + (a^3*Sech[(c + d*x)/2]^4)/(64*d) - (b^3*Sinh[2*(c + d*x)])/(4*
d) + (b^3*Sinh[4*(c + d*x)])/(32*d) - (3*a^2*b*Tanh[(c + d*x)/2])/(2*d)
```

Maple [A] time = 0.078, size = 108, normalized size = 0.7

$$\frac{1}{d}\left(a^3\left(\left(-\frac{(\mathrm{csch}\,(dx+c))^3}{4} + \frac{3\,\mathrm{csch}\,(dx+c)}{8}\right)\coth(dx+c) - \frac{3\,\mathrm{Artanh}\left(e^{dx+c}\right)}{4}\right) - 3\,a^2b\coth(dx+c) + 3\,ab^2\cosh(dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(
d*x+c)))-3*a^2*b*coth(d*x+c)+3*a*b^2*cosh(d*x+c)+b^3*((1/4*sinh(d*x+c)^3-3/
8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.16093, size = 344, normalized size = 2.32

$$\frac{1}{64}\,b^3\left(24\,x+\frac{e^{(4\,dx+4\,c)}}{d}-\frac{8\,e^{(2\,dx+2\,c)}}{d}+\frac{8\,e^{(-2\,dx-2\,c)}}{d}-\frac{e^{(-4\,dx-4\,c)}}{d}\right)+\frac{3}{2}\,ab^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)-\frac{1}{8}\,a^3\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3/2*a*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) - 1
/8*a^3*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*
x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d
*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x
- 8*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 2.60223, size = 11867, normalized size = 80.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*cosh(d*x + c)^16 + 16*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + b^3*si
nh(d*x + c)^16 - 12*b^3*cosh(d*x + c)^14 + 96*a*b^2*cosh(d*x + c)^13 + 12*(
10*b^3*cosh(d*x + c)^2 - b^3)*sinh(d*x + c)^14 + 8*(70*b^3*cosh(d*x + c)^3
- 21*b^3*cosh(d*x + c) + 12*a*b^2)*sinh(d*x + c)^13 + 2*(12*b^3*d*x + 19*b^
3)*cosh(d*x + c)^12 + 2*(910*b^3*cosh(d*x + c)^4 + 12*b^3*d*x - 546*b^3*cos
h(d*x + c)^2 + 624*a*b^2*cosh(d*x + c) + 19*b^3)*sinh(d*x + c)^12 + 48*(a^3
 - 6*a*b^2)*cosh(d*x + c)^11 + 24*(182*b^3*cosh(d*x + c)^5 - 182*b^3*cosh(d
*x + c)^3 + 312*a*b^2*cosh(d*x + c)^2 + 2*a^3 - 12*a*b^2 + (12*b^3*d*x + 19
*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 4*(24*b^3*d*x + 96*a^2*b + 11*b^3)*
cosh(d*x + c)^10 + 4*(2002*b^3*cosh(d*x + c)^6 - 3003*b^3*cosh(d*x + c)^4 +
 6864*a*b^2*cosh(d*x + c)^3 - 24*b^3*d*x - 96*a^2*b - 11*b^3 + 33*(12*b^3*d
*x + 19*b^3)*cosh(d*x + c)^2 + 132*(a^3 - 6*a*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^10 - 16*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^9 + 8*(1430*b^3*cosh(d*x + c
)^7 - 3003*b^3*cosh(d*x + c)^5 + 8580*a*b^2*cosh(d*x + c)^4 + 55*(12*b^3*d*
x + 19*b^3)*cosh(d*x + c)^3 - 22*a^3 + 24*a*b^2 + 330*(a^3 - 6*a*b^2)*cosh(
d*x + c)^2 - 5*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c))*sinh(d*x + c
)^9 + 144*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^8 + 18*(715*b^3*cosh(d*x + c)^8
 - 2002*b^3*cosh(d*x + c)^6 + 6864*a*b^2*cosh(d*x + c)^5 + 8*b^3*d*x + 55*(
12*b^3*d*x + 19*b^3)*cosh(d*x + c)^4 + 440*(a^3 - 6*a*b^2)*cosh(d*x + c)^3
+ 64*a^2*b - 10*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^2 - 8*(11*a^
3 - 12*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^8 - 16*(11*a^3 - 12*a*b^2)*cosh(
d*x + c)^7 + 16*(715*b^3*cosh(d*x + c)^9 - 2574*b^3*cosh(d*x + c)^7 + 10296
*a*b^2*cosh(d*x + c)^6 + 99*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^5 + 990*(a^
3 - 6*a*b^2)*cosh(d*x + c)^4 - 30*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x
 + c)^3 - 11*a^3 + 12*a*b^2 - 36*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^2 + 72*(
b^3*d*x + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 4*(24*b^3*d*x + 288*a^2
*b - 11*b^3)*cosh(d*x + c)^6 + 4*(2002*b^3*cosh(d*x + c)^10 - 9009*b^3*cosh
(d*x + c)^8 + 41184*a*b^2*cosh(d*x + c)^7 + 462*(12*b^3*d*x + 19*b^3)*cosh(
d*x + c)^6 + 5544*(a^3 - 6*a*b^2)*cosh(d*x + c)^5 - 24*b^3*d*x - 210*(24*b^
3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^4 - 336*(11*a^3 - 12*a*b^2)*cosh(d
*x + c)^3 - 288*a^2*b + 11*b^3 + 1008*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^2 -
```

28*(11*a^3 - 12*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 + 96*a*b^2*cosh(d*x + c)^3 + 48*(a^3 - 6*a*b^2)*cosh(d*x + c)^5 + 24*(182*b^3*cosh(d*x + c)^11 - 1001*b^3*cosh(d*x + c)^9 + 5148*a*b^2*cosh(d*x + c)^8 + 66*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^7 + 924*(a^3 - 6*a*b^2)*cosh(d*x + c)^6 - 42*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^5 - 84*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^4 + 336*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^3 + 2*a^3 - 12*a*b^2 - 14*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^2 - (24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 12*b^3*cosh(d*x + c)^2 + 2*(12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)^4 + 2*(910*b^3*cosh(d*x + c)^12 - 6006*b^3*cosh(d*x + c)^10 + 34320*a*b^2*cosh(d*x + c)^9 + 495*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^8 + 7920*(a^3 - 6*a*b^2)*cosh(d*x + c)^7 - 420*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^6 - 1008*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^5 + 12*b^3*d*x + 5040*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^4 - 280*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^3 + 192*a^2*b - 19*b^3 - 30*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^2 + 120*(a^3 - 6*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(70*b^3*cosh(d*x + c)^13 - 546*b^3*cosh(d*x + c)^11 + 3432*a*b^2*cosh(d*x + c)^10 + 55*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^9 + 990*(a^3 - 6*a*b^2)*cosh(d*x + c)^8 - 60*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^7 - 168*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^6 + 1008*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^5 - 70*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^4 - 10*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^3 + 12*a*b^2 + 60*(a^3 - 6*a*b^2)*cosh(d*x + c)^2 + (12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - b^3 + 12*(10*b^3*cosh(d*x + c)^14 - 91*b^3*cosh(d*x + c)^12 + 624*a*b^2*cosh(d*x + c)^11 + 11*(12*b^3*d*x + 19*b^3)*cosh(d*x + c)^10 + 220*(a^3 - 6*a*b^2)*cosh(d*x + c)^9 - 15*(24*b^3*d*x + 96*a^2*b + 11*b^3)*cosh(d*x + c)^8 - 48*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^7 + 336*(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^6 - 28*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^5 - 5*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^4 + 24*a*b^2*cosh(d*x + c) + 40*(a^3 - 6*a*b^2)*cosh(d*x + c)^3 + b^3 + (12*b^3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 24*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*sinh(d*x + c)^12 - 4*a^3*cosh(d*x + c)^10 + 6*a^3*cosh(d*x + c)^8 + 2*(33*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 4*a^3*cosh(d*x + c)^6 + 3*(165*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + 2*a^3)*sinh(d*x + c)^8 + 24*(33*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + 2*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 - 210*a^3*cosh(d*x + c)^4 + 42*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^6 + 24*(33*a^3*cosh(d*x + c)^7 - 42*a^3*cosh(d*x + c)^5 + 14*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c)^5 + (495*a^3*cosh(d*x + c)^8 - 840*a^3*cosh(d*x + c)^6 + 420*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 - 120*a^3*cosh(d*x + c)^7 + 84*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(11*a^3*cosh(d*x + c)^10 - 30*a^3*cosh(d*x + c)^8 + 28*a^3*cosh(d*x + c)^6 - 10*a^3*cosh(d*x + c)^4 + a^3*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*a^3*cosh(d*x + c)^11 - 10*a^3*cosh(d*x + c)^9 + 12*a^3*cosh(d*x + c)^7 - 6*a^3*cosh(d*x + c)^5 + a^3*cosh(d*x + c)^3)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 24*(a^3*cosh(d*x + c)^12 + 12*a^3*cosh(d*x + c)*sinh(d*x + c)^11 + a^3*sinh(d*x + c)^12 - 4*a^3*cosh(d*x + c)^10 + 6*a^3*cosh(d*x + c)^8 + 2*(33*a^3*cosh(d*x + c)^2 - 2*a^3)*sinh(d*x + c)^10 + 20*(11*a^3*cosh(d*x + c)^3 - 2*a^3*cosh(d*x + c))*sinh(d*x + c)^9 - 4*a^3*cosh(d*x + c)^6 + 3*(165*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + 2*a^3)*sinh(d*x + c)^8 + 24*(33*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + 2*a^3*cosh(d*x + c))*sinh(d*x + c)^7 + a^3*cosh(d*x + c)^4 + 4*(231*a^3*cosh(d*x + c)^6 - 210*a^3*cosh(d*x + c)^4 + 42*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^6 + 24*(33*a^3*cosh(d*x + c)^7 - 42*a^3*cosh(d*x + c)^5 + 14*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c)^5 + (495*a^3*cosh(d*x + c)^8 - 840*a^3*cosh(d*x + c)^6 + 420*a^3*cosh(d*x + c)^4 - 60*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^4 + 4*(55*a^3*cosh(d*x + c)^9 - 120*a^3*cosh(d*x + c)^7 + 84*a^3*cosh(d*x + c)^5 - 20*a^3*cosh(d*x + c)^3 + a^3*cosh(d*x + c))*si

```
nh(d*x + c)^3 + 6*(11*a^3*cosh(d*x + c)^10 - 30*a^3*cosh(d*x + c)^8 + 28*a^
3*cosh(d*x + c)^6 - 10*a^3*cosh(d*x + c)^4 + a^3*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 4*(3*a^3*cosh(d*x + c)^11 - 10*a^3*cosh(d*x + c)^9 + 12*a^3*cosh(d
*x + c)^7 - 6*a^3*cosh(d*x + c)^5 + a^3*cosh(d*x + c)^3)*sinh(d*x + c))*log
(cosh(d*x + c) + sinh(d*x + c) - 1) + 8*(2*b^3*cosh(d*x + c)^15 - 21*b^3*co
sh(d*x + c)^13 + 156*a*b^2*cosh(d*x + c)^12 + 3*(12*b^3*d*x + 19*b^3)*cosh(
d*x + c)^11 + 66*(a^3 - 6*a*b^2)*cosh(d*x + c)^10 - 5*(24*b^3*d*x + 96*a^2*
b + 11*b^3)*cosh(d*x + c)^9 - 18*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^8 + 144*
(b^3*d*x + 8*a^2*b)*cosh(d*x + c)^7 - 14*(11*a^3 - 12*a*b^2)*cosh(d*x + c)^
6 - 3*(24*b^3*d*x + 288*a^2*b - 11*b^3)*cosh(d*x + c)^5 + 36*a*b^2*cosh(d*x
 + c)^2 + 30*(a^3 - 6*a*b^2)*cosh(d*x + c)^4 + 3*b^3*cosh(d*x + c) + (12*b^
3*d*x + 192*a^2*b - 19*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c
)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh(d*x + c)^12 - 4*d*cosh(
d*x + c)^10 + 2*(33*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^10 + 20*(11*d*co
sh(d*x + c)^3 - 2*d*cosh(d*x + c))*sinh(d*x + c)^9 + 6*d*cosh(d*x + c)^8 +
3*(165*d*cosh(d*x + c)^4 - 60*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^8 + 24
*(33*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x
 + c)^7 - 4*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c)^6 - 210*d*cosh(d*x +
 c)^4 + 42*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^
7 - 42*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x
 + c)^5 + d*cosh(d*x + c)^4 + (495*d*cosh(d*x + c)^8 - 840*d*cosh(d*x + c)^
6 + 420*d*cosh(d*x + c)^4 - 60*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 4*(
55*d*cosh(d*x + c)^9 - 120*d*cosh(d*x + c)^7 + 84*d*cosh(d*x + c)^5 - 20*d*
cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(11*d*cosh(d*x + c)^
10 - 30*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 - 10*d*cosh(d*x + c)^4 + d
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 4*(3*d*cosh(d*x + c)^11 - 10*d*cosh(d*x
 + c)^9 + 12*d*cosh(d*x + c)^7 - 6*d*cosh(d*x + c)^5 + d*cosh(d*x + c)^3)*s
inh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.4095, size = 475, normalized size = 3.21

$$\frac{3\,(dx+c)b^3}{8\,d}-\frac{3\,a^3\log\left(e^{(dx+c)}+1\right)}{8\,d}+\frac{3\,a^3\log\left(\left|e^{(dx+c)}-1\right|\right)}{8\,d}+\frac{b^3d^3e^{(4\,dx+4\,c)}-8\,b^3d^3e^{(2\,dx+2\,c)}+96\,ab^2d^3e^{(dx+c)}}{64\,d^4}+\frac{(96\,ab^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 3/8*(d*x + c)*b^3/d - 3/8*a^3*log(e^(d*x + c) + 1)/d + 3/8*a^3*log(abs(e^(d
*x + c) - 1))/d + 1/64*(b^3*d^3*e^(4*d*x + 4*c) - 8*b^3*d^3*e^(2*d*x + 2*c)
 + 96*a*b^2*d^3*e^(d*x + c))/d^4 + 1/64*(96*a*b^2*e^(3*d*x + 3*c) + 12*b^3*
e^(2*d*x + 2*c) - b^3 + 48*(a^3 + 2*a*b^2)*e^(11*d*x + 11*c) - 8*(48*a^2*b
- b^3)*e^(10*d*x + 10*c) - 16*(11*a^3 + 24*a*b^2)*e^(9*d*x + 9*c) + 3*(384*
a^2*b - 11*b^3)*e^(8*d*x + 8*c) - 16*(11*a^3 - 36*a*b^2)*e^(7*d*x + 7*c) -
```

```
4*(288*a^2*b - 13*b^3)*e^(6*d*x + 6*c) + 48*(a^3 - 8*a*b^2)*e^(5*d*x + 5*c)
 + 2*(192*a^2*b - 19*b^3)*e^(4*d*x + 4*c))*e^(-4*d*x - 4*c)/(d*(e^(d*x + c)
 + 1)^4*(e^(d*x + c) - 1)^4)
```

3.169 $\int \mathbf{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3 dx$

Optimal. Leaf size=131

$$\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3a^2b\coth(c+dx)\mathbf{csch}(c+dx)}{2d}-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)}{d}$$

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^3*Cosh[c + d*x])/d
+ (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x])/d + (2*a^3*Coth[c + d*x
]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) - (3*a^2*b*Coth[c + d*x]*Csch[c +
d*x])/(2*d)
```

Rubi [A] time = 0.124965, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3768, 3770, 3767, 2633}

$$\frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{3a^2b\coth(c+dx)\mathbf{csch}(c+dx)}{2d}-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}-\frac{a^3\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] 3*a*b^2*x + (3*a^2*b*ArcTanh[Cosh[c + d*x]])/(2*d) - (b^3*Cosh[c + d*x])/d
+ (b^3*Cosh[c + d*x]^3)/(3*d) - (a^3*Coth[c + d*x])/d + (2*a^3*Coth[c + d*x
]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) - (3*a^2*b*Coth[c + d*x]*Csch[c +
d*x])/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
```

```
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\int\left(-3ab^2-3a^2b\operatorname{csch}^3(c+dx)-a^3\operatorname{csch}^6(c+dx)-b^3\sinh^3(c+dx)\right)$$
$$= 3ab^2x + a^3\int \operatorname{csch}^6(c+dx)\,dx + \left(3a^2b\right)\int \operatorname{csch}^3(c+dx)\,dx + b^3\int \sin$$
$$= 3ab^2x - \frac{3a^2b\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{1}{2}\left(3a^2b\right)\int \operatorname{csch}(c+dx)\,dx -$$
$$= 3ab^2x + \frac{3a^2b\tanh^{-1}(\cosh(c+dx))}{2d} - \frac{b^3\cosh(c+dx)}{d} + \frac{b^3\cosh^3(c+d}{3d}$$

Mathematica [A] time = 1.84643, size = 225, normalized size = 1.72

$$\frac{1}{2}a\left(8\left(-32a^2\tanh\left(\frac{1}{2}(c+dx)\right)-24a^2\sinh^6\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^5(c+dx)-38a^2\sinh^4\left(\frac{1}{2}(c+dx)\right)\operatorname{csch}^3(c+dx)-45ab\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] (-360*b^3*Cosh[c + d*x] + 40*b^3*Cosh[3*(c + d*x)] + (a*(-256*a^2*Coth[(c +
 d*x)/2] - 360*a*b*Csch[(c + d*x)/2]^2 + 19*a^2*Csch[(c + d*x)/2]^4*Sinh[c
+ d*x] - 3*a^2*Csch[(c + d*x)/2]^6*Sinh[c + d*x] + 8*(180*b*(2*b*(c + d*x)
- a*Log[Tanh[(c + d*x)/2]]) - 45*a*b*Sech[(c + d*x)/2]^2 - 38*a^2*Csch[c +
d*x]^3*Sinh[(c + d*x)/2]^4 - 24*a^2*Csch[c + d*x]^5*Sinh[(c + d*x)/2]^6 - 3
2*a^2*Tanh[(c + d*x)/2])))/2)/(480*d)
```

Maple [A] time = 0.085, size = 99, normalized size = 0.8

$$\frac{1}{d}\left(a^3\left(-\frac{8}{15}-\frac{(\operatorname{csch}(dx+c))^4}{5}+\frac{4(\operatorname{csch}(dx+c))^2}{15}\right)\coth(dx+c)+3a^2b\left(-1/2\operatorname{csch}(dx+c)\coth(dx+c)+\operatorname{Artanh}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(
-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a*b^2*(d*x+c)+b^3*(-2/3
+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maximا [B] time = 1.08746, size = 493, normalized size = 3.76

$$3ab^2x+\frac{1}{24}b^3\left(\frac{e^{(3dx+3c)}}{d}-\frac{9e^{(dx+c)}}{d}-\frac{9e^{(-dx-c)}}{d}+\frac{e^{(-3dx-3c)}}{d}\right)+\frac{3}{2}a^2b\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2(e}{d(2e^{(-}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] 3*a*b^2*x + 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/
d + e^(-3*d*x - 3*c)/d) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x
- c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) - 1))) - 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2
*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-
10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-
4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c
) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6
*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)))
```

Fricas [B] time = 2.36594, size = 12388, normalized size = 94.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/120*(5*b^3*cosh(d*x + c)^16 + 80*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 5*b
^3*sinh(d*x + c)^16 + 360*a*b^2*d*x*cosh(d*x + c)^13 - 70*b^3*cosh(d*x + c)
^14 - 1800*a*b^2*d*x*cosh(d*x + c)^11 + 10*(60*b^3*cosh(d*x + c)^2 - 7*b^3)
*sinh(d*x + c)^14 + 3600*a*b^2*d*x*cosh(d*x + c)^9 + 20*(140*b^3*cosh(d*x +
 c)^3 + 18*a*b^2*d*x - 49*b^3*cosh(d*x + c))*sinh(d*x + c)^13 - 10*(36*a^2*
b - 23*b^3)*cosh(d*x + c)^12 + 10*(910*b^3*cosh(d*x + c)^4 + 468*a*b^2*d*x*
cosh(d*x + c) - 637*b^3*cosh(d*x + c)^2 - 36*a^2*b + 23*b^3)*sinh(d*x + c)^
12 + 40*(546*b^3*cosh(d*x + c)^5 + 702*a*b^2*d*x*cosh(d*x + c)^2 - 637*b^3*
cosh(d*x + c)^3 - 45*a*b^2*d*x - 3*(36*a^2*b - 23*b^3)*cosh(d*x + c))*sinh(
d*x + c)^11 + 90*(8*a^2*b - 3*b^3)*cosh(d*x + c)^10 + 10*(4004*b^3*cosh(d*x
 + c)^6 + 10296*a*b^2*d*x*cosh(d*x + c)^3 - 7007*b^3*cosh(d*x + c)^4 - 1980
*a*b^2*d*x*cosh(d*x + c) + 72*a^2*b - 27*b^3 - 66*(36*a^2*b - 23*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^10 + 20*(2860*b^3*cosh(d*x + c)^7 + 12870*a*b^2*d
*x*cosh(d*x + c)^4 - 7007*b^3*cosh(d*x + c)^5 - 4950*a*b^2*d*x*cosh(d*x + c
)^2 + 180*a*b^2*d*x - 110*(36*a^2*b - 23*b^3)*cosh(d*x + c)^3 + 45*(8*a^2*b
 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 30*(2145*b^3*cosh(d*x + c)^8 + 1
5444*a*b^2*d*x*cosh(d*x + c)^5 - 7007*b^3*cosh(d*x + c)^6 - 9900*a*b^2*d*x*
cosh(d*x + c)^3 + 1080*a*b^2*d*x*cosh(d*x + c) - 165*(36*a^2*b - 23*b^3)*co
sh(d*x + c)^4 + 135*(8*a^2*b - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 - 80
*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^7 + 80*(715*b^3*cosh(d*x + c)^9 + 77
22*a*b^2*d*x*cosh(d*x + c)^6 - 3003*b^3*cosh(d*x + c)^7 - 7425*a*b^2*d*x*co
sh(d*x + c)^4 + 1620*a*b^2*d*x*cosh(d*x + c)^2 - 99*(36*a^2*b - 23*b^3)*cos
h(d*x + c)^5 - 45*a*b^2*d*x + 135*(8*a^2*b - 3*b^3)*cosh(d*x + c)^3 - 16*a^
3)*sinh(d*x + c)^7 - 90*(8*a^2*b - 3*b^3)*cosh(d*x + c)^6 + 10*(4004*b^3*co
sh(d*x + c)^10 + 61776*a*b^2*d*x*cosh(d*x + c)^7 - 21021*b^3*cosh(d*x + c)^
8 - 83160*a*b^2*d*x*cosh(d*x + c)^5 + 30240*a*b^2*d*x*cosh(d*x + c)^3 - 924
*(36*a^2*b - 23*b^3)*cosh(d*x + c)^6 + 1890*(8*a^2*b - 3*b^3)*cosh(d*x + c)
^4 - 72*a^2*b + 27*b^3 - 56*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*sinh(d*x
 + c)^6 + 40*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^5 + 20*(1092*b^3*cosh(d*
x + c)^11 + 23166*a*b^2*d*x*cosh(d*x + c)^8 - 7007*b^3*cosh(d*x + c)^9 - 41
580*a*b^2*d*x*cosh(d*x + c)^6 + 22680*a*b^2*d*x*cosh(d*x + c)^4 - 396*(36*a
^2*b - 23*b^3)*cosh(d*x + c)^7 + 1134*(8*a^2*b - 3*b^3)*cosh(d*x + c)^5 + 9
0*a*b^2*d*x + 32*a^3 - 84*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^2 - 27*(8*a
^2*b - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 70*b^3*cosh(d*x + c)^2 + 10*
(36*a^2*b - 23*b^3)*cosh(d*x + c)^4 + 10*(910*b^3*cosh(d*x + c)^12 + 25740*
a*b^2*d*x*cosh(d*x + c)^9 - 7007*b^3*cosh(d*x + c)^10 - 59400*a*b^2*d*x*cos
h(d*x + c)^7 + 45360*a*b^2*d*x*cosh(d*x + c)^5 - 495*(36*a^2*b - 23*b^3)*co
sh(d*x + c)^8 + 1890*(8*a^2*b - 3*b^3)*cosh(d*x + c)^6 - 280*(45*a*b^2*d*x
+ 16*a^3)*cosh(d*x + c)^3 + 36*a^2*b - 23*b^3 - 135*(8*a^2*b - 3*b^3)*cosh(
```

```
d*x + c)^2 + 20*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 8*
(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^3 + 8*(350*b^3*cosh(d*x + c)^13 + 128
70*a*b^2*d*x*cosh(d*x + c)^10 - 3185*b^3*cosh(d*x + c)^11 - 37125*a*b^2*d*x
*cosh(d*x + c)^8 + 37800*a*b^2*d*x*cosh(d*x + c)^6 - 275*(36*a^2*b - 23*b^3
)*cosh(d*x + c)^9 + 1350*(8*a^2*b - 3*b^3)*cosh(d*x + c)^7 - 45*a*b^2*d*x -
 350*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^4 - 225*(8*a^2*b - 3*b^3)*cosh(d
*x + c)^3 - 16*a^3 + 50*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^2 + 5*(36*a^2
*b - 23*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 5*b^3 + 2*(300*b^3*cosh(d*x +
 c)^14 + 14040*a*b^2*d*x*cosh(d*x + c)^11 - 3185*b^3*cosh(d*x + c)^12 - 495
00*a*b^2*d*x*cosh(d*x + c)^9 + 64800*a*b^2*d*x*cosh(d*x + c)^7 - 330*(36*a^
2*b - 23*b^3)*cosh(d*x + c)^10 + 2025*(8*a^2*b - 3*b^3)*cosh(d*x + c)^8 - 8
40*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^5 - 675*(8*a^2*b - 3*b^3)*cosh(d*x
 + c)^4 + 200*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^3 + 35*b^3 + 30*(36*a^2
*b - 23*b^3)*cosh(d*x + c)^2 - 12*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c))*si
nh(d*x + c)^2 + 180*(a^2*b*cosh(d*x + c)^13 + 13*a^2*b*cosh(d*x + c)*sinh(d
*x + c)^12 + a^2*b*sinh(d*x + c)^13 - 5*a^2*b*cosh(d*x + c)^11 + 10*a^2*b*c
osh(d*x + c)^9 + (78*a^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^11 + 11
*(26*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^10 - 10*a
^2*b*cosh(d*x + c)^7 + 5*(143*a^2*b*cosh(d*x + c)^4 - 55*a^2*b*cosh(d*x + c
)^2 + 2*a^2*b)*sinh(d*x + c)^9 + 3*(429*a^2*b*cosh(d*x + c)^5 - 275*a^2*b*c
osh(d*x + c)^3 + 30*a^2*b*cosh(d*x + c))*sinh(d*x + c)^8 + 5*a^2*b*cosh(d*x
 + c)^5 + 2*(858*a^2*b*cosh(d*x + c)^6 - 825*a^2*b*cosh(d*x + c)^4 + 180*a^
2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^7 + 2*(858*a^2*b*cosh(d*x + c)
^7 - 1155*a^2*b*cosh(d*x + c)^5 + 420*a^2*b*cosh(d*x + c)^3 - 35*a^2*b*cosh
(d*x + c))*sinh(d*x + c)^6 - a^2*b*cosh(d*x + c)^3 + (1287*a^2*b*cosh(d*x +
 c)^8 - 2310*a^2*b*cosh(d*x + c)^6 + 1260*a^2*b*cosh(d*x + c)^4 - 210*a^2*b
*cosh(d*x + c)^2 + 5*a^2*b)*sinh(d*x + c)^5 + 5*(143*a^2*b*cosh(d*x + c)^9
- 330*a^2*b*cosh(d*x + c)^7 + 252*a^2*b*cosh(d*x + c)^5 - 70*a^2*b*cosh(d*x
 + c)^3 + 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^4 + (286*a^2*b*cosh(d*x + c)
^10 - 825*a^2*b*cosh(d*x + c)^8 + 840*a^2*b*cosh(d*x + c)^6 - 350*a^2*b*cos
h(d*x + c)^4 + 50*a^2*b*cosh(d*x + c)^2 - a^2*b)*sinh(d*x + c)^3 + (78*a^2*
b*cosh(d*x + c)^11 - 275*a^2*b*cosh(d*x + c)^9 + 360*a^2*b*cosh(d*x + c)^7
- 210*a^2*b*cosh(d*x + c)^5 + 50*a^2*b*cosh(d*x + c)^3 - 3*a^2*b*cosh(d*x +
 c))*sinh(d*x + c)^2 + (13*a^2*b*cosh(d*x + c)^12 - 55*a^2*b*cosh(d*x + c)^
10 + 90*a^2*b*cosh(d*x + c)^8 - 70*a^2*b*cosh(d*x + c)^6 + 25*a^2*b*cosh(d*
x + c)^4 - 3*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh
(d*x + c) + 1) - 180*(a^2*b*cosh(d*x + c)^13 + 13*a^2*b*cosh(d*x + c)*sinh(
d*x + c)^12 + a^2*b*sinh(d*x + c)^13 - 5*a^2*b*cosh(d*x + c)^11 + 10*a^2*b*
cosh(d*x + c)^9 + (78*a^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^11 + 1
1*(26*a^2*b*cosh(d*x + c)^3 - 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^10 - 10*
a^2*b*cosh(d*x + c)^7 + 5*(143*a^2*b*cosh(d*x + c)^4 - 55*a^2*b*cosh(d*x +
c)^2 + 2*a^2*b)*sinh(d*x + c)^9 + 3*(429*a^2*b*cosh(d*x + c)^5 - 275*a^2*b*
cosh(d*x + c)^3 + 30*a^2*b*cosh(d*x + c))*sinh(d*x + c)^8 + 5*a^2*b*cosh(d*
x + c)^5 + 2*(858*a^2*b*cosh(d*x + c)^6 - 825*a^2*b*cosh(d*x + c)^4 + 180*a
^2*b*cosh(d*x + c)^2 - 5*a^2*b)*sinh(d*x + c)^7 + 2*(858*a^2*b*cosh(d*x + c
)^7 - 1155*a^2*b*cosh(d*x + c)^5 + 420*a^2*b*cosh(d*x + c)^3 - 35*a^2*b*cos
h(d*x + c))*sinh(d*x + c)^6 - a^2*b*cosh(d*x + c)^3 + (1287*a^2*b*cosh(d*x
+ c)^8 - 2310*a^2*b*cosh(d*x + c)^6 + 1260*a^2*b*cosh(d*x + c)^4 - 210*a^2*
b*cosh(d*x + c)^2 + 5*a^2*b)*sinh(d*x + c)^5 + 5*(143*a^2*b*cosh(d*x + c)^9
 - 330*a^2*b*cosh(d*x + c)^7 + 252*a^2*b*cosh(d*x + c)^5 - 70*a^2*b*cosh(d*
x + c)^3 + 5*a^2*b*cosh(d*x + c))*sinh(d*x + c)^4 + (286*a^2*b*cosh(d*x + c
)^10 - 825*a^2*b*cosh(d*x + c)^8 + 840*a^2*b*cosh(d*x + c)^6 - 350*a^2*b*co
sh(d*x + c)^4 + 50*a^2*b*cosh(d*x + c)^2 - a^2*b)*sinh(d*x + c)^3 + (78*a^2
*b*cosh(d*x + c)^11 - 275*a^2*b*cosh(d*x + c)^9 + 360*a^2*b*cosh(d*x + c)^7
 - 210*a^2*b*cosh(d*x + c)^5 + 50*a^2*b*cosh(d*x + c)^3 - 3*a^2*b*cosh(d*x
+ c))*sinh(d*x + c)^2 + (13*a^2*b*cosh(d*x + c)^12 - 55*a^2*b*cosh(d*x + c)
^10 + 90*a^2*b*cosh(d*x + c)^8 - 70*a^2*b*cosh(d*x + c)^6 + 25*a^2*b*cosh(d
*x + c)^4 - 3*a^2*b*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sin
h(d*x + c) - 1) + 4*(20*b^3*cosh(d*x + c)^15 + 1170*a*b^2*d*x*cosh(d*x + c)
```

```
^12 - 245*b^3*cosh(d*x + c)^13 - 4950*a*b^2*d*x*cosh(d*x + c)^10 + 8100*a*b
^2*d*x*cosh(d*x + c)^8 - 30*(36*a^2*b - 23*b^3)*cosh(d*x + c)^11 + 225*(8*a
^2*b - 3*b^3)*cosh(d*x + c)^9 - 140*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^6
 - 135*(8*a^2*b - 3*b^3)*cosh(d*x + c)^5 + 50*(45*a*b^2*d*x + 16*a^3)*cosh(
d*x + c)^4 + 35*b^3*cosh(d*x + c) + 10*(36*a^2*b - 23*b^3)*cosh(d*x + c)^3
- 6*(45*a*b^2*d*x + 16*a^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c
)^13 + 13*d*cosh(d*x + c)*sinh(d*x + c)^12 + d*sinh(d*x + c)^13 - 5*d*cosh(
d*x + c)^11 + (78*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^11 + 11*(26*d*cosh
(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^10 + 10*d*cosh(d*x + c)^9 +
5*(143*d*cosh(d*x + c)^4 - 55*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^9 + 3*
(429*d*cosh(d*x + c)^5 - 275*d*cosh(d*x + c)^3 + 30*d*cosh(d*x + c))*sinh(d
*x + c)^8 - 10*d*cosh(d*x + c)^7 + 2*(858*d*cosh(d*x + c)^6 - 825*d*cosh(d*
x + c)^4 + 180*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c)^7 + 2*(858*d*cosh(d*x
 + c)^7 - 1155*d*cosh(d*x + c)^5 + 420*d*cosh(d*x + c)^3 - 35*d*cosh(d*x +
c))*sinh(d*x + c)^6 + 5*d*cosh(d*x + c)^5 + (1287*d*cosh(d*x + c)^8 - 2310*
d*cosh(d*x + c)^6 + 1260*d*cosh(d*x + c)^4 - 210*d*cosh(d*x + c)^2 + 5*d)*s
inh(d*x + c)^5 + 5*(143*d*cosh(d*x + c)^9 - 330*d*cosh(d*x + c)^7 + 252*d*c
osh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^4
- d*cosh(d*x + c)^3 + (286*d*cosh(d*x + c)^10 - 825*d*cosh(d*x + c)^8 + 840
*d*cosh(d*x + c)^6 - 350*d*cosh(d*x + c)^4 + 50*d*cosh(d*x + c)^2 - d)*sinh
(d*x + c)^3 + (78*d*cosh(d*x + c)^11 - 275*d*cosh(d*x + c)^9 + 360*d*cosh(d
*x + c)^7 - 210*d*cosh(d*x + c)^5 + 50*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c
))*sinh(d*x + c)^2 + (13*d*cosh(d*x + c)^12 - 55*d*cosh(d*x + c)^10 + 90*d*
cosh(d*x + c)^8 - 70*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 - 3*d*cosh(d*
x + c)^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.44204, size = 389, normalized size = 2.97

$$\frac{3\,(dx+c)ab^2}{d}+\frac{3\,a^2b\log\left(e^{(dx+c)}+1\right)}{2\,d}-\frac{3\,a^2b\log\left(\left|e^{(dx+c)}-1\right|\right)}{2\,d}+\frac{b^3d^2e^{(3\,dx+3\,c)}-9\,b^3d^2e^{(dx+c)}}{24\,d^3}-\frac{\left(475\,b^3e^{(8\,dx+8\,c)}+1280\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] 3*(d*x + c)*a*b^2/d + 3/2*a^2*b*log(e^(d*x + c) + 1)/d - 3/2*a^2*b*log(abs(
e^(d*x + c) - 1))/d + 1/24*(b^3*d^2*e^(3*d*x + 3*c) - 9*b^3*d^2*e^(d*x + c)
)/d^3 - 1/120*(475*b^3*e^(8*d*x + 8*c) + 1280*a^3*e^(7*d*x + 7*c) - 640*a^3
*e^(5*d*x + 5*c) + 128*a^3*e^(3*d*x + 3*c) - 70*b^3*e^(2*d*x + 2*c) + 5*b^3
 + 45*(8*a^2*b + b^3)*e^(12*d*x + 12*c) - 10*(72*a^2*b + 23*b^3)*e^(10*d*x
+ 10*c) + 20*(36*a^2*b - 25*b^3)*e^(6*d*x + 6*c) - 5*(72*a^2*b - 55*b^3)*e^
(4*d*x + 4*c))*e^(-3*d*x - 3*c)/(d*(e^(d*x + c) + 1)^5*(e^(d*x + c) - 1)^5)
```

3.170 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx$

Optimal. Leaf size=166

$$-\frac{a^2b\coth^3(c+dx)}{d}+\frac{3a^2b\coth(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3\coth(c+$$

```
[Out] -(b^3*x)/2 + (5*a^3*ArcTanh[Cosh[c + d*x]])/(16*d) - (3*a*b^2*ArcTanh[Cosh[
c + d*x]])/d + (3*a^2*b*Coth[c + d*x])/d - (a^2*b*Coth[c + d*x]^3)/d - (5*a
^3*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^3*Coth[c + d*x]*Csch[c + d*x]
^3)/(24*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d) + (b^3*Cosh[c + d*x]
*Sinh[c + d*x])/(2*d)
```

Rubi [A] time = 0.197102, antiderivative size = 166, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3770, 3767, 3768, 2635, 8}

$$-\frac{a^2b\coth^3(c+dx)}{d}+\frac{3a^2b\coth(c+dx)}{d}+\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3\coth(c+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] -(b^3*x)/2 + (5*a^3*ArcTanh[Cosh[c + d*x]])/(16*d) - (3*a*b^2*ArcTanh[Cosh[
c + d*x]])/d + (3*a^2*b*Coth[c + d*x])/d - (a^2*b*Coth[c + d*x]^3)/d - (5*a
^3*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^3*Coth[c + d*x]*Csch[c + d*x]
^3)/(24*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d) + (b^3*Cosh[c + d*x]
*Sinh[c + d*x])/(2*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \text{csch}^7(c+dx)\left(a+b\sinh^3(c+dx)\right)^3\,dx = -\left(i\int\left(3iab^2\text{csch}(c+dx)+3ia^2b\text{csch}^4(c+dx)+ia^3\text{csch}^7(c+dx)+ib^3\text{si}\right.\right.$$

$$= a^3\int\text{csch}^7(c+dx)\,dx+\left(3a^2b\right)\int\text{csch}^4(c+dx)\,dx+\left(3ab^2\right)\int\text{csch}(c+d$$

$$= -\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{a^3\coth(c+dx)\text{csch}^5(c+dx)}{6d}+\frac{b^3\cosh(c-}{}$$

$$= -\frac{b^3x}{2}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\coth(c+dx)}{d}-\frac{a^2b\coth^3(c+d}{d}$$

$$= -\frac{b^3x}{2}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\coth(c+dx)}{d}-\frac{a^2b\coth^3(c+d}{d}$$

$$= -\frac{b^3x}{2}+\frac{5a^3\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{3ab^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{3a^2b\,\text{co}}{}$$

Mathematica [A] time = 1.55176, size = 236, normalized size = 1.42

$$-\frac{-384a^2b\tanh\left(\frac{1}{2}(c+dx)\right)-384a^2b\coth\left(\frac{1}{2}(c+dx)\right)-384a^2b\sinh^4\left(\frac{1}{2}(c+dx)\right)\text{csch}^3(c+dx)-6a^2\text{csch}^4\left(\frac{1}{2}(c+dx)\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^3)^3,x]
```

```
[Out] -(192*b^3*c + 192*b^3*d*x - 384*a^2*b*Coth[(c + d*x)/2] + 30*a^3*Csch[(c +
d*x)/2]^2 + a^3*Csch[(c + d*x)/2]^6 + 120*a^3*Log[Tanh[(c + d*x)/2]] - 1152
*a*b^2*Log[Tanh[(c + d*x)/2]] + 30*a^3*Sech[(c + d*x)/2]^2 + 6*a^3*Sech[(c
+ d*x)/2]^4 + a^3*Sech[(c + d*x)/2]^6 - 384*a^2*b*Csch[c + d*x]^3*Sinh[(c +
 d*x)/2]^4 - 6*a^2*Csch[(c + d*x)/2]^4*(a - 4*b*Sinh[c + d*x]) - 96*b^3*Sin
h[2*(c + d*x)] - 384*a^2*b*Tanh[(c + d*x)/2])/(384*d)
```

Maple [A] time = 0.083, size = 119, normalized size = 0.7

$$\frac{1}{d}\left(a^3\left(\left(-\frac{(\text{csch}\,(dx+c))^5}{6}+\frac{5\,(\text{csch}\,(dx+c))^3}{24}-\frac{5\,\text{csch}\,(dx+c)}{16}\right)\coth\,(dx+c)+\frac{5\,\text{Artanh}\left(e^{dx+c}\right)}{8}\right)+3\,a^2b\left(2/3-1/3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x)
```

```
[Out] 1/d*(a^3*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x
+c)+5/8*arctanh(exp(d*x+c)))+3*a^2*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)-6*
a*b^2*arctanh(exp(d*x+c))+b^3*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [B] time = 1.08364, size = 479, normalized size = 2.89

$$-\frac{1}{8}b^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)+\frac{1}{48}a^3\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}\right.}{d\left(6e^{(-2dx-2c)}-15e^{(-4dx-4\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 1/48*a^3*(15*log(
e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*
e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d
*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x -
4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e
^(-12*d*x - 12*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x -
 c) - 1)/d) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*
x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 2.78234, size = 16188, normalized size = 97.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^3*cosh(d*x + c)^16 + 96*b^3*cosh(d*x + c)*sinh(d*x + c)^15 + 6*b^
3*sinh(d*x + c)^16 - 30*a^3*cosh(d*x + c)^13 - 12*(2*b^3*d*x + 3*b^3)*cosh(
d*x + c)^14 - 12*(2*b^3*d*x - 60*b^3*cosh(d*x + c)^2 + 3*b^3)*sinh(d*x + c)
^14 + 170*a^3*cosh(d*x + c)^11 + 6*(560*b^3*cosh(d*x + c)^3 - 5*a^3 - 28*(2
*b^3*d*x + 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 + 12*(12*b^3*d*x + 7*b^3)
*cosh(d*x + c)^12 + 6*(1820*b^3*cosh(d*x + c)^4 + 24*b^3*d*x - 65*a^3*cosh(
d*x + c) + 14*b^3 - 182*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
12 - 396*a^3*cosh(d*x + c)^9 + 2*(13104*b^3*cosh(d*x + c)^5 - 1170*a^3*cosh
(d*x + c)^2 - 2184*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^3 + 85*a^3 + 72*(12*b^
3*d*x + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 12*(30*b^3*d*x + 48*a^2*b
+ 7*b^3)*cosh(d*x + c)^10 + 2*(24024*b^3*cosh(d*x + c)^6 - 4290*a^3*cosh(d*
x + c)^3 - 180*b^3*d*x - 6006*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^4 + 935*a^3
*cosh(d*x + c) - 288*a^2*b - 42*b^3 + 396*(12*b^3*d*x + 7*b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^10 - 396*a^3*cosh(d*x + c)^7 + 2*(34320*b^3*cosh(d*x + c
)^7 - 10725*a^3*cosh(d*x + c)^4 - 12012*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^5
 + 4675*a^3*cosh(d*x + c)^2 + 1320*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^3 - 1
98*a^3 - 60*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^9
+ 480*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^8 + 6*(12870*b^3*cosh(d*x + c)^8 -
6435*a^3*cosh(d*x + c)^5 - 6006*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^6 + 4675*
a^3*cosh(d*x + c)^3 + 80*b^3*d*x + 990*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^4
 - 594*a^3*cosh(d*x + c) + 320*a^2*b - 90*(30*b^3*d*x + 48*a^2*b + 7*b^3)*c
osh(d*x + c)^2)*sinh(d*x + c)^8 + 170*a^3*cosh(d*x + c)^5 + 12*(5720*b^3*co
sh(d*x + c)^9 - 4290*a^3*cosh(d*x + c)^6 - 3432*(2*b^3*d*x + 3*b^3)*cosh(d*
x + c)^7 + 4675*a^3*cosh(d*x + c)^4 + 792*(12*b^3*d*x + 7*b^3)*cosh(d*x + c
)^5 - 1188*a^3*cosh(d*x + c)^2 - 120*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d
*x + c)^3 - 33*a^3 + 320*(b^3*d*x + 4*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^7
 - 12*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^6 + 12*(4004*b^3*cosh(
d*x + c)^10 - 4290*a^3*cosh(d*x + c)^7 - 3003*(2*b^3*d*x + 3*b^3)*cosh(d*x
```

+ c)^8 + 6545*a^3*cosh(d*x + c)^5 + 924*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^
6 - 2772*a^3*cosh(d*x + c)^3 - 30*b^3*d*x - 210*(30*b^3*d*x + 48*a^2*b + 7*
b^3)*cosh(d*x + c)^4 - 231*a^3*cosh(d*x + c) - 192*a^2*b + 7*b^3 + 1120*(b^
3*d*x + 4*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 30*a^3*cosh(d*x + c)^3
+ 2*(13104*b^3*cosh(d*x + c)^11 - 19305*a^3*cosh(d*x + c)^8 - 12012*(2*b^3*
d*x + 3*b^3)*cosh(d*x + c)^9 + 39270*a^3*cosh(d*x + c)^6 + 4752*(12*b^3*d*x
+ 7*b^3)*cosh(d*x + c)^7 - 24948*a^3*cosh(d*x + c)^4 - 1512*(30*b^3*d*x +
48*a^2*b + 7*b^3)*cosh(d*x + c)^5 - 4158*a^3*cosh(d*x + c)^2 + 13440*(b^3*d
*x + 4*a^2*b)*cosh(d*x + c)^3 + 85*a^3 - 36*(30*b^3*d*x + 192*a^2*b - 7*b^3
)*cosh(d*x + c))*sinh(d*x + c)^5 + 12*(12*b^3*d*x + 96*a^2*b - 7*b^3)*cosh(
d*x + c)^4 + 2*(5460*b^3*cosh(d*x + c)^12 - 10725*a^3*cosh(d*x + c)^9 - 600
6*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^10 + 28050*a^3*cosh(d*x + c)^7 + 2970*(
12*b^3*d*x + 7*b^3)*cosh(d*x + c)^8 - 24948*a^3*cosh(d*x + c)^5 - 1260*(30*
b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^6 - 6930*a^3*cosh(d*x + c)^3 + 72
*b^3*d*x + 16800*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^4 + 425*a^3*cosh(d*x + c
) + 576*a^2*b - 42*b^3 - 90*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + 2*(1680*b^3*cosh(d*x + c)^13 - 4290*a^3*cosh(d*x + c)^
10 - 2184*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^11 + 14025*a^3*cosh(d*x + c)^8
+ 1320*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^9 - 16632*a^3*cosh(d*x + c)^6 - 7
20*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^7 - 6930*a^3*cosh(d*x + c)
^4 + 13440*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^5 + 850*a^3*cosh(d*x + c)^2 -
120*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x + c)^3 - 15*a^3 + 24*(12*b^3*
d*x + 96*a^2*b - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*b^3 - 12*(2*b^3*
d*x + 16*a^2*b - 3*b^3)*cosh(d*x + c)^2 + 2*(360*b^3*cosh(d*x + c)^14 - 117
0*a^3*cosh(d*x + c)^11 - 546*(2*b^3*d*x + 3*b^3)*cosh(d*x + c)^12 + 4675*a^
3*cosh(d*x + c)^9 + 396*(12*b^3*d*x + 7*b^3)*cosh(d*x + c)^10 - 7128*a^3*co
sh(d*x + c)^7 - 270*(30*b^3*d*x + 48*a^2*b + 7*b^3)*cosh(d*x + c)^8 - 4158*
a^3*cosh(d*x + c)^5 + 6720*(b^3*d*x + 4*a^2*b)*cosh(d*x + c)^6 + 850*a^3*co
sh(d*x + c)^3 - 12*b^3*d*x - 90*(30*b^3*d*x + 192*a^2*b - 7*b^3)*cosh(d*x +
c)^4 - 45*a^3*cosh(d*x + c) - 96*a^2*b + 18*b^3 + 36*(12*b^3*d*x + 96*a^2*
b - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((5*a^3 - 48*a*b^2)*cosh(d*
x + c)^14 + 14*(5*a^3 - 48*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^13 + (5*a^3 -
48*a*b^2)*sinh(d*x + c)^14 - 6*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^12 - (30*a
^3 - 288*a*b^2 - 91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^12 +
4*(91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 18*(5*a^3 - 48*a*b^2)*cosh(d*x +
c))*sinh(d*x + c)^11 + 15*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 + (1001*(5*a
^3 - 48*a*b^2)*cosh(d*x + c)^4 + 75*a^3 - 720*a*b^2 - 396*(5*a^3 - 48*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(5*a^3 - 48*a*b^2)*cosh(d*x +
c)^5 - 660*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 + 75*(5*a^3 - 48*a*b^2)*cosh
(d*x + c))*sinh(d*x + c)^9 - 20*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 + (3003*
(5*a^3 - 48*a*b^2)*cosh(d*x + c)^6 - 2970*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^
4 - 100*a^3 + 960*a*b^2 + 675*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c)^8 + 8*(429*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 - 594*(5*a^3 - 48*a*b^2)
*cosh(d*x + c)^5 + 225*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 20*(5*a^3 - 48*
a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 15*(5*a^3 - 48*a*b^2)*cosh(d*x + c)
^6 + (3003*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 - 5544*(5*a^3 - 48*a*b^2)*cos
h(d*x + c)^6 + 3150*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 + 75*a^3 - 720*a*b^2
- 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 2*(1001*(5*a^3
- 48*a*b^2)*cosh(d*x + c)^9 - 2376*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 + 18
90*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^5 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c
)^3 + 45*(5*a^3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 6*(5*a^3 - 48*
a*b^2)*cosh(d*x + c)^4 + (1001*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 - 2970*(
5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 + 3150*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^6
- 1400*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 - 30*a^3 + 288*a*b^2 + 225*(5*a^
3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(91*(5*a^3 - 48*a*b^2)*c
osh(d*x + c)^11 - 330*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^9 + 450*(5*a^3 - 48*
a*b^2)*cosh(d*x + c)^7 - 280*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^5 + 75*(5*a^3
- 48*a*b^2)*cosh(d*x + c)^3 - 6*(5*a^3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^3 + (5*a^3 - 48*a*b^2)*cosh(d*x + c)^2 + (91*(5*a^3 - 48*a*b^2)*cosh(

```
d*x + c)^12 - 396*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 + 675*(5*a^3 - 48*a*b
^2)*cosh(d*x + c)^8 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^6 + 225*(5*a^3 -
 48*a*b^2)*cosh(d*x + c)^4 + 5*a^3 - 48*a*b^2 - 36*(5*a^3 - 48*a*b^2)*cosh(
d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^13 - 36
*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^11 + 75*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^
9 - 80*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 + 45*(5*a^3 - 48*a*b^2)*cosh(d*x
+ c)^5 - 12*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 + (5*a^3 - 48*a*b^2)*cosh(d*
x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 3*((5*a^3 -
 48*a*b^2)*cosh(d*x + c)^14 + 14*(5*a^3 - 48*a*b^2)*cosh(d*x + c)*sinh(d*x
+ c)^13 + (5*a^3 - 48*a*b^2)*sinh(d*x + c)^14 - 6*(5*a^3 - 48*a*b^2)*cosh(d
*x + c)^12 - (30*a^3 - 288*a*b^2 - 91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*s
inh(d*x + c)^12 + 4*(91*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 18*(5*a^3 - 48
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^11 + 15*(5*a^3 - 48*a*b^2)*cosh(d*x +
c)^10 + (1001*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 + 75*a^3 - 720*a*b^2 - 396
*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 2*(1001*(5*a^3 - 48
*a*b^2)*cosh(d*x + c)^5 - 660*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 + 75*(5*a^
3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 20*(5*a^3 - 48*a*b^2)*cosh(d
*x + c)^8 + (3003*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^6 - 2970*(5*a^3 - 48*a*b
^2)*cosh(d*x + c)^4 - 100*a^3 + 960*a*b^2 + 675*(5*a^3 - 48*a*b^2)*cosh(d*x
 + c)^2)*sinh(d*x + c)^8 + 8*(429*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 - 594*
(5*a^3 - 48*a*b^2)*cosh(d*x + c)^5 + 225*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3
 - 20*(5*a^3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 15*(5*a^3 - 48*a*
b^2)*cosh(d*x + c)^6 + (3003*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 - 5544*(5*a
^3 - 48*a*b^2)*cosh(d*x + c)^6 + 3150*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 +
75*a^3 - 720*a*b^2 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
6 + 2*(1001*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^9 - 2376*(5*a^3 - 48*a*b^2)*co
sh(d*x + c)^7 + 1890*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^5 - 560*(5*a^3 - 48*a
*b^2)*cosh(d*x + c)^3 + 45*(5*a^3 - 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 - 6*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 + (1001*(5*a^3 - 48*a*b^2)*cosh(d*
x + c)^10 - 2970*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 + 3150*(5*a^3 - 48*a*b^
2)*cosh(d*x + c)^6 - 1400*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 - 30*a^3 + 288
*a*b^2 + 225*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(91*(5
*a^3 - 48*a*b^2)*cosh(d*x + c)^11 - 330*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^9
+ 450*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 - 280*(5*a^3 - 48*a*b^2)*cosh(d*x
+ c)^5 + 75*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 - 6*(5*a^3 - 48*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^3 + (5*a^3 - 48*a*b^2)*cosh(d*x + c)^2 + (91*(5*a^3
 - 48*a*b^2)*cosh(d*x + c)^12 - 396*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^10 + 6
75*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^8 - 560*(5*a^3 - 48*a*b^2)*cosh(d*x + c
)^6 + 225*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^4 + 5*a^3 - 48*a*b^2 - 36*(5*a^3
 - 48*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(7*(5*a^3 - 48*a*b^2)*cos
h(d*x + c)^13 - 36*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^11 + 75*(5*a^3 - 48*a*b
^2)*cosh(d*x + c)^9 - 80*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^7 + 45*(5*a^3 - 4
8*a*b^2)*cosh(d*x + c)^5 - 12*(5*a^3 - 48*a*b^2)*cosh(d*x + c)^3 + (5*a^3 -
 48*a*b^2)*cosh(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c)
- 1) + 2*(48*b^3*cosh(d*x + c)^15 - 195*a^3*cosh(d*x + c)^12 - 84*(2*b^3*d*
x + 3*b^3)*cosh(d*x + c)^13 + 935*a^3*cosh(d*x + c)^10 + 72*(12*b^3*d*x + 7
*b^3)*cosh(d*x + c)^11 - 1782*a^3*cosh(d*x + c)^8 - 60*(30*b^3*d*x + 48*a^2
*b + 7*b^3)*cosh(d*x + c)^9 - 1386*a^3*cosh(d*x + c)^6 + 1920*(b^3*d*x + 4*
a^2*b)*cosh(d*x + c)^7 + 425*a^3*cosh(d*x + c)^4 - 36*(30*b^3*d*x + 192*a^2
*b - 7*b^3)*cosh(d*x + c)^5 - 45*a^3*cosh(d*x + c)^2 + 24*(12*b^3*d*x + 96*
a^2*b - 7*b^3)*cosh(d*x + c)^3 - 12*(2*b^3*d*x + 16*a^2*b - 3*b^3)*cosh(d*x
 + c))*sinh(d*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x + c)*sinh(d*x + c
)^13 + d*sinh(d*x + c)^14 - 6*d*cosh(d*x + c)^12 + (91*d*cosh(d*x + c)^2 -
6*d)*sinh(d*x + c)^12 + 4*(91*d*cosh(d*x + c)^3 - 18*d*cosh(d*x + c))*sinh(
d*x + c)^11 + 15*d*cosh(d*x + c)^10 + (1001*d*cosh(d*x + c)^4 - 396*d*cosh(
d*x + c)^2 + 15*d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d*x + c)^5 - 660*d*cos
h(d*x + c)^3 + 75*d*cosh(d*x + c))*sinh(d*x + c)^9 - 20*d*cosh(d*x + c)^8 +
 (3003*d*cosh(d*x + c)^6 - 2970*d*cosh(d*x + c)^4 + 675*d*cosh(d*x + c)^2 -
 20*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 - 594*d*cosh(d*x + c)^5 +
```

```
225*d*cosh(d*x + c)^3 - 20*d*cosh(d*x + c))*sinh(d*x + c)^7 + 15*d*cosh(d*
x + c)^6 + (3003*d*cosh(d*x + c)^8 - 5544*d*cosh(d*x + c)^6 + 3150*d*cosh(d
*x + c)^4 - 560*d*cosh(d*x + c)^2 + 15*d)*sinh(d*x + c)^6 + 2*(1001*d*cosh(
d*x + c)^9 - 2376*d*cosh(d*x + c)^7 + 1890*d*cosh(d*x + c)^5 - 560*d*cosh(d
*x + c)^3 + 45*d*cosh(d*x + c))*sinh(d*x + c)^5 - 6*d*cosh(d*x + c)^4 + (10
01*d*cosh(d*x + c)^10 - 2970*d*cosh(d*x + c)^8 + 3150*d*cosh(d*x + c)^6 - 1
400*d*cosh(d*x + c)^4 + 225*d*cosh(d*x + c)^2 - 6*d)*sinh(d*x + c)^4 + 4*(9
1*d*cosh(d*x + c)^11 - 330*d*cosh(d*x + c)^9 + 450*d*cosh(d*x + c)^7 - 280*
d*cosh(d*x + c)^5 + 75*d*cosh(d*x + c)^3 - 6*d*cosh(d*x + c))*sinh(d*x + c)
^3 + d*cosh(d*x + c)^2 + (91*d*cosh(d*x + c)^12 - 396*d*cosh(d*x + c)^10 +
675*d*cosh(d*x + c)^8 - 560*d*cosh(d*x + c)^6 + 225*d*cosh(d*x + c)^4 - 36*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)^13 - 36*d*cos
h(d*x + c)^11 + 75*d*cosh(d*x + c)^9 - 80*d*cosh(d*x + c)^7 + 45*d*cosh(d*x
 + c)^5 - 12*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**3)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.46106, size = 455, normalized size = 2.74

$$-\frac{(dx+c)b^3}{2d}+\frac{b^3e^{(2dx+2c)}}{8d}+\frac{\left(5a^3-48ab^2\right)\log\left(e^{(dx+c)}+1\right)}{16d}-\frac{\left(5a^3-48ab^2\right)\log\left(\left|e^{(dx+c)}-1\right|\right)}{16d}-\frac{\left(15a^3e^{(13dx+13c)}+3b^3\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^3)^3,x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*b^3/d + 1/8*b^3*e^(2*d*x + 2*c)/d + 1/16*(5*a^3 - 48*a*b^2)*
log(e^(d*x + c) + 1)/d - 1/16*(5*a^3 - 48*a*b^2)*log(abs(e^(d*x + c) - 1))/
d - 1/24*(15*a^3*e^(13*d*x + 13*c) + 3*b^3*e^(12*d*x + 12*c) - 85*a^3*e^(11
*d*x + 11*c) + 198*a^3*e^(9*d*x + 9*c) + 198*a^3*e^(7*d*x + 7*c) - 85*a^3*e
^(5*d*x + 5*c) + 15*a^3*e^(3*d*x + 3*c) + 3*b^3 + 18*(16*a^2*b - b^3)*e^(10
*d*x + 10*c) - 15*(64*a^2*b - 3*b^3)*e^(8*d*x + 8*c) + 12*(96*a^2*b - 5*b^3
)*e^(6*d*x + 6*c) - 9*(64*a^2*b - 5*b^3)*e^(4*d*x + 4*c) + 6*(16*a^2*b - 3*
b^3)*e^(2*d*x + 2*c))*e^(-2*d*x - 2*c)/(d*(e^(d*x + c) + 1)^6*(e^(d*x + c)
- 1)^6)
```

3.171 $\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=328

$$-\frac{2a^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}((-1)}{}\right.}{3b^2d\sqrt{\sqrt[3]{-1}a}}$$

```
[Out] -((a*x)/b^2) - (2*(-1)^(2/3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3)
 + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/
3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - (2*(-1)^(2/
3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x
)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^
(2/3)]*b^2*d) - (2*a^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sq
rt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^2*d) - Cosh[c + d*x]/(
b*d) + Cosh[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.738527, antiderivative size = 328, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 2633, 3213, 2660, 618, 204}

$$-\frac{2a^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}((-1)}{}\right.}{3b^2d\sqrt{\sqrt[3]{-1}a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3),x]
```



```
[Out] -((a*x)/b^2) - (2*(-1)^(2/3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3)
 + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/
3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - (2*(-1)^(2/
3)*a^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x
)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^
(2/3)]*b^2*d) - (2*a^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sq
rt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^2*d) - Cosh[c + d*x]/(
b*d) + Cosh[c + d*x]^3/(3*b*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sinh^6(c+dx)}{a+b\sinh^3(c+dx)}\,dx &= -\int \left(\frac{a}{b^2}-\frac{\sinh^3(c+dx)}{b}-\frac{a^2}{b^2\left(a+b\sinh^3(c+dx)\right)}\right)dx\\
&= -\frac{ax}{b^2}+\frac{a^2\int \frac{1}{a+b\sinh^3(c+dx)}\,dx}{b^2}+\frac{\int \sinh^3(c+dx)\,dx}{b}\\
&= -\frac{ax}{b^2}+\frac{a^2\int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\ldots\right.}\ldots\right.}{b^2}\\
&= -\frac{ax}{b^2}-\frac{\cosh(c+dx)}{bd}+\frac{\cosh^3(c+dx)}{3bd}+\frac{\left(\sqrt[6]{-1}a^{4/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^2}+\frac{\left(\sqrt[6]{-1}a^{4/3}\right)\int \ldots}{\ldots}\\
&= -\frac{ax}{b^2}-\frac{\cosh(c+dx)}{bd}+\frac{\cosh^3(c+dx)}{3bd}-\frac{\left(2(-1)^{2/3}a^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx,x,\ldots\right.}{3b^2d}\\
&= -\frac{ax}{b^2}-\frac{\cosh(c+dx)}{bd}+\frac{\cosh^3(c+dx)}{3bd}+\frac{\left(4(-1)^{2/3}a^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\ldots\right.}{3b^2d}\\
&= -\frac{ax}{b^2}+\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^2d}-\frac{2(-1)^{2/3}a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tan\ldots}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)\ldots}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}b^2\ldots}}
\end{aligned}
$$

Mathematica [C] time = 0.349243, size = 168, normalized size = 0.51

$$
\frac{8a^2\operatorname{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\#1c+\ldots}{\#1^4b-2\#1^2b+4\#1a+b}\ldots\right.}{12b^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-12*a*c - 12*a*d*x - 9*b*Cosh[c + d*x] + b*Cosh[3*(c + d*x)] + 8*a^2*RootS
um[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1 + 2*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(12*b^2*d)
```

Maple [C] time = 0.066, size = 259, normalized size = 0.8

$$\frac{1}{3bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}-\frac{1}{2bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{1}{2bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{a}{db^2}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b
/(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/3/d/b/(tanh(
1/2*d*x+1/2*c)-1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*d*x
+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-1/3/d*a^2/b^2*sum((_R^4-2*_R
^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(
_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$8a^2\int\frac{e^{(3dx+3c)}}{b^3e^{(6dx+6c)}-3b^3e^{(4dx+4c)}+8ab^2e^{(3dx+3c)}+3b^3e^{(2dx+2c)}-b^3}\,dx-\frac{\left(24adxe^{(3dx+3c)}-be^{(6dx+6c)}+9be^{(4dx+4c)}\right.}{24b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 8*a^2*integrate(e^(3*d*x + 3*c)/(b^3*e^(6*d*x + 6*c) - 3*b^3*e^(4*d*x + 4*c
) + 8*a*b^2*e^(3*d*x + 3*c) + 3*b^3*e^(2*d*x + 2*c) - b^3), x) - 1/24*(24*a
*d*x*e^(3*d*x + 3*c) - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c) + 9*b*e^(2*d
*x + 2*c) - b)*e^(-3*d*x - 3*c)/(b^2*d)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^6}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^6/(b*sinh(d*x + c)^3 + a), x)
```

3.172 $\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=295

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2a\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2a\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] -x/(2*b) + (2*a*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
+ d*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^
(2/3)) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3)
+ I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqr
t[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - a^(1/3
)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b
^(5/3)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.558601, antiderivative size = 295, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 2635, 8, 2660, 618, 206, 204}

$$\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2a\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2a\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]
```


```
[Out] -x/(2*b) + (2*a*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
+ d*x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^
(2/3)) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3)
+ I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqr
t[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - a^(1/3
)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b
^(5/3)*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\left(i\int\left(\frac{i\sinh^2(c+dx)}{b}-\frac{ia\sinh^2(c+dx)}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx\right)$$

$$=\frac{\int\sinh^2(c+dx)\,dx}{b}-\frac{a\int\frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{b}$$

$$=\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}-\frac{\int 1\,dx}{2b}+\frac{a\int\left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx\right.}\right.}{b}$$

$$=-\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}+\frac{(ia)\int\frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{5/3}}+\frac{(ia)\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}}{3b^{5/3}}$$

$$=-\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}+\frac{(2a)\,\mathrm{Subst}\left(\int\frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}x-i\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{5/3}d}$$

$$=-\frac{x}{2b}+\frac{\cosh(c+dx)\sinh(c+dx)}{2bd}-\frac{(4a)\,\mathrm{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\right.}{3b^{5/3}d}$$

$$=-\frac{x}{2b}-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{5/3}d}-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{5/3}d}+\frac{2a\tanh^{-}}{3\sqrt{}}$$

Mathematica [C] time = 0.316158, size = 299, normalized size = 1.01

$$-2a\mathrm{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)-4\#1}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-6*(c + d*x) - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &
, (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*
x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1
^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cos
h[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3
 + b*#1^5) & ] + 3*Sinh[2*(c + d*x)])/(12*b*d)
```

Maple [C] time = 0.06, size = 207, normalized size = 0.7

$$-\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{2\,bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)+\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] -1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/2/d/b*
ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1
/2*d*x+1/2*c)-1)+1/2/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+4/3/d*a/b*sum(_R^2/(_R^5
*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z
^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(4\,dxe^{(2\,dx+2\,c)}-e^{(4\,dx+4\,c)}+1\right)e^{(-2\,dx-2\,c)}}{8\,bd}-\frac{1}{32}\int\frac{64\left(ae^{(5\,dx+5\,c)}-2\,ae^{(3\,dx+3\,c)}+ae^{(dx+c)}\right)}{b^2e^{(6\,dx+6\,c)}-3\,b^2e^{(4\,dx+4\,c)}+8\,abe^{(3\,dx+3\,c)}+3\,b^2e^{(2\,dx+2\,c)}-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/8*(4*d*x*e^(2*d*x + 2*c) - e^(4*d*x + 4*c) + 1)*e^(-2*d*x - 2*c)/(b*d) -
 1/32*integrate(64*(a*e^(5*d*x + 5*c) - 2*a*e^(3*d*x + 3*c) + a*e^(d*x + c)
)/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c) + 8*a*b*e^(3*d*x + 3*c) + 3*
b^2*e^(2*d*x + 2*c) - b^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^5}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^5/(b*sinh(d*x + c)^3 + a), x)
```

3.173 $\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=303

$$-\frac{2a^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] (-2*a^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*
a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(4/3)*d) + (2*(-1)^(1/3)*a^(2/3)*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3
)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(4/3)*d) - (
2*a^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2
/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(4/3)*d) + Cosh[c + d*x]/(b*d)
```

Rubi [A] time = 0.544964, antiderivative size = 303, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 2638, 2660, 618, 204}

$$-\frac{2a^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}a^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*a^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*
a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(4/3)*d) + (2*(-1)^(1/3)*a^(2/3)*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3
)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(4/3)*d) - (
2*a^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2
/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(4/3)*d) + Cosh[c + d*x]/(b*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \left(\frac{\sinh(c+dx)}{b} - \frac{a\sinh(c+dx)}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \sinh(c+dx)\,dx}{b} - \frac{a\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{b}$$

$$= \frac{\cosh(c+dx)}{bd} + \frac{(ia)\int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)} - \frac{\ }{3\sqrt[3]{a}\sqrt[3]{b}}\right.}{b}$$

$$= \frac{\cosh(c+dx)}{bd} - \frac{\left(ia^{2/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{4/3}} + \frac{\left(\sqrt[6]{-1}a^{2/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)}\,d}{3b^{4/3}}$$

$$= \frac{\cosh(c+dx)}{bd} - \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{4/3}d} + \frac{\left(2\sqrt[3]{-\right.}{}$$

$$= \frac{\cosh(c+dx)}{bd} + \frac{\left(4a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{-1}\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(i\right.\right.}{3b^{4/3}d}$$

$$= -\frac{2\sqrt[3]{-1}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{4/3}d} - \frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d} - \cdots$$

Mathematica [C] time = 0.34527, size = 214, normalized size = 0.71

$$\frac{3\cosh(c+dx) - a\,\text{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&, \frac{2\#1^2\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\text{co}\right.}{\ }\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*Cosh[c + d*x] - a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 &
 , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x
)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2
)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(3*b*d)
```

Maple [C] time = 0.052, size = 128, normalized size = 0.4

$$\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{-1}-\frac{2a}{3bd}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^3-}{\_R^5a-2\_R^3a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(tanh(1/2*d*x+1/2*c)-1)-2/3/d*a/b*sum((
_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Root
Of(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2dx+2c)}+1\right)e^{(-dx-c)}}{2bd}-\frac{1}{16}\int\frac{64\left(ae^{(4dx+4c)}-ae^{(2dx+2c)}\right)}{b^2e^{(6dx+6c)}-3b^2e^{(4dx+4c)}+8abe^{(3dx+3c)}+3b^2e^{(2dx+2c)}-b^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/16*integrate(64*(a*e^(4*d*
x + 4*c) - a*e^(2*d*x + 2*c))/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c)
+ 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^4}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)
```

3.174 $\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{2\sqrt[3]{a}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\right.}{\sqrt{}}\right.}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}}}$$

```
[Out] x/b + (2*(-1)^(2/3)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1
/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*
Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d) + (2*(-1)^(2/3)*a^(1/3)*
ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt
[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b*d)
 + (2*a^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) +
b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b*d)
```

Rubi [A] time = 0.439897, antiderivative size = 294, normalized size of antiderivative = 1., number of steps used = 13, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 3213, 2660, 618, 204}

$$\frac{2\sqrt[3]{a}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\right.}{\sqrt{}}\right.}{3bd\sqrt{\sqrt[3]{-1}a^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] x/b + (2*(-1)^(2/3)*a^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1
/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*
Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d) + (2*(-1)^(2/3)*a^(1/3)*
ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt
[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b*d)
 + (2*a^(1/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) +
b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = i\int \left(-\frac{i}{b}+\frac{ia}{b\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sinh^3(c+dx)}\,dx}{b}$$

$$= \frac{x}{b}-\frac{a\int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(\right.}}{b}$$

$$= \frac{x}{b}-\frac{\left(\sqrt[6]{-1}\sqrt[3]{a}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b}-\frac{\left(\sqrt[6]{-1}\sqrt[3]{a}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b}-\frac{\left(\sqrt[6]{-1}\sqrt[3]{a}\right.}{}$$

$$= \frac{x}{b}+\frac{\left(2(-1)^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3bd}+\frac{\left(2(-1)^{2/3}\sqrt[3]{a}\right)\text{S}}{}$$

$$= \frac{x}{b}-\frac{\left(4(-1)^{2/3}\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3bd}-$$

$$= \frac{x}{b}-\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}bd}+\frac{2(-1)^{2/3}\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/}}}\right.}{3\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}bd}$$

Mathematica [C] time = 0.230597, size = 145, normalized size = 0.49

$$\frac{-2a\text{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\#1c+}{\#1^4b-2\#1^2b+4\#1a+b}\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (3*c + 3*d*x - 2*a*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & ,
 (c*#1 + d*x*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ]
)/(3*b*d)
```

Maple [C] time = 0.043, size = 129, normalized size = 0.4

$$\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{a}{3bd}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^4-2}{\_R^5a-2\_R^3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)+1/3/d*a/b*s
um((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_
R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-8a\int\frac{e^{(3dx+3c)}}{b^2e^{(6dx+6c)}-3b^2e^{(4dx+4c)}+8abe^{(3dx+3c)}+3b^2e^{(2dx+2c)}-b^2}\,dx+\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -8*a*integrate(e^(3*d*x + 3*c)/(b^2*e^(6*d*x + 6*c) - 3*b^2*e^(4*d*x + 4*c)
 + 8*a*b*e^(3*d*x + 3*c) + 3*b^2*e^(2*d*x + 2*c) - b^2), x) + x/b
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\sinh\left(dx+c\right)^3}{b\sinh\left(dx+c\right)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^3 + a), x)
```

3.175 $\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=262

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] (-2*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/
Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(
2/3)]*b^(2/3)*d) - (2*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Ta
nh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a
^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)
/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Rubi [A] time = 0.270745, antiderivative size = 262, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 2660, 618, 206, 204}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/
Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(
2/3)]*b^(2/3)*d) - (2*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Ta
nh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a
^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)
/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*Sqrt[a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\int \left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)} + \frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)} + \frac{}{3b^{2/3}\left((-1)^{5/6}\right.}\right.$$

$$= -\frac{i\int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{2/3}} - \frac{i\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{2/3}} - \frac{i\int \frac{1}{(-1)^{5/6}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3b^{2/3}}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}x-i\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d} - \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx\right.}{3b^{2/3}d}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3b^{2/3}d} + \frac{4\,\text{Subst}\left(\int \frac{}{4\left(a^{2/3}+\right.}\right.}{}$$

$$= \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,b^{2/3}d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,b^{2/3}d} - \frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh}{\sqrt{a^{2/}}}\right.}{3\sqrt{a^{2/3}+b^{2/3}}}$$

Mathematica [C] time = 0.17752, size = 275, normalized size = 1.05

$$\text{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)-4\#1^2\log}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*
x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1
^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) & ]/(6*d)
```

Maple [C] time = 0.042, size = 78, normalized size = 0.3

$$-\frac{4}{3d}\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] -4/3/d*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),
_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

3.176 $\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=290

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] (2*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/S
qrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a
^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(1/3)*d) - (2*(-1)^(1/3)*ArcTan[((-1)^(1/6)*
((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3)
 - b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2
*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3
*a^(1/3)*Sqrt[a^(2/3) + b^(2/3)]*b^(1/3)*d)
```

Rubi [A] time = 0.335756, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3220, 2660, 618, 204}

$$\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/S
qrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a
^(2/3) - (-1)^(2/3)*b^(2/3)]*b^(1/3)*d) - (2*(-1)^(1/3)*ArcTan[((-1)^(1/6)*
((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3)
 - b^(2/3)]])/(3*a^(1/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2
*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3
*a^(1/3)*Sqrt[a^(2/3) + b^(2/3)]*b^(1/3)*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\left(i\int\left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}\right.\right.$$

$$=\frac{i\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{\sqrt[6]{-1}\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}}-\frac{(-1)^{5/6}\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{}}}{3\sqrt[3]{a}\sqrt[3]{}}$$

$$=\frac{2\,\mathrm{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}-\frac{\left(2\sqrt[3]{-1}\right)\mathrm{Subst}\left(\int\frac{}{\sqrt[6]{-1}\sqrt[3]{a}}\right.}{}$$

$$=-\frac{4\,\mathrm{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx,x,2\sqrt[3]{-1}\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}d}+\frac{\left(4\sqrt[3]{}\right.}{}$$

$$=\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\sqrt[3]{b}d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\sqrt[3]{b}d}+\frac{2\tan}{3}$$

Mathematica [C] time = 0.236425, size = 199, normalized size = 0.69

$$\frac{\mathrm{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1^2\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\#1^2c}{\#1^4b-2}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-c - d*x - 2*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/
2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2)/(b + 4*a*#1 - 2*b*#
1^2 + b*#1^4) & ]/(3*d)
```

Maple [C] time = 0.038, size = 82, normalized size = 0.3

$$\frac{2}{3d}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^3-\_R}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 2/3/d*sum((_R^3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\sinh(dx+c)}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sinh(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

3.177 $\int \frac{1}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=280

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] (-2*(-1)^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c +
 d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[
(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - (2*(-1)^(2/3)*ArcTan[((-1)^(1
/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(
2/3) - b^(2/3)]])/(3*a^(2/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (2*Arc
Tanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*a^(
2/3)*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Rubi [A] time = 0.250961, antiderivative size = 280, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2660, 618, 204}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^3)^(-1),x]
```

```
[Out] (-2*(-1)^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c +
 d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[
(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - (2*(-1)^(2/3)*ArcTan[((-1)^(1
/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(
2/3) - b^(2/3)]])/(3*a^(2/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (2*Arc
Tanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*a^(
2/3)*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx = \int \left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{}{3a^{2/3}\left(\sqrt[6]{\cdot}\right.}\right.$$

$$= \frac{\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3a^{2/3}} + \frac{\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3a^{2/3}} + \frac{\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\ldots}}{3a^{2/3}}$$

$$= -\frac{\left(2(-1)^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3a^{2/3}d} - \frac{\left(2(-1)^{2/3}\right)\text{Subst}\left(\int \ldots\right.}{}$$

$$= \frac{\left(4(-1)^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3a^{2/3}d} + \frac{\left(4(-1)^{2/3}\ldots\right.}{}$$

$$= \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d} - \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} - \ldots$$

Mathematica [C] time = 0.173833, size = 131, normalized size = 0.47

$$\frac{2\text{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&, \frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\#1c+\#1\ldots}{\#1^4b-2\#1^2b+4\#1a+b}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^3)^(-1),x]
```

```
[Out] (2*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x*#1
+ 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(3*d)
```

Maple [C] time = 0.04, size = 87, normalized size = 0.3

$$\frac{1}{3d}\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{-\_R^4+2\_R^2-1}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/3/d*sum((-_R^4+2*_R^2-1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+
1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(d*x + c)^3 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a+b\sinh^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sinh(c + d*x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sinh(d*x + c)^3 + a), x)
```

3.178 $\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=286

$$\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}+b^{2/3}}}+\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3ad\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] (2*b^(1/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x
)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/
3)) - b^(2/3)]*d) + (2*b^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a
^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a*Sqrt[(
-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*b^(1/3)
*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3
*a*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Rubi [A] time = 0.432414, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3220, 3770, 2660, 618, 206, 204}

$$\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}+b^{2/3}}}+\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3ad\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*b^(1/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x
)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/
3)) - b^(2/3)]*d) + (2*b^(1/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a
^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a*Sqrt[(
-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (2*b^(1/3)
*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3
*a*Sqrt[a^(2/3) + b^(2/3)]*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{a+b\sinh^3(c+dx)}\,dx = i\int \left(-\frac{i\operatorname{csch}(c+dx)}{a}+\frac{ib\sinh^2(c+dx)}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$=\frac{\int \operatorname{csch}(c+dx)\,dx}{a}-\frac{b\int \frac{\sinh^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$=-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{b\int \left(\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{}{3b^{2/3}}\right.}{a}$$

$$=-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\left(i\sqrt[3]{b}\right)\int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3a}+\frac{\left(i\sqrt[3]{b}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,d}{3a}$$

$$=-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\left(2\sqrt[3]{b}\right)\operatorname{Subst}\left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}x-i\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3ad}+\frac{(2}{}$$

$$=-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}-\frac{\left(4\sqrt[3]{b}\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan\right.}{3ad}$$

$$=-\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d}-\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}-\frac{\tanh^{-1}(}{}$$

Mathematica [C] time = 0.254159, size = 295, normalized size = 1.03

$$\frac{6\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-b\text{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-s\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (6*Log[Tanh[(c + d*x)/2]] - b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 +
 b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*c*#1^2 - 2*d*x*#1^2 - 4*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2
- 2*b*#1^3 + b*#1^5) & ])/(6*a*d)
```

Maple [C] time = 0.072, size = 100, normalized size = 0.4

$$\frac{1}{da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4b}{3da}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^2}{\_R^5a-2\_R^3a-4\_R^2b+\_Ra}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/d/a*ln(tanh(1/2*d*x+1/2*c))+4/3/d/a*b*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+
_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*
a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad}+\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad}-2\int\frac{be^{(5\,dx+5\,c)}-2\,be^{(3\,dx+3\,c)}+be^{(dx+c)}}{abe^{(6\,dx+6\,c)}-3\,abe^{(4\,dx+4\,c)}+8\,a^2e^{(3\,dx+3\,c)}+3\,abe^{(2\,dx+2\,c)}-ab}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(5*d*x + 5*c) - 2*b*e^(3*d*x + 3*c) + b*e^(d*x + c))/(a*
b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e
^(2*d*x + 2*c) - a*b), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)/(b*sinh(d*x + c)^3 + a), x)
```

3.179 $\int \frac{\operatorname{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=304

$$-\frac{2b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tan\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2}}}\right.}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] (-2*b^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1
)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(1/3)*b^(2/3)*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3
)*a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (
2*b^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2
/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - Coth[c + d*x]/(a*d)
```

Rubi [A] time = 0.478469, antiderivative size = 304, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3767, 8, 2660, 618, 204}

$$-\frac{2b^{2/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\sqrt[3]{-1}b^{2/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tan\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2}}}\right.}{3a^{4/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (-2*b^(2/3)*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1
)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(1/3)*b^(2/3)*ArcTan[((-
1)^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3
)*a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) - (
2*b^(2/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^(2
/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - Coth[c + d*x]/(a*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{csch}^2(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\int \left(-\frac{\text{csch}^2(c+dx)}{a}+\frac{b\sinh(c+dx)}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx$$

$$=\frac{\int \text{csch}^2(c+dx)\,dx}{a}-\frac{b\int \frac{\sinh(c+dx)}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$=\frac{(ib)\int \left(\frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\right.}}{a}$$

$$=-\frac{\coth(c+dx)}{ad}-\frac{\left(ib^{2/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+(-1)^{5/6}\sqrt[3]{b}\sinh(c+dx)}\,dx}{3a^{4/3}}+\frac{\left(\sqrt[6]{-1}b^{2/3}\right)\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+\sqrt[6]{-1}\sqrt[3]{b}\sinh(c}}{3a^{4/3}}$$

$$=-\frac{\coth(c+dx)}{ad}-\frac{\left(2b^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x+\sqrt[6]{-1}\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(ic+idx)\right)\right)}{3a^{4/3}d}+$$

$$=-\frac{\coth(c+dx)}{ad}+\frac{\left(4b^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}\right)-x^2}\,dx,x,2\sqrt[3]{-1}\sqrt[3]{b}+2\sqrt[6]{-1}\sqrt[3]{a}\tan}{3a^{4/3}d}$$

$$=-\frac{2\sqrt[3]{-1}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}$$

Mathematica [C] time = 0.390087, size = 230, normalized size = 0.76

$$-\frac{2b\text{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1^2\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)}{\#1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -(3*Coth[(c + d*x)/2] + 2*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b
*#1^6 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
```

```
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + c*#1^2 + d*x*#1^2 + 2*Log[-Cosh[(c
 + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^2)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*Tanh[(c + d*x)/2])/(6*a*d
)
```

Maple [C] time = 0.075, size = 123, normalized size = 0.4

$$-\frac{1}{2da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{2b}{3da}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{\_R^3-\_R}{\_R^5a-2\_R^3a-4\_R^2b+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-2/3/d/a*b*sum((_R^
3-_R)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(
_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{ade^{(2\,dx+2\,c)}-ad}-4\int\frac{be^{(4\,dx+4\,c)}-be^{(2\,dx+2\,c)}}{abe^{(6\,dx+6\,c)}-3\,abe^{(4\,dx+4\,c)}+8\,a^2e^{(3\,dx+3\,c)}+3\,abe^{(2\,dx+2\,c)}-ab}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(4*d*x + 4*c) - b*e^(2*d*
x + 2*c))/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c) + 8*a^2*e^(3*d*x + 3
*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)^2}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^2/(b*sinh(d*x + c)^3 + a), x)
```

3.180 $\int \frac{\mathrm{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=322

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tan\ldots\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2\ldots}}}\right.}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] (2*(-1)^(2/3)*b*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
+ d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(5/3)*Sqrt
[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(2/3)*b*ArcTan[((-1)
^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*
a^(2/3) - b^(2/3)]])/(3*a^(5/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + Arc
Tanh[Cosh[c + d*x]]/(2*a*d) + (2*b*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x
)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*a^(5/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - (Co
th[c + d*x]*Csch[c + d*x])/(2*a*d)
```

Rubi [A] time = 0.457326, antiderivative size = 322, normalized size of antiderivative = 1., number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 3768, 3770, 3213, 2660, 618, 204}

$$\frac{2b\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+b^{2/3}}}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[6]{-1}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tan\ldots\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2\ldots}}}\right.}{3a^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*b*ArcTan[((-1)^(1/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c
+ d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(5/3)*Sqrt
[(-1)^(1/3)*a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) + (2*(-1)^(2/3)*b*ArcTan[((-1)
^(1/6)*((-1)^(5/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*
a^(2/3) - b^(2/3)]])/(3*a^(5/3)*Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + Arc
Tanh[Cosh[c + d*x]]/(2*a*d) + (2*b*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x
)/2])/Sqrt[a^(2/3) + b^(2/3)]])/(3*a^(5/3)*Sqrt[a^(2/3) + b^(2/3)]*d) - (Co
th[c + d*x]*Csch[c + d*x])/(2*a*d)
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\text{csch}^3(c+dx)}{a+b\sinh^3(c+dx)}\,dx = -\left(i\int\left(\frac{i\,\text{csch}^3(c+dx)}{a}-\frac{ib}{a\left(a+b\sinh^3(c+dx)\right)}\right)dx\right)$$

$$=\frac{\int \text{csch}^3(c+dx)\,dx}{a}-\frac{b\int \frac{1}{a+b\sinh^3(c+dx)}\,dx}{a}$$

$$=-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}-\frac{\int \text{csch}(c+dx)\,dx}{2a}-\frac{b\int\left(\frac{\sqrt[6]{-1}}{3a^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}+\frac{}{3a^{2/3}(}\right.}{}$$

$$=\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}-\frac{\left(\sqrt[6]{-1}b\right)\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx}{3a^{5/3}}-$$

$$=\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}+\frac{\left(2(-1)^{2/3}b\right)\text{Subst}\left(\int\frac{1}{\sqrt[6]{-1}\sqrt[3]{a}-2\sqrt[3]{b}x+}\right.}{3a^{5/3}}$$

$$=\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}-\frac{\coth(c+dx)\text{csch}(c+dx)}{2ad}-\frac{\left(4(-1)^{2/3}b\right)\text{Subst}\left(\int\frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b^{2/3}\right.}\right.}{}$$

$$=-\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}+\frac{2(-1)^{2/3}b\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}+(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+d\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}}\right.}{3a^{5/3}\sqrt{\sqrt[3]{-1}a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}$$

Mathematica [C] time = 0.497428, size = 178, normalized size = 0.55

$$-\frac{16b\text{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)+\#1c}{\#1^4b-2\#1^2b+4\#1a+b}\right.}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] -(16*b*RootSum[-b + 3*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c*#1 + d*x
*#1 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1)/(b + 4*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*(Csch[(
c + d*x)/2]^2 + 4*Log[Tanh[(c + d*x)/2]] + Sech[(c + d*x)/2]^2))/(24*a*d)
```

Maple [C] time = 0.089, size = 146, normalized size = 0.5

$$\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-2}-\frac{1}{2da}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b}{3da}\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/8/d/a/tanh(1/2*d*x+1/2*c)^2-1/2/d/a*ln(tanh
(1/2*d*x+1/2*c))+1/3/d/a*b*sum((_R^4-2*_R^2+1)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R
*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-
a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-8b\int\frac{e^{(3dx+3c)}}{abe^{(6dx+6c)}-3abe^{(4dx+4c)}+8a^2e^{(3dx+3c)}+3abe^{(2dx+2c)}-ab}\,dx-\frac{e^{(3dx+3c)}+e^{(dx+c)}}{ade^{(4dx+4c)}-2ade^{(2dx+2c)}+ad}+\frac{\log\left(\left(e^{(dx+c}\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -8*b*integrate(e^(3*d*x + 3*c)/(a*b*e^(6*d*x + 6*c) - 3*a*b*e^(4*d*x + 4*c)
 + 8*a^2*e^(3*d*x + 3*c) + 3*a*b*e^(2*d*x + 2*c) - a*b), x) - (e^(3*d*x + 3
*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*c) + a*d) + 1/
2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c) - 1)*e^(-c))/(
a*d)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)^3}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^3/(b*sinh(d*x + c)^3 + a), x)
```

3.181 $\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx$

Optimal. Leaf size=317

$$-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2b^{4/3}\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3a^2d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

```
[Out] (-2*b^(4/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^
(2/3)) - b^(2/3)]*d) - (2*b^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) +
I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a^2*S
qrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) -
 (2*b^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^
(2/3)]])/(3*a^2*Sqrt[a^(2/3) + b^(2/3)]*d) + Coth[c + d*x]/(a*d) - Coth[c +
 d*x]^3/(3*a*d)
```

Rubi [A] time = 0.432501, antiderivative size = 317, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 3770, 3767, 2660, 618, 206, 204}

$$-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}+b^{2/3}}}-\frac{2b^{4/3}\tan^{-1}\left(\frac{(-1)^{5/6}\left(\sqrt[6]{-1}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[6]{-1}\left((-1)^{5/6}\sqrt[3]{b}+i\sqrt[3]{a}\tanh\left(\frac{1}{2}\right.\right.}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right.}{3a^2d\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```


```
[Out] (-2*b^(4/3)*ArcTan[((-1)^(5/6)*((-1)^(1/6)*b^(1/3) + I*a^(1/3)*Tanh[(c + d*
x)/2]))/Sqrt[-((-1)^(2/3)*a^(2/3)) - b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^
(2/3)) - b^(2/3)]*d) - (2*b^(4/3)*ArcTan[((-1)^(1/6)*((-1)^(5/6)*b^(1/3) +
I*a^(1/3)*Tanh[(c + d*x)/2]))/Sqrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]])/(3*a^2*S
qrt[(-1)^(1/3)*a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cosh[c + d*x]])/(a^2*d) -
 (2*b^(4/3)*ArcTanh[(b^(1/3) - a^(1/3)*Tanh[(c + d*x)/2])/Sqrt[a^(2/3) + b^
(2/3)]])/(3*a^2*Sqrt[a^(2/3) + b^(2/3)]*d) + Coth[c + d*x]/(a*d) - Coth[c +
 d*x]^3/(3*a*d)
```


Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a+b\sinh^3(c+dx)}\,dx = \int \left(-\frac{b\operatorname{csch}(c+dx)}{a^2}+\frac{\operatorname{csch}^4(c+dx)}{a}-\frac{b^2\sinh^2(c+dx)}{a^2\left(-a-b\sinh^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \operatorname{csch}^4(c+dx)\,dx}{a}-\frac{b\int \operatorname{csch}(c+dx)\,dx}{a^2}-\frac{b^2\int \frac{\sinh^2(c+dx)}{-a-b\sinh^3(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{b^2\int \left(-\frac{i}{3b^{2/3}\left(-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}-\frac{i}{3b^{2/3}\left(\sqrt[6]{-1}\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)\right)}-\cdots\right.}{a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{\left(ib^{4/3}\right)\int \frac{1}{-i\sqrt[3]{a}-i\sqrt[3]{b}\sinh(c+dx)}\,dx \cdots}{3a^2}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{\left(2b^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{-i\sqrt[3]{a}-2\sqrt[3]{b}x-i\cdots}\right.}{3a\cdots}$$

$$= \frac{b\tanh^{-1}(\cosh(c+dx))}{a^2d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}+\frac{\left(4b^{4/3}\right)\operatorname{Subst}\left(\int \frac{1}{-4\left(\sqrt[3]{-1}a^{2/3}-b\cdots\right.}\right.}{\cdots}$$

$$= \frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{-1}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}-b^{2/3}}\,d}+\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}-(-1)^{2/3}\sqrt[3]{a}\tanh\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{\sqrt[3]{-1}a^{2/3}-b^{2/3}}\,d}+\frac{b\tanh^{-1}\cdots}{\cdots}$$

Mathematica [C] time = 6.06156, size = 370, normalized size = 1.17

$$4b^2\operatorname{RootSum}\left[8\#1^3a+\#1^6b-3\#1^4b+3\#1^2b-b\&,\frac{2\#1^4\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)-\cdots}{\cdots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a + b*Sinh[c + d*x]^3),x]
```

```
[Out] (8*a*Coth[(c + d*x)/2] - 24*b*Log[Tanh[(c + d*x)/2]] + 4*b^2*RootSum[-b + 3
*b*#1^2 + 8*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (c + d*x + 2*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 2*
c*#1^2 - 2*d*x*#1^2 - 4*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + c*#1^4 + d*x*#1^4 + 2*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^4)/(b*#1 + 4*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + 8*a*Csch[c + d*x]
^3*Sinh[(c + d*x)/2]^4 - (a*Csch[(c + d*x)/2]^4*Sinh[c + d*x])/2 + 8*a*Tanh
[(c + d*x)/2])/(24*a^2*d)
```

Maple [C] time = 0.087, size = 178, normalized size = 0.6

$$-\frac{1}{24\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{3}+\frac{3}{8\,da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{24\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-3}+\frac{3}{8\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{b}{da^{2}}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)-1/24/d/a/tanh(1
/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tanh(1/2*d*x+1/2*c
))-4/3/d/a^2*b^2*sum(_R^2/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(tanh(1/2*d*x+1
/2*c)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{4\left(3\,e^{(2\,dx+2\,c)}-1\right)}{3\left(ade^{(6\,dx+6\,c)}-3\,ade^{(4\,dx+4\,c)}+3\,ade^{(2\,dx+2\,c)}-ad\right)}+\frac{b\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{a^{2}d}-\frac{b\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{a^{2}d}+16\int\frac{}{8\left(a^{2}b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x + 4*c) +
 3*a*d*e^(2*d*x + 2*c) - a*d) + b*log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) - b
*log((e^(d*x + c) - 1)*e^(-c))/(a^2*d) + 16*integrate(1/8*(b^2*e^(5*d*x + 5
*c) - 2*b^2*e^(3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b*e^(6*d*x + 6*c) - 3*a
^2*b*e^(4*d*x + 4*c) + 8*a^3*e^(3*d*x + 3*c) + 3*a^2*b*e^(2*d*x + 2*c) - a^
2*b), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a+b*sinh(d*x+c)**3),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{csch}(dx+c)^4}{b\sinh(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a+b*sinh(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csch(d*x + c)^4/(b*sinh(d*x + c)^3 + a), x)
```

3.182 $\int \frac{1}{1+\sinh^3(x)}\,dx$

Optimal. Leaf size=139

$$-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\log\left(-\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}\sqrt[6]{-1}\log\left(\sqrt[3]{-1}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-1}+1\right)-$$

```
[Out] (-2*(-1)^(1/6)*ArcTan[(I + (-1)^(1/6)*Tanh[x/2])/Sqrt[1 - (-1)^(1/3)]])/(3*
Sqrt[1 - (-1)^(1/3)]) - (Sqrt[2]*ArcTanh[(1 - Tanh[x/2])/Sqrt[2]])/3 - ((-1
)^(1/6)*Log[1 + (-1)^(5/6) - (-1)^(1/6)*Tanh[x/2]])/3 + ((-1)^(1/6)*Log[1 +
 (-1)^(1/6) + (-1)^(1/3)*Tanh[x/2]])/3
```

Rubi [A] time = 0.187819, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 1., Rules used = {3213, 2660, 618, 204, 617, 206, 616, 31}

$$-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\log\left(-\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}\sqrt[6]{-1}\log\left(\sqrt[3]{-1}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-1}+1\right)-$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^3)^(-1),x]
```


```
[Out] (-2*(-1)^(1/6)*ArcTan[(I + (-1)^(1/6)*Tanh[x/2])/Sqrt[1 - (-1)^(1/3)]])/(3*
Sqrt[1 - (-1)^(1/3)]) - (Sqrt[2]*ArcTanh[(1 - Tanh[x/2])/Sqrt[2]])/3 - ((-1
)^(1/6)*Log[1 + (-1)^(5/6) - (-1)^(1/6)*Tanh[x/2]])/3 + ((-1)^(1/6)*Log[1 +
 (-1)^(1/6) + (-1)^(1/3)*Tanh[x/2]])/3
```


Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sinh^3(x)}\,dx &= \int \left(\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}-i\sinh(x)\right)}+\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}+\sqrt[6]{-1}\sinh(x)\right)}+\frac{\sqrt[6]{-1}}{3\left(\sqrt[6]{-1}+(-1)^{5/6}\sinh(x)\right)}\right)dx\\
&= \frac{1}{3}\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}-i\sinh(x)}\,dx+\frac{1}{3}\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}+\sqrt[6]{-1}\sinh(x)}\,dx+\frac{1}{3}\sqrt[6]{-1}\int \frac{1}{\sqrt[6]{-1}+(-1)^{5/6}\sinh
\\
&= \frac{1}{3}\left(2\sqrt[6]{-1}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}-2ix-\sqrt[6]{-1}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{3}\left(2\sqrt[6]{-1}\right)\text{Subst}\left(\int \frac{1}{\sqrt[6]{-1}+2\sqrt[6]{-1}
\\
&= -\left(\frac{2}{3}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,1-\tanh\left(\frac{x}{2}\right)\right)\right)-\frac{1}{3}\left(4\sqrt[6]{-1}\right)\text{Subst}\left(\int \frac{1}{-4\left(1-\sqrt[3]{-1}\right)-x^2}\,dx,x,-2
\\
&= -\frac{2\sqrt[6]{-1}\tan^{-1}\left(\frac{i+\sqrt[6]{-1}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[3]{-1}}}\right)}{3\sqrt{1-\sqrt[3]{-1}}}-\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{1}{3}\sqrt[6]{-1}\log\left(1+(-1)^{5/6}-\sqrt[6]{-1}\tan
\end{aligned}$$

Mathematica [A] time = 1.39943, size = 156, normalized size = 1.12

$$\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)+i\sqrt{-1-i\sqrt{3}}\left(\sqrt{3}+i\right)\tan^{-1}\left(\frac{2+\left(1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2+2i\sqrt{3}}}\right)+\left(-1-i\sqrt{3}\right)\sqrt{-1+i\sqrt{3}}\tan^{-1}\left(\frac{2+\left(1+i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2-2i\sqrt{3}}}
}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^3)^(-1),x]
```

```
[Out] (I*Sqrt[-1 - I*Sqrt[3]]*(I + Sqrt[3])*ArcTan[(2 + (1 - I*Sqrt[3])*Tanh[x/2]
)/Sqrt[-2 + (2*I)*Sqrt[3]]] + (-1 - I*Sqrt[3])*Sqrt[-1 + I*Sqrt[3]]*ArcTan[
```

```
(2 + (1 + I*Sqrt[3])*Tanh[x/2])/Sqrt[-2 - (2*I)*Sqrt[3]]] + 2*ArcTanh[(-1 +
 Tanh[x/2])/Sqrt[2]])/(3*Sqrt[2])
```

Maple [C] time = 0.029, size = 82, normalized size = 0.6

$$\frac{2}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4+2\_Z^3+2\_Z^2-2\_Z+1\right)}\frac{-\_R^2-\_R+1}{2\_R^3+3\_R^2+2\_R-1}\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)+\frac{\sqrt{2}}{3}\mathrm{Artanh}\left(\frac{\sqrt{2}}{4}\left(2\tanh\left(x/2\right)-2\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^3),x)
```

```
[Out] 2/3*sum((-_R^2-_R+1)/(2*_R^3+3*_R^2+2*_R-1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z
^4+2*_Z^3+2*_Z^2-2*_Z+1))+1/3*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{6}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x-1}{\sqrt{2}+e^x+1}\right)-\int\frac{2\left(e^{(3x)}-4e^{(2x)}-e^x\right)}{3\left(e^{(4x)}-2e^{(3x)}+2e^{(2x)}+2e^x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(2)*log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - integrate(2/3*(
e^(3*x) - 4*e^(2*x) - e^x)/(e^(4*x) - 2*e^(3*x) + 2*e^(2*x) + 2*e^x + 1), x
)
```

Fricas [B] time = 1.93521, size = 591, normalized size = 4.25

$$-\frac{1}{6}\sqrt{3}\log\left(-4\left(\sqrt{3}+1\right)e^x+4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{3}\log\left(4\left(\sqrt{3}-1\right)e^x-4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{2}\log\left(-\frac{2\left(\sqrt{2}-1\right)e}{e^{(2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*log(-4*(sqrt(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sq
rt(3)*log(4*(sqrt(3) - 1)*e^x - 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqrt(2)*lo
g(-(2*(sqrt(2) - 1)*e^x + 2*sqrt(2) - e^(2*x) - 3)/(e^(2*x) + 2*e^x - 1)) +
 2/3*arctan(-(sqrt(3) + 1)*e^x + sqrt((sqrt(3) - 1)*e^x - sqrt(3) + e^(2*x)
 + 2)*(sqrt(3) + 1) - 1) - 2/3*arctan(-(sqrt(3) - 1)*e^x + 1/2*sqrt(-4*(sqr
t(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8)*(sqrt(3) - 1) + 1)
```

Sympy [B] time = 174.633, size = 1423, normalized size = 10.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**3),x)
```

```
[Out] 167721543*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1 + sqrt(2))/(711545940*sqrt(
1 + sqrt(3)*I) + 503164629*sqrt(2)*sqrt(1 + sqrt(3)*I) + 711545940*sqrt(3)*
I*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 821622480
*sqrt(6)*I + 1162008936*sqrt(3)*I) + 118590990*sqrt(2)*sqrt(1 + sqrt(3)*I)*
log(tanh(x/2) - 1 + sqrt(2))/(711545940*sqrt(1 + sqrt(3)*I) + 503164629*sqr
t(2)*sqrt(1 + sqrt(3)*I) + 711545940*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 503164
629*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 821622480*sqrt(6)*I + 1162008936*sqrt(3
)*I) + 193668156*sqrt(6)*I*log(tanh(x/2) - 1 + sqrt(2))/(711545940*sqrt(1 +
 sqrt(3)*I) + 503164629*sqrt(2)*sqrt(1 + sqrt(3)*I) + 711545940*sqrt(3)*I*s
qrt(1 + sqrt(3)*I) + 503164629*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 821622480*sq
rt(6)*I + 1162008936*sqrt(3)*I) + 273874160*sqrt(3)*I*log(tanh(x/2) - 1 + s
qrt(2))/(711545940*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(2)*sqrt(1 + sqrt(3)
*I) + 711545940*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(6)*I*sqrt(1
+ sqrt(3)*I) + 821622480*sqrt(6)*I + 1162008936*sqrt(3)*I) + 118590990*sqrt
(6)*I*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1 + sqrt(2))/(711545940*sqrt(1 +
sqrt(3)*I) + 503164629*sqrt(2)*sqrt(1 + sqrt(3)*I) + 711545940*sqrt(3)*I*sq
rt(1 + sqrt(3)*I) + 503164629*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 821622480*sqr
t(6)*I + 1162008936*sqrt(3)*I) + 167721543*sqrt(3)*I*sqrt(1 + sqrt(3)*I)*lo
g(tanh(x/2) - 1 + sqrt(2))/(711545940*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(
2)*sqrt(1 + sqrt(3)*I) + 711545940*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 50316462
9*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 821622480*sqrt(6)*I + 1162008936*sqrt(3)*
I) - 167721543*sqrt(3)*I*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - sqrt(2) - 1)/(
711545940*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(2)*sqrt(1 + sqrt(3)*I) + 711
545940*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(6)*I*sqrt(1 + sqrt(3)
*I) + 821622480*sqrt(6)*I + 1162008936*sqrt(3)*I) - 118590990*sqrt(6)*I*sqr
t(1 + sqrt(3)*I)*log(tanh(x/2) - sqrt(2) - 1)/(711545940*sqrt(1 + sqrt(3)*I
) + 503164629*sqrt(2)*sqrt(1 + sqrt(3)*I) + 711545940*sqrt(3)*I*sqrt(1 + sq
rt(3)*I) + 503164629*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 821622480*sqrt(6)*I +
1162008936*sqrt(3)*I) - 273874160*sqrt(3)*I*log(tanh(x/2) - sqrt(2) - 1)/(7
11545940*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(2)*sqrt(1 + sqrt(3)*I) + 7115
45940*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(6)*I*sqrt(1 + sqrt(3)*
I) + 821622480*sqrt(6)*I + 1162008936*sqrt(3)*I) - 193668156*sqrt(6)*I*log(
tanh(x/2) - sqrt(2) - 1)/(711545940*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(2)
*sqrt(1 + sqrt(3)*I) + 711545940*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 503164629*
sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 821622480*sqrt(6)*I + 1162008936*sqrt(3)*I)
 - 118590990*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - sqrt(2) - 1)/(7115
45940*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(2)*sqrt(1 + sqrt(3)*I) + 7115459
40*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 503164629*sqrt(6)*I*sqrt(1 + sqrt(3)*I)
+ 821622480*sqrt(6)*I + 1162008936*sqrt(3)*I) - 167721543*sqrt(1 + sqrt(3)*
I)*log(tanh(x/2) - sqrt(2) - 1)/(711545940*sqrt(1 + sqrt(3)*I) + 503164629*
sqrt(2)*sqrt(1 + sqrt(3)*I) + 711545940*sqrt(3)*I*sqrt(1 + sqrt(3)*I) + 503
164629*sqrt(6)*I*sqrt(1 + sqrt(3)*I) + 821622480*sqrt(6)*I + 1162008936*sqr
t(3)*I)
```

Giac [A] time = 1.16328, size = 140, normalized size = 1.01

$$\frac{1}{6}\left(\sqrt{3}+i\right)\log\left(\sqrt{3}+(i+1)\,e^{x}-1\right)+\frac{1}{6}\left(\sqrt{3}-i\right)\log\left(i\sqrt{3}+(i+1)\,e^{x}-i\right)-\frac{1}{6}\left(\sqrt{3}+i\right)\log\left(-i\sqrt{3}+(i+1)\,e^{x}-i\right)-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^3),x, algorithm="giac")
```

```
[Out] 1/6*(sqrt(3) + I)*log(sqrt(3) + (I + 1)*e^x - 1) + 1/6*(sqrt(3) - I)*log(I*
sqrt(3) + (I + 1)*e^x - I) - 1/6*(sqrt(3) + I)*log(-I*sqrt(3) + (I + 1)*e^x
 - I) - 1/6*(sqrt(3) - I)*log(-sqrt(3) + (I + 1)*e^x - 1) + 1/6*sqrt(2)*log
(1/2*abs(-2*sqrt(2) + 2*e^x + 2)/(sqrt(2) + e^x + 1))
```

3.183 $\int \frac{1}{1-\sinh^3(x)}\,dx$

Optimal. Leaf size=133

$$\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)-\frac{1}{3}(-1)^{5/6}\log\left((-1)^{2/3}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}(-1)^{5/6}\log\left((-1)^{5/6}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-1}\right.$$

```
[Out] (2*(-1)^(5/6)*ArcTan[(I - (-1)^(5/6)*Tanh[x/2])/Sqrt[1 + (-1)^(2/3)]])/(3*S
qrt[1 + (-1)^(2/3)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]])/3 - ((-1)
^(5/6)*Log[1 + (-1)^(5/6) + (-1)^(2/3)*Tanh[x/2]])/3 + ((-1)^(5/6)*Log[1 +
(-1)^(1/6) + (-1)^(5/6)*Tanh[x/2]])/3
```

Rubi [A] time = 0.188952, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 8, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.8, Rules used = {3213, 2660, 618, 204, 616, 31, 617, 206}

$$\frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)-\frac{1}{3}(-1)^{5/6}\log\left((-1)^{2/3}\tanh\left(\frac{x}{2}\right)+(-1)^{5/6}+1\right)+\frac{1}{3}(-1)^{5/6}\log\left((-1)^{5/6}\tanh\left(\frac{x}{2}\right)+\sqrt[6]{-1}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^3)^(-1),x]
```


```
[Out] (2*(-1)^(5/6)*ArcTan[(I - (-1)^(5/6)*Tanh[x/2])/Sqrt[1 + (-1)^(2/3)]])/(3*S
qrt[1 + (-1)^(2/3)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]])/3 - ((-1)
^(5/6)*Log[1 + (-1)^(5/6) + (-1)^(2/3)*Tanh[x/2]])/3 + ((-1)^(5/6)*Log[1 +
(-1)^(1/6) + (-1)^(5/6)*Tanh[x/2]])/3
```


Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 616

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = Rt[b^2
 - 4*a*c, 2]}, Dist[c/q, Int[1/Simp[b/2 - q/2 + c*x, x], x], x] - Dist[c/q,
 Int[1/Simp[b/2 + q/2 + c*x, x], x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2
- 4*a*c, 0] && PosQ[b^2 - 4*a*c] && PerfectSquareQ[b^2 - 4*a*c]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-\sinh^3(x)}\,dx = \int \left(-\frac{(-1)^{5/6}}{3\left(-(-1)^{5/6}-i\sinh(x)\right)} - \frac{(-1)^{5/6}}{3\left(-(-1)^{5/6}+\sqrt[6]{-1}\sinh(x)\right)} - \frac{(-1)^{5/6}}{3\left(-(-1)^{5/6}+(-1)^{5/6}\sinh(x)\right)}\right) dx$$

$$= -\left(\frac{1}{3}(-1)^{5/6}\int \frac{1}{-(-1)^{5/6}-i\sinh(x)}\,dx\right) - \frac{1}{3}(-1)^{5/6}\int \frac{1}{-(-1)^{5/6}+\sqrt[6]{-1}\sinh(x)}\,dx - \frac{1}{3}(-1)^{5/6}\int \frac{}{-(-}$$

$$= -\left(\frac{1}{3}\left(2(-1)^{5/6}\right)\operatorname{Subst}\left(\int \frac{1}{-(-1)^{5/6}-2ix+(-1)^{5/6}x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)\right) - \frac{1}{3}\left(2(-1)^{5/6}\right)\operatorname{Subst}\left(\int \frac{}{-(}\right.$$

$$= \frac{2}{3}\operatorname{Subst}\left(\int \frac{1}{2-x^2}\,dx, x, 1+\tanh\left(\frac{x}{2}\right)\right) + \frac{1}{3}(-1)^{2/3}\operatorname{Subst}\left(\int \frac{1}{-1+\sqrt[6]{-1}+(-1)^{5/6}x}\,dx, x, \tanh\left(\frac{x}{2}\right.\right.$$

$$= \frac{2(-1)^{5/6}\tan^{-1}\left(\frac{i-(-1)^{5/6}\tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{2/3}}}\right)}{3\sqrt{1+(-1)^{2/3}}} + \frac{1}{3}\sqrt{2}\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right) - \frac{1}{3}(-1)^{5/6}\log\left(1+(-1)^{5/6}+(-1)^{2/3}\right.$$

Mathematica [A] time = 1.21926, size = 156, normalized size = 1.17

$$\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\sqrt{-1+i\sqrt{3}}\left(1+i\sqrt{3}\right)\tan^{-1}\left(\frac{2+\left(-1-i\sqrt{3}\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2-2i\sqrt{3}}}\right)+\sqrt{-1-i\sqrt{3}}\left(1-i\sqrt{3}\right)\tan^{-1}\left(\frac{2+i\left(\sqrt{3}+i\right)\tanh\left(\frac{x}{2}\right)}{\sqrt{-2+2i\sqrt{3}}}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^3)^(-1),x]
```

```
[Out] (Sqrt[-1 + I*Sqrt[3]]*(1 + I*Sqrt[3])*ArcTan[(2 + (-1 - I*Sqrt[3])*Tanh[x/2
])/Sqrt[-2 - (2*I)*Sqrt[3]]] + Sqrt[-1 - I*Sqrt[3]]*(1 - I*Sqrt[3])*ArcTan[
(2 + I*(I + Sqrt[3])*Tanh[x/2])/Sqrt[-2 + (2*I)*Sqrt[3]]] + 2*ArcTanh[(1 +
Tanh[x/2])/Sqrt[2]])/(3*Sqrt[2])
```

Maple [C] time = 0.029, size = 80, normalized size = 0.6

$$\frac{2}{3}\sum_{\_R=\mathrm{RootOf}\left(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1\right)}\frac{-\_R^2+\_R+1}{2\_R^3-3\_R^2+2\_R+1}\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)+\frac{\sqrt{2}}{3}\mathrm{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^3),x)
```

```
[Out] 2/3*sum((-_R^2+_R+1)/(2*_R^3-3*_R^2+2*_R+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z
^4-2*_Z^3+2*_Z^2+2*_Z+1))+1/3*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2)
)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{6}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x+1}{\sqrt{2}+e^x-1}\right)+\int\frac{2\left(e^{(3x)}+4e^{(2x)}-e^x\right)}{3\left(e^{(4x)}+2e^{(3x)}+2e^{(2x)}-2e^x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="maxima")
```

```
[Out] -1/6*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + integrate(2/3*
(e^(3*x) + 4*e^(2*x) - e^x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1),
x)
```

Fricas [B] time = 1.85063, size = 590, normalized size = 4.44

$$-\frac{1}{6}\sqrt{3}\log\left(4\left(\sqrt{3}+1\right)e^x+4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{3}\log\left(-4\left(\sqrt{3}-1\right)e^x-4\sqrt{3}+4e^{(2x)}+8\right)+\frac{1}{6}\sqrt{2}\log\left(\frac{2\left(\sqrt{2}-1\right.}{e}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/6*sqrt(3)*log(4*(sqrt(3) + 1)*e^x + 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqr
t(3)*log(-4*(sqrt(3) - 1)*e^x - 4*sqrt(3) + 4*e^(2*x) + 8) + 1/6*sqrt(2)*lo
g((2*(sqrt(2) - 1)*e^x - 2*sqrt(2) + e^(2*x) + 3)/(e^(2*x) - 2*e^x - 1)) -
2/3*arctan(-(sqrt(3) + 1)*e^x + 1/2*sqrt(-4*(sqrt(3) - 1)*e^x - 4*sqrt(3) +
 4*e^(2*x) + 8)*(sqrt(3) + 1) + 1) + 2/3*arctan(-(sqrt(3) - 1)*e^x + sqrt((
sqrt(3) + 1)*e^x + sqrt(3) + e^(2*x) + 2)*(sqrt(3) - 1) - 1)
```

Sympy [B] time = 72.4185, size = 3742, normalized size = 28.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**3),x)
```

```
[Out] 831721548108794611356*sqrt(6)*I*log(tanh(x/2) + 1 + sqrt(2))/(-105860870410
80284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt
(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665
022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 3528695
680360094803440*log(tanh(x/2) + 1 + sqrt(2))/(-10586087041080284410320*sqrt
(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqr
t(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1
 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 2037293400944170367740*
sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) + 1 + sqrt(2))/(-1058608704108028
4410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*
sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 122237604056650222
06440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 28811679581
48549067306*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) + 1 + sqrt(2))/(-105860870410
80284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt
(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665
022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 1176231
893453364934480*sqrt(3)*I*log(tanh(x/2) + 1 + sqrt(2))/(-105860870410802844
10320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sq
rt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206
440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 2495164644326
383834068*sqrt(2)*log(tanh(x/2) + 1 + sqrt(2))/(-10586087041080284410320*sq
rt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + s
qrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt
(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 249516464432638383406
8*sqrt(2)*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) +
14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I
) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqr
t(3)*I) + 4990329288652767668136*sqrt(3)*I) + 1176231893453364934480*sqrt(3
)*I*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) + 149709
87865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 35
28695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I
) + 4990329288652767668136*sqrt(3)*I) + 2881167958148549067306*sqrt(1 + sqr
t(3)*I)*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) + 14
970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I)
- 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(
3)*I) + 4990329288652767668136*sqrt(3)*I) - 2037293400944170367740*sqrt(2)*
sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*
sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 +
 sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sq
rt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 3528695680360094803
440*log(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) + 149709
87865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 35
28695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I
) + 4990329288652767668136*sqrt(3)*I) - 831721548108794611356*sqrt(6)*I*log
(tanh(x/2) - sqrt(2) + 1)/(-10586087041080284410320*sqrt(2) + 1497098786595
8303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 352869568
0360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 499
0329288652767668136*sqrt(3)*I) + 1440583979074274533653*sqrt(3)*I*sqrt(1 -
sqrt(3)*I)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(2)*sqrt(1 - sqrt(
3)*I)/2 + sqrt(3)*I/2)/(-10586087041080284410320*sqrt(2) + 1497098786595830
3004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 352869568036
0094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 499032
9288652767668136*sqrt(3)*I) + 2352463786906729868960*sqrt(3)*I*sqrt(1 - sqr
t(3)*I)*log(tanh(x/2) - 1/2 - sqrt(2)*sqrt(1 - sqrt(3)*I)/2 + sqrt(3)*I/2)/
(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445
647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I +
12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3
```

```
)*I) - 1018646700472085183870*sqrt(2)*sqrt(1 - sqrt(3)*I)*sqrt(1 + sqrt(3)*
I)*log(tanh(x/2) - 1/2 - sqrt(2)*sqrt(1 - sqrt(3)*I)/2 + sqrt(3)*I/2)/(-105
86087041080284410320*sqrt(2) + 149709878659583030044080 - 864350387444564720
1918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223
760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I)
+ 1440583979074274533653*sqrt(1 - sqrt(3)*I)*sqrt(1 + sqrt(3)*I)*log(tanh(x
/2) - 1/2 - sqrt(2)*sqrt(1 - sqrt(3)*I)/2 + sqrt(3)*I/2)/(-1058608704108028
4410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*
sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 122237604056650222
06440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 16634430962
17589222712*sqrt(6)*I*sqrt(1 - sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(2)*sqr
t(1 - sqrt(3)*I)/2 + sqrt(3)*I/2)/(-10586087041080284410320*sqrt(2) + 14970
987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3
528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*
I) + 4990329288652767668136*sqrt(3)*I) - 1018646700472085183870*sqrt(6)*I*s
qrt(1 - sqrt(3)*I)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(2)*sqrt(1
 - sqrt(3)*I)/2 + sqrt(3)*I/2)/(-10586087041080284410320*sqrt(2) + 14970987
865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528
695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I)
+ 4990329288652767668136*sqrt(3)*I) + 1018646700472085183870*sqrt(6)*I*sqrt
(1 - sqrt(3)*I)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 + sqrt(2)*sqrt(1 -
sqrt(3)*I)/2 + sqrt(3)*I/2)/(-10586087041080284410320*sqrt(2) + 14970987865
958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695
680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4
990329288652767668136*sqrt(3)*I) + 1663443096217589222712*sqrt(6)*I*sqrt(1
- sqrt(3)*I)*log(tanh(x/2) - 1/2 + sqrt(2)*sqrt(1 - sqrt(3)*I)/2 + sqrt(3)*
I/2)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 86435038
74445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)
*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*s
qrt(3)*I) - 1440583979074274533653*sqrt(1 - sqrt(3)*I)*sqrt(1 + sqrt(3)*I)*
log(tanh(x/2) - 1/2 + sqrt(2)*sqrt(1 - sqrt(3)*I)/2 + sqrt(3)*I/2)/(-105860
87041080284410320*sqrt(2) + 14970987865958303004408 - 864350387444564720191
8*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760
405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 1
018646700472085183870*sqrt(2)*sqrt(1 - sqrt(3)*I)*sqrt(1 + sqrt(3)*I)*log(t
anh(x/2) - 1/2 + sqrt(2)*sqrt(1 - sqrt(3)*I)/2 + sqrt(3)*I/2)/(-10586087041
080284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqr
t(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 1222376040566
5022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 235246
3786906729868960*sqrt(3)*I*sqrt(1 - sqrt(3)*I)*log(tanh(x/2) - 1/2 + sqrt(2
)*sqrt(1 - sqrt(3)*I)/2 + sqrt(3)*I/2)/(-10586087041080284410320*sqrt(2) +
14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I
) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqr
t(3)*I) + 4990329288652767668136*sqrt(3)*I) - 1440583979074274533653*sqrt(3
)*I*sqrt(1 - sqrt(3)*I)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 + sqrt(2)*s
qrt(1 - sqrt(3)*I)/2 + sqrt(3)*I/2)/(-10586087041080284410320*sqrt(2) + 149
70987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) -
 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3
)*I) + 4990329288652767668136*sqrt(3)*I) + 831721548108794611356*sqrt(6)*I*
sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 - sqrt(2)*sqrt(1 + sq
rt(3)*I)/2)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 8
643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*
sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 49903292886527676
68136*sqrt(3)*I) - 4074586801888340735480*sqrt(2)*log(tanh(x/2) - 1/2 - sqr
t(3)*I/2 - sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-10586087041080284410320*sqrt(2)
 + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3
)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 +
sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 3528695680360094803440*sqr
t(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 - sqrt(2)*sqrt(1 + sqrt(
```

```
3)*I)/2)/(-1058608704108028441032 0*sqrt(2) + 14970987865958303004408 - 8643
503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqr
t(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 499032928865276766 81
36*sqrt(3)*I) - 2495164644326383834068*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh
(x/2) - 1/2 - sqrt(3)*I/2 - sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-10586087041080
284410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2
)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 122237604056650 2
2206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 117623189
3453364934480*sqrt(3)*I*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I
/2 - sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-10586087041080284410320*sqrt(2) + 149
70987865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) -
 3528695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3
)*I) + 4990329288652767668136*sqrt(3)*I) + 5762335916297098134612*log(tanh(
x/2) - 1/2 - sqrt(3)*I/2 - sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-105860870410802
84410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)
*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 12223760405665022
206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) - 5762335916
297098134612*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 + sqrt(2)*sqrt(1 + sqrt(3)*I
)/2)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 86435038
74445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)
*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*s
qrt(3)*I) + 1176231893453364934480*sqrt(3)*I*sqrt(1 + sqrt(3)*I)*log(tanh(x
/2) - 1/2 - sqrt(3)*I/2 + sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-1058608704108028
4410320*sqrt(2) + 14970987865958303004408 - 8643503874445647201918*sqrt(2)*
sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 122237604056650222
06440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)*I) + 24951646443
26383834068*sqrt(2)*sqrt(1 + sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 +
 sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(-10586087041080284410320*sqrt(2) + 1497098
7865958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 352
8695680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I)
 + 4990329288652767668136*sqrt(3)*I) - 3528695680360094803440*sqrt(1 + sqrt
(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 + sqrt(2)*sqrt(1 + sqrt(3)*I)/2)/(
-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 86435038744456
47201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)*I + 1
2223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*sqrt(3)
*I) + 4074586801888340735480*sqrt(2)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 + sq
rt(2)*sqrt(1 + sqrt(3)*I)/2)/(-10586087041080284410320*sqrt(2) + 1497098786
5958303004408 - 8643503874445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 352869
5680360094803440*sqrt(6)*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) +
4990329288652767668136*sqrt(3)*I) - 8317215481087946113 56*sqrt(6)*I*sqrt(1
+ sqrt(3)*I)*log(tanh(x/2) - 1/2 - sqrt(3)*I/2 + sqrt(2)*sqrt(1 + sqrt(3)*I
)/2)/(-10586087041080284410320*sqrt(2) + 14970987865958303004408 - 86435038
74445647201918*sqrt(2)*sqrt(1 + sqrt(3)*I) - 3528695680360094803440*sqrt(6)
*I + 12223760405665022206440*sqrt(1 + sqrt(3)*I) + 4990329288652767668136*s
qrt(3)*I)
```

Giac [A] time = 1.18007, size = 146, normalized size = 1.1

$$-\frac{1}{6}\left(\sqrt{3}-i\right)\log\left(\sqrt{3}+(i+1)\,e^{x}+1\right)-\frac{1}{6}\left(\sqrt{3}+i\right)\log\left(i\sqrt{3}+(i+1)\,e^{x}+i\right)+\frac{1}{6}\left(\sqrt{3}-i\right)\log\left(-i\sqrt{3}+(i+1)\,e^{x}+i\right)+\frac{1}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^3),x, algorithm="giac")
```

```
[Out] -1/6*(sqrt(3) - I)*log(sqrt(3) + (I + 1)*e^x + 1) - 1/6*(sqrt(3) + I)*log(I
*sqrt(3) + (I + 1)*e^x + I) + 1/6*(sqrt(3) - I)*log(-I*sqrt(3) + (I + 1)*e^
```

```
x + I) + 1/6*(sqrt(3) + I)*log(-sqrt(3) + (I + 1)*e^x + 1) - 1/6*sqrt(2)*lo
g(abs(-2*sqrt(2) + 2*e^x - 2)/abs(2*sqrt(2) + 2*e^x - 2))
```

3.184 $\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$

Optimal. Leaf size=111

$$\frac{(48a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{(80a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x(48a+35b)+\frac{b\sinh(c+dx)\,\text{c}}{8d}$$

```
[Out] ((48*a + 35*b)*x)/128 - ((80*a + 93*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d)
 + ((48*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b*Cosh[c +
d*x]^5*Sinh[c + d*x])/(48*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x])/(8*d)
```

Rubi [A] time = 0.142796, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{(48a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{(80a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x(48a+35b)+\frac{b\sinh(c+dx)\,\text{c}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] ((48*a + 35*b)*x)/128 - ((80*a + 93*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d)
 + ((48*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b*Cosh[c +
d*x]^5*Sinh[c + d*x])/(48*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x])/(8*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
```

```
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^4(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^4\left(a-2ax^2+(a+b)x^4\right)}{\left(1-x^2\right)^5}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)}{8d} - \frac{\text{Subst}\left(\int \frac{b+8bx^2-8(a-b)x^4+8(a+b)x^6}{\left(1-x^2\right)^4}dx, x, \tan\right.}{8d}$$

$$= -\frac{25b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b\cosh^7(c+dx)\sinh(c+dx)}{8d} + \frac{\text{Subs}}{}$$

$$= \frac{(48a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} - \frac{25b\cosh^5(c+dx)\sinh(c+d}{48d}$$

$$= -\frac{(80a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{(48a+163b)\cosh^3(c+dx)\sin}{192d}$$

$$= \frac{1}{128}(48a+35b)x - \frac{(80a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{(48a+163b}{}$$

Mathematica [A] time = 0.141628, size = 82, normalized size = 0.74

$$\frac{-96(8a+7b)\sinh(2(c+dx))+24(4a+7b)\sinh(4(c+dx))+1152ac+1152adx-32b\sinh(6(c+dx))+3b\sinh(8(c}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (1152*a*c + 840*b*c + 1152*a*d*x + 840*b*d*x - 96*(8*a + 7*b)*Sinh[2*(c + d
*x)] + 24*(4*a + 7*b)*Sinh[4*(c + d*x)] - 32*b*Sinh[6*(c + d*x)] + 3*b*Sinh
[8*(c + d*x)])/(3072*d)
```

Maple [A] time = 0.014, size = 98, normalized size = 0.9

$$\frac{1}{d}\left(b\left(\left(\frac{(\sinh(dx+c))^7}{8}-\frac{7(\sinh(dx+c))^5}{48}+\frac{35(\sinh(dx+c))^3}{192}-\frac{35\sinh(dx+c)}{128}\right)\cosh(dx+c)+\frac{35dx}{128}+\frac{35c}{128}\right)+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(b*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*s
inh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+a*((1/4*sinh(d*x+c)^3-3/8*sinh
(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.14058, size = 236, normalized size = 2.13

$$\frac{1}{64}a\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{6144}b\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+672e^{(-6dx-6c)}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) - 1/6144*b*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x -
 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (6
72*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*
d*x - 8*c))/d)
```

Fricas [A] time = 1.62985, size = 467, normalized size = 4.21

$$\frac{3b\cosh(dx+c)\sinh(dx+c)^7+3\left(7b\cosh(dx+c)^3-8b\cosh(dx+c)\right)\sinh(dx+c)^5+\left(21b\cosh(dx+c)^5-80b\,\mathrm{co}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/384*(3*b*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b*cosh(d*x + c)^3 - 8*b*cos
h(d*x + c))*sinh(d*x + c)^5 + (21*b*cosh(d*x + c)^5 - 80*b*cosh(d*x + c)^3
+ 12*(4*a + 7*b)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(48*a + 35*b)*d*x + 3*(
b*cosh(d*x + c)^7 - 8*b*cosh(d*x + c)^5 + 4*(4*a + 7*b)*cosh(d*x + c)^3 - 8
*(8*a + 7*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 14.1661, size = 306, normalized size = 2.76

$$\begin{cases}\frac{3ax\sinh^4(c+dx)}{8}-\frac{3ax\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3ax\cosh^4(c+dx)}{8}+\frac{5a\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{3a\sinh(c+dx)\cosh^3(c+dx)}{8d}+\frac{35bx\sinh^8(c+d}{128}\\ x\left(a+b\sinh^4(c)\right)\sinh^4(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 + 5*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
- 3*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 35*b*x*sinh(c + d*x)**8/128 -
35*b*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*b*x*sinh(c + d*x)**4*cosh
(c + d*x)**4/64 - 35*b*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 35*b*x*cosh
(c + d*x)**8/128 + 93*b*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*b*sinh
(c + d*x)**5*cosh(c + d*x)**3/(384*d) + 385*b*sinh(c + d*x)**3*cosh(c + d*x
)**5/(384*d) - 35*b*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(
a + b*sinh(c)**4)*sinh(c)**4, True))
```

Giac [A] time = 1.20316, size = 258, normalized size = 2.32

$$48\,(dx+c)(48\,a+35\,b)+3\,be^{(8\,dx+8\,c)}-32\,be^{(6\,dx+6\,c)}+96\,ae^{(4\,dx+4\,c)}+168\,be^{(4\,dx+4\,c)}-768\,ae^{(2\,dx+2\,c)}-672\,be^{(2\,dx+2\,c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/6144*(48*(d*x + c)*(48*a + 35*b) + 3*b*e^(8*d*x + 8*c) - 32*b*e^(6*d*x +
6*c) + 96*a*e^(4*d*x + 4*c) + 168*b*e^(4*d*x + 4*c) - 768*a*e^(2*d*x + 2*c)
 - 672*b*e^(2*d*x + 2*c) - (2400*a*e^(8*d*x + 8*c) + 1750*b*e^(8*d*x + 8*c)
 - 768*a*e^(6*d*x + 6*c) - 672*b*e^(6*d*x + 6*c) + 96*a*e^(4*d*x + 4*c) + 1
68*b*e^(4*d*x + 4*c) - 32*b*e^(2*d*x + 2*c) + 3*b)*e^(-8*d*x - 8*c))/d
```

3.185 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=67

$$\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}$$

```
[Out] -(((a + b)*Cosh[c + d*x])/d) + ((a + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b*Cos
h[c + d*x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0661279, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3215, 1153}

$$\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^7(c+dx)}{7d}-\frac{3b\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -(((a + b)*Cosh[c + d*x])/d) + ((a + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b*Cos
h[c + d*x]^5)/(5*d) + (b*Cosh[c + d*x]^7)/(7*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx&=-\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)\,dx,x,\cosh(c+dx)\right)}{d}\\&=-\frac{\text{Subst}\left(\int\left(a\left(1+\frac{b}{a}\right)-(a+3b)x^2+3bx^4-bx^6\right)\,dx,x,\cosh(c+dx)\right)}{d}\\&=-\frac{(a+b)\cosh(c+dx)}{d}+\frac{(a+3b)\cosh^3(c+dx)}{3d}-\frac{3b\cosh^5(c+dx)}{5d}+\frac{b\cosh}{}\end{aligned}$$

Mathematica [A] time = 0.0288673, size = 93, normalized size = 1.39

$$-\frac{3a\cosh(c+dx)}{4d}+\frac{a\cosh(3(c+dx))}{12d}-\frac{35b\cosh(c+dx)}{64d}+\frac{7b\cosh(3(c+dx))}{64d}-\frac{7b\cosh(5(c+dx))}{320d}+\frac{b\cosh(7(c+dx}{448d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-3*a*Cosh[c + d*x])/(4*d) - (35*b*Cosh[c + d*x])/(64*d) + (a*Cosh[3*(c + d
*x)])/(12*d) + (7*b*Cosh[3*(c + d*x)])/(64*d) - (7*b*Cosh[5*(c + d*x)])/(32
0*d) + (b*Cosh[7*(c + d*x)])/(448*d)
```

Maple [A] time = 0.014, size = 66, normalized size = 1.

$$\frac{1}{d}\left(b\left(-\frac{16}{35}+\frac{(\sinh(dx+c))^6}{7}-\frac{6(\sinh(dx+c))^4}{35}+\frac{8(\sinh(dx+c))^2}{35}\right)\cosh(dx+c)+a\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\mathrm{co}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(b*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cos
h(d*x+c)+a*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.12945, size = 212, normalized size = 3.16

$$-\frac{1}{4480}b\left(\frac{\left(49e^{(-2dx-2c)}-245e^{(-4dx-4c)}+1225e^{(-6dx-6c)}-5\right)e^{(7dx+7c)}}{d}+\frac{1225e^{(-dx-c)}-245e^{(-3dx-3c)}+49e^{(-5dx-5c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/4480*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x - 6
*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) + 49
*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a*(e^(3*d*x + 3*c)/d - 9*
e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.6346, size = 439, normalized size = 6.55

$$15b\cosh(dx+c)^7+105b\cosh(dx+c)\sinh(dx+c)^6-147b\cosh(dx+c)^5+105\left(5b\cosh(dx+c)^3-7b\cosh(dx\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b*cosh(d*x + c)^7 + 105*b*cosh(d*x + c)*sinh(d*x + c)^6 - 147*b*
cosh(d*x + c)^5 + 105*(5*b*cosh(d*x + c)^3 - 7*b*cosh(d*x + c))*sinh(d*x +
c)^4 + 35*(16*a + 21*b)*cosh(d*x + c)^3 + 105*(3*b*cosh(d*x + c)^5 - 14*b*c
osh(d*x + c)^3 + (16*a + 21*b)*cosh(d*x + c))*sinh(d*x + c)^2 - 105*(48*a +
 35*b)*cosh(d*x + c))/d
```

Sympy [A] time = 8.48735, size = 128, normalized size = 1.91

$$\begin{cases} \frac{a\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a\cosh^3(c+dx)}{3d} + \frac{b\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{2b\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{8b\sinh^2(c+dx)\cosh^5(c+dx)}{5d} - \frac{16b\cosh \\ x\left(a+b\sinh^4(c)\right)\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a*cosh(c + d*x)**3/(3*d)
+ b*sinh(c + d*x)**6*cosh(c + d*x)/d - 2*b*sinh(c + d*x)**4*cosh(c + d*x)**
3/d + 8*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 16*b*cosh(c + d*x)**7/(
35*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**3, True))
```

Giac [B] time = 1.16218, size = 198, normalized size = 2.96

$$\frac{15\,be^{(7\,dx+7\,c)} - 147\,be^{(5\,dx+5\,c)} + 560\,ae^{(3\,dx+3\,c)} + 735\,be^{(3\,dx+3\,c)} - 5040\,ae^{(dx+c)} - 3675\,be^{(dx+c)} - \left(5040\,ae^{(6\,dx+6\,c)} + 3675\,b}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/13440*(15*b*e^(7*d*x + 7*c) - 147*b*e^(5*d*x + 5*c) + 560*a*e^(3*d*x + 3*
c) + 735*b*e^(3*d*x + 3*c) - 5040*a*e^(d*x + c) - 3675*b*e^(d*x + c) - (504
0*a*e^(6*d*x + 6*c) + 3675*b*e^(6*d*x + 6*c) - 560*a*e^(4*d*x + 4*c) - 735*
b*e^(4*d*x + 4*c) + 147*b*e^(2*d*x + 2*c) - 15*b)*e^(-7*d*x - 7*c))/d
```

3.186 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=83

$$\frac{(8a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}x(8a+5b)+\frac{b\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b\sinh(c+dx)\cosh^3(c+dx}{24d}$$

```
[Out] -((8*a + 5*b)*x)/16 + ((8*a + 11*b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (
13*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*Cosh[c + d*x]^5*Sinh[c + d*
x])/(6*d)
```

Rubi [A] time = 0.0976626, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3217, 1257, 1157, 385, 206}

$$\frac{(8a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}x(8a+5b)+\frac{b\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b\sinh(c+dx)\cosh^3(c+dx}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -((8*a + 5*b)*x)/16 + ((8*a + 11*b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) - (
13*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*Cosh[c + d*x]^5*Sinh[c + d*
x])/(6*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-2ax^2+(a+b)x^4\right)}{\left(1-x^2\right)^4}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d} + \frac{\text{Subst}\left(\int \frac{-b+6(a-b)x^2-6(a+b)x^4}{\left(1-x^2\right)^3}dx, x, \tanh(c+ \right.}{6d}$$

$$= -\frac{13b\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d} - \frac{\text{Subst}\left(\int \right.}{}$$

$$= \frac{(8a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\,c}{}$$

$$= -\frac{1}{16}(8a+5b)x + \frac{(8a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b\cosh^3(c+dx)\,s}{24d}$$

Mathematica [A] time = 0.0771824, size = 63, normalized size = 0.76

$$\frac{(48a+45b)\sinh(2(c+dx)) - 96ac - 96adx - 9b\sinh(4(c+dx)) + b\sinh(6(c+dx)) - 60bc - 60bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-96*a*c - 60*b*c - 96*a*d*x - 60*b*d*x + (48*a + 45*b)*Sinh[2*(c + d*x)] -
 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.014, size = 76, normalized size = 0.9

$$\frac{1}{d}\left(b\left(\left(\frac{(\sinh(dx+c))^5}{6} - \frac{5(\sinh(dx+c))^3}{24} + \frac{5\sinh(dx+c)}{16}\right)\cosh(dx+c) - \frac{5dx}{16} - \frac{5c}{16}\right) + a\left(\frac{\cosh(dx+c)\sinh(dx+}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(b*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)
-5/16*d*x-5/16*c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.11428, size = 165, normalized size = 1.99

$$-\frac{1}{8}a\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{384}b\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b*((9*e^(-2*d
*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d +
(45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [A] time = 1.60365, size = 292, normalized size = 3.52

$$\frac{3b\cosh(dx+c)\sinh(dx+c)^5+2\left(5b\cosh(dx+c)^3-9b\cosh(dx+c)\right)\sinh(dx+c)^3-6(8a+5b)dx+3\left(b\cosh\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 - 9*b*cosh
(d*x + c))*sinh(d*x + c)^3 - 6*(8*a + 5*b)*d*x + 3*(b*cosh(d*x + c)^5 - 6*b
*cosh(d*x + c)^3 + (16*a + 15*b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 4.93975, size = 206, normalized size = 2.48

$$\begin{cases}\frac{ax\sinh^2(c+dx)}{2}-\frac{ax\cosh^2(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}+\frac{5bx\sinh^6(c+dx)}{16}-\frac{15bx\sinh^4(c+dx)\cosh^2(c+dx)}{16}+\frac{15bx\sinh^2(c+dx)\cosh^4(c}{16} \\ x\left(a+b\sinh^4(c)\right)\sinh^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*x*sinh(c + d*x)**2/2 - a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*x
)*cosh(c + d*x)/(2*d) + 5*b*x*sinh(c + d*x)**6/16 - 15*b*x*sinh(c + d*x)**4
*cosh(c + d*x)**2/16 + 15*b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 5*b*x*
cosh(c + d*x)**6/16 + 11*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) - 5*b*sinh
(c + d*x)**3*cosh(c + d*x)**3/(6*d) + 5*b*sinh(c + d*x)*cosh(c + d*x)**5/(1
6*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)*sinh(c)**2, True))
```

Giac [A] time = 1.16648, size = 193, normalized size = 2.33

$$-\frac{24(dx+c)(8a+5b)-be^{(6dx+6c)}+9be^{(4dx+4c)}-48ae^{(2dx+2c)}-45be^{(2dx+2c)}-\left(176ae^{(6dx+6c)}+110be^{(6dx+6c)}-48\right.}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/384*(24*(d*x + c)*(8*a + 5*b) - b*e^(6*d*x + 6*c) + 9*b*e^(4*d*x + 4*c)
- 48*a*e^(2*d*x + 2*c) - 45*b*e^(2*d*x + 2*c) - (176*a*e^(6*d*x + 6*c) + 11
0*b*e^(6*d*x + 6*c) - 48*a*e^(4*d*x + 4*c) - 45*b*e^(4*d*x + 4*c) + 9*b*e^(
2*d*x + 2*c) - b)*e^(-6*d*x - 6*c))/d
```

3.187 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}$$

```
[Out] ((a + b)*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c + d*x]^
5)/(5*d)
```

Rubi [A] time = 0.0342356, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3215}

$$\frac{(a+b)\cosh(c+dx)}{d}+\frac{b\cosh^5(c+dx)}{5d}-\frac{2b\cosh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] ((a + b)*Cosh[c + d*x])/d - (2*b*Cosh[c + d*x]^3)/(3*d) + (b*Cosh[c + d*x]^
5)/(5*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int\left(a+b-2bx^2+bx^4\right)\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{(a+b)\cosh(c+dx)}{d}-\frac{2b\cosh^3(c+dx)}{3d}+\frac{b\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0223954, size = 69, normalized size = 1.5

$$\frac{a\sinh(c)\sinh(dx)}{d}+\frac{a\cosh(c)\cosh(dx)}{d}+\frac{5b\cosh(c+dx)}{8d}-\frac{5b\cosh(3(c+dx))}{48d}+\frac{b\cosh(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (a*Cosh[c]*Cosh[d*x])/d + (5*b*Cosh[c + d*x])/(8*d) - (5*b*Cosh[3*(c + d*x)
])/(48*d) + (b*Cosh[5*(c + d*x)])/(80*d) + (a*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.013, size = 44, normalized size = 1.

$$\frac{1}{d}\left(b\left(\frac{8}{15}+\frac{(\sinh(dx+c))^4}{5}-\frac{4(\sinh(dx+c))^2}{15}\right)\cosh(dx+c)+a\cosh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a*cosh(d*x+c
))
```

Maxima [B] time = 1.05633, size = 131, normalized size = 2.85

$$\frac{1}{480}\,b\left(\frac{3\,e^{(5\,dx+5\,c)}}{d}-\frac{25\,e^{(3\,dx+3\,c)}}{d}+\frac{150\,e^{(dx+c)}}{d}+\frac{150\,e^{(-dx-c)}}{d}-\frac{25\,e^{(-3\,dx-3\,c)}}{d}+\frac{3\,e^{(-5\,dx-5\,c)}}{d}\right)+\frac{a\cosh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/480*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 1
50*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*cosh(
d*x + c)/d
```

Fricas [B] time = 1.66974, size = 250, normalized size = 5.43

$$\frac{3\,b\cosh(dx+c)^5+15\,b\cosh(dx+c)\sinh(dx+c)^4-25\,b\cosh(dx+c)^3+15\left(2\,b\cosh(dx+c)^3-5\,b\cosh(dx+c)\right)\mathrm{si}\ldots}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*cosh(d*x + c)^5 + 15*b*cosh(d*x + c)*sinh(d*x + c)^4 - 25*b*cosh
(d*x + c)^3 + 15*(2*b*cosh(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c)^2
+ 30*(8*a + 5*b)*cosh(d*x + c))/d
```

Sympy [A] time = 2.53825, size = 80, normalized size = 1.74

$$\begin{cases}\frac{a\cosh(c+dx)}{d}+\frac{b\sinh^4(c+dx)\cosh(c+dx)}{d}-\frac{4b\sinh^2(c+dx)\cosh^3(c+dx)}{3d}+\frac{8b\cosh^5(c+dx)}{15d} & \text{for } d\neq 0\\ x\left(a+b\sinh^4(c)\right)\sinh(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Piecewise((a*cosh(c + d*x)/d + b*sinh(c + d*x)**4*cosh(c + d*x)/d - 4*b*sin
h(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 8*b*cosh(c + d*x)**5/(15*d), Ne(d, 0
)), (x*(a + b*sinh(c)**4)*sinh(c), True))
```

Giac [B] time = 1.18581, size = 132, normalized size = 2.87

$$\frac{3\, be^{(5\, dx+5\, c)} - 25\, be^{(3\, dx+3\, c)} + 240\, ae^{(dx+c)} + 150\, be^{(dx+c)} + \left(240\, ae^{(4\, dx+4\, c)} + 150\, be^{(4\, dx+4\, c)} - 25\, be^{(2\, dx+2\, c)} + 3\, b\right)e^{(-5\, dx-}}{480\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/480*(3*b*e^(5*d*x + 5*c) - 25*b*e^(3*d*x + 3*c) + 240*a*e^(d*x + c) + 150
*b*e^(d*x + c) + (240*a*e^(4*d*x + 4*c) + 150*b*e^(4*d*x + 4*c) - 25*b*e^(2
*d*x + 2*c) + 3*b)*e^(-5*d*x - 5*c))/d
```

3.188 $\int (a + b\sinh^4(c + dx))\, dx$

Optimal. Leaf size=52

$$ax + \frac{b\sinh^3(c + dx)\cosh(c + dx)}{4d} - \frac{3b\sinh(c + dx)\cosh(c + dx)}{8d} + \frac{3bx}{8}$$

```
[Out] a*x + (3*b*x)/8 - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*x
]*Sinh[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0339566, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2635, 8}

$$ax + \frac{b\sinh^3(c + dx)\cosh(c + dx)}{4d} - \frac{3b\sinh(c + dx)\cosh(c + dx)}{8d} + \frac{3bx}{8}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sinh[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*x)/8 - (3*b*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c + d*x
]*Sinh[c + d*x]^3)/(4*d)
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int (a + b\sinh^4(c + dx))\, dx &= ax + b\int \sinh^4(c + dx)\, dx\\ &= ax + \frac{b\cosh(c + dx)\sinh^3(c + dx)}{4d} - \frac{1}{4}(3b)\int \sinh^2(c + dx)\, dx\\ &= ax - \frac{3b\cosh(c + dx)\sinh(c + dx)}{8d} + \frac{b\cosh(c + dx)\sinh^3(c + dx)}{4d} + \frac{1}{8}(3b)\int 1\, dx\\ &= ax + \frac{3bx}{8} - \frac{3b\cosh(c + dx)\sinh(c + dx)}{8d} + \frac{b\cosh(c + dx)\sinh^3(c + dx)}{4d}\end{aligned}$$

Mathematica [A] time = 0.0539226, size = 49, normalized size = 0.94

$$ax + \frac{3b(c + dx)}{8d} - \frac{b\sinh(2(c + dx))}{4d} + \frac{b\sinh(4(c + dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sinh[c + d*x]^4,x]
```

```
[Out] a*x + (3*b*(c + d*x))/(8*d) - (b*Sinh[2*(c + d*x)])/(4*d) + (b*Sinh[4*(c +
d*x)])/(32*d)
```

Maple [A] time = 0.005, size = 44, normalized size = 0.9

$$ax+\frac{b}{d}\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c)+\frac{3dx}{8}+\frac{3c}{8}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sinh(d*x+c)^4,x)
```

```
[Out] a*x+b/d*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)
```

Maxima [A] time = 1.16404, size = 89, normalized size = 1.71

$$\frac{1}{64}b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] 1/64*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + a*x
```

Fricas [A] time = 1.61935, size = 155, normalized size = 2.98

$$\frac{b\cosh(dx+c)\sinh(dx+c)^3+(8a+3b)dx+\left(b\cosh(dx+c)^3-4b\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (8*a + 3*b)*d*x + (b*cosh(d*x + c)^3
 - 4*b*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 1.24768, size = 100, normalized size = 1.92

$$ax+b\begin{cases}\frac{3x\sinh^4(c+dx)}{8}-\frac{3x\sinh^2(c+dx)\cosh^2(c+dx)}{4}+\frac{3x\cosh^4(c+dx)}{8}+\frac{5\sinh^3(c+dx)\cosh(c+dx)}{8d}-\frac{3\sinh(c+dx)\cosh^3(c+dx)}{8d} & \text{for } d \\ x\sinh^4(c) & \text{other}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)**4,x)
```

```
[Out] a*x + b*Piecewise((3*x*sinh(c + d*x)**4/8 - 3*x*sinh(c + d*x)**2*cosh(c + d
*x)**2/4 + 3*x*cosh(c + d*x)**4/8 + 5*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
```

```
- 3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*sinh(c)**4, True))
```

Giac [A] time = 1.12907, size = 99, normalized size = 1.9

$$ax + \frac{\left(24\,dx - \left(18\,e^{(4\,dx+4\,c)} - 8\,e^{(2\,dx+2\,c)} + 1\right)e^{(-4\,dx-4\,c)} + 24\,c + e^{(4\,dx+4\,c)} - 8\,e^{(2\,dx+2\,c)}\right)b}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sinh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] a*x + 1/64*(24*d*x - (18*e^(4*d*x + 4*c) - 8*e^(2*d*x + 2*c) + 1)*e^(-4*d*x
 - 4*c) + 24*c + e^(4*d*x + 4*c) - 8*e^(2*d*x + 2*c))*b/d
```

3.189 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$

Optimal. Leaf size=42

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)
/(3*d)
```

Rubi [A] time = 0.0433106, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3215, 1153, 206}

$$-\frac{a\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b\cosh^3(c+dx)}{3d}-\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -((a*ArcTanh[Cosh[c + d*x]])/d) - (b*Cosh[c + d*x])/d + (b*Cosh[c + d*x]^3)
/(3*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx = -\frac{\operatorname{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(b-bx^2+\frac{a}{1-x^2}\right)dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}-\frac{a\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{a\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b\cosh(c+dx)}{d}+\frac{b\cosh^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0240189, size = 70, normalized size = 1.67

$$\frac{a\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{3b\cosh(c+dx)}{4d}+\frac{b\cosh(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-3*b*Cosh[c + d*x])/(4*d) + (b*Cosh[3*(c + d*x)])/(12*d) - (a*Log[Cosh[c/2
 + (d*x)/2]])/d + (a*Log[Sinh[c/2 + (d*x)/2]])/d
```

Maple [A] time = 0.029, size = 36, normalized size = 0.9

$$\frac{1}{d}\left(-2\,a\operatorname{Artanh}\left(e^{dx+c}\right)+b\left(-\frac{2}{3}+\frac{(\sinh(dx+c))^2}{3}\right)\cosh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(-2*a*arctanh(exp(d*x+c))+b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [A] time = 1.16071, size = 96, normalized size = 2.29

$$\frac{1}{24}\,b\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)+\frac{a\log\left(\tanh\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/24*b*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x
- 3*c)/d) + a*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 1.68997, size = 1100, normalized size = 26.19

$$\frac{b\cosh(dx+c)^6 + 6\,b\cosh(dx+c)\sinh(dx+c)^5 + b\sinh(dx+c)^6 - 9\,b\cosh(dx+c)^4 + 3\left(5\,b\cosh(dx+c)^2 - 3\,b\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/24*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x +
c)^6 - 9*b*cosh(d*x + c)^4 + 3*(5*b*cosh(d*x + c)^2 - 3*b)*sinh(d*x + c)^4
+ 4*(5*b*cosh(d*x + c)^3 - 9*b*cosh(d*x + c))*sinh(d*x + c)^3 - 9*b*cosh(d*
x + c)^2 + 3*(5*b*cosh(d*x + c)^4 - 18*b*cosh(d*x + c)^2 - 3*b)*sinh(d*x +
c)^2 - 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*cosh
(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*log(cosh(d*x + c) + sinh(d*x
 + c) + 1) + 24*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)^2*sinh(d*x + c) + 3*
a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3)*log(cosh(d*x + c) + si
nh(d*x + c) - 1) + 6*(b*cosh(d*x + c)^5 - 6*b*cosh(d*x + c)^3 - 3*b*cosh(d*
x + c))*sinh(d*x + c) + b)/(d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c)^2*sinh(d*
x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + d*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18077, size = 126, normalized size = 3.

$$-\frac{\left(9\,be^{(2\,dx+2\,c)} - b\right)e^{(-3\,dx-3\,c)}}{24\,d} - \frac{a\log\left(e^{(dx+c)} + 1\right)}{d} + \frac{a\log\left(\left|e^{(dx+c)} - 1\right|\right)}{d} + \frac{bd^2e^{(3\,dx+3\,c)} - 9\,bd^2e^{(dx+c)}}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/24*(9*b*e^(2*d*x + 2*c) - b)*e^(-3*d*x - 3*c)/d - a*log(e^(d*x + c) + 1)
/d + a*log(abs(e^(d*x + c) - 1))/d + 1/24*(b*d^2*e^(3*d*x + 3*c) - 9*b*d^2*
e^(d*x + c))/d^3
```

3.190 $\int \mathrm{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=39

$$-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

```
[Out] -(b*x)/2 - (a*Coth[c + d*x])/d + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rubi [A] time = 0.0522995, antiderivative size = 39, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3217, 1259, 453, 206}

$$-\frac{a\coth(c+dx)}{d}+\frac{b\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -(b*x)/2 - (a*Coth[c + d*x])/d + (b*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) && !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^2\left(1-x^2\right)^2}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{b\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-2a+(2a+b)x^2}{x^2(1-x^2)}dx, x, \tanh(c+dx)\right)}{2d}$$
$$= -\frac{a\coth(c+dx)}{d} + \frac{b\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{b\,\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, t\right.}{2d}$$
$$= -\frac{bx}{2} - \frac{a\coth(c+dx)}{d} + \frac{b\cosh(c+dx)\sinh(c+dx)}{2d}$$

Mathematica [A] time = 0.119255, size = 45, normalized size = 1.15

$$-\frac{a\coth(c+dx)}{d} + \frac{b(-c-dx)}{2d} + \frac{b\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (b*(-c - d*x))/(2*d) - (a*Coth[c + d*x])/d + (b*Sinh[2*(c + d*x)])/(4*d)
```

Maple [A] time = 0.03, size = 39, normalized size = 1.

$$\frac{1}{d}\left(-\coth(dx+c)\,a + b\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} - \frac{dx}{2} - \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(-coth(d*x+c)*a+b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.1604, size = 73, normalized size = 1.87

$$-\frac{1}{8}b\left(4x - \frac{e^{(2dx+2c)}}{d} + \frac{e^{(-2dx-2c)}}{d}\right) + \frac{2a}{d\left(e^{(-2dx-2c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) + 2*a/(d*(e^(-2*d*x -
 2*c) - 1))
```

Fricas [A] time = 1.6723, size = 185, normalized size = 4.74

$$\frac{b\cosh(dx+c)^3 + 3b\cosh(dx+c)\sinh(dx+c)^2 - (8a+b)\cosh(dx+c) - 4(bdx-2a)\sinh(dx+c)}{8d\sinh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 - (8*a + b)*cosh
(d*x + c) - 4*(b*d*x - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15407, size = 124, normalized size = 3.18

$$-\frac{(dx+c)b}{2d}+\frac{be^{(2dx+2c)}}{8d}+\frac{be^{(4dx+4c)}-16ae^{(2dx+2c)}-2be^{(2dx+2c)}+b}{8d\left(e^{(4dx+4c)}-e^{(2dx+2c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*b/d + 1/8*b*e^(2*d*x + 2*c)/d + 1/8*(b*e^(4*d*x + 4*c) - 16*
a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)/(d*(e^(4*d*x + 4*c) - e^(2*d*x
 + 2*c)))
```

3.191 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=47

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\cosh(c+dx)}{d}$$

```
[Out] (a*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*Cosh[c + d*x])/d - (a*Coth[c + d*x]*C
sch[c + d*x])/(2*d)
```

Rubi [A] time = 0.0539296, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3215, 1157, 388, 206}

$$\frac{a\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{b\cosh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (a*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*Cosh[c + d*x])/d - (a*Coth[c + d*x]*C
sch[c + d*x])/(2*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^2}dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{a\coth(c+dx)\text{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-a-2b+2bx^2}{1-x^2}dx, x, \cosh(c+dx)\right)}{2d}$$
$$= \frac{b\cosh(c+dx)}{d} - \frac{a\coth(c+dx)\text{csch}(c+dx)}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-x^2}dx, x, \cosh(\right.}{2d}$$
$$= \frac{a\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b\cosh(c+dx)}{d} - \frac{a\coth(c+dx)\text{csch}(c+dx)}{2d}$$

Mathematica [A] time = 0.0260933, size = 82, normalized size = 1.74

$$-\frac{a\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{b\sinh(c)\sinh(dx)}{d} + \frac{b\cosh(c)\cosh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (b*Cosh[c]*Cosh[d*x])/d - (a*Csch[(c + d*x)/2]^2)/(8*d) - (a*Log[Tanh[(c +
d*x)/2]])/(2*d) - (a*Sech[(c + d*x)/2]^2)/(8*d) + (b*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.04, size = 38, normalized size = 0.8

$$\frac{1}{d}\left(a\left(-\frac{\text{csch}\,(dx+c)\coth\,(dx+c)}{2} + \text{Artanh}\left(e^{dx+c}\right)\right) + b\cosh\,(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+b*cosh(d*x+c))
```

Maxima [B] time = 1.23746, size = 155, normalized size = 3.3

$$\frac{1}{2}b\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{1}{2}a\left(\frac{\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(e^{(-dx-c)}+e^{(-3\,dx-3\,c)}\right)}{d\left(2\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a*(log(e^(-d*x - c) + 1)/d - l
og(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*
x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 1.7422, size = 1867, normalized size = 39.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^6 + 6*b*cosh(d*x + c)*sinh(d*x + c)^5 + b*sinh(d*x + c
)^6 - (2*a + b)*cosh(d*x + c)^4 + (15*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x
 + c)^4 + 4*(5*b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c)^3
 - (2*a + b)*cosh(d*x + c)^2 + (15*b*cosh(d*x + c)^4 - 6*(2*a + b)*cosh(d*x
 + c)^2 - 2*a - b)*sinh(d*x + c)^2 + (a*cosh(d*x + c)^5 + 5*a*cosh(d*x + c)
*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 - 2*a*cosh(d*x + c)^3 + 2*(5*a*cosh(d*
x + c)^2 - a)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^3 - 3*a*cosh(d*x + c))
*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh(d*x + c)^4 - 6*a*cosh(d*x +
c)^2 + a)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - (a*cosh(d
*x + c)^5 + 5*a*cosh(d*x + c)*sinh(d*x + c)^4 + a*sinh(d*x + c)^5 - 2*a*cos
h(d*x + c)^3 + 2*(5*a*cosh(d*x + c)^2 - a)*sinh(d*x + c)^3 + 2*(5*a*cosh(d*
x + c)^3 - 3*a*cosh(d*x + c))*sinh(d*x + c)^2 + a*cosh(d*x + c) + (5*a*cosh
(d*x + c)^4 - 6*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))*log(cosh(d*x + c) + s
inh(d*x + c) - 1) + 2*(3*b*cosh(d*x + c)^5 - 2*(2*a + b)*cosh(d*x + c)^3 -
(2*a + b)*cosh(d*x + c))*sinh(d*x + c) + b)/(d*cosh(d*x + c)^5 + 5*d*cosh(d
*x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5 - 2*d*cosh(d*x + c)^3 + 2*(5*d*
cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^3 - 3*d*cosh(d*
x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (5*d*cosh(d*x + c)^4 - 6*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19865, size = 155, normalized size = 3.3

$$\frac{b\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{2\,d}+\frac{a\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{4\,d}-\frac{a\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{4\,d}-\frac{a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^2-4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*b*(e^(d*x + c) + e^(-d*x - c))/d + 1/4*a*log(e^(d*x + c) + e^(-d*x - c)
 + 2)/d - 1/4*a*log(e^(d*x + c) + e^(-d*x - c) - 2)/d - a*(e^(d*x + c) + e^
(-d*x - c))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)*d)
```

3.192 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}+bx$$

```
[Out] b*x + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0498847, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3217, 1261, 207}

$$-\frac{a\coth^3(c+dx)}{3d}+\frac{a\coth(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] b*x + (a*Coth[c + d*x])/d - (a*Coth[c + d*x]^3)/(3*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^4\left(1-x^2\right)}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(\frac{a}{x^4}-\frac{a}{x^2}-\frac{b}{-1+x^2}\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}-\frac{b\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= bx+\frac{a\coth(c+dx)}{d}-\frac{a\coth^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0154378, size = 40, normalized size = 1.29

$$\frac{2a\coth(c+dx)}{3d}-\frac{a\coth(c+dx)\operatorname{csch}^2(c+dx)}{3d}+bx$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] b*x + (2*a*Coth[c + d*x])/(3*d) - (a*Coth[c + d*x]*Csch[c + d*x]^2)/(3*d)
```

Maple [A] time = 0.039, size = 33, normalized size = 1.1

$$\frac{1}{d}\left(a\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)+(dx+c)b\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+(d*x+c)*b)
```

Maxima [B] time = 1.16455, size = 131, normalized size = 4.23

$$bx+\frac{4}{3}a\left(\frac{3\,e^{(-2\,dx-2\,c)}}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}-\frac{1}{d\left(3\,e^{(-2\,dx-2\,c)}-3\,e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}-1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] b*x + 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
 + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
 e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 1.67282, size = 333, normalized size = 10.74

$$\frac{2\,a\cosh(dx+c)^3+6\,a\cosh(dx+c)\sinh(dx+c)^2+(3\,bdx-2\,a)\sinh(dx+c)^3-6\,a\cosh(dx+c)-3\left(3\,bdx-(3\,b\right.}{3\left(d\sinh(dx+c)^3+3\left(d\cosh(dx+c)^2-d\right)\sinh(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/3*(2*a*cosh(d*x + c)^3 + 6*a*cosh(d*x + c)*sinh(d*x + c)^2 + (3*b*d*x - 2
*a)*sinh(d*x + c)^3 - 6*a*cosh(d*x + c) - 3*(3*b*d*x - (3*b*d*x - 2*a)*cosh
(d*x + c)^2 - 2*a)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(d*cosh(d*x + c)^2
 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.16184, size = 61, normalized size = 1.97

$$\frac{(dx+c)b}{d} - \frac{4\left(3\,ae^{(2\,dx+2\,c)} - a\right)}{3\,d\left(e^{(2\,dx+2\,c)} - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] (d*x + c)*b/d - 4/3*(3*a*e^(2*d*x + 2*c) - a)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.193 $\int \mathbf{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)dx$

Optimal. Leaf size=64

$$-\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a\coth(c+dx)\mathbf{csch}^3(c+dx)}{4d}+\frac{3a\coth(c+dx)\mathbf{csch}(c+dx)}{8d}$$

```
[Out] -((3*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*Coth[c + d*x]*Csch[c + d
*x])/(8*d) - (a*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.0627252, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3215, 1157, 385, 206}

$$-\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a\coth(c+dx)\mathbf{csch}^3(c+dx)}{4d}+\frac{3a\coth(c+dx)\mathbf{csch}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4),x]
```


```
[Out] -((3*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(8*d) + (3*a*Coth[c + d*x]*Csch[c + d
*x])/(8*d) - (a*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \text{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = -\frac{\text{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^3}dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{a\coth(c+dx)\text{csch}^3(c+dx)}{4d} + \frac{\text{Subst}\left(\int \frac{-3a-4b+4bx^2}{(1-x^2)^2}dx, x, \cosh(c+dx)\right)}{4d}$$
$$= \frac{3a\coth(c+dx)\text{csch}(c+dx)}{8d} - \frac{a\coth(c+dx)\text{csch}^3(c+dx)}{4d} - \frac{(3a+8b)\,\text{Su}}{}$$
$$= -\frac{(3a+8b)\tanh^{-1}(\cosh(c+dx))}{8d} + \frac{3a\coth(c+dx)\text{csch}(c+dx)}{8d} - \frac{a\coth(}{}$$

Mathematica [B] time = 0.0291577, size = 139, normalized size = 2.17

$$-\frac{a\text{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a\text{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a\text{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a\text{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (3*a*Csch[(c + d*x)/2]^2)/(32*d) - (a*Csch[(c + d*x)/2]^4)/(64*d) - (b*Log[
Cosh[c/2 + (d*x)/2]])/d + (b*Log[Sinh[c/2 + (d*x)/2]])/d + (3*a*Log[Tanh[(c
 + d*x)/2]])/(8*d) + (3*a*Sech[(c + d*x)/2]^2)/(32*d) + (a*Sech[(c + d*x)/2
]^4)/(64*d)
```

Maple [A] time = 0.039, size = 54, normalized size = 0.8

$$\frac{1}{d}\left(a\left(\left(-\frac{(\text{csch}\,(dx+c))^3}{4} + \frac{3\,\text{csch}\,(dx+c)}{8}\right)\coth(dx+c) - \frac{3\,\text{Artanh}\left(e^{dx+c}\right)}{4}\right) - 2\,b\text{Artanh}\left(e^{dx+c}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*
x+c)))-2*b*arctanh(exp(d*x+c)))
```

Maxima [B] time = 1.04026, size = 235, normalized size = 3.67

$$-\frac{1}{8}a\left(\frac{3\log\left(e^{(-dx-c)}+1\right)}{d} - \frac{3\log\left(e^{(-dx-c)}-1\right)}{d} + \frac{2\left(3e^{(-dx-c)} - 11e^{(-3dx-3c)} - 11e^{(-5dx-5c)} + 3e^{(-7dx-7c)}\right)}{d\left(4e^{(-2dx-2c)} - 6e^{(-4dx-4c)} + 4e^{(-6dx-6c)} - e^{(-8dx-8c)} - 1\right)}\right) - b\left(\frac{\log\left(e^{(}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/8*a*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*
x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d
*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x
- 8*c) - 1))) - b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Fricas [B] time = 1.86017, size = 3906, normalized size = 61.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(6*a*cosh(d*x + c)^7 + 42*a*cosh(d*x + c)*sinh(d*x + c)^6 + 6*a*sinh(d*
x + c)^7 - 22*a*cosh(d*x + c)^5 + 2*(63*a*cosh(d*x + c)^2 - 11*a)*sinh(d*x
+ c)^5 + 10*(21*a*cosh(d*x + c)^3 - 11*a*cosh(d*x + c))*sinh(d*x + c)^4 - 2
2*a*cosh(d*x + c)^3 + 2*(105*a*cosh(d*x + c)^4 - 110*a*cosh(d*x + c)^2 - 11
*a)*sinh(d*x + c)^3 + 2*(63*a*cosh(d*x + c)^5 - 110*a*cosh(d*x + c)^3 - 33*
a*cosh(d*x + c))*sinh(d*x + c)^2 + 6*a*cosh(d*x + c) - ((3*a + 8*b)*cosh(d*
x + c)^8 + 8*(3*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sinh(d
*x + c)^8 - 4*(3*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^
2 - 3*a - 8*b)*sinh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3 - 3*(3*a
+ 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x + c)^4 + 2*(
35*(3*a + 8*b)*cosh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2 + 9*a + 24*
b)*sinh(d*x + c)^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a + 8*b)*cosh
(d*x + c)^3 + 3*(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a + 8*b)*
cosh(d*x + c)^2 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a + 8*b)*cosh(d*
x + c)^4 + 9*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^2 + 8*(
(3*a + 8*b)*cosh(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5 + 3*(3*a + 8*b)
*cosh(d*x + c)^3 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + 8*b)*lo
g(cosh(d*x + c) + sinh(d*x + c) + 1) + ((3*a + 8*b)*cosh(d*x + c)^8 + 8*(3*
a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + 8*b)*sinh(d*x + c)^8 - 4*(3
*a + 8*b)*cosh(d*x + c)^6 + 4*(7*(3*a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*s
inh(d*x + c)^6 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^3 - 3*(3*a + 8*b)*cosh(d*x
+ c))*sinh(d*x + c)^5 + 6*(3*a + 8*b)*cosh(d*x + c)^4 + 2*(35*(3*a + 8*b)*c
osh(d*x + c)^4 - 30*(3*a + 8*b)*cosh(d*x + c)^2 + 9*a + 24*b)*sinh(d*x + c)
^4 + 8*(7*(3*a + 8*b)*cosh(d*x + c)^5 - 10*(3*a + 8*b)*cosh(d*x + c)^3 + 3*
(3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(3*a + 8*b)*cosh(d*x + c)^2
+ 4*(7*(3*a + 8*b)*cosh(d*x + c)^6 - 15*(3*a + 8*b)*cosh(d*x + c)^4 + 9*(3*
a + 8*b)*cosh(d*x + c)^2 - 3*a - 8*b)*sinh(d*x + c)^2 + 8*((3*a + 8*b)*cosh
(d*x + c)^7 - 3*(3*a + 8*b)*cosh(d*x + c)^5 + 3*(3*a + 8*b)*cosh(d*x + c)^3
 - (3*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 3*a + 8*b)*log(cosh(d*x + c)
+ sinh(d*x + c) - 1) + 2*(21*a*cosh(d*x + c)^6 - 55*a*cosh(d*x + c)^4 - 33*
a*cosh(d*x + c)^2 + 3*a)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x +
 c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 - 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh
(d*x + c)^2 - d)*sinh(d*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 - 3*d*cosh(d*x +
c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 - 30*d*
cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 - 10*d*cosh
(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^3 - 4*d*cosh(d*x + c)^2 + 4*
(7*d*cosh(d*x + c)^6 - 15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 - d)*sinh
(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 - 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c
)^3 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16487, size = 174, normalized size = 2.72

$$-\frac{(3\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{16\,d}+\frac{(3\,a+8\,b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{16\,d}+\frac{3\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{3}-20\,a\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{4\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right)^{2}-4\right)^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/16*(3*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d + 1/16*(3*a + 8*b)*
log(e^(d*x + c) + e^(-d*x - c) - 2)/d + 1/4*(3*a*(e^(d*x + c) + e^(-d*x - c
))^3 - 20*a*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))^2
- 4)^2*d)
```

3.194 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=47

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^5(c+dx)}{5d}+\frac{2a\coth^3(c+dx)}{3d}$$

```
[Out] -(((a + b)*Coth[c + d*x])/d) + (2*a*Coth[c + d*x]^3)/(3*d) - (a*Coth[c + d*
x]^5)/(5*d)
```

Rubi [A] time = 0.0434493, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3217, 14}

$$-\frac{(a+b)\coth(c+dx)}{d}-\frac{a\coth^5(c+dx)}{5d}+\frac{2a\coth^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] -(((a + b)*Coth[c + d*x])/d) + (2*a*Coth[c + d*x]^3)/(3*d) - (a*Coth[c + d*
x]^5)/(5*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\begin{aligned}\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx&=\frac{\text{Subst}\left(\int \frac{a-2ax^2+(a+b)x^4}{x^6}\,dx,x,\tanh(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int \left(\frac{a}{x^6}-\frac{2a}{x^4}+\frac{a+b}{x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\&=-\frac{(a+b)\coth(c+dx)}{d}+\frac{2a\coth^3(c+dx)}{3d}-\frac{a\coth^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0342195, size = 71, normalized size = 1.51

$$-\frac{8a\coth(c+dx)}{15d}-\frac{a\coth(c+dx)\operatorname{csch}^4(c+dx)}{5d}+\frac{4a\coth(c+dx)\operatorname{csch}^2(c+dx)}{15d}-\frac{b\coth(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-8*a*Coth[c + d*x])/(15*d) - (b*Coth[c + d*x])/d + (4*a*Coth[c + d*x]*Csch
[c + d*x]^2)/(15*d) - (a*Coth[c + d*x]*Csch[c + d*x]^4)/(5*d)
```

Maple [A] time = 0.036, size = 45, normalized size = 1.

$$\frac{1}{d}\left(a\left(-\frac{8}{15}-\frac{(\operatorname{csch}(dx+c))^4}{5}+\frac{4(\operatorname{csch}(dx+c))^2}{15}\right)\coth(dx+c)-b\coth(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-b*coth(d*x+
c))
```

Maxima [B] time = 1.04262, size = 308, normalized size = 6.55

$$-\frac{16}{15}a\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-1\right)}-\frac{10}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16/15*a*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) +
 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e
^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1))) + 2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.66127, size = 903, normalized size = 19.21

$$-\frac{4\left((4a+15b)\cosh(dx+c)^4-16a\cosh(dx+c)\sinh(dx+c)^3\right.}{15\left(d\cosh(dx+c)^6+6d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6-6d\cosh(dx+c)^4+3\left(5d\cosh(dx+c)^2-2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -4/15*((4*a + 15*b)*cosh(d*x + c)^4 - 16*a*cosh(d*x + c)*sinh(d*x + c)^3 +
(4*a + 15*b)*sinh(d*x + c)^4 - 20*(a + 3*b)*cosh(d*x + c)^2 + 2*(3*(4*a + 1
5*b)*cosh(d*x + c)^2 - 10*a - 30*b)*sinh(d*x + c)^2 - 8*(2*a*cosh(d*x + c)^
3 - 5*a*cosh(d*x + c))*sinh(d*x + c) + 40*a + 45*b)/(d*cosh(d*x + c)^6 + 6*
d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)^6 - 6*d*cosh(d*x + c)^4 +
 3*(5*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^4 + 4*(5*d*cosh(d*x + c)^3 - 4
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x + c)^2 + 3*(5*d*cosh(d*x
+ c)^4 - 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)
```

```
^5 - 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c) - 10*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14916, size = 131, normalized size = 2.79

$$-\frac{2\left(15\,be^{(8\,dx+8\,c)} - 60\,be^{(6\,dx+6\,c)} + 80\,ae^{(4\,dx+4\,c)} + 90\,be^{(4\,dx+4\,c)} - 40\,ae^{(2\,dx+2\,c)} - 60\,be^{(2\,dx+2\,c)} + 8\,a + 15\,b\right)}{15\,d\left(e^{(2\,dx+2\,c)} - 1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -2/15*(15*b*e^(8*d*x + 8*c) - 60*b*e^(6*d*x + 6*c) + 80*a*e^(4*d*x + 4*c) +
 90*b*e^(4*d*x + 4*c) - 40*a*e^(2*d*x + 2*c) - 60*b*e^(2*d*x + 2*c) + 8*a +
 15*b)/(d*(e^(2*d*x + 2*c) - 1)^5)
```

3.195 $\int \mathbf{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)\,dx$

Optimal. Leaf size=92

$$\frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{(5a+8b)\coth(c+dx)\mathbf{csch}(c+dx)}{16d}-\frac{a\coth(c+dx)\mathbf{csch}^5(c+dx)}{6d}+\frac{5a\coth(c+dx)}{24}$$

```
[Out] ((5*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(16*d) - ((5*a + 8*b)*Coth[c + d*x]*Cs
ch[c + d*x])/(16*d) + (5*a*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a*Coth[
c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.0837145, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3215, 1157, 385, 199, 206}

$$\frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{(5a+8b)\coth(c+dx)\mathbf{csch}(c+dx)}{16d}-\frac{a\coth(c+dx)\mathbf{csch}^5(c+dx)}{6d}+\frac{5a\coth(c+dx)}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] ((5*a + 8*b)*ArcTanh[Cosh[c + d*x]])/(16*d) - ((5*a + 8*b)*Coth[c + d*x]*Cs
ch[c + d*x])/(16*d) + (5*a*Coth[c + d*x]*Csch[c + d*x]^3)/(24*d) - (a*Coth[
c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
```

```
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)dx = \frac{\operatorname{Subst}\left(\int \frac{a+b-2bx^2+bx^4}{(1-x^2)^4}dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-5a-6b+6bx^2}{(1-x^2)^3}dx, x, \cosh(c+dx\right.}{6d}$$

$$= \frac{5a\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} - \frac{a\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} + \frac{(5a+8b}{}$$

$$= -\frac{(5a+8b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} -$$

$$= \frac{(5a+8b)\tanh^{-1}(\cosh(c+dx))}{16d} - \frac{(5a+8b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} +$$

Mathematica [B] time = 0.035123, size = 199, normalized size = 2.16

$$-\frac{a\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d} + \frac{a\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5a\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{a\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d} - \frac{a\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} - \frac{5as}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4),x]
```

```
[Out] (-5*a*Csch[(c + d*x)/2]^2)/(64*d) - (b*Csch[(c + d*x)/2]^2)/(8*d) + (a*Csch
[(c + d*x)/2]^4)/(64*d) - (a*Csch[(c + d*x)/2]^6)/(384*d) - (5*a*Log[Tanh[(
c + d*x)/2]])/(16*d) - (b*Log[Tanh[(c + d*x)/2]])/(2*d) - (5*a*Sech[(c + d*
x)/2]^2)/(64*d) - (b*Sech[(c + d*x)/2]^2)/(8*d) - (a*Sech[(c + d*x)/2]^4)/(
64*d) - (a*Sech[(c + d*x)/2]^6)/(384*d)
```

Maple [A] time = 0.041, size = 78, normalized size = 0.9

$$\frac{1}{d}\left(a\left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6} + \frac{5(\operatorname{csch}(dx+c))^3}{24} - \frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c) + \frac{5\operatorname{Artanh}\left(e^{dx+c}\right)}{8}\right) + b\left(-\frac{\operatorname{csch}(dx+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x)
```

```
[Out] 1/d*(a*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x+c
)+5/8*arctanh(exp(d*x+c)))+b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+
```

c))))

Maxima [B] time = 1.05621, size = 362, normalized size = 3.93

$$\frac{1}{48}a\left(\frac{15\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15e^{(-dx-c)}-85e^{(-3dx-3c)}+198e^{(-5dx-5c)}+198e^{(-7dx-7c)}-85e^{(-9}\right.}{d\left(6e^{(-2dx-2c)}-15e^{(-4dx-4c)}+20e^{(-6dx-6c)}-15e^{(-8dx-8c)}+6e^{(-10d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/48*a*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(
-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*
c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) -
15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*
d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 1/2*b*(log(e^(-d*x - c) + 1)/d -
log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d
*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 1.88138, size = 8357, normalized size = 90.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/48*(6*(5*a + 8*b)*cosh(d*x + c)^11 + 66*(5*a + 8*b)*cosh(d*x + c)*sinh(d
*x + c)^10 + 6*(5*a + 8*b)*sinh(d*x + c)^11 - 2*(85*a + 72*b)*cosh(d*x + c)
^9 + 2*(165*(5*a + 8*b)*cosh(d*x + c)^2 - 85*a - 72*b)*sinh(d*x + c)^9 + 18
*(55*(5*a + 8*b)*cosh(d*x + c)^3 - (85*a + 72*b)*cosh(d*x + c))*sinh(d*x +
c)^8 + 12*(33*a + 8*b)*cosh(d*x + c)^7 + 12*(165*(5*a + 8*b)*cosh(d*x + c)^
4 - 6*(85*a + 72*b)*cosh(d*x + c)^2 + 33*a + 8*b)*sinh(d*x + c)^7 + 84*(33*
(5*a + 8*b)*cosh(d*x + c)^5 - 2*(85*a + 72*b)*cosh(d*x + c)^3 + (33*a + 8*b
)*cosh(d*x + c))*sinh(d*x + c)^6 + 12*(33*a + 8*b)*cosh(d*x + c)^5 + 12*(23
1*(5*a + 8*b)*cosh(d*x + c)^6 - 21*(85*a + 72*b)*cosh(d*x + c)^4 + 21*(33*a
 + 8*b)*cosh(d*x + c)^2 + 33*a + 8*b)*sinh(d*x + c)^5 + 12*(165*(5*a + 8*b)
*cosh(d*x + c)^7 - 21*(85*a + 72*b)*cosh(d*x + c)^5 + 35*(33*a + 8*b)*cosh(
d*x + c)^3 + 5*(33*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(85*a + 72*b
)*cosh(d*x + c)^3 + 2*(495*(5*a + 8*b)*cosh(d*x + c)^8 - 84*(85*a + 72*b)*c
osh(d*x + c)^6 + 210*(33*a + 8*b)*cosh(d*x + c)^4 + 60*(33*a + 8*b)*cosh(d*
x + c)^2 - 85*a - 72*b)*sinh(d*x + c)^3 + 6*(55*(5*a + 8*b)*cosh(d*x + c)^9
 - 12*(85*a + 72*b)*cosh(d*x + c)^7 + 42*(33*a + 8*b)*cosh(d*x + c)^5 + 20*
(33*a + 8*b)*cosh(d*x + c)^3 - (85*a + 72*b)*cosh(d*x + c))*sinh(d*x + c)^2
 + 6*(5*a + 8*b)*cosh(d*x + c) - 3*((5*a + 8*b)*cosh(d*x + c)^12 + 12*(5*a
+ 8*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a + 8*b)*sinh(d*x + c)^12 - 6*(5
*a + 8*b)*cosh(d*x + c)^10 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)
*sinh(d*x + c)^10 + 20*(11*(5*a + 8*b)*cosh(d*x + c)^3 - 3*(5*a + 8*b)*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*(5*a + 8*b)*cosh(d*x + c)^8 + 15*(33*(5*a +
 8*b)*cosh(d*x + c)^4 - 18*(5*a + 8*b)*cosh(d*x + c)^2 + 5*a + 8*b)*sinh(d*
x + c)^8 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^5 - 30*(5*a + 8*b)*cosh(d*x + c
)^3 + 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a + 8*b)*cosh(d*
x + c)^6 + 4*(231*(5*a + 8*b)*cosh(d*x + c)^6 - 315*(5*a + 8*b)*cosh(d*x +
c)^4 + 105*(5*a + 8*b)*cosh(d*x + c)^2 - 25*a - 40*b)*sinh(d*x + c)^6 + 24*
```

```
(33*(5*a + 8*b)*cosh(d*x + c)^7 - 63*(5*a + 8*b)*cosh(d*x + c)^5 + 35*(5*a
+ 8*b)*cosh(d*x + c)^3 - 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 15*
(5*a + 8*b)*cosh(d*x + c)^4 + 15*(33*(5*a + 8*b)*cosh(d*x + c)^8 - 84*(5*a
+ 8*b)*cosh(d*x + c)^6 + 70*(5*a + 8*b)*cosh(d*x + c)^4 - 20*(5*a + 8*b)*co
sh(d*x + c)^2 + 5*a + 8*b)*sinh(d*x + c)^4 + 20*(11*(5*a + 8*b)*cosh(d*x +
c)^9 - 36*(5*a + 8*b)*cosh(d*x + c)^7 + 42*(5*a + 8*b)*cosh(d*x + c)^5 - 20
*(5*a + 8*b)*cosh(d*x + c)^3 + 3*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^3
 - 6*(5*a + 8*b)*cosh(d*x + c)^2 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^10 - 45*
(5*a + 8*b)*cosh(d*x + c)^8 + 70*(5*a + 8*b)*cosh(d*x + c)^6 - 50*(5*a + 8*
b)*cosh(d*x + c)^4 + 15*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)*sinh(d*x +
 c)^2 + 12*((5*a + 8*b)*cosh(d*x + c)^11 - 5*(5*a + 8*b)*cosh(d*x + c)^9 +
10*(5*a + 8*b)*cosh(d*x + c)^7 - 10*(5*a + 8*b)*cosh(d*x + c)^5 + 5*(5*a +
8*b)*cosh(d*x + c)^3 - (5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) + 5*a + 8*b
)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((5*a + 8*b)*cosh(d*x + c)^12
+ 12*(5*a + 8*b)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a + 8*b)*sinh(d*x + c)
^12 - 6*(5*a + 8*b)*cosh(d*x + c)^10 + 6*(11*(5*a + 8*b)*cosh(d*x + c)^2 -
5*a - 8*b)*sinh(d*x + c)^10 + 20*(11*(5*a + 8*b)*cosh(d*x + c)^3 - 3*(5*a +
 8*b)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a + 8*b)*cosh(d*x + c)^8 + 15*
(33*(5*a + 8*b)*cosh(d*x + c)^4 - 18*(5*a + 8*b)*cosh(d*x + c)^2 + 5*a + 8*
b)*sinh(d*x + c)^8 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^5 - 30*(5*a + 8*b)*co
sh(d*x + c)^3 + 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c)^7 - 20*(5*a + 8*
b)*cosh(d*x + c)^6 + 4*(231*(5*a + 8*b)*cosh(d*x + c)^6 - 315*(5*a + 8*b)*c
osh(d*x + c)^4 + 105*(5*a + 8*b)*cosh(d*x + c)^2 - 25*a - 40*b)*sinh(d*x +
c)^6 + 24*(33*(5*a + 8*b)*cosh(d*x + c)^7 - 63*(5*a + 8*b)*cosh(d*x + c)^5
+ 35*(5*a + 8*b)*cosh(d*x + c)^3 - 5*(5*a + 8*b)*cosh(d*x + c))*sinh(d*x +
c)^5 + 15*(5*a + 8*b)*cosh(d*x + c)^4 + 15*(33*(5*a + 8*b)*cosh(d*x + c)^8
- 84*(5*a + 8*b)*cosh(d*x + c)^6 + 70*(5*a + 8*b)*cosh(d*x + c)^4 - 20*(5*a
 + 8*b)*cosh(d*x + c)^2 + 5*a + 8*b)*sinh(d*x + c)^4 + 20*(11*(5*a + 8*b)*c
osh(d*x + c)^9 - 36*(5*a + 8*b)*cosh(d*x + c)^7 + 42*(5*a + 8*b)*cosh(d*x +
 c)^5 - 20*(5*a + 8*b)*cosh(d*x + c)^3 + 3*(5*a + 8*b)*cosh(d*x + c))*sinh(
d*x + c)^3 - 6*(5*a + 8*b)*cosh(d*x + c)^2 + 6*(11*(5*a + 8*b)*cosh(d*x + c
)^10 - 45*(5*a + 8*b)*cosh(d*x + c)^8 + 70*(5*a + 8*b)*cosh(d*x + c)^6 - 50
*(5*a + 8*b)*cosh(d*x + c)^4 + 15*(5*a + 8*b)*cosh(d*x + c)^2 - 5*a - 8*b)*
sinh(d*x + c)^2 + 12*((5*a + 8*b)*cosh(d*x + c)^11 - 5*(5*a + 8*b)*cosh(d*x
 + c)^9 + 10*(5*a + 8*b)*cosh(d*x + c)^7 - 10*(5*a + 8*b)*cosh(d*x + c)^5 +
 5*(5*a + 8*b)*cosh(d*x + c)^3 - (5*a + 8*b)*cosh(d*x + c))*sinh(d*x + c) +
 5*a + 8*b)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 6*(11*(5*a + 8*b)*cosh
(d*x + c)^10 - 3*(85*a + 72*b)*cosh(d*x + c)^8 + 14*(33*a + 8*b)*cosh(d*x +
 c)^6 + 10*(33*a + 8*b)*cosh(d*x + c)^4 - (85*a + 72*b)*cosh(d*x + c)^2 + 5
*a + 8*b)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x
+ c)^11 + d*sinh(d*x + c)^12 - 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)
^2 - d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*si
nh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 - 18*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 - 30*d*cosh(d*x
 + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 - 20*d*cosh(d*x + c)^6 + 4*(23
1*d*cosh(d*x + c)^6 - 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 - 5*d)*
sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 35*d*co
sh(d*x + c)^3 - 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 +
 15*(33*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 - 2
0*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 - 36*d*
cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 - 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*
x + c))*sinh(d*x + c)^3 - 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 -
45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 - 50*d*cosh(d*x + c)^4 + 15*d*c
osh(d*x + c)^2 - d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 - 5*d*cosh(d*x
 + c)^9 + 10*d*cosh(d*x + c)^7 - 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3
 - d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17426, size = 285, normalized size = 3.1

$$\frac{(5a+8b)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{32d}-\frac{(5a+8b)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{32d}-\frac{15a\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+24b\left(e^{(dx+c)}+e^{(-dx-c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/32*(5*a + 8*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/32*(5*a + 8*b)*l
og(e^(d*x + c) + e^(-d*x - c) - 2)/d - 1/24*(15*a*(e^(d*x + c) + e^(-d*x -
c))^5 + 24*b*(e^(d*x + c) + e^(-d*x - c))^5 - 160*a*(e^(d*x + c) + e^(-d*x
- c))^3 - 192*b*(e^(d*x + c) + e^(-d*x - c))^3 + 528*a*(e^(d*x + c) + e^(-d
*x - c)) + 384*b*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c
))^2 - 4)^3*d)
```

3.196 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=120

$$\frac{2b(a+5b)\cosh^7(c+dx)}{7d}-\frac{2b(3a+5b)\cosh^5(c+dx)}{5d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2 c}{}$$

```
[Out] -(((a + b)^2*Cosh[c + d*x])/d) + ((a + b)*(a + 5*b)*Cosh[c + d*x]^3)/(3*d)
- (2*b*(3*a + 5*b)*Cosh[c + d*x]^5)/(5*d) + (2*b*(a + 5*b)*Cosh[c + d*x]^7)
/(7*d) - (5*b^2*Cosh[c + d*x]^9)/(9*d) + (b^2*Cosh[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.129451, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3215, 1153}

$$\frac{2b(a+5b)\cosh^7(c+dx)}{7d}-\frac{2b(3a+5b)\cosh^5(c+dx)}{5d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(((a + b)^2*Cosh[c + d*x])/d) + ((a + b)*(a + 5*b)*Cosh[c + d*x]^3)/(3*d)
- (2*b*(3*a + 5*b)*Cosh[c + d*x]^5)/(5*d) + (2*b*(a + 5*b)*Cosh[c + d*x]^7)
/(7*d) - (5*b^2*Cosh[c + d*x]^9)/(9*d) + (b^2*Cosh[c + d*x]^11)/(11*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\begin{aligned}\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)^2\,dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left((a+b)^2+(-a-5b)(a+b)x^2+2b(3a+5b)x^4-2b(a+5b)x\right.\right.}{d}\\ &= -\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{(a+b)(a+5b)\cosh^3(c+dx)}{3d}-\frac{2b(3a+5b)\cosh}{5d}\end{aligned}$$

Mathematica [A] time = 0.0635944, size = 207, normalized size = 1.72

$$-\frac{3a^2\cosh(c+dx)}{4d}+\frac{a^2\cosh(3(c+dx))}{12d}-\frac{35ab\cosh(c+dx)}{32d}+\frac{7ab\cosh(3(c+dx))}{32d}-\frac{7ab\cosh(5(c+dx))}{160d}+\frac{ab\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*a^2*Cosh[c + d*x])/(4*d) - (35*a*b*Cosh[c + d*x])/(32*d) - (231*b^2*Cos
h[c + d*x])/(512*d) + (a^2*Cosh[3*(c + d*x)])/(12*d) + (7*a*b*Cosh[3*(c + d
*x)])/(32*d) + (55*b^2*Cosh[3*(c + d*x)])/(512*d) - (7*a*b*Cosh[5*(c + d*x)
])/(160*d) - (33*b^2*Cosh[5*(c + d*x)])/(1024*d) + (a*b*Cosh[7*(c + d*x)])/
(224*d) + (55*b^2*Cosh[7*(c + d*x)])/(7168*d) - (11*b^2*Cosh[9*(c + d*x)])/
(9216*d) + (b^2*Cosh[11*(c + d*x)])/(11264*d)
```

Maple [A] time = 0.023, size = 132, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(-\frac{256}{693}+\frac{(\sinh(dx+c))^{10}}{11}-\frac{10\,(\sinh(dx+c))^8}{99}+\frac{80\,(\sinh(dx+c))^6}{693}-\frac{32\,(\sinh(dx+c))^4}{231}+\frac{128\,(\sinh(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(b^2*(-256/693+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/693*sinh(d*x+
c)^6-32/231*sinh(d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(-16/35+
1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+a^2*(-
2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.04615, size = 414, normalized size = 3.45

$$-\frac{1}{1419264}b^2\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\,e^{(-8\,dx-8\,c)}+320166\,e^{(-10\,dx-10\,c)}-63\right)e^{(11\,dx+11}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/1419264*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6
*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(1
1*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(
-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*
x - 11*c))/d) - 1/2240*a*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1
225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-
3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a^2*(e^(
3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.63243, size = 1111, normalized size = 9.26

$$\frac{315\,b^2\cosh(dx+c)^{11}+3465\,b^2\cosh(dx+c)\sinh(dx+c)^{10}-4235\,b^2\cosh(dx+c)^9+3465\left(15\,b^2\cosh(dx+c)^3-11\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/3548160*(315*b^2*cosh(d*x + c)^11 + 3465*b^2*cosh(d*x + c)*sinh(d*x + c)^
10 - 4235*b^2*cosh(d*x + c)^9 + 3465*(15*b^2*cosh(d*x + c)^3 - 11*b^2*cosh(
d*x + c))*sinh(d*x + c)^8 + 495*(32*a*b + 55*b^2)*cosh(d*x + c)^7 + 1155*(1
26*b^2*cosh(d*x + c)^5 - 308*b^2*cosh(d*x + c)^3 + 3*(32*a*b + 55*b^2)*cosh
(d*x + c))*sinh(d*x + c)^6 - 693*(224*a*b + 165*b^2)*cosh(d*x + c)^5 + 3465
*(30*b^2*cosh(d*x + c)^7 - 154*b^2*cosh(d*x + c)^5 + 5*(32*a*b + 55*b^2)*co
sh(d*x + c)^3 - (224*a*b + 165*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 2310*(
128*a^2 + 336*a*b + 165*b^2)*cosh(d*x + c)^3 + 3465*(5*b^2*cosh(d*x + c)^9
- 44*b^2*cosh(d*x + c)^7 + 3*(32*a*b + 55*b^2)*cosh(d*x + c)^5 - 2*(224*a*b
 + 165*b^2)*cosh(d*x + c)^3 + 2*(128*a^2 + 336*a*b + 165*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^2 - 6930*(384*a^2 + 560*a*b + 231*b^2)*cosh(d*x + c))/d
```

Sympy [A] time = 74.8908, size = 280, normalized size = 2.33

$$\begin{cases} \frac{a^2\sinh^2(c+dx)\cosh(c+dx)}{d} - \frac{2a^2\cosh^3(c+dx)}{3d} + \frac{2ab\sinh^6(c+dx)\cosh(c+dx)}{d} - \frac{4ab\sinh^4(c+dx)\cosh^3(c+dx)}{d} + \frac{16ab\sinh^2(c+dx)\cosh^5(c+dx)}{5d} \\ x\left(a+b\sinh^4(c)\right)^2\sinh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)**2*cosh(c + d*x)/d - 2*a**2*cosh(c + d*x)**3/
(3*d) + 2*a*b*sinh(c + d*x)**6*cosh(c + d*x)/d - 4*a*b*sinh(c + d*x)**4*cos
h(c + d*x)**3/d + 16*a*b*sinh(c + d*x)**2*cosh(c + d*x)**5/(5*d) - 32*a*b*c
osh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**10*cosh(c + d*x)/d - 10*b**2*s
inh(c + d*x)**8*cosh(c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**6*cosh(c +
d*x)**5/(3*d) - 32*b**2*sinh(c + d*x)**4*cosh(c + d*x)**7/(7*d) + 128*b**2*
sinh(c + d*x)**2*cosh(c + d*x)**9/(63*d) - 256*b**2*cosh(c + d*x)**11/(693*
d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**3, True))
```

Giac [B] time = 1.31993, size = 443, normalized size = 3.69

$$\frac{315\,b^2e^{(11\,dx+11\,c)} - 4235\,b^2e^{(9\,dx+9\,c)} + 15840\,abe^{(7\,dx+7\,c)} + 27225\,b^2e^{(7\,dx+7\,c)} - 155232\,abe^{(5\,dx+5\,c)} - 114345\,b^2e^{(5\,dx+5\,c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/7096320*(315*b^2*e^(11*d*x + 11*c) - 4235*b^2*e^(9*d*x + 9*c) + 15840*a*b
*e^(7*d*x + 7*c) + 27225*b^2*e^(7*d*x + 7*c) - 155232*a*b*e^(5*d*x + 5*c) -
 114345*b^2*e^(5*d*x + 5*c) + 295680*a^2*e^(3*d*x + 3*c) + 776160*a*b*e^(3*
d*x + 3*c) + 381150*b^2*e^(3*d*x + 3*c) - 2661120*a^2*e^(d*x + c) - 3880800
*a*b*e^(d*x + c) - 1600830*b^2*e^(d*x + c) - (2661120*a^2*e^(10*d*x + 10*c)
 + 3880800*a*b*e^(10*d*x + 10*c) + 1600830*b^2*e^(10*d*x + 10*c) - 295680*a
^2*e^(8*d*x + 8*c) - 776160*a*b*e^(8*d*x + 8*c) - 381150*b^2*e^(8*d*x + 8*c
) + 155232*a*b*e^(6*d*x + 6*c) + 114345*b^2*e^(6*d*x + 6*c) - 15840*a*b*e^(
4*d*x + 4*c) - 27225*b^2*e^(4*d*x + 4*c) + 4235*b^2*e^(2*d*x + 2*c) - 315*b
^2)*e^(-11*d*x - 11*c))/d
```

3.197 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx$

Optimal. Leaf size=161

$$\frac{\left(128a^2+352ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{1}{256}x\left(128a^2+160ab+63b^2\right)+\frac{b(160a+513b)\sinh(c+dx)\cosh}{480d}$$

```
[Out] -((128*a^2 + 160*a*b + 63*b^2)*x)/256 + ((128*a^2 + 352*a*b + 193*b^2)*Cosh
[c + d*x]*Sinh[c + d*x])/(256*d) - (b*(416*a + 447*b)*Cosh[c + d*x]^3*Sinh[
c + d*x])/(384*d) + (b*(160*a + 513*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(480*
d) - (41*b^2*Cosh[c + d*x]^7*Sinh[c + d*x])/(80*d) + (b^2*Cosh[c + d*x]^9*S
inh[c + d*x])/(10*d)
```

Rubi [A] time = 0.280985, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{\left(128a^2+352ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{1}{256}x\left(128a^2+160ab+63b^2\right)+\frac{b(160a+513b)\sinh(c+dx)\cosh}{480d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```


```
[Out] -((128*a^2 + 160*a*b + 63*b^2)*x)/256 + ((128*a^2 + 352*a*b + 193*b^2)*Cosh
[c + d*x]*Sinh[c + d*x])/(256*d) - (b*(416*a + 447*b)*Cosh[c + d*x]^3*Sinh[
c + d*x])/(384*d) + (b*(160*a + 513*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(480*
d) - (41*b^2*Cosh[c + d*x]^7*Sinh[c + d*x])/(80*d) + (b^2*Cosh[c + d*x]^9*S
inh[c + d*x])/(10*d)
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{x^2\left(a-2ax^2+(a+b)x^4\right)^2}{\left(1-x^2\right)^6}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b^2\cosh^9(c+dx)\sinh(c+dx)}{10d}+\frac{\text{Subst}\left(\int\frac{-b^2+10\left(a^2-b^2\right)x^2-10\left(3a^2+b^2\right)x^4+10(}{\left(1-x^2\right)^5}\right.}{1}$$

$$=-\frac{41b^2\cosh^7(c+dx)\sinh(c+dx)}{80d}+\frac{b^2\cosh^9(c+dx)\sinh(c+dx)}{10d}-\frac{\text{Su}}{}$$

$$=\frac{b(160a+513b)\cosh^5(c+dx)\sinh(c+dx)}{480d}-\frac{41b^2\cosh^7(c+dx)\sinh(c}{80d}$$

$$=-\frac{b(416a+447b)\cosh^3(c+dx)\sinh(c+dx)}{384d}+\frac{b(160a+513b)\cosh^5(c}{480d}$$

$$=\frac{\left(128a^2+352ab+193b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d}-\frac{b(416a+447b)\,\text{c}}{}$$

$$=-\frac{1}{256}\left(128a^2+160ab+63b^2\right)x+\frac{\left(128a^2+352ab+193b^2\right)\cosh(c+dx}{256d}$$

Mathematica [A] time = 0.352497, size = 139, normalized size = 0.86

$$-\frac{-60\left(128a^2+240ab+105b^2\right)\sinh(2(c+dx))+15360a^2c+15360a^2dx-320ab\sinh(6(c+dx))+360b(8a+5b)\,\text{sin}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(15360*a^2*c + 19200*a*b*c + 7560*b^2*c + 15360*a^2*d*x + 19200*a*b*d*x +
7560*b^2*d*x - 60*(128*a^2 + 240*a*b + 105*b^2)*Sinh[2*(c + d*x)] + 360*b*(
8*a + 5*b)*Sinh[4*(c + d*x)] - 320*a*b*Sinh[6*(c + d*x)] - 450*b^2*Sinh[6*(
c + d*x)] + 75*b^2*Sinh[8*(c + d*x)] - 6*b^2*Sinh[10*(c + d*x)])/(30720*d)
```

Maple [A] time = 0.022, size = 148, normalized size = 0.9

$$\frac{1}{d}\left(b^{2}\left(\left(\frac{(\sinh(dx+c))^{9}}{10}-\frac{9\,(\sinh(dx+c))^{7}}{80}+\frac{21\,(\sinh(dx+c))^{5}}{160}-\frac{21\,(\sinh(dx+c))^{3}}{128}+\frac{63\sinh(dx+c)}{256}\right)\cosh(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(b^2*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/12
8*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+2*a*b*
((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d
*x-5/16*c)+a^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [A] time = 1.10221, size = 351, normalized size = 2.18

$$-\frac{1}{8}\,a^{2}\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{20480}\,b^{2}\left(\frac{\left(25\,e^{(-2\,dx-2\,c)}-150\,e^{(-4\,dx-4\,c)}+600\,e^{(-6\,dx-6\,c)}-2100\,e^{(-8\,dx-8\,c)}-2\right)e^{(10\,}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/8*a^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^2*((25*
e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(-8
*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x -
 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) +
 2*e^(-10*d*x - 10*c))/d) - 1/192*a*b*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d*x -
 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e
^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)
```

Fricas [B] time = 1.65095, size = 802, normalized size = 4.98

$$\frac{15\,b^{2}\cosh(dx+c)\sinh(dx+c)^{9}+30\left(6\,b^{2}\cosh(dx+c)^{3}-5\,b^{2}\cosh(dx+c)\right)\sinh(dx+c)^{7}+3\left(126\,b^{2}\cosh(dx+c)^{5}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/7680*(15*b^2*cosh(d*x + c)*sinh(d*x + c)^9 + 30*(6*b^2*cosh(d*x + c)^3 -
5*b^2*cosh(d*x + c))*sinh(d*x + c)^7 + 3*(126*b^2*cosh(d*x + c)^5 - 350*b^2
*cosh(d*x + c)^3 + 5*(32*a*b + 45*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 10*
(18*b^2*cosh(d*x + c)^7 - 105*b^2*cosh(d*x + c)^5 + 5*(32*a*b + 45*b^2)*cos
h(d*x + c)^3 - 36*(8*a*b + 5*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 30*(128*
a^2 + 160*a*b + 63*b^2)*d*x + 15*(b^2*cosh(d*x + c)^9 - 10*b^2*cosh(d*x + c
```

```
)^7 + (32*a*b + 45*b^2)*cosh(d*x + c)^5 - 24*(8*a*b + 5*b^2)*cosh(d*x + c)^
3 + 2*(128*a^2 + 240*a*b + 105*b^2)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 60.8387, size = 484, normalized size = 3.01

$$\begin{cases} \frac{a^2x\sinh^2(c+dx)}{2} - \frac{a^2x\cosh^2(c+dx)}{2} + \frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d} + \frac{5abx\sinh^6(c+dx)}{8} - \frac{15abx\sinh^4(c+dx)\cosh^2(c+dx)}{8} + \frac{15abx\sinh^2(c+dx)\cosh \\ x\left(a+b\sinh^4(c)\right)^2\sinh^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*x*sinh(c + d*x)**2/2 - a**2*x*cosh(c + d*x)**2/2 + a**2*sin
h(c + d*x)*cosh(c + d*x)/(2*d) + 5*a*b*x*sinh(c + d*x)**6/8 - 15*a*b*x*sinh
(c + d*x)**4*cosh(c + d*x)**2/8 + 15*a*b*x*sinh(c + d*x)**2*cosh(c + d*x)**
4/8 - 5*a*b*x*cosh(c + d*x)**6/8 + 11*a*b*sinh(c + d*x)**5*cosh(c + d*x)/(8
*d) - 5*a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + 5*a*b*sinh(c + d*x)*c
osh(c + d*x)**5/(8*d) + 63*b**2*x*sinh(c + d*x)**10/256 - 315*b**2*x*sinh(c
 + d*x)**8*cosh(c + d*x)**2/256 + 315*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)
**4/128 - 315*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**6/128 + 315*b**2*x*sin
h(c + d*x)**2*cosh(c + d*x)**8/256 - 63*b**2*x*cosh(c + d*x)**10/256 + 193*
b**2*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) - 237*b**2*sinh(c + d*x)**7*cos
h(c + d*x)**3/(128*d) + 21*b**2*sinh(c + d*x)**5*cosh(c + d*x)**5/(10*d) -
147*b**2*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 63*b**2*sinh(c + d*x)*
cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c)**2, T
rue))
```

Giac [B] time = 1.34792, size = 428, normalized size = 2.66

$$\frac{6\,b^2e^{(10\,dx+10\,c)} - 75\,b^2e^{(8\,dx+8\,c)} + 320\,abe^{(6\,dx+6\,c)} + 450\,b^2e^{(6\,dx+6\,c)} - 2880\,abe^{(4\,dx+4\,c)} - 1800\,b^2e^{(4\,dx+4\,c)} + 7680\,a^2e^{(2\,d}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/61440*(6*b^2*e^(10*d*x + 10*c) - 75*b^2*e^(8*d*x + 8*c) + 320*a*b*e^(6*d*
x + 6*c) + 450*b^2*e^(6*d*x + 6*c) - 2880*a*b*e^(4*d*x + 4*c) - 1800*b^2*e^
(4*d*x + 4*c) + 7680*a^2*e^(2*d*x + 2*c) + 14400*a*b*e^(2*d*x + 2*c) + 6300
*b^2*e^(2*d*x + 2*c) - 240*(128*a^2 + 160*a*b + 63*b^2)*(d*x + c) + (35072*
a^2*e^(10*d*x + 10*c) + 43840*a*b*e^(10*d*x + 10*c) + 17262*b^2*e^(10*d*x +
 10*c) - 7680*a^2*e^(8*d*x + 8*c) - 14400*a*b*e^(8*d*x + 8*c) - 6300*b^2*e^
(8*d*x + 8*c) + 2880*a*b*e^(6*d*x + 6*c) + 1800*b^2*e^(6*d*x + 6*c) - 320*a
*b*e^(4*d*x + 4*c) - 450*b^2*e^(4*d*x + 4*c) + 75*b^2*e^(2*d*x + 2*c) - 6*b
^2)*e^(-10*d*x - 10*c))/d
```

3.198 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=92

$$\frac{2b(a+3b)\cosh^5(c+dx)}{5d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^9(c+dx)}{9d}-\frac{4b^2\cosh^7(c+dx)}{7d}$$

```
[Out] ((a + b)^2*Cosh[c + d*x])/d - (4*b*(a + b)*Cosh[c + d*x]^3)/(3*d) + (2*b*(a
 + 3*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[
c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0860677, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3215, 1090}

$$\frac{2b(a+3b)\cosh^5(c+dx)}{5d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{(a+b)^2\cosh(c+dx)}{d}+\frac{b^2\cosh^9(c+dx)}{9d}-\frac{4b^2\cosh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((a + b)^2*Cosh[c + d*x])/d - (4*b*(a + b)*Cosh[c + d*x]^3)/(3*d) + (2*b*(a
 + 3*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*Cosh[c + d*x]^7)/(7*d) + (b^2*Cosh[
c + d*x]^9)/(9*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx&=\frac{\text{Subst}\left(\int\left(a+b-2bx^2+bx^4\right)^2\,dx,x,\cosh(c+dx)\right)}{d}\\&=\frac{\text{Subst}\left(\int\left(a^2\left(1+\frac{b(2a+b)}{a^2}\right)-4ab\left(1+\frac{b}{a}\right)x^2+2ab\left(1+\frac{3b}{a}\right)x^4-4b^2x^6+b^2x^8\right.\right.}{d}\\&=\frac{(a+b)^2\cosh(c+dx)}{d}-\frac{4b(a+b)\cosh^3(c+dx)}{3d}+\frac{2b(a+3b)\cosh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0440235, size = 164, normalized size = 1.78

$$\frac{a^2\sinh(c)\sinh(dx)}{d}+\frac{a^2\cosh(c)\cosh(dx)}{d}+\frac{5ab\cosh(c+dx)}{4d}-\frac{5ab\cosh(3(c+dx))}{24d}+\frac{ab\cosh(5(c+dx))}{40d}+\frac{63b^2\cosh}{128}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (a^2*Cosh[c]*Cosh[d*x])/d + (5*a*b*Cosh[c + d*x])/(4*d) + (63*b^2*Cosh[c +
d*x])/(128*d) - (5*a*b*Cosh[3*(c + d*x)])/(24*d) - (7*b^2*Cosh[3*(c + d*x)]
)/(64*d) + (a*b*Cosh[5*(c + d*x)])/(40*d) + (9*b^2*Cosh[5*(c + d*x)])/(320*
d) - (9*b^2*Cosh[7*(c + d*x)])/(1792*d) + (b^2*Cosh[9*(c + d*x)])/(2304*d)
+ (a^2*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.021, size = 100, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(\frac{128}{315}+\frac{(\sinh(dx+c))^8}{9}-\frac{8(\sinh(dx+c))^6}{63}+\frac{16(\sinh(dx+c))^4}{105}-\frac{64(\sinh(dx+c))^2}{315}\right)\cosh(dx+c)+2ab\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4
-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+2*a*b*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(
d*x+c)^2)*cosh(d*x+c)+a^2*cosh(d*x+c))
```

Maxima [B] time = 1.04689, size = 305, normalized size = 3.32

$$-\frac{1}{161280}b^2\left(\frac{\left(405e^{(-2dx-2c)}-2268e^{(-4dx-4c)}+8820e^{(-6dx-6c)}-39690e^{(-8dx-8c)}-35\right)e^{(9dx+9c)}}{d}-\frac{39690e^{(-dx-c)}-88\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/161280*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) + 1/240*a*b*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x
+ 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d +
 3*e^(-5*d*x - 5*c)/d) + a^2*cosh(d*x + c)/d
```

Fricas [B] time = 1.63248, size = 740, normalized size = 8.04

$$\frac{35b^2\cosh(dx+c)^9+315b^2\cosh(dx+c)\sinh(dx+c)^8-405b^2\cosh(dx+c)^7+105\left(28b^2\cosh(dx+c)^3-27b^2\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^2*cosh(d*x + c)^9 + 315*b^2*cosh(d*x + c)*sinh(d*x + c)^8 - 4
05*b^2*cosh(d*x + c)^7 + 105*(28*b^2*cosh(d*x + c)^3 - 27*b^2*cosh(d*x + c)
)*sinh(d*x + c)^6 + 252*(8*a*b + 9*b^2)*cosh(d*x + c)^5 + 315*(14*b^2*cosh(
```

```
d*x + c)^5 - 45*b^2*cosh(d*x + c)^3 + 4*(8*a*b + 9*b^2)*cosh(d*x + c))*sinh
(d*x + c)^4 - 420*(40*a*b + 21*b^2)*cosh(d*x + c)^3 + 315*(4*b^2*cosh(d*x +
 c)^7 - 27*b^2*cosh(d*x + c)^5 + 8*(8*a*b + 9*b^2)*cosh(d*x + c)^3 - 4*(40*
a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 630*(128*a^2 + 160*a*b + 63*
b^2)*cosh(d*x + c))/d
```

Sympy [A] time = 43.7265, size = 204, normalized size = 2.22

$$\begin{cases} \frac{a^2\cosh(c+dx)}{d} + \frac{2ab\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{8ab\sinh^2(c+dx)\cosh^3(c+dx)}{3d} + \frac{16ab\cosh^5(c+dx)}{15d} + \frac{b^2\sinh^8(c+dx)\cosh(c+dx)}{d} - \frac{8b^2\sinh^6(c+dx)}{3d} \\ x\left(a+b\sinh^4(c)\right)^2\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*cosh(c + d*x)/d + 2*a*b*sinh(c + d*x)**4*cosh(c + d*x)/d -
8*a*b*sinh(c + d*x)**2*cosh(c + d*x)**3/(3*d) + 16*a*b*cosh(c + d*x)**5/(15
*d) + b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*b**2*sinh(c + d*x)**6*cosh(
c + d*x)**3/(3*d) + 16*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d) - 64*b*
*2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*b**2*cosh(c + d*x)**9/(31
5*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2*sinh(c), True))
```

Giac [B] time = 1.27739, size = 331, normalized size = 3.6

$$\frac{35\,b^2e^{(9\,dx+9\,c)} - 405\,b^2e^{(7\,dx+7\,c)} + 2016\,abe^{(5\,dx+5\,c)} + 2268\,b^2e^{(5\,dx+5\,c)} - 16800\,abe^{(3\,dx+3\,c)} - 8820\,b^2e^{(3\,dx+3\,c)} + 80640\,a^2e^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/161280*(35*b^2*e^(9*d*x + 9*c) - 405*b^2*e^(7*d*x + 7*c) + 2016*a*b*e^(5*
d*x + 5*c) + 2268*b^2*e^(5*d*x + 5*c) - 16800*a*b*e^(3*d*x + 3*c) - 8820*b^
2*e^(3*d*x + 3*c) + 80640*a^2*e^(d*x + c) + 100800*a*b*e^(d*x + c) + 39690*
b^2*e^(d*x + c) + (80640*a^2*e^(8*d*x + 8*c) + 100800*a*b*e^(8*d*x + 8*c) +
 39690*b^2*e^(8*d*x + 8*c) - 16800*a*b*e^(6*d*x + 6*c) - 8820*b^2*e^(6*d*x
+ 6*c) + 2016*a*b*e^(4*d*x + 4*c) + 2268*b^2*e^(4*d*x + 4*c) - 405*b^2*e^(2
*d*x + 2*c) + 35*b^2)*e^(-9*d*x - 9*c))/d
```

3.199 $\int \left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=125

$$\frac{1}{128}x\left(128a^2+96ab+35b^2\right)+\frac{b(96a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{b(160a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}$$

```
[Out] ((128*a^2 + 96*a*b + 35*b^2)*x)/128 - (b*(160*a + 93*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(128*d) + (b*(96*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d
) - (25*b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^2*Cosh[c + d*x]^7*Si
nh[c + d*x])/(8*d)
```

Rubi [A] time = 0.161943, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3209, 1157, 1814, 385, 206}

$$\frac{1}{128}x\left(128a^2+96ab+35b^2\right)+\frac{b(96a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-\frac{b(160a+93b)\sinh(c+dx)\cosh(c+dx)}{128d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((128*a^2 + 96*a*b + 35*b^2)*x)/128 - (b*(160*a + 93*b)*Cosh[c + d*x]*Sinh[
c + d*x])/(128*d) + (b*(96*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d
) - (25*b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^2*Cosh[c + d*x]^7*Si
nh[c + d*x])/(8*d)
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sinh^4(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{\left(1-x^2\right)^5} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d} - \frac{\text{Subst}\left(\int \frac{-8a^2+b^2+8\left(3a^2+b^2\right)x^2-8(3a-b)(a+b)x^4+8(a+b)^2x^6}{\left(1-x^2\right)^4} dx, x, \text{t}\right.}{8d}$$

$$= -\frac{25b^2\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^2\cosh^7(c+dx)\sinh(c+dx)}{8d} + \frac{\text{Subst}\left(\int \frac{48a^2+19b^2-}{}\right.}{}$$

$$= \frac{b(96a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} - \frac{25b^2\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^2\cosh^7(}{}$$

$$= -\frac{b(160a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b(96a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} -$$

$$= \frac{1}{128}\left(128a^2+96ab+35b^2\right)x - \frac{b(160a+93b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b(96a+163b)}{}$$

Mathematica [A] time = 0.165761, size = 92, normalized size = 0.74

$$\frac{24\left(128a^2+96ab+35b^2\right)(c+dx) - 96b(16a+7b)\sinh(2(c+dx)) + 24b(8a+7b)\sinh(4(c+dx)) - 32b^2\sinh(6(c+dx)}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (24*(128*a^2 + 96*a*b + 35*b^2)*(c + d*x) - 96*b*(16*a + 7*b)*Sinh[2*(c + d
*x)] + 24*b*(8*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*(c + d*x)] + 3*b^
2*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A] time = 0.017, size = 111, normalized size = 0.9

$$\frac{1}{d}\left(b^2\left(\left(\frac{(\sinh(dx+c))^7}{8} - \frac{7(\sinh(dx+c))^5}{48} + \frac{35(\sinh(dx+c))^3}{192} - \frac{35\sinh(dx+c)}{128}\right)\cosh(dx+c) + \frac{35dx}{128} + \frac{35c}{128}\right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128
*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+2*a*b*((1/4*sinh(d*x+c)^3-3/
8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^2*(d*x+c))
```

Maxima [A] time = 1.11707, size = 247, normalized size = 1.98

$$\frac{1}{32}ab\left(24x + \frac{e^{(4dx+4c)}}{d} - \frac{8e^{(2dx+2c)}}{d} + \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + a^2x - \frac{1}{6144}b^2\left(\frac{\left(32e^{(-2dx-2c)} - 168e^{(-4dx-4c)} + 672\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/32*a*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + a^2*x - 1/6144*b^2*((32*e^(-2*d*x - 2*c) - 168
*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 1680*(d*x
 + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x - 6*c
) - 3*e^(-8*d*x - 8*c))/d)
```

Fricas [A] time = 1.62932, size = 522, normalized size = 4.18

$$\frac{3b^2\cosh(dx+c)\sinh(dx+c)^7 + 3\left(7b^2\cosh(dx+c)^3 - 8b^2\cosh(dx+c)\right)\sinh(dx+c)^5 + \left(21b^2\cosh(dx+c)^5 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 - 8*b
^2*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 - 80*b^2*cosh(d
*x + c)^3 + 12*(8*a*b + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(128*a^2
+ 96*a*b + 35*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 - 8*b^2*cosh(d*x + c)^5 + 4
*(8*a*b + 7*b^2)*cosh(d*x + c)^3 - 8*(16*a*b + 7*b^2)*cosh(d*x + c))*sinh(d
*x + c))/d
```

Sympy [A] time = 19.2012, size = 332, normalized size = 2.66

$$\begin{cases} a^2x + \frac{3abx\sinh^4(c+dx)}{4} - \frac{3abx\sinh^2(c+dx)\cosh^2(c+dx)}{2} + \frac{3abx\cosh^4(c+dx)}{4} + \frac{5ab\sinh^3(c+dx)\cosh(c+dx)}{4d} - \frac{3ab\sinh(c+dx)\cosh^3(c+dx)}{4d} + \\ x\left(a+b\sinh^4(c)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Piecewise((a**2*x + 3*a*b*x*sinh(c + d*x)**4/4 - 3*a*b*x*sinh(c + d*x)**2*c
osh(c + d*x)**2/2 + 3*a*b*x*cosh(c + d*x)**4/4 + 5*a*b*sinh(c + d*x)**3*cos
h(c + d*x)/(4*d) - 3*a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) + 35*b**2*x*s
inh(c + d*x)**8/128 - 35*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 105*
b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 35*b**2*x*sinh(c + d*x)**2*co
sh(c + d*x)**6/32 + 35*b**2*x*cosh(c + d*x)**8/128 + 93*b**2*sinh(c + d*x)*
*7*cosh(c + d*x)/(128*d) - 511*b**2*sinh(c + d*x)**5*cosh(c + d*x)**3/(384*
```

```
d) + 385*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(384*d) - 35*b**2*sinh(c +
d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)), (x*(a + b*sinh(c)**4)**2, True))
```

Giac [B] time = 1.18992, size = 319, normalized size = 2.55

$$\frac{3\,b^2e^{(8\,dx+8\,c)} - 32\,b^2e^{(6\,dx+6\,c)} + 192\,abe^{(4\,dx+4\,c)} + 168\,b^2e^{(4\,dx+4\,c)} - 1536\,abe^{(2\,dx+2\,c)} - 672\,b^2e^{(2\,dx+2\,c)} + 48\,\big(128\,a^2 + 96\,ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/6144*(3*b^2*e^(8*d*x + 8*c) - 32*b^2*e^(6*d*x + 6*c) + 192*a*b*e^(4*d*x +
 4*c) + 168*b^2*e^(4*d*x + 4*c) - 1536*a*b*e^(2*d*x + 2*c) - 672*b^2*e^(2*d
*x + 2*c) + 48*(128*a^2 + 96*a*b + 35*b^2)*(d*x + c) - (6400*a^2*e^(8*d*x +
 8*c) + 4800*a*b*e^(8*d*x + 8*c) + 1750*b^2*e^(8*d*x + 8*c) - 1536*a*b*e^(6
*d*x + 6*c) - 672*b^2*e^(6*d*x + 6*c) + 192*a*b*e^(4*d*x + 4*c) + 168*b^2*e
^(4*d*x + 4*c) - 32*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-8*d*x - 8*c))/d
```

3.200 $\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx$

Optimal. Leaf size=92

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}-\frac{3b^2\cosh^5(c+dx)}{5d}$$

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) - (b*(2*a + b)*Cosh[c + d*x])/d + (b*(2*a
 + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[
c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.092699, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3215, 1153, 206}

$$-\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^7(c+dx)}{7d}-\frac{3b^2\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((a^2*ArcTanh[Cosh[c + d*x]])/d) - (b*(2*a + b)*Cosh[c + d*x])/d + (b*(2*a
 + 3*b)*Cosh[c + d*x]^3)/(3*d) - (3*b^2*Cosh[c + d*x]^5)/(5*d) + (b^2*Cosh[
c + d*x]^7)/(7*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\operatorname{Subst}\left(\int \left(b(2a+b)-b(2a+3b)x^2+3b^2x^4-b^2x^6+\frac{a^2}{1-x^2}\right)dx, x, \cosh(c+\right.}{d}$$
$$= -\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b(2a+3b)\cosh^3(c+dx)}{3d}-\frac{3b^2\cosh^5(c+dx)}{5d}+$$
$$= -\frac{a^2\tanh^{-1}(\cosh(c+dx))}{d}-\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b(2a+3b)\cosh^3(c+}{3d}$$

Mathematica [A] time = 0.0411324, size = 146, normalized size = 1.59

$$\frac{a^2\log\left(\sinh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a^2\log\left(\cosh\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{3ab\cosh(c+dx)}{2d}+\frac{ab\cosh(3(c+dx))}{6d}-\frac{35b^2\cosh(c+dx)}{64d}+\frac{7b^2\text{co}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*a*b*Cosh[c + d*x])/(2*d) - (35*b^2*Cosh[c + d*x])/(64*d) + (a*b*Cosh[3*
(c + d*x)])/(6*d) + (7*b^2*Cosh[3*(c + d*x)])/(64*d) - (7*b^2*Cosh[5*(c + d
*x)])/(320*d) + (b^2*Cosh[7*(c + d*x)])/(448*d) - (a^2*Log[Cosh[c/2 + (d*x)
/2]])/d + (a^2*Log[Sinh[c/2 + (d*x)/2]])/d
```

Maple [A] time = 0.039, size = 82, normalized size = 0.9

$$\frac{1}{d}\left(-2\,a^2\operatorname{Artanh}\left(e^{dx+c}\right)+2\,ab\left(-2/3+1/3\,(\sinh(dx+c))^2\right)\cosh(dx+c)+b^2\left(-\frac{16}{35}+\frac{(\sinh(dx+c))^6}{7}-\frac{6\,(\sinh(dx+}{35}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(-2*a^2*arctanh(exp(d*x+c))+2*a*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)+
b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*cosh(d
*x+c))
```

Maxima [B] time = 1.05125, size = 239, normalized size = 2.6

$$-\frac{1}{4480}b^2\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
```

```
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/12*a*b*(e^(3*d*x + 3*c)/d
- 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) + a^2*log(tanh(1
/2*d*x + 1/2*c))/d
```

Fricas [B] time = 1.92082, size = 4263, normalized size = 46.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/13440*(15*b^2*cosh(d*x + c)^14 + 210*b^2*cosh(d*x + c)*sinh(d*x + c)^13 +
 15*b^2*sinh(d*x + c)^14 - 147*b^2*cosh(d*x + c)^12 + 21*(65*b^2*cosh(d*x +
 c)^2 - 7*b^2)*sinh(d*x + c)^12 + 84*(65*b^2*cosh(d*x + c)^3 - 21*b^2*cosh(
d*x + c))*sinh(d*x + c)^11 + 35*(32*a*b + 21*b^2)*cosh(d*x + c)^10 + 7*(214
5*b^2*cosh(d*x + c)^4 - 1386*b^2*cosh(d*x + c)^2 + 160*a*b + 105*b^2)*sinh(
d*x + c)^10 + 70*(429*b^2*cosh(d*x + c)^5 - 462*b^2*cosh(d*x + c)^3 + 5*(32
*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 105*(96*a*b + 35*b^2)*cosh(
d*x + c)^8 + 105*(429*b^2*cosh(d*x + c)^6 - 693*b^2*cosh(d*x + c)^4 + 15*(3
2*a*b + 21*b^2)*cosh(d*x + c)^2 - 96*a*b - 35*b^2)*sinh(d*x + c)^8 + 24*(21
45*b^2*cosh(d*x + c)^7 - 4851*b^2*cosh(d*x + c)^5 + 175*(32*a*b + 21*b^2)*c
osh(d*x + c)^3 - 35*(96*a*b + 35*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 105*
(96*a*b + 35*b^2)*cosh(d*x + c)^6 + 21*(2145*b^2*cosh(d*x + c)^8 - 6468*b^2
*cosh(d*x + c)^6 + 350*(32*a*b + 21*b^2)*cosh(d*x + c)^4 - 140*(96*a*b + 35
*b^2)*cosh(d*x + c)^2 - 480*a*b - 175*b^2)*sinh(d*x + c)^6 + 42*(715*b^2*co
sh(d*x + c)^9 - 2772*b^2*cosh(d*x + c)^7 + 210*(32*a*b + 21*b^2)*cosh(d*x +
 c)^5 - 140*(96*a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a*b + 35*b^2)*cosh(d
*x + c))*sinh(d*x + c)^5 + 35*(32*a*b + 21*b^2)*cosh(d*x + c)^4 + 35*(429*b
^2*cosh(d*x + c)^10 - 2079*b^2*cosh(d*x + c)^8 + 210*(32*a*b + 21*b^2)*cosh
(d*x + c)^6 - 210*(96*a*b + 35*b^2)*cosh(d*x + c)^4 - 45*(96*a*b + 35*b^2)*
cosh(d*x + c)^2 + 32*a*b + 21*b^2)*sinh(d*x + c)^4 - 147*b^2*cosh(d*x + c)^
2 + 140*(39*b^2*cosh(d*x + c)^11 - 231*b^2*cosh(d*x + c)^9 + 30*(32*a*b + 2
1*b^2)*cosh(d*x + c)^7 - 42*(96*a*b + 35*b^2)*cosh(d*x + c)^5 - 15*(96*a*b
+ 35*b^2)*cosh(d*x + c)^3 + (32*a*b + 21*b^2)*cosh(d*x + c))*sinh(d*x + c)^
3 + 21*(65*b^2*cosh(d*x + c)^12 - 462*b^2*cosh(d*x + c)^10 + 75*(32*a*b + 2
1*b^2)*cosh(d*x + c)^8 - 140*(96*a*b + 35*b^2)*cosh(d*x + c)^6 - 75*(96*a*b
 + 35*b^2)*cosh(d*x + c)^4 + 10*(32*a*b + 21*b^2)*cosh(d*x + c)^2 - 7*b^2)*
sinh(d*x + c)^2 + 15*b^2 - 13440*(a^2*cosh(d*x + c)^7 + 7*a^2*cosh(d*x + c)
^6*sinh(d*x + c) + 21*a^2*cosh(d*x + c)^5*sinh(d*x + c)^2 + 35*a^2*cosh(d*x
 + c)^4*sinh(d*x + c)^3 + 35*a^2*cosh(d*x + c)^3*sinh(d*x + c)^4 + 21*a^2*c
osh(d*x + c)^2*sinh(d*x + c)^5 + 7*a^2*cosh(d*x + c)*sinh(d*x + c)^6 + a^2*
sinh(d*x + c)^7)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 13440*(a^2*cosh(d
*x + c)^7 + 7*a^2*cosh(d*x + c)^6*sinh(d*x + c) + 21*a^2*cosh(d*x + c)^5*si
nh(d*x + c)^2 + 35*a^2*cosh(d*x + c)^4*sinh(d*x + c)^3 + 35*a^2*cosh(d*x +
c)^3*sinh(d*x + c)^4 + 21*a^2*cosh(d*x + c)^2*sinh(d*x + c)^5 + 7*a^2*cosh(
d*x + c)*sinh(d*x + c)^6 + a^2*sinh(d*x + c)^7)*log(cosh(d*x + c) + sinh(d*
x + c) - 1) + 14*(15*b^2*cosh(d*x + c)^13 - 126*b^2*cosh(d*x + c)^11 + 25*(
32*a*b + 21*b^2)*cosh(d*x + c)^9 - 60*(96*a*b + 35*b^2)*cosh(d*x + c)^7 - 4
5*(96*a*b + 35*b^2)*cosh(d*x + c)^5 + 10*(32*a*b + 21*b^2)*cosh(d*x + c)^3
- 21*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x +
c)^6*sinh(d*x + c) + 21*d*cosh(d*x + c)^5*sinh(d*x + c)^2 + 35*d*cosh(d*x +
 c)^4*sinh(d*x + c)^3 + 35*d*cosh(d*x + c)^3*sinh(d*x + c)^4 + 21*d*cosh(d*
x + c)^2*sinh(d*x + c)^5 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*sinh(d*x +
 c)^7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30251, size = 301, normalized size = 3.27

$$-\frac{a^2\log\left(e^{(dx+c)}+1\right)}{d}+\frac{a^2\log\left(\left|e^{(dx+c)}-1\right|\right)}{d}-\frac{\left(10080\,abe^{(6\,dx+6\,c)}+3675\,b^2e^{(6\,dx+6\,c)}-1120\,abe^{(4\,dx+4\,c)}-735\,b^2e^{(4\,dx+4\,c)}+\right.}{13440\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -a^2*log(e^(d*x + c) + 1)/d + a^2*log(abs(e^(d*x + c) - 1))/d - 1/13440*(10
080*a*b*e^(6*d*x + 6*c) + 3675*b^2*e^(6*d*x + 6*c) - 1120*a*b*e^(4*d*x + 4*
c) - 735*b^2*e^(4*d*x + 4*c) + 147*b^2*e^(2*d*x + 2*c) - 15*b^2)*e^(-7*d*x
- 7*c)/d + 1/13440*(15*b^2*d^6*e^(7*d*x + 7*c) - 147*b^2*d^6*e^(5*d*x + 5*c
) + 1120*a*b*d^6*e^(3*d*x + 3*c) + 735*b^2*d^6*e^(3*d*x + 3*c) - 10080*a*b*
d^6*e^(d*x + c) - 3675*b^2*d^6*e^(d*x + c))/d^7
```

3.201 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=103

$$-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}bx(16a+5b)+\frac{b^2\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b^2}{}$$

```
[Out] -(b*(16*a + 5*b)*x)/16 - (a^2*Coth[c + d*x])/d + (b*(16*a + 11*b)*Cosh[c +
d*x]*Sinh[c + d*x])/(16*d) - (13*b^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d)
+ (b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(6*d)
```

Rubi [A] time = 0.194466, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 453, 206}

$$-\frac{a^2\coth(c+dx)}{d}+\frac{b(16a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}bx(16a+5b)+\frac{b^2\sinh(c+dx)\cosh^5(c+dx)}{6d}-\frac{13b^2}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(b*(16*a + 5*b)*x)/16 - (a^2*Coth[c + d*x])/d + (b*(16*a + 11*b)*Cosh[c +
d*x]*Sinh[c + d*x])/(16*d) - (13*b^2*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d)
+ (b^2*Cosh[c + d*x]^5*Sinh[c + d*x])/(6*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^2\left(1-x^2\right)^4}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^5(c+dx)\sinh(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-6a^2+\left(18a^2+b^2\right)x^2-6(3a-b)(a+b)x^4+6(a+}{x^2\left(1-x^2\right)^3}\right.}{6d}$$

$$= -\frac{13b^2\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b^2\cosh^5(c+dx)\sinh(c+dx)}{6d} + \frac{\text{Subs}}{}$$

$$= \frac{b(16a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b^2\cosh^3(c+dx)\sinh(c+dx)}{24d}$$

$$= -\frac{a^2\coth(c+dx)}{d} + \frac{b(16a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b^2\cosh^3(c}{}$$

$$= -\frac{1}{16}b(16a+5b)x - \frac{a^2\coth(c+dx)}{d} + \frac{b(16a+11b)\cosh(c+dx)\sinh(c+a}{16d}$$

Mathematica [A] time = 0.306401, size = 77, normalized size = 0.75

$$\frac{b((96a+45b)\sinh(2(c+dx))-192ac-192adx-9b\sinh(4(c+dx))+b\sinh(6(c+dx))-60bc-60bdx)-192a^2\coth(}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-192*a^2*Coth[c + d*x] + b*(-192*a*c - 60*b*c - 192*a*d*x - 60*b*d*x + (96
*a + 45*b)*Sinh[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])
)/(192*d)
```

Maple [A] time = 0.038, size = 91, normalized size = 0.9

$$\frac{1}{d}\left(-a^2\coth(dx+c)+2\,ab\,(1/2\,\cosh(dx+c)\sinh(dx+c)-1/2\,dx-c/2)+b^2\left(\left(\left(\frac{(\sinh(dx+c))^5}{6}-\frac{5\,(\sinh(dx+c))^3}{24}+\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(-a^2*coth(d*x+c)+2*a*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^2
*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*
d*x-5/16*c))
```

Maxima [A] time = 1.08813, size = 197, normalized size = 1.91

$$-\frac{1}{4}\,ab\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{384}\,b^2\left(\frac{\left(9\,e^{(-2\,dx-2\,c)}-45\,e^{(-4\,dx-4\,c)}-1\right)e^{(6\,dx+6\,c)}}{d}+\frac{120\,(dx+c)}{d}+\frac{45\,e^{(-2\,dx-2\,c)}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/4*a*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^2*((9*e^(
-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/
d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^
2/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.70794, size = 562, normalized size = 5.46

$$\frac{b^2\cosh\left(dx+c\right)^7+7\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6-10\,b^2\cosh\left(dx+c\right)^5+5\left(7\,b^2\cosh\left(dx+c\right)^3-10\,b^2\cosh\left(dx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(b^2*cosh(d*x + c)^7 + 7*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 10*b^2*c
osh(d*x + c)^5 + 5*(7*b^2*cosh(d*x + c)^3 - 10*b^2*cosh(d*x + c))*sinh(d*x
+ c)^4 + 6*(16*a*b + 9*b^2)*cosh(d*x + c)^3 + (21*b^2*cosh(d*x + c)^5 - 100
*b^2*cosh(d*x + c)^3 + 18*(16*a*b + 9*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 -
 3*(128*a^2 + 32*a*b + 15*b^2)*cosh(d*x + c) - 24*((16*a*b + 5*b^2)*d*x - 1
6*a^2)*sinh(d*x + c))/(d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29338, size = 273, normalized size = 2.65

$$-\frac{\left(16\,ab+5\,b^2\right)(dx+c)}{16\,d}+\frac{\left(352\,abe^{(6\,dx+6\,c)}+110\,b^2e^{(6\,dx+6\,c)}-96\,abe^{(4\,dx+4\,c)}-45\,b^2e^{(4\,dx+4\,c)}+9\,b^2e^{(2\,dx+2\,c)}-b^2\right)e^{(-6\,\ldots}}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/16*(16*a*b + 5*b^2)*(d*x + c)/d + 1/384*(352*a*b*e^(6*d*x + 6*c) + 110*b
^2*e^(6*d*x + 6*c) - 96*a*b*e^(4*d*x + 4*c) - 45*b^2*e^(4*d*x + 4*c) + 9*b^
2*e^(2*d*x + 2*c) - b^2)*e^(-6*d*x - 6*c)/d - 2*a^2/(d*(e^(2*d*x + 2*c) - 1
)) + 1/384*(b^2*d^2*e^(6*d*x + 6*c) - 9*b^2*d^2*e^(4*d*x + 4*c) + 96*a*b*d^
2*e^(2*d*x + 2*c) + 45*b^2*d^2*e^(2*d*x + 2*c))/d^3
```

3.202 $\int \mathrm{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=92

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\mathrm{csch}(c+dx)}{2d}+\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c}{3d}$$

```
[Out] (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(2*a + b)*Cosh[c + d*x])/d - (2*b^2
*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) - (a^2*Coth[c + d*x]*
Csch[c + d*x])/(2*d)
```

Rubi [A] time = 0.134862, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3215, 1157, 1810, 206}

$$\frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^2\coth(c+dx)\mathrm{csch}(c+dx)}{2d}+\frac{b(2a+b)\cosh(c+dx)}{d}+\frac{b^2\cosh^5(c+dx)}{5d}-\frac{2b^2\cosh^3(c}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (a^2*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(2*a + b)*Cosh[c + d*x])/d - (2*b^2
*Cosh[c + d*x]^3)/(3*d) + (b^2*Cosh[c + d*x]^5)/(5*d) - (a^2*Coth[c + d*x]*
Csch[c + d*x])/(2*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-a^2-4ab-2b^2+2b(2a+3b)x^2-6b^2x^4+2b^2x^6}{1-x^2}\right.}{2d}$$

$$= -\frac{a^2\coth(c+dx)\operatorname{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int\left(-2b(2a+b)+4b^2x^2-2b^2x^4-\right.\right.}{2d}$$

$$= \frac{b(2a+b)\cosh(c+dx)}{d} - \frac{2b^2\cosh^3(c+dx)}{3d} + \frac{b^2\cosh^5(c+dx)}{5d} - \frac{a^2\coth(}{}$$

$$= \frac{a^2\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b(2a+b)\cosh(c+dx)}{d} - \frac{2b^2\cosh^3(c+dx)}{3d} + $$

Mathematica [A] time = 0.0457128, size = 144, normalized size = 1.57

$$-\frac{a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{8d} - \frac{a^2\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{2d} + \frac{2ab\sinh(c)\sinh(dx)}{d} + \frac{2ab\cosh(c)\cosh(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (2*a*b*Cosh[c]*Cosh[d*x])/d + (5*b^2*Cosh[c + d*x])/(8*d) - (5*b^2*Cosh[3*(
c + d*x)])/(48*d) + (b^2*Cosh[5*(c + d*x)])/(80*d) - (a^2*Csch[(c + d*x)/2]
^2)/(8*d) - (a^2*Log[Tanh[(c + d*x)/2]])/(2*d) - (a^2*Sech[(c + d*x)/2]^2)/
(8*d) + (2*a*b*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.048, size = 74, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(-\frac{\operatorname{csch}(dx+c)\coth(dx+c)}{2} + \operatorname{Artanh}\left(e^{dx+c}\right)\right) + 2ab\cosh(dx+c) + b^2\left(\frac{8}{15} + \frac{(\sinh(dx+c))^4}{5} - \frac{4(\sinh(dx+}{15}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+2*a*b*cosh(d*x+
c)+b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.1179, size = 275, normalized size = 2.99

$$\frac{1}{480}b^2\left(\frac{3e^{(5dx+5c)}}{d} - \frac{25e^{(3dx+3c)}}{d} + \frac{150e^{(dx+c)}}{d} + \frac{150e^{(-dx-c)}}{d} - \frac{25e^{(-3dx-3c)}}{d} + \frac{3e^{(-5dx-5c)}}{d}\right) + ab\left(\frac{e^{(dx+c)}}{d} + \frac{e^{(-dx-c)}}{d}\right) + \frac{1}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/480*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a*b*(
e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2*a^2*(log(e^(-d*x - c) + 1)/d - log(e^
(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2
*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 1.98212, size = 6047, normalized size = 65.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^2*cosh(d*x + c)^14 + 42*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 3*b
^2*sinh(d*x + c)^14 - 31*b^2*cosh(d*x + c)^12 + (273*b^2*cosh(d*x + c)^2 -
31*b^2)*sinh(d*x + c)^12 + 12*(91*b^2*cosh(d*x + c)^3 - 31*b^2*cosh(d*x + c
))*sinh(d*x + c)^11 + (480*a*b + 203*b^2)*cosh(d*x + c)^10 + (3003*b^2*cosh
(d*x + c)^4 - 2046*b^2*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x + c)^1
0 + 2*(3003*b^2*cosh(d*x + c)^5 - 3410*b^2*cosh(d*x + c)^3 + 5*(480*a*b + 2
03*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(
d*x + c)^8 + (9009*b^2*cosh(d*x + c)^6 - 15345*b^2*cosh(d*x + c)^4 + 45*(48
0*a*b + 203*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*a*b - 175*b^2)*sinh(d*x +
c)^8 + 8*(1287*b^2*cosh(d*x + c)^7 - 3069*b^2*cosh(d*x + c)^5 + 15*(480*a*b
 + 203*b^2)*cosh(d*x + c)^3 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c))*s
inh(d*x + c)^7 - 5*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^6 + (9009*b^2*c
osh(d*x + c)^8 - 28644*b^2*cosh(d*x + c)^6 + 210*(480*a*b + 203*b^2)*cosh(d
*x + c)^4 - 140*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^2 - 480*a^2 - 480*
a*b - 175*b^2)*sinh(d*x + c)^6 + 2*(3003*b^2*cosh(d*x + c)^9 - 12276*b^2*co
sh(d*x + c)^7 + 126*(480*a*b + 203*b^2)*cosh(d*x + c)^5 - 140*(96*a^2 + 96*
a*b + 35*b^2)*cosh(d*x + c)^3 - 15*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^5 + (480*a*b + 203*b^2)*cosh(d*x + c)^4 + (3003*b^2*cosh(d*
x + c)^10 - 15345*b^2*cosh(d*x + c)^8 + 210*(480*a*b + 203*b^2)*cosh(d*x +
c)^6 - 350*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^4 - 75*(96*a^2 + 96*a*b
 + 35*b^2)*cosh(d*x + c)^2 + 480*a*b + 203*b^2)*sinh(d*x + c)^4 - 31*b^2*co
sh(d*x + c)^2 + 4*(273*b^2*cosh(d*x + c)^11 - 1705*b^2*cosh(d*x + c)^9 + 30
*(480*a*b + 203*b^2)*cosh(d*x + c)^7 - 70*(96*a^2 + 96*a*b + 35*b^2)*cosh(d
*x + c)^5 - 25*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x + c)^3 + (480*a*b + 203*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + (273*b^2*cosh(d*x + c)^12 - 2046*b^2*
cosh(d*x + c)^10 + 45*(480*a*b + 203*b^2)*cosh(d*x + c)^8 - 140*(96*a^2 + 9
6*a*b + 35*b^2)*cosh(d*x + c)^6 - 75*(96*a^2 + 96*a*b + 35*b^2)*cosh(d*x +
c)^4 + 6*(480*a*b + 203*b^2)*cosh(d*x + c)^2 - 31*b^2)*sinh(d*x + c)^2 + 3*
b^2 + 240*(a^2*cosh(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*
sinh(d*x + c)^9 - 2*a^2*cosh(d*x + c)^7 + 2*(18*a^2*cosh(d*x + c)^2 - a^2)*
sinh(d*x + c)^7 + a^2*cosh(d*x + c)^5 + 14*(6*a^2*cosh(d*x + c)^3 - a^2*cos
h(d*x + c))*sinh(d*x + c)^6 + (126*a^2*cosh(d*x + c)^4 - 42*a^2*cosh(d*x +
c)^2 + a^2)*sinh(d*x + c)^5 + (126*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x +
c)^3 + 5*a^2*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(42*a^2*cosh(d*x + c)^6 - 3
5*a^2*cosh(d*x + c)^4 + 5*a^2*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*a^2*
cosh(d*x + c)^7 - 21*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3)*sinh(d*x
+ c)^2 + (9*a^2*cosh(d*x + c)^8 - 14*a^2*cosh(d*x + c)^6 + 5*a^2*cosh(d*x +
 c)^4)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 240*(a^2*cos
h(d*x + c)^9 + 9*a^2*cosh(d*x + c)*sinh(d*x + c)^8 + a^2*sinh(d*x + c)^9 -
2*a^2*cosh(d*x + c)^7 + 2*(18*a^2*cosh(d*x + c)^2 - a^2)*sinh(d*x + c)^7 +
a^2*cosh(d*x + c)^5 + 14*(6*a^2*cosh(d*x + c)^3 - a^2*cosh(d*x + c))*sinh(d
*x + c)^6 + (126*a^2*cosh(d*x + c)^4 - 42*a^2*cosh(d*x + c)^2 + a^2)*sinh(d
*x + c)^5 + (126*a^2*cosh(d*x + c)^5 - 70*a^2*cosh(d*x + c)^3 + 5*a^2*cosh(
```

```
d*x + c))*sinh(d*x + c)^4 + 2*(42*a^2*cosh(d*x + c)^6 - 35*a^2*cosh(d*x + c
)^4 + 5*a^2*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*a^2*cosh(d*x + c)^7 -
21*a^2*cosh(d*x + c)^5 + 5*a^2*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (9*a^2*co
sh(d*x + c)^8 - 14*a^2*cosh(d*x + c)^6 + 5*a^2*cosh(d*x + c)^4)*sinh(d*x +
c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(21*b^2*cosh(d*x + c)^13 - 1
86*b^2*cosh(d*x + c)^11 + 5*(480*a*b + 203*b^2)*cosh(d*x + c)^9 - 20*(96*a^
2 + 96*a*b + 35*b^2)*cosh(d*x + c)^7 - 15*(96*a^2 + 96*a*b + 35*b^2)*cosh(d
*x + c)^5 + 2*(480*a*b + 203*b^2)*cosh(d*x + c)^3 - 31*b^2*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^9 + 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*si
nh(d*x + c)^9 - 2*d*cosh(d*x + c)^7 + 2*(18*d*cosh(d*x + c)^2 - d)*sinh(d*x
 + c)^7 + 14*(6*d*cosh(d*x + c)^3 - d*cosh(d*x + c))*sinh(d*x + c)^6 + d*co
sh(d*x + c)^5 + (126*d*cosh(d*x + c)^4 - 42*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^5 + (126*d*cosh(d*x + c)^5 - 70*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c)
)*sinh(d*x + c)^4 + 2*(42*d*cosh(d*x + c)^6 - 35*d*cosh(d*x + c)^4 + 5*d*co
sh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(18*d*cosh(d*x + c)^7 - 21*d*cosh(d*x +
c)^5 + 5*d*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (9*d*cosh(d*x + c)^8 - 14*d*c
osh(d*x + c)^6 + 5*d*cosh(d*x + c)^4)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.28386, size = 275, normalized size = 2.99

$$\frac{a^2 \log\left(e^{(dx+c)} + e^{(-dx-c)} + 2\right)}{4\,d} - \frac{a^2 \log\left(e^{(dx+c)} + e^{(-dx-c)} - 2\right)}{4\,d} - \frac{a^2\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 - 4\right)d} + \frac{3\,b^2 d^4\left(e^{(dx+c)} + e^{(-dx-c)}\right)^5 - 4$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/4*a^2*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/4*a^2*log(e^(d*x + c) + e
^(-d*x - c) - 2)/d - a^2*(e^(d*x + c) + e^(-d*x - c))/(((e^(d*x + c) + e^(-
d*x - c))^2 - 4)*d) + 1/480*(3*b^2*d^4*(e^(d*x + c) + e^(-d*x - c))^5 - 40*
b^2*d^4*(e^(d*x + c) + e^(-d*x - c))^3 + 480*a*b*d^4*(e^(d*x + c) + e^(-d*x
 - c)) + 240*b^2*d^4*(e^(d*x + c) + e^(-d*x - c)))/d^5
```

3.203 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=91

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}+\frac{1}{8}bx(16a+3b)+\frac{b^2\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{5b^2\sinh(c+dx)\cosh(c+dx)}{8d}$$

```
[Out] (b*(16*a + 3*b)*x)/8 + (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
- (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]^3*Sinh[c +
 d*x])/(4*d)
```

Rubi [A] time = 0.164625, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 1261, 207}

$$-\frac{a^2\coth^3(c+dx)}{3d}+\frac{a^2\coth(c+dx)}{d}+\frac{1}{8}bx(16a+3b)+\frac{b^2\sinh(c+dx)\cosh^3(c+dx)}{4d}-\frac{5b^2\sinh(c+dx)\cosh(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (b*(16*a + 3*b)*x)/8 + (a^2*Coth[c + d*x])/d - (a^2*Coth[c + d*x]^3)/(3*d)
- (5*b^2*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^2*Cosh[c + d*x]^3*Sinh[c +
 d*x])/(4*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^4\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{4a^2-12a^2x^2+\left(12a^2+8ab-b^2\right)x^4-4(a+b)^2x^6}{x^4\left(1-x^2\right)^2}\right.}{4d}$$

$$= -\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^2\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^2\cosh^3(c+dx)\sinh(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\right.}{}$$

$$= \frac{a^2\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{5b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^2\,\mathrm{c}}{}$$

$$= \frac{1}{8}b(16a+3b)x + \frac{a^2\coth(c+dx)}{d} - \frac{a^2\coth^3(c+dx)}{3d} - \frac{5b^2\cosh(c+dx)\,\mathrm{si}}{8d}$$

Mathematica [A] time = 0.324689, size = 68, normalized size = 0.75

$$\frac{3b(64adx - 8b\sinh(2(c+dx)) + b\sinh(4(c+dx)) + 12bc + 12bdx) - 32a^2\coth(c+dx)\left(\operatorname{csch}^2(c+dx)-2\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-32*a^2*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + 3*b*(12*b*c + 64*a*d*x + 12
*b*d*x - 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(96*d)
```

Maple [A] time = 0.044, size = 75, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c) + 2ab(dx+c) + b^2\left(\left(\frac{(\sinh(dx+c))^3}{4}-\frac{3\sinh(dx+c)}{8}\right)\cosh(dx+c) + \frac{3\,dx}{8}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+2*a*b*(d*x+c)+b^2*((1/4*sinh(d
*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.04836, size = 223, normalized size = 2.45

$$\frac{1}{64}b^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+2abx+\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}-3e^{(-4dx-4c)}+e^{(-6dx-\cdots}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/64*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 2*a*b*x + 4/3*a^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^
(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-
2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 1.74776, size = 761, normalized size = 8.36

$$\frac{3b^2\cosh(dx+c)^7+21b^2\cosh(dx+c)\sinh(dx+c)^6-33b^2\cosh(dx+c)^5+15\left(7b^2\cosh(dx+c)^3-11b^2\cosh(d\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/192*(3*b^2*cosh(d*x + c)^7 + 21*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - 33*b^
2*cosh(d*x + c)^5 + 15*(7*b^2*cosh(d*x + c)^3 - 11*b^2*cosh(d*x + c))*sinh(
d*x + c)^4 + (128*a^2 + 81*b^2)*cosh(d*x + c)^3 + 8*(3*(16*a*b + 3*b^2)*d*x
 - 16*a^2)*sinh(d*x + c)^3 + 3*(21*b^2*cosh(d*x + c)^5 - 110*b^2*cosh(d*x +
 c)^3 + (128*a^2 + 81*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 3*(128*a^2 + 17
*b^2)*cosh(d*x + c) - 24*(3*(16*a*b + 3*b^2)*d*x - (3*(16*a*b + 3*b^2)*d*x
- 16*a^2)*cosh(d*x + c)^2 - 16*a^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3 + 3*(
d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.31744, size = 207, normalized size = 2.27

$$\frac{\left(16ab+3b^2\right)(dx+c)}{8d}-\frac{\left(96abe^{(4dx+4c)}+18b^2e^{(4dx+4c)}-8b^2e^{(2dx+2c)}+b^2\right)e^{(-4dx-4c)}}{64d}+\frac{b^2de^{(4dx+4c)}-8b^2de^{(2dx+2c)}\cdots}{64d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*(16*a*b + 3*b^2)*(d*x + c)/d - 1/64*(96*a*b*e^(4*d*x + 4*c) + 18*b^2*e^
(4*d*x + 4*c) - 8*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/d + 1/64*(b^2
*d*e^(4*d*x + 4*c) - 8*b^2*d*e^(2*d*x + 2*c))/d^2 - 4/3*(3*a^2*e^(2*d*x + 2
*c) - a^2)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.204 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=101

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2\cosh^3(c+}{3d}$$

```
[Out] -(a*(3*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(8*d) - (b^2*Cosh[c + d*x])/d + (b
^2*Cosh[c + d*x]^3)/(3*d) + (3*a^2*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (a^
2*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.151209, antiderivative size = 101, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 1153, 206}

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d}-\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d}+\frac{b^2\cosh^3(c+}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(a*(3*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(8*d) - (b^2*Cosh[c + d*x])/d + (b
^2*Cosh[c + d*x]^3)/(3*d) + (3*a^2*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (a^
2*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
```

```
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^3} dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-(a+2b)(3a+2b)+4b(2a+3b)x^2-12b^2x^4+}{\left(1-x^2\right)^2}\right.}{4d}$$

$$= \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{\operatorname{Subst}\left(\int\right.}{}$$

$$= -\frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cosh^3(c+dx)}{3d} + \frac{3a^2\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^2}{}$$

$$= -\frac{a(3a+16b)\tanh^{-1}(\cosh(c+dx))}{8d} - \frac{b^2\cosh(c+dx)}{d} + \frac{b^2\cosh^3(c+dx)}{3d}$$

Mathematica [A] time = 0.0434006, size = 186, normalized size = 1.84

$$-\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d} + \frac{3a^2\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)}{32d} + \frac{3a^2\log\left(\tanh\left(\frac{1}{2}(c+d\right.\right.}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-3*b^2*Cosh[c + d*x])/(4*d) + (b^2*Cosh[3*(c + d*x)])/(12*d) + (3*a^2*Csch
[(c + d*x)/2]^2)/(32*d) - (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (2*a*b*Log[Cos
h[c/2 + (d*x)/2]])/d + (2*a*b*Log[Sinh[c/2 + (d*x)/2]])/d + (3*a^2*Log[Tanh
[(c + d*x)/2]])/(8*d) + (3*a^2*Sech[(c + d*x)/2]^2)/(32*d) + (a^2*Sech[(c +
 d*x)/2]^4)/(64*d)
```

Maple [A] time = 0.057, size = 79, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(\left(-\frac{(\operatorname{csch}(dx+c))^3}{4}+\frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c)-\frac{3\operatorname{Artanh}\left(e^{dx+c}\right)}{4}\right)-4ab\operatorname{Artanh}\left(e^{dx+c}\right)+b^2\left(-\frac{2}{3}+\frac{(\sinh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(
d*x+c)))-4*a*b*arctanh(exp(d*x+c))+b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c)
)
```

Maxima [B] time = 1.12611, size = 316, normalized size = 3.13

$$\frac{1}{24}\,b^2\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)-\frac{1}{8}\,a^2\left(\frac{3\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{3\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(3\,e^{(-dx-c)}\right.}{d\left(4\,e^{(-2\,dx-2\,c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/24*b^2*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) - 1/8*a^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/
d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7
*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*
c) - e^(-8*d*x - 8*c) - 1))) - 2*a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x
 - c) - 1)/d)
```

Fricas [B] time = 2.0716, size = 8741, normalized size = 86.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^14 + 14*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + b^2*si
nh(d*x + c)^14 - 13*b^2*cosh(d*x + c)^12 + 13*(7*b^2*cosh(d*x + c)^2 - b^2)
*sinh(d*x + c)^12 + 52*(7*b^2*cosh(d*x + c)^3 - 3*b^2*cosh(d*x + c))*sinh(d
*x + c)^11 + 3*(6*a^2 + 11*b^2)*cosh(d*x + c)^10 + (1001*b^2*cosh(d*x + c)^
4 - 858*b^2*cosh(d*x + c)^2 + 18*a^2 + 33*b^2)*sinh(d*x + c)^10 + 2*(1001*b
^2*cosh(d*x + c)^5 - 1430*b^2*cosh(d*x + c)^3 + 15*(6*a^2 + 11*b^2)*cosh(d*
x + c))*sinh(d*x + c)^9 - 3*(22*a^2 + 7*b^2)*cosh(d*x + c)^8 + 3*(1001*b^2*
cosh(d*x + c)^6 - 2145*b^2*cosh(d*x + c)^4 + 45*(6*a^2 + 11*b^2)*cosh(d*x +
 c)^2 - 22*a^2 - 7*b^2)*sinh(d*x + c)^8 + 24*(143*b^2*cosh(d*x + c)^7 - 429
*b^2*cosh(d*x + c)^5 + 15*(6*a^2 + 11*b^2)*cosh(d*x + c)^3 - (22*a^2 + 7*b^
2)*cosh(d*x + c))*sinh(d*x + c)^7 - 3*(22*a^2 + 7*b^2)*cosh(d*x + c)^6 + 3*
(1001*b^2*cosh(d*x + c)^8 - 4004*b^2*cosh(d*x + c)^6 + 210*(6*a^2 + 11*b^2)
*cosh(d*x + c)^4 - 28*(22*a^2 + 7*b^2)*cosh(d*x + c)^2 - 22*a^2 - 7*b^2)*si
nh(d*x + c)^6 + 2*(1001*b^2*cosh(d*x + c)^9 - 5148*b^2*cosh(d*x + c)^7 + 37
8*(6*a^2 + 11*b^2)*cosh(d*x + c)^5 - 84*(22*a^2 + 7*b^2)*cosh(d*x + c)^3 -
9*(22*a^2 + 7*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 3*(6*a^2 + 11*b^2)*cosh
(d*x + c)^4 + (1001*b^2*cosh(d*x + c)^10 - 6435*b^2*cosh(d*x + c)^8 + 630*(
6*a^2 + 11*b^2)*cosh(d*x + c)^6 - 210*(22*a^2 + 7*b^2)*cosh(d*x + c)^4 - 45
*(22*a^2 + 7*b^2)*cosh(d*x + c)^2 + 18*a^2 + 33*b^2)*sinh(d*x + c)^4 - 13*b
^2*cosh(d*x + c)^2 + 4*(91*b^2*cosh(d*x + c)^11 - 715*b^2*cosh(d*x + c)^9 +
 90*(6*a^2 + 11*b^2)*cosh(d*x + c)^7 - 42*(22*a^2 + 7*b^2)*cosh(d*x + c)^5
- 15*(22*a^2 + 7*b^2)*cosh(d*x + c)^3 + 3*(6*a^2 + 11*b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + (91*b^2*cosh(d*x + c)^12 - 858*b^2*cosh(d*x + c)^10 + 135*
(6*a^2 + 11*b^2)*cosh(d*x + c)^8 - 84*(22*a^2 + 7*b^2)*cosh(d*x + c)^6 - 45
*(22*a^2 + 7*b^2)*cosh(d*x + c)^4 + 18*(6*a^2 + 11*b^2)*cosh(d*x + c)^2 - 1
3*b^2)*sinh(d*x + c)^2 + b^2 - 3*((3*a^2 + 16*a*b)*cosh(d*x + c)^11 + 11*(3
```

*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^2 + 16*a*b)*sinh(d*x + c)^11 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 12*a^2 - 64*a*b)*sinh(d*x + c)^9 + 3*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 12*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^8 + 6*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 6*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 24*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^7 + 42*(11*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 8*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 + 2*(231*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 252*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 + 63*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 6*a^2 - 32*a*b)*sinh(d*x + c)^5 + 2*(165*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 - 252*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 + 105*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 10*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (165*(3*a^2 + 16*a*b)*cosh(d*x + c)^8 - 336*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 + 210*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^3 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 - 144*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 126*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (11*(3*a^2 + 16*a*b)*cosh(d*x + c)^10 - 36*(3*a^2 + 16*a*b)*cosh(d*x + c)^8 + 42*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 20*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 3*((3*a^2 + 16*a*b)*cosh(d*x + c)^11 + 11*(3*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^10 + (3*a^2 + 16*a*b)*sinh(d*x + c)^11 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 12*a^2 - 64*a*b)*sinh(d*x + c)^9 + 3*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 12*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^8 + 6*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 6*(55*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 24*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^7 + 42*(11*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 8*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 - 4*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 + 2*(231*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 252*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 + 63*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 - 6*a^2 - 32*a*b)*sinh(d*x + c)^5 + 2*(165*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 - 252*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 + 105*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 - 10*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 + (3*a^2 + 16*a*b)*cosh(d*x + c)^3 + (165*(3*a^2 + 16*a*b)*cosh(d*x + c)^8 - 336*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 + 210*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^2 + 3*a^2 + 16*a*b)*sinh(d*x + c)^3 + (55*(3*a^2 + 16*a*b)*cosh(d*x + c)^9 - 144*(3*a^2 + 16*a*b)*cosh(d*x + c)^7 + 126*(3*a^2 + 16*a*b)*cosh(d*x + c)^5 - 40*(3*a^2 + 16*a*b)*cosh(d*x + c)^3 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (11*(3*a^2 + 16*a*b)*cosh(d*x + c)^10 - 36*(3*a^2 + 16*a*b)*cosh(d*x + c)^8 + 42*(3*a^2 + 16*a*b)*cosh(d*x + c)^6 - 20*(3*a^2 + 16*a*b)*cosh(d*x + c)^4 + 3*(3*a^2 + 16*a*b)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(7*b^2*cosh(d*x + c)^13 - 78*b^2*cosh(d*x + c)^11 + 15*(6*a^2 + 11*b^2)*cosh(d*x + c)^9 - 12*(22*a^2 + 7*b^2)*cosh(d*x + c)^7 - 9*(22*a^2 + 7*b^2)*cosh(d*x + c)^5 + 6*(6*a^2 + 11*b^2)*cosh(d*x + c)^3 - 13*b^2*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^11 - 4*d*cosh(d*x + c)^9 + (55*d*cosh(d*x + c)^2 - 4*d)*sinh(d*x + c)^9 + 3*(55*d*cosh(d*x + c)^3 - 12*d*cosh(d*x + c))*sinh(d*x + c)^8 + 6*d*cosh(d*x + c)^7 + 6*(55*d*cosh(d*x + c)^4 - 24*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 42*(11*d*cosh(d*x + c)^5 - 8*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^6 - 4*d*cosh(d*x + c)^5 + 2*(231*d*cosh(d*x + c)^6 - 252*d*cosh(d*x + c)^4 + 63*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^5 + 2*(165*d*cosh(d*x + c)^7 - 252*d*cosh(d*x + c)^5 + 105*d*cosh(d*x + c)^3 - 10*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*cosh(d*x + c)^3 + (165*d*cosh(d*x + c)^8 - 336*d*cosh(d*x + c)^6 + 210*d*cosh(d*x + c)^4 - 40*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 + (55*d*cosh(d*x + c)^9 - 144*d*cosh(d*x + c)^7 + 126*d*cosh(d*x + c)^5 - 40*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + (11*d*cosh(d*x + c)^10 - 36*d*c

```
osh(d*x + c)^8 + 42*d*cosh(d*x + c)^6 - 20*d*cosh(d*x + c)^4 + 3*d*cosh(d*x
 + c)^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.31755, size = 262, normalized size = 2.59

$$-\frac{\left(3\,a^2+16\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{16\,d}+\frac{\left(3\,a^2+16\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{16\,d}+\frac{b^2d^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-12\,b}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/16*(3*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d + 1/16*(3*a^2
+ 16*a*b)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d + 1/24*(b^2*d^2*(e^(d*x + c
) + e^(-d*x - c))^3 - 12*b^2*d^2*(e^(d*x + c) + e^(-d*x - c)))/d^3 + 1/4*(3
*a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 20*a^2*(e^(d*x + c) + e^(-d*x - c)))/
(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^2*d)
```

3.205 $\int \mathrm{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=84

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

```
[Out] -(b^2*x)/2 - (a*(a + 2*b)*Coth[c + d*x])/d + (2*a^2*Coth[c + d*x]^3)/(3*d)
- (a^2*Coth[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rubi [A] time = 0.145736, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3217, 1259, 1802, 207}

$$-\frac{a^2\coth^5(c+dx)}{5d}+\frac{2a^2\coth^3(c+dx)}{3d}-\frac{a(a+2b)\coth(c+dx)}{d}+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{b^2x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(b^2*x)/2 - (a*(a + 2*b)*Coth[c + d*x])/d + (2*a^2*Coth[c + d*x]^3)/(3*d)
- (a^2*Coth[c + d*x]^5)/(5*d) + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^2 dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^2}{x^6\left(1-x^2\right)^2} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int \frac{-2a^2+6a^2x^2-2a(3a+2b)x^4+\left(2a^2+4ab+\right.}{x^6\left(1-x^2\right)}\right.}{2d}$$

$$= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\operatorname{Subst}\left(\int\left(-\frac{2a^2}{x^6}+\frac{4a^2}{x^4}-\frac{2a(a+2b)}{x^2}-\frac{b^2}{-1+x^2}\right.\right.}{2d}$$

$$= -\frac{a(a+2b)\coth(c+dx)}{d} + \frac{2a^2\coth^3(c+dx)}{3d} - \frac{a^2\coth^5(c+dx)}{5d} + \frac{b^2 c}{}$$

$$= -\frac{b^2x}{2} - \frac{a(a+2b)\coth(c+dx)}{d} + \frac{2a^2\coth^3(c+dx)}{3d} - \frac{a^2\coth^5(c+dx)}{5d}$$

Mathematica [A] time = 0.635186, size = 67, normalized size = 0.8

$$\frac{15b^2(\sinh(2(c+dx))-2(c+dx))-4a\coth(c+dx)\left(3a\operatorname{csch}^4(c+dx)-4a\operatorname{csch}^2(c+dx)+8a+30b\right)}{60d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (-4*a*Coth[c + d*x]*(8*a + 30*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x]^4
) + 15*b^2*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(60*d)
```

Maple [A] time = 0.049, size = 74, normalized size = 0.9

$$\frac{1}{d}\left(a^2\left(-\frac{8}{15}-\frac{(\operatorname{csch}(dx+c))^4}{5}+\frac{4(\operatorname{csch}(dx+c))^2}{15}\right)\coth(dx+c)-2ab\coth(dx+c)+b^2\left(\frac{\cosh(dx+c)\sinh(dx+}{2}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-2*a*b*cot
h(d*x+c)+b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))
```

Maxima [B] time = 1.06529, size = 360, normalized size = 4.29

$$-\frac{1}{8}b^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{16}{15}a^2\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}-10e^{(-4dx-4c)}+10e^{(-6dx-6c)}-5e^{(-8dx-8c)}+e^{(-10dx-10c)}-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/8*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 16/15*a^2*(5*e^(-
2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
```

```
6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/
(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) - 1))) + 4*a*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.71391, size = 1185, normalized size = 14.11

$$\frac{15\,b^2\cosh\left(dx+c\right)^7 + 105\,b^2\cosh\left(dx+c\right)\sinh\left(dx+c\right)^6 - \left(64\,a^2+240\,ab+75\,b^2\right)\cosh\left(dx+c\right)^5 - 4\left(15\,b^2dx-16\,a^2 -\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/120*(15*b^2*cosh(d*x + c)^7 + 105*b^2*cosh(d*x + c)*sinh(d*x + c)^6 - (64
*a^2 + 240*a*b + 75*b^2)*cosh(d*x + c)^5 - 4*(15*b^2*d*x - 16*a^2 - 60*a*b)
*sinh(d*x + c)^5 + 5*(105*b^2*cosh(d*x + c)^3 - (64*a^2 + 240*a*b + 75*b^2)
*cosh(d*x + c))*sinh(d*x + c)^4 + 5*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*x +
c)^3 + 20*(15*b^2*d*x - 2*(15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^2 -
16*a^2 - 60*a*b)*sinh(d*x + c)^3 + 5*(63*b^2*cosh(d*x + c)^5 - 2*(64*a^2 +
240*a*b + 75*b^2)*cosh(d*x + c)^3 + 3*(64*a^2 + 144*a*b + 27*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^2 - 5*(128*a^2 + 96*a*b + 15*b^2)*cosh(d*x + c) - 20*((
15*b^2*d*x - 16*a^2 - 60*a*b)*cosh(d*x + c)^4 + 30*b^2*d*x - 3*(15*b^2*d*x
- 16*a^2 - 60*a*b)*cosh(d*x + c)^2 - 32*a^2 - 120*a*b)*sinh(d*x + c))/(d*si
nh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*cosh(d*x
 + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27883, size = 234, normalized size = 2.79

$$-\frac{(dx+c)b^2}{2\,d} + \frac{b^2e^{(2\,dx+2\,c)}}{8\,d} + \frac{\left(2\,b^2e^{(2\,dx+2\,c)} - b^2\right)e^{(-2\,dx-2\,c)}}{8\,d} - \frac{4\left(15\,abe^{(8\,dx+8\,c)} - 60\,abe^{(6\,dx+6\,c)} + 40\,a^2e^{(4\,dx+4\,c)} + 90\,abe^{(4\right.}}{15\,d\left(e^{(2\,dx+2\,c}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*b^2/d + 1/8*b^2*e^(2*d*x + 2*c)/d + 1/8*(2*b^2*e^(2*d*x + 2*
c) - b^2)*e^(-2*d*x - 2*c)/d - 4/15*(15*a*b*e^(8*d*x + 8*c) - 60*a*b*e^(6*d
*x + 6*c) + 40*a^2*e^(4*d*x + 4*c) + 90*a*b*e^(4*d*x + 4*c) - 20*a^2*e^(2*d
*x + 2*c) - 60*a*b*e^(2*d*x + 2*c) + 4*a^2 + 15*a*b)/(d*(e^(2*d*x + 2*c) -
1)^5)
```

3.206 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx$

Optimal. Leaf size=111

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}+\frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a(5a+16b)}{}$$

```
[Out] (a*(5*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*Cosh[c + d*x])/d - (a
*(5*a + 16*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Cs
ch[c + d*x]^3)/(24*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.174123, antiderivative size = 111, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 388, 206}

$$-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}+\frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a(5a+16b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (a*(5*a + 16*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*Cosh[c + d*x])/d - (a
*(5*a + 16*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^2*Coth[c + d*x]*Cs
ch[c + d*x]^3)/(24*d) - (a^2*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
```

```
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^2\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+b-2bx^2+bx^4\right)^2}{\left(1-x^2\right)^4}\,dx,x,\cosh(c+dx)\right)}{d}$$

$$=-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}-\frac{\text{Subst}\left(\int\frac{-5a^2-12ab-6b^2+6b(2a+3b)x^2-18b^2x^4+\cdots}{\left(1-x^2\right)^3}\right.}{6d}$$

$$=\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}-\frac{a^2\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{\text{Subst}\left(\cdots\right.}{\cdots}$$

$$=-\frac{a(5a+16b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}+\frac{5a^2\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}-\cdots$$

$$=\frac{b^2\cosh(c+dx)}{d}-\frac{a(5a+16b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}+\frac{5a^2\coth(c+d\cdots}{24\cdots}$$

$$=\frac{a(5a+16b)\tanh^{-1}(\cosh(c+dx))}{16d}+\frac{b^2\cosh(c+dx)}{d}-\frac{a(5a+16b)\coth(c\cdots}{16\cdots}$$

Mathematica [B] time = 0.0355098, size = 240, normalized size = 2.16

$$-\frac{a^2\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)}{384d}+\frac{a^2\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{5a^2\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{a^2\operatorname{sech}^6\left(\frac{1}{2}(c+dx)\right)}{384d}-\frac{a^2\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right)}{64d}-\frac{5a\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (b^2*Cosh[c]*Cosh[d*x])/d - (5*a^2*Csch[(c + d*x)/2]^2)/(64*d) - (a*b*Csch[
(c + d*x)/2]^2)/(4*d) + (a^2*Csch[(c + d*x)/2]^4)/(64*d) - (a^2*Csch[(c + d
*x)/2]^6)/(384*d) - (5*a^2*Log[Tanh[(c + d*x)/2]])/(16*d) - (a*b*Log[Tanh[(
c + d*x)/2]])/d - (5*a^2*Sech[(c + d*x)/2]^2)/(64*d) - (a*b*Sech[(c + d*x)/
2]^2)/(4*d) - (a^2*Sech[(c + d*x)/2]^4)/(64*d) - (a^2*Sech[(c + d*x)/2]^6)/
(384*d) + (b^2*Sinh[c]*Sinh[d*x])/d
```

Maple [A] time = 0.048, size = 92, normalized size = 0.8

$$\frac{1}{d}\left(a^2\left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6}+\frac{5\,(\operatorname{csch}(dx+c))^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\,\text{Artanh}\left(e^{dx+c}\right)}{8}\right)+2\,ab\left(-1/2\operatorname{csch}(\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d*(a^2*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x
+c)+5/8*arctanh(exp(d*x+c)))+2*a*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(ex
p(d*x+c)))+b^2*cosh(d*x+c))
```

Maxima [B] time = 1.04059, size = 404, normalized size = 3.64

$$\frac{1}{2}\,b^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right)+\frac{1}{48}\,a^2\left(\frac{15\,\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{15\,\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2\left(15\,e^{(-dx-c)}-85\,e^{(-3\,dx-3\,c)}+198\,e^{(-5}\right.}{d\left(6\,e^{(-2\,dx-2\,c)}-15\,e^{(-4\,dx-4\,c)}+20\,e^{(-6}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^2*(15*log(e^(-d*x - c) +
1)/d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c
) + 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*
e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6
*d*x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*
c) - 1))) + a*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(
-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1
)))
```

Fricas [B] time = 2.11855, size = 11957, normalized size = 107.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(24*b^2*cosh(d*x + c)^14 + 336*b^2*cosh(d*x + c)*sinh(d*x + c)^13 + 24
*b^2*sinh(d*x + c)^14 - 6*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^12 + 6*(3
64*b^2*cosh(d*x + c)^2 - 5*a^2 - 16*a*b - 20*b^2)*sinh(d*x + c)^12 + 24*(36
4*b^2*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^11 + 2*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^10 + 2*(12012*b^2*c
osh(d*x + c)^4 - 198*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^2 + 85*a^2 + 1
44*a*b + 108*b^2)*sinh(d*x + c)^10 + 4*(12012*b^2*cosh(d*x + c)^5 - 330*(5*
a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^3 + 5*(85*a^2 + 144*a*b + 108*b^2)*cos
h(d*x + c))*sinh(d*x + c)^9 - 12*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^8
 + 6*(12012*b^2*cosh(d*x + c)^6 - 495*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x +
c)^4 + 15*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^2 - 66*a^2 - 32*a*b -
20*b^2)*sinh(d*x + c)^8 + 48*(1716*b^2*cosh(d*x + c)^7 - 99*(5*a^2 + 16*a*b
 + 20*b^2)*cosh(d*x + c)^5 + 5*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^3
 - 2*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 12*(33*a^2
 + 16*a*b + 10*b^2)*cosh(d*x + c)^6 + 12*(6006*b^2*cosh(d*x + c)^8 - 462*(5
*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^6 + 35*(85*a^2 + 144*a*b + 108*b^2)*c
osh(d*x + c)^4 - 28*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^2 - 33*a^2 - 1
6*a*b - 10*b^2)*sinh(d*x + c)^6 + 24*(2002*b^2*cosh(d*x + c)^9 - 198*(5*a^2
 + 16*a*b + 20*b^2)*cosh(d*x + c)^7 + 21*(85*a^2 + 144*a*b + 108*b^2)*cosh(
d*x + c)^5 - 28*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^3 - 3*(33*a^2 + 16
*a*b + 10*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(85*a^2 + 144*a*b + 108*b
^2)*cosh(d*x + c)^4 + 2*(12012*b^2*cosh(d*x + c)^10 - 1485*(5*a^2 + 16*a*b
```

+ 20*b^2)*cosh(d*x + c)^8 + 210*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^
6 - 420*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^4 - 90*(33*a^2 + 16*a*b +
10*b^2)*cosh(d*x + c)^2 + 85*a^2 + 144*a*b + 108*b^2)*sinh(d*x + c)^4 + 8*(
1092*b^2*cosh(d*x + c)^11 - 165*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^9 +
 30*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^7 - 84*(33*a^2 + 16*a*b + 10
*b^2)*cosh(d*x + c)^5 - 30*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^3 + (85
*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 6*(5*a^2 + 16*a*
b + 20*b^2)*cosh(d*x + c)^2 + 6*(364*b^2*cosh(d*x + c)^12 - 66*(5*a^2 + 16*
a*b + 20*b^2)*cosh(d*x + c)^10 + 15*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x +
 c)^8 - 56*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^6 - 30*(33*a^2 + 16*a*b
 + 10*b^2)*cosh(d*x + c)^4 + 2*(85*a^2 + 144*a*b + 108*b^2)*cosh(d*x + c)^2
 - 5*a^2 - 16*a*b - 20*b^2)*sinh(d*x + c)^2 + 24*b^2 + 3*((5*a^2 + 16*a*b)*
cosh(d*x + c)^13 + 13*(5*a^2 + 16*a*b)*cosh(d*x + c)*sinh(d*x + c)^12 + (5*
a^2 + 16*a*b)*sinh(d*x + c)^13 - 6*(5*a^2 + 16*a*b)*cosh(d*x + c)^11 + 6*(1
3*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 5*a^2 - 16*a*b)*sinh(d*x + c)^11 + 22*
(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*si
nh(d*x + c)^10 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 + 5*(143*(5*a^2 + 16*a
*b)*cosh(d*x + c)^4 - 66*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 + 15*a^2 + 48*a*b
)*sinh(d*x + c)^9 + 9*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 - 110*(5*a^2 +
16*a*b)*cosh(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^
8 - 20*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*cosh(d*x
+ c)^6 - 495*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 + 135*(5*a^2 + 16*a*b)*cosh(d
*x + c)^2 - 25*a^2 - 80*a*b)*sinh(d*x + c)^7 + 4*(429*(5*a^2 + 16*a*b)*cosh
(d*x + c)^7 - 693*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 + 315*(5*a^2 + 16*a*b)*c
osh(d*x + c)^3 - 35*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 15*(5
*a^2 + 16*a*b)*cosh(d*x + c)^5 + 3*(429*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 -
924*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 + 630*(5*a^2 + 16*a*b)*cosh(d*x + c)^4
 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 + 25*a^2 + 80*a*b)*sinh(d*x + c)^5
+ 5*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 - 396*(5*a^2 + 16*a*b)*cosh(d*x +
 c)^7 + 378*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 - 140*(5*a^2 + 16*a*b)*cosh(d*
x + c)^3 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^4 - 6*(5*a^2 +
16*a*b)*cosh(d*x + c)^3 + 2*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^10 - 495*(5
*a^2 + 16*a*b)*cosh(d*x + c)^8 + 630*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 - 350
*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 + 75*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 1
5*a^2 - 48*a*b)*sinh(d*x + c)^3 + 6*(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^11 -
 55*(5*a^2 + 16*a*b)*cosh(d*x + c)^9 + 90*(5*a^2 + 16*a*b)*cosh(d*x + c)^7
- 70*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 + 25*(5*a^2 + 16*a*b)*cosh(d*x + c)^3
 - 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^2 + 16*a*b)*cos
h(d*x + c) + (13*(5*a^2 + 16*a*b)*cosh(d*x + c)^12 - 66*(5*a^2 + 16*a*b)*co
sh(d*x + c)^10 + 135*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 - 140*(5*a^2 + 16*a*b
)*cosh(d*x + c)^6 + 75*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 - 18*(5*a^2 + 16*a*
b)*cosh(d*x + c)^2 + 5*a^2 + 16*a*b)*sinh(d*x + c))*log(cosh(d*x + c) + sin
h(d*x + c) + 1) - 3*((5*a^2 + 16*a*b)*cosh(d*x + c)^13 + 13*(5*a^2 + 16*a*b
)*cosh(d*x + c)*sinh(d*x + c)^12 + (5*a^2 + 16*a*b)*sinh(d*x + c)^13 - 6*(5
*a^2 + 16*a*b)*cosh(d*x + c)^11 + 6*(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 -
5*a^2 - 16*a*b)*sinh(d*x + c)^11 + 22*(13*(5*a^2 + 16*a*b)*cosh(d*x + c)^3
- 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^10 + 15*(5*a^2 + 16*a*b)*
cosh(d*x + c)^9 + 5*(143*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 - 66*(5*a^2 + 16*
a*b)*cosh(d*x + c)^2 + 15*a^2 + 48*a*b)*sinh(d*x + c)^9 + 9*(143*(5*a^2 + 1
6*a*b)*cosh(d*x + c)^5 - 110*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 + 15*(5*a^2 +
 16*a*b)*cosh(d*x + c))*sinh(d*x + c)^8 - 20*(5*a^2 + 16*a*b)*cosh(d*x + c)
^7 + 4*(429*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 - 495*(5*a^2 + 16*a*b)*cosh(d*
x + c)^4 + 135*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 25*a^2 - 80*a*b)*sinh(d*x
 + c)^7 + 4*(429*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 - 693*(5*a^2 + 16*a*b)*co
sh(d*x + c)^5 + 315*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 - 35*(5*a^2 + 16*a*b)*
cosh(d*x + c))*sinh(d*x + c)^6 + 15*(5*a^2 + 16*a*b)*cosh(d*x + c)^5 + 3*(4
29*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 - 924*(5*a^2 + 16*a*b)*cosh(d*x + c)^6
+ 630*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)
^2 + 25*a^2 + 80*a*b)*sinh(d*x + c)^5 + 5*(143*(5*a^2 + 16*a*b)*cosh(d*x +

```
c)^9 - 396*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 + 378*(5*a^2 + 16*a*b)*cosh(d*x
 + c)^5 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 + 15*(5*a^2 + 16*a*b)*cosh(d
*x + c))*sinh(d*x + c)^4 - 6*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 + 2*(143*(5*a
^2 + 16*a*b)*cosh(d*x + c)^10 - 495*(5*a^2 + 16*a*b)*cosh(d*x + c)^8 + 630*
(5*a^2 + 16*a*b)*cosh(d*x + c)^6 - 350*(5*a^2 + 16*a*b)*cosh(d*x + c)^4 + 7
5*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 - 15*a^2 - 48*a*b)*sinh(d*x + c)^3 + 6*(
13*(5*a^2 + 16*a*b)*cosh(d*x + c)^11 - 55*(5*a^2 + 16*a*b)*cosh(d*x + c)^9
+ 90*(5*a^2 + 16*a*b)*cosh(d*x + c)^7 - 70*(5*a^2 + 16*a*b)*cosh(d*x + c)^5
 + 25*(5*a^2 + 16*a*b)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b)*cosh(d*x + c))*
sinh(d*x + c)^2 + (5*a^2 + 16*a*b)*cosh(d*x + c) + (13*(5*a^2 + 16*a*b)*cos
h(d*x + c)^12 - 66*(5*a^2 + 16*a*b)*cosh(d*x + c)^10 + 135*(5*a^2 + 16*a*b)
*cosh(d*x + c)^8 - 140*(5*a^2 + 16*a*b)*cosh(d*x + c)^6 + 75*(5*a^2 + 16*a*
b)*cosh(d*x + c)^4 - 18*(5*a^2 + 16*a*b)*cosh(d*x + c)^2 + 5*a^2 + 16*a*b)*
sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(84*b^2*cosh(d*x
+ c)^13 - 18*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c)^11 + 5*(85*a^2 + 144*a
*b + 108*b^2)*cosh(d*x + c)^9 - 24*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)
^7 - 18*(33*a^2 + 16*a*b + 10*b^2)*cosh(d*x + c)^5 + 2*(85*a^2 + 144*a*b +
108*b^2)*cosh(d*x + c)^3 - 3*(5*a^2 + 16*a*b + 20*b^2)*cosh(d*x + c))*sinh(
d*x + c))/(d*cosh(d*x + c)^13 + 13*d*cosh(d*x + c)*sinh(d*x + c)^12 + d*sin
h(d*x + c)^13 - 6*d*cosh(d*x + c)^11 + 6*(13*d*cosh(d*x + c)^2 - d)*sinh(d*
x + c)^11 + 22*(13*d*cosh(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^10
+ 15*d*cosh(d*x + c)^9 + 5*(143*d*cosh(d*x + c)^4 - 66*d*cosh(d*x + c)^2 +
3*d)*sinh(d*x + c)^9 + 9*(143*d*cosh(d*x + c)^5 - 110*d*cosh(d*x + c)^3 + 1
5*d*cosh(d*x + c))*sinh(d*x + c)^8 - 20*d*cosh(d*x + c)^7 + 4*(429*d*cosh(d
*x + c)^6 - 495*d*cosh(d*x + c)^4 + 135*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x +
 c)^7 + 4*(429*d*cosh(d*x + c)^7 - 693*d*cosh(d*x + c)^5 + 315*d*cosh(d*x +
 c)^3 - 35*d*cosh(d*x + c))*sinh(d*x + c)^6 + 15*d*cosh(d*x + c)^5 + 3*(429
*d*cosh(d*x + c)^8 - 924*d*cosh(d*x + c)^6 + 630*d*cosh(d*x + c)^4 - 140*d*
cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(143*d*cosh(d*x + c)^9 - 396*d*c
osh(d*x + c)^7 + 378*d*cosh(d*x + c)^5 - 140*d*cosh(d*x + c)^3 + 15*d*cosh(
d*x + c))*sinh(d*x + c)^4 - 6*d*cosh(d*x + c)^3 + 2*(143*d*cosh(d*x + c)^10
 - 495*d*cosh(d*x + c)^8 + 630*d*cosh(d*x + c)^6 - 350*d*cosh(d*x + c)^4 +
75*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^3 + 6*(13*d*cosh(d*x + c)^11 - 55
*d*cosh(d*x + c)^9 + 90*d*cosh(d*x + c)^7 - 70*d*cosh(d*x + c)^5 + 25*d*cos
h(d*x + c)^3 - 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c) + (13*d
*cosh(d*x + c)^12 - 66*d*cosh(d*x + c)^10 + 135*d*cosh(d*x + c)^8 - 140*d*c
osh(d*x + c)^6 + 75*d*cosh(d*x + c)^4 - 18*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.27771, size = 338, normalized size = 3.05

$$\frac{b^2\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{2\,d}+\frac{\left(5\,a^2+16\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{32\,d}-\frac{\left(5\,a^2+16\,ab\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{32\,d}-\frac{15\,a^2\left(e^{(d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*(e^(d*x + c) + e^(-d*x - c))/d + 1/32*(5*a^2 + 16*a*b)*log(e^(d*x +
 c) + e^(-d*x - c) + 2)/d - 1/32*(5*a^2 + 16*a*b)*log(e^(d*x + c) + e^(-d*x
 - c) - 2)/d - 1/24*(15*a^2*(e^(d*x + c) + e^(-d*x - c))^5 + 48*a*b*(e^(d*x
 + c) + e^(-d*x - c))^5 - 160*a^2*(e^(d*x + c) + e^(-d*x - c))^3 - 384*a*b*
(e^(d*x + c) + e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) + e^(-d*x - c)) + 768
*a*b*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3*
d)
```

3.207 $\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=220

$$\frac{b\left(3a^2+45ab+70b^2\right)\cosh^9(c+dx)}{9d}-\frac{4b\left(3a^2+15ab+14b^2\right)\cosh^7(c+dx)}{7d}+\frac{(a+b)\left(a^2+17ab+28b^2\right)\cosh^5(c+}{5d}$$

```
[Out] ((a + b)^3*Cosh[c + d*x])/d - (2*(a + b)^2*(a + 4*b)*Cosh[c + d*x]^3)/(3*d)
 + ((a + b)*(a^2 + 17*a*b + 28*b^2)*Cosh[c + d*x]^5)/(5*d) - (4*b*(3*a^2 +
15*a*b + 14*b^2)*Cosh[c + d*x]^7)/(7*d) + (b*(3*a^2 + 45*a*b + 70*b^2)*Cosh
[c + d*x]^9)/(9*d) - (2*b^2*(9*a + 28*b)*Cosh[c + d*x]^11)/(11*d) + (b^2*(3
*a + 28*b)*Cosh[c + d*x]^13)/(13*d) - (8*b^3*Cosh[c + d*x]^15)/(15*d) + (b^
3*Cosh[c + d*x]^17)/(17*d)
```

Rubi [A] time = 0.218072, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3215, 1153}

$$\frac{b\left(3a^2+45ab+70b^2\right)\cosh^9(c+dx)}{9d}-\frac{4b\left(3a^2+15ab+14b^2\right)\cosh^7(c+dx)}{7d}+\frac{(a+b)\left(a^2+17ab+28b^2\right)\cosh^5(c+}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((a + b)^3*Cosh[c + d*x])/d - (2*(a + b)^2*(a + 4*b)*Cosh[c + d*x]^3)/(3*d)
 + ((a + b)*(a^2 + 17*a*b + 28*b^2)*Cosh[c + d*x]^5)/(5*d) - (4*b*(3*a^2 +
15*a*b + 14*b^2)*Cosh[c + d*x]^7)/(7*d) + (b*(3*a^2 + 45*a*b + 70*b^2)*Cosh
[c + d*x]^9)/(9*d) - (2*b^2*(9*a + 28*b)*Cosh[c + d*x]^11)/(11*d) + (b^2*(3
*a + 28*b)*Cosh[c + d*x]^13)/(13*d) - (8*b^3*Cosh[c + d*x]^15)/(15*d) + (b^
3*Cosh[c + d*x]^17)/(17*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \sinh^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a+b-2bx^2+bx^4\right)^3dx,x,\cosh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int\left((a+b)^3-2(a+b)^2(a+4b)x^2+(a+b)\left(a^2+17ab+28b^2\right)x^4-\right.\right.}{}$$

$$= \frac{(a+b)^3\cosh(c+dx)}{d} - \frac{2(a+b)^2(a+4b)\cosh^3(c+dx)}{3d} + \frac{(a+b)\left(a^2+17a\right.}{}$$

Mathematica [A] time = 2.26629, size = 288, normalized size = 1.31

$$\frac{1531530\left(48384a^2b+20480a^3+41184ab^2+12155b^3\right)\cosh(c+dx)-2042040\left(8064a^2b+2560a^3+7722ab^2+2431b^3\right)}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (1531530*(20480*a^3 + 48384*a^2*b + 41184*a*b^2 + 12155*b^3)*Cosh[c + d*x]
- 2042040*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*Cosh[3*(c + d*x)]
 + 627314688*a^3*Cosh[5*(c + d*x)] + 4234374144*a^2*b*Cosh[5*(c + d*x)] + 5
256210960*a*b^2*Cosh[5*(c + d*x)] + 1895421528*b^3*Cosh[5*(c + d*x)] - 7561
38240*a^2*b*Cosh[7*(c + d*x)] - 1501774560*a*b^2*Cosh[7*(c + d*x)] - 676936
260*b^3*Cosh[7*(c + d*x)] + 65345280*a^2*b*Cosh[9*(c + d*x)] + 318558240*a*
b^2*Cosh[9*(c + d*x)] + 202502300*b^3*Cosh[9*(c + d*x)] - 43439760*a*b^2*Co
sh[11*(c + d*x)] - 47338200*b^3*Cosh[11*(c + d*x)] + 2827440*a*b^2*Cosh[13*
(c + d*x)] + 8011080*b^3*Cosh[13*(c + d*x)] - 867867*b^3*Cosh[15*(c + d*x)]
 + 45045*b^3*Cosh[17*(c + d*x)])/(50185175040*d)
```

Maple [A] time = 0.096, size = 258, normalized size = 1.2

$$\frac{1}{d}\left(b^3\left(\frac{32768}{109395}+\frac{(\sinh(dx+c))^{16}}{17}-\frac{16\,(\sinh(dx+c))^{14}}{255}+\frac{224\,(\sinh(dx+c))^{12}}{3315}-\frac{896\,(\sinh(dx+c))^{10}}{12155}+\frac{1792\,(\sinh}{218}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*(32768/109395+1/17*sinh(d*x+c)^16-16/255*sinh(d*x+c)^14+224/3315*s
inh(d*x+c)^12-896/12155*sinh(d*x+c)^10+1792/21879*sinh(d*x+c)^8-2048/21879*
sinh(d*x+c)^6+4096/36465*sinh(d*x+c)^4-16384/109395*sinh(d*x+c)^2)*cosh(d*x
+c)+3*a*b^2*(1024/3003+1/13*sinh(d*x+c)^12-12/143*sinh(d*x+c)^10+40/429*sin
h(d*x+c)^8-320/3003*sinh(d*x+c)^6+128/1001*sinh(d*x+c)^4-512/3003*sinh(d*x+
c)^2)*cosh(d*x+c)+3*a^2*b*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/
105*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(8/15+1/5*sinh(d*x+
c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.07614, size = 810, normalized size = 3.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/14338621440*b^3*((123981*e^(-2*d*x - 2*c) - 1144440*e^(-4*d*x - 4*c) + 6
762600*e^(-6*d*x - 6*c) - 28928900*e^(-8*d*x - 8*c) + 96705180*e^(-10*d*x -
 10*c) - 270774504*e^(-12*d*x - 12*c) + 709171320*e^(-14*d*x - 14*c) - 2659
392450*e^(-16*d*x - 16*c) - 6435)*e^(17*d*x + 17*c)/d - (2659392450*e^(-d*x
 - c) - 709171320*e^(-3*d*x - 3*c) + 270774504*e^(-5*d*x - 5*c) - 96705180*
e^(-7*d*x - 7*c) + 28928900*e^(-9*d*x - 9*c) - 6762600*e^(-11*d*x - 11*c) +
 1144440*e^(-13*d*x - 13*c) - 123981*e^(-15*d*x - 15*c) + 6435*e^(-17*d*x -
 17*c))/d) - 1/8200192*a*b^2*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*d*x - 4*
c) + 122694*e^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e^(-10*d*x
 - 10*c) - 5153148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d - (5153148
*e^(-d*x - c) - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*c) - 122694
*e^(-7*d*x - 7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x - 11*c) + 231*
e^(-13*d*x - 13*c))/d) - 1/53760*a^2*b*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*
d*x - 4*c) + 8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x
+ 9*c)/d - (39690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5
*c) - 405*e^(-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/480*a^3*(3*e^(5*d*
x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d
- 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d)
```

Fricas [B] time = 1.78734, size = 2898, normalized size = 13.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/50185175040*(45045*b^3*cosh(d*x + c)^17 + 765765*b^3*cosh(d*x + c)*sinh(d
*x + c)^16 - 867867*b^3*cosh(d*x + c)^15 + 765765*(40*b^3*cosh(d*x + c)^3 -
 17*b^3*cosh(d*x + c))*sinh(d*x + c)^14 + 471240*(6*a*b^2 + 17*b^3)*cosh(d*
x + c)^13 + 255255*(1092*b^3*cosh(d*x + c)^5 - 1547*b^3*cosh(d*x + c)^3 + 2
4*(6*a*b^2 + 17*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 - 556920*(78*a*b^2 + 8
5*b^3)*cosh(d*x + c)^11 + 153153*(5720*b^3*cosh(d*x + c)^7 - 17017*b^3*cosh
(d*x + c)^5 + 880*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^3 - 40*(78*a*b^2 + 85*b^
3)*cosh(d*x + c))*sinh(d*x + c)^10 + 340340*(192*a^2*b + 936*a*b^2 + 595*b^
3)*cosh(d*x + c)^9 + 765765*(1430*b^3*cosh(d*x + c)^9 - 7293*b^3*cosh(d*x +
 c)^7 + 792*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^5 - 120*(78*a*b^2 + 85*b^3)*co
sh(d*x + c)^3 + 4*(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^8 - 437580*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*cosh(d*x + c)^7 + 255
255*(2184*b^3*cosh(d*x + c)^11 - 17017*b^3*cosh(d*x + c)^9 + 3168*(6*a*b^2
+ 17*b^3)*cosh(d*x + c)^7 - 1008*(78*a*b^2 + 85*b^3)*cosh(d*x + c)^5 + 112*
(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c)^3 - 12*(1728*a^2*b + 3432*a
*b^2 + 1547*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 1225224*(512*a^3 + 3456*a
^2*b + 4290*a*b^2 + 1547*b^3)*cosh(d*x + c)^5 + 765765*(140*b^3*cosh(d*x +
c)^13 - 1547*b^3*cosh(d*x + c)^11 + 440*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^9
- 240*(78*a*b^2 + 85*b^3)*cosh(d*x + c)^7 + 56*(192*a^2*b + 936*a*b^2 + 595
*b^3)*cosh(d*x + c)^5 - 20*(1728*a^2*b + 3432*a*b^2 + 1547*b^3)*cosh(d*x +
c)^3 + 8*(512*a^3 + 3456*a^2*b + 4290*a*b^2 + 1547*b^3)*cosh(d*x + c))*sinh
(d*x + c)^4 - 2042040*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 + 2431*b^3)*cosh(
d*x + c)^3 + 765765*(8*b^3*cosh(d*x + c)^15 - 119*b^3*cosh(d*x + c)^13 + 48
*(6*a*b^2 + 17*b^3)*cosh(d*x + c)^11 - 40*(78*a*b^2 + 85*b^3)*cosh(d*x + c)
^9 + 16*(192*a^2*b + 936*a*b^2 + 595*b^3)*cosh(d*x + c)^7 - 12*(1728*a^2*b
+ 3432*a*b^2 + 1547*b^3)*cosh(d*x + c)^5 + 16*(512*a^3 + 3456*a^2*b + 4290*
a*b^2 + 1547*b^3)*cosh(d*x + c)^3 - 8*(2560*a^3 + 8064*a^2*b + 7722*a*b^2 +
 2431*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 1531530*(20480*a^3 + 48384*a^2*
```

```
b + 41184*a*b^2 + 12155*b^3)*cosh(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.7945, size = 934, normalized size = 4.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/100370350080*(45045*b^3*e^(17*d*x + 17*c) - 867867*b^3*e^(15*d*x + 15*c)
+ 2827440*a*b^2*e^(13*d*x + 13*c) + 8011080*b^3*e^(13*d*x + 13*c) - 4343976
0*a*b^2*e^(11*d*x + 11*c) - 47338200*b^3*e^(11*d*x + 11*c) + 65345280*a^2*b
*e^(9*d*x + 9*c) + 318558240*a*b^2*e^(9*d*x + 9*c) + 202502300*b^3*e^(9*d*x
 + 9*c) - 756138240*a^2*b*e^(7*d*x + 7*c) - 1501774560*a*b^2*e^(7*d*x + 7*c
) - 676936260*b^3*e^(7*d*x + 7*c) + 627314688*a^3*e^(5*d*x + 5*c) + 4234374
144*a^2*b*e^(5*d*x + 5*c) + 5256210960*a*b^2*e^(5*d*x + 5*c) + 1895421528*b
^3*e^(5*d*x + 5*c) - 5227622400*a^3*e^(3*d*x + 3*c) - 16467010560*a^2*b*e^(
3*d*x + 3*c) - 15768632880*a*b^2*e^(3*d*x + 3*c) - 4964199240*b^3*e^(3*d*x
+ 3*c) + 31365734400*a^3*e^(d*x + c) + 74101547520*a^2*b*e^(d*x + c) + 6307
4531520*a*b^2*e^(d*x + c) + 18615747150*b^3*e^(d*x + c) + (31365734400*a^3*
e^(16*d*x + 16*c) + 74101547520*a^2*b*e^(16*d*x + 16*c) + 63074531520*a*b^2
*e^(16*d*x + 16*c) + 18615747150*b^3*e^(16*d*x + 16*c) - 5227622400*a^3*e^(
14*d*x + 14*c) - 16467010560*a^2*b*e^(14*d*x + 14*c) - 15768632880*a*b^2*e^
(14*d*x + 14*c) - 4964199240*b^3*e^(14*d*x + 14*c) + 627314688*a^3*e^(12*d*
x + 12*c) + 4234374144*a^2*b*e^(12*d*x + 12*c) + 5256210960*a*b^2*e^(12*d*x
 + 12*c) + 1895421528*b^3*e^(12*d*x + 12*c) - 756138240*a^2*b*e^(10*d*x + 1
0*c) - 1501774560*a*b^2*e^(10*d*x + 10*c) - 676936260*b^3*e^(10*d*x + 10*c)
 + 65345280*a^2*b*e^(8*d*x + 8*c) + 318558240*a*b^2*e^(8*d*x + 8*c) + 20250
2300*b^3*e^(8*d*x + 8*c) - 43439760*a*b^2*e^(6*d*x + 6*c) - 47338200*b^3*e^
(6*d*x + 6*c) + 2827440*a*b^2*e^(4*d*x + 4*c) + 8011080*b^3*e^(4*d*x + 4*c)
 - 867867*b^3*e^(2*d*x + 2*c) + 45045*b^3)*e^(-17*d*x - 17*c))/d
```

3.208 $\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=183

$$\frac{b\left(3a^2+30ab+35b^2\right)\cosh^7(c+dx)}{7d}+\frac{3b^2(a+7b)\cosh^{11}(c+dx)}{11d}-\frac{5b^2(3a+7b)\cosh^9(c+dx)}{9d}-\frac{3b(a+b)(3a+7b}{5d}$$

```
[Out] -(((a + b)^3*Cosh[c + d*x])/d) + ((a + b)^2*(a + 7*b)*Cosh[c + d*x]^3)/(3*d
) - (3*b*(a + b)*(3*a + 7*b)*Cosh[c + d*x]^5)/(5*d) + (b*(3*a^2 + 30*a*b +
35*b^2)*Cosh[c + d*x]^7)/(7*d) - (5*b^2*(3*a + 7*b)*Cosh[c + d*x]^9)/(9*d)
+ (3*b^2*(a + 7*b)*Cosh[c + d*x]^11)/(11*d) - (7*b^3*Cosh[c + d*x]^13)/(13*
d) + (b^3*Cosh[c + d*x]^15)/(15*d)
```

Rubi [A] time = 0.175103, antiderivative size = 183, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3215, 1153}

$$\frac{b\left(3a^2+30ab+35b^2\right)\cosh^7(c+dx)}{7d}+\frac{3b^2(a+7b)\cosh^{11}(c+dx)}{11d}-\frac{5b^2(3a+7b)\cosh^9(c+dx)}{9d}-\frac{3b(a+b)(3a+7b}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(((a + b)^3*Cosh[c + d*x])/d) + ((a + b)^2*(a + 7*b)*Cosh[c + d*x]^3)/(3*d
) - (3*b*(a + b)*(3*a + 7*b)*Cosh[c + d*x]^5)/(5*d) + (b*(3*a^2 + 30*a*b +
35*b^2)*Cosh[c + d*x]^7)/(7*d) - (5*b^2*(3*a + 7*b)*Cosh[c + d*x]^9)/(9*d)
+ (3*b^2*(a + 7*b)*Cosh[c + d*x]^11)/(11*d) - (7*b^3*Cosh[c + d*x]^13)/(13*
d) + (b^3*Cosh[c + d*x]^15)/(15*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \sinh^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=-\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-2bx^2+bx^4\right)^3\,dx,x,\cosh(c+dx)\right)}{d}$$

$$=-\frac{\text{Subst}\left(\int\left((a+b)^3-(a+b)^2(a+7b)x^2+3b(a+b)(3a+7b)x^4-b\left(3a\right.\right.\right.}{}$$

$$=-\frac{(a+b)^3\cosh(c+dx)}{d}+\frac{(a+b)^2(a+7b)\cosh^3(c+dx)}{3d}-\frac{3b(a+b)(3a}{}$$

Mathematica [A] time = 2.47, size = 185, normalized size = 1.01

$$\frac{-135135\left(8960a^2b+4096a^3+7392ab^2+2145b^3\right)\cosh(c+dx)+15015\left(16128a^2b+4096a^3+15840ab^2+5005b^3\right)\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-135135*(4096*a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*Cosh[c + d*x] + 15
015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5005*b^3)*Cosh[3*(c + d*x)] + b
*(-27027*(1792*a^2 + 2640*a*b + 1001*b^2)*Cosh[5*(c + d*x)] + 19305*(256*a^
2 + 880*a*b + 455*b^2)*Cosh[7*(c + d*x)] - 7*b*(715*(528*a + 455*b)*Cosh[9*
(c + d*x)] - 1755*(16*a + 35*b)*Cosh[11*(c + d*x)] + 7425*b*Cosh[13*(c + d*
x)] - 429*b*Cosh[15*(c + d*x)])))/(738017280*d)
```

Maple [A] time = 0.059, size = 218, normalized size = 1.2

$$\frac{1}{d}\left(b^3\left(-\frac{2048}{6435}+\frac{(\sinh(dx+c))^{14}}{15}-\frac{14(\sinh(dx+c))^{12}}{195}+\frac{56(\sinh(dx+c))^{10}}{715}-\frac{112(\sinh(dx+c))^8}{1287}+\frac{128(\sinh(dx}{1287}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*(-2048/6435+1/15*sinh(d*x+c)^14-14/195*sinh(d*x+c)^12+56/715*sinh(
d*x+c)^10-112/1287*sinh(d*x+c)^8+128/1287*sinh(d*x+c)^6-256/2145*sinh(d*x+c
)^4+1024/6435*sinh(d*x+c)^2)*cosh(d*x+c)+3*a*b^2*(-256/693+1/11*sinh(d*x+c)
^10-10/99*sinh(d*x+c)^8+80/693*sinh(d*x+c)^6-32/231*sinh(d*x+c)^4+128/693*s
inh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c
)^4+8/35*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c
))
```

Maxima [B] time = 1.07978, size = 676, normalized size = 3.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/210862080*b^3*((7425*e^(-2*d*x - 2*c) - 61425*e^(-4*d*x - 4*c) + 325325*
e^(-6*d*x - 6*c) - 1254825*e^(-8*d*x - 8*c) + 3864861*e^(-10*d*x - 10*c) -
10735725*e^(-12*d*x - 12*c) + 41409225*e^(-14*d*x - 14*c) - 429)*e^(15*d*x
+ 15*c)/d + (41409225*e^(-d*x - c) - 10735725*e^(-3*d*x - 3*c) + 3864861*e^
(-5*d*x - 5*c) - 1254825*e^(-7*d*x - 7*c) + 325325*e^(-9*d*x - 9*c) - 61425
*e^(-11*d*x - 11*c) + 7425*e^(-13*d*x - 13*c) - 429*e^(-15*d*x - 15*c))/d)
- 1/473088*a*b^2*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(
-6*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^
(11*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e
^(-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*
d*x - 11*c))/d) - 3/4480*a^2*b*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c)
 + 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*
e^(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/24*a^3*
```

```
(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/
d)
```

Fricas [B] time = 1.73061, size = 2207, normalized size = 12.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/738017280*(3003*b^3*cosh(d*x + c)^15 + 45045*b^3*cosh(d*x + c)*sinh(d*x +
 c)^14 - 51975*b^3*cosh(d*x + c)^13 + 15015*(91*b^3*cosh(d*x + c)^3 - 45*b^
3*cosh(d*x + c))*sinh(d*x + c)^12 + 12285*(16*a*b^2 + 35*b^3)*cosh(d*x + c)
^11 + 9009*(1001*b^3*cosh(d*x + c)^5 - 1650*b^3*cosh(d*x + c)^3 + 15*(16*a*
b^2 + 35*b^3)*cosh(d*x + c))*sinh(d*x + c)^10 - 5005*(528*a*b^2 + 455*b^3)*
cosh(d*x + c)^9 + 45045*(429*b^3*cosh(d*x + c)^7 - 1485*b^3*cosh(d*x + c)^5
 + 45*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^3 - (528*a*b^2 + 455*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^8 + 19305*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x +
c)^7 + 15015*(1001*b^3*cosh(d*x + c)^9 - 5940*b^3*cosh(d*x + c)^7 + 378*(16
*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 28*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^3
 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 270
27*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 + 45045*(91*b^3*cos
h(d*x + c)^11 - 825*b^3*cosh(d*x + c)^9 + 90*(16*a*b^2 + 35*b^3)*cosh(d*x +
 c)^7 - 14*(528*a*b^2 + 455*b^3)*cosh(d*x + c)^5 + 15*(256*a^2*b + 880*a*b^
2 + 455*b^3)*cosh(d*x + c)^3 - 3*(1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(
d*x + c))*sinh(d*x + c)^4 + 15015*(4096*a^3 + 16128*a^2*b + 15840*a*b^2 + 5
005*b^3)*cosh(d*x + c)^3 + 45045*(7*b^3*cosh(d*x + c)^13 - 90*b^3*cosh(d*x
+ c)^11 + 15*(16*a*b^2 + 35*b^3)*cosh(d*x + c)^9 - 4*(528*a*b^2 + 455*b^3)*
cosh(d*x + c)^7 + 9*(256*a^2*b + 880*a*b^2 + 455*b^3)*cosh(d*x + c)^5 - 6*(
1792*a^2*b + 2640*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 16128*a^2
*b + 15840*a*b^2 + 5005*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 135135*(4096*
a^3 + 8960*a^2*b + 7392*a*b^2 + 2145*b^3)*cosh(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.72125, size = 779, normalized size = 4.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/1476034560*(3003*b^3*e^(15*d*x + 15*c) - 51975*b^3*e^(13*d*x + 13*c) + 19
6560*a*b^2*e^(11*d*x + 11*c) + 429975*b^3*e^(11*d*x + 11*c) - 2642640*a*b^2
*e^(9*d*x + 9*c) - 2277275*b^3*e^(9*d*x + 9*c) + 4942080*a^2*b*e^(7*d*x + 7
*c) + 16988400*a*b^2*e^(7*d*x + 7*c) + 8783775*b^3*e^(7*d*x + 7*c) - 484323
84*a^2*b*e^(5*d*x + 5*c) - 71351280*a*b^2*e^(5*d*x + 5*c) - 27054027*b^3*e^
(5*d*x + 5*c) + 61501440*a^3*e^(3*d*x + 3*c) + 242161920*a^2*b*e^(3*d*x + 3
*c) + 237837600*a*b^2*e^(3*d*x + 3*c) + 75150075*b^3*e^(3*d*x + 3*c) - 5535
12960*a^3*e^(d*x + c) - 1210809600*a^2*b*e^(d*x + c) - 998917920*a*b^2*e^(d
*x + c) - 289864575*b^3*e^(d*x + c) - (553512960*a^3*e^(14*d*x + 14*c) + 12
10809600*a^2*b*e^(14*d*x + 14*c) + 998917920*a*b^2*e^(14*d*x + 14*c) + 2898
64575*b^3*e^(14*d*x + 14*c) - 61501440*a^3*e^(12*d*x + 12*c) - 242161920*a^
2*b*e^(12*d*x + 12*c) - 237837600*a*b^2*e^(12*d*x + 12*c) - 75150075*b^3*e^
(12*d*x + 12*c) + 48432384*a^2*b*e^(10*d*x + 10*c) + 71351280*a*b^2*e^(10*d
*x + 10*c) + 27054027*b^3*e^(10*d*x + 10*c) - 4942080*a^2*b*e^(8*d*x + 8*c)
 - 16988400*a*b^2*e^(8*d*x + 8*c) - 8783775*b^3*e^(8*d*x + 8*c) + 2642640*a
*b^2*e^(6*d*x + 6*c) + 2277275*b^3*e^(6*d*x + 6*c) - 196560*a*b^2*e^(4*d*x
+ 4*c) - 429975*b^3*e^(4*d*x + 4*c) + 51975*b^3*e^(2*d*x + 2*c) - 3003*b^3)
*e^(-15*d*x - 15*c))/d
```

3.209 $\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=143

$$\frac{b^2(a+5b)\cosh^9(c+dx)}{3d}-\frac{4b^2(3a+5b)\cosh^7(c+dx)}{7d}+\frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}$$

```
[Out] ((a + b)^3*Cosh[c + d*x])/d - (2*b*(a + b)^2*Cosh[c + d*x]^3)/d + (3*b*(a +
 b)*(a + 5*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*(3*a + 5*b)*Cosh[c + d*x]^7)/
(7*d) + (b^2*(a + 5*b)*Cosh[c + d*x]^9)/(3*d) - (6*b^3*Cosh[c + d*x]^11)/(1
1*d) + (b^3*Cosh[c + d*x]^13)/(13*d)
```

Rubi [A] time = 0.15744, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3215, 1090}

$$\frac{b^2(a+5b)\cosh^9(c+dx)}{3d}-\frac{4b^2(3a+5b)\cosh^7(c+dx)}{7d}+\frac{3b(a+b)(a+5b)\cosh^5(c+dx)}{5d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] ((a + b)^3*Cosh[c + d*x])/d - (2*b*(a + b)^2*Cosh[c + d*x]^3)/d + (3*b*(a +
 b)*(a + 5*b)*Cosh[c + d*x]^5)/(5*d) - (4*b^2*(3*a + 5*b)*Cosh[c + d*x]^7)/
(7*d) + (b^2*(a + 5*b)*Cosh[c + d*x]^9)/(3*d) - (6*b^3*Cosh[c + d*x]^11)/(1
1*d) + (b^3*Cosh[c + d*x]^13)/(13*d)
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1090

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[ExpandInteg
rand[(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*
c, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \sinh(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int\left(a+b-2bx^2+bx^4\right)^3\,dx,x,\cosh(c+dx)\right)}{d}$$

$$=\frac{\text{Subst}\left(\int\left(a^3\left(1+\frac{b(3a^2+3ab+b^2)}{a^3}\right)-6b(a+b)^2x^2+12b^2(a+b)\left(1+\frac{a+b}{4b}\right)x\right.\right.}{}$$

$$=\frac{(a+b)^3\cosh(c+dx)}{d}-\frac{2b(a+b)^2\cosh^3(c+dx)}{d}+\frac{3b(a+b)(a+5b)\cos}{5d}$$

Mathematica [A] time = 0.90112, size = 157, normalized size = 1.1

$$\frac{-15015b\left(1280a^2+1344ab+429b^2\right)\cosh(3(c+dx))+3003b\left(768a^2+1728ab+715b^2\right)\cosh(5(c+dx))+60060\left(1920a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*Cosh[c + d*x] - 15015
*b*(1280*a^2 + 1344*a*b + 429*b^2)*Cosh[3*(c + d*x)] + 3003*b*(768*a^2 + 17
28*a*b + 715*b^2)*Cosh[5*(c + d*x)] - 4290*b^2*(216*a + 143*b)*Cosh[7*(c +
d*x)] + 10010*b^2*(8*a + 13*b)*Cosh[9*(c + d*x)] - 17745*b^3*Cosh[11*(c + d
*x)] + 1155*b^3*Cosh[13*(c + d*x)])/(61501440*d)
```

Maple [A] time = 0.02, size = 176, normalized size = 1.2

$$\frac{1}{d}\left(b^3\left(\frac{1024}{3003}+\frac{(\sinh(dx+c))^{12}}{13}-\frac{12\,(\sinh(dx+c))^{10}}{143}+\frac{40\,(\sinh(dx+c))^{8}}{429}-\frac{320\,(\sinh(dx+c))^{6}}{3003}+\frac{128\,(\sinh(dx+c\right.\right.}{1001}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*(1024/3003+1/13*sinh(d*x+c)^12-12/143*sinh(d*x+c)^10+40/429*sinh(d
*x+c)^8-320/3003*sinh(d*x+c)^6+128/1001*sinh(d*x+c)^4-512/3003*sinh(d*x+c)^
2)*cosh(d*x+c)+3*a*b^2*(128/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105
*sinh(d*x+c)^4-64/315*sinh(d*x+c)^2)*cosh(d*x+c)+3*a^2*b*(8/15+1/5*sinh(d*x
+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+a^3*cosh(d*x+c))
```

Maxima [B] time = 1.04483, size = 539, normalized size = 3.77

$$-\frac{1}{24600576}b^3\left(\frac{\left(3549\,e^{(-2\,dx-2\,c)}-26026\,e^{(-4\,dx-4\,c)}+122694\,e^{(-6\,dx-6\,c)}-429429\,e^{(-8\,dx-8\,c)}+1288287\,e^{(-10\,dx-10\,c)}-5153\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/24600576*b^3*((3549*e^(-2*d*x - 2*c) - 26026*e^(-4*d*x - 4*c) + 122694*e
^(-6*d*x - 6*c) - 429429*e^(-8*d*x - 8*c) + 1288287*e^(-10*d*x - 10*c) - 51
53148*e^(-12*d*x - 12*c) - 231)*e^(13*d*x + 13*c)/d - (5153148*e^(-d*x - c)
 - 1288287*e^(-3*d*x - 3*c) + 429429*e^(-5*d*x - 5*c) - 122694*e^(-7*d*x -
7*c) + 26026*e^(-9*d*x - 9*c) - 3549*e^(-11*d*x - 11*c) + 231*e^(-13*d*x -
13*c))/d) - 1/53760*a*b^2*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) +
8820*e^(-6*d*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (3
9690*e^(-d*x - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(
-7*d*x - 7*c) + 35*e^(-9*d*x - 9*c))/d) + 1/160*a^2*b*(3*e^(5*d*x + 5*c)/d
- 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*
d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + a^3*cosh(d*x + c)/d
```

Fricas [B] time = 1.73622, size = 1620, normalized size = 11.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/61501440*(1155*b^3*cosh(d*x + c)^13 + 15015*b^3*cosh(d*x + c)*sinh(d*x +
c)^12 - 17745*b^3*cosh(d*x + c)^11 + 15015*(22*b^3*cosh(d*x + c)^3 - 13*b^3
*cosh(d*x + c))*sinh(d*x + c)^10 + 10010*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^9
 + 45045*(33*b^3*cosh(d*x + c)^5 - 65*b^3*cosh(d*x + c)^3 + 2*(8*a*b^2 + 13
*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 - 4290*(216*a*b^2 + 143*b^3)*cosh(d*x
+ c)^7 + 30030*(66*b^3*cosh(d*x + c)^7 - 273*b^3*cosh(d*x + c)^5 + 28*(8*a*
b^2 + 13*b^3)*cosh(d*x + c)^3 - (216*a*b^2 + 143*b^3)*cosh(d*x + c))*sinh(d
*x + c)^6 + 3003*(768*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x + c)^5 + 15015
*(55*b^3*cosh(d*x + c)^9 - 390*b^3*cosh(d*x + c)^7 + 84*(8*a*b^2 + 13*b^3)*
cosh(d*x + c)^5 - 10*(216*a*b^2 + 143*b^3)*cosh(d*x + c)^3 + (768*a^2*b + 1
728*a*b^2 + 715*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 15015*(1280*a^2*b + 1
344*a*b^2 + 429*b^3)*cosh(d*x + c)^3 + 15015*(6*b^3*cosh(d*x + c)^11 - 65*b
^3*cosh(d*x + c)^9 + 24*(8*a*b^2 + 13*b^3)*cosh(d*x + c)^7 - 6*(216*a*b^2 +
 143*b^3)*cosh(d*x + c)^5 + 2*(768*a^2*b + 1728*a*b^2 + 715*b^3)*cosh(d*x +
 c)^3 - 3*(1280*a^2*b + 1344*a*b^2 + 429*b^3)*cosh(d*x + c))*sinh(d*x + c)^
2 + 60060*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*cosh(d*x + c))/d
```

Sympy [A] time = 169.693, size = 377, normalized size = 2.64

$$\begin{cases} \frac{a^3\cosh(c+dx)}{d} + \frac{3a^2b\sinh^4(c+dx)\cosh(c+dx)}{d} - \frac{4a^2b\sinh^2(c+dx)\cosh^3(c+dx)}{d} + \frac{8a^2b\cosh^5(c+dx)}{5d} + \frac{3ab^2\sinh^8(c+dx)\cosh(c+dx)}{d} - 8ab^2\sin \\ x\left(a+b\sinh^4(c)\right)^3\sinh(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*cosh(c + d*x)/d + 3*a**2*b*sinh(c + d*x)**4*cosh(c + d*x)/d
 - 4*a**2*b*sinh(c + d*x)**2*cosh(c + d*x)**3/d + 8*a**2*b*cosh(c + d*x)**5
/(5*d) + 3*a*b**2*sinh(c + d*x)**8*cosh(c + d*x)/d - 8*a*b**2*sinh(c + d*x)
**6*cosh(c + d*x)**3/d + 48*a*b**2*sinh(c + d*x)**4*cosh(c + d*x)**5/(5*d)
- 192*a*b**2*sinh(c + d*x)**2*cosh(c + d*x)**7/(35*d) + 128*a*b**2*cosh(c +
 d*x)**9/(105*d) + b**3*sinh(c + d*x)**12*cosh(c + d*x)/d - 4*b**3*sinh(c +
 d*x)**10*cosh(c + d*x)**3/d + 8*b**3*sinh(c + d*x)**8*cosh(c + d*x)**5/d -
 64*b**3*sinh(c + d*x)**6*cosh(c + d*x)**7/(7*d) + 128*b**3*sinh(c + d*x)**
4*cosh(c + d*x)**9/(21*d) - 512*b**3*sinh(c + d*x)**2*cosh(c + d*x)**11/(23
1*d) + 1024*b**3*cosh(c + d*x)**13/(3003*d), Ne(d, 0)), (x*(a + b*sinh(c)**
4)**3*sinh(c), True))
```

Giac [B] time = 1.67781, size = 621, normalized size = 4.34

$$\frac{1155\,b^3e^{(13\,dx+13\,c)} - 17745\,b^3e^{(11\,dx+11\,c)} + 80080\,ab^2e^{(9\,dx+9\,c)} + 130130\,b^3e^{(9\,dx+9\,c)} - 926640\,ab^2e^{(7\,dx+7\,c)} - 613470\,b^3e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/123002880*(1155*b^3*e^(13*d*x + 13*c) - 17745*b^3*e^(11*d*x + 11*c) + 800
80*a*b^2*e^(9*d*x + 9*c) + 130130*b^3*e^(9*d*x + 9*c) - 926640*a*b^2*e^(7*d
*x + 7*c) - 613470*b^3*e^(7*d*x + 7*c) + 2306304*a^2*b*e^(5*d*x + 5*c) + 51
89184*a*b^2*e^(5*d*x + 5*c) + 2147145*b^3*e^(5*d*x + 5*c) - 19219200*a^2*b*
e^(3*d*x + 3*c) - 20180160*a*b^2*e^(3*d*x + 3*c) - 6441435*b^3*e^(3*d*x + 3
*c) + 61501440*a^3*e^(d*x + c) + 115315200*a^2*b*e^(d*x + c) + 90810720*a*b
^2*e^(d*x + c) + 25765740*b^3*e^(d*x + c) + (61501440*a^3*e^(12*d*x + 12*c)
 + 115315200*a^2*b*e^(12*d*x + 12*c) + 90810720*a*b^2*e^(12*d*x + 12*c) + 2
5765740*b^3*e^(12*d*x + 12*c) - 19219200*a^2*b*e^(10*d*x + 10*c) - 20180160
*a*b^2*e^(10*d*x + 10*c) - 6441435*b^3*e^(10*d*x + 10*c) + 2306304*a^2*b*e^
(8*d*x + 8*c) + 5189184*a*b^2*e^(8*d*x + 8*c) + 2147145*b^3*e^(8*d*x + 8*c)
 - 926640*a*b^2*e^(6*d*x + 6*c) - 613470*b^3*e^(6*d*x + 6*c) + 80080*a*b^2*
e^(4*d*x + 4*c) + 130130*b^3*e^(4*d*x + 4*c) - 17745*b^3*e^(2*d*x + 2*c) +
1155*b^3)*e^(-13*d*x - 13*c))/d
```

3.210 $\int \mathbf{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=158

$$\frac{b\left(3a^2+9ab+5b^2\right)\cosh^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2(3a+10b)\cosh}{7d}$$

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x])
/d + (b*(3*a^2 + 9*a*b + 5*b^2)*Cosh[c + d*x]^3)/(3*d) - (b^2*(9*a + 10*b)*
Cosh[c + d*x]^5)/(5*d) + (b^2*(3*a + 10*b)*Cosh[c + d*x]^7)/(7*d) - (5*b^3*
Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.135437, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3215, 1153, 206}

$$\frac{b\left(3a^2+9ab+5b^2\right)\cosh^3(c+dx)}{3d}-\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}-\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d}+\frac{b^2(3a+10b)\cosh}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((a^3*ArcTanh[Cosh[c + d*x]])/d) - (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x])
/d + (b*(3*a^2 + 9*a*b + 5*b^2)*Cosh[c + d*x]^3)/(3*d) - (b^2*(9*a + 10*b)*
Cosh[c + d*x]^5)/(5*d) + (b^2*(3*a + 10*b)*Cosh[c + d*x]^7)/(7*d) - (5*b^3*
Cosh[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]^11)/(11*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{1-x^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(b\left(3a^2+3ab+b^2\right)-b\left(3a^2+9ab+5b^2\right)x^2+b^2(9a+10b)x^4-\right.\right.}{d}$$

$$= -\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d} + \frac{b\left(3a^2+9ab+5b^2\right)\cosh^3(c+dx)}{3d} - \frac{b^2}{}$$

$$= -\frac{a^3\tanh^{-1}(\cosh(c+dx))}{d} - \frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d} + \frac{b\left(3a^2+9a\right.}{}$$

Mathematica [A] time = 0.380613, size = 139, normalized size = 0.88

$$\frac{-20790b\left(384a^2+280ab+77b^2\right)\cosh(c+dx)+3548160a^3\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-2079b^2(112a+55b)\cosh(5(c+dx}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-20790*b*(384*a^2 + 280*a*b + 77*b^2)*Cosh[c + d*x] + 6930*b*(8*a + 5*b)*(
16*a + 11*b)*Cosh[3*(c + d*x)] - 2079*b^2*(112*a + 55*b)*Cosh[5*(c + d*x)]
+ 495*b^2*(48*a + 55*b)*Cosh[7*(c + d*x)] - 4235*b^3*Cosh[9*(c + d*x)] + 31
5*b^3*Cosh[11*(c + d*x)] + 3548160*a^3*Log[Tanh[(c + d*x)/2]])/(3548160*d)
```

Maple [A] time = 0.039, size = 148, normalized size = 0.9

$$\frac{1}{d}\left(-2\,a^3\mathrm{Artanh}\left(e^{dx+c}\right)+3\,a^2b\left(-2/3+1/3\,(\sinh(dx+c))^2\right)\cosh(dx+c)+3\,ab^2\left(-\frac{16}{35}+1/7\,(\sinh(dx+c))^6-\frac{6\,(\sin}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(-2*a^3*arctanh(exp(d*x+c))+3*a^2*b*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c
)+3*a*b^2*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)*
cosh(d*x+c)+b^3*(-256/693+1/11*sinh(d*x+c)^10-10/99*sinh(d*x+c)^8+80/693*si
nh(d*x+c)^6-32/231*sinh(d*x+c)^4+128/693*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.06644, size = 441, normalized size = 2.79

$$-\frac{1}{1419264}\,b^3\left(\frac{\left(847\,e^{(-2\,dx-2\,c)}-5445\,e^{(-4\,dx-4\,c)}+22869\,e^{(-6\,dx-6\,c)}-76230\,e^{(-8\,dx-8\,c)}+320166\,e^{(-10\,dx-10\,c)}-63\right)e^{(11\,dx+11}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/1419264*b^3*((847*e^(-2*d*x - 2*c) - 5445*e^(-4*d*x - 4*c) + 22869*e^(-6
*d*x - 6*c) - 76230*e^(-8*d*x - 8*c) + 320166*e^(-10*d*x - 10*c) - 63)*e^(1
1*d*x + 11*c)/d + (320166*e^(-d*x - c) - 76230*e^(-3*d*x - 3*c) + 22869*e^(
-5*d*x - 5*c) - 5445*e^(-7*d*x - 7*c) + 847*e^(-9*d*x - 9*c) - 63*e^(-11*d*
x - 11*c))/d) - 3/4480*a*b^2*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) +
 1225*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^
(-3*d*x - 3*c) + 49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/8*a^2*b*(
e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d
) + a^3*log(tanh(1/2*d*x + 1/2*c))/d
```

Fricas [B] time = 2.04014, size = 10637, normalized size = 67.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/7096320*(315*b^3*cosh(d*x + c)^22 + 6930*b^3*cosh(d*x + c)*sinh(d*x + c)^
21 + 315*b^3*sinh(d*x + c)^22 - 4235*b^3*cosh(d*x + c)^20 + 385*(189*b^3*co
sh(d*x + c)^2 - 11*b^3)*sinh(d*x + c)^20 + 7700*(63*b^3*cosh(d*x + c)^3 - 1
1*b^3*cosh(d*x + c))*sinh(d*x + c)^19 + 495*(48*a*b^2 + 55*b^3)*cosh(d*x +
c)^18 + 55*(41895*b^3*cosh(d*x + c)^4 - 14630*b^3*cosh(d*x + c)^2 + 432*a*b
^2 + 495*b^3)*sinh(d*x + c)^18 + 330*(25137*b^3*cosh(d*x + c)^5 - 14630*b^3
*cosh(d*x + c)^3 + 27*(48*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 -
 2079*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^16 + 33*(712215*b^3*cosh(d*x + c)^
6 - 621775*b^3*cosh(d*x + c)^4 - 7056*a*b^2 - 3465*b^3 + 2295*(48*a*b^2 + 5
5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 528*(101745*b^3*cosh(d*x + c)^7
- 124355*b^3*cosh(d*x + c)^5 + 765*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^3 - 63
*(112*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 6930*(128*a^2*b + 1
68*a*b^2 + 55*b^3)*cosh(d*x + c)^14 + 330*(305235*b^3*cosh(d*x + c)^8 - 497
420*b^3*cosh(d*x + c)^6 + 4590*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^4 + 2688*a
^2*b + 3528*a*b^2 + 1155*b^3 - 756*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^14 + 4620*(33915*b^3*cosh(d*x + c)^9 - 71060*b^3*cosh(d*x + c)^
7 + 918*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^5 - 252*(112*a*b^2 + 55*b^3)*cosh
(d*x + c)^3 + 21*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^13 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^12 + 2310*(88
179*b^3*cosh(d*x + c)^10 - 230945*b^3*cosh(d*x + c)^8 + 3978*(48*a*b^2 + 55
*b^3)*cosh(d*x + c)^6 - 1638*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^4 - 3456*a^
2*b - 2520*a*b^2 - 693*b^3 + 273*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^12 + 8*(27776385*b^3*cosh(d*x + c)^11 - 88913825*b^3*
cosh(d*x + c)^9 + 1969110*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 1135134*(11
2*a*b^2 + 55*b^3)*cosh(d*x + c)^5 + 315315*(128*a^2*b + 168*a*b^2 + 55*b^3)
*cosh(d*x + c)^3 - 31185*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*si
nh(d*x + c)^11 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^10 +
22*(9258795*b^3*cosh(d*x + c)^12 - 35565530*b^3*cosh(d*x + c)^10 + 984555*(
48*a*b^2 + 55*b^3)*cosh(d*x + c)^8 - 756756*(112*a*b^2 + 55*b^3)*cosh(d*x +
 c)^6 + 315315*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^4 - 362880*a^
2*b - 264600*a*b^2 - 72765*b^3 - 62370*(384*a^2*b + 280*a*b^2 + 77*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^10 + 220*(712215*b^3*cosh(d*x + c)^13 - 3233230
*b^3*cosh(d*x + c)^11 + 109395*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^9 - 108108
*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^7 + 63063*(128*a^2*b + 168*a*b^2 + 55*b
^3)*cosh(d*x + c)^5 - 20790*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^
3 - 945*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 6
930*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^8 + 330*(305235*b^3*cosh
(d*x + c)^14 - 1616615*b^3*cosh(d*x + c)^12 + 65637*(48*a*b^2 + 55*b^3)*cos
h(d*x + c)^10 - 81081*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^8 + 63063*(128*a^2
```

```
*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^6 - 31185*(384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^4 + 2688*a^2*b + 3528*a*b^2 + 1155*b^3 - 2835*(384*a^2
*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 2640*(20349*b^3
*cosh(d*x + c)^15 - 124355*b^3*cosh(d*x + c)^13 + 5967*(48*a*b^2 + 55*b^3)*
cosh(d*x + c)^11 - 9009*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^9 + 9009*(128*a^
2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 6237*(384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^5 - 945*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)
^3 + 21*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 2
079*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^6 + 231*(101745*b^3*cosh(d*x + c)^16
 - 710600*b^3*cosh(d*x + c)^14 + 39780*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^12
 - 72072*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^10 + 90090*(128*a^2*b + 168*a*b
^2 + 55*b^3)*cosh(d*x + c)^8 - 83160*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(
d*x + c)^6 - 18900*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^4 - 1008*
a*b^2 - 495*b^3 + 840*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^6 + 462*(17955*b^3*cosh(d*x + c)^17 - 142120*b^3*cosh(d*x + c)^1
5 + 9180*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^13 - 19656*(112*a*b^2 + 55*b^3)*
cosh(d*x + c)^11 + 30030*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^9 -
 35640*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^7 - 11340*(384*a^2*b
+ 280*a*b^2 + 77*b^3)*cosh(d*x + c)^5 + 840*(128*a^2*b + 168*a*b^2 + 55*b^3
)*cosh(d*x + c)^3 - 27*(112*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^5
- 4235*b^3*cosh(d*x + c)^2 + 495*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^4 + 165*
(13965*b^3*cosh(d*x + c)^18 - 124355*b^3*cosh(d*x + c)^16 + 9180*(48*a*b^2
+ 55*b^3)*cosh(d*x + c)^14 - 22932*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^12 +
42042*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^10 - 62370*(384*a^2*b
+ 280*a*b^2 + 77*b^3)*cosh(d*x + c)^8 - 26460*(384*a^2*b + 280*a*b^2 + 77*b
^3)*cosh(d*x + c)^6 + 2940*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^4
 + 144*a*b^2 + 165*b^3 - 189*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 660*(735*b^3*cosh(d*x + c)^19 - 7315*b^3*cosh(d*x + c)^17 + 612*(
48*a*b^2 + 55*b^3)*cosh(d*x + c)^15 - 1764*(112*a*b^2 + 55*b^3)*cosh(d*x +
c)^13 + 3822*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^11 - 6930*(384*
a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^9 - 3780*(384*a^2*b + 280*a*b^2 +
 77*b^3)*cosh(d*x + c)^7 + 588*(128*a^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x +
c)^5 - 63*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^3 + 3*(48*a*b^2 + 55*b^3)*cosh
(d*x + c))*sinh(d*x + c)^3 + 315*b^3 + 55*(1323*b^3*cosh(d*x + c)^20 - 1463
0*b^3*cosh(d*x + c)^18 + 1377*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^16 - 4536*(
112*a*b^2 + 55*b^3)*cosh(d*x + c)^14 + 11466*(128*a^2*b + 168*a*b^2 + 55*b^
3)*cosh(d*x + c)^12 - 24948*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^
10 - 17010*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^8 + 3528*(128*a^2
*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^6 - 567*(112*a*b^2 + 55*b^3)*cosh(d*
x + c)^4 - 77*b^3 + 54*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
 - 7096320*(a^3*cosh(d*x + c)^11 + 11*a^3*cosh(d*x + c)^10*sinh(d*x + c) +
55*a^3*cosh(d*x + c)^9*sinh(d*x + c)^2 + 165*a^3*cosh(d*x + c)^8*sinh(d*x +
 c)^3 + 330*a^3*cosh(d*x + c)^7*sinh(d*x + c)^4 + 462*a^3*cosh(d*x + c)^6*s
inh(d*x + c)^5 + 462*a^3*cosh(d*x + c)^5*sinh(d*x + c)^6 + 330*a^3*cosh(d*x
 + c)^4*sinh(d*x + c)^7 + 165*a^3*cosh(d*x + c)^3*sinh(d*x + c)^8 + 55*a^3*
cosh(d*x + c)^2*sinh(d*x + c)^9 + 11*a^3*cosh(d*x + c)*sinh(d*x + c)^10 + a
^3*sinh(d*x + c)^11)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 7096320*(a^3*
cosh(d*x + c)^11 + 11*a^3*cosh(d*x + c)^10*sinh(d*x + c) + 55*a^3*cosh(d*x
+ c)^9*sinh(d*x + c)^2 + 165*a^3*cosh(d*x + c)^8*sinh(d*x + c)^3 + 330*a^3*
cosh(d*x + c)^7*sinh(d*x + c)^4 + 462*a^3*cosh(d*x + c)^6*sinh(d*x + c)^5 +
 462*a^3*cosh(d*x + c)^5*sinh(d*x + c)^6 + 330*a^3*cosh(d*x + c)^4*sinh(d*x
 + c)^7 + 165*a^3*cosh(d*x + c)^3*sinh(d*x + c)^8 + 55*a^3*cosh(d*x + c)^2*
sinh(d*x + c)^9 + 11*a^3*cosh(d*x + c)*sinh(d*x + c)^10 + a^3*sinh(d*x + c)
^11)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 22*(315*b^3*cosh(d*x + c)^21
- 3850*b^3*cosh(d*x + c)^19 + 405*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^17 - 15
12*(112*a*b^2 + 55*b^3)*cosh(d*x + c)^15 + 4410*(128*a^2*b + 168*a*b^2 + 55
*b^3)*cosh(d*x + c)^13 - 11340*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x +
c)^11 - 9450*(384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^9 + 2520*(128*a
^2*b + 168*a*b^2 + 55*b^3)*cosh(d*x + c)^7 - 567*(112*a*b^2 + 55*b^3)*cosh(
```

```
d*x + c)^5 - 385*b^3*cosh(d*x + c) + 90*(48*a*b^2 + 55*b^3)*cosh(d*x + c)^3
)*sinh(d*x + c))/(d*cosh(d*x + c)^11 + 11*d*cosh(d*x + c)^10*sinh(d*x + c)
+ 55*d*cosh(d*x + c)^9*sinh(d*x + c)^2 + 165*d*cosh(d*x + c)^8*sinh(d*x + c
)^3 + 330*d*cosh(d*x + c)^7*sinh(d*x + c)^4 + 462*d*cosh(d*x + c)^6*sinh(d*
x + c)^5 + 462*d*cosh(d*x + c)^5*sinh(d*x + c)^6 + 330*d*cosh(d*x + c)^4*si
nh(d*x + c)^7 + 165*d*cosh(d*x + c)^3*sinh(d*x + c)^8 + 55*d*cosh(d*x + c)^
2*sinh(d*x + c)^9 + 11*d*cosh(d*x + c)*sinh(d*x + c)^10 + d*sinh(d*x + c)^1
1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out
```

Giac [B] time = 1.65024, size = 570, normalized size = 3.61

$$-\frac{a^3 \log\left(e^{(dx+c)}+1\right)}{d}+\frac{a^3 \log\left(\left|e^{(dx+c)}-1\right|\right)}{d}-\frac{\left(7983360\,a^2be^{(10\,dx+10\,c)}+5821200\,ab^2e^{(10\,dx+10\,c)}+1600830\,b^3e^{(10\,dx+10\,c)}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] -a^3*log(e^(d*x + c) + 1)/d + a^3*log(abs(e^(d*x + c) - 1))/d - 1/7096320*(
7983360*a^2*b*e^(10*d*x + 10*c) + 5821200*a*b^2*e^(10*d*x + 10*c) + 1600830
*b^3*e^(10*d*x + 10*c) - 887040*a^2*b*e^(8*d*x + 8*c) - 1164240*a*b^2*e^(8*
d*x + 8*c) - 381150*b^3*e^(8*d*x + 8*c) + 232848*a*b^2*e^(6*d*x + 6*c) + 11
4345*b^3*e^(6*d*x + 6*c) - 23760*a*b^2*e^(4*d*x + 4*c) - 27225*b^3*e^(4*d*x
 + 4*c) + 4235*b^3*e^(2*d*x + 2*c) - 315*b^3)*e^(-11*d*x - 11*c)/d + 1/7096
320*(315*b^3*d^10*e^(11*d*x + 11*c) - 4235*b^3*d^10*e^(9*d*x + 9*c) + 23760
*a*b^2*d^10*e^(7*d*x + 7*c) + 27225*b^3*d^10*e^(7*d*x + 7*c) - 232848*a*b^2
*d^10*e^(5*d*x + 5*c) - 114345*b^3*d^10*e^(5*d*x + 5*c) + 887040*a^2*b*d^10
*e^(3*d*x + 3*c) + 1164240*a*b^2*d^10*e^(3*d*x + 3*c) + 381150*b^3*d^10*e^(
3*d*x + 3*c) - 7983360*a^2*b*d^10*e^(d*x + c) - 5821200*a*b^2*d^10*e^(d*x +
 c) - 1600830*b^3*d^10*e^(d*x + c))/d^11
```

3.211 $\int \operatorname{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=148

$$\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{3b^2(a+2b)\cosh^5(c+dx)}{5d}$$

```
[Out] (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]
)/d - (2*b^2*(3*a + 2*b)*Cosh[c + d*x]^3)/(3*d) + (3*b^2*(a + 2*b)*Cosh[c +
 d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d
) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rubi [A] time = 0.209365, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3215, 1157, 1810, 206}

$$\frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d}+\frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d}-\frac{a^3\coth(c+dx)\operatorname{csch}(c+dx)}{2d}+\frac{3b^2(a+2b)\cosh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (a^3*ArcTanh[Cosh[c + d*x]])/(2*d) + (b*(3*a^2 + 3*a*b + b^2)*Cosh[c + d*x]
)/d - (2*b^2*(3*a + 2*b)*Cosh[c + d*x]^3)/(3*d) + (3*b^2*(a + 2*b)*Cosh[c +
 d*x]^5)/(5*d) - (4*b^3*Cosh[c + d*x]^7)/(7*d) + (b^3*Cosh[c + d*x]^9)/(9*d
) - (a^3*Coth[c + d*x]*Csch[c + d*x])/(2*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \mathrm{csch}^3(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\mathrm{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-a^3-6a^2b-6ab^2-2b^3+2b\left(3a^2+9ab+5b \right.}{}\right.}{}$$

$$= -\frac{a^3\coth(c+dx)\mathrm{csch}(c+dx)}{2d} - \frac{\text{Subst}\left(\int \left(-2b\left(3a^2+3ab+b^2\right)+4b^2\right.\right.}{}$$

$$= \frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d} - \frac{2b^2(3a+2b)\cosh^3(c+dx)}{3d} + \frac{3b^2(a}{}$$

$$= \frac{a^3\tanh^{-1}(\cosh(c+dx))}{2d} + \frac{b\left(3a^2+3ab+b^2\right)\cosh(c+dx)}{d} - \frac{2b^2(3a+}{}$$

Mathematica [A] time = 0.340198, size = 155, normalized size = 1.05

$$\frac{1890b\left(128a^2+80ab+21b^2\right)\cosh(c+dx) - 10080a^3\mathrm{csch}^2\left(\frac{1}{2}(c+dx)\right) - 10080a^3\mathrm{sech}^2\left(\frac{1}{2}(c+dx)\right) - 40320a^3\log\left(t\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (1890*b*(128*a^2 + 80*a*b + 21*b^2)*Cosh[c + d*x] - 1260*b^2*(20*a + 7*b)*C
osh[3*(c + d*x)] + 3024*a*b^2*Cosh[5*(c + d*x)] + 2268*b^3*Cosh[5*(c + d*x)
] - 405*b^3*Cosh[7*(c + d*x)] + 35*b^3*Cosh[9*(c + d*x)] - 10080*a^3*Csch[(
c + d*x)/2]^2 - 40320*a^3*Log[Tanh[(c + d*x)/2]] - 10080*a^3*Sech[(c + d*x)
/2]^2)/(80640*d)
```

Maple [A] time = 0.049, size = 130, normalized size = 0.9

$$\frac{1}{d}\left(a^3\left(-\frac{\mathrm{csch}\,(dx+c)\coth\,(dx+c)}{2} + \mathrm{Artanh}\left(e^{dx+c}\right)\right) + 3\,a^2b\cosh\,(dx+c) + 3\,ab^2\left(\frac{8}{15} + 1/5\,(\sinh\,(dx+c))^4 - \frac{4\,(s}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(exp(d*x+c)))+3*a^2*b*cosh(d*
x+c)+3*a*b^2*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+c)^2)*cosh(d*x+c)+b^3*(1
28/315+1/9*sinh(d*x+c)^8-8/63*sinh(d*x+c)^6+16/105*sinh(d*x+c)^4-64/315*sin
h(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.07909, size = 451, normalized size = 3.05

$$-\frac{1}{161280}b^3\left(\frac{\left(405\,e^{(-2\,dx-2\,c)} - 2268\,e^{(-4\,dx-4\,c)} + 8820\,e^{(-6\,dx-6\,c)} - 39690\,e^{(-8\,dx-8\,c)} - 35\right)e^{(9\,dx+9\,c)}}{d} - \frac{39690\,e^{(-dx-c)} - 88}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/161280*b^3*((405*e^(-2*d*x - 2*c) - 2268*e^(-4*d*x - 4*c) + 8820*e^(-6*d
*x - 6*c) - 39690*e^(-8*d*x - 8*c) - 35)*e^(9*d*x + 9*c)/d - (39690*e^(-d*x
 - c) - 8820*e^(-3*d*x - 3*c) + 2268*e^(-5*d*x - 5*c) - 405*e^(-7*d*x - 7*c
) + 35*e^(-9*d*x - 9*c))/d) + 1/160*a*b^2*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*
x + 3*c)/d + 150*e^(d*x + c)/d + 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d
 + 3*e^(-5*d*x - 5*c)/d) + 3/2*a^2*b*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/2
*a^3*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) +
 e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [B] time = 2.12155, size = 13697, normalized size = 92.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/161280*(35*b^3*cosh(d*x + c)^22 + 770*b^3*cosh(d*x + c)*sinh(d*x + c)^21
+ 35*b^3*sinh(d*x + c)^22 - 475*b^3*cosh(d*x + c)^20 + 5*(1617*b^3*cosh(d*x
 + c)^2 - 95*b^3)*sinh(d*x + c)^20 + 100*(539*b^3*cosh(d*x + c)^3 - 95*b^3*
cosh(d*x + c))*sinh(d*x + c)^19 + (3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^18
+ (256025*b^3*cosh(d*x + c)^4 - 90250*b^3*cosh(d*x + c)^2 + 3024*a*b^2 + 31
13*b^3)*sinh(d*x + c)^18 + 6*(153615*b^3*cosh(d*x + c)^5 - 90250*b^3*cosh(d
*x + c)^3 + 3*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 9*(
3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^16 + 3*(870485*b^3*cosh(d*x + c)^6 - 7
67125*b^3*cosh(d*x + c)^4 - 10416*a*b^2 - 4587*b^3 + 51*(3024*a*b^2 + 3113*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 48*(124355*b^3*cosh(d*x + c)^7 - 1
53425*b^3*cosh(d*x + c)^5 + 17*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^3 - 3*
(3472*a*b^2 + 1529*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 126*(1920*a^2*b +
 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^14 + 6*(1865325*b^3*cosh(d*x + c)^8 -
3068500*b^3*cosh(d*x + c)^6 + 510*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^4 +
 40320*a^2*b + 34104*a*b^2 + 9933*b^3 - 180*(3472*a*b^2 + 1529*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^14 + 4*(4352425*b^3*cosh(d*x + c)^9 - 9205500*b^3*c
osh(d*x + c)^7 + 2142*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^5 - 1260*(3472*
a*b^2 + 1529*b^3)*cosh(d*x + c)^3 + 441*(1920*a^2*b + 1624*a*b^2 + 473*b^3)
*cosh(d*x + c))*sinh(d*x + c)^13 - 630*(256*a^3 + 384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^12 + 2*(11316305*b^3*cosh(d*x + c)^10 - 29917875*b^3*c
osh(d*x + c)^8 + 9282*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^6 - 8190*(3472*
a*b^2 + 1529*b^3)*cosh(d*x + c)^4 - 80640*a^3 - 120960*a^2*b - 88200*a*b^2
- 24255*b^3 + 5733*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^2)*sin
h(d*x + c)^12 + 8*(3086265*b^3*cosh(d*x + c)^11 - 9972625*b^3*cosh(d*x + c)
^9 + 3978*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^7 - 4914*(3472*a*b^2 + 1529
*b^3)*cosh(d*x + c)^5 + 5733*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x +
 c)^3 - 945*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(
d*x + c)^11 - 630*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^
10 + 2*(11316305*b^3*cosh(d*x + c)^12 - 43879550*b^3*cosh(d*x + c)^10 + 218
79*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^8 - 36036*(3472*a*b^2 + 1529*b^3)*
cosh(d*x + c)^6 + 63063*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^4
 - 80640*a^3 - 120960*a^2*b - 88200*a*b^2 - 24255*b^3 - 20790*(256*a^3 + 38
4*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(435242
5*b^3*cosh(d*x + c)^13 - 19945250*b^3*cosh(d*x + c)^11 + 12155*(3024*a*b^2
+ 3113*b^3)*cosh(d*x + c)^9 - 25740*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^7
 + 63063*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^5 - 34650*(256*a
```

^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^3 - 1575*(256*a^3 + 384*
a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 126*(1920*a^2*
b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^8 + 6*(1865325*b^3*cosh(d*x + c)^14
 - 9972625*b^3*cosh(d*x + c)^12 + 7293*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c
)^10 - 19305*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^8 + 63063*(1920*a^2*b +
1624*a*b^2 + 473*b^3)*cosh(d*x + c)^6 - 51975*(256*a^3 + 384*a^2*b + 280*a*
b^2 + 77*b^3)*cosh(d*x + c)^4 + 40320*a^2*b + 34104*a*b^2 + 9933*b^3 - 4725
*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^
8 + 48*(124355*b^3*cosh(d*x + c)^15 - 767125*b^3*cosh(d*x + c)^13 + 663*(30
24*a*b^2 + 3113*b^3)*cosh(d*x + c)^11 - 2145*(3472*a*b^2 + 1529*b^3)*cosh(d
x + c)^9 + 9009(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^7 - 1039
5*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^5 - 1575*(256*a^
3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^3 + 21*(1920*a^2*b + 1624
*a*b^2 + 473*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 9*(3472*a*b^2 + 1529*b^3
)*cosh(d*x + c)^6 + 3*(870485*b^3*cosh(d*x + c)^16 - 6137000*b^3*cosh(d*x +
 c)^14 + 6188*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^12 - 24024*(3472*a*b^2
+ 1529*b^3)*cosh(d*x + c)^10 + 126126*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*c
osh(d*x + c)^8 - 194040*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x
 + c)^6 - 44100*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^4
- 10416*a*b^2 - 4587*b^3 + 1176*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^6 + 6*(153615*b^3*cosh(d*x + c)^17 - 1227400*b^3*co
sh(d*x + c)^15 + 1428*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^13 - 6552*(3472
*a*b^2 + 1529*b^3)*cosh(d*x + c)^11 + 42042*(1920*a^2*b + 1624*a*b^2 + 473*
b^3)*cosh(d*x + c)^9 - 83160*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cos
h(d*x + c)^7 - 26460*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x +
c)^5 + 1176*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^3 - 9*(3472*a
*b^2 + 1529*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 475*b^3*cosh(d*x + c)^2 +
 (3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^4 + (256025*b^3*cosh(d*x + c)^18 - 2
301375*b^3*cosh(d*x + c)^16 + 3060*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^14
 - 16380*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^12 + 126126*(1920*a^2*b + 16
24*a*b^2 + 473*b^3)*cosh(d*x + c)^10 - 311850*(256*a^3 + 384*a^2*b + 280*a*
b^2 + 77*b^3)*cosh(d*x + c)^8 - 132300*(256*a^3 + 384*a^2*b + 280*a*b^2 + 7
7*b^3)*cosh(d*x + c)^6 + 8820*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x
+ c)^4 + 3024*a*b^2 + 3113*b^3 - 135*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^4 + 4*(13475*b^3*cosh(d*x + c)^19 - 135375*b^3*cosh(d*x +
c)^17 + 204*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^15 - 1260*(3472*a*b^2 + 1
529*b^3)*cosh(d*x + c)^13 + 11466*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(
d*x + c)^11 - 34650*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c
)^9 - 18900*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^7 + 17
64*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^5 - 45*(3472*a*b^2 + 1
529*b^3)*cosh(d*x + c)^3 + (3024*a*b^2 + 3113*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 35*b^3 + (8085*b^3*cosh(d*x + c)^20 - 90250*b^3*cosh(d*x + c)^18 +
 153*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^16 - 1080*(3472*a*b^2 + 1529*b^3
)*cosh(d*x + c)^14 + 11466*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c
)^12 - 41580*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^10 -
28350*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh(d*x + c)^8 + 3528*(19
20*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^6 - 135*(3472*a*b^2 + 1529*b
^3)*cosh(d*x + c)^4 - 475*b^3 + 6*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 80640*(a^3*cosh(d*x + c)^13 + 13*a^3*cosh(d*x + c)*sinh(d
*x + c)^12 + a^3*sinh(d*x + c)^13 - 2*a^3*cosh(d*x + c)^11 + a^3*cosh(d*x +
 c)^9 + 2*(39*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^11 + 22*(13*a^3*cosh
(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c)^10 + (715*a^3*cosh(d*x + c)^
4 - 110*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^9 + 3*(429*a^3*cosh(d*x +
c)^5 - 110*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d*x + c)^8 + 12*
(143*a^3*cosh(d*x + c)^6 - 55*a^3*cosh(d*x + c)^4 + 3*a^3*cosh(d*x + c)^2)*
sinh(d*x + c)^7 + 12*(143*a^3*cosh(d*x + c)^7 - 77*a^3*cosh(d*x + c)^5 + 7*
a^3*cosh(d*x + c)^3)*sinh(d*x + c)^6 + 3*(429*a^3*cosh(d*x + c)^8 - 308*a^3
*cosh(d*x + c)^6 + 42*a^3*cosh(d*x + c)^4)*sinh(d*x + c)^5 + (715*a^3*cosh(
d*x + c)^9 - 660*a^3*cosh(d*x + c)^7 + 126*a^3*cosh(d*x + c)^5)*sinh(d*x +

```
c)^4 + 2*(143*a^3*cosh(d*x + c)^10 - 165*a^3*cosh(d*x + c)^8 + 42*a^3*cosh(
d*x + c)^6)*sinh(d*x + c)^3 + 2*(39*a^3*cosh(d*x + c)^11 - 55*a^3*cosh(d*x
+ c)^9 + 18*a^3*cosh(d*x + c)^7)*sinh(d*x + c)^2 + (13*a^3*cosh(d*x + c)^12
 - 22*a^3*cosh(d*x + c)^10 + 9*a^3*cosh(d*x + c)^8)*sinh(d*x + c))*log(cosh
(d*x + c) + sinh(d*x + c) + 1) - 80640*(a^3*cosh(d*x + c)^13 + 13*a^3*cosh(
d*x + c)*sinh(d*x + c)^12 + a^3*sinh(d*x + c)^13 - 2*a^3*cosh(d*x + c)^11 +
 a^3*cosh(d*x + c)^9 + 2*(39*a^3*cosh(d*x + c)^2 - a^3)*sinh(d*x + c)^11 +
22*(13*a^3*cosh(d*x + c)^3 - a^3*cosh(d*x + c))*sinh(d*x + c)^10 + (715*a^3
*cosh(d*x + c)^4 - 110*a^3*cosh(d*x + c)^2 + a^3)*sinh(d*x + c)^9 + 3*(429*
a^3*cosh(d*x + c)^5 - 110*a^3*cosh(d*x + c)^3 + 3*a^3*cosh(d*x + c))*sinh(d
*x + c)^8 + 12*(143*a^3*cosh(d*x + c)^6 - 55*a^3*cosh(d*x + c)^4 + 3*a^3*co
sh(d*x + c)^2)*sinh(d*x + c)^7 + 12*(143*a^3*cosh(d*x + c)^7 - 77*a^3*cosh(
d*x + c)^5 + 7*a^3*cosh(d*x + c)^3)*sinh(d*x + c)^6 + 3*(429*a^3*cosh(d*x +
 c)^8 - 308*a^3*cosh(d*x + c)^6 + 42*a^3*cosh(d*x + c)^4)*sinh(d*x + c)^5 +
 (715*a^3*cosh(d*x + c)^9 - 660*a^3*cosh(d*x + c)^7 + 126*a^3*cosh(d*x + c)
^5)*sinh(d*x + c)^4 + 2*(143*a^3*cosh(d*x + c)^10 - 165*a^3*cosh(d*x + c)^8
 + 42*a^3*cosh(d*x + c)^6)*sinh(d*x + c)^3 + 2*(39*a^3*cosh(d*x + c)^11 - 5
5*a^3*cosh(d*x + c)^9 + 18*a^3*cosh(d*x + c)^7)*sinh(d*x + c)^2 + (13*a^3*c
osh(d*x + c)^12 - 22*a^3*cosh(d*x + c)^10 + 9*a^3*cosh(d*x + c)^8)*sinh(d*x
 + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(385*b^3*cosh(d*x + c)^21
 - 4750*b^3*cosh(d*x + c)^19 + 9*(3024*a*b^2 + 3113*b^3)*cosh(d*x + c)^17 -
 72*(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^15 + 882*(1920*a^2*b + 1624*a*b^2
 + 473*b^3)*cosh(d*x + c)^13 - 3780*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b
^3)*cosh(d*x + c)^11 - 3150*(256*a^3 + 384*a^2*b + 280*a*b^2 + 77*b^3)*cosh
(d*x + c)^9 + 504*(1920*a^2*b + 1624*a*b^2 + 473*b^3)*cosh(d*x + c)^7 - 27*
(3472*a*b^2 + 1529*b^3)*cosh(d*x + c)^5 - 475*b^3*cosh(d*x + c) + 2*(3024*a
*b^2 + 3113*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c)^13 + 13*d
*cosh(d*x + c)*sinh(d*x + c)^12 + d*sinh(d*x + c)^13 - 2*d*cosh(d*x + c)^11
 + 2*(39*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^11 + 22*(13*d*cosh(d*x + c)^3
 - d*cosh(d*x + c))*sinh(d*x + c)^10 + d*cosh(d*x + c)^9 + (715*d*cosh(d*x
+ c)^4 - 110*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^9 + 3*(429*d*cosh(d*x + c
)^5 - 110*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^8 + 12*(143*
d*cosh(d*x + c)^6 - 55*d*cosh(d*x + c)^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x +
c)^7 + 12*(143*d*cosh(d*x + c)^7 - 77*d*cosh(d*x + c)^5 + 7*d*cosh(d*x + c)
^3)*sinh(d*x + c)^6 + 3*(429*d*cosh(d*x + c)^8 - 308*d*cosh(d*x + c)^6 + 42
*d*cosh(d*x + c)^4)*sinh(d*x + c)^5 + (715*d*cosh(d*x + c)^9 - 660*d*cosh(d
*x + c)^7 + 126*d*cosh(d*x + c)^5)*sinh(d*x + c)^4 + 2*(143*d*cosh(d*x + c)
^10 - 165*d*cosh(d*x + c)^8 + 42*d*cosh(d*x + c)^6)*sinh(d*x + c)^3 + 2*(39
*d*cosh(d*x + c)^11 - 55*d*cosh(d*x + c)^9 + 18*d*cosh(d*x + c)^7)*sinh(d*x
 + c)^2 + (13*d*cosh(d*x + c)^12 - 22*d*cosh(d*x + c)^10 + 9*d*cosh(d*x + c
)^8)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.66912, size = 455, normalized size = 3.07

$$\frac{a^3 \log \left(e^{(dx+c)} + e^{(-dx-c)} + 2\right)}{4\,d} - \frac{a^3 \log \left(e^{(dx+c)} + e^{(-dx-c)} - 2\right)}{4\,d} - \frac{a^3\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)} + e^{(-dx-c)}\right)^2 - 4\right)d} + \frac{35\,b^3d^8\left(e^{(dx+c)} + e^{(-dx-c)}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/4*a^3*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/4*a^3*log(e^(d*x + c) + e
^(-d*x - c) - 2)/d - a^3*(e^(d*x + c) + e^(-d*x - c))/(((e^(d*x + c) + e^(-
d*x - c))^2 - 4)*d) + 1/161280*(35*b^3*d^8*(e^(d*x + c) + e^(-d*x - c))^9 -
 720*b^3*d^8*(e^(d*x + c) + e^(-d*x - c))^7 + 3024*a*b^2*d^8*(e^(d*x + c) +
 e^(-d*x - c))^5 + 6048*b^3*d^8*(e^(d*x + c) + e^(-d*x - c))^5 - 40320*a*b^
2*d^8*(e^(d*x + c) + e^(-d*x - c))^3 - 26880*b^3*d^8*(e^(d*x + c) + e^(-d*x
 - c))^3 + 241920*a^2*b*d^8*(e^(d*x + c) + e^(-d*x - c)) + 241920*a*b^2*d^8
*(e^(d*x + c) + e^(-d*x - c)) + 80640*b^3*d^8*(e^(d*x + c) + e^(-d*x - c)))
/d^9
```

3.212 $\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=142

$$-\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}+\frac{b^2(a+b)\cosh^3(\cdots}{d}$$

```
[Out] (-3*a^2*(a + 8*b)*ArcTanh[Cosh[c + d*x]])/(8*d) - (b^2*(3*a + b)*Cosh[c + d
*x])/d + (b^2*(a + b)*Cosh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) +
(b^3*Cosh[c + d*x]^7)/(7*d) + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (
a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rubi [A] time = 0.272746, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 1810, 206}

$$-\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d}-\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d}+\frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d}+\frac{b^2(a+b)\cosh^3(\cdots}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*a^2*(a + 8*b)*ArcTanh[Cosh[c + d*x]])/(8*d) - (b^2*(3*a + b)*Cosh[c + d
*x])/d + (b^2*(a + b)*Cosh[c + d*x]^3)/d - (3*b^3*Cosh[c + d*x]^5)/(5*d) +
(b^3*Cosh[c + d*x]^7)/(7*d) + (3*a^3*Coth[c + d*x]*Csch[c + d*x])/(8*d) - (
a^3*Coth[c + d*x]*Csch[c + d*x]^3)/(4*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^5(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{-3a^3-12a^2b-12ab^2-4b^3+4b\left(3a^2+9a\right.}{}\right.}{}$$

$$= \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{\operatorname{Subst}}{}$$

$$= \frac{3a^3\coth(c+dx)\operatorname{csch}(c+dx)}{8d} - \frac{a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{4d} - \frac{\operatorname{Subst}}{}$$

$$= -\frac{b^2(3a+b)\cosh(c+dx)}{d} + \frac{b^2(a+b)\cosh^3(c+dx)}{d} - \frac{3b^3\cosh^5(c+dx}{5d}$$

$$= -\frac{3a^2(a+8b)\tanh^{-1}(\cosh(c+dx))}{8d} - \frac{b^2(3a+b)\cosh(c+dx)}{d} + \frac{b^2(a+}{}$$

Mathematica [A] time = 0.382854, size = 173, normalized size = 1.22

$$\frac{6720a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - 35a^3\operatorname{csch}^4\left(\frac{1}{2}(c+dx)\right) + 210a^3\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right) + 35a^3\operatorname{sech}^4\left(\frac{1}{2}(c+dx)\right) + 210a^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^5*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-35*b^2*(144*a + 35*b)*Cosh[c + d*x] + 35*b^2*(16*a + 7*b)*Cosh[3*(c + d*x
)] - 49*b^3*Cosh[5*(c + d*x)] + 5*b^3*Cosh[7*(c + d*x)] + 210*a^3*Csch[(c +
 d*x)/2]^2 - 35*a^3*Csch[(c + d*x)/2]^4 + 840*a^3*Log[Tanh[(c + d*x)/2]] +
6720*a^2*b*Log[Tanh[(c + d*x)/2]] + 210*a^3*Sech[(c + d*x)/2]^2 + 35*a^3*Se
ch[(c + d*x)/2]^4)/(2240*d)
```

Maple [A] time = 0.046, size = 125, normalized size = 0.9

$$\frac{1}{d}\left(a^3\left(\left(-\frac{(\operatorname{csch}(dx+c))^3}{4} + \frac{3\operatorname{csch}(dx+c)}{8}\right)\coth(dx+c) - \frac{3\operatorname{Artanh}\left(e^{dx+c}\right)}{4}\right) - 6a^2b\operatorname{Artanh}\left(e^{dx+c}\right) + 3ab^2\left(-2/3\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(
d*x+c)))-6*a^2*b*arctanh(exp(d*x+c))+3*a*b^2*(-2/3+1/3*sinh(d*x+c)^2)*cosh(
d*x+c)+b^3*(-16/35+1/7*sinh(d*x+c)^6-6/35*sinh(d*x+c)^4+8/35*sinh(d*x+c)^2)
*cosh(d*x+c))
```

Maxima [B] time = 1.05803, size = 459, normalized size = 3.23

$$-\frac{1}{4480}\,b^3\left(\frac{\left(49\,e^{(-2\,dx-2\,c)}-245\,e^{(-4\,dx-4\,c)}+1225\,e^{(-6\,dx-6\,c)}-5\right)e^{(7\,dx+7\,c)}}{d}+\frac{1225\,e^{(-dx-c)}-245\,e^{(-3\,dx-3\,c)}+49\,e^{(-5\,dx-5\,c)}-}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/4480*b^3*((49*e^(-2*d*x - 2*c) - 245*e^(-4*d*x - 4*c) + 1225*e^(-6*d*x -
 6*c) - 5)*e^(7*d*x + 7*c)/d + (1225*e^(-d*x - c) - 245*e^(-3*d*x - 3*c) +
49*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/d) + 1/8*a*b^2*(e^(3*d*x + 3*c)/d
 - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*x - 3*c)/d) - 1/8*a^3*(3*lo
g(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*
e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*
x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))
) - 3*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d)
```

Fricas [B] time = 2.4282, size = 17403, normalized size = 122.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/4480*(5*b^3*cosh(d*x + c)^22 + 110*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 5
*b^3*sinh(d*x + c)^22 - 69*b^3*cosh(d*x + c)^20 + 3*(385*b^3*cosh(d*x + c)^
2 - 23*b^3)*sinh(d*x + c)^20 + 20*(385*b^3*cosh(d*x + c)^3 - 69*b^3*cosh(d*
x + c))*sinh(d*x + c)^19 + (560*a*b^2 + 471*b^3)*cosh(d*x + c)^18 + (36575*
b^3*cosh(d*x + c)^4 - 13110*b^3*cosh(d*x + c)^2 + 560*a*b^2 + 471*b^3)*sinh
(d*x + c)^18 + 18*(7315*b^3*cosh(d*x + c)^5 - 4370*b^3*cosh(d*x + c)^3 + (5
60*a*b^2 + 471*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - (7280*a*b^2 + 2519*b^
3)*cosh(d*x + c)^16 + (373065*b^3*cosh(d*x + c)^6 - 334305*b^3*cosh(d*x + c
)^4 - 7280*a*b^2 - 2519*b^3 + 153*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^16 + 16*(53295*b^3*cosh(d*x + c)^7 - 66861*b^3*cosh(d*x + c)^5
+ 51*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^3 - (7280*a*b^2 + 2519*b^3)*cosh(d
*x + c))*sinh(d*x + c)^15 + 6*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c
)^14 + 6*(266475*b^3*cosh(d*x + c)^8 - 445740*b^3*cosh(d*x + c)^6 + 510*(56
0*a*b^2 + 471*b^3)*cosh(d*x + c)^4 + 560*a^3 + 3080*a*b^2 + 891*b^3 - 20*(7
280*a*b^2 + 2519*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14 + 4*(621775*b^3*cos
h(d*x + c)^9 - 1337220*b^3*cosh(d*x + c)^7 + 2142*(560*a*b^2 + 471*b^3)*cos
h(d*x + c)^5 - 140*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^3 + 21*(560*a^3 +
3080*a*b^2 + 891*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 14*(880*a^3 + 840*a
*b^2 + 231*b^3)*cosh(d*x + c)^12 + 2*(1616615*b^3*cosh(d*x + c)^10 - 434596
5*b^3*cosh(d*x + c)^8 + 9282*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^6 - 910*(7
280*a*b^2 + 2519*b^3)*cosh(d*x + c)^4 - 6160*a^3 - 5880*a*b^2 - 1617*b^3 +
273*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^12 + 24
*(146965*b^3*cosh(d*x + c)^11 - 482885*b^3*cosh(d*x + c)^9 + 1326*(560*a*b^
```

```
2 + 471*b^3)*cosh(d*x + c)^7 - 182*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^5
+ 91*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^3 - 7*(880*a^3 + 840*a*
b^2 + 231*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 - 14*(880*a^3 + 840*a*b^2 +
231*b^3)*cosh(d*x + c)^10 + 2*(1616615*b^3*cosh(d*x + c)^12 - 6374082*b^3*c
osh(d*x + c)^10 + 21879*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^8 - 4004*(7280*
a*b^2 + 2519*b^3)*cosh(d*x + c)^6 + 3003*(560*a^3 + 3080*a*b^2 + 891*b^3)*c
osh(d*x + c)^4 - 6160*a^3 - 5880*a*b^2 - 1617*b^3 - 462*(880*a^3 + 840*a*b^
2 + 231*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(621775*b^3*cosh(d*x + c
)^13 - 2897310*b^3*cosh(d*x + c)^11 + 12155*(560*a*b^2 + 471*b^3)*cosh(d*x
+ c)^9 - 2860*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^7 + 3003*(560*a^3 + 308
0*a*b^2 + 891*b^3)*cosh(d*x + c)^5 - 770*(880*a^3 + 840*a*b^2 + 231*b^3)*co
sh(d*x + c)^3 - 35*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^9 + 6*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^8 + 6*(266475*b^3
*cosh(d*x + c)^14 - 1448655*b^3*cosh(d*x + c)^12 + 7293*(560*a*b^2 + 471*b^
3)*cosh(d*x + c)^10 - 2145*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^8 + 3003*(
560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^6 - 1155*(880*a^3 + 840*a*b^2
 + 231*b^3)*cosh(d*x + c)^4 + 560*a^3 + 3080*a*b^2 + 891*b^3 - 105*(880*a^3
 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(53295*b^3*co
sh(d*x + c)^15 - 334305*b^3*cosh(d*x + c)^13 + 1989*(560*a*b^2 + 471*b^3)*c
osh(d*x + c)^11 - 715*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^9 + 1287*(560*a
^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^7 - 693*(880*a^3 + 840*a*b^2 + 231
*b^3)*cosh(d*x + c)^5 - 105*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^3
 + 3*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - (728
0*a*b^2 + 2519*b^3)*cosh(d*x + c)^6 + (373065*b^3*cosh(d*x + c)^16 - 267444
0*b^3*cosh(d*x + c)^14 + 18564*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^12 - 800
8*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^10 + 18018*(560*a^3 + 3080*a*b^2 +
891*b^3)*cosh(d*x + c)^8 - 12936*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x +
 c)^6 - 2940*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^4 - 7280*a*b^2 -
 2519*b^3 + 168*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 6*(21945*b^3*cosh(d*x + c)^17 - 178296*b^3*cosh(d*x + c)^15 + 1428
*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^13 - 728*(7280*a*b^2 + 2519*b^3)*cosh(
d*x + c)^11 + 2002*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^9 - 1848*
(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^7 - 588*(880*a^3 + 840*a*b^2
+ 231*b^3)*cosh(d*x + c)^5 + 56*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x +
 c)^3 - (7280*a*b^2 + 2519*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 69*b^3*cos
h(d*x + c)^2 + (560*a*b^2 + 471*b^3)*cosh(d*x + c)^4 + (36575*b^3*cosh(d*x
+ c)^18 - 334305*b^3*cosh(d*x + c)^16 + 3060*(560*a*b^2 + 471*b^3)*cosh(d*x
 + c)^14 - 1820*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^12 + 6006*(560*a^3 +
3080*a*b^2 + 891*b^3)*cosh(d*x + c)^10 - 6930*(880*a^3 + 840*a*b^2 + 231*b^
3)*cosh(d*x + c)^8 - 2940*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^6 +
 420*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^4 + 560*a*b^2 + 471*b^3
 - 15*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(1925*b^
3*cosh(d*x + c)^19 - 19665*b^3*cosh(d*x + c)^17 + 204*(560*a*b^2 + 471*b^3)
*cosh(d*x + c)^15 - 140*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^13 + 546*(560
*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^11 - 770*(880*a^3 + 840*a*b^2 +
231*b^3)*cosh(d*x + c)^9 - 420*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c
)^7 + 84*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^5 - 5*(7280*a*b^2 +
 2519*b^3)*cosh(d*x + c)^3 + (560*a*b^2 + 471*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^3 + 5*b^3 + 3*(385*b^3*cosh(d*x + c)^20 - 4370*b^3*cosh(d*x + c)^18 +
51*(560*a*b^2 + 471*b^3)*cosh(d*x + c)^16 - 40*(7280*a*b^2 + 2519*b^3)*cosh
(d*x + c)^14 + 182*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^12 - 308*
(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)^10 - 210*(880*a^3 + 840*a*b^2
 + 231*b^3)*cosh(d*x + c)^8 + 56*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x
+ c)^6 - 5*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^4 - 23*b^3 + 2*(560*a*b^2
+ 471*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 1680*((a^3 + 8*a^2*b)*cosh(d*
x + c)^15 + 15*(a^3 + 8*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^14 + (a^3 + 8*a^
2*b)*sinh(d*x + c)^15 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c)^13 - (4*a^3 + 32*a^
2*b - 105*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(35*(a^3 +
 8*a^2*b)*cosh(d*x + c)^3 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^
```

```
12 + 6*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 + 3*(455*(a^3 + 8*a^2*b)*cosh(d*x +
 c)^4 + 2*a^3 + 16*a^2*b - 104*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
c)^11 + 11*(273*(a^3 + 8*a^2*b)*cosh(d*x + c)^5 - 104*(a^3 + 8*a^2*b)*cosh(
d*x + c)^3 + 6*(a^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^10 - 4*(a^3 + 8
*a^2*b)*cosh(d*x + c)^9 + (5005*(a^3 + 8*a^2*b)*cosh(d*x + c)^6 - 2860*(a^3
 + 8*a^2*b)*cosh(d*x + c)^4 - 4*a^3 - 32*a^2*b + 330*(a^3 + 8*a^2*b)*cosh(d
*x + c)^2)*sinh(d*x + c)^9 + 9*(715*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 - 572*(
a^3 + 8*a^2*b)*cosh(d*x + c)^5 + 110*(a^3 + 8*a^2*b)*cosh(d*x + c)^3 - 4*(a
^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^8 + (a^3 + 8*a^2*b)*cosh(d*x + c
)^7 + (6435*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 - 6864*(a^3 + 8*a^2*b)*cosh(d*x
 + c)^6 + 1980*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 + a^3 + 8*a^2*b - 144*(a^3 +
 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (5005*(a^3 + 8*a^2*b)*cosh(d*x
 + c)^9 - 6864*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 + 2772*(a^3 + 8*a^2*b)*cosh(
d*x + c)^5 - 336*(a^3 + 8*a^2*b)*cosh(d*x + c)^3 + 7*(a^3 + 8*a^2*b)*cosh(d
*x + c))*sinh(d*x + c)^6 + 3*(1001*(a^3 + 8*a^2*b)*cosh(d*x + c)^10 - 1716*
(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 924*(a^3 + 8*a^2*b)*cosh(d*x + c)^6 - 168
*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 + 7*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(
d*x + c)^5 + (1365*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 - 2860*(a^3 + 8*a^2*b)*
cosh(d*x + c)^9 + 1980*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 - 504*(a^3 + 8*a^2*b
)*cosh(d*x + c)^5 + 35*(a^3 + 8*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c)^4 + (
455*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 - 1144*(a^3 + 8*a^2*b)*cosh(d*x + c)^1
0 + 990*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 - 336*(a^3 + 8*a^2*b)*cosh(d*x + c)
^6 + 35*(a^3 + 8*a^2*b)*cosh(d*x + c)^4)*sinh(d*x + c)^3 + 3*(35*(a^3 + 8*a
^2*b)*cosh(d*x + c)^13 - 104*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 + 110*(a^3 +
8*a^2*b)*cosh(d*x + c)^9 - 48*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 + 7*(a^3 + 8*
a^2*b)*cosh(d*x + c)^5)*sinh(d*x + c)^2 + (15*(a^3 + 8*a^2*b)*cosh(d*x + c)
^14 - 52*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 + 66*(a^3 + 8*a^2*b)*cosh(d*x + c
)^10 - 36*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 7*(a^3 + 8*a^2*b)*cosh(d*x + c)
^6)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 1680*((a^3 + 8*
a^2*b)*cosh(d*x + c)^15 + 15*(a^3 + 8*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^14
 + (a^3 + 8*a^2*b)*sinh(d*x + c)^15 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c)^13 -
(4*a^3 + 32*a^2*b - 105*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^13 +
 13*(35*(a^3 + 8*a^2*b)*cosh(d*x + c)^3 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c))*
sinh(d*x + c)^12 + 6*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 + 3*(455*(a^3 + 8*a^2
*b)*cosh(d*x + c)^4 + 2*a^3 + 16*a^2*b - 104*(a^3 + 8*a^2*b)*cosh(d*x + c)^
2)*sinh(d*x + c)^11 + 11*(273*(a^3 + 8*a^2*b)*cosh(d*x + c)^5 - 104*(a^3 +
8*a^2*b)*cosh(d*x + c)^3 + 6*(a^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^1
0 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c)^9 + (5005*(a^3 + 8*a^2*b)*cosh(d*x + c)
^6 - 2860*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 - 4*a^3 - 32*a^2*b + 330*(a^3 + 8
*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(715*(a^3 + 8*a^2*b)*cosh(d*x
+ c)^7 - 572*(a^3 + 8*a^2*b)*cosh(d*x + c)^5 + 110*(a^3 + 8*a^2*b)*cosh(d*x
 + c)^3 - 4*(a^3 + 8*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^8 + (a^3 + 8*a^2*b
)*cosh(d*x + c)^7 + (6435*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 - 6864*(a^3 + 8*a
^2*b)*cosh(d*x + c)^6 + 1980*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 + a^3 + 8*a^2*
b - 144*(a^3 + 8*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + (5005*(a^3 + 8*a
^2*b)*cosh(d*x + c)^9 - 6864*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 + 2772*(a^3 +
8*a^2*b)*cosh(d*x + c)^5 - 336*(a^3 + 8*a^2*b)*cosh(d*x + c)^3 + 7*(a^3 + 8
*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^6 + 3*(1001*(a^3 + 8*a^2*b)*cosh(d*x +
 c)^10 - 1716*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 924*(a^3 + 8*a^2*b)*cosh(d*
x + c)^6 - 168*(a^3 + 8*a^2*b)*cosh(d*x + c)^4 + 7*(a^3 + 8*a^2*b)*cosh(d*x
 + c)^2)*sinh(d*x + c)^5 + (1365*(a^3 + 8*a^2*b)*cosh(d*x + c)^11 - 2860*(a
^3 + 8*a^2*b)*cosh(d*x + c)^9 + 1980*(a^3 + 8*a^2*b)*cosh(d*x + c)^7 - 504*
(a^3 + 8*a^2*b)*cosh(d*x + c)^5 + 35*(a^3 + 8*a^2*b)*cosh(d*x + c)^3)*sinh(
d*x + c)^4 + (455*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 - 1144*(a^3 + 8*a^2*b)*c
osh(d*x + c)^10 + 990*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 - 336*(a^3 + 8*a^2*b)
*cosh(d*x + c)^6 + 35*(a^3 + 8*a^2*b)*cosh(d*x + c)^4)*sinh(d*x + c)^3 + 3*
(35*(a^3 + 8*a^2*b)*cosh(d*x + c)^13 - 104*(a^3 + 8*a^2*b)*cosh(d*x + c)^11
 + 110*(a^3 + 8*a^2*b)*cosh(d*x + c)^9 - 48*(a^3 + 8*a^2*b)*cosh(d*x + c)^7
 + 7*(a^3 + 8*a^2*b)*cosh(d*x + c)^5)*sinh(d*x + c)^2 + (15*(a^3 + 8*a^2*b)
```

```
*cosh(d*x + c)^14 - 52*(a^3 + 8*a^2*b)*cosh(d*x + c)^12 + 66*(a^3 + 8*a^2*b
)*cosh(d*x + c)^10 - 36*(a^3 + 8*a^2*b)*cosh(d*x + c)^8 + 7*(a^3 + 8*a^2*b)
*cosh(d*x + c)^6)*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 2
*(55*b^3*cosh(d*x + c)^21 - 690*b^3*cosh(d*x + c)^19 + 9*(560*a*b^2 + 471*b
^3)*cosh(d*x + c)^17 - 8*(7280*a*b^2 + 2519*b^3)*cosh(d*x + c)^15 + 42*(560
*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^13 - 84*(880*a^3 + 840*a*b^2 + 2
31*b^3)*cosh(d*x + c)^11 - 70*(880*a^3 + 840*a*b^2 + 231*b^3)*cosh(d*x + c)
^9 + 24*(560*a^3 + 3080*a*b^2 + 891*b^3)*cosh(d*x + c)^7 - 3*(7280*a*b^2 +
2519*b^3)*cosh(d*x + c)^5 - 69*b^3*cosh(d*x + c) + 2*(560*a*b^2 + 471*b^3)*
cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c)^15 + 15*d*cosh(d*x + c)*si
nh(d*x + c)^14 + d*sinh(d*x + c)^15 - 4*d*cosh(d*x + c)^13 + (105*d*cosh(d*
x + c)^2 - 4*d)*sinh(d*x + c)^13 + 13*(35*d*cosh(d*x + c)^3 - 4*d*cosh(d*x
+ c))*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^11 + 3*(455*d*cosh(d*x + c)^4 -
104*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^11 + 11*(273*d*cosh(d*x + c)^5 -
 104*d*cosh(d*x + c)^3 + 6*d*cosh(d*x + c))*sinh(d*x + c)^10 - 4*d*cosh(d*x
 + c)^9 + (5005*d*cosh(d*x + c)^6 - 2860*d*cosh(d*x + c)^4 + 330*d*cosh(d*x
 + c)^2 - 4*d)*sinh(d*x + c)^9 + 9*(715*d*cosh(d*x + c)^7 - 572*d*cosh(d*x
+ c)^5 + 110*d*cosh(d*x + c)^3 - 4*d*cosh(d*x + c))*sinh(d*x + c)^8 + d*cos
h(d*x + c)^7 + (6435*d*cosh(d*x + c)^8 - 6864*d*cosh(d*x + c)^6 + 1980*d*co
sh(d*x + c)^4 - 144*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + (5005*d*cosh(d
*x + c)^9 - 6864*d*cosh(d*x + c)^7 + 2772*d*cosh(d*x + c)^5 - 336*d*cosh(d*
x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)^6 + 3*(1001*d*cosh(d*x + c)^10
- 1716*d*cosh(d*x + c)^8 + 924*d*cosh(d*x + c)^6 - 168*d*cosh(d*x + c)^4 +
7*d*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (1365*d*cosh(d*x + c)^11 - 2860*d*co
sh(d*x + c)^9 + 1980*d*cosh(d*x + c)^7 - 504*d*cosh(d*x + c)^5 + 35*d*cosh(
d*x + c)^3)*sinh(d*x + c)^4 + (455*d*cosh(d*x + c)^12 - 1144*d*cosh(d*x + c
)^10 + 990*d*cosh(d*x + c)^8 - 336*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4
)*sinh(d*x + c)^3 + 3*(35*d*cosh(d*x + c)^13 - 104*d*cosh(d*x + c)^11 + 110
*d*cosh(d*x + c)^9 - 48*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5)*sinh(d*x +
 c)^2 + (15*d*cosh(d*x + c)^14 - 52*d*cosh(d*x + c)^12 + 66*d*cosh(d*x + c)
^10 - 36*d*cosh(d*x + c)^8 + 7*d*cosh(d*x + c)^6)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**5*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.74344, size = 404, normalized size = 2.85

$$-\frac{3\left(a^3+8\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{16\,d}+\frac{3\left(a^3+8\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{16\,d}+\frac{3\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^3-20\,a^3 \cdots}{4\left(\left(e^{(dx+c)}+e^{(-dx-c)}\right) \cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^5*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -3/16*(a^3 + 8*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d + 3/16*(a^3 + 8
*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d + 1/4*(3*a^3*(e^(d*x + c) + e
```

```
^(-d*x - c))^3 - 20*a^3*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-
d*x - c))^2 - 4)^2*d) + 1/4480*(5*b^3*d^6*(e^(d*x + c) + e^(-d*x - c))^7 -
84*b^3*d^6*(e^(d*x + c) + e^(-d*x - c))^5 + 560*a*b^2*d^6*(e^(d*x + c) + e^
(-d*x - c))^3 + 560*b^3*d^6*(e^(d*x + c) + e^(-d*x - c))^3 - 6720*a*b^2*d^6
*(e^(d*x + c) + e^(-d*x - c)) - 2240*b^3*d^6*(e^(d*x + c) + e^(-d*x - c)))/
d^7
```

3.213 $\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=156

$$\frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3 c}{}$$

```
[Out] (a^2*(5*a + 24*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*(3*a + b)*Cosh[c +
d*x])/d - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) - (a^
2*(5*a + 24*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^3*Coth[c + d*x]*C
sch[c + d*x]^3)/(24*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```

Rubi [A] time = 0.304455, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 1810, 206}

$$\frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d}-\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d}-\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d}+\frac{5a^3 c}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] (a^2*(5*a + 24*b)*ArcTanh[Cosh[c + d*x]])/(16*d) + (b^2*(3*a + b)*Cosh[c +
d*x])/d - (2*b^3*Cosh[c + d*x]^3)/(3*d) + (b^3*Cosh[c + d*x]^5)/(5*d) - (a^
2*(5*a + 24*b)*Coth[c + d*x]*Csch[c + d*x])/(16*d) + (5*a^3*Coth[c + d*x]*C
sch[c + d*x]^3)/(24*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(6*d)
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1810

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[Pq*
(a + b*x^2)^p, x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^7(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^4}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} - \frac{\operatorname{Subst}\left(\int \frac{-5a^3-18a^2b-18ab^2-6b^3+6b\left(3a^2+9ab+5\right.}{}\right.}{}$$

$$= \frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d} - \frac{a^3\coth(c+dx)\operatorname{csch}^5(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\right.}{}$$

$$= -\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}$$

$$= -\frac{a^2(5a+24b)\coth(c+dx)\operatorname{csch}(c+dx)}{16d} + \frac{5a^3\coth(c+dx)\operatorname{csch}^3(c+dx)}{24d}$$

$$= \frac{b^2(3a+b)\cosh(c+dx)}{d} - \frac{2b^3\cosh^3(c+dx)}{3d} + \frac{b^3\cosh^5(c+dx)}{5d} - \frac{a^2(5a+}{}$$

$$= \frac{a^2(5a+24b)\tanh^{-1}(\cosh(c+dx))}{16d} + \frac{b^2(3a+b)\cosh(c+dx)}{d} - \frac{2b^3\cosh^3}{3d}$$

Mathematica [A] time = 0.702756, size = 223, normalized size = 1.43

$$-\frac{720a^2b\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+720a^2b\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+2880a^2b\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+5a^3\operatorname{csch}^6\left(\frac{1}{2}(c+dx)\right)-30a^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^7*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(-240*b^2*(24*a + 5*b)*Cosh[c + d*x] + 200*b^3*Cosh[3*(c + d*x)] - 24*b^3*
Cosh[5*(c + d*x)] + 150*a^3*Csch[(c + d*x)/2]^2 + 720*a^2*b*Csch[(c + d*x)/
2]^2 - 30*a^3*Csch[(c + d*x)/2]^4 + 5*a^3*Csch[(c + d*x)/2]^6 + 600*a^3*Log
[Tanh[(c + d*x)/2]] + 2880*a^2*b*Log[Tanh[(c + d*x)/2]] + 150*a^3*Sech[(c +
 d*x)/2]^2 + 720*a^2*b*Sech[(c + d*x)/2]^2 + 30*a^3*Sech[(c + d*x)/2]^4 + 5
*a^3*Sech[(c + d*x)/2]^6)/(1920*d)
```

Maple [A] time = 0.047, size = 128, normalized size = 0.8

$$\frac{1}{d}\left(a^3\left(\left(-\frac{(\operatorname{csch}(dx+c))^5}{6}+\frac{5\,(\operatorname{csch}(dx+c))^3}{24}-\frac{5\operatorname{csch}(dx+c)}{16}\right)\coth(dx+c)+\frac{5\operatorname{Artanh}\left(e^{dx+c}\right)}{8}\right)+3\,a^2b\left(-1/2\operatorname{csch}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*x
+c)+5/8*arctanh(exp(d*x+c)))+3*a^2*b*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh(
exp(d*x+c)))+3*a*b^2*cosh(d*x+c)+b^3*(8/15+1/5*sinh(d*x+c)^4-4/15*sinh(d*x+
c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.11351, size = 527, normalized size = 3.38

$$\frac{1}{480}b^3\left(\frac{3e^{(5dx+5c)}}{d}-\frac{25e^{(3dx+3c)}}{d}+\frac{150e^{(dx+c)}}{d}+\frac{150e^{(-dx-c)}}{d}-\frac{25e^{(-3dx-3c)}}{d}+\frac{3e^{(-5dx-5c)}}{d}\right)+\frac{3}{2}ab^2\left(\frac{e^{(dx+c)}}{d}+\frac{e^{(-dx-c)}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/480*b^3*(3*e^(5*d*x + 5*c)/d - 25*e^(3*d*x + 3*c)/d + 150*e^(d*x + c)/d +
 150*e^(-d*x - c)/d - 25*e^(-3*d*x - 3*c)/d + 3*e^(-5*d*x - 5*c)/d) + 3/2*a
*b^2*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/48*a^3*(15*log(e^(-d*x - c) + 1)/
d - 15*log(e^(-d*x - c) - 1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) +
 198*e^(-5*d*x - 5*c) + 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) - 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c)
- 1))) + 3/2*a^2*b*(log(e^(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(
e^(-d*x - c) + e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c)
- 1)))
```

Fricas [B] time = 2.39735, size = 22631, normalized size = 145.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^22 + 66*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 3*b
^3*sinh(d*x + c)^22 - 43*b^3*cosh(d*x + c)^20 + (693*b^3*cosh(d*x + c)^2 -
43*b^3)*sinh(d*x + c)^20 + 20*(231*b^3*cosh(d*x + c)^3 - 43*b^3*cosh(d*x +
c))*sinh(d*x + c)^19 + 15*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^18 + 5*(4389*b^
3*cosh(d*x + c)^4 - 1634*b^3*cosh(d*x + c)^2 + 144*a*b^2 + 69*b^3)*sinh(d*x
 + c)^18 + 6*(13167*b^3*cosh(d*x + c)^5 - 8170*b^3*cosh(d*x + c)^3 + 45*(48
*a*b^2 + 23*b^3)*cosh(d*x + c))*sinh(d*x + c)^17 - 15*(20*a^3 + 96*a^2*b +
240*a*b^2 + 79*b^3)*cosh(d*x + c)^16 + 3*(74613*b^3*cosh(d*x + c)^6 - 69445
*b^3*cosh(d*x + c)^4 - 100*a^3 - 480*a^2*b - 1200*a*b^2 - 395*b^3 + 765*(48
*a*b^2 + 23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^16 + 48*(10659*b^3*cosh(d*x
 + c)^7 - 13889*b^3*cosh(d*x + c)^5 + 255*(48*a*b^2 + 23*b^3)*cosh(d*x + c)
^3 - 5*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c))*sinh(d*x + c
)^15 + 10*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^14 + 10
*(95931*b^3*cosh(d*x + c)^8 - 166668*b^3*cosh(d*x + c)^6 + 4590*(48*a*b^2 +
 23*b^3)*cosh(d*x + c)^4 + 170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3 - 180*
(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^14
+ 20*(74613*b^3*cosh(d*x + c)^9 - 166668*b^3*cosh(d*x + c)^7 + 6426*(48*a*b
```

^2 + 23*b^3)*cosh(d*x + c)^5 - 420*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)
*cosh(d*x + c)^3 + 7*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x +
c))*sinh(d*x + c)^13 - 90*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x
+ c)^12 + 2*(969969*b^3*cosh(d*x + c)^10 - 2708355*b^3*cosh(d*x + c)^8 + 1
39230*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^6 - 13650*(20*a^3 + 96*a^2*b + 240*
a*b^2 + 79*b^3)*cosh(d*x + c)^4 - 1980*a^3 - 1440*a^2*b - 1800*a*b^2 - 495*
b^3 + 455*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^12 + 8*(264537*b^3*cosh(d*x + c)^11 - 902785*b^3*cosh(d*x + c)^9
+ 59670*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^7 - 8190*(20*a^3 + 96*a^2*b + 240
*a*b^2 + 79*b^3)*cosh(d*x + c)^5 + 455*(170*a^3 + 432*a^2*b + 648*a*b^2 + 1
87*b^3)*cosh(d*x + c)^3 - 135*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(
d*x + c))*sinh(d*x + c)^11 - 90*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cos
h(d*x + c)^10 + 2*(969969*b^3*cosh(d*x + c)^12 - 3972254*b^3*cosh(d*x + c)^
10 + 328185*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^8 - 60060*(20*a^3 + 96*a^2*b
+ 240*a*b^2 + 79*b^3)*cosh(d*x + c)^6 + 5005*(170*a^3 + 432*a^2*b + 648*a*b
^2 + 187*b^3)*cosh(d*x + c)^4 - 1980*a^3 - 1440*a^2*b - 1800*a*b^2 - 495*b^
3 - 2970*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^10 + 20*(74613*b^3*cosh(d*x + c)^13 - 361114*b^3*cosh(d*x + c)^11 + 36
465*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^9 - 8580*(20*a^3 + 96*a^2*b + 240*a*b
^2 + 79*b^3)*cosh(d*x + c)^7 + 1001*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*
b^3)*cosh(d*x + c)^5 - 990*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x
+ c)^3 - 45*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*
x + c)^9 + 10*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^8 +
10*(95931*b^3*cosh(d*x + c)^14 - 541671*b^3*cosh(d*x + c)^12 + 65637*(48*a
*b^2 + 23*b^3)*cosh(d*x + c)^10 - 19305*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79
*b^3)*cosh(d*x + c)^8 + 3003*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*co
sh(d*x + c)^6 - 4455*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^
4 + 170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3 - 405*(44*a^3 + 32*a^2*b + 40
*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(31977*b^3*cosh(d*x
+ c)^15 - 208335*b^3*cosh(d*x + c)^13 + 29835*(48*a*b^2 + 23*b^3)*cosh(d*x
+ c)^11 - 10725*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^9 +
2145*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^7 - 4455*(44
*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 675*(44*a^3 + 32*a^2
*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^3 + 5*(170*a^3 + 432*a^2*b + 648*a*b^
2 + 187*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 - 15*(20*a^3 + 96*a^2*b + 240*a
*b^2 + 79*b^3)*cosh(d*x + c)^6 + (223839*b^3*cosh(d*x + c)^16 - 1666680*b^3
*cosh(d*x + c)^14 + 278460*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^12 - 120120*(2
0*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^10 + 30030*(170*a^3 +
432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^8 - 83160*(44*a^3 + 32*a^2*b
+ 40*a*b^2 + 11*b^3)*cosh(d*x + c)^6 - 18900*(44*a^3 + 32*a^2*b + 40*a*b^2
+ 11*b^3)*cosh(d*x + c)^4 - 300*a^3 - 1440*a^2*b - 3600*a*b^2 - 1185*b^3 +
280*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 + 2*(39501*b^3*cosh(d*x + c)^17 - 333336*b^3*cosh(d*x + c)^15 + 6426
0*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^13 - 32760*(20*a^3 + 96*a^2*b + 240*a*b
^2 + 79*b^3)*cosh(d*x + c)^11 + 10010*(170*a^3 + 432*a^2*b + 648*a*b^2 + 18
7*b^3)*cosh(d*x + c)^9 - 35640*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh
(d*x + c)^7 - 11340*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^5
+ 280*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^3 - 45*(20
*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 43*b
^3*cosh(d*x + c)^2 + 15*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^4 + 5*(4389*b^3*c
osh(d*x + c)^18 - 41667*b^3*cosh(d*x + c)^16 + 9180*(48*a*b^2 + 23*b^3)*cos
h(d*x + c)^14 - 5460*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)
^12 + 2002*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^10 - 8
910*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^8 - 3780*(44*a^3
+ 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^6 + 140*(170*a^3 + 432*a^2*b
+ 648*a*b^2 + 187*b^3)*cosh(d*x + c)^4 + 144*a*b^2 + 69*b^3 - 45*(20*a^3 +
96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 20*(231*b
^3*cosh(d*x + c)^19 - 2451*b^3*cosh(d*x + c)^17 + 612*(48*a*b^2 + 23*b^3)*c
osh(d*x + c)^15 - 420*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c

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)^13 + 182*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^11 - 9
90*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^9 - 540*(44*a^3 +
32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^7 + 28*(170*a^3 + 432*a^2*b + 6
48*a*b^2 + 187*b^3)*cosh(d*x + c)^5 - 15*(20*a^3 + 96*a^2*b + 240*a*b^2 + 7
9*b^3)*cosh(d*x + c)^3 + 3*(48*a*b^2 + 23*b^3)*cosh(d*x + c))*sinh(d*x + c)
^3 + 3*b^3 + (693*b^3*cosh(d*x + c)^20 - 8170*b^3*cosh(d*x + c)^18 + 2295*(
48*a*b^2 + 23*b^3)*cosh(d*x + c)^16 - 1800*(20*a^3 + 96*a^2*b + 240*a*b^2 +
 79*b^3)*cosh(d*x + c)^14 + 910*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)
*cosh(d*x + c)^12 - 5940*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x +
 c)^10 - 4050*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*cosh(d*x + c)^8 + 280
*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c)^6 - 225*(20*a^3
+ 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^4 - 43*b^3 + 90*(48*a*b^2 +
23*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 30*((5*a^3 + 24*a^2*b)*cosh(d*x
+ c)^17 + 17*(5*a^3 + 24*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^16 + (5*a^3 + 2
4*a^2*b)*sinh(d*x + c)^17 - 6*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^15 - 2*(15*a
^3 + 72*a^2*b - 68*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 1
0*(68*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 9*(5*a^3 + 24*a^2*b)*cosh(d*x +
c))*sinh(d*x + c)^14 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^13 + 5*(476*(5*a
^3 + 24*a^2*b)*cosh(d*x + c)^4 + 15*a^3 + 72*a^2*b - 126*(5*a^3 + 24*a^2*b)
*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(476*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^5 - 210*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 + 15*(5*a^3 + 24*a^2*b)*cosh(
d*x + c))*sinh(d*x + c)^12 - 20*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 + 2*(61
88*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 - 4095*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^4 - 50*a^3 - 240*a^2*b + 585*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*
x + c)^11 + 22*(884*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 819*(5*a^3 + 24*a^
2*b)*cosh(d*x + c)^5 + 195*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 10*(5*a^3 +
 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^10 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x
 + c)^9 + 5*(4862*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 6006*(5*a^3 + 24*a^2
*b)*cosh(d*x + c)^6 + 2145*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 + 15*a^3 + 72
*a^2*b - 220*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 5*(4862*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 - 7722*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
7 + 3861*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 - 660*(5*a^3 + 24*a^2*b)*cosh(d
*x + c)^3 + 27*(5*a^3 + 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^8 - 6*(5*a^3
 + 24*a^2*b)*cosh(d*x + c)^7 + 2*(9724*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10
- 19305*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 + 12870*(5*a^3 + 24*a^2*b)*cosh(
d*x + c)^6 - 3300*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 - 15*a^3 - 72*a^2*b +
270*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 2*(6188*(5*a^3 +
24*a^2*b)*cosh(d*x + c)^11 - 15015*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 + 128
70*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 4620*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^5 + 630*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 21*(5*a^3 + 24*a^2*b)*cosh(
d*x + c))*sinh(d*x + c)^6 + (5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 + (6188*(5*a
^3 + 24*a^2*b)*cosh(d*x + c)^12 - 18018*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10
 + 19305*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 9240*(5*a^3 + 24*a^2*b)*cosh(
d*x + c)^6 + 1890*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 + 5*a^3 + 24*a^2*b - 1
26*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*(476*(5*a^3 + 24
*a^2*b)*cosh(d*x + c)^13 - 1638*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 + 2145*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 - 1320*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
7 + 378*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 - 42*(5*a^3 + 24*a^2*b)*cosh(d*x
 + c)^3 + (5*a^3 + 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^4 + 10*(68*(5*a^3
 + 24*a^2*b)*cosh(d*x + c)^14 - 273*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^12 + 4
29*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 - 330*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^8 + 126*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 - 21*(5*a^3 + 24*a^2*b)*cosh(
d*x + c)^4 + (5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(68*(5
*a^3 + 24*a^2*b)*cosh(d*x + c)^15 - 315*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^13
 + 585*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 - 550*(5*a^3 + 24*a^2*b)*cosh(d*
x + c)^9 + 270*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 63*(5*a^3 + 24*a^2*b)*c
osh(d*x + c)^5 + 5*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (1
7*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^16 - 90*(5*a^3 + 24*a^2*b)*cosh(d*x + c)
^14 + 195*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^12 - 220*(5*a^3 + 24*a^2*b)*cosh
```

(d*x + c)^10 + 135*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 42*(5*a^3 + 24*a^2*
b)*cosh(d*x + c)^6 + 5*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4)*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) + 1) - 30*((5*a^3 + 24*a^2*b)*cosh(d*x + c
)^17 + 17*(5*a^3 + 24*a^2*b)*cosh(d*x + c)*sinh(d*x + c)^16 + (5*a^3 + 24*a
^2*b)*sinh(d*x + c)^17 - 6*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^15 - 2*(15*a^3
+ 72*a^2*b - 68*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 10*(
68*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 9*(5*a^3 + 24*a^2*b)*cosh(d*x + c))
*sinh(d*x + c)^14 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^13 + 5*(476*(5*a^3
+ 24*a^2*b)*cosh(d*x + c)^4 + 15*a^3 + 72*a^2*b - 126*(5*a^3 + 24*a^2*b)*co
sh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(476*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
5 - 210*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x
 + c))*sinh(d*x + c)^12 - 20*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 + 2*(6188*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 - 4095*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
4 - 50*a^3 - 240*a^2*b + 585*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
 c)^11 + 22*(884*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 819*(5*a^3 + 24*a^2*b
)*cosh(d*x + c)^5 + 195*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 10*(5*a^3 + 24
*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^10 + 15*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^9 + 5*(4862*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 6006*(5*a^3 + 24*a^2*b)
*cosh(d*x + c)^6 + 2145*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 + 15*a^3 + 72*a^
2*b - 220*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 5*(4862*(5*
a^3 + 24*a^2*b)*cosh(d*x + c)^9 - 7722*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 +
 3861*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 - 660*(5*a^3 + 24*a^2*b)*cosh(d*x
+ c)^3 + 27*(5*a^3 + 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^8 - 6*(5*a^3 +
24*a^2*b)*cosh(d*x + c)^7 + 2*(9724*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 - 1
9305*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 + 12870*(5*a^3 + 24*a^2*b)*cosh(d*x
 + c)^6 - 3300*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 - 15*a^3 - 72*a^2*b + 270
*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 2*(6188*(5*a^3 + 24*
a^2*b)*cosh(d*x + c)^11 - 15015*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^9 + 12870*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 4620*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
5 + 630*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3 - 21*(5*a^3 + 24*a^2*b)*cosh(d*x
 + c))*sinh(d*x + c)^6 + (5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 + (6188*(5*a^3
+ 24*a^2*b)*cosh(d*x + c)^12 - 18018*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 +
19305*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 9240*(5*a^3 + 24*a^2*b)*cosh(d*x
 + c)^6 + 1890*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4 + 5*a^3 + 24*a^2*b - 126*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*(476*(5*a^3 + 24*a^
2*b)*cosh(d*x + c)^13 - 1638*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 + 2145*(5*
a^3 + 24*a^2*b)*cosh(d*x + c)^9 - 1320*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 +
 378*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^5 - 42*(5*a^3 + 24*a^2*b)*cosh(d*x +
c)^3 + (5*a^3 + 24*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^4 + 10*(68*(5*a^3 +
24*a^2*b)*cosh(d*x + c)^14 - 273*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^12 + 429*
(5*a^3 + 24*a^2*b)*cosh(d*x + c)^10 - 330*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^
8 + 126*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^6 - 21*(5*a^3 + 24*a^2*b)*cosh(d*x
 + c)^4 + (5*a^3 + 24*a^2*b)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(68*(5*a^
3 + 24*a^2*b)*cosh(d*x + c)^15 - 315*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^13 +
585*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^11 - 550*(5*a^3 + 24*a^2*b)*cosh(d*x +
 c)^9 + 270*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^7 - 63*(5*a^3 + 24*a^2*b)*cosh
(d*x + c)^5 + 5*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^3)*sinh(d*x + c)^2 + (17*(
5*a^3 + 24*a^2*b)*cosh(d*x + c)^16 - 90*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^14
 + 195*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^12 - 220*(5*a^3 + 24*a^2*b)*cosh(d*
x + c)^10 + 135*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^8 - 42*(5*a^3 + 24*a^2*b)*
cosh(d*x + c)^6 + 5*(5*a^3 + 24*a^2*b)*cosh(d*x + c)^4)*sinh(d*x + c))*log(
cosh(d*x + c) + sinh(d*x + c) - 1) + 2*(33*b^3*cosh(d*x + c)^21 - 430*b^3*c
osh(d*x + c)^19 + 135*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^17 - 120*(20*a^3 +
96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^15 + 70*(170*a^3 + 432*a^2*b +
 648*a*b^2 + 187*b^3)*cosh(d*x + c)^13 - 540*(44*a^3 + 32*a^2*b + 40*a*b^2
+ 11*b^3)*cosh(d*x + c)^11 - 450*(44*a^3 + 32*a^2*b + 40*a*b^2 + 11*b^3)*co
sh(d*x + c)^9 + 40*(170*a^3 + 432*a^2*b + 648*a*b^2 + 187*b^3)*cosh(d*x + c
)^7 - 45*(20*a^3 + 96*a^2*b + 240*a*b^2 + 79*b^3)*cosh(d*x + c)^5 - 43*b^3*
cosh(d*x + c) + 30*(48*a*b^2 + 23*b^3)*cosh(d*x + c)^3)*sinh(d*x + c))/(d*c

```
osh(d*x + c)^17 + 17*d*cosh(d*x + c)*sinh(d*x + c)^16 + d*sinh(d*x + c)^17
- 6*d*cosh(d*x + c)^15 + 2*(68*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^15 +
10*(68*d*cosh(d*x + c)^3 - 9*d*cosh(d*x + c))*sinh(d*x + c)^14 + 15*d*cosh(
d*x + c)^13 + 5*(476*d*cosh(d*x + c)^4 - 126*d*cosh(d*x + c)^2 + 3*d)*sinh(
d*x + c)^13 + 13*(476*d*cosh(d*x + c)^5 - 210*d*cosh(d*x + c)^3 + 15*d*cosh
(d*x + c))*sinh(d*x + c)^12 - 20*d*cosh(d*x + c)^11 + 2*(6188*d*cosh(d*x +
c)^6 - 4095*d*cosh(d*x + c)^4 + 585*d*cosh(d*x + c)^2 - 10*d)*sinh(d*x + c)
^11 + 22*(884*d*cosh(d*x + c)^7 - 819*d*cosh(d*x + c)^5 + 195*d*cosh(d*x +
c)^3 - 10*d*cosh(d*x + c))*sinh(d*x + c)^10 + 15*d*cosh(d*x + c)^9 + 5*(486
2*d*cosh(d*x + c)^8 - 6006*d*cosh(d*x + c)^6 + 2145*d*cosh(d*x + c)^4 - 220
*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^9 + 5*(4862*d*cosh(d*x + c)^9 - 772
2*d*cosh(d*x + c)^7 + 3861*d*cosh(d*x + c)^5 - 660*d*cosh(d*x + c)^3 + 27*d
*cosh(d*x + c))*sinh(d*x + c)^8 - 6*d*cosh(d*x + c)^7 + 2*(9724*d*cosh(d*x
+ c)^10 - 19305*d*cosh(d*x + c)^8 + 12870*d*cosh(d*x + c)^6 - 3300*d*cosh(d
*x + c)^4 + 270*d*cosh(d*x + c)^2 - 3*d)*sinh(d*x + c)^7 + 2*(6188*d*cosh(d
*x + c)^11 - 15015*d*cosh(d*x + c)^9 + 12870*d*cosh(d*x + c)^7 - 4620*d*cos
h(d*x + c)^5 + 630*d*cosh(d*x + c)^3 - 21*d*cosh(d*x + c))*sinh(d*x + c)^6
+ d*cosh(d*x + c)^5 + (6188*d*cosh(d*x + c)^12 - 18018*d*cosh(d*x + c)^10 +
 19305*d*cosh(d*x + c)^8 - 9240*d*cosh(d*x + c)^6 + 1890*d*cosh(d*x + c)^4
- 126*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 5*(476*d*cosh(d*x + c)^13 -
1638*d*cosh(d*x + c)^11 + 2145*d*cosh(d*x + c)^9 - 1320*d*cosh(d*x + c)^7 +
 378*d*cosh(d*x + c)^5 - 42*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x +
 c)^4 + 10*(68*d*cosh(d*x + c)^14 - 273*d*cosh(d*x + c)^12 + 429*d*cosh(d*x
 + c)^10 - 330*d*cosh(d*x + c)^8 + 126*d*cosh(d*x + c)^6 - 21*d*cosh(d*x +
c)^4 + d*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(68*d*cosh(d*x + c)^15 - 315*
d*cosh(d*x + c)^13 + 585*d*cosh(d*x + c)^11 - 550*d*cosh(d*x + c)^9 + 270*d
*cosh(d*x + c)^7 - 63*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3)*sinh(d*x + c
)^2 + (17*d*cosh(d*x + c)^16 - 90*d*cosh(d*x + c)^14 + 195*d*cosh(d*x + c)^
12 - 220*d*cosh(d*x + c)^10 + 135*d*cosh(d*x + c)^8 - 42*d*cosh(d*x + c)^6
+ 5*d*cosh(d*x + c)^4)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**7*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.68381, size = 463, normalized size = 2.97

$$\frac{\left(5\,a^3+24\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{32\,d}-\frac{\left(5\,a^3+24\,a^2b\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{32\,d}-\frac{15\,a^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)^5+72$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^7*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/32*(5*a^3 + 24*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 1/32*(5*a^3
 + 24*a^2*b)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d - 1/24*(15*a^3*(e^(d*x +
 c) + e^(-d*x - c))^5 + 72*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 - 160*a^3*(
```

```
e^(d*x + c) + e^(-d*x - c))^3 - 576*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 +
528*a^3*(e^(d*x + c) + e^(-d*x - c)) + 1152*a^2*b*(e^(d*x + c) + e^(-d*x -
c)))/(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^3*d) + 1/480*(3*b^3*d^4*(e^(d*x
+ c) + e^(-d*x - c))^5 - 40*b^3*d^4*(e^(d*x + c) + e^(-d*x - c))^3 + 720*a*
b^2*d^4*(e^(d*x + c) + e^(-d*x - c)) + 240*b^3*d^4*(e^(d*x + c) + e^(-d*x -
 c)))/d^5
```

3.214 $\int \mathbf{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=171

$$-\frac{a\left(35a^2+144ab+384b^2\right)\tanh^{-1}(\cosh(c+dx))}{128d}-\frac{a^2(35a+144b)\coth(c+dx)\text{csch}^3(c+dx)}{192d}+\frac{a^2(35a+144b)\cot}{1}$$

```
[Out] -(a*(35*a^2 + 144*a*b + 384*b^2)*ArcTanh[Cosh[c + d*x]])/(128*d) - (b^3*Cos
h[c + d*x])/d + (b^3*Cosh[c + d*x]^3)/(3*d) + (a^2*(35*a + 144*b)*Coth[c +
d*x]*Csch[c + d*x])/(128*d) - (a^2*(35*a + 144*b)*Coth[c + d*x]*Csch[c + d*
x]^3)/(192*d) + (7*a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(48*d) - (a^3*Coth[c
+ d*x]*Csch[c + d*x]^7)/(8*d)
```

Rubi [A] time = 0.331856, antiderivative size = 171, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 1153, 206}

$$-\frac{a\left(35a^2+144ab+384b^2\right)\tanh^{-1}(\cosh(c+dx))}{128d}-\frac{a^2(35a+144b)\coth(c+dx)\text{csch}^3(c+dx)}{192d}+\frac{a^2(35a+144b)\cot}{1}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(a*(35*a^2 + 144*a*b + 384*b^2)*ArcTanh[Cosh[c + d*x]])/(128*d) - (b^3*Cos
h[c + d*x])/d + (b^3*Cosh[c + d*x]^3)/(3*d) + (a^2*(35*a + 144*b)*Coth[c +
d*x]*Csch[c + d*x])/(128*d) - (a^2*(35*a + 144*b)*Coth[c + d*x]*Csch[c + d*
x]^3)/(192*d) + (7*a^3*Coth[c + d*x]*Csch[c + d*x]^5)/(48*d) - (a^3*Coth[c
+ d*x]*Csch[c + d*x]^7)/(8*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1153

```
Int[((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
 x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x],
x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e
 + a*e^2, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \text{csch}^9(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^5}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\text{csch}^7(c+dx)}{8d} + \frac{\text{Subst}\left(\int \frac{-(a+2b)\left(7a^2+10ab+4b^2\right)+8b\left(3a^2+9ab+5\right.}{}\right.}{}$$

$$= \frac{7a^3\coth(c+dx)\text{csch}^5(c+dx)}{48d} - \frac{a^3\coth(c+dx)\text{csch}^7(c+dx)}{8d} - \frac{\text{Subst}\left(\right.}{}$$

$$= -\frac{a^2(35a+144b)\coth(c+dx)\text{csch}^3(c+dx)}{192d} + \frac{7a^3\coth(c+dx)\text{csch}^5(c+}{48d}$$

$$= \frac{a^2(35a+144b)\coth(c+dx)\text{csch}(c+dx)}{128d} - \frac{a^2(35a+144b)\coth(c+dx)\text{cs}}{192d}$$

$$= \frac{a^2(35a+144b)\coth(c+dx)\text{csch}(c+dx)}{128d} - \frac{a^2(35a+144b)\coth(c+dx)\text{cs}}{192d}$$

$$= -\frac{b^3\cosh(c+dx)}{d} + \frac{b^3\cosh^3(c+dx)}{3d} + \frac{a^2(35a+144b)\coth(c+dx)\text{csch}(c}{128d}$$

$$= -\frac{a\left(35a^2+144ab+384b^2\right)\tanh^{-1}(\cosh(c+dx))}{128d} - \frac{b^3\cosh(c+dx)}{d} + \frac{b^3\,\text{c}}{}$$

Mathematica [A] time = 1.69414, size = 219, normalized size = 1.28

$$\frac{a\left(48\left(35a^2+144ab+384b^2\right)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-3a^2\text{csch}^8\left(\frac{1}{2}(c+dx)\right)+20a^2\text{csch}^6\left(\frac{1}{2}(c+dx)\right)+3a^2\text{sech}^8\left(\frac{1}{2}(c-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^9*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-4608*b^3*Cosh[c + d*x] + 512*b^3*Cosh[3*(c + d*x)] + a*(12*a*(35*a + 144*
b)*Csch[(c + d*x)/2]^2 - 18*a*(5*a + 16*b)*Csch[(c + d*x)/2]^4 + 20*a^2*Csc
h[(c + d*x)/2]^6 - 3*a^2*Csch[(c + d*x)/2]^8 + 48*(35*a^2 + 144*a*b + 384*b
^2)*Log[Tanh[(c + d*x)/2]] + 12*a*(35*a + 144*b)*Sech[(c + d*x)/2]^2 + 18*a
*(5*a + 16*b)*Sech[(c + d*x)/2]^4 + 20*a^2*Sech[(c + d*x)/2]^6 + 3*a^2*Sech
[(c + d*x)/2]^8))/(6144*d)
```

Maple [A] time = 0.083, size = 143, normalized size = 0.8

$$\frac{1}{d}\left(a^3\left(\left(-\frac{(\operatorname{csch}(dx+c))^7}{8}+\frac{7(\operatorname{csch}(dx+c))^5}{48}-\frac{35(\operatorname{csch}(dx+c))^3}{192}+\frac{35\operatorname{csch}(dx+c)}{128}\right)\coth(dx+c)-\frac{35\operatorname{Artanh}}{64}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/8*csch(d*x+c)^7+7/48*csch(d*x+c)^5-35/192*csch(d*x+c)^3+35/12
8*csch(d*x+c))*coth(d*x+c)-35/64*arctanh(exp(d*x+c)))+3*a^2*b*((-1/4*csch(d
*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctanh(exp(d*x+c)))-6*a*b^2*arcta
nh(exp(d*x+c))+b^3*(-2/3+1/3*sinh(d*x+c)^2)*cosh(d*x+c))
```

Maxima [B] time = 1.13245, size = 625, normalized size = 3.65

$$\frac{1}{24}b^3\left(\frac{e^{(3\,dx+3\,c)}}{d}-\frac{9\,e^{(dx+c)}}{d}-\frac{9\,e^{(-dx-c)}}{d}+\frac{e^{(-3\,dx-3\,c)}}{d}\right)-\frac{1}{384}a^3\left(\frac{105\log\left(e^{(-dx-c)}+1\right)}{d}-\frac{105\log\left(e^{(-dx-c)}-1\right)}{d}+\frac{2(105}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*(e^(3*d*x + 3*c)/d - 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d + e^(-3*d*
x - 3*c)/d) - 1/384*a^3*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*x - c)
 - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x - 5*c
) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*e^(-11*d*x - 11*c)
 - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x - 2*c)
- 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*d*x - 8*c) + 56*e^(-
10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) - e^(-16*d*x
- 16*c) - 1))) - 3/8*a^2*b*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x - c)
- 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c) + 3*
e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) - e^(-8*d*x - 8*c) - 1))) - 3*a*b^2*(log(e^(-d*x - c) + 1)/d - log(
e^(-d*x - c) - 1)/d)
```

Fricas [B] time = 2.76965, size = 30027, normalized size = 175.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(16*b^3*cosh(d*x + c)^22 + 352*b^3*cosh(d*x + c)*sinh(d*x + c)^21 + 1
6*b^3*sinh(d*x + c)^22 - 272*b^3*cosh(d*x + c)^20 + 16*(231*b^3*cosh(d*x +
c)^2 - 17*b^3)*sinh(d*x + c)^20 + 320*(77*b^3*cosh(d*x + c)^3 - 17*b^3*cosh
(d*x + c))*sinh(d*x + c)^19 + 2*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x +
c)^18 + 2*(58520*b^3*cosh(d*x + c)^4 - 25840*b^3*cosh(d*x + c)^2 + 105*a^3
+ 432*a^2*b + 728*b^3)*sinh(d*x + c)^18 + 12*(35112*b^3*cosh(d*x + c)^5 - 2
5840*b^3*cosh(d*x + c)^3 + 3*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c))
```

```
*sinh(d*x + c)^17 - 2*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^16 +
2*(596904*b^3*cosh(d*x + c)^6 - 658920*b^3*cosh(d*x + c)^4 - 805*a^3 - 3312
*a^2*b - 1880*b^3 + 153*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^2)*si
nh(d*x + c)^16 + 32*(85272*b^3*cosh(d*x + c)^7 - 131784*b^3*cosh(d*x + c)^5
 + 51*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^3 - (805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c))*sinh(d*x + c)^15 + 2*(2681*a^3 + 7344*a^2*b +
 2512*b^3)*cosh(d*x + c)^14 + 2*(2558160*b^3*cosh(d*x + c)^8 - 5271360*b^3*
cosh(d*x + c)^6 + 3060*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^4 + 26
81*a^3 + 7344*a^2*b + 2512*b^3 - 120*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^14 + 4*(1989680*b^3*cosh(d*x + c)^9 - 5271360*b^
3*cosh(d*x + c)^7 + 4284*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^5 -
280*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^3 + 7*(2681*a^3 + 7344*
a^2*b + 2512*b^3)*cosh(d*x + c))*sinh(d*x + c)^13 - 2*(5053*a^3 + 4464*a^2*
b + 1232*b^3)*cosh(d*x + c)^12 + 2*(5173168*b^3*cosh(d*x + c)^10 - 17131920
*b^3*cosh(d*x + c)^8 + 18564*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^
6 - 1820*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^4 - 5053*a^3 - 446
4*a^2*b - 1232*b^3 + 91*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^12 + 8*(1410864*b^3*cosh(d*x + c)^11 - 5710640*b^3*cosh(d*x
+ c)^9 + 7956*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^7 - 1092*(805*a
^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^5 + 91*(2681*a^3 + 7344*a^2*b + 2
512*b^3)*cosh(d*x + c)^3 - 3*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x +
c))*sinh(d*x + c)^11 - 2*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^1
0 + 2*(5173168*b^3*cosh(d*x + c)^12 - 25126816*b^3*cosh(d*x + c)^10 + 43758
*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^8 - 8008*(805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c)^6 + 1001*(2681*a^3 + 7344*a^2*b + 2512*b^3)*co
sh(d*x + c)^4 - 5053*a^3 - 4464*a^2*b - 1232*b^3 - 66*(5053*a^3 + 4464*a^2*
b + 1232*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 4*(1989680*b^3*cosh(d*x +
 c)^13 - 11421280*b^3*cosh(d*x + c)^11 + 24310*(105*a^3 + 432*a^2*b + 728*b
^3)*cosh(d*x + c)^9 - 5720*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^
7 + 1001*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^5 - 110*(5053*a^3
 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^3 - 5*(5053*a^3 + 4464*a^2*b + 1232
*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 2*(2681*a^3 + 7344*a^2*b + 2512*b^3)
*cosh(d*x + c)^8 + 2*(2558160*b^3*cosh(d*x + c)^14 - 17131920*b^3*cosh(d*x
+ c)^12 + 43758*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^10 - 12870*(8
05*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^8 + 3003*(2681*a^3 + 7344*a^2
*b + 2512*b^3)*cosh(d*x + c)^6 - 495*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cos
h(d*x + c)^4 + 2681*a^3 + 7344*a^2*b + 2512*b^3 - 45*(5053*a^3 + 4464*a^2*b
 + 1232*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 16*(170544*b^3*cosh(d*x + c
)^15 - 1317840*b^3*cosh(d*x + c)^13 + 3978*(105*a^3 + 432*a^2*b + 728*b^3)*
cosh(d*x + c)^11 - 1430*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^9 +
 429*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^7 - 99*(5053*a^3 + 44
64*a^2*b + 1232*b^3)*cosh(d*x + c)^5 - 15*(5053*a^3 + 4464*a^2*b + 1232*b^3
)*cosh(d*x + c)^3 + (2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c))*sinh(
d*x + c)^7 - 2*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^6 + 2*(59690
4*b^3*cosh(d*x + c)^16 - 5271360*b^3*cosh(d*x + c)^14 + 18564*(105*a^3 + 43
2*a^2*b + 728*b^3)*cosh(d*x + c)^12 - 8008*(805*a^3 + 3312*a^2*b + 1880*b^3
)*cosh(d*x + c)^10 + 3003*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^
8 - 924*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^6 - 210*(5053*a^3
+ 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^4 - 805*a^3 - 3312*a^2*b - 1880*b^3
+ 28*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 +
4*(105336*b^3*cosh(d*x + c)^17 - 1054272*b^3*cosh(d*x + c)^15 + 4284*(105*a
^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^13 - 2184*(805*a^3 + 3312*a^2*b + 1
880*b^3)*cosh(d*x + c)^11 + 1001*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*
x + c)^9 - 396*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^7 - 126*(50
53*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^5 + 28*(2681*a^3 + 7344*a^2*b
 + 2512*b^3)*cosh(d*x + c)^3 - 3*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x
 + c))*sinh(d*x + c)^5 - 272*b^3*cosh(d*x + c)^2 + 2*(105*a^3 + 432*a^2*b +
 728*b^3)*cosh(d*x + c)^4 + 2*(58520*b^3*cosh(d*x + c)^18 - 658920*b^3*cosh
(d*x + c)^16 + 3060*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^14 - 1820
```

```
*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^12 + 1001*(2681*a^3 + 7344
*a^2*b + 2512*b^3)*cosh(d*x + c)^10 - 495*(5053*a^3 + 4464*a^2*b + 1232*b^3
)*cosh(d*x + c)^8 - 210*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^6
 + 70*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^4 + 105*a^3 + 432*a^2
*b + 728*b^3 - 15*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c)^4 + 8*(3080*b^3*cosh(d*x + c)^19 - 38760*b^3*cosh(d*x + c)^17 + 204
*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^15 - 140*(805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c)^13 + 91*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cos
h(d*x + c)^11 - 55*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^9 - 30*
(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^7 + 14*(2681*a^3 + 7344*a^
2*b + 2512*b^3)*cosh(d*x + c)^5 - 5*(805*a^3 + 3312*a^2*b + 1880*b^3)*cosh(
d*x + c)^3 + (105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
 + 16*b^3 + 2*(1848*b^3*cosh(d*x + c)^20 - 25840*b^3*cosh(d*x + c)^18 + 153
*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^16 - 120*(805*a^3 + 3312*a^2
*b + 1880*b^3)*cosh(d*x + c)^14 + 91*(2681*a^3 + 7344*a^2*b + 2512*b^3)*cos
h(d*x + c)^12 - 66*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^10 - 45
*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^8 + 28*(2681*a^3 + 7344*a
^2*b + 2512*b^3)*cosh(d*x + c)^6 - 15*(805*a^3 + 3312*a^2*b + 1880*b^3)*cos
h(d*x + c)^4 - 136*b^3 + 6*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^2 - 3*((35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^19 + 1
9*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^18 + (35*a^3
 + 144*a^2*b + 384*a*b^2)*sinh(d*x + c)^19 - 8*(35*a^3 + 144*a^2*b + 384*a*
b^2)*cosh(d*x + c)^17 - (280*a^3 + 1152*a^2*b + 3072*a*b^2 - 171*(35*a^3 +
144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^17 + 17*(57*(35*a^3 +
 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 8*(35*a^3 + 144*a^2*b + 384*a*b^2
)*cosh(d*x + c))*sinh(d*x + c)^16 + 28*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^15 + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 2
45*a^3 + 1008*a^2*b + 2688*a*b^2 - 272*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^2)*sinh(d*x + c)^15 + 4*(2907*(35*a^3 + 144*a^2*b + 384*a*b^2)*c
osh(d*x + c)^5 - 1360*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 + 10
5*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^14 - 56*(35
*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^13 + 28*(969*(35*a^3 + 144*a^2*
b + 384*a*b^2)*cosh(d*x + c)^6 - 680*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(
d*x + c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 105*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 52*(969*(35*a^3 + 144*a^2*b + 3
84*a*b^2)*cosh(d*x + c)^7 - 952*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
 c)^5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 14*(35*a^3 +
 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^12 + 70*(35*a^3 + 144*
a^2*b + 384*a*b^2)*cosh(d*x + c)^11 + 2*(37791*(35*a^3 + 144*a^2*b + 384*a*
b^2)*cosh(d*x + c)^8 - 49504*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)
^6 + 19110*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 1225*a^3 + 50
40*a^2*b + 13440*a*b^2 - 2184*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^11 + 22*(4199*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
+ c)^9 - 7072*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 3822*(35*a
^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 - 728*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^3 + 35*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)
)*sinh(d*x + c)^10 - 56*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 +
2*(46189*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 97240*(35*a^3
+ 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 70070*(35*a^3 + 144*a^2*b + 384*
a*b^2)*cosh(d*x + c)^6 - 20020*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
c)^4 - 980*a^3 - 4032*a^2*b - 10752*a*b^2 + 1925*(35*a^3 + 144*a^2*b + 384*
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 2*(37791*(35*a^3 + 144*a^2*b + 38
4*a*b^2)*cosh(d*x + c)^11 - 97240*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^9 + 90090*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 - 36036*(3
5*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 + 5775*(35*a^3 + 144*a^2*b +
 384*a*b^2)*cosh(d*x + c)^3 - 252*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^8 + 28*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
7 + 4*(12597*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 - 38896*(35*
a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 + 45045*(35*a^3 + 144*a^2*b +
```

384*a*b^2)*cosh(d*x + c)^8 - 24024*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d
x + c)^6 + 5775(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 245*a^3
 + 1008*a^2*b + 2688*a*b^2 - 504*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
+ c)^2)*sinh(d*x + c)^7 + 28*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^13 - 3536*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 + 5005*(3
5*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 - 3432*(35*a^3 + 144*a^2*b +
 384*a*b^2)*cosh(d*x + c)^7 + 1155*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*
x + c)^5 - 168*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 + 7*(35*a^3
 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 8*(35*a^3 + 144*
a^2*b + 384*a*b^2)*cosh(d*x + c)^5 + 4*(2907*(35*a^3 + 144*a^2*b + 384*a*b^
2)*cosh(d*x + c)^14 - 12376*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
12 + 21021*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 18018*(35*a^
3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 8085*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^6 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 147*(35*a^3 + 144*a^2*b + 384*a*b^2
)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2
)*cosh(d*x + c)^15 - 4760*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^13
 + 9555*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 - 10010*(35*a^3 +
 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 + 5775*(35*a^3 + 144*a^2*b + 384*a*
b^2)*cosh(d*x + c)^7 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 10*(35*a^3 + 144
*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^3 + 144*a^2*b +
384*a*b^2)*cosh(d*x + c)^3 + (969*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^16 - 5440*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^14 + 12740*(
35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 - 16016*(35*a^3 + 144*a^2*
b + 384*a*b^2)*cosh(d*x + c)^10 + 11550*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c)^8 - 4704*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^6 + 980
*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 35*a^3 + 144*a^2*b + 38
4*a*b^2 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c
)^3 + (171*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^17 - 1088*(35*a^3
 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^15 + 2940*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^13 - 4368*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
 c)^11 + 3850*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 - 2016*(35*a
^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 588*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^5 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)
^3 + 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (1
9*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^18 - 136*(35*a^3 + 144*a^2
*b + 384*a*b^2)*cosh(d*x + c)^16 + 420*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^14 - 728*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 + 770
*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 504*(35*a^3 + 144*a^2*
b + 384*a*b^2)*cosh(d*x + c)^8 + 196*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(
d*x + c)^6 - 40*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 3*(35*a^
3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d*x + c
) + sinh(d*x + c) + 1) + 3*((35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
19 + 19*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^18 + (
35*a^3 + 144*a^2*b + 384*a*b^2)*sinh(d*x + c)^19 - 8*(35*a^3 + 144*a^2*b +
384*a*b^2)*cosh(d*x + c)^17 - (280*a^3 + 1152*a^2*b + 3072*a*b^2 - 171*(35*
a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^17 + 17*(57*(35
*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 8*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^16 + 28*(35*a^3 + 144*a^2*b + 384*a*b^
2)*cosh(d*x + c)^15 + 4*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)
^4 + 245*a^3 + 1008*a^2*b + 2688*a*b^2 - 272*(35*a^3 + 144*a^2*b + 384*a*b^
2)*cosh(d*x + c)^2)*sinh(d*x + c)^15 + 4*(2907*(35*a^3 + 144*a^2*b + 384*a*
b^2)*cosh(d*x + c)^5 - 1360*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^
3 + 105*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^14 -
56*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^13 + 28*(969*(35*a^3 + 14
4*a^2*b + 384*a*b^2)*cosh(d*x + c)^6 - 680*(35*a^3 + 144*a^2*b + 384*a*b^2)
*cosh(d*x + c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 105*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 52*(969*(35*a^3 + 144*a^2

*b + 384*a*b^2)*cosh(d*x + c)^7 - 952*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh
(d*x + c)^5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 14*(35
*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^12 + 70*(35*a^3
+ 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 + 2*(37791*(35*a^3 + 144*a^2*b +
384*a*b^2)*cosh(d*x + c)^8 - 49504*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*
x + c)^6 + 19110*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 1225*a^
3 + 5040*a^2*b + 13440*a*b^2 - 2184*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c)^11 + 22*(4199*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^9 - 7072*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 3822
*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 - 728*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^3 + 35*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*
x + c))*sinh(d*x + c)^10 - 56*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c
)^9 + 2*(46189*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 97240*(3
5*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 70070*(35*a^3 + 144*a^2*b
+ 384*a*b^2)*cosh(d*x + c)^6 - 20020*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(
d*x + c)^4 - 980*a^3 - 4032*a^2*b - 10752*a*b^2 + 1925*(35*a^3 + 144*a^2*b
+ 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 2*(37791*(35*a^3 + 144*a^2*
b + 384*a*b^2)*cosh(d*x + c)^11 - 97240*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c)^9 + 90090*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 - 36
036*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 + 5775*(35*a^3 + 144*a
^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 252*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c))*sinh(d*x + c)^8 + 28*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^7 + 4*(12597*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 - 3889
6*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 + 45045*(35*a^3 + 144*a
^2*b + 384*a*b^2)*cosh(d*x + c)^8 - 24024*(35*a^3 + 144*a^2*b + 384*a*b^2)*
cosh(d*x + c)^6 + 5775*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 2
45*a^3 + 1008*a^2*b + 2688*a*b^2 - 504*(35*a^3 + 144*a^2*b + 384*a*b^2)*cos
h(d*x + c)^2)*sinh(d*x + c)^7 + 28*(969*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c)^13 - 3536*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 + 5
005*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 - 3432*(35*a^3 + 144*a
^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 1155*(35*a^3 + 144*a^2*b + 384*a*b^2)*c
osh(d*x + c)^5 - 168*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 + 7*(
35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 8*(35*a^3
+ 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^5 + 4*(2907*(35*a^3 + 144*a^2*b + 38
4*a*b^2)*cosh(d*x + c)^14 - 12376*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^12 + 21021*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 18018*
(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 8085*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^6 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(
d*x + c)^4 - 70*a^3 - 288*a^2*b - 768*a*b^2 + 147*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 4*(969*(35*a^3 + 144*a^2*b + 384
*a*b^2)*cosh(d*x + c)^15 - 4760*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x +
 c)^13 + 9555*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^11 - 10010*(35
*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 + 5775*(35*a^3 + 144*a^2*b +
384*a*b^2)*cosh(d*x + c)^7 - 1764*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x
 + c)^5 + 245*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^3 - 10*(35*a^3
 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 + (35*a^3 + 144*a^
2*b + 384*a*b^2)*cosh(d*x + c)^3 + (969*(35*a^3 + 144*a^2*b + 384*a*b^2)*co
sh(d*x + c)^16 - 5440*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^14 + 1
2740*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12 - 16016*(35*a^3 + 14
4*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 + 11550*(35*a^3 + 144*a^2*b + 384*a*b
^2)*cosh(d*x + c)^8 - 4704*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^6
 + 980*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 35*a^3 + 144*a^2*
b + 384*a*b^2 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d
x + c)^3 + (171(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^17 - 1088*(
35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^15 + 2940*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^13 - 4368*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh
(d*x + c)^11 + 3850*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^9 - 2016
*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^7 + 588*(35*a^3 + 144*a^2*b
 + 384*a*b^2)*cosh(d*x + c)^5 - 80*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*

```
x + c)^3 + 3*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^
2 + (19*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^18 - 136*(35*a^3 + 1
44*a^2*b + 384*a*b^2)*cosh(d*x + c)^16 + 420*(35*a^3 + 144*a^2*b + 384*a*b^
2)*cosh(d*x + c)^14 - 728*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^12
 + 770*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^10 - 504*(35*a^3 + 14
4*a^2*b + 384*a*b^2)*cosh(d*x + c)^8 + 196*(35*a^3 + 144*a^2*b + 384*a*b^2)
*cosh(d*x + c)^6 - 40*(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^4 + 3*
(35*a^3 + 144*a^2*b + 384*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))*log(cosh(d
*x + c) + sinh(d*x + c) - 1) + 4*(88*b^3*cosh(d*x + c)^21 - 1360*b^3*cosh(d
*x + c)^19 + 9*(105*a^3 + 432*a^2*b + 728*b^3)*cosh(d*x + c)^17 - 8*(805*a^
3 + 3312*a^2*b + 1880*b^3)*cosh(d*x + c)^15 + 7*(2681*a^3 + 7344*a^2*b + 25
12*b^3)*cosh(d*x + c)^13 - 6*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x +
c)^11 - 5*(5053*a^3 + 4464*a^2*b + 1232*b^3)*cosh(d*x + c)^9 + 4*(2681*a^3
+ 7344*a^2*b + 2512*b^3)*cosh(d*x + c)^7 - 3*(805*a^3 + 3312*a^2*b + 1880*b
^3)*cosh(d*x + c)^5 - 136*b^3*cosh(d*x + c) + 2*(105*a^3 + 432*a^2*b + 728*
b^3)*cosh(d*x + c)^3)*sinh(d*x + c))/(d*cosh(d*x + c)^19 + 19*d*cosh(d*x +
c)*sinh(d*x + c)^18 + d*sinh(d*x + c)^19 - 8*d*cosh(d*x + c)^17 + (171*d*co
sh(d*x + c)^2 - 8*d)*sinh(d*x + c)^17 + 17*(57*d*cosh(d*x + c)^3 - 8*d*cosh
(d*x + c))*sinh(d*x + c)^16 + 28*d*cosh(d*x + c)^15 + 4*(969*d*cosh(d*x + c
)^4 - 272*d*cosh(d*x + c)^2 + 7*d)*sinh(d*x + c)^15 + 4*(2907*d*cosh(d*x +
c)^5 - 1360*d*cosh(d*x + c)^3 + 105*d*cosh(d*x + c))*sinh(d*x + c)^14 - 56*
d*cosh(d*x + c)^13 + 28*(969*d*cosh(d*x + c)^6 - 680*d*cosh(d*x + c)^4 + 10
5*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^13 + 52*(969*d*cosh(d*x + c)^7 - 9
52*d*cosh(d*x + c)^5 + 245*d*cosh(d*x + c)^3 - 14*d*cosh(d*x + c))*sinh(d*x
 + c)^12 + 70*d*cosh(d*x + c)^11 + 2*(37791*d*cosh(d*x + c)^8 - 49504*d*cos
h(d*x + c)^6 + 19110*d*cosh(d*x + c)^4 - 2184*d*cosh(d*x + c)^2 + 35*d)*sin
h(d*x + c)^11 + 22*(4199*d*cosh(d*x + c)^9 - 7072*d*cosh(d*x + c)^7 + 3822*
d*cosh(d*x + c)^5 - 728*d*cosh(d*x + c)^3 + 35*d*cosh(d*x + c))*sinh(d*x +
c)^10 - 56*d*cosh(d*x + c)^9 + 2*(46189*d*cosh(d*x + c)^10 - 97240*d*cosh(d
*x + c)^8 + 70070*d*cosh(d*x + c)^6 - 20020*d*cosh(d*x + c)^4 + 1925*d*cosh
(d*x + c)^2 - 28*d)*sinh(d*x + c)^9 + 2*(37791*d*cosh(d*x + c)^11 - 97240*d
*cosh(d*x + c)^9 + 90090*d*cosh(d*x + c)^7 - 36036*d*cosh(d*x + c)^5 + 5775
*d*cosh(d*x + c)^3 - 252*d*cosh(d*x + c))*sinh(d*x + c)^8 + 28*d*cosh(d*x +
 c)^7 + 4*(12597*d*cosh(d*x + c)^12 - 38896*d*cosh(d*x + c)^10 + 45045*d*co
sh(d*x + c)^8 - 24024*d*cosh(d*x + c)^6 + 5775*d*cosh(d*x + c)^4 - 504*d*co
sh(d*x + c)^2 + 7*d)*sinh(d*x + c)^7 + 28*(969*d*cosh(d*x + c)^13 - 3536*d*
cosh(d*x + c)^11 + 5005*d*cosh(d*x + c)^9 - 3432*d*cosh(d*x + c)^7 + 1155*d
*cosh(d*x + c)^5 - 168*d*cosh(d*x + c)^3 + 7*d*cosh(d*x + c))*sinh(d*x + c)
^6 - 8*d*cosh(d*x + c)^5 + 4*(2907*d*cosh(d*x + c)^14 - 12376*d*cosh(d*x +
c)^12 + 21021*d*cosh(d*x + c)^10 - 18018*d*cosh(d*x + c)^8 + 8085*d*cosh(d*
x + c)^6 - 1764*d*cosh(d*x + c)^4 + 147*d*cosh(d*x + c)^2 - 2*d)*sinh(d*x +
 c)^5 + 4*(969*d*cosh(d*x + c)^15 - 4760*d*cosh(d*x + c)^13 + 9555*d*cosh(d
*x + c)^11 - 10010*d*cosh(d*x + c)^9 + 5775*d*cosh(d*x + c)^7 - 1764*d*cosh
(d*x + c)^5 + 245*d*cosh(d*x + c)^3 - 10*d*cosh(d*x + c))*sinh(d*x + c)^4 +
 d*cosh(d*x + c)^3 + (969*d*cosh(d*x + c)^16 - 5440*d*cosh(d*x + c)^14 + 12
740*d*cosh(d*x + c)^12 - 16016*d*cosh(d*x + c)^10 + 11550*d*cosh(d*x + c)^8
 - 4704*d*cosh(d*x + c)^6 + 980*d*cosh(d*x + c)^4 - 80*d*cosh(d*x + c)^2 +
d)*sinh(d*x + c)^3 + (171*d*cosh(d*x + c)^17 - 1088*d*cosh(d*x + c)^15 + 29
40*d*cosh(d*x + c)^13 - 4368*d*cosh(d*x + c)^11 + 3850*d*cosh(d*x + c)^9 -
2016*d*cosh(d*x + c)^7 + 588*d*cosh(d*x + c)^5 - 80*d*cosh(d*x + c)^3 + 3*d
*cosh(d*x + c))*sinh(d*x + c)^2 + (19*d*cosh(d*x + c)^18 - 136*d*cosh(d*x +
 c)^16 + 420*d*cosh(d*x + c)^14 - 728*d*cosh(d*x + c)^12 + 770*d*cosh(d*x +
 c)^10 - 504*d*cosh(d*x + c)^8 + 196*d*cosh(d*x + c)^6 - 40*d*cosh(d*x + c)
^4 + 3*d*cosh(d*x + c)^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**9*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70297, size = 473, normalized size = 2.77

$$-\frac{\left(35\,a^{3}+144\,a^{2}b+384\,ab^{2}\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{256\,d}+\frac{\left(35\,a^{3}+144\,a^{2}b+384\,ab^{2}\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}-2\right)}{256\,d}+\frac{b^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^9*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/256*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x - c) + 2)
/d + 1/256*(35*a^3 + 144*a^2*b + 384*a*b^2)*log(e^(d*x + c) + e^(-d*x - c)
- 2)/d + 1/24*(b^3*d^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12*b^3*d^2*(e^(d*x
+ c) + e^(-d*x - c)))/d^3 + 1/192*(105*a^3*(e^(d*x + c) + e^(-d*x - c))^7 +
 432*a^2*b*(e^(d*x + c) + e^(-d*x - c))^7 - 1540*a^3*(e^(d*x + c) + e^(-d*x
 - c))^5 - 6336*a^2*b*(e^(d*x + c) + e^(-d*x - c))^5 + 8176*a^3*(e^(d*x + c
) + e^(-d*x - c))^3 + 29952*a^2*b*(e^(d*x + c) + e^(-d*x - c))^3 - 17856*a^
3*(e^(d*x + c) + e^(-d*x - c)) - 46080*a^2*b*(e^(d*x + c) + e^(-d*x - c)))/
(((e^(d*x + c) + e^(-d*x - c))^2 - 4)^4*d)
```

3.215 $\int \mathrm{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=189

$$\frac{3a\left(21a^2+80ab+128b^2\right)\tanh^{-1}(\cosh(c+dx))}{256d}-\frac{3a\left(21a^2+80ab+128b^2\right)\coth(c+dx)\mathrm{csch}(c+dx)}{256d}-\frac{a^2(21a+80b)}{}$$

```
[Out] (3*a*(21*a^2 + 80*a*b + 128*b^2)*ArcTanh[Cosh[c + d*x]])/(256*d) + (b^3*Cos
h[c + d*x])/d - (3*a*(21*a^2 + 80*a*b + 128*b^2)*Coth[c + d*x]*Csch[c + d*x
])/(256*d) + (a^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^3)/(128*d) - (a
^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(160*d) + (9*a^3*Coth[c + d
*x]*Csch[c + d*x]^7)/(80*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^9)/(10*d)
```

Rubi [A] time = 0.369044, antiderivative size = 189, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 388, 206}

$$\frac{3a\left(21a^2+80ab+128b^2\right)\tanh^{-1}(\cosh(c+dx))}{256d}-\frac{3a\left(21a^2+80ab+128b^2\right)\coth(c+dx)\mathrm{csch}(c+dx)}{256d}-\frac{a^2(21a+80b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] (3*a*(21*a^2 + 80*a*b + 128*b^2)*ArcTanh[Cosh[c + d*x]])/(256*d) + (b^3*Cos
h[c + d*x])/d - (3*a*(21*a^2 + 80*a*b + 128*b^2)*Coth[c + d*x]*Csch[c + d*x
])/(256*d) + (a^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^3)/(128*d) - (a
^2*(21*a + 80*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(160*d) + (9*a^3*Coth[c + d
*x]*Csch[c + d*x]^7)/(80*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^9)/(10*d)
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^{11}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^6}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{10d} - \frac{\operatorname{Subst}\left(\int \frac{-9a^3-30a^2b-30ab^2-10b^3+10b\left(3a^2\right.}{}\right.}{}$$

$$= \frac{9a^3\coth(c+dx)\operatorname{csch}^7(c+dx)}{80d} - \frac{a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{10d} + \frac{\operatorname{Sub}}{}$$

$$= -\frac{a^2(21a+80b)\coth(c+dx)\operatorname{csch}^5(c+dx)}{160d} + \frac{9a^3\coth(c+dx)\operatorname{csch}^7(c}{80d}$$

$$= \frac{a^2(21a+80b)\coth(c+dx)\operatorname{csch}^3(c+dx)}{128d} - \frac{a^2(21a+80b)\coth(c+dx}{160d}$$

$$= -\frac{3a\left(21a^2+80ab+128b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{256d} + \frac{a^2(21a+80b)}{}$$

$$= \frac{b^3\cosh(c+dx)}{d} - \frac{3a\left(21a^2+80ab+128b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{256d}$$

$$= \frac{3a\left(21a^2+80ab+128b^2\right)\tanh^{-1}(\cosh(c+dx))}{256d} + \frac{b^3\cosh(c+dx)}{d} - \frac{3}{}$$

Mathematica [A] time = 2.47304, size = 265, normalized size = 1.4

$$\frac{b^3\cosh(c+dx)}{d} - \frac{a\left(60\left(21a^2+80ab+128b^2\right)\operatorname{csch}^2\left(\frac{1}{2}(c+dx)\right)+60\left(21a^2+80ab+128b^2\right)\operatorname{sech}^2\left(\frac{1}{2}(c+dx)\right)+24}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^11*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b^3*Cosh[c + d*x])/d - (a*(60*(21*a^2 + 80*a*b + 128*b^2)*Csch[(c + d*x)/2
]^2 - 40*a*(7*a + 24*b)*Csch[(c + d*x)/2]^4 + 10*a*(7*a + 16*b)*Csch[(c + d
*x)/2]^6 - 15*a^2*Csch[(c + d*x)/2]^8 + 2*a^2*Csch[(c + d*x)/2]^10 + 240*(2
1*a^2 + 80*a*b + 128*b^2)*Log[Tanh[(c + d*x)/2]] + 60*(21*a^2 + 80*a*b + 12
8*b^2)*Sech[(c + d*x)/2]^2 + 40*a*(7*a + 24*b)*Sech[(c + d*x)/2]^4 + 10*a*(
7*a + 16*b)*Sech[(c + d*x)/2]^6 + 15*a^2*Sech[(c + d*x)/2]^8 + 2*a^2*Sech[(
```

```
c + d*x)/2]^10))/(20480*d)
```

Maple [A] time = 0.081, size = 166, normalized size = 0.9

$$\frac{1}{d}\left(a^3\left(\left(-\frac{(\operatorname{csch}(dx+c))^9}{10}+\frac{9(\operatorname{csch}(dx+c))^7}{80}-\frac{21(\operatorname{csch}(dx+c))^5}{160}+\frac{21(\operatorname{csch}(dx+c))^3}{128}-\frac{63\operatorname{csch}(dx+c)}{256}\right)\coth(dx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/10*csch(d*x+c)^9+9/80*csch(d*x+c)^7-21/160*csch(d*x+c)^5+21/1
28*csch(d*x+c)^3-63/256*csch(d*x+c))*coth(d*x+c)+63/128*arctanh(exp(d*x+c))
)+3*a^2*b*((-1/6*csch(d*x+c)^5+5/24*csch(d*x+c)^3-5/16*csch(d*x+c))*coth(d*
x+c)+5/8*arctanh(exp(d*x+c)))+3*a*b^2*(-1/2*csch(d*x+c)*coth(d*x+c)+arctanh
(exp(d*x+c)))+b^3*cosh(d*x+c))
```

Maxima [B] time = 1.14825, size = 774, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*(e^(d*x + c)/d + e^(-d*x - c)/d) + 1/1280*a^3*(315*log(e^(-d*x - c)
 + 1)/d - 315*log(e^(-d*x - c) - 1)/d + 2*(315*e^(-d*x - c) - 3045*e^(-3*d*
x - 3*c) + 13188*e^(-5*d*x - 5*c) - 33660*e^(-7*d*x - 7*c) + 55970*e^(-9*d*
x - 9*c) + 55970*e^(-11*d*x - 11*c) - 33660*e^(-13*d*x - 13*c) + 13188*e^(-
15*d*x - 15*c) - 3045*e^(-17*d*x - 17*c) + 315*e^(-19*d*x - 19*c))/(d*(10*e
^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) + 120*e^(-6*d*x - 6*c) - 210*e^(-8*d*
x - 8*c) + 252*e^(-10*d*x - 10*c) - 210*e^(-12*d*x - 12*c) + 120*e^(-14*d*x
 - 14*c) - 45*e^(-16*d*x - 16*c) + 10*e^(-18*d*x - 18*c) - e^(-20*d*x - 20*
c) - 1))) + 1/16*a^2*b*(15*log(e^(-d*x - c) + 1)/d - 15*log(e^(-d*x - c) -
1)/d + 2*(15*e^(-d*x - c) - 85*e^(-3*d*x - 3*c) + 198*e^(-5*d*x - 5*c) + 19
8*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) + 15*e^(-11*d*x - 11*c))/(d*(6*e^(
-2*d*x - 2*c) - 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) - 15*e^(-8*d*x -
8*c) + 6*e^(-10*d*x - 10*c) - e^(-12*d*x - 12*c) - 1))) + 3/2*a*b^2*(log(e^
(-d*x - c) + 1)/d - log(e^(-d*x - c) - 1)/d + 2*(e^(-d*x - c) + e^(-3*d*x -
 3*c))/(d*(2*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c) - 1)))
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**11*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70407, size = 653, normalized size = 3.46

$$\frac{b^3\left(e^{(dx+c)}+e^{(-dx-c)}\right)}{2\,d}+\frac{3\left(21\,a^3+80\,a^2b+128\,ab^2\right)\log\left(e^{(dx+c)}+e^{(-dx-c)}+2\right)}{512\,d}-\frac{3\left(21\,a^3+80\,a^2b+128\,ab^2\right)\log\left(e^{(dx+\cdots}\right.}{512\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^11*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*(e^(d*x + c) + e^(-d*x - c))/d + 3/512*(21*a^3 + 80*a^2*b + 128*a*b
^2)*log(e^(d*x + c) + e^(-d*x - c) + 2)/d - 3/512*(21*a^3 + 80*a^2*b + 128*
a*b^2)*log(e^(d*x + c) + e^(-d*x - c) - 2)/d - 1/640*(315*a^3*(e^(d*x + c)
+ e^(-d*x - c))^9 + 1200*a^2*b*(e^(d*x + c) + e^(-d*x - c))^9 + 1920*a*b^2*
(e^(d*x + c) + e^(-d*x - c))^9 - 5880*a^3*(e^(d*x + c) + e^(-d*x - c))^7 -
22400*a^2*b*(e^(d*x + c) + e^(-d*x - c))^7 - 30720*a*b^2*(e^(d*x + c) + e^(
-d*x - c))^7 + 43008*a^3*(e^(d*x + c) + e^(-d*x - c))^5 + 163840*a^2*b*(e^(
d*x + c) + e^(-d*x - c))^5 + 184320*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 -
151680*a^3*(e^(d*x + c) + e^(-d*x - c))^3 - 542720*a^2*b*(e^(d*x + c) + e^(
-d*x - c))^3 - 491520*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 + 247040*a^3*(e^
(d*x + c) + e^(-d*x - c)) + 675840*a^2*b*(e^(d*x + c) + e^(-d*x - c)) + 491
520*a*b^2*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))^2 -
4)^5*d)
```

3.216 $\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=220

$$-\frac{\left(840a^2b+231a^3+1152ab^2+1024b^3\right)\tanh^{-1}(\cosh(c+dx))}{1024d}-\frac{a\left(77a^2+280ab+384b^2\right)\coth(c+dx)\operatorname{csch}^3(c+dx)}{512d}+$$

```
[Out] -((231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*ArcTanh[Cosh[c + d*x]])/(10
24*d) + (3*a*(77*a^2 + 280*a*b + 384*b^2)*Coth[c + d*x]*Csch[c + d*x])/(102
4*d) - (a*(77*a^2 + 280*a*b + 384*b^2)*Coth[c + d*x]*Csch[c + d*x]^3)/(512*
d) + (7*a^2*(11*a + 40*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(640*d) - (3*a^2*(
11*a + 40*b)*Coth[c + d*x]*Csch[c + d*x]^7)/(320*d) + (11*a^3*Coth[c + d*x]
*Csch[c + d*x]^9)/(120*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^11)/(12*d)
```

Rubi [A] time = 0.397589, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3215, 1157, 1814, 385, 206}

$$-\frac{\left(840a^2b+231a^3+1152ab^2+1024b^3\right)\tanh^{-1}(\cosh(c+dx))}{1024d}-\frac{a\left(77a^2+280ab+384b^2\right)\coth(c+dx)\operatorname{csch}^3(c+dx)}{512d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] -((231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*ArcTanh[Cosh[c + d*x]])/(10
24*d) + (3*a*(77*a^2 + 280*a*b + 384*b^2)*Coth[c + d*x]*Csch[c + d*x])/(102
4*d) - (a*(77*a^2 + 280*a*b + 384*b^2)*Coth[c + d*x]*Csch[c + d*x]^3)/(512*
d) + (7*a^2*(11*a + 40*b)*Coth[c + d*x]*Csch[c + d*x]^5)/(640*d) - (3*a^2*(
11*a + 40*b)*Coth[c + d*x]*Csch[c + d*x]^7)/(320*d) + (11*a^3*Coth[c + d*x]
*Csch[c + d*x]^9)/(120*d) - (a^3*Coth[c + d*x]*Csch[c + d*x]^11)/(12*d)
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
```

```
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^{13}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-2bx^2+bx^4\right)^3}{\left(1-x^2\right)^7}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+dx)}{12d} + \frac{\text{Subst}\left(\int \frac{-11a^3-36a^2b-36ab^2-12b^3+12b(3}{}\right.}{}$$

$$= \frac{11a^3\coth(c+dx)\operatorname{csch}^9(c+dx)}{120d} - \frac{a^3\coth(c+dx)\operatorname{csch}^{11}(c+dx)}{12d} - \frac{\text{Su}}{}$$

$$= -\frac{3a^2(11a+40b)\coth(c+dx)\operatorname{csch}^7(c+dx)}{320d} + \frac{11a^3\coth(c+dx)\operatorname{csch}^9}{120d}$$

$$= \frac{7a^2(11a+40b)\coth(c+dx)\operatorname{csch}^5(c+dx)}{640d} - \frac{3a^2(11a+40b)\coth(c+}{320d}$$

$$= -\frac{a\left(77a^2+280ab+384b^2\right)\coth(c+dx)\operatorname{csch}^3(c+dx)}{512d} + \frac{7a^2(11a+40}{}$$

$$= \frac{3a\left(77a^2+280ab+384b^2\right)\coth(c+dx)\operatorname{csch}(c+dx)}{1024d} - \frac{a\left(77a^2+280\right.}{}$$

$$= -\frac{\left(231a^3+840a^2b+1152ab^2+1024b^3\right)\tanh^{-1}(\cosh(c+dx))}{1024d} + \frac{3a\left(7\right.}{}$$

Mathematica [A] time = 2.06945, size = 246, normalized size = 1.12

$$\frac{15360\left(840a^2b+231a^3+1152ab^2+1024b^3\right)\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+2a\left(750629a^2+2074200ab+1422720b^2\right)\cosh}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^13*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-30*a*(76555*a^2 + 75816*a*b + 45696*b^2)*Coth[c + d*x]*Csch[c + d*x]^11 +
 2*a*(750629*a^2 + 2074200*a*b + 1422720*b^2)*Cosh[3*(c + d*x)]*Csch[c + d*
```

```
x]^12 - 9*a*(77099*a^2 + 280360*a*b + 246400*b^2)*Cosh[5*(c + d*x)]*Csch[c
+ d*x]^12 + 63*a*(3421*a^2 + 12440*a*b + 14720*b^2)*Cosh[7*(c + d*x)]*Csch[
c + d*x]^12 - 525*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[9*(c + d*x)]*Csch[c +
 d*x]^12 + 45*a*(77*a^2 + 280*a*b + 384*b^2)*Cosh[11*(c + d*x)]*Csch[c + d*
x]^12 + 15360*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*Log[Tanh[(c + d
*x)/2]])/(15728640*d)
```

Maple [A] time = 0.085, size = 202, normalized size = 0.9

$$\frac{1}{d}\left(a^3\left(\left(-\frac{(\operatorname{csch}(dx+c))^{11}}{12}+\frac{11\,(\operatorname{csch}(dx+c))^{9}}{120}-\frac{33\,(\operatorname{csch}(dx+c))^{7}}{320}+\frac{77\,(\operatorname{csch}(dx+c))^{5}}{640}-\frac{77\,(\operatorname{csch}(dx+c))^{3}}{512}+2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*((-1/12*csch(d*x+c)^11+11/120*csch(d*x+c)^9-33/320*csch(d*x+c)^7+7
7/640*csch(d*x+c)^5-77/512*csch(d*x+c)^3+231/1024*csch(d*x+c))*coth(d*x+c)-
231/512*arctanh(exp(d*x+c)))+3*a^2*b*((-1/8*csch(d*x+c)^7+7/48*csch(d*x+c)^
5-35/192*csch(d*x+c)^3+35/128*csch(d*x+c))*coth(d*x+c)-35/64*arctanh(exp(d*
x+c)))+3*a*b^2*((-1/4*csch(d*x+c)^3+3/8*csch(d*x+c))*coth(d*x+c)-3/4*arctan
h(exp(d*x+c)))-2*b^3*arctanh(exp(d*x+c)))
```

Maxima [B] time = 1.15229, size = 972, normalized size = 4.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/15360*a^3*(3465*log(e^(-d*x - c) + 1)/d - 3465*log(e^(-d*x - c) - 1)/d +
 2*(3465*e^(-d*x - c) - 40425*e^(-3*d*x - 3*c) + 215523*e^(-5*d*x - 5*c) -
693891*e^(-7*d*x - 7*c) + 1501258*e^(-9*d*x - 9*c) - 2296650*e^(-11*d*x - 1
1*c) - 2296650*e^(-13*d*x - 13*c) + 1501258*e^(-15*d*x - 15*c) - 693891*e^(
-17*d*x - 17*c) + 215523*e^(-19*d*x - 19*c) - 40425*e^(-21*d*x - 21*c) + 34
65*e^(-23*d*x - 23*c))/(d*(12*e^(-2*d*x - 2*c) - 66*e^(-4*d*x - 4*c) + 220*
e^(-6*d*x - 6*c) - 495*e^(-8*d*x - 8*c) + 792*e^(-10*d*x - 10*c) - 924*e^(-
12*d*x - 12*c) + 792*e^(-14*d*x - 14*c) - 495*e^(-16*d*x - 16*c) + 220*e^(-
18*d*x - 18*c) - 66*e^(-20*d*x - 20*c) + 12*e^(-22*d*x - 22*c) - e^(-24*d*x
 - 24*c) - 1))) - 1/128*a^2*b*(105*log(e^(-d*x - c) + 1)/d - 105*log(e^(-d*
x - c) - 1)/d + 2*(105*e^(-d*x - c) - 805*e^(-3*d*x - 3*c) + 2681*e^(-5*d*x
 - 5*c) - 5053*e^(-7*d*x - 7*c) - 5053*e^(-9*d*x - 9*c) + 2681*e^(-11*d*x -
 11*c) - 805*e^(-13*d*x - 13*c) + 105*e^(-15*d*x - 15*c))/(d*(8*e^(-2*d*x -
 2*c) - 28*e^(-4*d*x - 4*c) + 56*e^(-6*d*x - 6*c) - 70*e^(-8*d*x - 8*c) + 5
6*e^(-10*d*x - 10*c) - 28*e^(-12*d*x - 12*c) + 8*e^(-14*d*x - 14*c) - e^(-1
6*d*x - 16*c) - 1))) - 3/8*a*b^2*(3*log(e^(-d*x - c) + 1)/d - 3*log(e^(-d*x
 - c) - 1)/d + 2*(3*e^(-d*x - c) - 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x - 5*c
) + 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) - 6*e^(-4*d*x - 4*c) + 4*e^(
-6*d*x - 6*c) - e^(-8*d*x - 8*c) - 1))) - b^3*(log(e^(-d*x - c) + 1)/d - lo
g(e^(-d*x - c) - 1)/d)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**13*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.65125, size = 730, normalized size = 3.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^13*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/2048*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log(e^(d*x + c) + e^(
-d*x - c) + 2)/d + 1/2048*(231*a^3 + 840*a^2*b + 1152*a*b^2 + 1024*b^3)*log
(e^(d*x + c) + e^(-d*x - c) - 2)/d + 1/7680*(3465*a^3*(e^(d*x + c) + e^(-d*
x - c))^11 + 12600*a^2*b*(e^(d*x + c) + e^(-d*x - c))^11 + 17280*a*b^2*(e^(
d*x + c) + e^(-d*x - c))^11 - 78540*a^3*(e^(d*x + c) + e^(-d*x - c))^9 - 28
5600*a^2*b*(e^(d*x + c) + e^(-d*x - c))^9 - 391680*a*b^2*(e^(d*x + c) + e^(
-d*x - c))^9 + 731808*a^3*(e^(d*x + c) + e^(-d*x - c))^7 + 2661120*a^2*b*(e
^(d*x + c) + e^(-d*x - c))^7 + 3502080*a*b^2*(e^(d*x + c) + e^(-d*x - c))^7
 - 3560832*a^3*(e^(d*x + c) + e^(-d*x - c))^5 - 12948480*a^2*b*(e^(d*x + c)
 + e^(-d*x - c))^5 - 15482880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^5 + 939136
0*a^3*(e^(d*x + c) + e^(-d*x - c))^3 + 32839680*a^2*b*(e^(d*x + c) + e^(-d*
x - c))^3 + 33914880*a*b^2*(e^(d*x + c) + e^(-d*x - c))^3 - 12180480*a^3*(e
^(d*x + c) + e^(-d*x - c)) - 34283520*a^2*b*(e^(d*x + c) + e^(-d*x - c)) -
29491200*a*b^2*(e^(d*x + c) + e^(-d*x - c)))/(((e^(d*x + c) + e^(-d*x - c))
^2 - 4)^6*d)
```

3.217 $\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=255

$$\frac{b\left(1920a^2+12312ab+10579b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{3840d}-\frac{b\left(4992a^2+10728ab+5549b^2\right)\sinh(c+dx)\cosh^3(c+d\ldots}{3072d}$$

```
[Out] -((1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x)/2048 + ((1024*a^3 + 422
4*a^2*b + 4632*a*b^2 + 1619*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(2048*d) - (b
*(4992*a^2 + 10728*a*b + 5549*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(3072*d)
+ (b*(1920*a^2 + 12312*a*b + 10579*b^2)*Cosh[c + d*x]^5*Sinh[c + d*x])/(384
0*d) - (b^2*(6888*a + 11821*b)*Cosh[c + d*x]^7*Sinh[c + d*x])/(4480*d) + (b
^2*(504*a + 2593*b)*Cosh[c + d*x]^9*Sinh[c + d*x])/(1680*d) - (85*b^3*Cosh[
c + d*x]^11*Sinh[c + d*x])/(168*d) + (b^3*Cosh[c + d*x]^13*Sinh[c + d*x])/(
14*d)
```

Rubi [A] time = 0.557087, antiderivative size = 255, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3217, 1257, 1814, 1157, 385, 206}

$$\frac{b\left(1920a^2+12312ab+10579b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{3840d}-\frac{b\left(4992a^2+10728ab+5549b^2\right)\sinh(c+dx)\cosh^3(c+d\ldots}{3072d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] -((1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*x)/2048 + ((1024*a^3 + 422
4*a^2*b + 4632*a*b^2 + 1619*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(2048*d) - (b
*(4992*a^2 + 10728*a*b + 5549*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(3072*d)
+ (b*(1920*a^2 + 12312*a*b + 10579*b^2)*Cosh[c + d*x]^5*Sinh[c + d*x])/(384
0*d) - (b^2*(6888*a + 11821*b)*Cosh[c + d*x]^7*Sinh[c + d*x])/(4480*d) + (b
^2*(504*a + 2593*b)*Cosh[c + d*x]^9*Sinh[c + d*x])/(1680*d) - (85*b^3*Cosh[
c + d*x]^11*Sinh[c + d*x])/(168*d) + (b^3*Cosh[c + d*x]^13*Sinh[c + d*x])/(
14*d)
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1257

```
Int[(x_)^(m_.)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)
^4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
 + e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[1/(2*e^(2*p + m/2)*
(q + 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*e^(2*p + m/2)*
(q + 1)*x^m*(a + b*x^2 + c*x^4)^p - (-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^
p*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; FreeQ[{a, b, c, d, e}
, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1] && IGtQ[m/2, 0]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \sinh^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(a-2ax^2+(a+b)x^4\right)^3}{\left(1-x^2\right)^8}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^{13}(c+dx)\sinh(c+dx)}{14d} + \frac{\text{Subst}\left(\int \frac{-b^3+14\left(a^3-b^3\right)x^2-14\left(5a^3+b^3\right)x^4+14(10a}{}\right.}{}$$

$$= -\frac{85b^3\cosh^{11}(c+dx)\sinh(c+dx)}{168d} + \frac{b^3\cosh^{13}(c+dx)\sinh(c+dx)}{14d} - \frac{\text{Sub}}{}$$

$$= \frac{b^2(504a+2593b)\cosh^9(c+dx)\sinh(c+dx)}{1680d} - \frac{85b^3\cosh^{11}(c+dx)\sinh(c}{168d}$$

$$= -\frac{b^2(6888a+11821b)\cosh^7(c+dx)\sinh(c+dx)}{4480d} + \frac{b^2(504a+2593b)\cosh}{168}$$

$$= \frac{b\left(1920a^2+12312ab+10579b^2\right)\cosh^5(c+dx)\sinh(c+dx)}{3840d} - \frac{b^2(6888a+}{}$$

$$= -\frac{b\left(4992a^2+10728ab+5549b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{3072d} + \frac{b\left(1920a^2+\right.}{}$$

$$= \frac{\left(1024a^3+4224a^2b+4632ab^2+1619b^3\right)\cosh(c+dx)\sinh(c+dx)}{2048d} - \frac{b\left(49\right.}{}$$

$$= -\frac{\left(1024a^3+1920a^2b+1512ab^2+429b^3\right)x}{2048} + \frac{\left(1024a^3+4224a^2b+4632ab\right.}{}$$

Mathematica [A] time = 0.680134, size = 189, normalized size = 0.74

$$\frac{-840\left(1920a^2b+1024a^3+1512ab^2+429b^3\right)(c+dx)-105b\left(2304a^2+2880ab+1001b^2\right)\sinh(4(c+dx))+35b\left(768a^2\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-840*(1024*a^3 + 1920*a^2*b + 1512*a*b^2 + 429*b^3)*(c + d*x) + 105*(4096*
a^3 + 11520*a^2*b + 10080*a*b^2 + 3003*b^3)*Sinh[2*(c + d*x)] - 105*b*(2304
*a^2 + 2880*a*b + 1001*b^2)*Sinh[4*(c + d*x)] + 35*b*(768*a^2 + 2160*a*b +
1001*b^2)*Sinh[6*(c + d*x)] - 105*b^2*(120*a + 91*b)*Sinh[8*(c + d*x)] + 21
*b^2*(48*a + 91*b)*Sinh[10*(c + d*x)] - 245*b^3*Sinh[12*(c + d*x)] + 15*b^3
*Sinh[14*(c + d*x)])/(1720320*d)
```

Maple [A] time = 0.054, size = 240, normalized size = 0.9

$$\frac{1}{d}\left(b^3\left(\left(\frac{(\sinh(dx+c))^{13}}{14} - \frac{13\,(\sinh(dx+c))^{11}}{168} + \frac{143\,(\sinh(dx+c))^9}{1680} - \frac{429\,(\sinh(dx+c))^7}{4480} + \frac{143\,(\sinh(dx+c))^5}{1280} - \right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)
```

[Out] 1/d*(b^3*((1/14*sinh(d*x+c)^13-13/168*sinh(d*x+c)^11+143/1680*sinh(d*x+c)^9 -429/4480*sinh(d*x+c)^7+143/1280*sinh(d*x+c)^5-143/1024*sinh(d*x+c)^3+429/2 048*sinh(d*x+c))*cosh(d*x+c)-429/2048*d*x-429/2048*c)+3*a*b^2*((1/10*sinh(d *x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d*x+c)^5-21/128*sinh(d*x+c)^3+63/256 *sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/256*c)+3*a^2*b*((1/6*sinh(d*x+c)^5- 5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*c)+a^3*(1/2* cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c))

Maxima [A] time = 1.21768, size = 597, normalized size = 2.34

$$-\frac{1}{8}a^3\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{3440640}b^3\left(\frac{\left(245e^{(-2dx-2c)}-1911e^{(-4dx-4c)}+9555e^{(-6dx-6c)}-35035e^{(-8dx-8}\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] -1/8*a^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/3440640*b^3*((2 45*e^(-2*d*x - 2*c) - 1911*e^(-4*d*x - 4*c) + 9555*e^(-6*d*x - 6*c) - 35035 *e^(-8*d*x - 8*c) + 105105*e^(-10*d*x - 10*c) - 315315*e^(-12*d*x - 12*c) - 15)*e^(14*d*x + 14*c)/d + 720720*(d*x + c)/d + (315315*e^(-2*d*x - 2*c) - 105105*e^(-4*d*x - 4*c) + 35035*e^(-6*d*x - 6*c) - 9555*e^(-8*d*x - 8*c) + 1911*e^(-10*d*x - 10*c) - 245*e^(-12*d*x - 12*c) + 15*e^(-14*d*x - 14*c))/d) - 3/20480*a*b^2*((25*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6* d*x - 6*c) - 2100*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 1/128*a^2*b*((9*e^(-2*d* x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d)

Fricas [B] time = 1.40475, size = 1688, normalized size = 6.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] 1/860160*(105*b^3*cosh(d*x + c)*sinh(d*x + c)^13 + 210*(13*b^3*cosh(d*x + c)^3 - 7*b^3*cosh(d*x + c))*sinh(d*x + c)^11 + 35*(429*b^3*cosh(d*x + c)^5 - 770*b^3*cosh(d*x + c)^3 + 3*(48*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d*x + c)^9 + 60*(429*b^3*cosh(d*x + c)^7 - 1617*b^3*cosh(d*x + c)^5 + 21*(48*a*b^ 2 + 91*b^3)*cosh(d*x + c)^3 - 7*(120*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh(d* x + c)^7 + 21*(715*b^3*cosh(d*x + c)^9 - 4620*b^3*cosh(d*x + c)^7 + 126*(48 *a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 140*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^3 + 5*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 7 0*(39*b^3*cosh(d*x + c)^11 - 385*b^3*cosh(d*x + c)^9 + 18*(48*a*b^2 + 91*b^ 3)*cosh(d*x + c)^7 - 42*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^5 + 5*(768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 - 3*(2304*a^2*b + 2880*a*b^2 + 10 01*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 420*(1024*a^3 + 1920*a^2*b + 1512* a*b^2 + 429*b^3)*d*x + 105*(b^3*cosh(d*x + c)^13 - 14*b^3*cosh(d*x + c)^11 + (48*a*b^2 + 91*b^3)*cosh(d*x + c)^9 - 4*(120*a*b^2 + 91*b^3)*cosh(d*x + c)^7 + (768*a^2*b + 2160*a*b^2 + 1001*b^3)*cosh(d*x + c)^5 - 2*(2304*a^2*b + 2880*a*b^2 + 1001*b^3)*cosh(d*x + c)^3 + (4096*a^3 + 11520*a^2*b + 10080*a

```
*b^2 + 3003*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70269, size = 756, normalized size = 2.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/3440640*(15*b^3*e^(14*d*x + 14*c) - 245*b^3*e^(12*d*x + 12*c) + 1008*a*b^
2*e^(10*d*x + 10*c) + 1911*b^3*e^(10*d*x + 10*c) - 12600*a*b^2*e^(8*d*x + 8
*c) - 9555*b^3*e^(8*d*x + 8*c) + 26880*a^2*b*e^(6*d*x + 6*c) + 75600*a*b^2*
e^(6*d*x + 6*c) + 35035*b^3*e^(6*d*x + 6*c) - 241920*a^2*b*e^(4*d*x + 4*c)
- 302400*a*b^2*e^(4*d*x + 4*c) - 105105*b^3*e^(4*d*x + 4*c) + 430080*a^3*e^
(2*d*x + 2*c) + 1209600*a^2*b*e^(2*d*x + 2*c) + 1058400*a*b^2*e^(2*d*x + 2*
c) + 315315*b^3*e^(2*d*x + 2*c) - 1680*(1024*a^3 + 1920*a^2*b + 1512*a*b^2
+ 429*b^3)*(d*x + c) + (2230272*a^3*e^(14*d*x + 14*c) + 4181760*a^2*b*e^(14
*d*x + 14*c) + 3293136*a*b^2*e^(14*d*x + 14*c) + 934362*b^3*e^(14*d*x + 14*
c) - 430080*a^3*e^(12*d*x + 12*c) - 1209600*a^2*b*e^(12*d*x + 12*c) - 10584
00*a*b^2*e^(12*d*x + 12*c) - 315315*b^3*e^(12*d*x + 12*c) + 241920*a^2*b*e^
(10*d*x + 10*c) + 302400*a*b^2*e^(10*d*x + 10*c) + 105105*b^3*e^(10*d*x + 1
0*c) - 26880*a^2*b*e^(8*d*x + 8*c) - 75600*a*b^2*e^(8*d*x + 8*c) - 35035*b^
3*e^(8*d*x + 8*c) + 12600*a*b^2*e^(6*d*x + 6*c) + 9555*b^3*e^(6*d*x + 6*c)
- 1008*a*b^2*e^(4*d*x + 4*c) - 1911*b^3*e^(4*d*x + 4*c) + 245*b^3*e^(2*d*x
+ 2*c) - 15*b^3)*e^(-14*d*x - 14*c))/d
```

3.218 $\int \left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=211

$$\frac{b\left(1152a^2+3912ab+2279b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{1536d}-\frac{b\left(1920a^2+2232ab+793b^2\right)\sinh(c+dx)\cosh(c+dx)}{1024d}$$

```
[Out] ((1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x)/1024 - (b*(1920*a^2 + 223
2*a*b + 793*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(1024*d) + (b*(1152*a^2 + 391
2*a*b + 2279*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(1536*d) - (b^2*(3000*a +
3481*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(1920*d) + (3*b^2*(40*a + 139*b)*Cos
h[c + d*x]^7*Sinh[c + d*x])/(320*d) - (61*b^3*Cosh[c + d*x]^9*Sinh[c + d*x]
)/(120*d) + (b^3*Cosh[c + d*x]^11*Sinh[c + d*x])/(12*d)
```

Rubi [A] time = 0.386844, antiderivative size = 211, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3209, 1157, 1814, 385, 206}

$$\frac{b\left(1152a^2+3912ab+2279b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{1536d}-\frac{b\left(1920a^2+2232ab+793b^2\right)\sinh(c+dx)\cosh(c+dx)}{1024d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^4)^3,x]
```



```
[Out] ((1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*x)/1024 - (b*(1920*a^2 + 223
2*a*b + 793*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(1024*d) + (b*(1152*a^2 + 391
2*a*b + 2279*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(1536*d) - (b^2*(3000*a +
3481*b)*Cosh[c + d*x]^5*Sinh[c + d*x])/(1920*d) + (3*b^2*(40*a + 139*b)*Cos
h[c + d*x]^7*Sinh[c + d*x])/(320*d) - (61*b^3*Cosh[c + d*x]^9*Sinh[c + d*x]
)/(120*d) + (b^3*Cosh[c + d*x]^11*Sinh[c + d*x])/(12*d)
```


Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 1814

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{Q = PolynomialQuot
ient[Pq, a + b*x^2, x], f = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x,
 0], g = Coeff[PolynomialRemainder[Pq, a + b*x^2, x], x, 1]}, Simp[((a*g -
b*f*x)*(a + b*x^2)^(p + 1))/(2*a*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int
[(a + b*x^2)^(p + 1)*ExpandToSum[2*a*(p + 1)*Q + f*(2*p + 3), x], x], x]] /
```

```
; FreeQ[{a, b}, x] && PolyQ[Pq, x] && LtQ[p, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \left(a+b\sinh^4(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{\left(1-x^2\right)^7} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^{11}(c+dx)\sinh(c+dx)}{12d} - \frac{\text{Subst}\left(\int \frac{-12a^3+b^3+12\left(5a^3+b^3\right)x^2-12\left(10a^3+3a^2b-b^3\right)x^4+12\left(10a^3+9\right.}{\left(1-x^2\right)^6}\right.}{12}$$

$$= -\frac{61b^3\cosh^9(c+dx)\sinh(c+dx)}{120d} + \frac{b^3\cosh^{11}(c+dx)\sinh(c+dx)}{12d} + \frac{\text{Subst}\left(\int \frac{3\left(40a^3+17\right.}{}\right.}{}$$

$$= \frac{3b^2(40a+139b)\cosh^7(c+dx)\sinh(c+dx)}{320d} - \frac{61b^3\cosh^9(c+dx)\sinh(c+dx)}{120d} + \frac{b^3\cosh}{}$$

$$= -\frac{b^2(3000a+3481b)\cosh^5(c+dx)\sinh(c+dx)}{1920d} + \frac{3b^2(40a+139b)\cosh^7(c+dx)\sinh(c}{320d}$$

$$= \frac{b\left(1152a^2+3912ab+2279b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{1536d} - \frac{b^2(3000a+3481b)\cosh^5(c-}{1920d}$$

$$= -\frac{b\left(1920a^2+2232ab+793b^2\right)\cosh(c+dx)\sinh(c+dx)}{1024d} + \frac{b\left(1152a^2+3912ab+2279b^{\cdot}\right.}{153}$$

$$= \frac{\left(1024a^3+1152a^2b+840ab^2+231b^3\right)x}{1024} - \frac{b\left(1920a^2+2232ab+793b^2\right)\cosh(c+dx)\sin}{1024d}$$

Mathematica [A] time = 0.389919, size = 156, normalized size = 0.74

$$\frac{120\left(1152a^2b+1024a^3+840ab^2+231b^3\right)(c+dx)-720b\left(128a^2+112ab+33b^2\right)\sinh(2(c+dx))+45b\left(256a^2+448ab\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (120*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*(c + d*x) - 720*b*(128*a
^2 + 112*a*b + 33*b^2)*Sinh[2*(c + d*x)] + 45*b*(256*a^2 + 448*a*b + 165*b^
2)*Sinh[4*(c + d*x)] - 40*b^2*(96*a + 55*b)*Sinh[6*(c + d*x)] + 45*b^2*(8*a
```

```
+ 11*b)*Sinh[8*(c + d*x)] - 72*b^3*Sinh[10*(c + d*x)] + 5*b^3*Sinh[12*(c +
d*x)])/(122880*d)
```

Maple [A] time = 0.056, size = 193, normalized size = 0.9

$$\frac{1}{d}\left(b^3\left(\left(\frac{(\sinh(dx+c))^{11}}{12}-\frac{11(\sinh(dx+c))^9}{120}+\frac{33(\sinh(dx+c))^7}{320}-\frac{77(\sinh(dx+c))^5}{640}+\frac{77(\sinh(dx+c))^3}{512}-2\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(b^3*((1/12*sinh(d*x+c)^11-11/120*sinh(d*x+c)^9+33/320*sinh(d*x+c)^7-77
/640*sinh(d*x+c)^5+77/512*sinh(d*x+c)^3-231/1024*sinh(d*x+c))*cosh(d*x+c)+2
31/1024*d*x+231/1024*c)+3*a*b^2*((1/8*sinh(d*x+c)^7-7/48*sinh(d*x+c)^5+35/1
92*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x+c)+35/128*d*x+35/128*c)+3*a^2
*b*((1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+a^3*(d*x
+c))
```

Maxima [A] time = 1.1122, size = 464, normalized size = 2.2

$$\frac{3}{64}a^2b\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+a^3x-\frac{1}{245760}b^3\left(\frac{\left(72e^{(-2dx-2c)}-495e^{(-4dx-4c)}+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a^2*b*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + a^3*x - 1/245760*b^3*((72*e^(-2*d*x - 2*c) -
 495*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 7425*e^(-8*d*x - 8*c) + 237
60*e^(-10*d*x - 10*c) - 5)*e^(12*d*x + 12*c)/d - 55440*(d*x + c)/d - (23760
*e^(-2*d*x - 2*c) - 7425*e^(-4*d*x - 4*c) + 2200*e^(-6*d*x - 6*c) - 495*e^(
-8*d*x - 8*c) + 72*e^(-10*d*x - 10*c) - 5*e^(-12*d*x - 12*c))/d) - 1/2048*a
*b^2*((32*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) -
3)*e^(8*d*x + 8*c)/d - 1680*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4
*d*x - 4*c) + 32*e^(-6*d*x - 6*c) - 3*e^(-8*d*x - 8*c))/d)
```

Fricas [B] time = 1.48421, size = 1200, normalized size = 5.69

$$\frac{15b^3\cosh(dx+c)\sinh(dx+c)^{11}+5\left(55b^3\cosh(dx+c)^3-36b^3\cosh(dx+c)\right)\sinh(dx+c)^9+90\left(11b^3\cosh(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/30720*(15*b^3*cosh(d*x + c)*sinh(d*x + c)^11 + 5*(55*b^3*cosh(d*x + c)^3
- 36*b^3*cosh(d*x + c))*sinh(d*x + c)^9 + 90*(11*b^3*cosh(d*x + c)^5 - 24*b
^3*cosh(d*x + c)^3 + (8*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 6*
(165*b^3*cosh(d*x + c)^7 - 756*b^3*cosh(d*x + c)^5 + 105*(8*a*b^2 + 11*b^3)
```

```
*cosh(d*x + c)^3 - 10*(96*a*b^2 + 55*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 +
5*(55*b^3*cosh(d*x + c)^9 - 432*b^3*cosh(d*x + c)^7 + 126*(8*a*b^2 + 11*b^3
)*cosh(d*x + c)^5 - 40*(96*a*b^2 + 55*b^3)*cosh(d*x + c)^3 + 9*(256*a^2*b +
 448*a*b^2 + 165*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(1024*a^3 + 1152*
a^2*b + 840*a*b^2 + 231*b^3)*d*x + 15*(b^3*cosh(d*x + c)^11 - 12*b^3*cosh(d
*x + c)^9 + 6*(8*a*b^2 + 11*b^3)*cosh(d*x + c)^7 - 4*(96*a*b^2 + 55*b^3)*co
sh(d*x + c)^5 + 3*(256*a^2*b + 448*a*b^2 + 165*b^3)*cosh(d*x + c)^3 - 24*(1
28*a^2*b + 112*a*b^2 + 33*b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 103.123, size = 666, normalized size = 3.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Piecewise((a**3*x + 9*a**2*b*x*sinh(c + d*x)**4/8 - 9*a**2*b*x*sinh(c + d*x
)**2*cosh(c + d*x)**2/4 + 9*a**2*b*x*cosh(c + d*x)**4/8 + 15*a**2*b*sinh(c
+ d*x)**3*cosh(c + d*x)/(8*d) - 9*a**2*b*sinh(c + d*x)*cosh(c + d*x)**3/(8*
d) + 105*a*b**2*x*sinh(c + d*x)**8/128 - 105*a*b**2*x*sinh(c + d*x)**6*cosh
(c + d*x)**2/32 + 315*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 105*a
*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 105*a*b**2*x*cosh(c + d*x)**
8/128 + 279*a*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) - 511*a*b**2*sinh
(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 385*a*b**2*sinh(c + d*x)**3*cosh(c
+ d*x)**5/(128*d) - 105*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 231
*b**3*x*sinh(c + d*x)**12/1024 - 693*b**3*x*sinh(c + d*x)**10*cosh(c + d*x)
**2/512 + 3465*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**4/1024 - 1155*b**3*x*
sinh(c + d*x)**6*cosh(c + d*x)**6/256 + 3465*b**3*x*sinh(c + d*x)**4*cosh(c
 + d*x)**8/1024 - 693*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**10/512 + 231*b
**3*x*cosh(c + d*x)**12/1024 + 793*b**3*sinh(c + d*x)**11*cosh(c + d*x)/(10
24*d) - 7337*b**3*sinh(c + d*x)**9*cosh(c + d*x)**3/(3072*d) + 9273*b**3*si
nh(c + d*x)**7*cosh(c + d*x)**5/(2560*d) - 7623*b**3*sinh(c + d*x)**5*cosh(
c + d*x)**7/(2560*d) + 1309*b**3*sinh(c + d*x)**3*cosh(c + d*x)**9/(1024*d)
 - 231*b**3*sinh(c + d*x)*cosh(c + d*x)**11/(1024*d), Ne(d, 0)), (x*(a + b*
sinh(c)**4)**3, True))
```

Giac [B] time = 1.14116, size = 601, normalized size = 2.85

$$\frac{5\,b^3e^{(12\,dx+12\,c)} - 72\,b^3e^{(10\,dx+10\,c)} + 360\,ab^2e^{(8\,dx+8\,c)} + 495\,b^3e^{(8\,dx+8\,c)} - 3840\,ab^2e^{(6\,dx+6\,c)} - 2200\,b^3e^{(6\,dx+6\,c)} + 11520\,a^2be}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/245760*(5*b^3*e^(12*d*x + 12*c) - 72*b^3*e^(10*d*x + 10*c) + 360*a*b^2*e^
(8*d*x + 8*c) + 495*b^3*e^(8*d*x + 8*c) - 3840*a*b^2*e^(6*d*x + 6*c) - 2200
*b^3*e^(6*d*x + 6*c) + 11520*a^2*b*e^(4*d*x + 4*c) + 20160*a*b^2*e^(4*d*x +
 4*c) + 7425*b^3*e^(4*d*x + 4*c) - 92160*a^2*b*e^(2*d*x + 2*c) - 80640*a*b^
2*e^(2*d*x + 2*c) - 23760*b^3*e^(2*d*x + 2*c) + 240*(1024*a^3 + 1152*a^2*b
+ 840*a*b^2 + 231*b^3)*(d*x + c) - (301056*a^3*e^(12*d*x + 12*c) + 338688*a
^2*b*e^(12*d*x + 12*c) + 246960*a*b^2*e^(12*d*x + 12*c) + 67914*b^3*e^(12*d
*x + 12*c) - 92160*a^2*b*e^(10*d*x + 10*c) - 80640*a*b^2*e^(10*d*x + 10*c)
```

```
- 23760*b^3*e^(10*d*x + 10*c) + 11520*a^2*b*e^(8*d*x + 8*c) + 20160*a*b^2*e
^(8*d*x + 8*c) + 7425*b^3*e^(8*d*x + 8*c) - 3840*a*b^2*e^(6*d*x + 6*c) - 22
00*b^3*e^(6*d*x + 6*c) + 360*a*b^2*e^(4*d*x + 4*c) + 495*b^3*e^(4*d*x + 4*c
) - 72*b^3*e^(2*d*x + 2*c) + 5*b^3)*e^(-12*d*x - 12*c))/d
```

3.219 $\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=181

$$\frac{b\left(384a^2+528ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{3}{256}bx\left(128a^2+80ab+21b^2\right)-\frac{a^3\coth(c+dx)}{d}+\frac{b^2(80a+171}{}$$

```
[Out] (-3*b*(128*a^2 + 80*a*b + 21*b^2)*x)/256 - (a^3*Coth[c + d*x])/d + (b*(384*
a^2 + 528*a*b + 193*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) - (b^2*(208*a
 + 149*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b^2*(80*a + 171*b)*Cosh
[c + d*x]^5*Sinh[c + d*x])/(160*d) - (41*b^3*Cosh[c + d*x]^7*Sinh[c + d*x])
/(80*d) + (b^3*Cosh[c + d*x]^9*Sinh[c + d*x])/(10*d)
```

Rubi [A] time = 0.438191, antiderivative size = 181, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 453, 206}

$$\frac{b\left(384a^2+528ab+193b^2\right)\sinh(c+dx)\cosh(c+dx)}{256d}-\frac{3}{256}bx\left(128a^2+80ab+21b^2\right)-\frac{a^3\coth(c+dx)}{d}+\frac{b^2(80a+171}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-3*b*(128*a^2 + 80*a*b + 21*b^2)*x)/256 - (a^3*Coth[c + d*x])/d + (b*(384*
a^2 + 528*a*b + 193*b^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) - (b^2*(208*a
 + 149*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b^2*(80*a + 171*b)*Cosh
[c + d*x]^5*Sinh[c + d*x])/(160*d) - (41*b^3*Cosh[c + d*x]^7*Sinh[c + d*x])
/(80*d) + (b^3*Cosh[c + d*x]^9*Sinh[c + d*x])/(10*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^2(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^2\left(1-x^2\right)^6}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b^3\cosh^9(c+dx)\sinh(c+dx)}{10d}-\frac{\text{Subst}\left(\int \frac{-10a^3+\left(50a^3+b^3\right)x^2-10\left(10a^3+3a^2b-\right.}{\ldots}\right.}{\ldots}$$

$$=-\frac{41b^3\cosh^7(c+dx)\sinh(c+dx)}{80d}+\frac{b^3\cosh^9(c+dx)\sinh(c+dx)}{10d}+\ldots$$

$$=\frac{b^2(80a+171b)\cosh^5(c+dx)\sinh(c+dx)}{160d}-\frac{41b^3\cosh^7(c+dx)\sinh(c}{80d}$$

$$=-\frac{b^2(208a+149b)\cosh^3(c+dx)\sinh(c+dx)}{128d}+\frac{b^2(80a+171b)\cosh^5(c}{160d}$$

$$=\frac{b\left(384a^2+528ab+193b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d}-\frac{b^2(208a+149b}{}$$

$$=-\frac{a^3\coth(c+dx)}{d}+\frac{b\left(384a^2+528ab+193b^2\right)\cosh(c+dx)\sinh(c+d}{256d}$$

$$=-\frac{3}{256}b\left(128a^2+80ab+21b^2\right)x-\frac{a^3\coth(c+dx)}{d}+\frac{b\left(384a^2+528ab\right.}{}$$

Mathematica [A] time = 0.770606, size = 134, normalized size = 0.74

$$\frac{-120b\left(128a^2+80ab+21b^2\right)(c+dx)+60b\left(128a^2+120ab+35b^2\right)\sinh(2(c+dx))-10240a^3\coth(c+dx)-120b^2}{10240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-120*b*(128*a^2 + 80*a*b + 21*b^2)*(c + d*x) - 10240*a^3*Coth[c + d*x] + 6
0*b*(128*a^2 + 120*a*b + 35*b^2)*Sinh[2*(c + d*x)] - 120*b^2*(12*a + 5*b)*S
```

```
inh[4*(c + d*x)] + 10*b^2*(16*a + 15*b)*Sinh[6*(c + d*x)] - 25*b^3*Sinh[8*(
c + d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)
```

Maple [A] time = 0.039, size = 163, normalized size = 0.9

$$\frac{1}{d}\left(-a^3\coth(dx+c)+3\,a^2b\,(1/2\,\cosh(dx+c)\sinh(dx+c)-1/2\,dx-c/2)+3\,ab^2\left(\left(1/6\,(\sinh(dx+c))^5-\frac{5\,(\sinh(dx+}{24}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(-a^3*coth(d*x+c)+3*a^2*b*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+3
*a*b^2*((1/6*sinh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)
-5/16*d*x-5/16*c)+b^3*((1/10*sinh(d*x+c)^9-9/80*sinh(d*x+c)^7+21/160*sinh(d
*x+c)^5-21/128*sinh(d*x+c)^3+63/256*sinh(d*x+c))*cosh(d*x+c)-63/256*d*x-63/
256*c))
```

Maxima [A] time = 1.09203, size = 383, normalized size = 2.12

$$-\frac{3}{8}\,a^2b\left(4\,x-\frac{e^{(2\,dx+2\,c)}}{d}+\frac{e^{(-2\,dx-2\,c)}}{d}\right)-\frac{1}{20480}\,b^3\left(\frac{\left(25\,e^{(-2\,dx-2\,c)}-150\,e^{(-4\,dx-4\,c)}+600\,e^{(-6\,dx-6\,c)}-2100\,e^{(-8\,dx-8\,c)}-2\right)e^{(1}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a^2*b*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/20480*b^3*((2
5*e^(-2*d*x - 2*c) - 150*e^(-4*d*x - 4*c) + 600*e^(-6*d*x - 6*c) - 2100*e^(
-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)/d + 5040*(d*x + c)/d + (2100*e^(-2*d*x
 - 2*c) - 600*e^(-4*d*x - 4*c) + 150*e^(-6*d*x - 6*c) - 25*e^(-8*d*x - 8*c)
 + 2*e^(-10*d*x - 10*c))/d) - 1/128*a*b^2*((9*e^(-2*d*x - 2*c) - 45*e^(-4*d
*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c)/d + (45*e^(-2*d*x - 2*c) -
 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) + 2*a^3/(d*(e^(-2*d*x - 2*c) - 1
))
```

Fricas [B] time = 1.44548, size = 1247, normalized size = 6.89

$$2\,b^3\cosh(dx+c)^{11}+22\,b^3\cosh(dx+c)\sinh(dx+c)^{10}-27\,b^3\cosh(dx+c)^9+3\left(110\,b^3\cosh(dx+c)^3-81\,b^3\cosh(d\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/20480*(2*b^3*cosh(d*x + c)^11 + 22*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 2
7*b^3*cosh(d*x + c)^9 + 3*(110*b^3*cosh(d*x + c)^3 - 81*b^3*cosh(d*x + c))*
sinh(d*x + c)^8 + 5*(32*a*b^2 + 35*b^3)*cosh(d*x + c)^7 + 7*(132*b^3*cosh(d
*x + c)^5 - 324*b^3*cosh(d*x + c)^3 + 5*(32*a*b^2 + 35*b^3)*cosh(d*x + c))*
sinh(d*x + c)^6 - 50*(32*a*b^2 + 15*b^3)*cosh(d*x + c)^5 + (660*b^3*cosh(d*
x + c)^7 - 3402*b^3*cosh(d*x + c)^5 + 175*(32*a*b^2 + 35*b^3)*cosh(d*x + c)
```

```
^3 - 250*(32*a*b^2 + 15*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 60*(128*a^2*b
 + 144*a*b^2 + 45*b^3)*cosh(d*x + c)^3 + (110*b^3*cosh(d*x + c)^9 - 972*b^3
*cosh(d*x + c)^7 + 105*(32*a*b^2 + 35*b^3)*cosh(d*x + c)^5 - 500*(32*a*b^2
+ 15*b^3)*cosh(d*x + c)^3 + 180*(128*a^2*b + 144*a*b^2 + 45*b^3)*cosh(d*x +
 c))*sinh(d*x + c)^2 - 20*(1024*a^3 + 384*a^2*b + 360*a*b^2 + 105*b^3)*cosh
(d*x + c) + 80*(256*a^3 - 3*(128*a^2*b + 80*a*b^2 + 21*b^3)*d*x)*sinh(d*x +
 c))/(d*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70399, size = 531, normalized size = 2.93

$$-\frac{3\left(128\,a^2b+80\,ab^2+21\,b^3\right)(dx+c)}{256\,d}-\frac{2\,a^3}{d\left(e^{(2\,dx+2\,c)}-1\right)}+\frac{\left(35072\,a^2be^{(10\,dx+10\,c)}+21920\,ab^2e^{(10\,dx+10\,c)}+5754\,b^3e^{(10}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -3/256*(128*a^2*b + 80*a*b^2 + 21*b^3)*(d*x + c)/d - 2*a^3/(d*(e^(2*d*x + 2
*c) - 1)) + 1/20480*(35072*a^2*b*e^(10*d*x + 10*c) + 21920*a*b^2*e^(10*d*x
+ 10*c) + 5754*b^3*e^(10*d*x + 10*c) - 7680*a^2*b*e^(8*d*x + 8*c) - 7200*a*
b^2*e^(8*d*x + 8*c) - 2100*b^3*e^(8*d*x + 8*c) + 1440*a*b^2*e^(6*d*x + 6*c)
 + 600*b^3*e^(6*d*x + 6*c) - 160*a*b^2*e^(4*d*x + 4*c) - 150*b^3*e^(4*d*x +
 4*c) + 25*b^3*e^(2*d*x + 2*c) - 2*b^3)*e^(-10*d*x - 10*c)/d + 1/20480*(2*b
^3*d^4*e^(10*d*x + 10*c) - 25*b^3*d^4*e^(8*d*x + 8*c) + 160*a*b^2*d^4*e^(6*
d*x + 6*c) + 150*b^3*d^4*e^(6*d*x + 6*c) - 1440*a*b^2*d^4*e^(4*d*x + 4*c) -
 600*b^3*d^4*e^(4*d*x + 4*c) + 7680*a^2*b*d^4*e^(2*d*x + 2*c) + 7200*a*b^2*
d^4*e^(2*d*x + 2*c) + 2100*b^3*d^4*e^(2*d*x + 2*c))/d^5
```

3.220 $\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=161

$$\frac{1}{128}bx\left(384a^2+144ab+35b^2\right)-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}+\frac{b^2(144a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-3b$$

```
[Out] (b*(384*a^2 + 144*a*b + 35*b^2)*x)/128 + (a^3*Coth[c + d*x])/d - (a^3*Coth[
c + d*x]^3)/(3*d) - (3*b^2*(80*a + 31*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*
d) + (b^2*(144*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b^3*
Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^3*Cosh[c + d*x]^7*Sinh[c + d*x])
/(8*d)
```

Rubi [A] time = 0.384284, antiderivative size = 161, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 1261, 207}

$$\frac{1}{128}bx\left(384a^2+144ab+35b^2\right)-\frac{a^3\coth^3(c+dx)}{3d}+\frac{a^3\coth(c+dx)}{d}+\frac{b^2(144a+163b)\sinh(c+dx)\cosh^3(c+dx)}{192d}-3b$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b*(384*a^2 + 144*a*b + 35*b^2)*x)/128 + (a^3*Coth[c + d*x])/d - (a^3*Coth[
c + d*x]^3)/(3*d) - (3*b^2*(80*a + 31*b)*Cosh[c + d*x]*Sinh[c + d*x])/(128*
d) + (b^2*(144*a + 163*b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) - (25*b^3*
Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) + (b^3*Cosh[c + d*x]^7*Sinh[c + d*x])
/(8*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
```

```
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^4(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^4\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^7(c+dx)\sinh(c+dx)}{8d} + \frac{\operatorname{Subst}\left(\int \frac{8a^3-40a^3x^2+\left(80a^3+24a^2b-b^3\right)x^4-8(1}{}\right.}{}$$

$$= -\frac{25b^3\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b^3\cosh^7(c+dx)\sinh(c+dx)}{8d} - \text{Su}$$

$$= \frac{b^2(144a+163b)\cosh^3(c+dx)\sinh(c+dx)}{192d} - \frac{25b^3\cosh^5(c+dx)\sinh(}{48d}$$

$$= -\frac{3b^2(80a+31b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b^2(144a+163b)\cosh^3(c}{192d}$$

$$= -\frac{3b^2(80a+31b)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b^2(144a+163b)\cosh^3(c}{192d}$$

$$= \frac{a^3\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} - \frac{3b^2(80a+31b)\cosh(c+dx)\sinh}{128d}$$

$$= \frac{1}{128}b\left(384a^2+144ab+35b^2\right)x + \frac{a^3\coth(c+dx)}{d} - \frac{a^3\coth^3(c+dx)}{3d} -$$

Mathematica [A] time = 0.782114, size = 131, normalized size = 0.81

$$\frac{b\left(9216a^2c+9216a^2dx-96b(24a+7b)\sinh(2(c+dx))+24b(12a+7b)\sinh(4(c+dx))+3456abc+3456abdx-32\right.}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-1024*a^3*Coth[c + d*x]*(-2 + Csch[c + d*x]^2) + b*(9216*a^2*c + 3456*a*b*
c + 840*b^2*c + 9216*a^2*d*x + 3456*a*b*d*x + 840*b^2*d*x - 96*b*(24*a + 7*
b)*Sinh[2*(c + d*x)] + 24*b*(12*a + 7*b)*Sinh[4*(c + d*x)] - 32*b^2*Sinh[6*
```

```
(c + d*x)] + 3*b^2*Sinh[8*(c + d*x)]))/(3072*d)
```

Maple [A] time = 0.048, size = 137, normalized size = 0.9

$$\frac{1}{d}\left(a^3\left(\frac{2}{3}-\frac{(\operatorname{csch}(dx+c))^2}{3}\right)\coth(dx+c)+3a^2b(dx+c)+3ab^2\left(\left(1/4\,(\sinh(dx+c))^3-3/8\sinh(dx+c)\right)\cosh(dx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(d*x+c)+3*a*b^2*((1/4*
sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c)+b^3*((1/8*sinh(d*
x+c)^7-7/48*sinh(d*x+c)^5+35/192*sinh(d*x+c)^3-35/128*sinh(d*x+c))*cosh(d*x
+c)+35/128*d*x+35/128*c))
```

Maxima [A] time = 1.10793, size = 381, normalized size = 2.37

$$\frac{3}{64}ab^2\left(24x+\frac{e^{(4dx+4c)}}{d}-\frac{8e^{(2dx+2c)}}{d}+\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)+3a^2bx-\frac{1}{6144}b^3\left(\frac{\left(32e^{(-2dx-2c)}-168e^{(-4dx-4c)}+67\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 3/64*a*b^2*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x -
2*c)/d - e^(-4*d*x - 4*c)/d) + 3*a^2*b*x - 1/6144*b^3*((32*e^(-2*d*x - 2*c)
 - 168*e^(-4*d*x - 4*c) + 672*e^(-6*d*x - 6*c) - 3)*e^(8*d*x + 8*c)/d - 168
0*(d*x + c)/d - (672*e^(-2*d*x - 2*c) - 168*e^(-4*d*x - 4*c) + 32*e^(-6*d*x
 - 6*c) - 3*e^(-8*d*x - 8*c))/d) + 4/3*a^3*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*
d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*
x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 1.36759, size = 1476, normalized size = 9.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/6144*(3*b^3*cosh(d*x + c)^11 + 33*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 41
*b^3*cosh(d*x + c)^9 + 9*(55*b^3*cosh(d*x + c)^3 - 41*b^3*cosh(d*x + c))*si
nh(d*x + c)^8 + 3*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^7 + 21*(66*b^3*cosh(d*x
 + c)^5 - 164*b^3*cosh(d*x + c)^3 + (96*a*b^2 + 91*b^3)*cosh(d*x + c))*sinh
(d*x + c)^6 - 3*(1056*a*b^2 + 425*b^3)*cosh(d*x + c)^5 + 3*(330*b^3*cosh(d*
x + c)^7 - 1722*b^3*cosh(d*x + c)^5 + 35*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^
3 - 5*(1056*a*b^2 + 425*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 8*(512*a^3 +
972*a*b^2 + 319*b^3)*cosh(d*x + c)^3 - 16*(256*a^3 - 3*(384*a^2*b + 144*a*b
^2 + 35*b^3)*d*x)*sinh(d*x + c)^3 + 3*(55*b^3*cosh(d*x + c)^9 - 492*b^3*cos
h(d*x + c)^7 + 21*(96*a*b^2 + 91*b^3)*cosh(d*x + c)^5 - 10*(1056*a*b^2 + 42
```

```
5*b^3)*cosh(d*x + c)^3 + 8*(512*a^3 + 972*a*b^2 + 319*b^3)*cosh(d*x + c))*s
inh(d*x + c)^2 - 24*(512*a^3 + 204*a*b^2 + 63*b^3)*cosh(d*x + c) + 48*(256*
a^3 - 3*(384*a^2*b + 144*a*b^2 + 35*b^3)*d*x - (256*a^3 - 3*(384*a^2*b + 14
4*a*b^2 + 35*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^3 +
 3*(d*cosh(d*x + c)^2 - d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70406, size = 423, normalized size = 2.63

$$\frac{\left(384\,a^2b+144\,ab^2+35\,b^3\right)(dx+c)}{128\,d}-\frac{\left(19200\,a^2be^{(8\,dx+8\,c)}+7200\,ab^2e^{(8\,dx+8\,c)}+1750\,b^3e^{(8\,dx+8\,c)}-2304\,ab^2e^{(6\,dx+6\,c)}\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/128*(384*a^2*b + 144*a*b^2 + 35*b^3)*(d*x + c)/d - 1/6144*(19200*a^2*b*e^
(8*d*x + 8*c) + 7200*a*b^2*e^(8*d*x + 8*c) + 1750*b^3*e^(8*d*x + 8*c) - 230
4*a*b^2*e^(6*d*x + 6*c) - 672*b^3*e^(6*d*x + 6*c) + 288*a*b^2*e^(4*d*x + 4*
c) + 168*b^3*e^(4*d*x + 4*c) - 32*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-8*d*x -
8*c)/d + 1/6144*(3*b^3*d^3*e^(8*d*x + 8*c) - 32*b^3*d^3*e^(6*d*x + 6*c) + 2
88*a*b^2*d^3*e^(4*d*x + 4*c) + 168*b^3*d^3*e^(4*d*x + 4*c) - 2304*a*b^2*d^3
*e^(2*d*x + 2*c) - 672*b^3*d^3*e^(2*d*x + 2*c))/d^4 - 4/3*(3*a^3*e^(2*d*x +
 2*c) - a^3)/(d*(e^(2*d*x + 2*c) - 1)^3)
```

3.221 $\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx$

Optimal. Leaf size=148

$$-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}+\frac{b^2(24a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}b^2x(2$$

```
[Out] -(b^2*(24*a + 5*b)*x)/16 - (a^2*(a + 3*b)*Coth[c + d*x])/d + (2*a^3*Coth[c
+ d*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) + (b^2*(24*a + 11*b)*Cosh[c +
 d*x]*Sinh[c + d*x])/(16*d) - (13*b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d)
 + (b^3*Cosh[c + d*x]^5*Sinh[c + d*x])/(6*d)
```

Rubi [A] time = 0.355256, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 1802, 207}

$$-\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^5(c+dx)}{5d}+\frac{2a^3\coth^3(c+dx)}{3d}+\frac{b^2(24a+11b)\sinh(c+dx)\cosh(c+dx)}{16d}-\frac{1}{16}b^2x(2$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(b^2*(24*a + 5*b)*x)/16 - (a^2*(a + 3*b)*Coth[c + d*x])/d + (2*a^3*Coth[c
+ d*x]^3)/(3*d) - (a^3*Coth[c + d*x]^5)/(5*d) + (b^2*(24*a + 11*b)*Cosh[c +
 d*x]*Sinh[c + d*x])/(16*d) - (13*b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d)
 + (b^3*Cosh[c + d*x]^5*Sinh[c + d*x])/(6*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^6(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^6\left(1-x^2\right)^4}dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh^5(c+dx)\sinh(c+dx)}{6d} - \frac{\text{Subst}\left(\int \frac{-6a^3+30a^3x^2-6a^2(10a+3b)x^4+\left(60a^3+\right.}{x^6}\right.}{}$$

$$= -\frac{13b^3\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b^3\cosh^5(c+dx)\sinh(c+dx)}{6d} + \frac{\text{Su}}{}$$

$$= \frac{b^2(24a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b^3\cosh^3(c+dx)\sinh(c+}{24d}$$

$$= \frac{b^2(24a+11b)\cosh(c+dx)\sinh(c+dx)}{16d} - \frac{13b^3\cosh^3(c+dx)\sinh(c+}{24d}$$

$$= -\frac{a^2(a+3b)\coth(c+dx)}{d} + \frac{2a^3\coth^3(c+dx)}{3d} - \frac{a^3\coth^5(c+dx)}{5d} + \frac{b^2(}{}$$

$$= -\frac{1}{16}b^2(24a+5b)x - \frac{a^2(a+3b)\coth(c+dx)}{d} + \frac{2a^3\coth^3(c+dx)}{3d} - \frac{a^3}{}$$

Mathematica [A] time = 1.15998, size = 110, normalized size = 0.74

$$\frac{5b^2(9(16a+5b)\sinh(2(c+dx)) - 288ac - 288adx - 9b\sinh(4(c+dx)) + b\sinh(6(c+dx)) - 60bc - 60bdx) - 64a^2\,\text{c}}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^6*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-64*a^2*Coth[c + d*x]*(8*a + 45*b - 4*a*Csch[c + d*x]^2 + 3*a*Csch[c + d*x
]^4) + 5*b^2*(-288*a*c - 60*b*c - 288*a*d*x - 60*b*d*x + 9*(16*a + 5*b)*Sin
h[2*(c + d*x)] - 9*b*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)]))/(960*d)
```

Maple [A] time = 0.046, size = 126, normalized size = 0.9

$$\frac{1}{d}\left(a^3\left(-\frac{8}{15} - \frac{(\operatorname{csch}(dx+c))^4}{5} + \frac{4\,(\operatorname{csch}(dx+c))^2}{15}\right)\coth(dx+c) - 3\,a^2b\coth(dx+c) + 3\,ab^2\,(1/2\,\cosh(dx+c)\sinh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-8/15-1/5*csch(d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-3*a^2*b*c
oth(d*x+c)+3*a*b^2*(1/2*cosh(d*x+c)*sinh(d*x+c)-1/2*d*x-1/2*c)+b^3*((1/6*si
nh(d*x+c)^5-5/24*sinh(d*x+c)^3+5/16*sinh(d*x+c))*cosh(d*x+c)-5/16*d*x-5/16*
c))
```

Maxima [B] time = 1.06571, size = 485, normalized size = 3.28

$$-\frac{3}{8}ab^2\left(4x-\frac{e^{(2dx+2c)}}{d}+\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{384}b^3\left(\frac{\left(9e^{(-2dx-2c)}-45e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}+\frac{120(dx+c)}{d}+\frac{45e^{(-2dx-2c)}-}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -3/8*a*b^2*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 1/384*b^3*((9*e
^(-2*d*x - 2*c) - 45*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d + 120*(d*x + c
)/d + (45*e^(-2*d*x - 2*c) - 9*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c))/d) - 16
/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x
- 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) - 1))) + 6*a^2*b/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.5227, size = 2006, normalized size = 13.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/1920*(5*b^3*cosh(d*x + c)^11 + 55*b^3*cosh(d*x + c)*sinh(d*x + c)^10 - 70
*b^3*cosh(d*x + c)^9 + 15*(55*b^3*cosh(d*x + c)^3 - 42*b^3*cosh(d*x + c))*s
inh(d*x + c)^8 + 20*(36*a*b^2 + 25*b^3)*cosh(d*x + c)^7 + 70*(33*b^3*cosh(d
*x + c)^5 - 84*b^3*cosh(d*x + c)^3 + 2*(36*a*b^2 + 25*b^3)*cosh(d*x + c))*s
inh(d*x + c)^6 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*cosh(d*x +
 c)^5 + 8*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*sinh(d*x + c)^5
 + 5*(330*b^3*cosh(d*x + c)^7 - 1764*b^3*cosh(d*x + c)^5 + 140*(36*a*b^2 +
25*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 1625*b^3)*c
osh(d*x + c))*sinh(d*x + c)^4 + 20*(256*a^3 + 864*a^2*b + 324*a*b^2 + 125*b
^3)*cosh(d*x + c)^3 - 40*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x -
 2*(128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(
d*x + c)^3 + 5*(55*b^3*cosh(d*x + c)^9 - 504*b^3*cosh(d*x + c)^7 + 84*(36*a
*b^2 + 25*b^3)*cosh(d*x + c)^5 - 2*(1024*a^3 + 5760*a^2*b + 3600*a*b^2 + 16
25*b^3)*cosh(d*x + c)^3 + 12*(256*a^3 + 864*a^2*b + 324*a*b^2 + 125*b^3)*co
sh(d*x + c))*sinh(d*x + c)^2 - 10*(1024*a^3 + 1152*a^2*b + 360*a*b^2 + 131*
b^3)*cosh(d*x + c) + 40*((128*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*
cosh(d*x + c)^4 + 256*a^3 + 1440*a^2*b - 30*(24*a*b^2 + 5*b^3)*d*x - 3*(128
*a^3 + 720*a^2*b - 15*(24*a*b^2 + 5*b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*sinh(d*x + c)^5 + 5*(2*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^3 + 5*(d*
```

```
cosh(d*x + c)^4 - 3*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**6*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70671, size = 414, normalized size = 2.8

$$-\frac{\left(24\,ab^{2}+5\,b^{3}\right)(dx+c)}{16\,d}+\frac{\left(528\,ab^{2}e^{(6\,dx+6\,c)}+110\,b^{3}e^{(6\,dx+6\,c)}-144\,ab^{2}e^{(4\,dx+4\,c)}-45\,b^{3}e^{(4\,dx+4\,c)}+9\,b^{3}e^{(2\,dx+2\,c)}-b^{3}\right)}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^6*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(24*a*b^2 + 5*b^3)*(d*x + c)/d + 1/384*(528*a*b^2*e^(6*d*x + 6*c) + 1
10*b^3*e^(6*d*x + 6*c) - 144*a*b^2*e^(4*d*x + 4*c) - 45*b^3*e^(4*d*x + 4*c)
 + 9*b^3*e^(2*d*x + 2*c) - b^3)*e^(-6*d*x - 6*c)/d + 1/384*(b^3*d^2*e^(6*d*
x + 6*c) - 9*b^3*d^2*e^(4*d*x + 4*c) + 144*a*b^2*d^2*e^(2*d*x + 2*c) + 45*b
^3*d^2*e^(2*d*x + 2*c))/d^3 - 2/15*(45*a^2*b*e^(8*d*x + 8*c) - 180*a^2*b*e^
(6*d*x + 6*c) + 80*a^3*e^(4*d*x + 4*c) + 270*a^2*b*e^(4*d*x + 4*c) - 40*a^3
*e^(2*d*x + 2*c) - 180*a^2*b*e^(2*d*x + 2*c) + 8*a^3 + 45*a^2*b)/(d*(e^(2*d
*x + 2*c) - 1)^5)
```

3.222 $\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=133

$$-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^7(c+dx)}{7d}+\frac{3a^3\coth^5(c+dx)}{5d}+\frac{3}{8}b^2x(8a+b)+\frac{b^3\sinh(c$$

```
[Out] (3*b^2*(8*a + b)*x)/8 + (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^2*(a + b)*Coth
[c + d*x]^3)/d + (3*a^3*Coth[c + d*x]^5)/(5*d) - (a^3*Coth[c + d*x]^7)/(7*d
) - (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c
 + d*x])/(4*d)
```

Rubi [A] time = 0.288891, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3217, 1259, 1805, 1802, 207}

$$-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^3\coth^7(c+dx)}{7d}+\frac{3a^3\coth^5(c+dx)}{5d}+\frac{3}{8}b^2x(8a+b)+\frac{b^3\sinh(c$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] (3*b^2*(8*a + b)*x)/8 + (a^2*(a + 3*b)*Coth[c + d*x])/d - (a^2*(a + b)*Coth
[c + d*x]^3)/d + (3*a^3*Coth[c + d*x]^5)/(5*d) - (a^3*Coth[c + d*x]^7)/(7*d
) - (5*b^3*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c
 + d*x])/(4*d)
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1805

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[
{Q = PolynomialQuotient[(c*x)^m*Pq, a + b*x^2, x], f = Coeff[PolynomialRema
inder[(c*x)^m*Pq, a + b*x^2, x], x, 0], g = Coeff[PolynomialRemainder[(c*x)
^m*Pq, a + b*x^2, x], x, 1]}, Simp[((a*g - b*f*x)*(a + b*x^2)^(p + 1))/(2*a
*b*(p + 1)), x] + Dist[1/(2*a*(p + 1)), Int[(c*x)^m*(a + b*x^2)^(p + 1)*Exp
andToSum[(2*a*(p + 1)*Q)/(c*x)^m + (f*(2*p + 3))/(c*x)^m, x], x], x]] /; Fr
eeQ[{a, b, c}, x] && PolyQ[Pq, x] && LtQ[p, -1] && ILtQ[m, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^8(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^8\left(1-x^2\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}+\frac{\text{Subst}\left(\int \frac{4a^3-20a^3x^2+4a^2(10a+3b)x^4-4a^2(10a}{}\right.}{}$$

$$=-\frac{5b^3\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{\text{Sub}}{}$$

$$=-\frac{5b^3\cosh(c+dx)\sinh(c+dx)}{8d}+\frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d}-\frac{\text{Sub}}{}$$

$$=\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{3a^3\coth^5(c+dx)}{5d}$$

$$=\frac{3}{8}b^2(8a+b)x+\frac{a^2(a+3b)\coth(c+dx)}{d}-\frac{a^2(a+b)\coth^3(c+dx)}{d}+\frac{3a}{}$$

Mathematica [A] time = 0.691789, size = 106, normalized size = 0.8

$$\frac{35b^2(12(8a+b)(c+dx)-8b\sinh(2(c+dx))+b\sinh(4(c+dx)))-32a^2\coth(c+dx)\left((8a+35b)\operatorname{csch}^2(c+dx)-2(8\right.}{1120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^8*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-32*a^2*Coth[c + d*x]*(-2*(8*a + 35*b) + (8*a + 35*b)*Csch[c + d*x]^2 - 6*
a*Csch[c + d*x]^4 + 5*a*Csch[c + d*x]^6) + 35*b^2*(12*(8*a + b)*(c + d*x) -
 8*b*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)]))/(1120*d)
```

Maple [A] time = 0.089, size = 121, normalized size = 0.9

$$\frac{1}{d}\left(a^3\left(\frac{16}{35}-\frac{(\operatorname{csch}(dx+c))^6}{7}+\frac{6\,(\operatorname{csch}(dx+c))^4}{35}-\frac{8\,(\operatorname{csch}(dx+c))^2}{35}\right)\coth(dx+c)+3\,a^2b\left(2/3-1/3\,(\operatorname{csch}(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(16/35-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*co
th(d*x+c)+3*a^2*b*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(d*x+c)+b^3*(
(1/4*sinh(d*x+c)^3-3/8*sinh(d*x+c))*cosh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [B] time = 1.14781, size = 725, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/64*b^3*(24*x + e^(4*d*x + 4*c)/d - 8*e^(2*d*x + 2*c)/d + 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) + 3*a*b^2*x + 32/35*a^3*(7*e^(-2*d*x - 2*c)/(d*(
7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d
*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*
c) - 1)) - 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c)
 + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^
(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(
-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x -
8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) -
1)) - 1/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c)
- 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-
14*d*x - 14*c) - 1))) + 4*a^2*b*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c)
- 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) -
3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 1.49259, size = 2464, normalized size = 18.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(35*b^3*cosh(d*x + c)^11 + 385*b^3*cosh(d*x + c)*sinh(d*x + c)^10 -
525*b^3*cosh(d*x + c)^9 + 525*(11*b^3*cosh(d*x + c)^3 - 9*b^3*cosh(d*x + c)
)*sinh(d*x + c)^8 + (1024*a^3 + 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^7 - 8*
(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*sinh(d*x + c)^7 + 7*(2310*b
^3*cosh(d*x + c)^5 - 6300*b^3*cosh(d*x + c)^3 + (1024*a^3 + 4480*a^2*b + 26
95*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 7*(1024*a^3 + 4480*a^2*b + 975*b^3
)*cosh(d*x + c)^5 + 56*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x - 3*(
128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x +
c)^5 + 35*(330*b^3*cosh(d*x + c)^7 - 1890*b^3*cosh(d*x + c)^5 + (1024*a^3 +
 4480*a^2*b + 2695*b^3)*cosh(d*x + c)^3 - (1024*a^3 + 4480*a^2*b + 975*b^3)
*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(
d*x + c)^3 - 56*(5*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x
 + c)^4 + 384*a^3 + 1680*a^2*b - 315*(8*a*b^2 + b^3)*d*x - 10*(128*a^3 + 56
0*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 7*(27
5*b^3*cosh(d*x + c)^9 - 2700*b^3*cosh(d*x + c)^7 + 3*(1024*a^3 + 4480*a^2*b
 + 2695*b^3)*cosh(d*x + c)^5 - 10*(1024*a^3 + 4480*a^2*b + 975*b^3)*cosh(d*
x + c)^3 + 18*(512*a^3 + 1600*a^2*b + 215*b^3)*cosh(d*x + c))*sinh(d*x + c)
^2 - 70*(512*a^3 + 576*a^2*b + 63*b^3)*cosh(d*x + c) - 56*((128*a^3 + 560*a
^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^6 - 5*(128*a^3 + 560*a^2*b -
105*(8*a*b^2 + b^3)*d*x)*cosh(d*x + c)^4 - 640*a^3 - 2800*a^2*b + 525*(8*a*
```

```
b^2 + b^3)*d*x + 9*(128*a^3 + 560*a^2*b - 105*(8*a*b^2 + b^3)*d*x)*cosh(d*x
 + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^7 + 7*(3*d*cosh(d*x + c)^2 - d)*si
nh(d*x + c)^5 + 7*(5*d*cosh(d*x + c)^4 - 10*d*cosh(d*x + c)^2 + 3*d)*sinh(d
*x + c)^3 + 7*(d*cosh(d*x + c)^6 - 5*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^
2 - 5*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**8*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70678, size = 356, normalized size = 2.68

$$\frac{3\left(8\,ab^2+b^3\right)(dx+c)}{8\,d}-\frac{\left(144\,ab^2e^{(4\,dx+4\,c)}+18\,b^3e^{(4\,dx+4\,c)}-8\,b^3e^{(2\,dx+2\,c)}+b^3\right)e^{(-4\,dx-4\,c)}}{64\,d}+\frac{b^3de^{(4\,dx+4\,c)}-8\,b^3de^{(2\,dx+2}}{64\,d^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^8*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 3/8*(8*a*b^2 + b^3)*(d*x + c)/d - 1/64*(144*a*b^2*e^(4*d*x + 4*c) + 18*b^3*
e^(4*d*x + 4*c) - 8*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c)/d + 1/64*(b
^3*d*e^(4*d*x + 4*c) - 8*b^3*d*e^(2*d*x + 2*c))/d^2 - 4/35*(105*a^2*b*e^(10
*d*x + 10*c) - 455*a^2*b*e^(8*d*x + 8*c) + 280*a^3*e^(6*d*x + 6*c) + 770*a^
2*b*e^(6*d*x + 6*c) - 168*a^3*e^(4*d*x + 4*c) - 630*a^2*b*e^(4*d*x + 4*c) +
 56*a^3*e^(2*d*x + 2*c) + 245*a^2*b*e^(2*d*x + 2*c) - 8*a^3 - 35*a^2*b)/(d*
(e^(2*d*x + 2*c) - 1)^7)
```

3.223 $\int \operatorname{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=140

$$-\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{3a^2(2a+b)\coth^5(c+dx)}{5d}+\frac{2a^2(2a+3b)\coth^3(c+dx)}{3d}-\frac{a^3\coth^9(c+dx)}{9d}+\frac{4a^3\,\mathrm{c}}{}$$

```
[Out] -(b^3*x)/2 - (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d + (2*a^2*(2*a + 3*b)
*Coth[c + d*x]^3)/(3*d) - (3*a^2*(2*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^3*
Coth[c + d*x]^7)/(7*d) - (a^3*Coth[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]*S
inh[c + d*x])/(2*d)
```

Rubi [A] time = 0.222652, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3217, 1259, 1802, 207}

$$-\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{3a^2(2a+b)\coth^5(c+dx)}{5d}+\frac{2a^2(2a+3b)\coth^3(c+dx)}{3d}-\frac{a^3\coth^9(c+dx)}{9d}+\frac{4a^3\,\mathrm{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]
```



```
[Out] -(b^3*x)/2 - (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d + (2*a^2*(2*a + 3*b)
*Coth[c + d*x]^3)/(3*d) - (3*a^2*(2*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^3*
Coth[c + d*x]^7)/(7*d) - (a^3*Coth[c + d*x]^9)/(9*d) + (b^3*Cosh[c + d*x]*S
inh[c + d*x])/(2*d)
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1259

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_.), x_Symbol] :> Simp[((-d)^(m/2 - 1)*(c*d^2 - b*d*e + a*e^2)^p*x*(d
+ e*x^2)^(q + 1))/(2*e^(2*p + m/2)*(q + 1)), x] + Dist[(-d)^(m/2 - 1)/(2*e^
(2*p)*(q + 1)), Int[x^m*(d + e*x^2)^(q + 1)*ExpandToSum[Together[(1*(2*(-d)
^(-(m/2) + 1)*e^(2*p)*(q + 1)*(a + b*x^2 + c*x^4)^p - ((c*d^2 - b*d*e + a*e
^2)^p/(e^(m/2)*x^m))*(d + e*(2*q + 3)*x^2)))/(d + e*x^2)], x], x], x] /; Fr
eeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && IGtQ[p, 0] && ILtQ[q, -1]
 && ILtQ[m/2, 0]
```

Rule 1802

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2)^(p_.), x_Symbol] :> Int[
ExpandIntegrand[(c*x)^m*Pq*(a + b*x^2)^p, x], x] /; FreeQ[{a, b, c, m}, x]
&& PolyQ[Pq, x] && IGtQ[p, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \text{csch}^{10}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{10}\left(1-x^2\right)^2} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int \frac{-2a^3+10a^3x^2-2a^2(10a+3b)x^4+2a^2(10}{}\right.}{}$$

$$= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} - \frac{\text{Subst}\left(\int\left(-\frac{2a^3}{x^{10}}+\frac{8a^3}{x^8}-\frac{6a^2(2a+b)}{x^6}+\frac{4a^2(}{}\right.\right.}{}$$

$$= -\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} + \frac{2a^2(2a+3b)\coth^3(c+dx)}{3d} - \frac{3a^2(}{}$$

$$= -\frac{b^3x}{2} - \frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} + \frac{2a^2(2a+3b)\coth^3(c+dx)}{3d}$$

Mathematica [A] time = 0.579417, size = 115, normalized size = 0.82

$$\frac{315b^3(\sinh(2(c+dx))-2(c+dx))-4a\coth(c+dx)\left(35a^2\text{csch}^8(c+dx)-40a^2\text{csch}^6(c+dx)+128a^2+3a(16a+63\right.}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^10*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (-4*a*Coth[c + d*x]*(128*a^2 + 504*a*b + 945*b^2 - 4*a*(16*a + 63*b)*Csch[c
 + d*x]^2 + 3*a*(16*a + 63*b)*Csch[c + d*x]^4 - 40*a^2*Csch[c + d*x]^6 + 35
*a^2*Csch[c + d*x]^8) + 315*b^3*(-2*(c + d*x) + Sinh[2*(c + d*x)]))/(1260*d
)
```

Maple [A] time = 0.084, size = 130, normalized size = 0.9

$$\frac{1}{d}\left(a^3\left(-\frac{128}{315}-\frac{(\text{csch}(dx+c))^8}{9}+\frac{8(\text{csch}(dx+c))^6}{63}-\frac{16(\text{csch}(dx+c))^4}{105}+\frac{64(\text{csch}(dx+c))^2}{315}\right)\coth(dx+c)+3a\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-16/105*csch(d*x+c)^
4+64/315*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(-8/15-1/5*csch(d*x+c)^4+4/15*c
sch(d*x+c)^2)*coth(d*x+c)-3*a*b^2*coth(d*x+c)+b^3*(1/2*cosh(d*x+c)*sinh(d*x
+c)-1/2*d*x-1/2*c))
```

Maxima [B] time = 1.21684, size = 1137, normalized size = 8.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*b^3*(4*x - e^(2*d*x + 2*c)/d + e^(-2*d*x - 2*c)/d) - 256/315*a^3*(9*e^
(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x
- 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12
*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1
)) - 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*
e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-1
2*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x -
 18*c) - 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x -
4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c)
- 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^
(-18*d*x - 18*c) - 1)) - 126*e^(-8*d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e
^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d
*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x -
 16*c) + e^(-18*d*x - 18*c) - 1)) - 1/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x
 - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*
c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) +
 e^(-18*d*x - 18*c) - 1))) - 16/5*a^2*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x
 - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1))) + 6*a*b^2/(d*(e^(-
2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.56726, size = 3646, normalized size = 26.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/2520*(315*b^3*cosh(d*x + c)^11 + 3465*b^3*cosh(d*x + c)*sinh(d*x + c)^10
- (1024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^9 - 4*(315*
b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*sinh(d*x + c)^9 + 9*(5775*b^3*
cosh(d*x + c)^3 - (1024*a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^8 + 9*(1024*a^3 + 4032*a^2*b + 5880*a*b^2 + 1225*b^3)*c
osh(d*x + c)^7 + 36*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2 - 4*(3
15*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^2)*sinh(d*x +
 c)^7 + 21*(6930*b^3*cosh(d*x + c)^5 - 4*(1024*a^3 + 4032*a^2*b + 7560*a*b^
2 + 2835*b^3)*cosh(d*x + c)^3 + 3*(1024*a^3 + 4032*a^2*b + 5880*a*b^2 + 122
5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 9*(4096*a^3 + 16128*a^2*b + 16800*a
*b^2 + 2625*b^3)*cosh(d*x + c)^5 - 36*(1260*b^3*d*x + 14*(315*b^3*d*x - 256
*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 - 1024*a^3 - 4032*a^2*b - 7
560*a*b^2 - 21*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x +
 c)^2)*sinh(d*x + c)^5 + 9*(11550*b^3*cosh(d*x + c)^7 - 14*(1024*a^3 + 4032
*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^5 + 35*(1024*a^3 + 4032*a^2*b
 + 5880*a*b^2 + 1225*b^3)*cosh(d*x + c)^3 - 5*(4096*a^3 + 16128*a^2*b + 168
00*a*b^2 + 2625*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 42*(2048*a^3 + 6144*a
^2*b + 5040*a*b^2 + 675*b^3)*cosh(d*x + c)^3 - 12*(28*(315*b^3*d*x - 256*a^
3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^6 - 8820*b^3*d*x - 105*(315*b^3*
d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 + 7168*a^3 + 28224
*a^2*b + 52920*a*b^2 + 120*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2
```

```
)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 9*(1925*b^3*cosh(d*x + c)^9 - 4*(1024*
a^3 + 4032*a^2*b + 7560*a*b^2 + 2835*b^3)*cosh(d*x + c)^7 + 21*(1024*a^3 +
4032*a^2*b + 5880*a*b^2 + 1225*b^3)*cosh(d*x + c)^5 - 10*(4096*a^3 + 16128*
a^2*b + 16800*a*b^2 + 2625*b^3)*cosh(d*x + c)^3 + 14*(2048*a^3 + 6144*a^2*b
 + 5040*a*b^2 + 675*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 126*(1024*a^3 + 1
152*a^2*b + 840*a*b^2 + 105*b^3)*cosh(d*x + c) - 36*((315*b^3*d*x - 256*a^3
 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^8 - 7*(315*b^3*d*x - 256*a^3 - 10
08*a^2*b - 1890*a*b^2)*cosh(d*x + c)^6 + 4410*b^3*d*x + 20*(315*b^3*d*x - 2
56*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d*x + c)^4 - 3584*a^3 - 14112*a^2*b
- 26460*a*b^2 - 28*(315*b^3*d*x - 256*a^3 - 1008*a^2*b - 1890*a*b^2)*cosh(d
*x + c)^2)*sinh(d*x + c))/(d*sinh(d*x + c)^9 + 9*(4*d*cosh(d*x + c)^2 - d)*
sinh(d*x + c)^7 + 9*(14*d*cosh(d*x + c)^4 - 21*d*cosh(d*x + c)^2 + 4*d)*sin
h(d*x + c)^5 + 3*(28*d*cosh(d*x + c)^6 - 105*d*cosh(d*x + c)^4 + 120*d*cosh
(d*x + c)^2 - 28*d)*sinh(d*x + c)^3 + 9*(d*cosh(d*x + c)^8 - 7*d*cosh(d*x +
 c)^6 + 20*d*cosh(d*x + c)^4 - 28*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**10*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.71514, size = 495, normalized size = 3.54

$$-\frac{(dx+c)b^3}{2d} + \frac{b^3e^{(2dx+2c)}}{8d} + \frac{\left(2b^3e^{(2dx+2c)} - b^3\right)e^{(-2dx-2c)}}{8d} - \frac{2\left(945ab^2e^{(16dx+16c)} - 7560ab^2e^{(14dx+14c)} + 5040a^2be^{(12}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^10*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*b^3/d + 1/8*b^3*e^(2*d*x + 2*c)/d + 1/8*(2*b^3*e^(2*d*x + 2*
c) - b^3)*e^(-2*d*x - 2*c)/d - 2/315*(945*a*b^2*e^(16*d*x + 16*c) - 7560*a*
b^2*e^(14*d*x + 14*c) + 5040*a^2*b*e^(12*d*x + 12*c) + 26460*a*b^2*e^(12*d*
x + 12*c) - 22680*a^2*b*e^(10*d*x + 10*c) - 52920*a*b^2*e^(10*d*x + 10*c) +
 16128*a^3*e^(8*d*x + 8*c) + 40824*a^2*b*e^(8*d*x + 8*c) + 66150*a*b^2*e^(8
*d*x + 8*c) - 10752*a^3*e^(6*d*x + 6*c) - 37296*a^2*b*e^(6*d*x + 6*c) - 529
20*a*b^2*e^(6*d*x + 6*c) + 4608*a^3*e^(4*d*x + 4*c) + 18144*a^2*b*e^(4*d*x
+ 4*c) + 26460*a*b^2*e^(4*d*x + 4*c) - 1152*a^3*e^(2*d*x + 2*c) - 4536*a^2*
b*e^(2*d*x + 2*c) - 7560*a*b^2*e^(2*d*x + 2*c) + 128*a^3 + 504*a^2*b + 945*
a*b^2)/(d*(e^(2*d*x + 2*c) - 1)^9)
```

3.224 $\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=147

$$-\frac{a\left(5a^2+9ab+3b^2\right)\coth^3(c+dx)}{3d}+\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{a^2(10a+3b)\coth^7(c+dx)}{7d}+\frac{a^2(10a+9b)\coth}{5d}$$

```
[Out] b^3*x + (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a*(5*a^2 + 9*a*b + 3*b
^2)*Coth[c + d*x]^3)/(3*d) + (a^2*(10*a + 9*b)*Coth[c + d*x]^5)/(5*d) - (a^
2*(10*a + 3*b)*Coth[c + d*x]^7)/(7*d) + (5*a^3*Coth[c + d*x]^9)/(9*d) - (a^
3*Coth[c + d*x]^11)/(11*d)
```

Rubi [A] time = 0.149097, antiderivative size = 147, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3217, 1261, 207}

$$-\frac{a\left(5a^2+9ab+3b^2\right)\coth^3(c+dx)}{3d}+\frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d}-\frac{a^2(10a+3b)\coth^7(c+dx)}{7d}+\frac{a^2(10a+9b)\coth}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] b^3*x + (a*(a^2 + 3*a*b + 3*b^2)*Coth[c + d*x])/d - (a*(5*a^2 + 9*a*b + 3*b
^2)*Coth[c + d*x]^3)/(3*d) + (a^2*(10*a + 9*b)*Coth[c + d*x]^5)/(5*d) - (a^
2*(10*a + 3*b)*Coth[c + d*x]^7)/(7*d) + (5*a^3*Coth[c + d*x]^9)/(9*d) - (a^
3*Coth[c + d*x]^11)/(11*d)
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{csch}^{12}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{12}(1-x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a^3}{x^{12}} - \frac{5a^3}{x^{10}} + \frac{a^2(10a+3b)}{x^8} - \frac{a^2(10a+9b)}{x^6} + \frac{a(5a^2+9ab+3b^2)}{x^4} - \frac{a(a^2+3ab+}{x^2}\right.\right.}{d}$$
$$= \frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} - \frac{a\left(5a^2+9ab+3b^2\right)\coth^3(c+dx)}{3d} +$$
$$= b^3x + \frac{a\left(a^2+3ab+3b^2\right)\coth(c+dx)}{d} - \frac{a\left(5a^2+9ab+3b^2\right)\coth^3(c+}{3d}$$

Mathematica [A] time = 6.10498, size = 239, normalized size = 1.63

$$\frac{\operatorname{csch}^3(c+dx)\left(-2376a^2b\cosh(c+dx) - 640a^3\cosh(c+dx) - 3465ab^2\cosh(c+dx)\right)}{3465d} + \frac{2\operatorname{csch}(c+dx)\left(2376a^2b\cosh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^12*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (b^3*(c + d*x))/d + (2*(640*a^3*Cosh[c + d*x] + 2376*a^2*b*Cosh[c + d*x] +
3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x])/(3465*d) + ((-640*a^3*Cosh[c + d*x
] - 2376*a^2*b*Cosh[c + d*x] - 3465*a*b^2*Cosh[c + d*x])*Csch[c + d*x]^3)/(
3465*d) + (2*(80*a^3*Cosh[c + d*x] + 297*a^2*b*Cosh[c + d*x])*Csch[c + d*x]
^5)/(1155*d) + ((-80*a^3*Cosh[c + d*x] - 297*a^2*b*Cosh[c + d*x])*Csch[c +
d*x]^7)/(693*d) + (10*a^3*Coth[c + d*x]*Csch[c + d*x]^8)/(99*d) - (a^3*Coth
[c + d*x]*Csch[c + d*x]^10)/(11*d)
```

Maple [A] time = 0.086, size = 145, normalized size = 1.

$$\frac{1}{d}\left(a^3\left(\frac{256}{693} - \frac{(\operatorname{csch}(dx+c))^{10}}{11} + \frac{10\,(\operatorname{csch}(dx+c))^8}{99} - \frac{80\,(\operatorname{csch}(dx+c))^6}{693} + \frac{32\,(\operatorname{csch}(dx+c))^4}{231} - \frac{128\,(\operatorname{csch}(dx+}{693}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(256/693-1/11*csch(d*x+c)^10+10/99*csch(d*x+c)^8-80/693*csch(d*x+c
)^6+32/231*csch(d*x+c)^4-128/693*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(16/35-
1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^
2*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c)+b^3*(d*x+c))
```

Maxima [B] time = 1.24004, size = 1743, normalized size = 11.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] b^3*x + 512/693*a^3*(11*e^(-2*d*x - 2*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4
*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x
- 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x
- 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c
) - 1)) - 55*e^(-4*d*x - 4*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c)
 + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 4
62*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 5
5*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 1
65*e^(-6*d*x - 6*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(
-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*
d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d
*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 330*e^(-8*d
*x - 8*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6
*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c
) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c)
 - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 462*e^(-10*d*x - 10*c
)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 33
0*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*
e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^
(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 1/(d*(11*e^(-2*d*x - 2*c) - 5
5*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-
10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-
16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x
 - 22*c) - 1))) + 96/35*a^2*b*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) -
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) - 21*e^(-4*d
*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) - 1/(d*(7*e^(-2*d
*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c)
 + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)))
 + 4*a*b^2*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c)
+ e^(-6*d*x - 6*c) - 1)) - 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) +
e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 1.72119, size = 4510, normalized size = 30.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/3465*(2*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^11 + 22*(640*a^
3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^10 + (3465*b^3*d*x
 - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*sinh(d*x + c)^11 - 22*(640*a^3 + 237
6*a^2*b + 3465*a*b^2)*cosh(d*x + c)^9 - 11*(3465*b^3*d*x - 1280*a^3 - 4752*
a^2*b - 6930*a*b^2 - 5*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*
cosh(d*x + c)^2)*sinh(d*x + c)^9 + 66*(5*(640*a^3 + 2376*a^2*b + 3465*a*b^2
)*cosh(d*x + c)^3 - 3*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c))*si
nh(d*x + c)^8 + 110*(640*a^3 + 2376*a^2*b + 3087*a*b^2)*cosh(d*x + c)^7 + 1
1*(17325*b^3*d*x + 30*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*c
osh(d*x + c)^4 - 6400*a^3 - 23760*a^2*b - 34650*a*b^2 - 36*(3465*b^3*d*x -
1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 154*
(6*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^5 - 12*(640*a^3 + 2376
```

```
*a^2*b + 3465*a*b^2)*cosh(d*x + c)^3 + 5*(640*a^3 + 2376*a^2*b + 3087*a*b^2
)*cosh(d*x + c))*sinh(d*x + c)^6 - 330*(640*a^3 + 2376*a^2*b + 2415*a*b^2)*
cosh(d*x + c)^5 + 33*(14*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2
)*cosh(d*x + c)^6 - 17325*b^3*d*x - 42*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*
b - 6930*a*b^2)*cosh(d*x + c)^4 + 6400*a^3 + 23760*a^2*b + 34650*a*b^2 + 35
*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(
d*x + c)^5 + 22*(30*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^7 - 1
26*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^5 + 175*(640*a^3 + 237
6*a^2*b + 3087*a*b^2)*cosh(d*x + c)^3 - 75*(640*a^3 + 2376*a^2*b + 2415*a*b
^2)*cosh(d*x + c))*sinh(d*x + c)^4 + 660*(640*a^3 + 1872*a^2*b + 1533*a*b^2
)*cosh(d*x + c)^3 + 11*(15*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b
^2)*cosh(d*x + c)^8 - 84*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2
)*cosh(d*x + c)^6 + 103950*b^3*d*x + 175*(3465*b^3*d*x - 1280*a^3 - 4752*a^
2*b - 6930*a*b^2)*cosh(d*x + c)^4 - 38400*a^3 - 142560*a^2*b - 207900*a*b^2
 - 150*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)
*sinh(d*x + c)^3 + 22*(5*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^
9 - 36*(640*a^3 + 2376*a^2*b + 3465*a*b^2)*cosh(d*x + c)^7 + 105*(640*a^3 +
 2376*a^2*b + 3087*a*b^2)*cosh(d*x + c)^5 - 150*(640*a^3 + 2376*a^2*b + 241
5*a*b^2)*cosh(d*x + c)^3 + 90*(640*a^3 + 1872*a^2*b + 1533*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^2 - 4620*(128*a^3 + 144*a^2*b + 105*a*b^2)*cosh(d*x + c
) + 11*((3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^1
0 - 9*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^8 +
 35*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^6 - 1
45530*b^3*d*x - 75*(3465*b^3*d*x - 1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh
(d*x + c)^4 + 53760*a^3 + 199584*a^2*b + 291060*a*b^2 + 90*(3465*b^3*d*x -
1280*a^3 - 4752*a^2*b - 6930*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*sinh
(d*x + c)^11 + 11*(5*d*cosh(d*x + c)^2 - d)*sinh(d*x + c)^9 + 11*(30*d*cosh
(d*x + c)^4 - 36*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^7 + 33*(14*d*cosh(d
*x + c)^6 - 42*d*cosh(d*x + c)^4 + 35*d*cosh(d*x + c)^2 - 5*d)*sinh(d*x + c
)^5 + 11*(15*d*cosh(d*x + c)^8 - 84*d*cosh(d*x + c)^6 + 175*d*cosh(d*x + c)
^4 - 150*d*cosh(d*x + c)^2 + 30*d)*sinh(d*x + c)^3 + 11*(d*cosh(d*x + c)^10
 - 9*d*cosh(d*x + c)^8 + 35*d*cosh(d*x + c)^6 - 75*d*cosh(d*x + c)^4 + 90*d
*cosh(d*x + c)^2 - 42*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**12*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out
```

Giac [B] time = 1.6987, size = 485, normalized size = 3.3

$$\frac{(dx+c)b^3}{d} - \frac{4\left(10395\,ab^2e^{(18\,dx+18\,c)} - 86625\,ab^2e^{(16\,dx+16\,c)} + 83160\,a^2be^{(14\,dx+14\,c)} + 318780\,ab^2e^{(14\,dx+14\,c)} - 382536\,a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^12*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^3/d - 4/3465*(10395*a*b^2*e^(18*d*x + 18*c) - 86625*a*b^2*e^(16
*d*x + 16*c) + 83160*a^2*b*e^(14*d*x + 14*c) + 318780*a*b^2*e^(14*d*x + 14*
c) - 382536*a^2*b*e^(12*d*x + 12*c) - 679140*a*b^2*e^(12*d*x + 12*c) + 2956
80*a^3*e^(10*d*x + 10*c) + 715176*a^2*b*e^(10*d*x + 10*c) + 921690*a*b^2*e^
(10*d*x + 10*c) - 211200*a^3*e^(8*d*x + 8*c) - 700920*a^2*b*e^(8*d*x + 8*c)
 - 824670*a*b^2*e^(8*d*x + 8*c) + 105600*a^3*e^(6*d*x + 6*c) + 392040*a^2*b
*e^(6*d*x + 6*c) + 485100*a*b^2*e^(6*d*x + 6*c) - 35200*a^3*e^(4*d*x + 4*c)
 - 130680*a^2*b*e^(4*d*x + 4*c) - 180180*a*b^2*e^(4*d*x + 4*c) + 7040*a^3*e
^(2*d*x + 2*c) + 26136*a^2*b*e^(2*d*x + 2*c) + 38115*a*b^2*e^(2*d*x + 2*c)
- 640*a^3 - 2376*a^2*b - 3465*a*b^2)/(d*(e^(2*d*x + 2*c) - 1)^11)
```

3.225 $\int \mathrm{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=144

$$-\frac{a^2(5a+b)\coth^9(c+dx)}{3d}+\frac{4a^2(5a+3b)\coth^7(c+dx)}{7d}-\frac{a^3\coth^{13}(c+dx)}{13d}+\frac{6a^3\coth^{11}(c+dx)}{11d}-\frac{3a(a+b)(5a+}{5}$$

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*a*(a + b)^2*Coth[c + d*x]^3)/d - (3*a*(
a + b)*(5*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^2*(5*a + 3*b)*Coth[c + d*x]^
7)/(7*d) - (a^2*(5*a + b)*Coth[c + d*x]^9)/(3*d) + (6*a^3*Coth[c + d*x]^11)
/(11*d) - (a^3*Coth[c + d*x]^13)/(13*d)
```

Rubi [A] time = 0.130776, antiderivative size = 144, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3217, 1108}

$$-\frac{a^2(5a+b)\coth^9(c+dx)}{3d}+\frac{4a^2(5a+3b)\coth^7(c+dx)}{7d}-\frac{a^3\coth^{13}(c+dx)}{13d}+\frac{6a^3\coth^{11}(c+dx)}{11d}-\frac{3a(a+b)(5a+}{5}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*a*(a + b)^2*Coth[c + d*x]^3)/d - (3*a*(
a + b)*(5*a + b)*Coth[c + d*x]^5)/(5*d) + (4*a^2*(5*a + 3*b)*Coth[c + d*x]^
7)/(7*d) - (a^2*(5*a + b)*Coth[c + d*x]^9)/(3*d) + (6*a^3*Coth[c + d*x]^11)
/(11*d) - (a^3*Coth[c + d*x]^13)/(13*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1108

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_Symbol
] :> Int[ExpandIntegrand[(d*x)^m*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a,
 b, c, d, m}, x] && IGtQ[p, 0] &&  !IntegerQ[(m + 1)/2]
```

Rubi steps

$$\begin{aligned}\int \mathrm{csch}^{14}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx &= \frac{\mathrm{Subst}\left(\int \frac{\left(a-2ax^2+(a+b)x^4\right)^3}{x^{14}}\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\mathrm{Subst}\left(\int \left(\frac{a^3}{x^{14}}-\frac{6a^3}{x^{12}}+\frac{3a^2(5a+b)}{x^{10}}-\frac{4a^2(5a+3b)}{x^8}+\frac{3a(a+b)(5a+b)}{x^6}-\frac{6a(a+b)^2}{x^4}+\right.\right.}{d}\\ &= -\frac{(a+b)^3\coth(c+dx)}{d}+\frac{2a(a+b)^2\coth^3(c+dx)}{d}-\frac{3a(a+b)(5a+b)}{5d}\end{aligned}$$

Mathematica [B] time = 3.20677, size = 350, normalized size = 2.43

$$-\frac{\operatorname{csch}^{13}(c+dx)\left(8580\left(1152a^2b+1024a^3+840ab^2+231b^3\right)\cosh(c+dx)-6435\left(2944a^2b+1024a^3+2408ab^2+693b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^14*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((8580*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c + d*x] - 6435*
(1024*a^3 + 2944*a^2*b + 2408*a*b^2 + 693*b^3)*Cosh[3*(c + d*x)] + 3660800*
a^3*Cosh[5*(c + d*x)] + 13087360*a^2*b*Cosh[5*(c + d*x)] + 13093080*a*b^2*C
osh[5*(c + d*x)] + 4129125*b^3*Cosh[5*(c + d*x)] - 1464320*a^3*Cosh[7*(c +
d*x)] - 5234944*a^2*b*Cosh[7*(c + d*x)] - 6390384*a*b^2*Cosh[7*(c + d*x)] -
 2312310*b^3*Cosh[7*(c + d*x)] + 399360*a^3*Cosh[9*(c + d*x)] + 1427712*a^2
*b*Cosh[9*(c + d*x)] + 1873872*a*b^2*Cosh[9*(c + d*x)] + 810810*b^3*Cosh[9*
(c + d*x)] - 66560*a^3*Cosh[11*(c + d*x)] - 237952*a^2*b*Cosh[11*(c + d*x)]
 - 312312*a*b^2*Cosh[11*(c + d*x)] - 165165*b^3*Cosh[11*(c + d*x)] + 5120*a
^3*Cosh[13*(c + d*x)] + 18304*a^2*b*Cosh[13*(c + d*x)] + 24024*a*b^2*Cosh[1
3*(c + d*x)] + 15015*b^3*Cosh[13*(c + d*x)])*Csch[c + d*x]^13)/(61501440*d)
```

Maple [A] time = 0.098, size = 177, normalized size = 1.2

$$\frac{1}{d}\left(a^3\left(-\frac{1024}{3003}-\frac{(\operatorname{csch}(dx+c))^{12}}{13}+\frac{12\,(\operatorname{csch}(dx+c))^{10}}{143}-\frac{40\,(\operatorname{csch}(dx+c))^{8}}{429}+\frac{320\,(\operatorname{csch}(dx+c))^{6}}{3003}-\frac{128\,(\operatorname{csch}(dx\right.\right.}{1001}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-1024/3003-1/13*csch(d*x+c)^12+12/143*csch(d*x+c)^10-40/429*csch(
d*x+c)^8+320/3003*csch(d*x+c)^6-128/1001*csch(d*x+c)^4+512/3003*csch(d*x+c)
^2)*coth(d*x+c)+3*a^2*b*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-16/1
05*csch(d*x+c)^4+64/315*csch(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(-8/15-1/5*csch(
d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c)-b^3*coth(d*x+c))
```

Maxima [B] time = 1.30355, size = 2587, normalized size = 17.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -2048/3003*a^3*(13*e^(-2*d*x - 2*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x
- 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10
*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x -
 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x -
22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 78*e^(-4*d*x - 4
*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) -
715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) +
1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c)
- 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) +
e^(-26*d*x - 26*c) - 1)) + 286*e^(-6*d*x - 6*c)/(d*(13*e^(-2*d*x - 2*c) - 7
```

```
8*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(
-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*
e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e
^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 715*
e^(-8*d*x - 8*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*
d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d
*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18
*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d
*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 1287*e^(-10*d*x - 10*c)/(d*(13*e^(-
2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x -
 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x
 - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*
x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26
*c) - 1)) - 1716*e^(-12*d*x - 12*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x
- 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10
*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x -
 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x -
22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 1/(d*(13*e^(-2*d
*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*
c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x -
14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x -
 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c)
 - 1))) - 256/105*a^2*b*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(
-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x
 - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 1
6*c) + e^(-18*d*x - 18*c) - 1)) - 36*e^(-4*d*x - 4*c)/(d*(9*e^(-2*d*x - 2*c
) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) + 126*
e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*e^(-
16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) + 84*e^(-6*d*x - 6*c)/(d*(9*e^(-2
*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8
*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c
) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 126*e^(-8*d*x - 8*c)/
(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^
(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14
*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 1/(d*(9*e^
(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x
- 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 1
4*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1))) - 16/5*a*b^2*(5*e^(
-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(-4*d*x - 4*c)
/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x - 2*c) - 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) - 1))) + 2*b^3/(d*(e^(-2*d*x - 2*c) - 1))
```

Fricas [B] time = 1.80262, size = 6778, normalized size = 47.07

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -4/15015*((2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^1
2 - 48*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^11 +
 (2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*sinh(d*x + c)^12 - 52*(6
40*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^10 - 2*(16640*a^
```

3 + 59488*a^2*b + 78078*a*b^2 + 90090*b^3 - 33*(2560*a^3 + 9152*a^2*b + 120
12*a*b^2 + 15015*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 - 40*(22*(640*a^3 +
 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^3 - 13*(640*a^3 + 2288*a^2*b + 3003
*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 78*(2560*a^3 + 9152*a^2*b + 13552*
a*b^2 + 12705*b^3)*cosh(d*x + c)^8 + 3*(165*(2560*a^3 + 9152*a^2*b + 12012*
a*b^2 + 15015*b^3)*cosh(d*x + c)^4 + 66560*a^3 + 237952*a^2*b + 352352*a*b^
2 + 330330*b^3 - 780*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*
x + c)^2)*sinh(d*x + c)^8 - 96*(33*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh
(d*x + c)^5 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^3 + 52*(
320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^7 - 572*(12
80*a^3 + 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^6 + 4*(231*(2560
*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^6 - 2730*(640*a^
3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^4 - 183040*a^3 - 6543
68*a^2*b - 1084083*a*b^2 - 825825*b^3 + 546*(2560*a^3 + 9152*a^2*b + 13552*
a*b^2 + 12705*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 - 24*(132*(640*a^3 + 22
88*a^2*b + 3003*a*b^2)*cosh(d*x + c)^7 - 546*(640*a^3 + 2288*a^2*b + 3003*a
*b^2)*cosh(d*x + c)^5 + 1456*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x +
 c)^3 - 143*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x + c))*sinh(d*x +
c)^5 + 143*(12800*a^3 + 53824*a^2*b + 79884*a*b^2 + 51975*b^3)*cosh(d*x + c
)^4 + (495*(2560*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^
8 - 10920*(640*a^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^6 +
5460*(2560*a^3 + 9152*a^2*b + 13552*a*b^2 + 12705*b^3)*cosh(d*x + c)^4 + 18
30400*a^3 + 7696832*a^2*b + 11423412*a*b^2 + 7432425*b^3 - 8580*(1280*a^3 +
 4576*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 16*
(55*(640*a^3 + 2288*a^2*b + 3003*a*b^2)*cosh(d*x + c)^9 - 390*(640*a^3 + 22
88*a^2*b + 3003*a*b^2)*cosh(d*x + c)^7 + 2184*(320*a^3 + 1144*a^2*b + 1309*
a*b^2)*cosh(d*x + c)^5 - 715*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x
+ c)^3 + 143*(3200*a^3 + 9424*a^2*b + 6489*a*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4392960*a^3 + 10323456*a^2*b + 12108096*a*b^2 + 6936930*b^3 - 3432*
(960*a^3 + 4664*a^2*b + 5859*a*b^2 + 3465*b^3)*cosh(d*x + c)^2 + 6*(11*(256
0*a^3 + 9152*a^2*b + 12012*a*b^2 + 15015*b^3)*cosh(d*x + c)^10 - 390*(640*a
^3 + 2288*a^2*b + 3003*a*b^2 + 3465*b^3)*cosh(d*x + c)^8 + 364*(2560*a^3 +
9152*a^2*b + 13552*a*b^2 + 12705*b^3)*cosh(d*x + c)^6 - 1430*(1280*a^3 + 45
76*a^2*b + 7581*a*b^2 + 5775*b^3)*cosh(d*x + c)^4 - 549120*a^3 - 2667808*a^
2*b - 3351348*a*b^2 - 1981980*b^3 + 143*(12800*a^3 + 53824*a^2*b + 79884*a*
b^2 + 51975*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 8*(6*(640*a^3 + 2288*a^
2*b + 3003*a*b^2)*cosh(d*x + c)^11 - 65*(640*a^3 + 2288*a^2*b + 3003*a*b^2)
*cosh(d*x + c)^9 + 624*(320*a^3 + 1144*a^2*b + 1309*a*b^2)*cosh(d*x + c)^7
- 429*(1280*a^3 + 4576*a^2*b + 4011*a*b^2)*cosh(d*x + c)^5 + 286*(3200*a^3
+ 9424*a^2*b + 6489*a*b^2)*cosh(d*x + c)^3 - 858*(960*a^3 + 1528*a^2*b + 90
3*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^14 + 14*d*cosh(d*x
+ c)*sinh(d*x + c)^13 + d*sinh(d*x + c)^14 - 14*d*cosh(d*x + c)^12 + 7*(13*
d*cosh(d*x + c)^2 - 2*d)*sinh(d*x + c)^12 + 4*(91*d*cosh(d*x + c)^3 - 36*d*
cosh(d*x + c))*sinh(d*x + c)^11 + 91*d*cosh(d*x + c)^10 + 7*(143*d*cosh(d*x
 + c)^4 - 132*d*cosh(d*x + c)^2 + 13*d)*sinh(d*x + c)^10 + 2*(1001*d*cosh(d
*x + c)^5 - 1320*d*cosh(d*x + c)^3 + 325*d*cosh(d*x + c))*sinh(d*x + c)^9 -
 364*d*cosh(d*x + c)^8 + 7*(429*d*cosh(d*x + c)^6 - 990*d*cosh(d*x + c)^4 +
 585*d*cosh(d*x + c)^2 - 52*d)*sinh(d*x + c)^8 + 8*(429*d*cosh(d*x + c)^7 -
 1188*d*cosh(d*x + c)^5 + 975*d*cosh(d*x + c)^3 - 208*d*cosh(d*x + c))*sinh
(d*x + c)^7 + 1001*d*cosh(d*x + c)^6 + 7*(429*d*cosh(d*x + c)^8 - 1848*d*co
sh(d*x + c)^6 + 2730*d*cosh(d*x + c)^4 - 1456*d*cosh(d*x + c)^2 + 143*d)*si
nh(d*x + c)^6 + 2*(1001*d*cosh(d*x + c)^9 - 4752*d*cosh(d*x + c)^7 + 8190*d
*cosh(d*x + c)^5 - 5824*d*cosh(d*x + c)^3 + 1287*d*cosh(d*x + c))*sinh(d*x
+ c)^5 - 2002*d*cosh(d*x + c)^4 + 7*(143*d*cosh(d*x + c)^10 - 990*d*cosh(d*
x + c)^8 + 2730*d*cosh(d*x + c)^6 - 3640*d*cosh(d*x + c)^4 + 2145*d*cosh(d*
x + c)^2 - 286*d)*sinh(d*x + c)^4 + 4*(91*d*cosh(d*x + c)^11 - 660*d*cosh(d
*x + c)^9 + 1950*d*cosh(d*x + c)^7 - 2912*d*cosh(d*x + c)^5 + 2145*d*cosh(d
*x + c)^3 - 572*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3003*d*cosh(d*x + c)^2 +
 7*(13*d*cosh(d*x + c)^12 - 132*d*cosh(d*x + c)^10 + 585*d*cosh(d*x + c)^8

```
- 1456*d*cosh(d*x + c)^6 + 2145*d*cosh(d*x + c)^4 - 1716*d*cosh(d*x + c)^2
+ 429*d)*sinh(d*x + c)^2 + 2*(7*d*cosh(d*x + c)^13 - 72*d*cosh(d*x + c)^11
+ 325*d*cosh(d*x + c)^9 - 832*d*cosh(d*x + c)^7 + 1287*d*cosh(d*x + c)^5 -
1144*d*cosh(d*x + c)^3 + 429*d*cosh(d*x + c))*sinh(d*x + c) - 1716*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**14*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.66735, size = 760, normalized size = 5.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^14*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -2/15015*(15015*b^3*e^(24*d*x + 24*c) - 180180*b^3*e^(22*d*x + 22*c) + 2402
40*a*b^2*e^(20*d*x + 20*c) + 990990*b^3*e^(20*d*x + 20*c) - 2042040*a*b^2*e
^(18*d*x + 18*c) - 3303300*b^3*e^(18*d*x + 18*c) + 2306304*a^2*b*e^(16*d*x
+ 16*c) + 7711704*a*b^2*e^(16*d*x + 16*c) + 7432425*b^3*e^(16*d*x + 16*c) -
 10762752*a^2*b*e^(14*d*x + 14*c) - 17008992*a*b^2*e^(14*d*x + 14*c) - 1189
1880*b^3*e^(14*d*x + 14*c) + 8785920*a^3*e^(12*d*x + 12*c) + 20646912*a^2*b
*e^(12*d*x + 12*c) + 24216192*a*b^2*e^(12*d*x + 12*c) + 13873860*b^3*e^(12*
d*x + 12*c) - 6589440*a^3*e^(10*d*x + 10*c) - 21250944*a^2*b*e^(10*d*x + 10
*c) - 23207184*a*b^2*e^(10*d*x + 10*c) - 11891880*b^3*e^(10*d*x + 10*c) + 3
660800*a^3*e^(8*d*x + 8*c) + 13087360*a^2*b*e^(8*d*x + 8*c) + 15135120*a*b^
2*e^(8*d*x + 8*c) + 7432425*b^3*e^(8*d*x + 8*c) - 1464320*a^3*e^(6*d*x + 6*
c) - 5234944*a^2*b*e^(6*d*x + 6*c) - 6630624*a*b^2*e^(6*d*x + 6*c) - 330330
0*b^3*e^(6*d*x + 6*c) + 399360*a^3*e^(4*d*x + 4*c) + 1427712*a^2*b*e^(4*d*x
 + 4*c) + 1873872*a*b^2*e^(4*d*x + 4*c) + 990990*b^3*e^(4*d*x + 4*c) - 6656
0*a^3*e^(2*d*x + 2*c) - 237952*a^2*b*e^(2*d*x + 2*c) - 312312*a*b^2*e^(2*d*
x + 2*c) - 180180*b^3*e^(2*d*x + 2*c) + 5120*a^3 + 18304*a^2*b + 24024*a*b^
2 + 15015*b^3)/(d*(e^(2*d*x + 2*c) - 1)^13)
```

3.226 $\int \mathrm{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=182

$$-\frac{a\left(35a^2+30ab+3b^2\right)\coth^7(c+dx)}{7d}-\frac{3a^2(7a+b)\coth^{11}(c+dx)}{11d}+\frac{5a^2(7a+3b)\coth^9(c+dx)}{9d}-\frac{a^3\coth^{15}(c+dx)}{15d}+$$

```
[Out] ((a + b)^3*Coth[c + d*x])/d - ((a + b)^2*(7*a + b)*Coth[c + d*x]^3)/(3*d) +
 (3*a*(a + b)*(7*a + 3*b)*Coth[c + d*x]^5)/(5*d) - (a*(35*a^2 + 30*a*b + 3*
b^2)*Coth[c + d*x]^7)/(7*d) + (5*a^2*(7*a + 3*b)*Coth[c + d*x]^9)/(9*d) - (
3*a^2*(7*a + b)*Coth[c + d*x]^11)/(11*d) + (7*a^3*Coth[c + d*x]^13)/(13*d)
- (a^3*Coth[c + d*x]^15)/(15*d)
```

Rubi [A] time = 0.158821, antiderivative size = 182, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3217, 1261}

$$-\frac{a\left(35a^2+30ab+3b^2\right)\coth^7(c+dx)}{7d}-\frac{3a^2(7a+b)\coth^{11}(c+dx)}{11d}+\frac{5a^2(7a+3b)\coth^9(c+dx)}{9d}-\frac{a^3\coth^{15}(c+dx)}{15d}+$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] ((a + b)^3*Coth[c + d*x])/d - ((a + b)^2*(7*a + b)*Coth[c + d*x]^3)/(3*d) +
 (3*a*(a + b)*(7*a + 3*b)*Coth[c + d*x]^5)/(5*d) - (a*(35*a^2 + 30*a*b + 3*
b^2)*Coth[c + d*x]^7)/(7*d) + (5*a^2*(7*a + 3*b)*Coth[c + d*x]^9)/(9*d) - (
3*a^2*(7*a + b)*Coth[c + d*x]^11)/(11*d) + (7*a^3*Coth[c + d*x]^13)/(13*d)
- (a^3*Coth[c + d*x]^15)/(15*d)
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \mathrm{csch}^{16}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx=\frac{\mathrm{Subst}\left(\int\frac{\left(1-x^2\right)\left(a-2ax^2+(a+b)x^4\right)^3}{x^{16}}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=\frac{\mathrm{Subst}\left(\int\left(\frac{a^3}{x^{16}}-\frac{7a^3}{x^{14}}+\frac{3a^2(7a+b)}{x^{12}}-\frac{5a^2(7a+3b)}{x^{10}}+\frac{a\left(35a^2+30ab+3b^2\right)}{x^8}+\frac{3a(-7a-3b)(a-}{x^6}\right.\right.}{d}$$

$$=\frac{(a+b)^3\coth(c+dx)}{d}-\frac{(a+b)^2(7a+b)\coth^3(c+dx)}{3d}+\frac{3a(a+b)(7a+3}{5}$$

Mathematica [B] time = 4.50951, size = 404, normalized size = 2.22

$$-\frac{\operatorname{csch}^{15}(c+dx)\left(45045\left(1152a^2b+1024a^3+840ab^2+231b^3\right)\cosh(c+dx)-5005\left(20352a^2b+7168a^3+16632ab^2\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^16*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((45045*(1024*a^3 + 1152*a^2*b + 840*a*b^2 + 231*b^3)*Cosh[c + d*x] - 5005
*(7168*a^3 + 20352*a^2*b + 16632*a*b^2 + 4785*b^3)*Cosh[3*(c + d*x)] + 2152
5504*a^3*Cosh[5*(c + d*x)] + 74954880*a^2*b*Cosh[5*(c + d*x)] + 74162088*a*
b^2*Cosh[5*(c + d*x)] + 23288265*b^3*Cosh[5*(c + d*x)] - 9784320*a^3*Cosh[7
*(c + d*x)] - 34070400*a^2*b*Cosh[7*(c + d*x)] - 39999960*a*b^2*Cosh[7*(c +
 d*x)] - 14189175*b^3*Cosh[7*(c + d*x)] + 3261440*a^3*Cosh[9*(c + d*x)] + 1
1356800*a^2*b*Cosh[9*(c + d*x)] + 14054040*a*b^2*Cosh[9*(c + d*x)] + 572071
5*b^3*Cosh[9*(c + d*x)] - 752640*a^3*Cosh[11*(c + d*x)] - 2620800*a^2*b*Cos
h[11*(c + d*x)] - 3243240*a*b^2*Cosh[11*(c + d*x)] - 1486485*b^3*Cosh[11*(c
 + d*x)] + 107520*a^3*Cosh[13*(c + d*x)] + 374400*a^2*b*Cosh[13*(c + d*x)]
+ 463320*a*b^2*Cosh[13*(c + d*x)] + 225225*b^3*Cosh[13*(c + d*x)] - 7168*a^
3*Cosh[15*(c + d*x)] - 24960*a^2*b*Cosh[15*(c + d*x)] - 30888*a*b^2*Cosh[15
*(c + d*x)] - 15015*b^3*Cosh[15*(c + d*x)])*Csch[c + d*x]^15)/(369008640*d)
```

Maple [A] time = 0.085, size = 218, normalized size = 1.2

$$\frac{1}{d}\left(a^3\left(\frac{2048}{6435}-\frac{(\operatorname{csch}(dx+c))^{14}}{15}+\frac{14\,(\operatorname{csch}(dx+c))^{12}}{195}-\frac{56\,(\operatorname{csch}(dx+c))^{10}}{715}+\frac{112\,(\operatorname{csch}(dx+c))^{8}}{1287}-\frac{128\,(\operatorname{csch}(d}{1287}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(2048/6435-1/15*csch(d*x+c)^14+14/195*csch(d*x+c)^12-56/715*csch(d
*x+c)^10+112/1287*csch(d*x+c)^8-128/1287*csch(d*x+c)^6+256/2145*csch(d*x+c)
^4-1024/6435*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(256/693-1/11*csch(d*x+c)^1
0+10/99*csch(d*x+c)^8-80/693*csch(d*x+c)^6+32/231*csch(d*x+c)^4-128/693*csc
h(d*x+c)^2)*coth(d*x+c)+3*a*b^2*(16/35-1/7*csch(d*x+c)^6+6/35*csch(d*x+c)^4
-8/35*csch(d*x+c)^2)*coth(d*x+c)+b^3*(2/3-1/3*csch(d*x+c)^2)*coth(d*x+c))
```

Maxima [B] time = 1.38143, size = 3687, normalized size = 20.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 4096/6435*a^3*(15*e^(-2*d*x - 2*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x
- 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 1
0*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x
- 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d
*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*
x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 105*e^(-4*d*x - 4*c)/(d*(15*e^(-2*d*
x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8
```

*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 455*e^(-6*d*x - 6*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 1365*e^(-8*d*x - 8*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 3003*e^(-10*d*x - 10*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 5005*e^(-12*d*x - 12*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 6435*e^(-14*d*x - 14*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 1/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1))) + 512/231*a^2*b*(11*e^(-2*d*x - 2*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 55*e^(-4*d*x - 4*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 165*e^(-6*d*x - 6*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 330*e^(-8*d*x - 8*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) + 462*e^(-10*d*x - 10*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 1/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1))) + 96/35*a

```
*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) - 1)) - 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x
 - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) +
 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) +
35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6
*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x -
12*c) + e^(-14*d*x - 14*c) - 1)) - 1/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x
- 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c)
- 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1))) + 4/3*b^3*(3*e^(-2*d*x -
 2*c)/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1))
- 1/(d*(3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) - 1)))
```

Fricas [B] time = 1.88967, size = 8951, normalized size = 49.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 8/45045*((3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^1
3 + 13*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)*sin
h(d*x + c)^12 - 2*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*sinh(d*x
 + c)^13 - 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x +
 c)^11 + 6*(8960*a^3 + 31200*a^2*b + 38610*a*b^2 + 65065*b^3 - 26*(1792*a^3
 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 +
 11*(26*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^3
- 15*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c))*sinh
(d*x + c)^10 + 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x
 + c)^9 - 10*(143*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x
 + c)^4 + 37632*a^3 + 131040*a^2*b + 216216*a*b^2 + 234234*b^3 - 165*(1792*
a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9
 + 9*(143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^
5 - 275*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^3
+ 210*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c))*sinh(d
*x + c)^8 - 182*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x
 + c)^7 - 4*(858*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x
+ c)^6 - 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c
)^4 - 407680*a^3 - 1419600*a^2*b - 2918916*a*b^2 - 2147145*b^3 + 3780*(896*
a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7
+ 2*(858*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^7
 - 3465*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^5
+ 8820*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^3 - 63
7*(8960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c))*sinh(d*
x + c)^6 + 1365*(3584*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x +
c)^5 - 6*(429*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c
)^8 - 2310*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^6
 + 8820*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^4 + 81
5360*a^3 + 3800160*a^2*b + 6396390*a*b^2 + 3578575*b^3 - 1274*(4480*a^3 + 1
5600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 5*
(143*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^9 - 9
90*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^7 + 529
2*(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^5 - 1274*(8
960*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c)^3 + 1365*(35
84*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x + c))*sinh(d*x + c)^4
 - 429*(25088*a^3 + 24000*a^2*b + 3492*a*b^2 - 10395*b^3)*cosh(d*x + c)^3 -
```

```
2*(286*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^10 -
 2475*(1792*a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^8 + 17
640*(896*a^3 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^6 - 6370*(
4480*a^3 + 15600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^4 - 5381376
*a^3 - 32329440*a^2*b - 40980654*a*b^2 - 19324305*b^3 + 13650*(1792*a^3 + 8
352*a^2*b + 14058*a*b^2 + 7865*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(2
6*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 15015*b^3)*cosh(d*x + c)^11 - 275
*(3584*a^3 + 12480*a^2*b + 15444*a*b^2 - 11011*b^3)*cosh(d*x + c)^9 + 2520*
(1792*a^3 + 6240*a^2*b + 5148*a*b^2 - 3861*b^3)*cosh(d*x + c)^7 - 1274*(896
0*a^3 + 31200*a^2*b + 13068*a*b^2 - 12705*b^3)*cosh(d*x + c)^5 + 4550*(3584
*a^3 + 8256*a^2*b + 1980*a*b^2 - 3025*b^3)*cosh(d*x + c)^3 - 429*(25088*a^3
 + 24000*a^2*b + 3492*a*b^2 - 10395*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - 2
860*(1792*a^3 - 1248*a^2*b - 108*a*b^2 + 693*b^3)*cosh(d*x + c) - 2*(13*(17
92*a^3 + 6240*a^2*b + 7722*a*b^2 + 15015*b^3)*cosh(d*x + c)^12 - 165*(1792*
a^3 + 6240*a^2*b + 7722*a*b^2 + 13013*b^3)*cosh(d*x + c)^10 + 1890*(896*a^3
 + 3120*a^2*b + 5148*a*b^2 + 5577*b^3)*cosh(d*x + c)^8 - 1274*(4480*a^3 + 1
5600*a^2*b + 32076*a*b^2 + 23595*b^3)*cosh(d*x + c)^6 + 6825*(1792*a^3 + 83
52*a^2*b + 14058*a*b^2 + 7865*b^3)*cosh(d*x + c)^4 + 20500480*a^3 + 5491200
0*a^2*b + 59304960*a*b^2 + 25765740*b^3 - 1287*(12544*a^3 + 75360*a^2*b + 9
5526*a*b^2 + 45045*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^17
 + 17*d*cosh(d*x + c)*sinh(d*x + c)^16 + d*sinh(d*x + c)^17 - 15*d*cosh(d*x
 + c)^15 + (136*d*cosh(d*x + c)^2 - 15*d)*sinh(d*x + c)^15 + 5*(136*d*cosh(
d*x + c)^3 - 45*d*cosh(d*x + c))*sinh(d*x + c)^14 + 104*d*cosh(d*x + c)^13
+ (2380*d*cosh(d*x + c)^4 - 1575*d*cosh(d*x + c)^2 + 106*d)*sinh(d*x + c)^1
3 + 13*(476*d*cosh(d*x + c)^5 - 525*d*cosh(d*x + c)^3 + 104*d*cosh(d*x + c)
)*sinh(d*x + c)^12 - 440*d*cosh(d*x + c)^11 + (12376*d*cosh(d*x + c)^6 - 20
475*d*cosh(d*x + c)^4 + 8268*d*cosh(d*x + c)^2 - 470*d)*sinh(d*x + c)^11 +
11*(1768*d*cosh(d*x + c)^7 - 4095*d*cosh(d*x + c)^5 + 2704*d*cosh(d*x + c)^
3 - 440*d*cosh(d*x + c))*sinh(d*x + c)^10 + 1260*d*cosh(d*x + c)^9 + 5*(486
2*d*cosh(d*x + c)^8 - 15015*d*cosh(d*x + c)^6 + 15158*d*cosh(d*x + c)^4 - 5
170*d*cosh(d*x + c)^2 + 294*d)*sinh(d*x + c)^9 + (24310*d*cosh(d*x + c)^9 -
 96525*d*cosh(d*x + c)^7 + 133848*d*cosh(d*x + c)^5 - 72600*d*cosh(d*x + c)
^3 + 11340*d*cosh(d*x + c))*sinh(d*x + c)^8 - 2548*d*cosh(d*x + c)^7 + (194
48*d*cosh(d*x + c)^10 - 96525*d*cosh(d*x + c)^8 + 181896*d*cosh(d*x + c)^6
- 155100*d*cosh(d*x + c)^4 + 52920*d*cosh(d*x + c)^2 - 3458*d)*sinh(d*x + c
)^7 + (12376*d*cosh(d*x + c)^11 - 75075*d*cosh(d*x + c)^9 + 178464*d*cosh(d
*x + c)^7 - 203280*d*cosh(d*x + c)^5 + 105840*d*cosh(d*x + c)^3 - 17836*d*c
osh(d*x + c))*sinh(d*x + c)^6 + 3640*d*cosh(d*x + c)^5 + (6188*d*cosh(d*x +
 c)^12 - 45045*d*cosh(d*x + c)^10 + 136422*d*cosh(d*x + c)^8 - 217140*d*cos
h(d*x + c)^6 + 185220*d*cosh(d*x + c)^4 - 72618*d*cosh(d*x + c)^2 + 6370*d)
*sinh(d*x + c)^5 + 5*(476*d*cosh(d*x + c)^13 - 4095*d*cosh(d*x + c)^11 + 14
872*d*cosh(d*x + c)^9 - 29040*d*cosh(d*x + c)^7 + 31752*d*cosh(d*x + c)^5 -
 17836*d*cosh(d*x + c)^3 + 3640*d*cosh(d*x + c))*sinh(d*x + c)^4 - 3432*d*c
osh(d*x + c)^3 + (680*d*cosh(d*x + c)^14 - 6825*d*cosh(d*x + c)^12 + 30316*
d*cosh(d*x + c)^10 - 77550*d*cosh(d*x + c)^8 + 123480*d*cosh(d*x + c)^6 - 1
21030*d*cosh(d*x + c)^4 + 63700*d*cosh(d*x + c)^2 - 9438*d)*sinh(d*x + c)^3
 + (136*d*cosh(d*x + c)^15 - 1575*d*cosh(d*x + c)^13 + 8112*d*cosh(d*x + c)
^11 - 24200*d*cosh(d*x + c)^9 + 45360*d*cosh(d*x + c)^7 - 53508*d*cosh(d*x
+ c)^5 + 36400*d*cosh(d*x + c)^3 - 10296*d*cosh(d*x + c))*sinh(d*x + c)^2 +
 1430*d*cosh(d*x + c) + (17*d*cosh(d*x + c)^16 - 225*d*cosh(d*x + c)^14 + 1
378*d*cosh(d*x + c)^12 - 5170*d*cosh(d*x + c)^10 + 13230*d*cosh(d*x + c)^8
- 24206*d*cosh(d*x + c)^6 + 31850*d*cosh(d*x + c)^4 - 28314*d*cosh(d*x + c)
^2 + 11440*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**16*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.70699, size = 838, normalized size = 4.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^16*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -4/45045*(45045*b^3*e^(26*d*x + 26*c) - 555555*b^3*e^(24*d*x + 24*c) + 1081
080*a*b^2*e^(22*d*x + 22*c) + 3153150*b^3*e^(22*d*x + 22*c) - 9297288*a*b^2
*e^(20*d*x + 20*c) - 10900890*b^3*e^(20*d*x + 20*c) + 11531520*a^2*b*e^(18*
d*x + 18*c) + 35675640*a*b^2*e^(18*d*x + 18*c) + 25600575*b^3*e^(18*d*x + 1
8*c) - 54362880*a^2*b*e^(16*d*x + 16*c) - 80463240*a*b^2*e^(16*d*x + 16*c)
- 43108065*b^3*e^(16*d*x + 16*c) + 46126080*a^3*e^(14*d*x + 14*c) + 1062547
20*a^2*b*e^(14*d*x + 14*c) + 118301040*a*b^2*e^(14*d*x + 14*c) + 53513460*b
^3*e^(14*d*x + 14*c) - 35875840*a^3*e^(12*d*x + 12*c) - 113393280*a^2*b*e^(
12*d*x + 12*c) - 118918800*a*b^2*e^(12*d*x + 12*c) - 49549500*b^3*e^(12*d*x
 + 12*c) + 21525504*a^3*e^(10*d*x + 10*c) + 74954880*a^2*b*e^(10*d*x + 10*c
) + 83459376*a*b^2*e^(10*d*x + 10*c) + 34189155*b^3*e^(10*d*x + 10*c) - 978
4320*a^3*e^(8*d*x + 8*c) - 34070400*a^2*b*e^(8*d*x + 8*c) - 41081040*a*b^2*
e^(8*d*x + 8*c) - 17342325*b^3*e^(8*d*x + 8*c) + 3261440*a^3*e^(6*d*x + 6*c
) + 11356800*a^2*b*e^(6*d*x + 6*c) + 14054040*a*b^2*e^(6*d*x + 6*c) + 62762
70*b^3*e^(6*d*x + 6*c) - 752640*a^3*e^(4*d*x + 4*c) - 2620800*a^2*b*e^(4*d*
x + 4*c) - 3243240*a*b^2*e^(4*d*x + 4*c) - 1531530*b^3*e^(4*d*x + 4*c) + 10
7520*a^3*e^(2*d*x + 2*c) + 374400*a^2*b*e^(2*d*x + 2*c) + 463320*a*b^2*e^(2
*d*x + 2*c) + 225225*b^3*e^(2*d*x + 2*c) - 7168*a^3 - 24960*a^2*b - 30888*a
*b^2 - 15015*b^3)/(d*(e^(2*d*x + 2*c) - 1)^15)
```

3.227 $\int \mathbf{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=221

$$-\frac{a\left(70a^2+45ab+3b^2\right)\coth^9(c+dx)}{9d}+\frac{4a\left(14a^2+15ab+3b^2\right)\coth^7(c+dx)}{7d}-\frac{(a+b)\left(28a^2+17ab+b^2\right)\coth^5(c+d}{5d}$$

```
[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*(a + b)^2*(4*a + b)*Coth[c + d*x]^3)/(3
*d) - ((a + b)*(28*a^2 + 17*a*b + b^2)*Coth[c + d*x]^5)/(5*d) + (4*a*(14*a^
2 + 15*a*b + 3*b^2)*Coth[c + d*x]^7)/(7*d) - (a*(70*a^2 + 45*a*b + 3*b^2)*C
oth[c + d*x]^9)/(9*d) + (2*a^2*(28*a + 9*b)*Coth[c + d*x]^11)/(11*d) - (a^2
*(28*a + 3*b)*Coth[c + d*x]^13)/(13*d) + (8*a^3*Coth[c + d*x]^15)/(15*d) -
(a^3*Coth[c + d*x]^17)/(17*d)
```

Rubi [A] time = 0.191817, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3217, 1261}

$$-\frac{a\left(70a^2+45ab+3b^2\right)\coth^9(c+dx)}{9d}+\frac{4a\left(14a^2+15ab+3b^2\right)\coth^7(c+dx)}{7d}-\frac{(a+b)\left(28a^2+17ab+b^2\right)\coth^5(c+d}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] -(((a + b)^3*Coth[c + d*x])/d) + (2*(a + b)^2*(4*a + b)*Coth[c + d*x]^3)/(3
*d) - ((a + b)*(28*a^2 + 17*a*b + b^2)*Coth[c + d*x]^5)/(5*d) + (4*a*(14*a^
2 + 15*a*b + 3*b^2)*Coth[c + d*x]^7)/(7*d) - (a*(70*a^2 + 45*a*b + 3*b^2)*C
oth[c + d*x]^9)/(9*d) + (2*a^2*(28*a + 9*b)*Coth[c + d*x]^11)/(11*d) - (a^2
*(28*a + 3*b)*Coth[c + d*x]^13)/(13*d) + (8*a^3*Coth[c + d*x]^15)/(15*d) -
(a^3*Coth[c + d*x]^17)/(17*d)
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \operatorname{csch}^{18}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^2\left(a-2ax^2+(a+b)x^4\right)^3}{x^{18}} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{18}} - \frac{8a^3}{x^{16}} + \frac{a^2(28a+3b)}{x^{14}} - \frac{2a^2(28a+9b)}{x^{12}} + \frac{a(70a^2+45ab+3b^2)}{x^{10}} - \frac{4a(14a^2}{}\right.\right.}{}$$

$$= -\frac{(a+b)^3\coth(c+dx)}{d} + \frac{2(a+b)^2(4a+b)\coth^3(c+dx)}{3d} - \frac{(a+b)\left(28\right.}{}$$

Mathematica [B] time = 6.18111, size = 494, normalized size = 2.24

$$\frac{\operatorname{csch}^{17}(c+dx)\left(-784143360a^2b\cosh(c+dx) + 1568286720a^2b\cosh(3(c+dx)) - 1211857920a^2b\cosh(5(c+dx)) + \right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^18*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-697016320*a^3*Cosh[c + d*x] - 784143360*a^2*b*Cosh[c + d*x] - 571771200*
a*b^2*Cosh[c + d*x] - 157237080*b^3*Cosh[c + d*x] + 557613056*a^3*Cosh[3*(c
 + d*x)] + 1568286720*a^2*b*Cosh[3*(c + d*x)] + 1280767488*a*b^2*Cosh[3*(c
+ d*x)] + 368384016*b^3*Cosh[3*(c + d*x)] - 354844672*a^3*Cosh[5*(c + d*x)]
 - 1211857920*a^2*b*Cosh[5*(c + d*x)] - 1189284096*a*b^2*Cosh[5*(c + d*x)]
- 372263892*b^3*Cosh[5*(c + d*x)] + 177422336*a^3*Cosh[7*(c + d*x)] + 60592
8960*a^2*b*Cosh[7*(c + d*x)] + 692659968*a*b^2*Cosh[7*(c + d*x)] + 24228804
6*b^3*Cosh[7*(c + d*x)] - 68239360*a^3*Cosh[9*(c + d*x)] - 233049600*a^2*b*
Cosh[9*(c + d*x)] - 277717440*a*b^2*Cosh[9*(c + d*x)] - 108738630*b^3*Cosh[
9*(c + d*x)] + 19496960*a^3*Cosh[11*(c + d*x)] + 66585600*a^2*b*Cosh[11*(c
+ d*x)] + 79347840*a*b^2*Cosh[11*(c + d*x)] + 33693660*b^3*Cosh[11*(c + d*x
)] - 3899392*a^3*Cosh[13*(c + d*x)] - 13317120*a^2*b*Cosh[13*(c + d*x)] - 1
5869568*a*b^2*Cosh[13*(c + d*x)] - 6942936*b^3*Cosh[13*(c + d*x)] + 487424*
a^3*Cosh[15*(c + d*x)] + 1664640*a^2*b*Cosh[15*(c + d*x)] + 1983696*a*b^2*C
osh[15*(c + d*x)] + 867867*b^3*Cosh[15*(c + d*x)] - 28672*a^3*Cosh[17*(c +
d*x)] - 97920*a^2*b*Cosh[17*(c + d*x)] - 116688*a*b^2*Cosh[17*(c + d*x)] -
51051*b^3*Cosh[17*(c + d*x)])*Csch[c + d*x]^17)/(6273146880*d)
```

Maple [A] time = 0.085, size = 258, normalized size = 1.2

$$\frac{1}{d}\left(a^3\left(-\frac{32768}{109395} - \frac{(\operatorname{csch}(dx+c))^{16}}{17} + \frac{16\,(\operatorname{csch}(dx+c))^{14}}{255} - \frac{224\,(\operatorname{csch}(dx+c))^{12}}{3315} + \frac{896\,(\operatorname{csch}(dx+c))^{10}}{12155} - \frac{1792\,(}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(-32768/109395-1/17*csch(d*x+c)^16+16/255*csch(d*x+c)^14-224/3315*
csch(d*x+c)^12+896/12155*csch(d*x+c)^10-1792/21879*csch(d*x+c)^8+2048/21879
*csch(d*x+c)^6-4096/36465*csch(d*x+c)^4+16384/109395*csch(d*x+c)^2)*coth(d*
x+c)+3*a^2*b*(-1024/3003-1/13*csch(d*x+c)^12+12/143*csch(d*x+c)^10-40/429*c
sch(d*x+c)^8+320/3003*csch(d*x+c)^6-128/1001*csch(d*x+c)^4+512/3003*csch(d*
x+c)^2)*coth(d*x+c)+3*a*b^2*(-128/315-1/9*csch(d*x+c)^8+8/63*csch(d*x+c)^6-
16/105*csch(d*x+c)^4+64/315*csch(d*x+c)^2)*coth(d*x+c)+b^3*(-8/15-1/5*csch(
```

```
d*x+c)^4+4/15*csch(d*x+c)^2)*coth(d*x+c))
```

Maxima [B] time = 1.29654, size = 5021, normalized size = 22.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -65536/109395*a^3*(17*e^(-2*d*x - 2*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*
d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x
 - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-
16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 1237
6*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) -
680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e
^(-34*d*x - 34*c) - 1)) - 136*e^(-4*d*x - 4*c)/(d*(17*e^(-2*d*x - 2*c) - 13
6*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^
(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24
310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c
) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x -
26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 3
2*c) + e^(-34*d*x - 34*c) - 1)) + 680*e^(-6*d*x - 6*c)/(d*(17*e^(-2*d*x - 2
*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) +
 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14
*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*
x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-2
6*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32
*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 2380*e^(-8*d*x - 8*c)/(d*(17*e^(-
2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x
 - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14
*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*
e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2
380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) -
17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) + 6188*e^(-10*d*x - 10*c)/
(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x - 6*c) - 238
0*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x - 12*c) + 1
9448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-18*d*x - 18*
c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188*e^(-24*d*x
- 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 136*e^(-30*d*x
 - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) - 12376*e^(-12*
d*x - 12*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 680*e^(-6*d*x
- 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 12376*e^(-12*d*x
 - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c) + 24310*e^(-
18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x - 22*c) - 6188
*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x - 28*c) + 13
6*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*c) - 1)) + 1
9448*e^(-14*d*x - 14*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 68
0*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 1237
6*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c)
+ 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x -
22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x
- 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34*
c) - 1)) - 24310*e^(-16*d*x - 16*c)/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x
 - 4*c) + 680*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x -
10*c) - 12376*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*
d*x - 16*c) + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e
```

```
^(-22*d*x - 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680
*e^(-28*d*x - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-
34*d*x - 34*c) - 1)) - 1/(d*(17*e^(-2*d*x - 2*c) - 136*e^(-4*d*x - 4*c) + 6
80*e^(-6*d*x - 6*c) - 2380*e^(-8*d*x - 8*c) + 6188*e^(-10*d*x - 10*c) - 123
76*e^(-12*d*x - 12*c) + 19448*e^(-14*d*x - 14*c) - 24310*e^(-16*d*x - 16*c)
 + 24310*e^(-18*d*x - 18*c) - 19448*e^(-20*d*x - 20*c) + 12376*e^(-22*d*x -
 22*c) - 6188*e^(-24*d*x - 24*c) + 2380*e^(-26*d*x - 26*c) - 680*e^(-28*d*x
 - 28*c) + 136*e^(-30*d*x - 30*c) - 17*e^(-32*d*x - 32*c) + e^(-34*d*x - 34
*c) - 1))) - 2048/1001*a^2*b*(13*e^(-2*d*x - 2*c)/(d*(13*e^(-2*d*x - 2*c) -
 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e
^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 128
7*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78
*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 78
*e^(-4*d*x - 4*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6
*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*
d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-1
8*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*
d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 286*e^(-6*d*x - 6*c)/(d*(13*e^(-2*
d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8
*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x -
 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x
- 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c
) - 1)) - 715*e^(-8*d*x - 8*c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c
) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) -
 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c
) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c)
 - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) + 1287*e^(-10*d*x - 10*
c)/(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 7
15*e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1
716*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) -
 286*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e
^(-26*d*x - 26*c) - 1)) - 1716*e^(-12*d*x - 12*c)/(d*(13*e^(-2*d*x - 2*c) -
 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*e^(-8*d*x - 8*c) + 1287*e
^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716*e^(-14*d*x - 14*c) - 128
7*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 286*e^(-20*d*x - 20*c) + 78
*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-26*d*x - 26*c) - 1)) - 1/
(d*(13*e^(-2*d*x - 2*c) - 78*e^(-4*d*x - 4*c) + 286*e^(-6*d*x - 6*c) - 715*
e^(-8*d*x - 8*c) + 1287*e^(-10*d*x - 10*c) - 1716*e^(-12*d*x - 12*c) + 1716
*e^(-14*d*x - 14*c) - 1287*e^(-16*d*x - 16*c) + 715*e^(-18*d*x - 18*c) - 28
6*e^(-20*d*x - 20*c) + 78*e^(-22*d*x - 22*c) - 13*e^(-24*d*x - 24*c) + e^(-
26*d*x - 26*c) - 1))) - 256/105*a*b^2*(9*e^(-2*d*x - 2*c)/(d*(9*e^(-2*d*x -
 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x - 8*c) +
126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x - 14*c) - 9*
e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 36*e^(-4*d*x - 4*c)/(d*(9*e
^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126*e^(-8*d*x
 - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(-14*d*x -
14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) + 84*e^(-6*d*x - 6*
c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) - 126
*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*e^(
-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)) - 126*e^(
-8*d*x - 8*c)/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x -
 6*c) - 126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*
c) + 36*e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1)
) - 1/(d*(9*e^(-2*d*x - 2*c) - 36*e^(-4*d*x - 4*c) + 84*e^(-6*d*x - 6*c) -
126*e^(-8*d*x - 8*c) + 126*e^(-10*d*x - 10*c) - 84*e^(-12*d*x - 12*c) + 36*
e^(-14*d*x - 14*c) - 9*e^(-16*d*x - 16*c) + e^(-18*d*x - 18*c) - 1))) - 16/
15*b^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 10*e^(
-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x -
```

```
 6*c) - 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) - 1)) - 1/(d*(5*e^(-2*d*x -
 2*c) - 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) - 1)))
```

Fricas [B] time = 1.90939, size = 11491, normalized size = 52.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -16/765765*((28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x
+ c)^14 - 14*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x
 + c)*sinh(d*x + c)^13 + (28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b
^3)*sinh(d*x + c)^14 - 34*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b
^3)*cosh(d*x + c)^12 - (487424*a^3 + 1664640*a^2*b + 1983696*a*b^2 + 724924
2*b^3 - 91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^12 - 4*(91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 4
59459*b^3)*cosh(d*x + c)^3 - 204*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81
081*b^3)*cosh(d*x + c))*sinh(d*x + c)^11 + 17*(229376*a^3 + 783360*a^2*b +
1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^10 + (1001*(28672*a^3 + 97920*a^
2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^4 + 3899392*a^3 + 13317120*a
^2*b + 30572256*a*b^2 + 43750707*b^3 - 2244*(14336*a^3 + 48960*a^2*b + 5834
4*a*b^2 + 213213*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 - 2*(1001*(28672*a^
3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh(d*x + c)^5 - 7480*(7168*a
^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*cosh(d*x + c)^3 + 85*(229376*a^
3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)*cosh(d*x + c))*sinh(d*x + c)^
9 - 68*(286720*a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x +
 c)^8 + (3003*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*
x + c)^6 - 16830*(14336*a^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(
d*x + c)^4 - 19496960*a^3 - 66585600*a^2*b - 206771136*a*b^2 - 163975812*b^
3 + 765*(229376*a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^8 - 8*(429*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 -
459459*b^3)*cosh(d*x + c)^7 - 6732*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 -
81081*b^3)*cosh(d*x + c)^5 + 255*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 -
 1756755*b^3)*cosh(d*x + c)^3 - 272*(71680*a^3 + 244800*a^2*b - 176748*a*b^
2 - 351351*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 17*(4014080*a^3 + 23592960
*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^6 + (3003*(28672*a^3
+ 97920*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^8 - 31416*(14336*a
^3 + 48960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^6 + 3570*(229376
*a^3 + 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^4 + 682393
60*a^3 + 401080320*a^2*b + 772007808*a*b^2 + 427347921*b^3 - 1904*(286720*a
^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^2)*sinh(d*x
+ c)^6 - 2*(1001*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*cosh
(d*x + c)^9 - 26928*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*cosh
(d*x + c)^7 + 2142*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^3)*
cosh(d*x + c)^5 - 7616*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351*b^
3)*cosh(d*x + c)^3 + 51*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 - 115
94583*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 442*(401408*a^3 + 3176640*a^2*b
 + 4162488*a*b^2 + 1857471*b^3)*cosh(d*x + c)^4 + (1001*(28672*a^3 + 97920*
a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^10 - 16830*(14336*a^3 + 48
960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^8 + 3570*(229376*a^3 +
783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^6 - 4760*(286720*
a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^4 - 1774223
36*a^3 - 1404074880*a^2*b - 1839819696*a*b^2 - 821002182*b^3 + 255*(4014080
*a^3 + 23592960*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^2)*sin
```

h(d*x + c)^4 - 4*(91*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459459*b^3)*
cosh(d*x + c)^11 - 3740*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 81081*b^3)*
cosh(d*x + c)^9 + 510*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 1756755*b^
3)*cosh(d*x + c)^7 - 3808*(71680*a^3 + 244800*a^2*b - 176748*a*b^2 - 351351
*b^3)*cosh(d*x + c)^5 + 85*(4014080*a^3 + 3824640*a^2*b - 12739584*a*b^2 -
11594583*b^3)*cosh(d*x + c)^3 - 884*(200704*a^3 - 217440*a^2*b - 480876*a*b
^2 - 297297*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 557613056*a^3 - 173631744
0*a^2*b - 1775057856*a*b^2 - 680611932*b^3 + 221*(4759552*a^3 + 12643200*a^
2*b + 13669392*a*b^2 + 5435199*b^3)*cosh(d*x + c)^2 + (91*(28672*a^3 + 9792
0*a^2*b + 116688*a*b^2 + 561561*b^3)*cosh(d*x + c)^12 - 2244*(14336*a^3 + 4
8960*a^2*b + 58344*a*b^2 + 213213*b^3)*cosh(d*x + c)^10 + 765*(229376*a^3 +
 783360*a^2*b + 1798368*a*b^2 + 2573571*b^3)*cosh(d*x + c)^8 - 1904*(286720
*a^3 + 979200*a^2*b + 3040752*a*b^2 + 2411409*b^3)*cosh(d*x + c)^6 + 255*(4
014080*a^3 + 23592960*a^2*b + 45412224*a*b^2 + 25138113*b^3)*cosh(d*x + c)^
4 + 1051860992*a^3 + 2794147200*a^2*b + 3020935632*a*b^2 + 1201178979*b^3 -
 2652*(401408*a^3 + 3176640*a^2*b + 4162488*a*b^2 + 1857471*b^3)*cosh(d*x +
 c)^2)*sinh(d*x + c)^2 - 2*(7*(28672*a^3 + 97920*a^2*b + 116688*a*b^2 - 459
459*b^3)*cosh(d*x + c)^13 - 408*(7168*a^3 + 24480*a^2*b + 29172*a*b^2 - 810
81*b^3)*cosh(d*x + c)^11 + 85*(229376*a^3 + 783360*a^2*b + 68640*a*b^2 - 17
56755*b^3)*cosh(d*x + c)^9 - 1088*(71680*a^3 + 244800*a^2*b - 176748*a*b^2
- 351351*b^3)*cosh(d*x + c)^7 + 51*(4014080*a^3 + 3824640*a^2*b - 12739584*
a*b^2 - 11594583*b^3)*cosh(d*x + c)^5 - 1768*(200704*a^3 - 217440*a^2*b - 4
80876*a*b^2 - 297297*b^3)*cosh(d*x + c)^3 - 5967*(57344*a^3 + 62080*a^2*b +
 64944*a*b^2 + 33033*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^20
 + 20*d*cosh(d*x + c)*sinh(d*x + c)^19 + d*sinh(d*x + c)^20 - 17*d*cosh(d*x
 + c)^18 + (190*d*cosh(d*x + c)^2 - 17*d)*sinh(d*x + c)^18 + 6*(190*d*cosh(
d*x + c)^3 - 51*d*cosh(d*x + c))*sinh(d*x + c)^17 + 136*d*cosh(d*x + c)^16
+ 17*(285*d*cosh(d*x + c)^4 - 153*d*cosh(d*x + c)^2 + 8*d)*sinh(d*x + c)^16
 + 272*(57*d*cosh(d*x + c)^5 - 51*d*cosh(d*x + c)^3 + 8*d*cosh(d*x + c))*si
nh(d*x + c)^15 - 681*d*cosh(d*x + c)^14 + 3*(12920*d*cosh(d*x + c)^6 - 1734
0*d*cosh(d*x + c)^4 + 5440*d*cosh(d*x + c)^2 - 227*d)*sinh(d*x + c)^14 + 2*
(38760*d*cosh(d*x + c)^7 - 72828*d*cosh(d*x + c)^5 + 38080*d*cosh(d*x + c)^
3 - 4753*d*cosh(d*x + c))*sinh(d*x + c)^13 + 2397*d*cosh(d*x + c)^12 + (125
970*d*cosh(d*x + c)^8 - 315588*d*cosh(d*x + c)^6 + 247520*d*cosh(d*x + c)^4
 - 61971*d*cosh(d*x + c)^2 + 2397*d)*sinh(d*x + c)^12 + 4*(41990*d*cosh(d*x
 + c)^9 - 135252*d*cosh(d*x + c)^7 + 148512*d*cosh(d*x + c)^5 - 61789*d*cos
h(d*x + c)^3 + 7089*d*cosh(d*x + c))*sinh(d*x + c)^11 - 6324*d*cosh(d*x + c
)^10 + (184756*d*cosh(d*x + c)^10 - 743886*d*cosh(d*x + c)^8 + 1089088*d*co
sh(d*x + c)^6 - 681681*d*cosh(d*x + c)^4 + 158202*d*cosh(d*x + c)^2 - 6324*
d)*sinh(d*x + c)^10 + 2*(83980*d*cosh(d*x + c)^11 - 413270*d*cosh(d*x + c)^
9 + 777920*d*cosh(d*x + c)^7 - 679679*d*cosh(d*x + c)^5 + 259930*d*cosh(d*x
 + c)^3 - 30260*d*cosh(d*x + c))*sinh(d*x + c)^9 + 13056*d*cosh(d*x + c)^8
+ 3*(41990*d*cosh(d*x + c)^12 - 247962*d*cosh(d*x + c)^10 + 583440*d*cosh(d
*x + c)^8 - 681681*d*cosh(d*x + c)^6 + 395505*d*cosh(d*x + c)^4 - 94860*d*c
osh(d*x + c)^2 + 4352*d)*sinh(d*x + c)^8 + 8*(9690*d*cosh(d*x + c)^13 - 676
26*d*cosh(d*x + c)^11 + 194480*d*cosh(d*x + c)^9 - 291291*d*cosh(d*x + c)^7
 + 233937*d*cosh(d*x + c)^5 - 90780*d*cosh(d*x + c)^3 + 11696*d*cosh(d*x +
c))*sinh(d*x + c)^7 - 21828*d*cosh(d*x + c)^6 + (38760*d*cosh(d*x + c)^14 -
 315588*d*cosh(d*x + c)^12 + 1089088*d*cosh(d*x + c)^10 - 2045043*d*cosh(d*
x + c)^8 + 2214828*d*cosh(d*x + c)^6 - 1328040*d*cosh(d*x + c)^4 + 365568*d
*cosh(d*x + c)^2 - 21828*d)*sinh(d*x + c)^6 + 2*(7752*d*cosh(d*x + c)^15 -
72828*d*cosh(d*x + c)^13 + 297024*d*cosh(d*x + c)^11 - 679679*d*cosh(d*x +
c)^9 + 935748*d*cosh(d*x + c)^7 - 762552*d*cosh(d*x + c)^5 + 327488*d*cosh(
d*x + c)^3 - 51204*d*cosh(d*x + c))*sinh(d*x + c)^5 + 30498*d*cosh(d*x + c)
^4 + (4845*d*cosh(d*x + c)^16 - 52020*d*cosh(d*x + c)^14 + 247520*d*cosh(d*
x + c)^12 - 681681*d*cosh(d*x + c)^10 + 1186515*d*cosh(d*x + c)^8 - 1328040
*d*cosh(d*x + c)^6 + 913920*d*cosh(d*x + c)^4 - 327420*d*cosh(d*x + c)^2 +
30498*d)*sinh(d*x + c)^4 + 4*(285*d*cosh(d*x + c)^17 - 3468*d*cosh(d*x + c)
^15 + 19040*d*cosh(d*x + c)^13 - 61789*d*cosh(d*x + c)^11 + 129965*d*cosh(d

```
*x + c)^9 - 181560*d*cosh(d*x + c)^7 + 163744*d*cosh(d*x + c)^5 - 85340*d*c
osh(d*x + c)^3 + 18122*d*cosh(d*x + c))*sinh(d*x + c)^3 - 36686*d*cosh(d*x
+ c)^2 + (190*d*cosh(d*x + c)^18 - 2601*d*cosh(d*x + c)^16 + 16320*d*cosh(d
*x + c)^14 - 61971*d*cosh(d*x + c)^12 + 158202*d*cosh(d*x + c)^10 - 284580*
d*cosh(d*x + c)^8 + 365568*d*cosh(d*x + c)^6 - 327420*d*cosh(d*x + c)^4 + 1
82988*d*cosh(d*x + c)^2 - 36686*d)*sinh(d*x + c)^2 + 2*(10*d*cosh(d*x + c)^
19 - 153*d*cosh(d*x + c)^17 + 1088*d*cosh(d*x + c)^15 - 4753*d*cosh(d*x + c
)^13 + 14178*d*cosh(d*x + c)^11 - 30260*d*cosh(d*x + c)^9 + 46784*d*cosh(d*
x + c)^7 - 51204*d*cosh(d*x + c)^5 + 36244*d*cosh(d*x + c)^3 - 11934*d*cosh
(d*x + c))*sinh(d*x + c) + 19448*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**18*(a+b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.6542, size = 917, normalized size = 4.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^18*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -16/765765*(510510*b^3*e^(28*d*x + 28*c) - 6381375*b^3*e^(26*d*x + 26*c) +
14702688*a*b^2*e^(24*d*x + 24*c) + 36807771*b^3*e^(24*d*x + 24*c) - 1274232
96*a*b^2*e^(22*d*x + 22*c) - 129771642*b^3*e^(22*d*x + 22*c) + 168030720*a^
2*b*e^(20*d*x + 20*c) + 494290368*a*b^2*e^(20*d*x + 20*c) + 312227916*b^3*e
^(20*d*x + 20*c) - 798145920*a^2*b*e^(18*d*x + 18*c) - 1132457040*a*b^2*e^(
18*d*x + 18*c) - 541906365*b^3*e^(18*d*x + 18*c) + 697016320*a^3*e^(16*d*x
+ 16*c) + 1582289280*a^2*b*e^(16*d*x + 16*c) + 1704228240*a*b^2*e^(16*d*x +
 16*c) + 699143445*b^3*e^(16*d*x + 16*c) - 557613056*a^3*e^(14*d*x + 14*c)
- 1736317440*a^2*b*e^(14*d*x + 14*c) - 1775057856*a*b^2*e^(14*d*x + 14*c) -
 680611932*b^3*e^(14*d*x + 14*c) + 354844672*a^3*e^(12*d*x + 12*c) + 121185
7920*a^2*b*e^(12*d*x + 12*c) + 1316707392*a*b^2*e^(12*d*x + 12*c) + 5020355
34*b^3*e^(12*d*x + 12*c) - 177422336*a^3*e^(10*d*x + 10*c) - 605928960*a^2*
b*e^(10*d*x + 10*c) - 707362656*a*b^2*e^(10*d*x + 10*c) - 279095817*b^3*e^(
10*d*x + 10*c) + 68239360*a^3*e^(8*d*x + 8*c) + 233049600*a^2*b*e^(8*d*x +
8*c) + 277717440*a*b^2*e^(8*d*x + 8*c) + 115120005*b^3*e^(8*d*x + 8*c) - 19
496960*a^3*e^(6*d*x + 6*c) - 66585600*a^2*b*e^(6*d*x + 6*c) - 79347840*a*b^
2*e^(6*d*x + 6*c) - 34204170*b^3*e^(6*d*x + 6*c) + 3899392*a^3*e^(4*d*x + 4
*c) + 13317120*a^2*b*e^(4*d*x + 4*c) + 15869568*a*b^2*e^(4*d*x + 4*c) + 694
2936*b^3*e^(4*d*x + 4*c) - 487424*a^3*e^(2*d*x + 2*c) - 1664640*a^2*b*e^(2*
d*x + 2*c) - 1983696*a*b^2*e^(2*d*x + 2*c) - 867867*b^3*e^(2*d*x + 2*c) + 2
8672*a^3 + 97920*a^2*b + 116688*a*b^2 + 51051*b^3)/(d*(e^(2*d*x + 2*c) - 1)
^17)
```

3.228 $\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3\,dx$

Optimal. Leaf size=248

$$-\frac{3a\left(42a^2+21ab+b^2\right)\coth^{11}(c+dx)}{11d}+\frac{a\left(42a^2+35ab+5b^2\right)\coth^9(c+dx)}{3d}-\frac{\left(105a^2b+84a^3+30ab^2+b^3\right)\coth^7}{7d}$$

```
[Out] ((a + b)^3*Coth[c + d*x])/d - ((a + b)^2*(3*a + b)*Coth[c + d*x]^3)/d + (3*
(a + b)*(12*a^2 + 9*a*b + b^2)*Coth[c + d*x]^5)/(5*d) - ((84*a^3 + 105*a^2*
b + 30*a*b^2 + b^3)*Coth[c + d*x]^7)/(7*d) + (a*(42*a^2 + 35*a*b + 5*b^2)*C
oth[c + d*x]^9)/(3*d) - (3*a*(42*a^2 + 21*a*b + b^2)*Coth[c + d*x]^11)/(11*
d) + (21*a^2*(4*a + b)*Coth[c + d*x]^13)/(13*d) - (a^2*(12*a + b)*Coth[c +
d*x]^15)/(5*d) + (9*a^3*Coth[c + d*x]^17)/(17*d) - (a^3*Coth[c + d*x]^19)/(
19*d)
```

Rubi [A] time = 0.222522, antiderivative size = 248, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3217, 1261}

$$-\frac{3a\left(42a^2+21ab+b^2\right)\coth^{11}(c+dx)}{11d}+\frac{a\left(42a^2+35ab+5b^2\right)\coth^9(c+dx)}{3d}-\frac{\left(105a^2b+84a^3+30ab^2+b^3\right)\coth^7}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] ((a + b)^3*Coth[c + d*x])/d - ((a + b)^2*(3*a + b)*Coth[c + d*x]^3)/d + (3*
(a + b)*(12*a^2 + 9*a*b + b^2)*Coth[c + d*x]^5)/(5*d) - ((84*a^3 + 105*a^2*
b + 30*a*b^2 + b^3)*Coth[c + d*x]^7)/(7*d) + (a*(42*a^2 + 35*a*b + 5*b^2)*C
oth[c + d*x]^9)/(3*d) - (3*a*(42*a^2 + 21*a*b + b^2)*Coth[c + d*x]^11)/(11*
d) + (21*a^2*(4*a + b)*Coth[c + d*x]^13)/(13*d) - (a^2*(12*a + b)*Coth[c +
d*x]^15)/(5*d) + (9*a^3*Coth[c + d*x]^17)/(17*d) - (a^3*Coth[c + d*x]^19)/(
19*d)
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1261

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (b_.)*(x_)^2 + (
c_.)*(x_)^4)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(f*x)^m*(d + e*x^2)^q*
(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, q}, x] && NeQ[
b^2 - 4*a*c, 0] && IGtQ[p, 0] && IGtQ[q, -2]
```

Rubi steps

$$\int \operatorname{csch}^{20}(c+dx)\left(a+b\sinh^4(c+dx)\right)^3 dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3(a-2ax^2+(a+b)x^4)^3}{x^{20}} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a^3}{x^{20}} - \frac{9a^3}{x^{18}} + \frac{3a^2(12a+b)}{x^{16}} - \frac{21a^2(4a+b)}{x^{14}} + \frac{3a(42a^2+21ab+b^2)}{x^{12}} - \frac{3a(42a^2+35a}{x^{10}}\right.\right.}{}$$

$$= \frac{(a+b)^3\coth(c+dx)}{d} - \frac{(a+b)^2(3a+b)\coth^3(c+dx)}{d} + \frac{3(a+b)\left(12a^2+\right.}{}$$

Mathematica [B] time = 6.16765, size = 548, normalized size = 2.21

$$\frac{\operatorname{csch}^{19}(c+dx)\left(-8939234304a^2b\cosh(c+dx)+18149354496a^2b\cosh(3(c+dx))-14582690304a^2b\cosh(5(c+dx))+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^20*(a + b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-7945986048*a^3*Cosh[c + d*x] - 8939234304*a^2*b*Cosh[c + d*x] - 65181916
80*a*b^2*Cosh[c + d*x] - 1792502712*b^3*Cosh[c + d*x] + 6501261312*a^3*Cosh
[3*(c + d*x)] + 18149354496*a^2*b*Cosh[3*(c + d*x)] + 14814072000*a*b^2*Cos
h[3*(c + d*x)] + 4260103848*b^3*Cosh[3*(c + d*x)] - 4334174208*a^3*Cosh[5*(
c + d*x)] - 14582690304*a^2*b*Cosh[5*(c + d*x)] - 14221509120*a*b^2*Cosh[5*
(c + d*x)] - 4440518082*b^3*Cosh[5*(c + d*x)] + 2333786112*a^3*Cosh[7*(c +
d*x)] + 7852217856*a^2*b*Cosh[7*(c + d*x)] + 8803791360*a*b^2*Cosh[7*(c + d
*x)] + 3047642598*b^3*Cosh[7*(c + d*x)] - 1000194048*a^3*Cosh[9*(c + d*x)]
- 3365236224*a^2*b*Cosh[9*(c + d*x)] - 3906077760*a*b^2*Cosh[9*(c + d*x)] -
 1489040982*b^3*Cosh[9*(c + d*x)] + 333398016*a^3*Cosh[11*(c + d*x)] + 1121
745408*a^2*b*Cosh[11*(c + d*x)] + 1302025920*a*b^2*Cosh[11*(c + d*x)] + 527
386002*b^3*Cosh[11*(c + d*x)] - 83349504*a^3*Cosh[13*(c + d*x)] - 280436352
*a^2*b*Cosh[13*(c + d*x)] - 325506480*a*b^2*Cosh[13*(c + d*x)] - 134271423*
b^3*Cosh[13*(c + d*x)] + 14708736*a^3*Cosh[15*(c + d*x)] + 49488768*a^2*b*C
osh[15*(c + d*x)] + 57442320*a*b^2*Cosh[15*(c + d*x)] + 23694957*b^3*Cosh[1
5*(c + d*x)] - 1634304*a^3*Cosh[17*(c + d*x)] - 5498752*a^2*b*Cosh[17*(c +
d*x)] - 6382480*a*b^2*Cosh[17*(c + d*x)] - 2632773*b^3*Cosh[17*(c + d*x)] +
 86016*a^3*Cosh[19*(c + d*x)] + 289408*a^2*b*Cosh[19*(c + d*x)] + 335920*a*
b^2*Cosh[19*(c + d*x)] + 138567*b^3*Cosh[19*(c + d*x)])*Csch[c + d*x]^19)/(
79459860480*d)
```

Maple [A] time = 0.09, size = 298, normalized size = 1.2

$$\frac{1}{d}\left(a^3\left(\frac{65536}{230945} - \frac{(\operatorname{csch}(dx+c))^{18}}{19} + \frac{18(\operatorname{csch}(dx+c))^{16}}{323} - \frac{96(\operatorname{csch}(dx+c))^{14}}{1615} + \frac{1344(\operatorname{csch}(dx+c))^{12}}{20995} - \frac{16128(\operatorname{csc}}{230}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/d*(a^3*(65536/230945-1/19*csch(d*x+c)^18+18/323*csch(d*x+c)^16-96/1615*cs
ch(d*x+c)^14+1344/20995*csch(d*x+c)^12-16128/230945*csch(d*x+c)^10+3584/461
89*csch(d*x+c)^8-4096/46189*csch(d*x+c)^6+24576/230945*csch(d*x+c)^4-32768/
230945*csch(d*x+c)^2)*coth(d*x+c)+3*a^2*b*(2048/6435-1/15*csch(d*x+c)^14+14
/195*csch(d*x+c)^12-56/715*csch(d*x+c)^10+112/1287*csch(d*x+c)^8-128/1287*c
```

```
sch(d*x+c)^6+256/2145*csch(d*x+c)^4-1024/6435*csch(d*x+c)^2)*coth(d*x+c)+3*
a*b^2*(256/693-1/11*csch(d*x+c)^10+10/99*csch(d*x+c)^8-80/693*csch(d*x+c)^6
+32/231*csch(d*x+c)^4-128/693*csch(d*x+c)^2)*coth(d*x+c)+b^3*(16/35-1/7*csc
h(d*x+c)^6+6/35*csch(d*x+c)^4-8/35*csch(d*x+c)^2)*coth(d*x+c))
```

Maxima [B] time = 1.29003, size = 6592, normalized size = 26.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 131072/230945*a^3*(19*e^(-2*d*x - 2*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*
d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*
x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(
-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 755
82*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c)
 - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32
*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) -
 1)) - 171*e^(-4*d*x - 4*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c)
+ 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) -
 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 1
6*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d
*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^
(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e
^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) + 969*
e^(-6*d*x - 6*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6
*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-
12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 9237
8*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c)
- 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x -
28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x -
 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) - 3876*e^(-8*d*x
- 8*c)/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c
) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 1
2*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d
*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^
(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 38
76*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 1
9*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) + 11628*e^(-10*d*x - 10*c)/
(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 387
6*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) +
50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18
*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*
x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-
30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-3
6*d*x - 36*c) + e^(-38*d*x - 38*c) - 1)) - 27132*e^(-12*d*x - 12*c)/(d*(19*
e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8
*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e
^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 9
2378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*
c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x
- 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x -
 36*c) + e^(-38*d*x - 38*c) - 1)) + 50388*e^(-14*d*x - 14*c)/(d*(19*e^(-2*d
*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x -
8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d
```

*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^
(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27
132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c)
 - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c)
 + e^(-38*d*x - 38*c) - 1)) - 75582*e^(-16*d*x - 16*c)/(d*(19*e^(-2*d*x - 2*
c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) +
11628*e^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14
*c) - 75582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*
x - 20*c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(
-26*d*x - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*
e^(-32*d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-3
8*d*x - 38*c) - 1)) + 92378*e^(-18*d*x - 18*c)/(d*(19*e^(-2*d*x - 2*c) - 17
1*e^(-4*d*x - 4*c) + 969*e^(-6*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e
^(-10*d*x - 10*c) - 27132*e^(-12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 7
5582*e^(-16*d*x - 16*c) + 92378*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*
c) + 75582*e^(-22*d*x - 22*c) - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x
 - 26*c) - 11628*e^(-28*d*x - 28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*
d*x - 32*c) + 171*e^(-34*d*x - 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x -
 38*c) - 1)) - 1/(d*(19*e^(-2*d*x - 2*c) - 171*e^(-4*d*x - 4*c) + 969*e^(-6
*d*x - 6*c) - 3876*e^(-8*d*x - 8*c) + 11628*e^(-10*d*x - 10*c) - 27132*e^(-
12*d*x - 12*c) + 50388*e^(-14*d*x - 14*c) - 75582*e^(-16*d*x - 16*c) + 9237
8*e^(-18*d*x - 18*c) - 92378*e^(-20*d*x - 20*c) + 75582*e^(-22*d*x - 22*c)
 - 50388*e^(-24*d*x - 24*c) + 27132*e^(-26*d*x - 26*c) - 11628*e^(-28*d*x -
28*c) + 3876*e^(-30*d*x - 30*c) - 969*e^(-32*d*x - 32*c) + 171*e^(-34*d*x -
 34*c) - 19*e^(-36*d*x - 36*c) + e^(-38*d*x - 38*c) - 1))) + 4096/2145*a^2*
b*(15*e^(-2*d*x - 2*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455
*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*
e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 50
05*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) -
 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) +
e^(-30*d*x - 30*c) - 1)) - 105*e^(-4*d*x - 4*c)/(d*(15*e^(-2*d*x - 2*c) - 1
05*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e
^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 643
5*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) +
1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) -
 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) + 455*e^(-6*d*x - 6*c)/(d
*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*
e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435
*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3
003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) +
 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1))
 - 1365*e^(-8*d*x - 8*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 45
5*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005
*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5
005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c)
 - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) +
 e^(-30*d*x - 30*c) - 1)) + 3003*e^(-10*d*x - 10*c)/(d*(15*e^(-2*d*x - 2*c)
 - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 30
03*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) -
 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c
) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*
c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c) - 1)) - 5005*e^(-12*d*x - 1
2*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x - 4*c) + 455*e^(-6*d*x - 6*c)
 - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*c) - 5005*e^(-12*d*x - 12*c)
 + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x - 16*c) + 5005*e^(-18*d*x - 18
*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x - 22*c) - 455*e^(-24*d*x -
24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x - 28*c) + e^(-30*d*x - 30*c)
 - 1)) + 6435*e^(-14*d*x - 14*c)/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*d*x -
4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x - 10*

```
c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*d*x -
16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-22*d*x
 - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-28*d*x
- 28*c) + e^(-30*d*x - 30*c) - 1)) - 1/(d*(15*e^(-2*d*x - 2*c) - 105*e^(-4*
d*x - 4*c) + 455*e^(-6*d*x - 6*c) - 1365*e^(-8*d*x - 8*c) + 3003*e^(-10*d*x
 - 10*c) - 5005*e^(-12*d*x - 12*c) + 6435*e^(-14*d*x - 14*c) - 6435*e^(-16*
d*x - 16*c) + 5005*e^(-18*d*x - 18*c) - 3003*e^(-20*d*x - 20*c) + 1365*e^(-
22*d*x - 22*c) - 455*e^(-24*d*x - 24*c) + 105*e^(-26*d*x - 26*c) - 15*e^(-2
8*d*x - 28*c) + e^(-30*d*x - 30*c) - 1))) + 512/231*a*b^2*(11*e^(-2*d*x - 2
*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) -
330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 33
0*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*
e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1)) - 55*e^(-4*d*x - 4*c)/(d*(11*
e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d
*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*
x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x
- 20*c) + e^(-22*d*x - 22*c) - 1)) + 165*e^(-6*d*x - 6*c)/(d*(11*e^(-2*d*x
- 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c)
+ 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c)
- 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) +
e^(-22*d*x - 22*c) - 1)) - 330*e^(-8*d*x - 8*c)/(d*(11*e^(-2*d*x - 2*c) - 5
5*e^(-4*d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-
10*d*x - 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-
16*d*x - 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x
 - 22*c) - 1)) + 462*e^(-10*d*x - 10*c)/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*
d*x - 4*c) + 165*e^(-6*d*x - 6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x -
 10*c) - 462*e^(-12*d*x - 12*c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x -
 16*c) + 55*e^(-18*d*x - 18*c) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c)
 - 1)) - 1/(d*(11*e^(-2*d*x - 2*c) - 55*e^(-4*d*x - 4*c) + 165*e^(-6*d*x -
6*c) - 330*e^(-8*d*x - 8*c) + 462*e^(-10*d*x - 10*c) - 462*e^(-12*d*x - 12*
c) + 330*e^(-14*d*x - 14*c) - 165*e^(-16*d*x - 16*c) + 55*e^(-18*d*x - 18*c
) - 11*e^(-20*d*x - 20*c) + e^(-22*d*x - 22*c) - 1))) + 32/35*b^3*(7*e^(-2*
d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*
c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e
^(-14*d*x - 14*c) - 1)) - 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e
^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
 - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) - 1)) + 35*e^(-6*d*x -
 6*c)/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) -
35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x - 12*c) + e^(-14
*d*x - 14*c) - 1)) - 1/(d*(7*e^(-2*d*x - 2*c) - 21*e^(-4*d*x - 4*c) + 35*e^
(-6*d*x - 6*c) - 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) - 7*e^(-12*d*x
 - 12*c) + e^(-14*d*x - 14*c) - 1)))
```

Fricas [B] time = 2.26097, size = 13925, normalized size = 56.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 64/4849845*((43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*
x + c)^15 + 15*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh
(d*x + c)*sinh(d*x + c)^14 - 2*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 124
7103*b^3)*sinh(d*x + c)^15 - 19*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 -
1538823*b^3)*cosh(d*x + c)^13 + 2*(408576*a^3 + 1374688*a^2*b + 1595620*a*b
^2 + 15935205*b^3 - 105*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^
```

3)*cosh(d*x + c)^2)*sinh(d*x + c)^13 + 13*(35*(43008*a^3 + 144704*a^2*b + 1
67960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^3 - 19*(43008*a^3 + 144704*a^2*b +
167960*a*b^2 - 1538823*b^3)*cosh(d*x + c))*sinh(d*x + c)^12 + 57*(129024*a
^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^11 - 6*(455*(
21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^4 + 1225
728*a^3 + 4124064*a^2*b + 17719780*a*b^2 + 31639465*b^3 - 494*(21504*a^3 +
72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^11 +
11*(273*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x +
c)^5 - 494*(43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*
x + c)^3 + 57*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh
(d*x + c))*sinh(d*x + c)^10 - 969*(43008*a^3 + 144704*a^2*b - 529880*a*b^2
- 586443*b^3)*cosh(d*x + c)^9 - 2*(5005*(21504*a^3 + 72352*a^2*b + 83980*a*
b^2 + 1247103*b^3)*cosh(d*x + c)^6 - 13585*(21504*a^3 + 72352*a^2*b + 83980
*a*b^2 + 838695*b^3)*cosh(d*x + c)^4 - 20837376*a^3 - 70109088*a^2*b - 4194
80100*a*b^2 - 351267345*b^3 + 3135*(64512*a^3 + 217056*a^2*b + 932620*a*b^2
+ 1665235*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(715*(43008*a^3 + 1447
04*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^7 - 2717*(43008*a^3 +
144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^5 + 1045*(129024*a
^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^3 - 969*(4300
8*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c))*sinh(d*x +
c)^8 + 6783*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x
+ c)^7 - 6*(2145*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cos
h(d*x + c)^8 - 10868*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*c
osh(d*x + c)^6 + 6270*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^
3)*cosh(d*x + c)^4 + 27783168*a^3 + 248673824*a^2*b + 549145220*a*b^2 + 303
230785*b^3 - 11628*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3)*co
sh(d*x + c)^2)*sinh(d*x + c)^7 + (5005*(43008*a^3 + 144704*a^2*b + 167960*a
*b^2 - 2355639*b^3)*cosh(d*x + c)^9 - 32604*(43008*a^3 + 144704*a^2*b + 167
960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^7 + 26334*(129024*a^3 + 434112*a^2*b
- 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^5 - 81396*(43008*a^3 + 144704*
a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c)^3 + 47481*(24576*a^3 - 545
92*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 323*
(1548288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)*cosh(d*x + c)^
5 - 2*(3003*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x
+ c)^10 - 24453*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d
x + c)^8 + 26334(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*c
osh(d*x + c)^6 - 122094*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^
3)*cosh(d*x + c)^4 - 250048512*a^3 - 3065771296*a^2*b - 4040697700*a*b^2 -
1763542209*b^3 + 20349*(86016*a^3 + 769888*a^2*b + 1700140*a*b^2 + 938795*b
^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + (1365*(43008*a^3 + 144704*a^2*b + 16
7960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^11 - 13585*(43008*a^3 + 144704*a^2*
b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^9 + 18810*(129024*a^3 + 43411
2*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^7 - 122094*(43008*a^3 +
144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x + c)^5 + 237405*(24576*
a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c)^3 - 1615*(1548
288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)*cosh(d*x + c))*sinh
(d*x + c)^4 - 323*(8687616*a^3 + 15456448*a^2*b + 15194920*a*b^2 + 6026163*
b^3)*cosh(d*x + c)^3 - 2*(455*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247
103*b^3)*cosh(d*x + c)^12 - 5434*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 8
38695*b^3)*cosh(d*x + c)^10 + 9405*(64512*a^3 + 217056*a^2*b + 932620*a*b^2
+ 1665235*b^3)*cosh(d*x + c)^8 - 81396*(21504*a^3 + 72352*a^2*b + 432900*a
*b^2 + 362505*b^3)*cosh(d*x + c)^6 + 33915*(86016*a^3 + 769888*a^2*b + 1700
140*a*b^2 + 938795*b^3)*cosh(d*x + c)^4 + 2569943040*a^3 + 6422325280*a^2*b
+ 6933472780*a*b^2 + 2675035935*b^3 - 3230*(774144*a^3 + 9491552*a^2*b + 1
2509900*a*b^2 + 5459883*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (105*(43008
*a^3 + 144704*a^2*b + 167960*a*b^2 - 2355639*b^3)*cosh(d*x + c)^13 - 1482*(
43008*a^3 + 144704*a^2*b + 167960*a*b^2 - 1538823*b^3)*cosh(d*x + c)^11 + 3
135*(129024*a^3 + 434112*a^2*b - 857480*a*b^2 - 2914769*b^3)*cosh(d*x + c)^
9 - 34884*(43008*a^3 + 144704*a^2*b - 529880*a*b^2 - 586443*b^3)*cosh(d*x +

c)^7 + 142443*(24576*a^3 - 54592*a^2*b - 293800*a*b^2 - 189761*b^3)*cosh(d*x + c)^5 - 3230*(1548288*a^3 - 8564416*a^2*b - 12926680*a*b^2 - 6120543*b^3)*cosh(d*x + c)^3 - 969*(8687616*a^3 + 15456448*a^2*b + 15194920*a*b^2 + 6026163*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 12597*(86016*a^3 + 215488*a^2*b + 179720*a*b^2 + 65703*b^3)*cosh(d*x + c) - 2*(15*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 1247103*b^3)*cosh(d*x + c)^14 - 247*(21504*a^3 + 72352*a^2*b + 83980*a*b^2 + 838695*b^3)*cosh(d*x + c)^12 + 627*(64512*a^3 + 217056*a^2*b + 932620*a*b^2 + 1665235*b^3)*cosh(d*x + c)^10 - 8721*(21504*a^3 + 72352*a^2*b + 432900*a*b^2 + 362505*b^3)*cosh(d*x + c)^8 + 6783*(86016*a^3 + 769888*a^2*b + 1700140*a*b^2 + 938795*b^3)*cosh(d*x + c)^6 - 1615*(774144*a^3 + 9491552*a^2*b + 12509900*a*b^2 + 5459883*b^3)*cosh(d*x + c)^4 - 2708858880*a^3 - 8648596320*a^2*b - 8918927940*a*b^2 - 3269488365*b^3 + 4845*(1591296*a^3 + 3976672*a^2*b + 4293172*a*b^2 + 1656369*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^23 + 23*d*cosh(d*x + c)*sinh(d*x + c)^22 + d*sinh(d*x + c)^23 - 19*d*cosh(d*x + c)^21 + (253*d*cosh(d*x + c)^2 - 19*d)*sinh(d*x + c)^21 + 7*(253*d*cosh(d*x + c)^3 - 57*d*cosh(d*x + c))*sinh(d*x + c)^20 + 171*d*cosh(d*x + c)^19 + (8855*d*cosh(d*x + c)^4 - 3990*d*cosh(d*x + c)^2 + 171*d)*sinh(d*x + c)^19 + 19*(1771*d*cosh(d*x + c)^5 - 1330*d*cosh(d*x + c)^3 + 171*d*cosh(d*x + c))*sinh(d*x + c)^18 - 969*d*cosh(d*x + c)^17 + 57*(1771*d*cosh(d*x + c)^6 - 1995*d*cosh(d*x + c)^4 + 513*d*cosh(d*x + c)^2 - 17*d)*sinh(d*x + c)^17 + 969*(253*d*cosh(d*x + c)^7 - 399*d*cosh(d*x + c)^5 + 171*d*cosh(d*x + c)^3 - 17*d*cosh(d*x + c))*sinh(d*x + c)^16 + 3875*d*cosh(d*x + c)^15 + (490314*d*cosh(d*x + c)^8 - 1031016*d*cosh(d*x + c)^6 + 662796*d*cosh(d*x + c)^4 - 131784*d*cosh(d*x + c)^2 + 3877*d)*sinh(d*x + c)^15 + (817190*d*cosh(d*x + c)^9 - 2209320*d*cosh(d*x + c)^7 + 1988388*d*cosh(d*x + c)^5 - 658920*d*cosh(d*x + c)^3 + 58125*d*cosh(d*x + c))*sinh(d*x + c)^14 - 11609*d*cosh(d*x + c)^13 + (1144066*d*cosh(d*x + c)^10 - 3866310*d*cosh(d*x + c)^8 + 4639572*d*cosh(d*x + c)^6 - 2306220*d*cosh(d*x + c)^4 + 407085*d*cosh(d*x + c)^2 - 11647*d)*sinh(d*x + c)^13 + 13*(104006*d*cosh(d*x + c)^11 - 429590*d*cosh(d*x + c)^9 + 662796*d*cosh(d*x + c)^7 - 461244*d*cosh(d*x + c)^5 + 135625*d*cosh(d*x + c)^3 - 11609*d*cosh(d*x + c))*sinh(d*x + c)^12 + 26961*d*cosh(d*x + c)^11 + (1352078*d*cosh(d*x + c)^12 - 6701604*d*cosh(d*x + c)^10 + 12924522*d*cosh(d*x + c)^8 - 11992344*d*cosh(d*x + c)^6 + 5292105*d*cosh(d*x + c)^4 - 908466*d*cosh(d*x + c)^2 + 27303*d)*sinh(d*x + c)^11 + (1144066*d*cosh(d*x + c)^13 - 6701604*d*cosh(d*x + c)^11 + 15796638*d*cosh(d*x + c)^9 - 18845112*d*cosh(d*x + c)^7 + 11636625*d*cosh(d*x + c)^5 - 3320174*d*cosh(d*x + c)^3 + 296571*d*cosh(d*x + c))*sinh(d*x + c)^10 - 49419*d*cosh(d*x + c)^9 + (817190*d*cosh(d*x + c)^14 - 5584670*d*cosh(d*x + c)^12 + 15796638*d*cosh(d*x + c)^10 - 23556390*d*cosh(d*x + c)^8 + 19404385*d*cosh(d*x + c)^6 - 8327605*d*cosh(d*x + c)^4 + 1501665*d*cosh(d*x + c)^2 - 51357*d)*sinh(d*x + c)^9 + 3*(163438*d*cosh(d*x + c)^15 - 1288770*d*cosh(d*x + c)^13 + 4308174*d*cosh(d*x + c)^11 - 7852130*d*cosh(d*x + c)^9 + 8311875*d*cosh(d*x + c)^7 - 4980261*d*cosh(d*x + c)^5 + 1482855*d*cosh(d*x + c)^3 - 148257*d*cosh(d*x + c))*sinh(d*x + c)^8 + 71706*d*cosh(d*x + c)^7 + 3*(81719*d*cosh(d*x + c)^16 - 736440*d*cosh(d*x + c)^14 + 2872116*d*cosh(d*x + c)^12 - 6281704*d*cosh(d*x + c)^10 + 8316165*d*cosh(d*x + c)^8 - 6662084*d*cosh(d*x + c)^6 + 3003330*d*cosh(d*x + c)^4 - 616284*d*cosh(d*x + c)^2 + 26486*d)*sinh(d*x + c)^7 + (100947*d*cosh(d*x + c)^17 - 1031016*d*cosh(d*x + c)^15 + 4639572*d*cosh(d*x + c)^13 - 11992344*d*cosh(d*x + c)^11 + 19394375*d*cosh(d*x + c)^9 - 19921044*d*cosh(d*x + c)^7 + 12455982*d*cosh(d*x + c)^5 - 4151196*d*cosh(d*x + c)^3 + 501942*d*cosh(d*x + c))*sinh(d*x + c)^6 - 80750*d*cosh(d*x + c)^5 + (33649*d*cosh(d*x + c)^18 - 386631*d*cosh(d*x + c)^16 + 1988388*d*cosh(d*x + c)^14 - 5996172*d*cosh(d*x + c)^12 + 11642631*d*cosh(d*x + c)^10 - 14989689*d*cosh(d*x + c)^8 + 12613986*d*cosh(d*x + c)^6 - 6470982*d*cosh(d*x + c)^4 + 1668618*d*cosh(d*x + c)^2 - 104006*d)*sinh(d*x + c)^5 + (8855*d*cosh(d*x + c)^19 - 113715*d*cosh(d*x + c)^17 + 662796*d*cosh(d*x + c)^15 - 2306220*d*cosh(d*x + c)^13 + 5289375*d*cosh(d*x + c)^11 - 8300435*d*cosh(d*x + c)^9 + 8897130*d*cosh(d*x + c)^7 - 6226794*d*cosh(d*x + c)^5 + 2509710*d*cosh(d*x + c)^3 - 403750*d*cosh(d*x + c

```
))*sinh(d*x + c)^4 + 65246*d*cosh(d*x + c)^3 + (1771*d*cosh(d*x + c)^20 - 2
5270*d*cosh(d*x + c)^18 + 165699*d*cosh(d*x + c)^16 - 658920*d*cosh(d*x + c
)^14 + 1764035*d*cosh(d*x + c)^12 - 3331042*d*cosh(d*x + c)^10 + 4504995*d*
cosh(d*x + c)^8 - 4313988*d*cosh(d*x + c)^6 + 2781030*d*cosh(d*x + c)^4 - 1
040060*d*cosh(d*x + c)^2 + 119510*d)*sinh(d*x + c)^3 + (253*d*cosh(d*x + c)
^21 - 3990*d*cosh(d*x + c)^19 + 29241*d*cosh(d*x + c)^17 - 131784*d*cosh(d*
x + c)^15 + 406875*d*cosh(d*x + c)^13 - 905502*d*cosh(d*x + c)^11 + 1482855
*d*cosh(d*x + c)^9 - 1779084*d*cosh(d*x + c)^7 + 1505826*d*cosh(d*x + c)^5
- 807500*d*cosh(d*x + c)^3 + 195738*d*cosh(d*x + c))*sinh(d*x + c)^2 - 2519
4*d*cosh(d*x + c) + (23*d*cosh(d*x + c)^22 - 399*d*cosh(d*x + c)^20 + 3249*
d*cosh(d*x + c)^18 - 16473*d*cosh(d*x + c)^16 + 58155*d*cosh(d*x + c)^14 -
151411*d*cosh(d*x + c)^12 + 300333*d*cosh(d*x + c)^10 - 462213*d*cosh(d*x +
 c)^8 + 556206*d*cosh(d*x + c)^6 - 520030*d*cosh(d*x + c)^4 + 358530*d*cosh
(d*x + c)^2 - 125970*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**20*(a+b*sinh(d*x+c)**4)**3,x)

[Out] Timed out
```

Giac [B] time = 1.66426, size = 995, normalized size = 4.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^20*(a+b*sinh(d*x+c)^4)^3,x, algorithm="giac")

[Out] -32/4849845*(4849845*b^3*e^(30*d*x + 30*c) - 61108047*b^3*e^(28*d*x + 28*c)
 + 155195040*a*b^2*e^(26*d*x + 26*c) + 355978623*b^3*e^(26*d*x + 26*c) - 13
52413920*a*b^2*e^(24*d*x + 24*c) - 1270797957*b^3*e^(24*d*x + 24*c) + 18623
40480*a^2*b*e^(22*d*x + 22*c) + 5287716720*a*b^2*e^(22*d*x + 22*c) + 310653
3573*b^3*e^(22*d*x + 22*c) - 8897848960*a^2*b*e^(20*d*x + 20*c) - 122567130
40*a*b^2*e^(20*d*x + 20*c) - 5504019807*b^3*e^(20*d*x + 20*c) + 7945986048*
a^3*e^(18*d*x + 18*c) + 17837083264*a^2*b*e^(18*d*x + 18*c) + 18774904720*a
*b^2*e^(18*d*x + 18*c) + 7296522519*b^3*e^(18*d*x + 18*c) - 6501261312*a^3*
e^(16*d*x + 16*c) - 20011694976*a^2*b*e^(16*d*x + 16*c) - 20101788720*a*b^2
*e^(16*d*x + 16*c) - 7366637421*b^3*e^(16*d*x + 16*c) + 4334174208*a^3*e^(1
4*d*x + 14*c) + 14582690304*a^2*b*e^(14*d*x + 14*c) + 15573923040*a*b^2*e^(
14*d*x + 14*c) + 5711316039*b^3*e^(14*d*x + 14*c) - 2333786112*a^3*e^(12*d*
x + 12*c) - 7852217856*a^2*b*e^(12*d*x + 12*c) - 8958986400*a*b^2*e^(12*d*x
 + 12*c) - 3403621221*b^3*e^(12*d*x + 12*c) + 1000194048*a^3*e^(10*d*x + 10
*c) + 3365236224*a^2*b*e^(10*d*x + 10*c) + 3906077760*a*b^2*e^(10*d*x + 10*
c) + 1550149029*b^3*e^(10*d*x + 10*c) - 333398016*a^3*e^(8*d*x + 8*c) - 112
1745408*a^2*b*e^(8*d*x + 8*c) - 1302025920*a*b^2*e^(8*d*x + 8*c) - 53223584
7*b^3*e^(8*d*x + 8*c) + 83349504*a^3*e^(6*d*x + 6*c) + 280436352*a^2*b*e^(6
*d*x + 6*c) + 325506480*a*b^2*e^(6*d*x + 6*c) + 134271423*b^3*e^(6*d*x + 6*
c) - 14708736*a^3*e^(4*d*x + 4*c) - 49488768*a^2*b*e^(4*d*x + 4*c) - 574423
20*a*b^2*e^(4*d*x + 4*c) - 23694957*b^3*e^(4*d*x + 4*c) + 1634304*a^3*e^(2*
```

```
d*x + 2*c) + 5498752*a^2*b*e^(2*d*x + 2*c) + 6382480*a*b^2*e^(2*d*x + 2*c)
+ 2632773*b^3*e^(2*d*x + 2*c) - 86016*a^3 - 289408*a^2*b - 335920*a*b^2 - 1
38567*b^3)/(d*(e^(2*d*x + 2*c) - 1)^19)
```

3.229 $\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh^3(c+dx)}{3bd}+\frac{\cosh(c+dx)}{bd}$$

```
[Out] -(a*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a
] - Sqrt[b]]*b^(7/4)*d) + (a*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)*d) + Cosh[c + d*x]/(b*d) - C
osh[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.254065, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1166, 205, 208}

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh^3(c+dx)}{3bd}+\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(a*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a
] - Sqrt[b]]*b^(7/4)*d) + (a*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)*d) + Cosh[c + d*x]/(b*d) - C
osh[c + d*x]^3/(3*b*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^7(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d}\\ &= -\frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{x^2}{b}+\frac{a-ax^2}{b\left(a-b+2bx^2-bx^4\right)}\right) dx, x, \cosh(c+dx)\right)}{d}\\ &= \frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+dx)}{3bd}-\frac{\text{Subst}\left(\int \frac{a-ax^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{bd}\\ &= \frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+dx)}{3bd}+\frac{a\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2bd}+\frac{a\,\text{Subst}}{}\\ &= -\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d}+\frac{\cosh(c+dx)}{bd}-\frac{\cosh^3(c+dx)}{3bd}\end{aligned}$$

Mathematica [C] time = 0.336444, size = 390, normalized size = 2.64

$$-3a\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{2\#1^6\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\text{co}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (18*Cosh[c + d*x] - 2*Cosh[3*(c + d*x)] - 3*a*RootSum[b - 4*b*#1^2 - 16*a*#
1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#
1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
 d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-C
osh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x
)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#
1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(24*b*d)
```

Maple [B] time = 0.082, size = 270, normalized size = 1.8

$$-\frac{1}{3\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-3}+\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}+\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{a}{2\,db^2}\sqrt{ab}\arctan\left(\frac{1}{4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/
b/(tanh(1/2*d*x+1/2*c)+1)-1/2/d*a/b^2*(a*b)^(1/2)/(-a*b-(a*b)^(1/2)*a)^(1/2
)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/
2)*a)^(1/2))-1/2/d*a/b^2*(a*b)^(1/2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*
(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+1
/3/d/b/(tanh(1/2*d*x+1/2*c)-1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b/
(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(e^{(6\,dx+6\,c)}-9\,e^{(4\,dx+4\,c)}-9\,e^{(2\,dx+2\,c)}+1\right)e^{(-3\,dx-3\,c)}}{24\,bd}-\frac{1}{128}\int\frac{256\,\left(ae^{(7\,dx+7\,c)}-3\,ae^{(5\,dx+5\,c)}+3\,ae^{(3\,dx+3\,c)}-ae\right.}{b^2e^{(8\,dx+8\,c)}-4\,b^2e^{(6\,dx+6\,c)}-4\,b^2e^{(2\,dx+2\,c)}+b^2-2\,\left(8\,abe^{(4\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/24*(e^(6*d*x + 6*c) - 9*e^(4*d*x + 4*c) - 9*e^(2*d*x + 2*c) + 1)*e^(-3*d
*x - 3*c)/(b*d) - 1/128*integrate(256*(a*e^(7*d*x + 7*c) - 3*a*e^(5*d*x + 5
*c) + 3*a*e^(3*d*x + 3*c) - a*e^(d*x + c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(
6*d*x + 6*c) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*
c))*e^(4*d*x)), x)
```

Fricas [B] time = 2.63877, size = 3729, normalized size = 25.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/24*(cosh(d*x + c)^6 + 6*cosh(d*x + c)*sinh(d*x + c)^5 + sinh(d*x + c)^6
+ 3*(5*cosh(d*x + c)^2 - 3)*sinh(d*x + c)^4 - 9*cosh(d*x + c)^4 + 4*(5*cosh
(d*x + c)^3 - 9*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*cosh(d*x + c)^4 - 18*
cosh(d*x + c)^2 - 3)*sinh(d*x + c)^2 - 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(
d*x + c)^2*sinh(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d
*x + c)^3)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4
)) + a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c
)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*cosh(d*x + c) +
a^2*b^2*d*sinh(d*x + c) - ((a*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*
d^3*sinh(d*x + c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3
 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)
*d^2))) + 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*
b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)*sqrt(-((a*b^3 - b^
4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2)
)*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*
x + c)^2 + a^3 - 2*(a^2*b^2*d*cosh(d*x + c) + a^2*b^2*d*sinh(d*x + c) - ((a
*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*d^3*sinh(d*x + c))*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^
7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) - 6*(b*d*cosh(d*x + c
```

```
)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x +
c)^2 + b*d*sinh(d*x + c)^3)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*
a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*
a^3*cosh(d*x + c)*sinh(d*x + c) + a^3*sinh(d*x + c)^2 + a^3 + 2*(a^2*b^2*d*
cosh(d*x + c) + a^2*b^2*d*sinh(d*x + c) + ((a*b^5 - b^6)*d^3*cosh(d*x + c)
+ (a*b^5 - b^6)*d^3*sinh(d*x + c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)
))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)
/((a*b^3 - b^4)*d^2))) + 6*(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sin
h(d*x + c) + 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)*sqr
t(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b
^3 - b^4)*d^2))*log(a^3*cosh(d*x + c)^2 + 2*a^3*cosh(d*x + c)*sinh(d*x + c)
 + a^3*sinh(d*x + c)^2 + a^3 - 2*(a^2*b^2*d*cosh(d*x + c) + a^2*b^2*d*sinh(
d*x + c) + ((a*b^5 - b^6)*d^3*cosh(d*x + c) + (a*b^5 - b^6)*d^3*sinh(d*x +
c))*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sqrt
(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 9*cosh
(d*x + c)^2 + 6*(cosh(d*x + c)^5 - 6*cosh(d*x + c)^3 - 3*cosh(d*x + c))*sin
h(d*x + c) + 1)/(b*d*cosh(d*x + c)^3 + 3*b*d*cosh(d*x + c)^2*sinh(d*x + c)
+ 3*b*d*cosh(d*x + c)*sinh(d*x + c)^2 + b*d*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.230 $\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh(c+dx)}{bd}$$

```
[Out] (Sqrt[a]*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(5/4)*d) + (Sqrt[a]*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqr
t[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) - Cosh[c + d*x
]/(b*d)
```

Rubi [A] time = 0.197599, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1093, 205, 208}

$$\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cosh(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]
```


```
[Out] (Sqrt[a]*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(5/4)*d) + (Sqrt[a]*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqr
t[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) - Cosh[c + d*x
]/(b*d)
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\sinh^5(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a-b+2bx^2-bx^4}\,dx,x,\cosh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a}{b\left(a-b+2bx^2-bx^4\right)}\right)dx,x,\cosh(c+dx)\right)}{d}\\ &= -\frac{\cosh(c+dx)}{bd}+\frac{a\,\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx,x,\cosh(c+dx)\right)}{bd}\\ &= -\frac{\cosh(c+dx)}{bd}-\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx,x,\cosh(c+dx)\right)}{2\sqrt{b}d}+\frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-b\ldots}\right.}{2\sqrt{\ldots}}\\ &= \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{5/4}d}+\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{5/4}d}-\frac{\cosh(c+dx)}{bd}\end{aligned}$$

Mathematica [C] time = 0.259954, size = 235, normalized size = 1.69

$$-\frac{a\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{2\#1^3\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cos\ldots\right.}{\ldots}\right.}{2bd\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(2*Cosh[c + d*x] + a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^
6 + b*#1^8 & , (-(c*#1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3
+ 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ]
)/(2*b*d)
```

Maple [A] time = 0.048, size = 175, normalized size = 1.3

$$-\frac{1}{bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}-\frac{a}{2\,bd}\arctan\left(\frac{1}{4}\left(-2\,\left(\tanh\left(1/2\,dx+c/2\right)\right)^2a+4\,\sqrt{ab}+2\,a\right)\frac{1}{\sqrt{-ab-\sqrt{aba}}}\right)\frac{1}{\sqrt{-ab-\sqrt{\ldots}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/2/d*a/b/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(
1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/
2))+1/2/d*a/b/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^
2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+1/d/b/(tanh(1/2*d*x+1/2*
c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(e^{(2\,dx+2\,c)}+1\right)e^{(-dx-c)}}{2\,bd}-\frac{1}{32}\int\frac{256\left(ae^{(5\,dx+5\,c)}-ae^{(3\,dx+3\,c)}\right)}{b^2e^{(8\,dx+8\,c)}-4\,b^2e^{(6\,dx+6\,c)}-4\,b^2e^{(2\,dx+2\,c)}+b^2-2\left(8\,abe^{(4\,c)}-3\,b^2e^{(4\,c)}\right)e^{(4\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/2*(e^(2*d*x + 2*c) + 1)*e^(-d*x - c)/(b*d) - 1/32*integrate(256*(a*e^(5*
d*x + 5*c) - a*e^(3*d*x + 3*c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c
) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*
x)), x)
```

Fricas [B] time = 2.11141, size = 2755, normalized size = 19.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*((b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(a^2*cosh
(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2
 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) - ((a*b^4 - b^5)*d^3*co
sh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*a*b^6
 + b^7)*d^4)))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)
*d^4)) + a)/((a*b^2 - b^3)*d^2))) - (b*d*cosh(d*x + c) + b*d*sinh(d*x + c))
*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((
a*b^2 - b^3)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x +
 c) + a^2*sinh(d*x + c)^2 + a^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d
*x + c) - ((a*b^4 - b^5)*d^3*cosh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c
))*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)))*sqrt(-((a*b^2 - b^3)*d^2*sqrt
(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) + (b*d*cos
h(d*x + c) + b*d*sinh(d*x + c))*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5
- 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(a^2*cosh(d*x + c)^2 +
2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 + a^2 + 2*(a^2*b*d*
cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a*b^4 - b^5)*d^3*cosh(d*x + c) +
(a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)))
*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a
*b^2 - b^3)*d^2))) - (b*d*cosh(d*x + c) + b*d*sinh(d*x + c))*sqrt(((a*b^2 -
 b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2
))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d
*x + c)^2 + a^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a*b^
4 - b^5)*d^3*cosh(d*x + c) + (a*b^4 - b^5)*d^3*sinh(d*x + c))*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)))*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 -
2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - 2*cosh(d*x + c)^2 - 4*cos
```

```
h(d*x + c)*sinh(d*x + c) - 2*sinh(d*x + c)^2 - 2)/(b*d*cosh(d*x + c) + b*d*
sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.231 $\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

```
[Out] -ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[Sqrt[a] -
Sqrt[b]]*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)*d)
```

Rubi [A] time = 0.114616, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3215, 1166, 205, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[Sqrt[a] -
Sqrt[b]]*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2d}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^{3/4}d}$$

Mathematica [C] time = 0.177202, size = 365, normalized size = 3.17

$$-\frac{\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-c -
d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2 + 6*Log[-Cosh[(c + d*x)/2] -
 Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 3*
c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + c*#1^6 + d*x*#1^6 + 2*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ]/(8*d)
```

Maple [A] time = 0.029, size = 142, normalized size = 1.2

$$-\frac{1}{2\,bd}\sqrt{ab}\arctan\left(\frac{1}{4}\left(-2\,(\tanh(1/2\,dx+c/2))^2\,a+4\,\sqrt{ab}+2\,a\right)\frac{1}{\sqrt{-ab-\sqrt{ab}a}}\right)\frac{1}{\sqrt{-ab-\sqrt{ab}a}}-\frac{1}{2\,bd}\sqrt{ab}\arctan\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/2/d*(a*b)^(1/2)/b/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x
+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-1/2/d*(a*b)^(1/2
)/b/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b
)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh(dx+c)^3}{b\sinh(dx+c)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(d*x + c)^3/(b*sinh(d*x + c)^4 - a), x)
```

Fricas [B] time = 1.90177, size = 2202, normalized size = 19.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a
*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(
d*x + c)^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) - ((a*b^2 - b^3)*d^3*
cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4
 + b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4
)) + 1)/((a*b - b^2)*d^2)) + 1) - 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*c
osh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c) + b*d*s
inh(d*x + c) - ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*
x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(-((a*b - b^2)*d^2*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) + 1) + 1/4*sqr
t(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2
)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x + c) + ((a*b^2 - b^3)*d^3*cosh(d*x
 + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*
d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/(
(a*b - b^2)*d^2)) + 1) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b
^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x +
c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(b*d*cosh(d*x + c) + b*d*sinh(d*x +
c) + ((a*b^2 - b^3)*d^3*cosh(d*x + c) + (a*b^2 - b^3)*d^3*sinh(d*x + c))*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)))*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2)) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.232 $\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqr
t[a] - Sqrt[b]]*b^(1/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)
```

Rubi [A] time = 0.103533, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3215, 1093, 205, 208}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqr
t[a] - Sqrt[b]]*b^(1/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{d}$$
$$= -\frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2\sqrt{a}d} + \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c\right.}{2\sqrt{a}d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt[4]{b}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt[4]{b}d}$$

Mathematica [C] time = 0.167964, size = 221, normalized size = 1.77

$$-\frac{\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^3\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\right.}{\quad}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(c*#
1) - d*x*#1 - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log[-Cosh[(c + d*
x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1
^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ]/(2*d)
```

Maple [A] time = 0.04, size = 126, normalized size = 1.

$$-\frac{1}{2d}\arctan\left(\frac{1}{4}\left(-2\left(\tanh\left(1/2\,dx+c/2\right)\right)^2a+4\sqrt{ab}+2a\right)\frac{1}{\sqrt{-ab-\sqrt{ab}a}}\right)\frac{1}{\sqrt{-ab-\sqrt{ab}a}}+\frac{1}{2d}\arctan\left(\frac{1}{4}\left(2\left(\tanh\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/2/d/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*
(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/2/d/(-a*b+(a*b)^(1/2)*a)^(1/
2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/
2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh\left(dx+c\right)}{b\sinh\left(dx+c\right)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(d*x + c)/(b*sinh(d*x + c)^4 - a), x)
```

Fricas [B] time = 1.98088, size = 2256, normalized size = 18.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/(
(a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sin
h(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*
d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a
^2*b^2 + a*b^3)*d^4)))*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 +
a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt
(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(cosh(d*x
+ c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d*cosh(d*x
+ c) + a*d*sinh(d*x + c) - ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*
b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(-((
a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*
d^2)) + 1) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*
d^4)) - 1)/((a^2 - a*b)*d^2))*log(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*
x + c) + sinh(d*x + c)^2 + 2*(a*d*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2
*b - a*b^2)*d^3*cosh(d*x + c) + (a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/(
(a^3*b - 2*a^2*b^2 + a*b^3)*d^4)))*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2
*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + 1) - 1/4*sqrt(((a^2 - a*b
)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log
(cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*(a*d
*cosh(d*x + c) + a*d*sinh(d*x + c) + ((a^2*b - a*b^2)*d^3*cosh(d*x + c) + (
a^2*b - a*b^2)*d^3*sinh(d*x + c))*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))
)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^
2 - a*b)*d^2)) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.233 $\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a*Sqr
t[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (b^(1/4)*ArcTanh[(
b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b
]]*d)
```

Rubi [A] time = 0.163084, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3215, 1170, 207, 1166, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a*Sqr
t[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c + d*x]]/(a*d) + (b^(1/4)*ArcTanh[(
b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b
]]*d)
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{csch}(c+dx)}{a-b\sinh^4(c+dx)}\,dx &= -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)}\,dx, x, \cosh(c+dx)\right)}{d}\\
&= -\frac{\operatorname{Subst}\left(\int \left(-\frac{1}{a(-1+x^2)}+\frac{b-bx^2}{a(a-b+2bx^2-bx^4)}\right)dx, x, \cosh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{ad}-\frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{ad}\\
&= -\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{b\operatorname{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cosh(c+dx)\right)}{2ad}+\frac{b\operatorname{Subst}\left(\int \frac{}{\sqrt{a}\sqrt{}}\right.}{}\\
&= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}d}-\frac{\tanh^{-1}(\cosh(c+dx))}{ad}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}d}\end{aligned}$$

Mathematica [C] time = 0.248951, size = 385, normalized size = 2.83

$$\frac{8\log\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)-b\operatorname{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{2\#1^6\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cos\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (8*Log[Tanh[(c + d*x)/2]] - b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 3*c*#1^2 + 3*d*x*#1^2
 + 6*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Si
nh[(c + d*x)/2]*#1]*#1^2 - 3*c*#1^4 - 3*d*x*#1^4 - 6*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 +
c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c
+ d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
 3*b*#1^5 + b*#1^7) & ])/(8*a*d)
```

Maple [A] time = 0.053, size = 159, normalized size = 1.2

$$-\frac{1}{2da}\sqrt{ab}\arctan\left(\frac{1}{4}\left(-2\,(\tanh(1/2\,dx+c/2))^2\,a+4\,\sqrt{ab}+2\,a\right)\frac{1}{\sqrt{-ab-\sqrt{ab}a}}\right)\frac{1}{\sqrt{-ab-\sqrt{ab}a}}-\frac{1}{2da}\sqrt{ab}\arctan\left(\frac{1}{4}\left(2\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/2/d*(a*b)^(1/2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x
+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-1/2/d*(a*b)^(1/2
)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b
)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+1/d/a*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{ad}+\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{ad}-2\int\frac{be^{(7\,dx+7\,c)}-3\,be^{(5\,dx+5\,c)}+3\,be^{(3\,dx+3\,c)}-be^{(dx+c)}}{abe^{(8\,dx+8\,c)}-4\,abe^{(6\,dx+6\,c)}-4\,abe^{(2\,dx+2\,c)}+ab-2\left(8\,a^2e^{(4\,c)}-3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -log((e^(d*x + c) + 1)*e^(-c))/(a*d) + log((e^(d*x + c) - 1)*e^(-c))/(a*d)
- 2*integrate((b*e^(7*d*x + 7*c) - 3*b*e^(5*d*x + 5*c) + 3*b*e^(3*d*x + 3*c
) - b*e^(d*x + c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*e^(
2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)
```

Fricas [B] time = 2.26058, size = 2431, normalized size = 17.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
 b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
 + c) + b*sinh(d*x + c)^2 + 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c) -
((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt(b/
((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2
*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)) + b) - a*d*sqrt(-((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)
)*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)
^2 - 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c) - ((a^4 - a^3*b)*d^3*cosh
(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4))
 + b)/((a^3 - a^2*b)*d^2)) + b) + a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5
- 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2
+ 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*(a*b*d*cosh(d*x +
 c) + a*b*d*sinh(d*x + c) + ((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b
```

```
)*d^3*sinh(d*x + c))*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)
) + b) - a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)
) - b)/((a^3 - a^2*b)*d^2))*log(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(
d*x + c) + b*sinh(d*x + c)^2 - 2*(a*b*d*cosh(d*x + c) + a*b*d*sinh(d*x + c)
 + ((a^4 - a^3*b)*d^3*cosh(d*x + c) + (a^4 - a^3*b)*d^3*sinh(d*x + c))*sqrt
(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)))*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 -
 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)) + b) - 4*log(cosh(d*x +
 c) + sinh(d*x + c) + 1) + 4*log(cosh(d*x + c) + sinh(d*x + c) - 1))/(a*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.234 $\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(\cosh(c+dx)+1)}+\frac{\tanh^{-1}(\cosh}{2ad}$$

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2
)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + ArcTanh[Cosh[c + d*x]]/(2*a*d) + (b^(3/4)*Ar
cTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqr
t[a] + Sqrt[b]]*d) + 1/(4*a*d*(1 - Cosh[c + d*x])) - 1/(4*a*d*(1 + Cosh[c +
 d*x]))
```

Rubi [A] time = 0.200735, antiderivative size = 184, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1170, 207, 1093, 205, 208}

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(\cosh(c+dx)+1)}+\frac{\tanh^{-1}(\cosh}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```


```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2
)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + ArcTanh[Cosh[c + d*x]]/(2*a*d) + (b^(3/4)*Ar
cTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqr
t[a] + Sqrt[b]]*d) + 1/(4*a*d*(1 - Cosh[c + d*x])) - 1/(4*a*d*(1 + Cosh[c +
 d*x]))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^3(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)^2(a-b+2bx^2-bx^4)}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}-\frac{1}{2a(-1+x^2)}+\frac{b}{a(a-b+2bx^2-bx^4)}\right)dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(1+\cosh(c+dx))}-\frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{2ad}+$$

$$= \frac{\tanh^{-1}(\cosh(c+dx))}{2ad}+\frac{1}{4ad(1-\cosh(c+dx))}-\frac{1}{4ad(1+\cosh(c+dx))}-\frac{b^{3/2}\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\tanh^{-1}(\cosh(c+dx))}{2ad}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}\,d}+\frac{}{4ad(1}$$

Mathematica [C] time = 0.378545, size = 265, normalized size = 1.44

$$-\frac{4b\operatorname{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^3\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\text{co}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^3/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(Csch[(c + d*x)/2]^2 + 4*Log[Tanh[(c + d*x)/2]] + 4*b*RootSum[b - 4*b*#1^2
 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(c*#1) - d*x*#1 - 2*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*
x)/2]*#1]*#1 + c*#1^3 + d*x*#1^3 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^3)/(-b - 8*a*#1^2 +
3*b*#1^2 - 3*b*#1^4 + b*#1^6) & ] + Sech[(c + d*x)/2]^2)/(8*a*d)
```

Maple [A] time = 0.066, size = 190, normalized size = 1.

$$\frac{1}{8da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2-\frac{b}{2da}\arctan\left(\frac{1}{4}\left(-2\left(\tanh\left(1/2\,dx+c/2\right)\right)^2a+4\sqrt{ab}+2a\right)\frac{1}{\sqrt{-ab-\sqrt{aba}}}\right)\frac{1}{\sqrt{-ab-\sqrt{aba}}}+\frac{}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] 1/8/d/a*tanh(1/2*d*x+1/2*c)^2-1/2/d*b/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1
/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2
))+1/2/d*b/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2
*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/8/d/a/tanh(1/2*d*x+1/2*
c)^2-1/2/d/a*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{(3\,dx+3\,c)}+e^{(dx+c)}}{ade^{(4\,dx+4\,c)}-2\,ade^{(2\,dx+2\,c)}+ad}+\frac{\log\left(\left(e^{(dx+c)}+1\right)e^{(-c)}\right)}{2\,ad}-\frac{\log\left(\left(e^{(dx+c)}-1\right)e^{(-c)}\right)}{2\,ad}-8\int\frac{be}{abe^{(8\,dx+8\,c)}-4\,abe^{(6\,dx+6\,c)}-4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -(e^(3*d*x + 3*c) + e^(d*x + c))/(a*d*e^(4*d*x + 4*c) - 2*a*d*e^(2*d*x + 2*
c) + a*d) + 1/2*log((e^(d*x + c) + 1)*e^(-c))/(a*d) - 1/2*log((e^(d*x + c)
- 1)*e^(-c))/(a*d) - 8*integrate((b*e^(5*d*x + 5*c) - b*e^(3*d*x + 3*c))/(a
*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c) - 4*a*b*e^(2*d*x + 2*c) + a*b -
2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*x)), x)
```

Fricas [B] time = 2.58645, size = 4716, normalized size = 25.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(4*cosh(d*x + c)^3 + 12*cosh(d*x + c)*sinh(d*x + c)^2 + 4*sinh(d*x + c
)^3 - (a*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh
(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(
d*x + c)^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c
))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2
)/((a^4 - a^3*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d
*x + c) + b^2*sinh(d*x + c)^2 + b^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*si
nh(d*x + c) - ((a^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x
 + c))*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*
sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) + (a
*d*cosh(d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c
)^4 - 2*a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)
^2 + a*d + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(
-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4
```

```
- a^3*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c)
+ b^2*sinh(d*x + c)^2 + b^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x +
 c) - ((a^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*s
qrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3
/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) - (a*d*cosh(
d*x + c)^4 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*
a*d*cosh(d*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d
 + 4*(a*d*cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 -
a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*
d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sin
h(d*x + c)^2 + b^2 + 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a
^5 - a^4*b)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*sqrt(b^3/(
(a^7 - 2*a^6*b + a^5*b^2)*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2
*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) + (a*d*cosh(d*x + c)^4
 + 4*a*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*a*d*cosh(d
*x + c)^2 + 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d + 4*(a*d*
cosh(d*x + c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*d^2
*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(
b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)
^2 + b^2 - 2*(a^2*b*d*cosh(d*x + c) + a^2*b*d*sinh(d*x + c) + ((a^5 - a^4*b
)*d^3*cosh(d*x + c) + (a^5 - a^4*b)*d^3*sinh(d*x + c))*sqrt(b^3/((a^7 - 2*a
^6*b + a^5*b^2)*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a
^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 2*(cosh(d*x + c)^4 + 4*cosh(d*
x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1)*sinh(d
*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))*sinh(d*
x + c) + 1)*log(cosh(d*x + c) + sinh(d*x + c) + 1) + 2*(cosh(d*x + c)^4 + 4
*cosh(d*x + c)*sinh(d*x + c)^3 + sinh(d*x + c)^4 + 2*(3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c)^2 - 2*cosh(d*x + c)^2 + 4*(cosh(d*x + c)^3 - cosh(d*x + c))
*sinh(d*x + c) + 1)*log(cosh(d*x + c) + sinh(d*x + c) - 1) + 4*(3*cosh(d*x
+ c)^2 + 1)*sinh(d*x + c) + 4*cosh(d*x + c))/(a*d*cosh(d*x + c)^4 + 4*a*d*c
osh(d*x + c)*sinh(d*x + c)^3 + a*d*sinh(d*x + c)^4 - 2*a*d*cosh(d*x + c)^2
+ 2*(3*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^2 + a*d + 4*(a*d*cosh(d*x +
 c)^3 - a*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**3/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^3/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.235 $\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=175

$$-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(\tanh(c+dx)+1)}+$$

```
[Out] x/(2*b) - (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]
)/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) + (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] +
Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) - 1
/(4*b*d*(1 - Tanh[c + d*x])) + 1/(4*b*d*(1 + Tanh[c + d*x]))
```

Rubi [A] time = 0.258026, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1287, 207, 1130, 208}

$$-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(\tanh(c+dx)+1)}+$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] x/(2*b) - (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]
)/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) + (a^(3/4)*ArcTanh[(Sqrt[Sqrt[a] +
Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) - 1
/(4*b*d*(1 - Tanh[c + d*x])) + 1/(4*b*d*(1 + Tanh[c + d*x]))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1-x^2)^2(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{4b(-1+x)^2}-\frac{1}{4b(1+x)^2}-\frac{1}{2b(-1+x^2)}+\frac{ax^2}{b(a-2ax^2+(a-b)x^4)}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(1+\tanh(c+dx))}-\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{2bd}$$

$$= \frac{x}{2b}-\frac{1}{4bd(1-\tanh(c+dx))}+\frac{1}{4bd(1+\tanh(c+dx))}+\frac{\left(a\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subst}\left(\int \frac{}{-a-\sqrt{a}\sqrt{}}\right.}{2b^{3/2}}$$

$$= \frac{x}{2b}-\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/2}d}+\frac{a^{3/4}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/2}d}-\frac{1}{4bd(1-\tanh(c}$$

Mathematica [A] time = 0.867199, size = 158, normalized size = 0.9

$$\frac{\frac{2a\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{2a\tan^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}+2\sqrt{b}(c+dx)-\sqrt{b}\sinh(2(c+dx))}{4b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (2*Sqrt[b]*(c + d*x) + (2*a*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt
[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*a*ArcTanh[((Sqrt[a
] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sq
rt[b]] - Sqrt[b]*Sinh[2*(c + d*x)])/(4*b^(3/2)*d)
```

Maple [C] time = 0.051, size = 223, normalized size = 1.3

$$\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-2}-\frac{1}{2\,bd}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^{-1}+\frac{1}{2\,bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{a}{bd}\sum_{\_R=\text{RootOf}(a\_Z^8-4a\_Z}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] 1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b*l
n(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-
8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16
*b)*_Z^4-4*a*_Z^2+a))-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b/(tanh(1/2*d
*x+1/2*c)-1)-1/2/d/b*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(4\,dxe^{(2\,dx+2\,c)}-e^{(4\,dx+4\,c)}+1\right)e^{(-2\,dx-2\,c)}}{8\,bd}-\frac{1}{64}\int\frac{256\left(ae^{(6\,dx+6\,c)}-2\,ae^{(4\,dx+4\,c)}+ae^{(2\,dx+2\,c)}\right)}{b^2e^{(8\,dx+8\,c)}-4\,b^2e^{(6\,dx+6\,c)}-4\,b^2e^{(2\,dx+2\,c)}+b^2-2\left(8\,abe^{(4\,c)}-3\,b^2e^{(4\,c)}\right)e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/8*(4*d*x*e^(2*d*x + 2*c) - e^(4*d*x + 4*c) + 1)*e^(-2*d*x - 2*c)/(b*d) -
1/64*integrate(256*(a*e^(6*d*x + 6*c) - 2*a*e^(4*d*x + 4*c) + a*e^(2*d*x +
2*c))/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c) - 4*b^2*e^(2*d*x + 2*c)
+ b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*x)), x)
```

Fricas [B] time = 2.39824, size = 3123, normalized size = 17.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(4*d*x*cosh(d*x + c)^2 - cosh(d*x + c)^4 - 4*cosh(d*x + c)*sinh(d*x + c
)^3 - sinh(d*x + c)^4 + 2*(2*d*x - 3*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(
b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)
^2)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2
)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d
*x + c) + a^2*sinh(d*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5
- 2*a*b^6 + b^7)*d^4)) - a^2 + 2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*
a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 -
2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + 2*(b*d*cosh(d*x + c)^2
+ 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt(((a*b^3 - b
^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2
))*log(a^2*cosh(d*x + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d
*x + c)^2 + 2*(a^2*b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4
)) - a^2 - 2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) -
 a^2*b*d)*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
 + a^2)/((a*b^3 - b^4)*d^2))) + 2*(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c
)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((
a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x
 + c)^2 + 2*a^2*cosh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 - 2*(a^2*
b^2 - a*b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2 + 2*((a*b^
4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(-((a
*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 -
b^4)*d^2))) - 2*(b*d*cosh(d*x + c)^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) +
b*d*sinh(d*x + c)^2)*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6
```

```
+ b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(a^2*cosh(d*x + c)^2 + 2*a^2*co
sh(d*x + c)*sinh(d*x + c) + a^2*sinh(d*x + c)^2 - 2*(a^2*b^2 - a*b^3)*d^2*s
qrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2 - 2*((a*b^4 - b^5)*d^3*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(-((a*b^3 - b^4)*d^2*sq
rt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) + 4*(2
*d*x*cosh(d*x + c) - cosh(d*x + c)^3)*sinh(d*x + c) + 1)/(b*d*cosh(d*x + c)
^2 + 2*b*d*cosh(d*x + c)*sinh(d*x + c) + b*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30782, size = 82, normalized size = 0.47

$$-\frac{\left(2\,e^{(2\,dx+2\,c)}-1\right)e^{(-2\,dx-2\,c)}}{8\,bd}+\frac{dx+c}{2\,bd}-\frac{e^{(2\,dx+2\,c)}}{8\,bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/8*(2*e^(2*d*x + 2*c) - 1)*e^(-2*d*x - 2*c)/(b*d) + 1/2*(d*x + c)/(b*d) -
 1/8*e^(2*d*x + 2*c)/(b*d)
```

3.236 $\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

```
[Out] -(x/b) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])
/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b*d)
```

Rubi [A] time = 0.206337, antiderivative size = 127, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1287, 207, 1166, 208}

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(x/b) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])
/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b(-1+x^2)} + \frac{a(1-x^2)}{b(a-2ax^2+(a-b)x^4)}\right) dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+dx)\right)}{bd} + \frac{a\,\text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{bd}$$

$$= -\frac{x}{b} - \frac{\left(\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2bd} - \frac{\left(a\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\right)\text{Subs}}{}$$

$$= -\frac{x}{b} + \frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}bd} + \frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}bd}$$

Mathematica [A] time = 0.449812, size = 143, normalized size = 1.13

$$\frac{\frac{\sqrt{a}\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\sqrt{a}\tan^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} - 2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) - (Sqrt[a]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a
 + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[a]*ArcTanh[((Sqrt[
a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*S
qrt[b]])/(2*b*d)
```

Maple [C] time = 0.033, size = 144, normalized size = 1.1

$$-\frac{1}{bd}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right) - \frac{a}{4bd}\sum_{\_R=\text{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)} \frac{\_R^6-3\_R^4+3\_R^2-1}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/4/d*a/b*sum((_R^6-3*_R^4+3*_R^2-1)/(_R^7
*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*
_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))+1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16\,a\int\frac{e^{(4\,dx+4\,c)}}{b^2e^{(8\,dx+8\,c)}-4\,b^2e^{(6\,dx+6\,c)}-4\,b^2e^{(2\,dx+2\,c)}+b^2-2\left(8\,abe^{(4\,c)}-3\,b^2e^{(4\,c)}\right)e^{(4\,dx)}}\,dx-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16*a*integrate(e^(4*d*x + 4*c)/(b^2*e^(8*d*x + 8*c) - 4*b^2*e^(6*d*x + 6*c
) - 4*b^2*e^(2*d*x + 2*c) + b^2 - 2*(8*a*b*e^(4*c) - 3*b^2*e^(4*c))*e^(4*d*
x)), x) - x/b
```

Fricas [B] time = 2.16358, size = 2128, normalized size = 16.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a
)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b
^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)
^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d)*sq
rt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((a*b^2
- b^3)*d^2)) - 1) - b*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 +
b^5)*d^4)) + a)/((a*b^2 - b^3)*d^2))*log(2*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3
 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) +
 sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d
^4)) - b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
 + a)/((a*b^2 - b^3)*d^2)) - 1) - b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b
^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2*(a*b - b^2)*d^2*
sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)
*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 -
2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a
*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + b*sqrt(-((a*b^2 - b^3)*d
^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-2
*(a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + cosh(d*x + c)^2
+ 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*((a*b^2 - b^3)*d^3*sq
rt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(
a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2)) - 1) + 4*x)/b
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.32317, size = 18, normalized size = 0.14

$$-\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -(d*x + c)/(b*d)
```

3.237 $\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

```
[Out] -ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[S
qrt[a] - Sqrt[b]]*Sqrt[b]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*
x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)
```

Rubi [A] time = 0.120626, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3217, 1130, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{b}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[S
qrt[a] - Sqrt[b]]*Sqrt[b]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*
x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2d} + \frac{\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-}\right.}{2d}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}d}$$

Mathematica [A] time = 0.349201, size = 127, normalized size = 1.02

$$\frac{\frac{\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\tan^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/Sqr
t[-a + Sqrt[a]*Sqrt[b]] + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[
a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[b]*d)
```

Maple [C] time = 0.031, size = 94, normalized size = 0.8

$$-\frac{1}{d}\sum_{\_R=\text{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\_R^4-\_R^2}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/d*sum((_R^4-_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d
*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{\sinh(dx+c)^2}{b\sinh(dx+c)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sinh(d*x + c)^2/(b*sinh(d*x + c)^4 - a), x)
```

Fricas [B] time = 2.21265, size = 2130, normalized size = 17.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/(
(a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)
*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2
+ 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*d)*s
qrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b -
 b^2)*d^2)) - 1) + 1/4*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a
*b^3)*d^4)) + 1)/((a*b - b^2)*d^2))*log(2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b -
2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c)
+ sinh(d*x + c)^2 - 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b
^3)*d^4)) - a*d)*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*
d^4)) + 1)/((a*b - b^2)*d^2)) - 1) + 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^
3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))*log(-2*(a^2 - a*b)*d
^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*x + c)^2 + 2*cosh(d*x
 + c)*sinh(d*x + c) + sinh(d*x + c)^2 + 2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3
*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b
- 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2)) - 1) - 1/4*sqrt(-((a*b -
 b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))
*log(-2*(a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + cosh(d*
x + c)^2 + 2*cosh(d*x + c)*sinh(d*x + c) + sinh(d*x + c)^2 - 2*((a^2*b - a*
b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(-((a*b - b^2
)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2)) - 1
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.616, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 0
```

3.238 $\int \frac{1}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sq
rt[a] - Sqrt[b]]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/
4)]/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)
```

Rubi [A] time = 0.0976941, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3209, 1166, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-1),x]
```


```
[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sq
rt[a] - Sqrt[b]]*d) + ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/
4)]/(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)
```


Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2d} - \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\right.}{2d}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}d} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}d}$$

Mathematica [A] time = 0.2393, size = 128, normalized size = 1.11

$$\frac{\frac{\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\tan^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{a}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-1),x]
```

```
[Out] (-(ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/S
qrt[-a + Sqrt[a]*Sqrt[b]]) + ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sq
rt[a + Sqrt[a]*Sqrt[b]]]/Sqrt[a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*d)
```

Maple [C] time = 0.033, size = 102, normalized size = 0.9

$$\frac{1}{4d}\sum_{\_R=\text{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{-\_R^6+3\_R^4-3\_R^2+1}{\_R^7a-3\_R^5a+3\_R^3a-8\_R^3b-\_Ra}\ln\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] 1/4/d*sum((-_R^6+3*_R^4-3*_R^2+1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*
ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z
^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\int \frac{1}{b\sinh(dx+c)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sinh(d*x + c)^4 - a), x)
```

Fricas [B] time = 2.16271, size = 2130, normalized size = 18.52

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a
^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c
)^2 + 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)
*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 -
 a*b)*d^2)) - b) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*
a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 - 2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b
^2)*d^4)) - a*b*d)*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*
d^4)) + 1)/((a^2 - a*b)*d^2)) - b) + 1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^
5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-2*(a^3 - a^2*b)*d
^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*x + c)^2 + 2*b*cosh(d
*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*((a^4 - a^3*b)*d^3*sqrt(b/((a
^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5
- 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b) - 1/4*sqrt(-((a^2 -
 a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*l
og(-2*(a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 - 2*((a^4 -
a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(-((a^2 - a
*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2)) - b
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{b \sinh(dx+c)^4 - a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sinh(d*x + c)^4 - a), x)
```

3.239 $\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth(c+dx)}{ad}$$

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(5
/4)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + (Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*
Tanh[c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Coth[c + d
*x]/(a*d)
```

Rubi [A] time = 0.181358, antiderivative size = 139, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1130, 208}

$$-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] -(Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(5
/4)*Sqrt[Sqrt[a] - Sqrt[b]]*d) + (Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*
Tanh[c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Coth[c + d
*x]/(a*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{x^2(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{bx^2}{a(a-2ax^2+(a-b)x^4)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= -\frac{\coth(c+dx)}{ad}+\frac{b\,\text{Subst}\left(\int \frac{x^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= -\frac{\coth(c+dx)}{ad}+\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{2ad}+\frac{\left(\left(1-\right.\right.}{}$$
$$= -\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\coth(c+dx)}{ad}$$

Mathematica [A] time = 0.813646, size = 143, normalized size = 1.03

$$\frac{\frac{\sqrt{b}\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\sqrt{b}\tan^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}-2\coth(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ((Sqrt[b]*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt
[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] + Sqrt[b])*T
anh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - 2*Cot
h[c + d*x])/(2*a*d)
```

Maple [C] time = 0.059, size = 135, normalized size = 1.

$$-\frac{1}{2da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{-1}-\frac{b}{da}\sum_{\_R=\text{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\_R^4}{\_R^7a-3\_R^5a+3\_}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/2/d/a*tanh(1/2*d*x+1/2*c)-1/2/d/a/tanh(1/2*d*x+1/2*c)-1/d/a*b*sum((_R^4-
_R^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_
R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{ade^{(2\,dx+2\,c)}-ad}-4\int\frac{be^{(6\,dx+6\,c)}-2\,be^{(4\,dx+4\,c)}+be^{(2\,dx+2\,c)}}{abe^{(8\,dx+8\,c)}-4\,abe^{(6\,dx+6\,c)}-4\,abe^{(2\,dx+2\,c)}+ab-2\left(8\,a^2e^{(4\,c)}-3\,abe^{(4\,c)}\right)e^{(4\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -2/(a*d*e^(2*d*x + 2*c) - a*d) - 4*integrate((b*e^(6*d*x + 6*c) - 2*b*e^(4*
d*x + 4*c) + b*e^(2*d*x + 2*c))/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c
) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*
x)), x)
```

Fricas [B] time = 2.28628, size = 2840, normalized size = 20.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d
*x + c)^2 - a*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2
)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x
+ c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a
^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 -
 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7
 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)
^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(((
a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2
*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2
*sinh(d*x + c)^2 + 2*(a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*
d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)
) - a^2*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^
4)) + b)/((a^3 - a^2*b)*d^2))) - (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)
*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b
^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(
d*x + c)^2 + 2*b^2*cosh(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - 2*(a
^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 + 2*((a^5 -
 a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(-((a^
3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b
)*d^2))) + (a*d*cosh(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*s
inh(d*x + c)^2 - a*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a
^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(b^2*cosh(d*x + c)^2 + 2*b^2*cos
h(d*x + c)*sinh(d*x + c) + b^2*sinh(d*x + c)^2 - 2*(a^4 - a^3*b)*d^2*sqrt(b
^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2 - 2*((a^5 - a^4*b)*d^3*sqrt(b^3/(
(a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^
3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) + 8)/(a*d*cos
h(d*x + c)^2 + 2*a*d*cosh(d*x + c)*sinh(d*x + c) + a*d*sinh(d*x + c)^2 - a*
d)
```

<u>**Sympy**</u> **[F(-1)]** time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

<u>**Giac**</u> **[A]** time = 2.00349, size = 28, normalized size = 0.2

$$-\frac{2}{ad\left(e^{(2\,dx+2\,c)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -2/(a*d*(e^(2*d*x + 2*c) - 1))
```

3.240 $\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth^3(c+dx)}{3ad}+\frac{\coth(c+dx)}{ad}$$

```
[Out] (b*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqr
t[Sqrt[a] - Sqrt[b]]*d) + (b*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x]
)/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) + Coth[c + d*x]/(a*d) - C
oth[c + d*x]^3/(3*a*d)
```

Rubi [A] time = 0.211268, antiderivative size = 148, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1166, 208}

$$\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\coth^3(c+dx)}{3ad}+\frac{\coth(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] (b*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqr
t[Sqrt[a] - Sqrt[b]]*d) + (b*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x]
)/a^(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) + Coth[c + d*x]/(a*d) - C
oth[c + d*x]^3/(3*a*d)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^4(c+dx)}{a-b\sinh^4(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{(1-x^2)^3}{x^4(a-2ax^2+(a-b)x^4)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(\frac{1}{ax^4}-\frac{1}{ax^2}+\frac{b-bx^2}{a(a-2ax^2+(a-b)x^4)}\right)dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}+\frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-2ax^2+(a-b)x^4}\,dx, x, \tanh(c+dx)\right)}{ad}$$
$$= \frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+dx)}{3ad}-\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b\right)\operatorname{Subst}\left(\int \frac{1}{-a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tanh(c+\right.}{2a^{3/2}d}$$
$$= \frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}d}+\frac{b\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}d}+\frac{\coth(c+dx)}{ad}-\frac{\coth^3(c+}{3ad}$$

Mathematica [A] time = 2.19296, size = 165, normalized size = 1.11

$$\frac{\frac{3b\tanh^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{3b\tan^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}+4\sqrt{a}\coth(c+dx)-2\sqrt{a}\coth(c+dx)\operatorname{csch}^2(c+dx)}{6a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^4/(a - b*Sinh[c + d*x]^4),x]
```

```
[Out] ((-3*b*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]
]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (3*b*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + 4*Sqrt[a]*Co
th[c + d*x] - 2*Sqrt[a]*Coth[c + d*x]*Csch[c + d*x]^2)/(6*a^(3/2)*d)
```

Maple [C] time = 0.075, size = 179, normalized size = 1.2

$$-\frac{1}{24\,da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3+\frac{3}{8\,da}\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{b}{4\,da}\sum_{\_R=\operatorname{RootOf}\left(a\_Z^8-4a\_Z^6+(6a-16b)\_Z^4-4a\_Z^2+a\right)}\frac{\_R^6-3\_}{\_R^7a-3\_R^5a+3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x)
```

```
[Out] -1/24/d/a*tanh(1/2*d*x+1/2*c)^3+3/8/d/a*tanh(1/2*d*x+1/2*c)-1/4/d/a*b*sum((
_R^6-3*_R^4+3*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*
d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/24/
```

```
d/a/tanh(1/2*d*x+1/2*c)^3+3/8/d/a/tanh(1/2*d*x+1/2*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-16\,b\int\frac{e^{(4\,dx+4\,c)}}{abe^{(8\,dx+8\,c)}-4\,abe^{(6\,dx+6\,c)}-4\,abe^{(2\,dx+2\,c)}+ab-2\left(8\,a^2e^{(4\,c)}-3\,abe^{(4\,c)}\right)e^{(4\,dx)}}\,dx-\frac{4\left(3\,e^{(2\,dx+2\,c)}\right.}{3\left(ade^{(6\,dx+6\,c)}-3\,ade^{(4\,dx+4\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -16*b*integrate(e^(4*d*x + 4*c)/(a*b*e^(8*d*x + 8*c) - 4*a*b*e^(6*d*x + 6*c
) - 4*a*b*e^(2*d*x + 2*c) + a*b - 2*(8*a^2*e^(4*c) - 3*a*b*e^(4*c))*e^(4*d*
x)), x) - 4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*e^(6*d*x + 6*c) - 3*a*d*e^(4*d*x
 + 4*c) + 3*a*d*e^(2*d*x + 2*c) - a*d)
```

Fricas [B] time = 2.21904, size = 5195, normalized size = 35.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*d*si
nh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sin
h(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cos
h(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c
)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x
+ c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5
/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^4*cosh(
d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x + c)^2 - b^4
+ 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 2*(a^
2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt
(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4
- a^3*b)*d^2))) - 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c
)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^
2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3
 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*
cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a
*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4 - a^3*b)*
d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*l
og(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b^4*sinh(d*x +
 c)^2 - b^4 + 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d
^4)) - 2*(a^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)
*d^4)))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) +
 b^2)/((a^4 - a^3*b)*d^2))) + 3*(a*d*cosh(d*x + c)^6 + 6*a*d*cosh(d*x + c)*
sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4 + 3*(5*a*d*co
sh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2 + 4*(5*a*d*cos
h(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*d*cosh(d*x + c
)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*(a*d*cosh(d*x
+ c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((
a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a
^3*b)*d^2))*log(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sinh(d*x + c) + b
```

```
^4*sinh(d*x + c)^2 - b^4 - 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/((a^9 - 2*a^8*b
 + a^7*b^2)*d^4)) + 2*(a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8
*b + a^7*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^
7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 3*(a*d*cosh(d*x + c)^6 + 6*a*d*
cosh(d*x + c)*sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d*cosh(d*x + c)^4
 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*cosh(d*x + c)^2
+ 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*a*
d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x + c)^2 - a*d + 6*
(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) -
b^2)/((a^4 - a^3*b)*d^2))*log(b^4*cosh(d*x + c)^2 + 2*b^4*cosh(d*x + c)*sin
h(d*x + c) + b^4*sinh(d*x + c)^2 - b^4 - 2*(a^5*b - a^4*b^2)*d^2*sqrt(b^5/(
(a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 2*(a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5
/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9
- 2*a^8*b + a^7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) - 48*cosh(d*x + c)^
2 - 96*cosh(d*x + c)*sinh(d*x + c) - 48*sinh(d*x + c)^2 + 16)/(a*d*cosh(d*x
 + c)^6 + 6*a*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*d*sinh(d*x + c)^6 - 3*a*d
*cosh(d*x + c)^4 + 3*(5*a*d*cosh(d*x + c)^2 - a*d)*sinh(d*x + c)^4 + 3*a*d*
cosh(d*x + c)^2 + 4*(5*a*d*cosh(d*x + c)^3 - 3*a*d*cosh(d*x + c))*sinh(d*x
+ c)^3 + 3*(5*a*d*cosh(d*x + c)^4 - 6*a*d*cosh(d*x + c)^2 + a*d)*sinh(d*x +
 c)^2 - a*d + 6*(a*d*cosh(d*x + c)^5 - 2*a*d*cosh(d*x + c)^3 + a*d*cosh(d*x
 + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**4/(a-b*sinh(d*x+c)**4),x)
```

```
[Out] Timed out
```

Giac [A] time = 2.85349, size = 46, normalized size = 0.31

$$-\frac{4\left(3\,e^{(2\,dx+2\,c)}-1\right)}{3\,ad\left(e^{(2\,dx+2\,c)}-1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^4/(a-b*sinh(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -4/3*(3*e^(2*d*x + 2*c) - 1)/(a*d*(e^(2*d*x + 2*c) - 1)^3)
```

3.241 $\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=235

$$\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tan}{8b^{9/4}d\left(\sqrt{a}\right.}$$

```
[Out] -(Sqrt[a]*(5*Sqrt[a] - 6*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[
a] - Sqrt[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(9/4)*d) - (Sqrt[a]*(5*Sqrt[
a] + 6*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(
8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(9/4)*d) + Cosh[c + d*x]/(b^2*d) + (a*Cosh[c
+ d*x]*(a + b - b*Cosh[c + d*x]^2))/(4*(a - b)*b^2*d*(a - b + 2*b*Cosh[c +
d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.485868, antiderivative size = 235, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1205, 1676, 1166, 205, 208}

$$\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tan}{8b^{9/4}d\left(\sqrt{a}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(Sqrt[a]*(5*Sqrt[a] - 6*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[
a] - Sqrt[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(9/4)*d) - (Sqrt[a]*(5*Sqrt[
a] + 6*Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(
8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(9/4)*d) + Cosh[c + d*x]/(b^2*d) + (a*Cosh[c
+ d*x]*(a + b - b*Cosh[c + d*x]^2))/(4*(a - b)*b^2*d*(a - b + 2*b*Cosh[c +
d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
```

```
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2a\left(a+\frac{a^2}{b}-4b\right)-2a(7a-}{a-b+2bx^2}\right.}{8a}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8a(a-b)}{b}+\frac{2\left(a^2(5\right.}{b(a-}\right.\right.}{8a}$$

$$= \frac{\cosh(c+dx)}{b^2d} + \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \text{Subst}\left(\int\right.$$

$$= \frac{\cosh(c+dx)}{b^2d} + \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \left(\sqrt{a}\left(5\sqrt{a}\right.\right.$$

$$= -\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d} - \frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{9/4}d} + \text{c}$$

Mathematica [C] time = 0.948725, size = 615, normalized size = 2.62

$$a\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{40\#1^4a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)-40\#1^2a\log\left(-\#1\sinh\left(\frac{1}{2}(c+d\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (32*Cosh[c + d*x] + (32*a*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/((
a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (a*Root
Sum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b
*d*x - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#
1 - Sinh[(c + d*x)/2]*#1] - 20*a*c*#1^2 + 27*b*c*#1^2 - 20*a*d*x*#1^2 + 27*
b*d*x*#1^2 - 40*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[
(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 20*a*c*#
1^4 - 27*b*c*#1^4 + 20*a*d*x*#1^4 - 27*b*d*x*#1^4 + 40*a*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
4 - 54*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32*
b^2*d)
```

Maple [B] time = 0.082, size = 1191, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d/b^2/(tanh(1/2*d*x+1/2*c)+1)-1/2/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^6+1/d*a/b/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^6
+3/2/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a
-b)*tanh(1/2*d*x+1/2*c)^4-4/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-3/2/d*a^2/b^2/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/d*a
/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2
*d*x+1/2*c)^2+1/2/d*a^2/b^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)/(a-b)+1/8/d*a/b^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-
2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a
*b)^(1/2)+5/8/d*a^2/b^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tan
h(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-3/4/d*a
/b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+
4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/8/d*a/b^2/(a-b)/(-a*b+(a*b
)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(
-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-5/8/d*a^2/b^2/(a-b)/(-a*b+(a*b)^(1/2
)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(
a*b)^(1/2)*a)^(1/2))+3/4/d*a/b/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*
(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1
/d/b^2/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b - b^2 + (a*b*e^(10*c) - b^2*e^(10*c))*e^(10*d*x) - (2*a*b*e^(8*c)
- 3*b^2*e^(8*c))*e^(8*d*x) - (20*a^2*e^(6*c) - 17*a*b*e^(6*c) + 2*b^2*e^(6*
c))*e^(6*d*x) - (20*a^2*e^(4*c) - 17*a*b*e^(4*c) + 2*b^2*e^(4*c))*e^(4*d*x)
 - (2*a*b*e^(2*c) - 3*b^2*e^(2*c))*e^(2*d*x))/((a*b^3*d*e^(9*c) - b^4*d*e^(
9*c))*e^(9*d*x) - 4*(a*b^3*d*e^(7*c) - b^4*d*e^(7*c))*e^(7*d*x) - 2*(8*a^2*
b^2*d*e^(5*c) - 11*a*b^3*d*e^(5*c) + 3*b^4*d*e^(5*c))*e^(5*d*x) - 4*(a*b^3*
d*e^(3*c) - b^4*d*e^(3*c))*e^(3*d*x) + (a*b^3*d*e^c - b^4*d*e^c)*e^(d*x)) +
 1/512*integrate(256*(a*b*e^(7*d*x + 7*c) - a*b*e^(d*x + c) + (20*a^2*e^(5*
c) - 27*a*b*e^(5*c))*e^(5*d*x) - (20*a^2*e^(3*c) - 27*a*b*e^(3*c))*e^(3*d*x
))/(a*b^3 - b^4 + (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*b^3*e^(6*c
) - b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(4*c) + 3*b^
4*e^(4*c))*e^(4*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.04473, size = 17667, normalized size = 75.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*(a*b - b^2)*cosh(d*x + c)^10 + 80*(a*b - b^2)*cosh(d*x + c)*sinh(d*
x + c)^9 + 8*(a*b - b^2)*sinh(d*x + c)^10 - 8*(2*a*b - 3*b^2)*cosh(d*x + c)
^8 + 8*(45*(a*b - b^2)*cosh(d*x + c)^2 - 2*a*b + 3*b^2)*sinh(d*x + c)^8 + 6
4*(15*(a*b - b^2)*cosh(d*x + c)^3 - (2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x
 + c)^7 - 8*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^6 + 8*(210*(a*b - b^2)*
cosh(d*x + c)^4 - 28*(2*a*b - 3*b^2)*cosh(d*x + c)^2 - 20*a^2 + 17*a*b - 2*
b^2)*sinh(d*x + c)^6 + 16*(126*(a*b - b^2)*cosh(d*x + c)^5 - 28*(2*a*b - 3*
b^2)*cosh(d*x + c)^3 - 3*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^5 - 8*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^4 + 8*(210*(a*b - b^2)*c
osh(d*x + c)^6 - 70*(2*a*b - 3*b^2)*cosh(d*x + c)^4 - 15*(20*a^2 - 17*a*b +
 2*b^2)*cosh(d*x + c)^2 - 20*a^2 + 17*a*b - 2*b^2)*sinh(d*x + c)^4 + 32*(30
*(a*b - b^2)*cosh(d*x + c)^7 - 14*(2*a*b - 3*b^2)*cosh(d*x + c)^5 - 5*(20*a
^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^3 - (20*a^2 - 17*a*b + 2*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 - 8*(2*a*b - 3*b^2)*cosh(d*x + c)^2 + 8*(45*(a*b - b^2
)*cosh(d*x + c)^8 - 28*(2*a*b - 3*b^2)*cosh(d*x + c)^6 - 15*(20*a^2 - 17*a*
b + 2*b^2)*cosh(d*x + c)^4 - 6*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^2 -
2*a*b + 3*b^2)*sinh(d*x + c)^2 + ((a*b^3 - b^4)*d*cosh(d*x + c)^9 + 9*(a*b^
3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a*b^3 - b^4)*d*sinh(d*x + c)^9
- 4*(a*b^3 - b^4)*d*cosh(d*x + c)^7 + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^2
- (a*b^3 - b^4)*d)*sinh(d*x + c)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos
h(d*x + c)^5 + 28*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh
(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 42*(a*
b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x +
 c)^5 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x
+ c)^5 - 70*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b
^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a*b^3 - b^4)*d*cosh(d*x + c)^
```

```
6 - 35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d
*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*cosh(
d*x + c) + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*cosh(d
*x + c)^5 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 -
 b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a*b^3 - b^4)*d*cosh(d*x + c)^8
 - 28*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d
*cosh(d*x + c)^4 - 12*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a*b^3 - b^4)*d)*si
nh(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7
 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a
^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4))
+ 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)
)*log(-625*a^5 + 2625*a^4*b - 3684*a^3*b^2 + 1728*a^2*b^3 - (625*a^5 - 2625
*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)^2 - 2*(625*a^5 - 2625*a
^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c) - (625*a^5
- 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*sinh(d*x + c)^2 + 2*((125*a^5*b
^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*cosh(d*x + c) + (125*a^5*b^
2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*sinh(d*x + c) - 2*((2*a^4*b^
7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*cosh(d*x + c) + (2*a^4
*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sinh(d*x + c))*sqrt
((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*
b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^1
5)*d^4)))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 -
3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*
b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 1
5*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2)))
- ((a*b^3 - b^4)*d*cosh(d*x + c)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d
*x + c)^8 + (a*b^3 - b^4)*d*sinh(d*x + c)^9 - 4*(a*b^3 - b^4)*d*cosh(d*x +
c)^7 + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c
)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^5 + 28*(3*(a*b^3 - b
^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*
(63*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*x + c)^2 -
(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*cosh(
d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*cos
h(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x +
 c)^4 + 4*(21*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*cosh(d*x
 + c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4
)*d)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*cosh(d*x + c) + 4*(9*(a*b^3 - b^4)*d
*cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2*b^2 - 11*a
*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x
 + c)^2 + (9*(a*b^3 - b^4)*d*cosh(d*x + c)^8 - 28*(a*b^3 - b^4)*d*cosh(d*x
+ c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 - 12*(a*b^3 -
b^4)*d*cosh(d*x + c)^2 + (a*b^3 - b^4)*d)*sinh(d*x + c))*sqrt(-((a^3*b^4 -
3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 -
6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*
b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)
/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-625*a^5 + 2625*a^4*b - 3
684*a^3*b^2 + 1728*a^2*b^3 - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^
2*b^3)*cosh(d*x + c)^2 - 2*(625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*
b^3)*cosh(d*x + c)*sinh(d*x + c) - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1
728*a^2*b^3)*sinh(d*x + c)^2 - 2*((125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4
- 336*a^2*b^5)*d*cosh(d*x + c) + (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 -
 336*a^2*b^5)*d*sinh(d*x + c) - 2*((2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11
*a*b^10 + 3*b^11)*d^3*cosh(d*x + c) + (2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 -
 11*a*b^10 + 3*b^11)*d^3*sinh(d*x + c))*sqrt((625*a^7 - 3450*a^6*b + 7161*a
^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11
- 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)))*sqrt(-((a^3*b^4 - 3*a
^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 662
4*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^1
2 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((
```

```
a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*cosh(d*x + c
)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a*b^3 - b^4)*d*sin
h(d*x + c)^9 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^7 + 4*(9*(a*b^3 - b^4)*d*cos
h(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^7 - 2*(8*a^2*b^2 - 11*a*b^3 +
 3*b^4)*d*cosh(d*x + c)^5 + 28*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (a*b^3
- b^4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x +
c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
d)*sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^3 + 2*(63*(a*b^3 - b^4
)*d*cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 5*(8*a^2*b^2 - 1
1*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a*b^3 - b^4)*d*c
osh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 5*(8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^3 + (a*b^3 -
 b^4)*d*cosh(d*x + c) + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^7 - 21*(a*b^3 -
b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3
 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a*b^3 - b^4)*d*co
sh(d*x + c)^8 - 28*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 10*(8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d*cosh(d*x + c)^4 - 12*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a*b^3
 - b^4)*d)*sinh(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*s
qrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a
^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 +
b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6
 - b^7)*d^2))*log(-625*a^5 + 2625*a^4*b - 3684*a^3*b^2 + 1728*a^2*b^3 - (62
5*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)^2 - 2*(625*
a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c)
 - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*sinh(d*x + c)^2 + 2
*((125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*cosh(d*x + c) +
 (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d*sinh(d*x + c) +
2*((2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*cosh(d*x +
 c) + (2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sinh(d*
x + c))*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3
*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*
a*b^14 + b^15)*d^4)))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt(
(625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b
^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15
)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b
^7)*d^2))) - ((a*b^3 - b^4)*d*cosh(d*x + c)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x
+ c)*sinh(d*x + c)^8 + (a*b^3 - b^4)*d*sinh(d*x + c)^9 - 4*(a*b^3 - b^4)*d*
cosh(d*x + c)^7 + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*s
inh(d*x + c)^7 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^5 + 28*(3
*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x
+ c)^6 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*
x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^5 - 4*(a*b^3 - b
^4)*d*cosh(d*x + c)^3 + 2*(63*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 70*(a*b^3 -
 b^4)*d*cosh(d*x + c)^3 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))
*sinh(d*x + c)^4 + 4*(21*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 35*(a*b^3 - b^4)
*d*cosh(d*x + c)^4 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (
a*b^3 - b^4)*d)*sinh(d*x + c)^3 + (a*b^3 - b^4)*d*cosh(d*x + c) + 4*(9*(a*b
^3 - b^4)*d*cosh(d*x + c)^7 - 21*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2
*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c
))*sinh(d*x + c)^2 + (9*(a*b^3 - b^4)*d*cosh(d*x + c)^8 - 28*(a*b^3 - b^4)*
d*cosh(d*x + c)^6 - 10*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 - 1
2*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + (a*b^3 - b^4)*d)*sinh(d*x + c))*sqrt(((
a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*
a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11
 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b -
 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-625*a^5 + 2625
*a^4*b - 3684*a^3*b^2 + 1728*a^2*b^3 - (625*a^5 - 2625*a^4*b + 3684*a^3*b^2
 - 1728*a^2*b^3)*cosh(d*x + c)^2 - 2*(625*a^5 - 2625*a^4*b + 3684*a^3*b^2 -
 1728*a^2*b^3)*cosh(d*x + c)*sinh(d*x + c) - (625*a^5 - 2625*a^4*b + 3684*a
```

```
^3*b^2 - 1728*a^2*b^3)*sinh(d*x + c)^2 - 2*((125*a^5*b^2 - 520*a^4*b^3 + 72
3*a^3*b^4 - 336*a^2*b^5)*d*cosh(d*x + c) + (125*a^5*b^2 - 520*a^4*b^3 + 723
*a^3*b^4 - 336*a^2*b^5)*d*sinh(d*x + c) + 2*((2*a^4*b^7 - 9*a^3*b^8 + 15*a^
2*b^9 - 11*a*b^10 + 3*b^11)*d^3*cosh(d*x + c) + (2*a^4*b^7 - 9*a^3*b^8 + 15
*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sinh(d*x + c))*sqrt((625*a^7 - 3450*a^6*
b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15
*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)))*sqrt(((a^3*
b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*
b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 2
0*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*
a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + 8*a*b - 8*b^2 + 16*(
5*(a*b - b^2)*cosh(d*x + c)^9 - 4*(2*a*b - 3*b^2)*cosh(d*x + c)^7 - 3*(20*a
^2 - 17*a*b + 2*b^2)*cosh(d*x + c)^5 - 2*(20*a^2 - 17*a*b + 2*b^2)*cosh(d*x
 + c)^3 - (2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 - b^4)*d*co
sh(d*x + c)^9 + 9*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^8 + (a*b^3 -
b^4)*d*sinh(d*x + c)^9 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^7 + 4*(9*(a*b^3 -
b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^7 - 2*(8*a^2*b^2 -
11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^5 + 28*(3*(a*b^3 - b^4)*d*cosh(d*x + c)^3
 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^6 + 2*(63*(a*b^3 - b^4)*d*c
osh(d*x + c)^4 - 42*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3
 + 3*b^4)*d)*sinh(d*x + c)^5 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^3 + 2*(63*(a
*b^3 - b^4)*d*cosh(d*x + c)^5 - 70*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 5*(8*a
^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(21*(a*b^3
- b^4)*d*cosh(d*x + c)^6 - 35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 5*(8*a^2*b^
2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^3
+ (a*b^3 - b^4)*d*cosh(d*x + c) + 4*(9*(a*b^3 - b^4)*d*cosh(d*x + c)^7 - 21
*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 5*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(
d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (9*(a*b^3 -
 b^4)*d*cosh(d*x + c)^8 - 28*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 10*(8*a^2*b^
2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 - 12*(a*b^3 - b^4)*d*cosh(d*x + c)^
2 + (a*b^3 - b^4)*d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.242 $\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4bd(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*A
rcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(Sqrt[a] + Sqrt
[b])^(3/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*
b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.370462, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1205, 1166, 205, 208}

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4bd(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*A
rcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(Sqrt[a] + Sqrt
[b])^(3/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*
b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx=-\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^2}\,dx,x,\cosh(c+dx)\right)}{d}$$

$$=-\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{4a(a-2b)-2a(3a-4b)x^2}{a-b+2bx^2-bx^4}\,d\ldots\right.}{8a(a-b)bd\ldots}$$

$$=-\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)}-\frac{\left(3a-\sqrt{a}\sqrt{b}-4b\right)\text{Subst}\left(\int \ldots\right.}{8(a\ldots}$$

$$=\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{a\,c\ldots}{4(a-b)bd\left(a\ldots\right.}$$

Mathematica [C] time = 0.663144, size = 737, normalized size = 3.51

$$-\frac{\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&,\frac{-6\#1^6a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\ldots\right)\ldots}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((-16*a*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c
 + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1
^4 - 4*b*#1^6 + b*#1^8 & , (3*a*c - 4*b*c + 3*a*d*x - 4*b*d*x + 6*a*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1] - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1] - 5*a*c*#1^2 + 12*b*c*#1^2 - 5*a*d*x*#1^2 + 12
*b*d*x*#1^2 - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 24*b*Log[-Cosh[(c + d*x)/2] - Sinh
[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 5*a*c*#
1^4 - 12*b*c*#1^4 + 5*a*d*x*#1^4 - 12*b*d*x*#1^4 + 10*a*Log[-Cosh[(c + d*x)
```

```
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4
 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^4 - 3*a*c*#1^6 + 4*b*c*#1^6 - 3*a*d*x*#1^6 + 4*b*
d*x*#1^6 - 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/
2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 + 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*
a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(32*(a - b)*b*d)
```

Maple [B] time = 0.184, size = 1200, normalized size = 5.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*ta
nh(1/2*d*x+1/2*c)^2+1)/(a-b)*tanh(1/2*d*x+1/2*c)^2*(a*b)^(1/2)-1/2/d/a/b/(t
anh(1/2*d*x+1/2*c)^4-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1
/2*c)^2+1)/(a-b)*tanh(1/2*d*x+1/2*c)^2*(a*b)^(1/2)-1/4/d/b/(tanh(1/2*d*x+1/
2*c)^4-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2+1)/(a-
b)*tanh(1/2*d*x+1/2*c)^2-1/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4-2*tanh(1/2*d*x+1/
2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)*(a*b)^(1/2)-1/4/d/b/(
tanh(1/2*d*x+1/2*c)^4-2*tanh(1/2*d*x+1/2*c)^2+4*(a*b)^(1/2)/a*tanh(1/2*d*x+
1/2*c)^2+1)/(a-b)+3/8/d*a/b^2/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(
2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a
*b)^(1/2)-1/2/d/b/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d
*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/
8/d*a/b/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^
2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/4/d/b^2/(tanh(1/2*d*x+
1/2*c)^4-4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)/(
a-b)*tanh(1/2*d*x+1/2*c)^2*(a*b)^(1/2)+1/2/d/a/b/(tanh(1/2*d*x+1/2*c)^4-4*(
a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)*tanh(1/
2*d*x+1/2*c)^2*(a*b)^(1/2)-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4-4*(a*b)^(1/2)/a*t
anh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)*tanh(1/2*d*x+1/2*c)^2
-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4-4*(a*b)^(1/2)/a*tanh(1/2*d*x+1/2*c)^2-2*tan
h(1/2*d*x+1/2*c)^2+1)/(a-b)+1/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4-4*(a*b)^(1/2)/
a*tanh(1/2*d*x+1/2*c)^2-2*tanh(1/2*d*x+1/2*c)^2+1)/(a-b)*(a*b)^(1/2)+3/8/d*
a/b^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2
*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/2/d/b/(a-b)
/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^
(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1/8/d*a/b/(a-b)/(-a*b-(a
*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a
)/(-a*b-(a*b)^(1/2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{ae^{(7\,dx+7\,c)} - 5\,ae^{(5\,dx+5\,c)} - 5\,ae^{(3\,dx+3\,c)} + ae^{(dx+c)}}{2\,(ab^2d - b^3d + (ab^2de^{(8\,c)} - b^3de^{(8\,c)})e^{(8\,dx)} - 4\,(ab^2de^{(6\,c)} - b^3de^{(6\,c)})e^{(6\,dx)} - 2\,(8\,a^2bde^{(4\,c)} - 11\,ab^2de^{(4\,c)} + 3\,b^3de^{(4\,c)})}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(a*e^(7*d*x + 7*c) - 5*a*e^(5*d*x + 5*c) - 5*a*e^(3*d*x + 3*c) + a*e^(
d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d*x) -
4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c) - 11*a
*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^3*d*e^
(2*c))*e^(2*d*x)) + 1/128*integrate(64*((3*a*e^(7*c) - 4*b*e^(7*c))*e^(7*d*
x) - (5*a*e^(5*c) - 12*b*e^(5*c))*e^(5*d*x) + (5*a*e^(3*c) - 12*b*e^(3*c))*
e^(3*d*x) - (3*a*e^c - 4*b*e^c)*e^(d*x))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^
3*e^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2
*b*e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c)
 - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.53825, size = 14307, normalized size = 68.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*cosh(d*x + c)^7 + 56*a*cosh(d*x + c)*sinh(d*x + c)^6 + 8*a*sinh(
d*x + c)^7 - 40*a*cosh(d*x + c)^5 + 8*(21*a*cosh(d*x + c)^2 - 5*a)*sinh(d*x
 + c)^5 + 40*(7*a*cosh(d*x + c)^3 - 5*a*cosh(d*x + c))*sinh(d*x + c)^4 - 40
*a*cosh(d*x + c)^3 + 40*(7*a*cosh(d*x + c)^4 - 10*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c)^3 + 8*(21*a*cosh(d*x + c)^5 - 50*a*cosh(d*x + c)^3 - 15*a*cosh(
d*x + c))*sinh(d*x + c)^2 - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b
^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(
a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*
b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x +
 c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 -
b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 -
4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 -
10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2
- b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2
 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*
cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 +
 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*s
qrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b +
1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9
 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16
*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-81*a^3 + 405*a^2*b
- 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x
 + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)*sinh(d
*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*sinh(d*x + c)^2 + 2*(2
*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*cosh(d*x + c) + 2*(9*a^3*b^
2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*sinh(d*x + c) - ((3*a^4*b^5 - 14*a^3*
b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*cosh(d*x + c) + (3*a^4*b^5 - 14*a^
3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*sinh(d*x + c))*sqrt((81*a^5 - 52
2*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 +
15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)))*sqrt(-((a^
3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*
b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3
*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a
^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)
^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh
(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh
```

```
(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*
b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 -
b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)
^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*s
inh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*c
osh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 +
 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*
d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*
((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a
^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a
^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5
*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*
a^2 - 15*a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-81
*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*b + 680*a*b^2 -
384*b^3)*cosh(d*x + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cos
h(d*x + c)*sinh(d*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*sinh(
d*x + c)^2 - 2*(2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*cosh(d*x +
 c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*sinh(d*x + c) - ((3*
a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*cosh(d*x + c) + (
3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*sinh(d*x + c))*
sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b
^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*
d^4)))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*
a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15
*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*
a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - ((a*b^2 - b^3
)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*
b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*
b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*
b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c
)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3
)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*
b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*
(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a
^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b
^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 1
1*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*
b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(
d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)
*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6
)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/(
(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 +
b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^
6)*d^2))*log(-81*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81*a^3 - 405*a^2*
b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)^2 - 2*(81*a^3 - 405*a^2*b + 680*a*b^
2 - 384*b^3)*cosh(d*x + c)*sinh(d*x + c) - (81*a^3 - 405*a^2*b + 680*a*b^2
- 384*b^3)*sinh(d*x + c)^2 + 2*(2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b
^5)*d*cosh(d*x + c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d*sinh
(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*c
osh(d*x + c) + (3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3
*sinh(d*x + c))*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 57
6*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6
*a*b^12 + b^13)*d^4)))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt
((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 -
 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)
) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)))
+ ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d
```

```
*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x +
c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c
)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)
*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (
8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x
 + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*
x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 -
 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d
*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2
 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3
- (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^3*b^3 - 3*a^2*b^4
+ 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3
 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^1
1 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4
 + 3*a*b^5 - b^6)*d^2))*log(-81*a^3 + 405*a^2*b - 680*a*b^2 + 384*b^3 - (81
*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)^2 - 2*(81*a^3 - 405*a
^2*b + 680*a*b^2 - 384*b^3)*cosh(d*x + c)*sinh(d*x + c) - (81*a^3 - 405*a^2
*b + 680*a*b^2 - 384*b^3)*sinh(d*x + c)^2 - 2*(2*(9*a^3*b^2 - 47*a^2*b^3 +
82*a*b^4 - 48*b^5)*d*cosh(d*x + c) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 -
 48*b^5)*d*sinh(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8
 + 5*b^9)*d^3*cosh(d*x + c) + (3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b
^8 + 5*b^9)*d^3*sinh(d*x + c))*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 13
92*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5
- b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b
^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^
12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5
 - b^6)*d^2))) + 8*a*cosh(d*x + c) + 8*(7*a*cosh(d*x + c)^6 - 25*a*cosh(d*x
 + c)^4 - 15*a*cosh(d*x + c)^2 + a)*sinh(d*x + c))/((a*b^2 - b^3)*d*cosh(d*
x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*
d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*
d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^
2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*
b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*
x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3
)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^
3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a
*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(
d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3
*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*
d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5
- (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x
 + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.243 $\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=217

$$-\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right.}{4bd(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c\right.}$$

```
[Out] -((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) - ((Sqrt[a] + 2*Sqrt[
b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(S
qrt[a] + Sqrt[b])^(3/2)*b^(5/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x
]^2))/(4*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.294387, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1205, 1166, 205, 208}

$$-\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right.}{4bd(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]
```


```
[Out] -((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) - ((Sqrt[a] + 2*Sqrt[
b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(S
qrt[a] + Sqrt[b])^(3/2)*b^(5/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c + d*x
]^2))/(4*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2a(a-3b)+2abx^2}{a-b+2bx^2-bx^4}\,dx, x\ldots\right.}{8a(a-b)bd}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{Subst}\left(\int \frac{\ldots}{-\sqrt{a}\sqrt{\ldots}}\right.}{8\sqrt{a}\left(\sqrt{a}-\ldots\right.}$$

$$= -\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d} - \frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/4}d} + \frac{\text{co}\ldots}{4(a-b)bd\ldots}$$

Mathematica [C] time = 0.551238, size = 597, normalized size = 2.75

$$\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{8\#1^4a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\ldots\right.\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2
*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 4*a*c*
#1^2 + 11*b*c*#1^2 - 4*a*d*x*#1^2 + 11*b*d*x*#1^2 - 8*a*Log[-Cosh[(c + d*x)
/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2
 + 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^2 + 4*a*c*#1^4 - 11*b*c*#1^4 + 4*a*d*x*#1^4 - 11*
b*d*x*#1^4 + 8*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x
)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 22*b*Log[-Cosh[(c + d*x)/2] - Sinh[(
```

```
c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^6
+ b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*
b*#1^5 + b*#1^7) & ])/(32*(a - b)*b*d)
```

Maple [B] time = 0.077, size = 1116, normalized size = 5.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*
x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*
tanh(1/2*d*x+1/2*c)^6+1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*
a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c
)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^6+3/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*t
anh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4
-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-3/
2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tan
h(1/2*d*x+1/2*c)^2-1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)*a/b/(a-b)+1/8/d/b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*a
rctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*
a)^(1/2))*(a*b)^(1/2)+1/8/d*a/b/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4
*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))
-1/4/d/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^
2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/8/d/b/(a-b)/(-a*b+(a*b
)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(
-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/8/d*a/b/(a-b)/(-a*b+(a*b)^(1/2)*a)
^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)
^(1/2)*a)^(1/2))+1/4/d/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(
1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(4\,ae^{(5\,c)}+be^{(5\,c)}\right)e^{(5\,dx)}+\left(4\,ae^{(3\,c)}+be^{(3\,c)}\right)e^{(3\,dx)}-be^{(7\,dx+7\,c)}-be^{(dx+c)}}{2\left(ab^{2}d-b^{3}d+\left(ab^{2}de^{(8\,c)}-b^{3}de^{(8\,c)}\right)e^{(8\,dx)}-4\left(ab^{2}de^{(6\,c)}-b^{3}de^{(6\,c)}\right)e^{(6\,dx)}-2\left(8\,a^{2}bde^{(4\,c)}-11\,ab^{2}de^{(4\,c)}+3\,b^{3}de^{(4\,c)}\right)e^{(4\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*e^(5*c) + b*e^(5*c))*e^(5*d*x) + (4*a*e^(3*c) + b*e^(3*c))*e^(3*
d*x) - b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*
c) - b^3*d*e^(8*c))*e^(8*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*
x) - 2*(8*a^2*b*d*e^(4*c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x)
 - 4*(a*b^2*d*e^(2*c) - b^3*d*e^(2*c))*e^(2*d*x)) + 1/32*integrate(16*((4*a
```

```
*e^(5*c) - 11*b*e^(5*c))*e^(5*d*x) - (4*a*e^(3*c) - 11*b*e^(3*c))*e^(3*d*x)
 + b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^3*e
^(8*c))*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*
e^(4*c) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c) -
b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.66042, size = 13905, normalized size = 64.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^7 + 56*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*b*sinh(d
*x + c)^7 - 8*(4*a + b)*cosh(d*x + c)^5 + 8*(21*b*cosh(d*x + c)^2 - 4*a - b
)*sinh(d*x + c)^5 + 40*(7*b*cosh(d*x + c)^3 - (4*a + b)*cosh(d*x + c))*sinh
(d*x + c)^4 - 8*(4*a + b)*cosh(d*x + c)^3 + 8*(35*b*cosh(d*x + c)^4 - 10*(4
*a + b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c)^3 + 8*(21*b*cosh(d*x + c)^
5 - 10*(4*a + b)*cosh(d*x + c)^3 - 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sin
h(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x
 + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x
+ c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b
^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 +
2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2
- (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(
d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh
(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^
4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sin
h(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*
b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)
^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b^2 - 3*a^3*b
^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 +
64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^
2*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b
^4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +
 28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)
*cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x
 + c)^2 + 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) +
(a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) - 2*((a^4*b^5 -
 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 +
3*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 8
0*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*
b^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5
 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*
d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))
) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh
(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x
+ c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x +
 c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^
3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 -
 (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d
*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(
```

d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4
- 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh
(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b
^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^
3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b^2 - 3*a^3*b^
3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 6
4*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2
*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +
28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*
cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x
+ c)^2 - 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) + (
a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) - 2*((a^4*b^5 -
3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + 3
*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80
*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b
^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a
*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5
- 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d
^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2)))
+ ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(
d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x +
c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x +
c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3
)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*
(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 -
(8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*
x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d
*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 +
4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4
- 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(
d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^
2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3
- (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b^2 - 3*a^3*b^
3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 6
4*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2
*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +
28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*
cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x
+ c)^2 + 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) + (
a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) + 2*((a^4*b^5 -
3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 + 3
*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80
*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b
^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5
- 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*
d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))
) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh
(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x
+ c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x +
c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^
3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2
(35(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 -
(8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d
x + c)^2 + 8(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(
d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3

```
+ 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4
 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh
(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b
^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^
3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b^2 - 3*a^3*b
^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 +
64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^
2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b
^4 - a*b^5)*d^2))*log(-a^3 + 9*a^2*b - 28*a*b^2 + 32*b^3 - (a^3 - 9*a^2*b +
 28*a*b^2 - 32*b^3)*cosh(d*x + c)^2 - 2*(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)
*cosh(d*x + c)*sinh(d*x + c) - (a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*sinh(d*x
 + c)^2 - 2*((a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*cosh(d*x + c) +
(a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d*sinh(d*x + c) + 2*((a^4*b^5 -
 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*cosh(d*x + c) + (a^4*b^5 - 3*a^3*b^6 +
3*a^2*b^7 - a*b^8)*d^3*sinh(d*x + c))*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 8
0*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*
b^9 - 6*a^2*b^10 + a*b^11)*d^4)))*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
 a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^
5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)
*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2)
)) + 8*b*cosh(d*x + c) + 8*(7*b*cosh(d*x + c)^6 - 5*(4*a + b)*cosh(d*x + c)
^4 - 3*(4*a + b)*cosh(d*x + c)^2 + b)*sinh(d*x + c))/((a*b^2 - b^3)*d*cosh(
d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3
)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3
)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*
b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(
a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(
d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b
^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 -
b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11
*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cos
h(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 +
 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3
)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^
5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d
*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.244 $\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

```
[Out] -ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a
] - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a
] + Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cosh[c + d
*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Co
sh[c + d*x]^4))
```

Rubi [A] time = 0.205211, antiderivative size = 186, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]
```


```
[Out] -ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a
] - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a
] + Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cosh[c + d
*x]*(2 - Cosh[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Co
sh[c + d*x]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-4ab+2abx^2}{a-b+2bx^2-bx^4}\,dx, x, \text{co}\right.}{8a(a-b)bd}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \text{co}\right.}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d} - \frac{\cosh(c+dx)\left(2-\cosh^2\right.}{4(a-b)d\left(a-b+2b\cosh^2(c+dx)\right.}$$

Mathematica [C] time = 0.485186, size = 422, normalized size = 2.27

$$-\frac{\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c\right.\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((16*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c +
 d*x)] + b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4
 - 4*b*#1^6 + b*#1^8 & , (-c - d*x - 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d
*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 7*c*#1^2 + 7*d*x*#1
^2 + 14*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
 Sinh[(c + d*x)/2]*#1]*#1^2 - 7*c*#1^4 - 7*d*x*#1^4 - 14*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
4 + c*#1^6 + d*x*#1^6 + 2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh
[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1
^3 - 3*b*#1^5 + b*#1^7) & ])/(32*(a - b)*d)
```

Maple [B] time = 0.058, size = 767, normalized size = 4.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh
(1/2*d*x+1/2*c)^6-3/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4+4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*
d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(
1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^4*b+5/2/d/(tanh(1/2*d*x+1
/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2
*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^2-1/2/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)+1/8/d/(a-
b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b
)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-1/8/d/b/(a-b)/(-a*b-(a*b)^(1/2)*a)
^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b
)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1/8/d/b/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arct
an(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(
1/2))*(a*b)^(1/2)-1/8/d/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh
(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{e^{(7\,dx+7\,c)} - 5\,e^{(5\,dx+5\,c)} - 5\,e^{(3\,dx+3\,c)} + e^{(dx+c)}}{2\,\left(abd - b^{2}d + \left(abde^{(8\,c)} - b^{2}de^{(8\,c)}\right)e^{(8\,dx)} - 4\,\left(abde^{(6\,c)} - b^{2}de^{(6\,c)}\right)e^{(6\,dx)} - 2\,\left(8\,a^{2}de^{(4\,c)} - 11\,abde^{(4\,c)} + 3\,b^{2}de^{(4\,c)}\right)e^{(4\,dx)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(e^(7*d*x + 7*c) - 5*e^(5*d*x + 5*c) - 5*e^(3*d*x + 3*c) + e^(d*x + c)
)/(a*b*d - b^2*d + (a*b*d*e^(8*c) - b^2*d*e^(8*c))*e^(8*d*x) - 4*(a*b*d*e^(
6*c) - b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*d*e^(4*c) - 11*a*b*d*e^(4*c) + 3
*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a*b*d*e^(2*c) - b^2*d*e^(2*c))*e^(2*d*x)) -
1/8*integrate(4*(e^(7*d*x + 7*c) - 7*e^(5*d*x + 5*c) + 7*e^(3*d*x + 3*c) -
e^(d*x + c))/(a*b - b^2 + (a*b*e^(8*c) - b^2*e^(8*c))*e^(8*d*x) - 4*(a*b*e^
(6*c) - b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c) + 3*b^2*
e^(4*c))*e^(4*d*x) - 4*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.36398, size = 11888, normalized size = 63.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*cosh(d*x + c)^7 + 56*cosh(d*x + c)*sinh(d*x + c)^6 + 8*sinh(d*x +
c)^7 + 8*(21*cosh(d*x + c)^2 - 5)*sinh(d*x + c)^5 - 40*cosh(d*x + c)^5 + 40
*(7*cosh(d*x + c)^3 - 5*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(7*cosh(d*x + c
)^4 - 10*cosh(d*x + c)^2 - 1)*sinh(d*x + c)^3 - 40*cosh(d*x + c)^3 + 8*(21*
cosh(d*x + c)^5 - 50*cosh(d*x + c)^3 - 15*cosh(d*x + c))*sinh(d*x + c)^2 -
((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c
)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(
7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2
 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 -
 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(
d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)
*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cos
h(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*
(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^
2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(
d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*c
osh(d*x + c)^3 - (a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 -
 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)
) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*c
osh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d
*x + c)^2 + 2*(2*(a^2*b + 3*a*b^2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*
sinh(d*x + c) - ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c
) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2
+ 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b
^7 - 6*a^2*b^8 + a*b^9)*d^4)))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^
4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a
^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d^2)) + a + 3*b) + ((a*b - b^2)*d*cosh(d*x + c)^8 +
 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c
)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2
- (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x +
c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*co
sh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)
*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*
cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3
 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 -
3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)
^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cos
h(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*c
osh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cosh(d*x + c)^2 + 2*(a + 3*b)*cos
h(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d*x + c)^2 - 2*(2*(a^2*b + 3*a*b^
2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*sinh(d*x + c) - ((a^5*b^2 - 2*a^
4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2
*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a
^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)))*s
qrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2
)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8
+ a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) +
a + 3*b) - ((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*s
inh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x +
 c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6
 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d
*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b
```

^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cosh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d*x + c)^2 + 2*(2*(a^2*b + 3*a*b^2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*sinh(d*x + c) + ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) + a + 3*b) + ((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cosh(d*x + c)^2 + 2*(a + 3*b)*cosh(d*x + c)*sinh(d*x + c) + (a + 3*b)*sinh(d*x + c)^2 - 2*(2*(a^2*b + 3*a*b^2)*d*cosh(d*x + c) + 2*(a^2*b + 3*a*b^2)*d*sinh(d*x + c) + ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*cosh(d*x + c) + (a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sinh(d*x + c))*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) + a + 3*b) + 8*(7*cosh(d*x + c)^6 - 25*cosh(d*x + c)^4 - 15*cosh(d*x + c)^2 + 1)*sinh(d*x + c) + 8*cosh(d*x + c))/((a*b - b^2)*d*cosh(d*x + c)^8 + 8*(a*b - b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*d*sinh(d*x + c)^8 - 4*(a*b - b^2)*d*cosh(d*x + c)^6 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^3 - 3*(a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b - b^2)*d*cosh(d*x + c)^4 - 30*(a*b - b^2)*d*cosh(d*x + c)^2 - (8*a^2 - 11*a*b + 3*b^2)*d)*sinh(d*x + c)^4 - 4*(a*b - b^2)*d*cosh(d*x + c)^2 + 8*(7*(a*b - b^2)*d*cosh(d*x + c)^5 - 10*(a*b - b^2)*d*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b - b^2)*d*cosh(d*x + c)^6 - 15*(a*b - b^2)*d*cosh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^2 - (a*b - b^2)*d)*sinh(d*x + c)^2 + (a*b - b^2)*d + 8*((a*b - b^2)*d*cosh(d*x + c)^7 - 3*(a*b - b^2)*d*cosh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*d*cosh(d*x + c)^3 - (a*b - b^2)*d*cosh(d*x + c))*sinh(d*x + c))

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.245 $\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=221

$$\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx\right.}{4ad(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh\right.}$$

```
[Out] ((3*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(1/4)*d) + ((3*Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(3/2)
*(Sqrt[a] + Sqrt[b])^(3/2)*b^(1/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c +
d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.298505, antiderivative size = 221, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3215, 1092, 1166, 205, 208}

$$\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\cosh(c+dx)\left(a-b\cosh^2(c+dx\right.}{4ad(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(1/4)*d) + ((3*Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(3/2)
*(Sqrt[a] + Sqrt[b])^(3/2)*b^(1/4)*d) + (Cosh[c + d*x]*(a + b - b*Cosh[c +
d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
```

```
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-2\left(4(a-b)b+4\right.}{a-b+2bx^2-bx^4}\right.}{8a(a-}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} - \frac{\left(\left(3\sqrt{a}-2\sqrt{b}\right)\sqrt{b}\right)\text{Subst}\left(\int\right.}{8a^{3/2}\left(\sqrt{a}\right.}$$

$$= \frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}d} + \frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt[4]{b}d} + \frac{\cosh}{4a(a-b)d\left(a\right.}$$

Mathematica [C] time = 0.35422, size = 597, normalized size = 2.7

$$\text{RootSum}\left[-16\#1^4a+\#1^8b-4\#1^6b+6\#1^4b-4\#1^2b+b\&, \frac{-24\#1^4a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}\right.\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((32*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2
*(c + d*x)] - b*Cosh[4*(c + d*x)]) + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b
*#1^4 - 4*b*#1^6 + b*#1^8 & , (-(b*c) - b*d*x - 2*b*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 12*a*c
*#1^2 - 5*b*c*#1^2 + 12*a*d*x*#1^2 - 5*b*d*x*#1^2 + 24*a*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^
2 - 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
- Sinh[(c + d*x)/2]*#1]*#1^2 - 12*a*c*#1^4 + 5*b*c*#1^4 - 12*a*d*x*#1^4 + 5
*b*d*x*#1^4 - 24*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh
[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + b*c*#1^
6 + b*d*x*#1^6 + 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c +
 d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
3*b*#1^5 + b*#1^7) & ])/(32*a*(a - b)*d)
```

Maple [B] time = 0.072, size = 1112, normalized size = 5.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh
(1/2*d*x+1/2*c)^6+1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)/(a-b)/a*tanh(1/2*d*x+1/2*c)^6*b+3/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4-4/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^4*b-3/
2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/
2*d*x+1/2*c)^2-1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tan
h(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a
)/a/(a-b)*tanh(1/2*d*x+1/2*c)^2*b+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)/(a-b)+1/8/d/(a-b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arcta
n(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(
1/2))*(a*b)^(1/2)-3/8/d/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tan
h(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+1/4/d/(
a-b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*
(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*b+1/8/d/(a-b)/a/(-a*b+(a*b)^(1
/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b
+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+3/8/d/(a-b)/(-a*b+(a*b)^(1/2)*a)^(1/2)*a
rctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a
)^(1/2))-1/4/d/(a-b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*
x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(4\,ae^{(5\,c)} + be^{(5\,c)}\right)e^{(5\,dx)} + \left(4\,ae^{(3\,c)} + be^{(3\,c)}\right)e^{(3\,dx)} - be^{(7\,dx+7\,c)} - be^{(dx+c)}}{2\left(a^2bd - ab^2d + \left(a^2bde^{(8\,c)} - ab^2de^{(8\,c)}\right)e^{(8\,dx)} - 4\left(a^2bde^{(6\,c)} - ab^2de^{(6\,c)}\right)e^{(6\,dx)} - 2\left(8\,a^3de^{(4\,c)} - 11\,a^2bde^{(4\,c)} + 3\,ab^2de^{(\cdots}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*e^(5*c) + b*e^(5*c))*e^(5*d*x) + (4*a*e^(3*c) + b*e^(3*c))*e^(3*
d*x) - b*e^(7*d*x + 7*c) - b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(
8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e
^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(
4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(-
((12*a*e^(5*c) - 5*b*e^(5*c))*e^(5*d*x) - (12*a*e^(3*c) - 5*b*e^(3*c))*e^(3
*d*x) - b*e^(7*d*x + 7*c) + b*e^(d*x + c))/(a^2*b - a*b^2 + (a^2*b*e^(8*c)
- a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(6*d*x) -
2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) - 4*(a^2*b
*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.73134, size = 13570, normalized size = 61.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^7 + 56*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*b*sinh(d
*x + c)^7 - 8*(4*a + b)*cosh(d*x + c)^5 + 8*(21*b*cosh(d*x + c)^2 - 4*a - b
)*sinh(d*x + c)^5 + 40*(7*b*cosh(d*x + c)^3 - (4*a + b)*cosh(d*x + c))*sinh
(d*x + c)^4 - 8*(4*a + b)*cosh(d*x + c)^3 + 8*(35*b*cosh(d*x + c)^4 - 10*(4
*a + b)*cosh(d*x + c)^2 - 4*a - b)*sinh(d*x + c)^3 + 8*(21*b*cosh(d*x + c)^
5 - 10*(4*a + b)*cosh(d*x + c)^3 - 3*(4*a + b)*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)
*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*
cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*
d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(
7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2
)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(
a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 -
 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a
^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x
 + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*
b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3
 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))
*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a
^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^
5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5
*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c
)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (81*a^2 -
81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 + 2*((27*a^4 -
24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*s
inh(d*x + c) - 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d
^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*
d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15
*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*sqrt(-((a^
6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^
9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^
7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*
d^2))) - ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2
)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b
^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 +
 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a
*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 -
 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)
^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 1
5*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh
(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((
a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x +
 c))*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((
81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 1
```

```
5*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3
*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cosh(d*x
 + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (81*a^
2 - 81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 - 2*((27*a^
4 - 24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)
*d*sinh(d*x + c) - 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^
5)*d^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b
^5)*d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2
+ 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*sqrt(-
((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/
((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^
3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b
^3)*d^2))) + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(
d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a
*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b -
 a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)
^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x
+ c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b
 - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)
^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x
+ c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6
 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*
cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d +
8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5
- (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d
*x + c))*sinh(d*x + c))*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqr
t((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4
+ 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6
- 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cosh(
d*x + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (81
*a^2 - 81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 + 2*((27
*a^4 - 24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2*b
^2)*d*sinh(d*x + c) + 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3
*b^5)*d^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^
3*b^5)*d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b
^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*sqr
t(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2
)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 +
a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3
*b^3)*d^2))) - ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cos
h(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b
 - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x +
c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*
x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2
*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x +
c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*
x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)
^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*
d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d
+ 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^
5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh
(d*x + c))*sinh(d*x + c))*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*s
qrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^
4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^
6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cos
h(d*x + c)^2 + 2*(81*a^2 - 81*a*b + 20*b^2)*cosh(d*x + c)*sinh(d*x + c) + (
```

```
81*a^2 - 81*a*b + 20*b^2)*sinh(d*x + c)^2 + 81*a^2 - 81*a*b + 20*b^2 - 2*((
27*a^4 - 24*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c) + (27*a^4 - 24*a^3*b + 5*a^2
*b^2)*d*sinh(d*x + c) + 2*((2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a
^3*b^5)*d^3*cosh(d*x + c) + (2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 +
a^3*b^5)*d^3*sinh(d*x + c))*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8
*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)))*s
qrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b
^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6
+ a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a
^3*b^3)*d^2))) + 8*b*cosh(d*x + c) + 8*(7*b*cosh(d*x + c)^6 - 5*(4*a + b)*c
osh(d*x + c)^4 - 3*(4*a + b)*cosh(d*x + c)^2 + b)*sinh(d*x + c))/((a^2*b -
a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^
7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6
 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c
)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^
2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 +
 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x +
c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*
d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*
b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh
(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d
*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*
b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*co
sh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3
*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.246 $\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\tan$$

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(3/
2)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/S
qrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c
 + d*x]]/(a^2*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1
/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]]
*d) - (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*C
osh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.361405, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\tan$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(3/
2)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/S
qrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c
 + d*x]]/(a^2*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1
/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]]
*d) - (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*C
osh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(-\frac{1}{a^2(-1+x^2)} + \frac{b-bx^2}{a\left(a-b+2bx^2-bx^4\right)^2} + \frac{b-bx^2}{a^2\left(a-b+2bx^2-bx^4\right)}\right) dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{a^2 d} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{a^2 d} - \ldots$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} - \frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)} + \ldots$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \ldots$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^2 d} + \ldots$$

Mathematica [C] time = 0.837766, size = 761, normalized size = 2.34

$$-\frac{b\operatorname{RootSum}\left[-16\#1^4 a+\#1^8 b-4\#1^6 b+6\#1^4 b-4\#1^2 b+b\&, \frac{10\#1^6 a\log\left(-\#1\sinh\left(\frac{1}{2}(c+dx)\right)+\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\sinh\left(\frac{1}{2}(c+dx)\right)-\cosh\left(\frac{1}{2}(c+dx)\right)\right)-38\#1^4 a\log\left(-\#1\sinh\left(\frac{1}{2}(c\ldots\right.\right.}{\ldots}\right.}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/((a - b)*(8*a - 3*b + 4*b*
Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + 32*Log[Tanh[(c + d*x)/2]] - (b*
RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-5*a*c
 + 4*b*c - 5*a*d*x + 4*b*d*x - 10*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 8*b*Log[-Cosh[(c + d*x
)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 1
9*a*c*#1^2 - 12*b*c*#1^2 + 19*a*d*x*#1^2 - 12*b*d*x*#1^2 + 38*a*Log[-Cosh[(
c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*
#1]*#1^2 - 24*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)
/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 19*a*c*#1^4 + 12*b*c*#1^4 - 19*a*d*x*
#1^4 + 12*b*d*x*#1^4 - 38*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Co
sh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 24*b*Log[-Cosh[(c + d*x)/
2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4
+ 5*a*c*#1^6 - 4*b*c*#1^6 + 5*a*d*x*#1^6 - 4*b*d*x*#1^6 + 10*a*Log[-Cosh[(c
 + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^6 - 8*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^
5 + b*#1^7) & ])/(a - b))/(32*a^2*d)
```

Maple [B] time = 0.096, size = 966, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)/a*ta
nh(1/2*d*x+1/2*c)^6*b-3/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^4*b+4/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)
^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+
1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^4+5/2/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x
+1/2*c)^2*b-1/2/d*b/a/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)/(a-b)-5/8/d/(a-b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2
*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+
1/2/d*b/a^2/(a-b)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/
2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1/8/d/(
a-b)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*
(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*b-5/8/d/(a-b)/a/(-a*b+(a*b)^(1
/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b
+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+1/2/d*b/a^2/(a-b)/(-a*b+(a*b)^(1/2)*a)^(
1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(
1/2)*a)^(1/2))*(a*b)^(1/2)-1/8/d/(a-b)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(
1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2
))*b+1/d/a^2*ln(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{be^{(7\,dx+7\,c)} - 5\,be^{(5\,dx+5\,c)} - 5\,be^{(3\,dx+3\,c)} + be^{(dx+c)}}{2\left(a^2bd - ab^2d + \left(a^2bde^{(8\,c)} - ab^2de^{(8\,c)}\right)e^{(8\,dx)} - 4\left(a^2bde^{(6\,c)} - ab^2de^{(6\,c)}\right)e^{(6\,dx)} - 2\left(8\,a^3de^{(4\,c)} - 11\,a^2bde^{(4\,c)} + 3\,ab^2de^{(4\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(b*e^(7*d*x + 7*c) - 5*b*e^(5*d*x + 5*c) - 5*b*e^(3*d*x + 3*c) + b*e^(
d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x
) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) -
11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*
b^2*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^2*d) + log((e^
(d*x + c) - 1)*e^(-c))/(a^2*d) - 2*integrate(1/4*((5*a*b*e^(7*c) - 4*b^2*e^
(7*c))*e^(7*d*x) - (19*a*b*e^(5*c) - 12*b^2*e^(5*c))*e^(5*d*x) + (19*a*b*e^
(3*c) - 12*b^2*e^(3*c))*e^(3*d*x) - (5*a*b*e^c - 4*b^2*e^c)*e^(d*x))/(a^3*b
 - a^2*b^2 + (a^3*b*e^(8*c) - a^2*b^2*e^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c)
 - a^2*b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^4*e^(4*c) - 11*a^3*b*e^(4*c) + 3*a^2
*b^2*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x
)
```

Fricas [B] time = 4.22233, size = 17785, normalized size = 54.72

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*a*b*cosh(d*x + c)^7 + 56*a*b*cosh(d*x + c)*sinh(d*x + c)^6 + 8*a*b
*sinh(d*x + c)^7 - 40*a*b*cosh(d*x + c)^5 + 8*(21*a*b*cosh(d*x + c)^2 - 5*a
*b)*sinh(d*x + c)^5 - 40*a*b*cosh(d*x + c)^3 + 40*(7*a*b*cosh(d*x + c)^3 -
5*a*b*cosh(d*x + c))*sinh(d*x + c)^4 + 40*(7*a*b*cosh(d*x + c)^4 - 10*a*b*c
osh(d*x + c)^2 - a*b)*sinh(d*x + c)^3 + 8*a*b*cosh(d*x + c) + 8*(21*a*b*cos
h(d*x + c)^5 - 50*a*b*cosh(d*x + c)^3 - 15*a*b*cosh(d*x + c))*sinh(d*x + c)
^2 + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*cosh(d*x
+ c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a^3*b - a^2
*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b
 - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*
x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b - a^2*b^2)*d
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4
 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*
d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (8*a^
4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 3*(8*
a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d
*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 -
3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*co
sh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a
^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 12
41*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*
b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^
3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b + 1125*a^2*b
^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)
*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cosh(
d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*s
inh(d*x + c)^2 + 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*
cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d*sinh
(d*x + c) - ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*
cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d
^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4
 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a
^8*b^5 + a^7*b^6)*d^4)))*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*s
qrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 -
 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d
^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)
*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*cosh
(d*x + c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a^3*b
- a^2*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (
a^3*b - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*co
sh(d*x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b - a^2*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^4 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*
b^2)*d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^3
*b - a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 -
(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3
*b - a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 -
3*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*s
inh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x + c)
```

^7 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)
*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt
(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2
 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 +
16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b + 1125*
a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128
*b^4)*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*
cosh(d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b
^4)*sinh(d*x + c)^2 - 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^
4)*d*cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d
*sinh(d*x + c) - ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)
*d^3*cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b
^4)*d^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
 - 6*a^8*b^5 + a^7*b^6)*d^4)))*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*
d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a
^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b
^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4
*b^3)*d^2))) + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d
*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a
^3*b - a^2*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^
2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)
*d*cosh(d*x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b -
a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(
d*x + c)^4 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3
*a^2*b^2)*d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)
^3 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)
^4 - 3*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)
*d)*sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x
 + c)^7 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2
*b^2)*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3
*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 -
 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^
2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b + 1
125*a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 -
 128*b^4)*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b
^4)*cosh(d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 1
28*b^4)*sinh(d*x + c)^2 + 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^
2*b^4)*d*cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^
4)*d*sinh(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*
b^4)*d^3*cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a
^6*b^4)*d^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 -
464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*
b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^
3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/
((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^
7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2
)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x +
c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b
^2)*d*cosh(d*x + c)^4 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b
 - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*co
sh(d*x + c)^4 - 30*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b
+ 3*a^2*b^2)*d)*sinh(d*x + c)^4 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8

```
*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^3 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^4 - 3*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b
^2)*d)*sinh(d*x + c)^2 + (a^3*b - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(
d*x + c)^7 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*
a^2*b^2)*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x +
c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*
a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^
2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a
*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-625*a^3*b
+ 1125*a^2*b^2 - 664*a*b^3 + 128*b^4 - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^
3 - 128*b^4)*cosh(d*x + c)^2 - 2*(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 12
8*b^4)*cosh(d*x + c)*sinh(d*x + c) - (625*a^3*b - 1125*a^2*b^2 + 664*a*b^3
- 128*b^4)*sinh(d*x + c)^2 - 2*(2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16
*a^2*b^4)*d*cosh(d*x + c) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2
*b^4)*d*sinh(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a
^6*b^4)*d^3*cosh(d*x + c) + (5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 +
3*a^6*b^4)*d^3*sinh(d*x + c))*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3
 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a
^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4
*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^
5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 +
 a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2
 - a^4*b^3)*d^2))) + 16*((a*b - b^2)*cosh(d*x + c)^8 + 8*(a*b - b^2)*cosh(d
*x + c)*sinh(d*x + c)^7 + (a*b - b^2)*sinh(d*x + c)^8 - 4*(a*b - b^2)*cosh(
d*x + c)^6 + 4*(7*(a*b - b^2)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c)^6
+ 8*(7*(a*b - b^2)*cosh(d*x + c)^3 - 3*(a*b - b^2)*cosh(d*x + c))*sinh(d*x
+ c)^5 - 2*(8*a^2 - 11*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a*b - b^2)*cos
h(d*x + c)^4 - 30*(a*b - b^2)*cosh(d*x + c)^2 - 8*a^2 + 11*a*b - 3*b^2)*sin
h(d*x + c)^4 + 8*(7*(a*b - b^2)*cosh(d*x + c)^5 - 10*(a*b - b^2)*cosh(d*x +
 c)^3 - (8*a^2 - 11*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b -
b^2)*cosh(d*x + c)^2 + 4*(7*(a*b - b^2)*cosh(d*x + c)^6 - 15*(a*b - b^2)*co
sh(d*x + c)^4 - 3*(8*a^2 - 11*a*b + 3*b^2)*cosh(d*x + c)^2 - a*b + b^2)*sin
h(d*x + c)^2 + a*b - b^2 + 8*((a*b - b^2)*cosh(d*x + c)^7 - 3*(a*b - b^2)*c
osh(d*x + c)^5 - (8*a^2 - 11*a*b + 3*b^2)*cosh(d*x + c)^3 - (a*b - b^2)*cos
h(d*x + c))*sinh(d*x + c))*log(cosh(d*x + c) + sinh(d*x + c) + 1) - 16*((a*
b - b^2)*cosh(d*x + c)^8 + 8*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a
*b - b^2)*sinh(d*x + c)^8 - 4*(a*b - b^2)*cosh(d*x + c)^6 + 4*(7*(a*b - b^2
)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c)^6 + 8*(7*(a*b - b^2)*cosh(d*x
+ c)^3 - 3*(a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 2*(8*a^2 - 11*a*b +
 3*b^2)*cosh(d*x + c)^4 + 2*(35*(a*b - b^2)*cosh(d*x + c)^4 - 30*(a*b - b^2
)*cosh(d*x + c)^2 - 8*a^2 + 11*a*b - 3*b^2)*sinh(d*x + c)^4 + 8*(7*(a*b - b
^2)*cosh(d*x + c)^5 - 10*(a*b - b^2)*cosh(d*x + c)^3 - (8*a^2 - 11*a*b + 3*
b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(a*b - b^2)*cosh(d*x + c)^2 + 4*(7*
(a*b - b^2)*cosh(d*x + c)^6 - 15*(a*b - b^2)*cosh(d*x + c)^4 - 3*(8*a^2 - 1
1*a*b + 3*b^2)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c)^2 + a*b - b^2 + 8
*((a*b - b^2)*cosh(d*x + c)^7 - 3*(a*b - b^2)*cosh(d*x + c)^5 - (8*a^2 - 11
*a*b + 3*b^2)*cosh(d*x + c)^3 - (a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*l
og(cosh(d*x + c) + sinh(d*x + c) - 1) + 8*(7*a*b*cosh(d*x + c)^6 - 25*a*b*c
osh(d*x + c)^4 - 15*a*b*cosh(d*x + c)^2 + a*b)*sinh(d*x + c))/((a^3*b - a^2
*b^2)*d*cosh(d*x + c)^8 + 8*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)
^7 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^8 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^6 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sin
h(d*x + c)^6 - 2*(8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a
^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 30*(a^3*b - a^2
*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d)*sinh(d*x + c)^4
 - 4*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^3*b - a^2*b^2)*d*cosh(d*
```

```
x + c)^5 - 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (8*a^4 - 11*a^3*b + 3*a
^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3*b - a^2*b^2)*d*cosh(d*
x + c)^6 - 15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^4 - 11*a^3*b + 3
*a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^2 + (a^3*b
 - a^2*b^2)*d + 8*((a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 3*(a^3*b - a^2*b^2
)*d*cosh(d*x + c)^5 - (8*a^4 - 11*a^3*b + 3*a^2*b^2)*d*cosh(d*x + c)^3 - (a
^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.247 $\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=320

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\sqrt{\sqrt{a}\cdots\right.}{2b^2d\sqrt{\sqrt{a}\cdots}}$$

```
[Out] x/b^2 - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1
/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[
a] + Sqrt[b]]*b^2*d) - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tanh[c + d*x]/(4*(
a - b)*b*d) + Tanh[c + d*x]^5/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tan
h[c + d*x]^4))
```

Rubi [A] time = 0.461176, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3217, 1313, 1275, 12, 1122, 1166, 208, 1287, 207}

$$\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tanh^{-1}\left(\sqrt{\sqrt{a}\cdots\right.}{2b^2d\sqrt{\sqrt{a}\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] x/b^2 - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1
/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[
a] + Sqrt[b]]*b^2*d) - (a^(1/4)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tanh[c + d*x]/(4*(
a - b)*b*d) + Tanh[c + d*x]^5/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tan
h[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
```

```
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1-x^2)\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a-ax^2)}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{x^4}{(1-x^2)\left(a-2ax^2+(a-b)x^4\right)}\,dx, x, \text{t}\right.}{bd}$$

$$= \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2abx^4}{a-2ax^2+(a-b)x^4}\,dx, x, \text{t}\right.}{8ab^2d}$$

$$= \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \tanh(c+\right.}{b^2d}$$

$$= \frac{x}{b^2} - \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\tanh^5(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\left(\sqrt{a}\left(\sqrt{a}\right.\right.}{}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^2d} - \frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}b^2d} - \frac{\tanh(c+dx)}{4(a-b)bd}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}b^2d} + \frac{\sqrt[4]{a}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\sqrt[4]{a}\tanh^{-1}\left(\right.}{2\sqrt{\sqrt{a}\ldots}}$$

Mathematica [A] time = 4.6367, size = 262, normalized size = 0.82

$$\frac{-\frac{\sqrt{a}\left(4\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{a}\left(4\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{2ab(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{(a-b)(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)} + 8(c\ldots}{8b^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (8*(c + d*x) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b])
*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a +
 Sqrt[a]*Sqrt[b]]) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTanh[((Sqrt[a] + S
qrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqr
t[a + Sqrt[a]*Sqrt[b]]) + (2*a*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)])
)/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])))/(8*b
^2*d)
```

Maple [C] time = 0.063, size = 574, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh
(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*
tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^7+5/2/d*a/b/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5
+5/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*
x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*
tanh(1/2*d*x+1/2*c)^3-1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)+1/16/d*a/b^2/(a-b)*sum(((4*a-5*b)*
_R^6+(-12*a+19*b)*_R^4+(12*a-19*b)*_R^2-4*a+5*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-
8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16
*b)*_Z^4-4*a*_Z^2+a))-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(a*b*d*e^(8*c) - b^2*d*e^(8*c))*x*e^(8*d*x) + a*b + 2*(a*b*d - b^2*d
)*x + (a*b*e^(6*c) - 8*(a*b*d*e^(6*c) - b^2*d*e^(6*c))*x)*e^(6*d*x) - (8*a^
2*e^(4*c) - 3*a*b*e^(4*c) + 4*(8*a^2*d*e^(4*c) - 11*a*b*d*e^(4*c) + 3*b^2*d
*e^(4*c))*x)*e^(4*d*x) - (5*a*b*e^(2*c) + 8*(a*b*d*e^(2*c) - b^2*d*e^(2*c))
*x)*e^(2*d*x))/(a*b^3*d - b^4*d + (a*b^3*d*e^(8*c) - b^4*d*e^(8*c))*e^(8*d*
x) - 4*(a*b^3*d*e^(6*c) - b^4*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b^2*d*e^(4*c)
 - 11*a*b^3*d*e^(4*c) + 3*b^4*d*e^(4*c))*e^(4*d*x) - 4*(a*b^3*d*e^(2*c) - b
^4*d*e^(2*c))*e^(2*d*x)) + 1/256*integrate(256*(a*b*e^(6*d*x + 6*c) + a*b*e
^(2*d*x + 2*c) + 2*(8*a^2*e^(4*c) - 11*a*b*e^(4*c))*e^(4*d*x))/(a*b^3 - b^4
 + (a*b^3*e^(8*c) - b^4*e^(8*c))*e^(8*d*x) - 4*(a*b^3*e^(6*c) - b^4*e^(6*c)
)*e^(6*d*x) - 2*(8*a^2*b^2*e^(4*c) - 11*a*b^3*e^(4*c) + 3*b^4*e^(4*c))*e^(4
*d*x) - 4*(a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.8798, size = 15757, normalized size = 49.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(16*(a*b - b^2)*d*x*cosh(d*x + c)^8 + 128*(a*b - b^2)*d*x*cosh(d*x + c
)*sinh(d*x + c)^7 + 16*(a*b - b^2)*d*x*sinh(d*x + c)^8 - 8*(8*(a*b - b^2)*d
*x - a*b)*cosh(d*x + c)^6 + 8*(56*(a*b - b^2)*d*x*cosh(d*x + c)^2 - 8*(a*b
- b^2)*d*x + a*b)*sinh(d*x + c)^6 + 16*(56*(a*b - b^2)*d*x*cosh(d*x + c)^3
- 3*(8*(a*b - b^2)*d*x - a*b)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(4*(8*a^2
- 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c)^4 + 8*(140*(a*b - b^2)
*d*x*cosh(d*x + c)^4 - 4*(8*a^2 - 11*a*b + 3*b^2)*d*x - 15*(8*(a*b - b^2)*d
*x - a*b)*cosh(d*x + c)^2 - 8*a^2 + 3*a*b)*sinh(d*x + c)^4 + 32*(28*(a*b -
b^2)*d*x*cosh(d*x + c)^5 - 5*(8*(a*b - b^2)*d*x - a*b)*cosh(d*x + c)^3 - (4
*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c))*sinh(d*x + c)
^3 + 16*(a*b - b^2)*d*x - 8*(8*(a*b - b^2)*d*x + 5*a*b)*cosh(d*x + c)^2 + 8
*(56*(a*b - b^2)*d*x*cosh(d*x + c)^6 - 15*(8*(a*b - b^2)*d*x - a*b)*cosh(d*
```

x + c)^4 - 8*(a*b - b^2)*d*x - 6*(4*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x + c)^2 - 5*a*b)*sinh(d*x + c)^2 + ((a*b^3 - b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)*sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*sinh(d*x + c)^2 + 2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - ((a*b^3 - b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)*sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*sinh(d*x + c)^2 - 2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^

4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - ((a*b^3 - b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)*sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*sinh(d*x + c)^2 + 2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2 + 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 - (a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3 - (128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cosh(d*x + c)^2 - 2*(128*a^3 - 664*a^2*b + 1125*

```
a*b^2 - 625*b^3)*cosh(d*x + c)*sinh(d*x + c) - (128*a^3 - 664*a^2*b + 1125*
a*b^2 - 625*b^3)*sinh(d*x + c)^2 - 2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7
 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2
*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2
*b^11 - 6*a*b^12 + b^13)*d^4)) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 12
5*b^5)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 46
4*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 +
15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 -
47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + 8*a*b
+ 16*(8*(a*b - b^2)*d*x*cosh(d*x + c)^7 - 3*(8*(a*b - b^2)*d*x - a*b)*cosh(
d*x + c)^5 - 2*(4*(8*a^2 - 11*a*b + 3*b^2)*d*x + 8*a^2 - 3*a*b)*cosh(d*x +
c)^3 - (8*(a*b - b^2)*d*x + 5*a*b)*cosh(d*x + c))*sinh(d*x + c))/((a*b^3 -
b^4)*d*cosh(d*x + c)^8 + 8*(a*b^3 - b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 +
(a*b^3 - b^4)*d*sinh(d*x + c)^8 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^6 + 4*(7*
(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x + c)^6 - 2*(8*a
^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a*b^3 - b^4)*d*cosh(d*
x + c)^3 - 3*(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^3
- b^4)*d*cosh(d*x + c)^4 - 30*(a*b^3 - b^4)*d*cosh(d*x + c)^2 - (8*a^2*b^2
- 11*a*b^3 + 3*b^4)*d)*sinh(d*x + c)^4 - 4*(a*b^3 - b^4)*d*cosh(d*x + c)^2
+ 8*(7*(a*b^3 - b^4)*d*cosh(d*x + c)^5 - 10*(a*b^3 - b^4)*d*cosh(d*x + c)^3
 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(
a*b^3 - b^4)*d*cosh(d*x + c)^6 - 15*(a*b^3 - b^4)*d*cosh(d*x + c)^4 - 3*(8*
a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^2 - (a*b^3 - b^4)*d)*sinh(d*x +
 c)^2 + (a*b^3 - b^4)*d + 8*((a*b^3 - b^4)*d*cosh(d*x + c)^7 - 3*(a*b^3 - b
^4)*d*cosh(d*x + c)^5 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cosh(d*x + c)^3 -
(a*b^3 - b^4)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.19348, size = 201, normalized size = 0.63

$$\frac{abe^{(6\,dx+6\,c)} - 8\,a^2e^{(4\,dx+4\,c)} + 3\,abe^{(4\,dx+4\,c)} - 5\,abe^{(2\,dx+2\,c)} + ab}{2\left(ab^2d - b^3d\right)\left(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b\right)} + \frac{dx + c}{b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a*b*e^(6*d*x + 6*c) - 8*a^2*e^(4*d*x + 4*c) + 3*a*b*e^(4*d*x + 4*c) -
5*a*b*e^(2*d*x + 2*c) + a*b)/((a*b^2*d - b^3*d)*(b*e^(8*d*x + 8*c) - 4*b*e^
(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x +
 2*c) + b)) + (d*x + c)/(b^2*d)
```

3.248 $\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=233

$$\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)}{4bd(a-b)}+\frac{\tanh}{4bd\left((a-b)\tanh\right.}$$

```
[Out] ((2*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - ((2*Sqrt[a] + 3*S
qrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(1/4
)*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) + Tanh[c + d*x]/(4*(a - b)*b*d) + (S
ech[c + d*x]^2*Tanh[c + d*x]^3)/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*T
anh[c + d*x]^4))
```

Rubi [A] time = 0.337798, antiderivative size = 233, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1120, 1279, 1166, 208}

$$\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)}{4bd(a-b)}+\frac{\tanh}{4bd\left((a-b)\tanh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((2*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - ((2*Sqrt[a] + 3*S
qrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(1/4
)*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) + Tanh[c + d*x]/(4*(a - b)*b*d) + (S
ech[c + d*x]^2*Tanh[c + d*x]^3)/(4*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*T
anh[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$= \frac{\text{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^2\left(6a-2ax^2\right)}{a-2ax^2+(a-b)x^4}\,dx,x,\text{ta}\ldots\right.}{8abd}$$

$$= \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\text{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2a^2-\ldots}{a-2ax\ldots}\right.}{\ldots}$$

$$= \frac{\tanh(c+dx)}{4(a-b)bd} + \frac{\text{sech}^2(c+dx)\tanh^3(c+dx)}{4bd\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(a-\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}-\ldots\right.}{\ldots}$$

$$= \frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d} - \ldots$$

Mathematica [A] time = 2.58959, size = 238, normalized size = 1.02

$$\frac{\frac{\sqrt{b}\left(-\sqrt{a}\sqrt{b}-2a+3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{b}\left(\sqrt{a}\sqrt{b}-2a+3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} - \frac{4b\sinh(2(c+dx))(-2a+b\cosh(2(c+dx))-b)}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}}{8b^2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[b]*(-2*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c
 + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]
*(-2*a - Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])
```

```
/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (4*b*(-2*a - b + b
*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] -
 b*Cosh[4*(c + d*x)]))/(8*(a - b)*b^2*d)
```

Maple [C] time = 0.054, size = 716, normalized size = 3.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x
+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*t
anh(1/2*d*x+1/2*c)^7-1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5-1/2/d*a/b/(tanh(1/
2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*t
anh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)
^3-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh
(1/2*d*x+1/2*c)^3+1/2/d*a/b/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/16/d/b/(a-b)*sum((-_R^6*a+(-5*a+12*b
)*_R^4+(5*a-12*b)*_R^2+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(
1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,ae^{(6\,c)} - be^{(6\,c)}\right)e^{(6\,dx)} - \left(8\,ae^{(4\,c)} - 3\,be^{(4\,c)}\right)e^{(4\,dx)} - \left(2\,ae^{(2\,c)} + 3\,be^{(2\,c)}\right)e^{(2\,dx)} + b}{2\left(ab^2d - b^3d + \left(ab^2de^{(8\,c)} - b^3de^{(8\,c)}\right)e^{(8\,dx)} - 4\left(ab^2de^{(6\,c)} - b^3de^{(6\,c)}\right)e^{(6\,dx)} - 2\left(8\,a^2bde^{(4\,c)} - 11\,ab^2de^{(4\,c)} + 3\,b^3de^{(4\,c)}\right)e^{(4}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - (8*a*e^(4*c) - 3*b*e^(4*c))*e^(
4*d*x) - (2*a*e^(2*c) + 3*b*e^(2*c))*e^(2*d*x) + b)/(a*b^2*d - b^3*d + (a*b
^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c
))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c) - 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))
*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^3*d*e^(2*c))*e^(2*d*x)) + 1/64*integrat
e(64*((2*a*e^(6*c) - 3*b*e^(6*c))*e^(6*d*x) + (2*a*e^(2*c) - 3*b*e^(2*c))*e
^(2*d*x) + 6*b*e^(4*d*x + 4*c))/(a*b^2 - b^3 + (a*b^2*e^(8*c) - b^3*e^(8*c)
)*e^(8*d*x) - 4*(a*b^2*e^(6*c) - b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*e^(4*c
) - 11*a*b^2*e^(4*c) + 3*b^3*e^(4*c))*e^(4*d*x) - 4*(a*b^2*e^(2*c) - b^3*e^
(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.5546, size = 13423, normalized size = 57.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(2*a - b)*cosh(d*x + c)^6 + 48*(2*a - b)*cosh(d*x + c)*sinh(d*x +
c)^5 + 8*(2*a - b)*sinh(d*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*
(2*a - b)*cosh(d*x + c)^2 - 8*a + 3*b)*sinh(d*x + c)^4 + 32*(5*(2*a - b)*co
sh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(2*a + 3*b)*
cosh(d*x + c)^2 + 8*(15*(2*a - b)*cosh(d*x + c)^4 - 6*(8*a - 3*b)*cosh(d*x
+ c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*
(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x +
c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*
cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30
*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x
 + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x
 + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 -
15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(
d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2
 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b -
11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d
*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90
*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7
 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^
4 + 3*a*b^5 - b^6)*d^2))*log(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*
b^4 + 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 +
15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 -
 81*a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x +
 c)*sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 8
1*a*b - 81*b^2 + 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^
7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 2*(5*a^3*b - 19*a^2*b
^2 + 18*a*b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25
*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 1
5*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(
a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c
)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c
)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*c
osh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*c
osh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*
(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x
+ c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x
+ c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 1
5*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d
*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2
- b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 1
1*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*
x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*
a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7
- 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4
 + 3*a*b^5 - b^6)*d^2))*log(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b
^4 + 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 -
81*a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x +
c)*sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 81
```

*a*b - 81*b^2 - 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7
)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 2*(5*a^3*b - 19*a^2*b^
2 + 18*a*b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*
a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15
*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 -
3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a
*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)
^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)
^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*co
sh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(
a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x +
c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x +
c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d
*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15
*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*
x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 -
b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11
*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x
+ c))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*
b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 -
6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 +
3*a*b^5 - b^6)*d^2))*log(-2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^
4 + 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15
*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 - 8
1*a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x + c
)*sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 81*
a*b - 81*b^2 + 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)
*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 2
0*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 2*(5*a^3*b - 19*a^2*b^2
+ 18*a*b^3)*d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^
2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a
^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*
a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b
^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8
- 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2
- (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh
(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh
(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*
b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c
)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c
)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*c
osh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(
a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x
+ c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b
^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a
*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x +
c))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b
+ 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*
a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3
*a*b^5 - b^6)*d^2))*log(-2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4
+ 9*a*b^5)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a
^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + (20*a^2 - 81*
a*b + 81*b^2)*cosh(d*x + c)^2 + 2*(20*a^2 - 81*a*b + 81*b^2)*cosh(d*x + c)*
sinh(d*x + c) + (20*a^2 - 81*a*b + 81*b^2)*sinh(d*x + c)^2 - 20*a^2 + 81*a*
b - 81*b^2 - 2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d
^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*
a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 2*(5*a^3*b - 19*a^2*b^2 +

```
18*a*b^3)*d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2
- 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3
*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^
2*b^4 + 3*a*b^5 - b^6)*d^2))) + 16*(3*(2*a - b)*cosh(d*x + c)^5 - 2*(8*a -
3*b)*cosh(d*x + c)^3 - (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a*
b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c
)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 +
 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 -
2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cos
h(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*
b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*
b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^
2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)
^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(
a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*
a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c
)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3
)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b
^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.43949, size = 205, normalized size = 0.88

$$-\frac{2\,ae^{(6\,dx+6\,c)} - be^{(6\,dx+6\,c)} - 8\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 2\,ae^{(2\,dx+2\,c)} - 3\,be^{(2\,dx+2\,c)} + b}{2\left(abd - b^2 d\right)\left(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*e^(6*d*x + 6*c) - b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e
^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + b)/((a*b*d - b
^2*d)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b
*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b))
```

3.249 $\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh^5(c+dx)}{4ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\tanh(}{4ad(}$$

```
[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a
] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tanh[c + d*
x]/(4*a*(a - b)*d) + Tanh[c + d*x]^5/(4*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a -
 b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.248514, antiderivative size = 195, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3217, 1275, 12, 1122, 1166, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh^5(c+dx)}{4ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-\frac{\tanh(}{4ad(}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]
```


```
[Out] ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a
] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d
*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tanh[c + d*
x]/(4*a*(a - b)*d) + Tanh[c + d*x]^5/(4*a*d*(a - 2*a*Tanh[c + d*x]^2 + (a -
 b)*Tanh[c + d*x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(1-x^2)}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2bx^4}{a-2ax^2+(a-b)x^4}\,dx, x, \text{t}\right.}{8abd}$$

$$= \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^4}{a-2ax^2+(a-b)x^4}\,dx, x, \text{ta}\right.}{4ad}$$

$$= -\frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{}{a-2a}\right.}{}$$

$$= -\frac{\tanh(c+dx)}{4a(a-b)d} + \frac{\tanh^5(c+dx)}{4ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(\sqrt{a}+\sqrt{b}\right)^2 \text{S}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}d} - \frac{\tanh(c+dx)}{4a(a-b)d} + \frac{}{4ad\left(a-2\right.}$$

Mathematica [A] time = 4.13686, size = 225, normalized size = 1.15

$$-\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}} - \frac{2(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}}{8d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -(((Sqrt[a] + Sqrt[b])*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a +
 Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((Sqrt[a
] - Sqrt[b])*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*S
qrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sinh[2*(c +
d*x)] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(
c + d*x)]))/(8*(a - b)*d)
```

Maple [C] time = 0.046, size = 490, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh
(1/2*d*x+1/2*c)^7+5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5+5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3-1/2/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/16/d/(
a-b)*sum((_R^6-7*_R^4+7*_R^2-1)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln
(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2
+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(8\,ae^{(4\,c)}-3\,be^{(4\,c)}\right)e^{(4\,dx)}-be^{(6\,dx+6\,c)}+5\,be^{(2\,dx+2\,c)}-b}{2\left(ab^2d-b^3d+\left(ab^2de^{(8\,c)}-b^3de^{(8\,c)}\right)e^{(8\,dx)}-4\left(ab^2de^{(6\,c)}-b^3de^{(6\,c)}\right)e^{(6\,dx)}-2\left(8\,a^2bde^{(4\,c)}-11\,ab^2de^{(4\,c)}+3\,b^3de^{(4\,c)}\right)e^{(4\,}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((8*a*e^(4*c) - 3*b*e^(4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) + 5*b*e^(2*
d*x + 2*c) - b)/(a*b^2*d - b^3*d + (a*b^2*d*e^(8*c) - b^3*d*e^(8*c))*e^(8*d
*x) - 4*(a*b^2*d*e^(6*c) - b^3*d*e^(6*c))*e^(6*d*x) - 2*(8*a^2*b*d*e^(4*c)
- 11*a*b^2*d*e^(4*c) + 3*b^3*d*e^(4*c))*e^(4*d*x) - 4*(a*b^2*d*e^(2*c) - b^
3*d*e^(2*c))*e^(2*d*x)) + 1/16*integrate(16*(e^(6*d*x + 6*c) - 6*e^(4*d*x +
 4*c) + e^(2*d*x + 2*c))/(a*b - b^2 + (a*b*e^(8*c) - b^2*e^(8*c))*e^(8*d*x)
 - 4*(a*b*e^(6*c) - b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^2*e^(4*c) - 11*a*b*e^(4
*c) + 3*b^2*e^(4*c))*e^(4*d*x) - 4*(a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)),
x)
```

Fricas [B] time = 2.72877, size = 12467, normalized size = 63.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^6 + 48*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*b*sinh(d
*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^2 - 8*a +
 3*b)*sinh(d*x + c)^4 + 32*(5*b*cosh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 40*b*cosh(d*x + c)^2 + 8*(15*b*cosh(d*x + c)^4 - 6*(8*a
 - 3*b)*cosh(d*x + c)^2 - 5*b)*sinh(d*x + c)^2 + ((a*b^2 - b^3)*d*cosh(d*x
+ c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*
sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*
cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2
+ 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^
2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x
+ c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*
d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)
*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b
^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*
x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b
^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d
+ 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 -
(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x +
 c))*sinh(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt(
(9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^
5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b
^3 - a*b^4)*d^2))*log(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a
^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^
5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*co
sh(d*x + c)*sinh(d*x + c) + (3*a + b)*sinh(d*x + c)^2 + 2*(2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (
3*a^3 + 4*a^2*b + a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d
^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4
 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 +
 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8
*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x +
 c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x +
 c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d
*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d
*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3
0*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*
x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*
x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 -
 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh
(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^
2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b -
 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(
d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6
*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*
a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*d^2))*log(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a^2 + 6*a*b
 + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*
b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*cosh(d*x + c
)*sinh(d*x + c) + (3*a + b)*sinh(d*x + c)^2 - 2*(2*(a^6*b - 3*a^5*b^2 + 3*a
^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a
^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (3*a^3 + 4*
a^2*b + a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9
*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*
```

b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) - ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*cosh(d*x + c)*sinh(d*x + c) + (3*a + b)*sinh(d*x + c)^2 + 2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a^3 + 4*a^2*b + a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) + ((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 - 15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a + b)*cosh(d*x + c)^2 + 2*(3*a + b)*cosh(d*x + c)*sinh(d*x + c) + (3*a + b)*sinh(d*x + c)^2 - 2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + (3*a^3 + 4*a^2*b + a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 3*a - b) + 16*(3*b*cosh(d*x + c)^5 - 2*(8*a - 3*b)*cosh(d*x + c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 - b^3)*d*sinh(d*x + c)^8 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^6 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^4 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - 3*(a*b^2 - b^3)*d*

```
cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 30
*(a*b^2 - b^3)*d*cosh(d*x + c)^2 - (8*a^2*b - 11*a*b^2 + 3*b^3)*d)*sinh(d*x
 + c)^4 - 4*(a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a*b^2 - b^3)*d*cosh(d*x
 + c)^5 - 10*(a*b^2 - b^3)*d*cosh(d*x + c)^3 - (8*a^2*b - 11*a*b^2 + 3*b^3)
*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a*b^2 - b^3)*d*cosh(d*x + c)^6 -
15*(a*b^2 - b^3)*d*cosh(d*x + c)^4 - 3*(8*a^2*b - 11*a*b^2 + 3*b^3)*d*cosh(
d*x + c)^2 - (a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a*b^2 - b^3)*d + 8*((a*b^2
 - b^3)*d*cosh(d*x + c)^7 - 3*(a*b^2 - b^3)*d*cosh(d*x + c)^5 - (8*a^2*b -
11*a*b^2 + 3*b^3)*d*cosh(d*x + c)^3 - (a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d
*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.33748, size = 171, normalized size = 0.88

$$\frac{be^{(6\,dx+6\,c)} - 8\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 5\,be^{(2\,dx+2\,c)} + b}{2\,\left(abd - b^{2}d\right)\left(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 5*b*e^
(2*d*x + 2*c) + b)/((a*b*d - b^2*d)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c
) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b))
```

3.250 $\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2} dx$

Optimal. Leaf size=220

$$-\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)\left(a-(a+b)\right.}{4ad(a-b)\left((a-b)\tanh^4(c+dx)\right.}$$

```
[Out] -((2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(
1/4)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) + ((2*Sqrt[a] + Sqrt
[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(5/4)*(
Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) + (Tanh[c + d*x]*(a - (a + b)*Tanh[c +
d*x]^2))/(4*a*(a - b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)
)
```

Rubi [A] time = 0.297621, antiderivative size = 220, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1333, 1166, 208}

$$-\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tanh(c+dx)\left(a-(a+b)\right.}{4ad(a-b)\left((a-b)\tanh^4(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^(
1/4)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) + ((2*Sqrt[a] + Sqrt
[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(5/4)*(
Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) + (Tanh[c + d*x]*(a - (a + b)*Tanh[c +
d*x]^2))/(4*a*(a - b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)
)
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
```

```
&& NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^2}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\frac{2a^2b}{a-b}-\frac{2a(3a-b)bx^2}{a-b}}{a-2ax^2+(a-b)x^4}\,d\right.}{8a^2bd}$$

$$= \frac{\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(2a-\sqrt{a}\sqrt{b}-b\right)\text{Subst}\left(\right.}{8a}$$

$$= -\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}d} + \frac{\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}d} +$$

Mathematica [A] time = 2.06416, size = 253, normalized size = 1.15

$$\frac{\frac{\sqrt{a}\left(-\sqrt{a}\sqrt{b}+2a-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}+2a-b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}+\frac{4\sqrt{a}\sinh(2(c+dx))(2a-b\cosh(2(c+dx))+b)}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c +
d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (
Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d
*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*S
qrt[a]*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*
Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

Maple [C] time = 0.069, size = 708, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] 1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(
1/2*d*x+1/2*c)^7-1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^5-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^5*b-1/2/d/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3-2/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d
*x+1/2*c)^3*b+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*ta
nh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+
a)/(a-b)*tanh(1/2*d*x+1/2*c)-1/16/d/a/(a-b)*sum((-_R^6*a+(11*a-4*b)*_R^4+(-
11*a+4*b)*_R^2+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+
1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(2\,ae^{(6\,c)}-be^{(6\,c)}\right)e^{(6\,dx)}-\left(8\,ae^{(4\,c)}-3\,be^{(4\,c)}\right)e^{(4\,dx)}-\left(2\,ae^{(2\,c)}+3\,be^{(2\,c)}\right)e^{(2\,dx)}+b}{2\left(a^2bd-ab^2d+\left(a^2bde^{(8\,c)}-ab^2de^{(8\,c)}\right)e^{(8\,dx)}-4\left(a^2bde^{(6\,c)}-ab^2de^{(6\,c)}\right)e^{(6\,dx)}-2\left(8\,a^3de^{(4\,c)}-11\,a^2bde^{(4\,c)}+3\,ab^2de^{(4\,c)}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - (8*a*e^(4*c) - 3*b*e^(4*c))*e^(
4*d*x) - (2*a*e^(2*c) + 3*b*e^(2*c))*e^(2*d*x) + b)/(a^2*b*d - a*b^2*d + (a
^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*
e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^
(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x)) - 1/4*i
ntegrate(4*((2*a*e^(6*c) - b*e^(6*c))*e^(6*d*x) - 2*(4*a*e^(4*c) - b*e^(4*c
))*e^(4*d*x) + (2*a*e^(2*c) - b*e^(2*c))*e^(2*d*x))/(a^2*b - a*b^2 + (a^2*b
*e^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(
6*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) -
 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.51966, size = 14390, normalized size = 65.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(2*a - b)*cosh(d*x + c)^6 + 48*(2*a - b)*cosh(d*x + c)*sinh(d*x +
c)^5 + 8*(2*a - b)*sinh(d*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*
(2*a - b)*cosh(d*x + c)^2 - 8*a + 3*b)*sinh(d*x + c)^4 + 32*(5*(2*a - b)*co
sh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c))*sinh(d*x + c)^3 - 8*(2*a + 3*b)*
cosh(d*x + c)^2 + 8*(15*(2*a - b)*cosh(d*x + c)^4 - 6*(8*a - 3*b)*cosh(d*x
```

```
+ c)^2 - 2*a - 3*b)*sinh(d*x + c)^2 + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 +
8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(
d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*c
osh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b +
 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a
^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*co
sh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b +
3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a
^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3
 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)
^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b -
a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 -
 (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^5*b - 3*a^4*b^2
+ 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3
+ b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6
*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3
- a^2*b^4)*d^2))*log(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
 + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*
cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)*sinh(
d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*sinh(d*x + c)^2 + 2*((3*a^8*
b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^
3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*
a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 2*(8*a^5 - 5*a^4*b + a^
3*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4
 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b
^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b
^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))) - ((a^2*b - a*b^2)*d*
cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*
b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(
a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(
8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh
(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a
^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8
*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*
x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*co
sh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x
 + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2
)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x +
c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d
*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((
a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a
^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 +
15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^
4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7
*a^4*b^3 + a^3*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b
^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^
6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b
 + 9*a*b^2 - b^3)*cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*c
osh(d*x + c)*sinh(d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*sinh(d*x +
 c)^2 - 2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sq
rt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2
+ 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 2*(8*
a^5 - 5*a^4*b + a^3*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
```

^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
 - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))) - (
(a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(
d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d
x + c)^6 + 4(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sin
h(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2
*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*co
sh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b
- a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a
^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^
2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*
b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*
a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(
d*x + c))*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4
- 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^
3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^
2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(-2*(4*a^7 - 13*a^6*
b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*
b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*
a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b + 9*a*b^2 - b^3 -
(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)^2 - 2*(32*a^3 - 28*a^2*b
+ 9*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c) - (32*a^3 - 28*a^2*b + 9*a*b^2
 - b^3)*sinh(d*x + c)^2 + 2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4
 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^
11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*
b^7)*d^4)) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d)*sqrt(((a^5*b - 3*a^4*b^2 + 3*
a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^
4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6
 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^
2*b^4)*d^2))) + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*co
sh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b
- a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*
b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x +
 c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d
*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^
2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x +
 c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d
x + c)^5 - 10(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*
b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c
)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)
*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d
 + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)
^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cos
h(d*x + c))*sinh(d*x + c))*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*
d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^1
0*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) +
 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(-2
*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*sqrt((64*a^4 - 8
0*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 -
 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 32*a^3 - 28*a^2*b +
 9*a*b^2 - b^3 - (32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)^2 - 2*(3
2*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c) - (32*a^3 - 2
8*a^2*b + 9*a*b^2 - b^3)*sinh(d*x + c)^2 - 2*((3*a^8*b - 10*a^7*b^2 + 12*a^
6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*
a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 -
 6*a^6*b^6 + a^5*b^7)*d^4)) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d)*sqrt(((a^5*b

```
 - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^
2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^
7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2
 + 3*a^3*b^3 - a^2*b^4)*d^2))) + 16*(3*(2*a - b)*cosh(d*x + c)^5 - 2*(8*a -
 3*b)*cosh(d*x + c)^3 - (2*a + 3*b)*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a
^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*
x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x
 + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(
d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b
 - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x +
 c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh
(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b -
a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2
*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a
*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2
- (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^
2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^
2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*
x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.2299, size = 205, normalized size = 0.93

$$-\frac{2\,ae^{(6\,dx+6\,c)} - be^{(6\,dx+6\,c)} - 8\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 2\,ae^{(2\,dx+2\,c)} - 3\,be^{(2\,dx+2\,c)} + b}{2\left(a^2d - abd\right)\left(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(2*a*e^(6*d*x + 6*c) - b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e
^(4*d*x + 4*c) - 2*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) + b)/((a^2*d - a
*b*d)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b
*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b))
```

3.251 $\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tanh(c+dx)\left(1-2\,\mathrm{t}\right.}{4ad(a-b)\left((a-b)\tanh^4(c+dx)\right.}$$

```
[Out] ((4*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(8*a^(7/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((4*Sqrt[a] + 3*Sqrt[b])*
ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(7/4)*(Sqrt[
a] + Sqrt[b])^(3/2)*d) - (b*Tanh[c + d*x]*(1 - 2*Tanh[c + d*x]^2))/(4*a*(a
- b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.246026, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3209, 1205, 1166, 208}

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tanh(c+dx)\left(1-2\,\mathrm{t}\right.}{4ad(a-b)\left((a-b)\tanh^4(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-2),x]
```

```
[Out] ((4*Sqrt[a] - 3*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(8*a^(7/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((4*Sqrt[a] + 3*Sqrt[b])*
ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(8*a^(7/4)*(Sqrt[
a] + Sqrt[b])^(3/2)*d) - (b*Tanh[c + d*x]*(1 - 2*Tanh[c + d*x]^2))/(4*a*(a
- b)*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{2a(4a-3b)b}{a-b}+\frac{4a(2a}{a}}{a-2ax^2+(a-b)}\right.}{8a}$$

$$= -\frac{b\tanh(c+dx)\left(1-2\tanh^2(c+dx)\right)}{4a(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\left(4a-\sqrt{a}\sqrt{b}-3b\right)\text{Subs}}{8}$$

$$= \frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}$$

Mathematica [A] time = 2.93151, size = 230, normalized size = 1.1

$$\frac{\frac{\left(-\sqrt{a}\sqrt{b}+4a-3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\left(\sqrt{a}\sqrt{b}+4a-3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{2\sqrt{a}b(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-2),x]
```

```
[Out] (-(((4*a + Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x]
)/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]]) + ((4*a - Sqrt[a
]*Sqrt[b] - 3*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[
a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sinh[2*(c + d*x)
] + Sinh[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c +
d*x)]))/(8*a^(3/2)*(a - b)*d)
```

Maple [C] time = 0.062, size = 534, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)*b/a/(a-b)*
tanh(1/2*d*x+1/2*c)^7+5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^5*b+5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^3*b-1/2/d/(tanh
(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*
b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)*b/a/(a-b)*tanh(1/2*d*x
+1/2*c)-1/16/d/a/(a-b)*sum(((4*a-3*b)*_R^6+(-12*a+5*b)*_R^4+(-5*b+12*a)*_R^
2-4*a+3*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-
_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(8\,ae^{(4\,c)}-3\,be^{(4\,c)}\right)e^{(4\,dx)}-be^{(6\,dx+6\,c)}+5\,be^{(2\,dx+2\,c)}-b}{2\left(a^2bd-ab^2d+\left(a^2bde^{(8\,c)}-ab^2de^{(8\,c)}\right)e^{(8\,dx)}-4\left(a^2bde^{(6\,c)}-ab^2de^{(6\,c)}\right)e^{(6\,dx)}-2\left(8\,a^3de^{(4\,c)}-11\,a^2bde^{(4\,c)}+3\,ab^2de^{(4\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((8*a*e^(4*c) - 3*b*e^(4*c))*e^(4*d*x) - b*e^(6*d*x + 6*c) + 5*b*e^(2*
d*x + 2*c) - b)/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(8*c) - a*b^2*d*e^(8*c))*e^
(8*d*x) - 4*(a^2*b*d*e^(6*c) - a*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^3*d*e^(4
*c) - 11*a^2*b*d*e^(4*c) + 3*a*b^2*d*e^(4*c))*e^(4*d*x) - 4*(a^2*b*d*e^(2*c
) - a*b^2*d*e^(2*c))*e^(2*d*x)) + integrate(-(2*(8*a*e^(4*c) - 5*b*e^(4*c))
*e^(4*d*x) - b*e^(6*d*x + 6*c) - b*e^(2*d*x + 2*c))/(a^2*b - a*b^2 + (a^2*b
*e^(8*c) - a*b^2*e^(8*c))*e^(8*d*x) - 4*(a^2*b*e^(6*c) - a*b^2*e^(6*c))*e^(
6*d*x) - 2*(8*a^3*e^(4*c) - 11*a^2*b*e^(4*c) + 3*a*b^2*e^(4*c))*e^(4*d*x) -
 4*(a^2*b*e^(2*c) - a*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.53477, size = 14862, normalized size = 70.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(8*b*cosh(d*x + c)^6 + 48*b*cosh(d*x + c)*sinh(d*x + c)^5 + 8*b*sinh(d
*x + c)^6 - 8*(8*a - 3*b)*cosh(d*x + c)^4 + 8*(15*b*cosh(d*x + c)^2 - 8*a +
 3*b)*sinh(d*x + c)^4 + 32*(5*b*cosh(d*x + c)^3 - (8*a - 3*b)*cosh(d*x + c)
)*sinh(d*x + c)^3 - 40*b*cosh(d*x + c)^2 + 8*(15*b*cosh(d*x + c)^4 - 6*(8*a
 - 3*b)*cosh(d*x + c)^2 - 5*b)*sinh(d*x + c)^2 - ((a^2*b - a*b^2)*d*cosh(d*
x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b
^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 -
 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x +
c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b -
a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 -
11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^
```

2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x
+ c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4
 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*si
nh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 -
3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d
*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^6 - 3
*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2
*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 +
15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 -
3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(384*a^3*b - 680*a^2*b^2 + 405*a*b^
3 - 81*b^4 + 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^
2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^1
3 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6
)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)^2 -
2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)*sinh(d*x + c
) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*sinh(d*x + c)^2 + 2*(2*(
2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1
392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^1
1*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + (120*a^5*b
 - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*
b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^
4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*
a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4
*b^2 - a^3*b^3)*d^2))) + ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*
b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 -
4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^
2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*c
osh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)
*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4
 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*s
inh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)
*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2
*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cos
h(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b +
 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b -
 a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh
(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b
^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3
*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^
5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 +
 a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^
3*b^3)*d^2))*log(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4 + 2*(16*a^8 -
 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*sqrt((576*a^4*b - 1392
*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b
^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (384*a^3*b - 6
80*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)^2 - 2*(384*a^3*b - 680*a^2*b
^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)*sinh(d*x + c) - (384*a^3*b - 680*a^2
*b^2 + 405*a*b^3 - 81*b^4)*sinh(d*x + c)^2 - 2*(2*(2*a^10 - 7*a^9*b + 9*a^8
*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b
^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15
*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + (120*a^5*b - 217*a^4*b^2 + 132*a^3*
b^3 - 27*a^2*b^4)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt(
(576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a
^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))
 - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) +
 ((a^2*b - a*b^2)*d*cosh(d*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sin
h(d*x + c)^7 + (a^2*b - a*b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh
(d*x + c)^6 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*s

```
inh(d*x + c)^6 - 2*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a
^2*b - a*b^2)*d*cosh(d*x + c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d
*x + c)^5 + 2*(35*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*
cosh(d*x + c)^2 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*
b - a*b^2)*d*cosh(d*x + c)^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*
(a^2*b - a*b^2)*d*cosh(d*x + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x
 + c))*sinh(d*x + c)^3 + 4*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b
 - a*b^2)*d*cosh(d*x + c)^4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c
)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b -
a*b^2)*d*cosh(d*x + c)^7 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 1
1*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sin
h(d*x + c))*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b
 - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2
- 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(384*a^3*
b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4 - 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 -
43*a^5*b^3 + 9*a^4*b^4)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 -
 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9
*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 -
81*b^4)*cosh(d*x + c)^2 - 2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*
cosh(d*x + c)*sinh(d*x + c) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4
)*sinh(d*x + c)^2 + 2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^
4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/
((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^
7*b^6)*d^4)) - (120*a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d)*sqrt
(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2
+ 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a
^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2
)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) - ((a^2*b - a*b^2)*d*cosh(d
*x + c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*
b^2)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b
- a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3
- 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x +
 c)^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b -
 a*b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 -
 11*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)
^2 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x
 + c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^
4 - 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*s
inh(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 -
 3*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(
d*x + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^6 - 3
*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2
*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 +
15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 -
3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(384*a^3*b - 680*a^2*b^2 + 405*a*b^
3 - 81*b^4 - 2*(16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^
2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^1
3 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6
)*d^4)) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)^2 -
2*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cosh(d*x + c)*sinh(d*x + c
) - (384*a^3*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*sinh(d*x + c)^2 - 2*(2*(
2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1
392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^1
1*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - (120*a^5*b
- 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b
^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4
 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a
```

```
^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*
b^2 - a^3*b^3)*d^2))) + 16*(3*b*cosh(d*x + c)^5 - 2*(8*a - 3*b)*cosh(d*x +
c)^3 - 5*b*cosh(d*x + c))*sinh(d*x + c) + 8*b)/((a^2*b - a*b^2)*d*cosh(d*x
+ c)^8 + 8*(a^2*b - a*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - a*b^2
)*d*sinh(d*x + c)^8 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 + 4*(7*(a^2*b - a
*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh(d*x + c)^6 - 2*(8*a^3 - 1
1*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^4 + 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)
^3 - 3*(a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^2*b - a*
b^2)*d*cosh(d*x + c)^4 - 30*(a^2*b - a*b^2)*d*cosh(d*x + c)^2 - (8*a^3 - 11
*a^2*b + 3*a*b^2)*d)*sinh(d*x + c)^4 - 4*(a^2*b - a*b^2)*d*cosh(d*x + c)^2
+ 8*(7*(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - 10*(a^2*b - a*b^2)*d*cosh(d*x +
c)^3 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7
*(a^2*b - a*b^2)*d*cosh(d*x + c)^6 - 15*(a^2*b - a*b^2)*d*cosh(d*x + c)^4 -
 3*(8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x + c)^2 - (a^2*b - a*b^2)*d)*sinh
(d*x + c)^2 + (a^2*b - a*b^2)*d + 8*((a^2*b - a*b^2)*d*cosh(d*x + c)^7 - 3*
(a^2*b - a*b^2)*d*cosh(d*x + c)^5 - (8*a^3 - 11*a^2*b + 3*a*b^2)*d*cosh(d*x
 + c)^3 - (a^2*b - a*b^2)*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 4.28869, size = 171, normalized size = 0.81

$$\frac{be^{(6\,dx+6\,c)} - 8\,ae^{(4\,dx+4\,c)} + 3\,be^{(4\,dx+4\,c)} - 5\,be^{(2\,dx+2\,c)} + b}{2\,\left(a^2d - abd\right)\left(be^{(8\,dx+8\,c)} - 4\,be^{(6\,dx+6\,c)} - 16\,ae^{(4\,dx+4\,c)} + 6\,be^{(4\,dx+4\,c)} - 4\,be^{(2\,dx+2\,c)} + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/2*(b*e^(6*d*x + 6*c) - 8*a*e^(4*d*x + 4*c) + 3*b*e^(4*d*x + 4*c) - 5*b*e^
(2*d*x + 2*c) + b)/((a^2*d - a*b*d)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c
) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b))
```

3.252 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=237

$$-\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{b\tanh(c+dx)\left(\right.}{4a^2d(a-b)\left((a-b)\tanh\right.}$$

```
[Out] -((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
 d*x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((6*Sqrt[a] + 5*
Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(8*a^(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Coth[c + d*x]/(a^2*d) + (b*Tanh[c
 + d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - 2*a*Tanh[c + d
*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.533692, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1334, 1664, 1166, 208}

$$-\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{b\tanh(c+dx)\left(\right.}{4a^2d(a-b)\left((a-b)\tanh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] -((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
 d*x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((6*Sqrt[a] + 5*
Sqrt[b])*Sqrt[b]*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/
(8*a^(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Coth[c + d*x]/(a^2*d) + (b*Tanh[c
 + d*x]*(a - (a + b)*Tanh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - 2*a*Tanh[c + d
*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1334

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{x^2\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab+\frac{2a(8a-7b)bx^2}{a-b}}{x^2\left(a-2ax^2\right.}\right.}{}$$

$$= \frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8b}{x^2}+\frac{2b^2(a}{(a-b)(a-}\right.\right.}{}$$

$$= -\frac{\coth(c+dx)}{a^2d} + \frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} - \frac{b\,\text{Subs}}{}$$

$$= -\frac{\coth(c+dx)}{a^2d} + \frac{b\tanh(c+dx)\left(a-(a+b)\tanh^2(c+dx)\right)}{4a^2(a-b)d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)} + \frac{\left(\left(7a+\right.\right.}{}$$

$$= -\frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan}{\sqrt[4]{a}}\right.}{8a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}$$

Mathematica [A] time = 1.87298, size = 272, normalized size = 1.15

$$\frac{\frac{\left(6a\sqrt{b}+5\sqrt{a}b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\left(6a\sqrt{b}-5\sqrt{a}b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{4\sqrt{a}b\sinh(2(c+dx))(2a-b\cosh(2(c+dx))+b)}{(a-b)(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)} - 8\sqrt{a}\,\text{co}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^2,x]
```

```
[Out] (((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c + d*x])/Sq
rt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]])
 + ((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])
/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]
) - 8*Sqrt[a]*Coth[c + d*x] + (4*Sqrt[a]*b*(2*a + b - b*Cosh[2*(c + d*x)])*
Sinh[2*(c + d*x)])/((a - b)*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(
c + d*x)])))/(8*a^(5/2)*d)
```

Maple [C] time = 0.09, size = 765, normalized size = 3.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/a^2*tanh(1/2*d*x+1/2*c)+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)*b/a/(a-b)*tanh(1/2*d*x+1/2*c)^7-1/2/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^5*b-2/
d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x
+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*t
anh(1/2*d*x+1/2*c)^5-1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)/a/(a-b)*tanh(1/2*d*x+1/2*c)^3*b-2/d*b^2/a^2/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)/(a-b)*tanh(1/2*d*x+1/2*c)^3+1/2/d/(tan
h(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16
*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)*b/a/(a-b)*tanh(1/2*d*
x+1/2*c)-1/16/d*b/a^2/(a-b)*sum((-_R^6*a+(27*a-20*b)*_R^4+(-27*a+20*b)*_R^2
+a)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=
RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))-1/2/d/a^2/tanh(1/2*d*x+
1/2*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b - 5*b^2 + (6*a*b*e^(8*c) - 5*b^2*e^(8*c))*e^(8*d*x) - 2*(13*a*b*
e^(6*c) - 10*b^2*e^(6*c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 47*a*b*e^(4*c) +
15*b^2*e^(4*c))*e^(4*d*x) - 2*(7*a*b*e^(2*c) - 10*b^2*e^(2*c))*e^(2*d*x))/(
a^3*b*d - a^2*b^2*d - (a^3*b*d*e^(10*c) - a^2*b^2*d*e^(10*c))*e^(10*d*x) +
5*(a^3*b*d*e^(8*c) - a^2*b^2*d*e^(8*c))*e^(8*d*x) + 2*(8*a^4*d*e^(6*c) - 13
*a^3*b*d*e^(6*c) + 5*a^2*b^2*d*e^(6*c))*e^(6*d*x) - 2*(8*a^4*d*e^(4*c) - 13
*a^3*b*d*e^(4*c) + 5*a^2*b^2*d*e^(4*c))*e^(4*d*x) - 5*(a^3*b*d*e^(2*c) - a^
2*b^2*d*e^(2*c))*e^(2*d*x)) - 4*integrate(1/4*((6*a*b*e^(6*c) - 5*b^2*e^(6*
c))*e^(6*d*x) - 2*(8*a*b*e^(4*c) - 5*b^2*e^(4*c))*e^(4*d*x) + (6*a*b*e^(2*c
```

```
) - 5*b^2*e^(2*c))*e^(2*d*x))/(a^3*b - a^2*b^2 + (a^3*b*e^(8*c) - a^2*b^2*e
^(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - a^2*b^2*e^(6*c))*e^(6*d*x) - 2*(8*a^
4*e^(4*c) - 11*a^3*b*e^(4*c) + 3*a^2*b^2*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2
*c) - a^2*b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 4.28504, size = 20254, normalized size = 85.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(8*(6*a*b - 5*b^2)*cosh(d*x + c)^8 + 64*(6*a*b - 5*b^2)*cosh(d*x + c)
*sinh(d*x + c)^7 + 8*(6*a*b - 5*b^2)*sinh(d*x + c)^8 - 16*(13*a*b - 10*b^2)
*cosh(d*x + c)^6 + 16*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^2 - 13*a*b + 10*b^2
)*sinh(d*x + c)^6 + 32*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^3 - 3*(13*a*b - 10
*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*(32*a^2 - 47*a*b + 15*b^2)*cosh(d
*x + c)^4 + 16*(35*(6*a*b - 5*b^2)*cosh(d*x + c)^4 - 15*(13*a*b - 10*b^2)*c
osh(d*x + c)^2 - 32*a^2 + 47*a*b - 15*b^2)*sinh(d*x + c)^4 + 64*(7*(6*a*b -
 5*b^2)*cosh(d*x + c)^5 - 5*(13*a*b - 10*b^2)*cosh(d*x + c)^3 - (32*a^2 - 4
7*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 16*(7*a*b - 10*b^2)*cosh(d
*x + c)^2 + 16*(14*(6*a*b - 5*b^2)*cosh(d*x + c)^6 - 15*(13*a*b - 10*b^2)*c
osh(d*x + c)^4 - 6*(32*a^2 - 47*a*b + 15*b^2)*cosh(d*x + c)^2 - 7*a*b + 10*
b^2)*sinh(d*x + c)^2 + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b -
a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c
)^10 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cos
h(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b +
 5*a^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 -
 (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b
^2)*d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 1
3*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*
d*cosh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b -
 a^2*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x +
 c))*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^
3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh
(d*x + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b
- a^2*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 -
35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d
*cosh(d*x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d
*cosh(d*x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12
*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*
sinh(d*x + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x +
 c)^9 - 20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^
2*b^2)*d*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)
^3 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^7 - 3*a^
6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2
*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3
+ 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)
/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b
^3 + 2625*a*b^4 - 625*b^5 + 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b
^3 + 25*a^5*b^4)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 345
0*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*
b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b
^4 - 625*b^5)*cosh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4
 - 625*b^5)*cosh(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 26
```

```
25*a*b^4 - 625*b^5)*sinh(d*x + c)^2 + 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 -
 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b
^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 +
15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 2*(144*a^6*b - 303*a^5*b^2 + 21
3*a^4*b^3 - 50*a^3*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2
*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/(
(a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a
^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b - a^2*
b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^10
 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cosh(d*
x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + 5*a
^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (a^
3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b^2)*
d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 13*a^
3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*co
sh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b - a^2
*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))
*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^3*b
- a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x
 + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b - a^
2*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 35*(
a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cos
h(d*x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c
)^3 + (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d*cos
h(d*x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12*(8*
a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*sinh
(d*x + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^
9 - 20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^2*b^
2)*d*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 +
 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(-((a^7 - 3*a^6*b
+ 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5
 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15
*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a
^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b^3 +
 2625*a*b^4 - 625*b^5 + 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 +
 25*a^5*b^4)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*
b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4
- 6*a^10*b^5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 -
 625*b^5)*cosh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 6
25*b^5)*cosh(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a
*b^4 - 625*b^5)*sinh(d*x + c)^2 - 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*
a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 -
 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a
^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 2*(144*a^6*b - 303*a^5*b^2 + 213*a^
4*b^3 - 50*a^3*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqr
t((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^1
5 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b
^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4
*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b - a^2*b^2)
*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^10 - 5
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cosh(d*x +
c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + 5*a^2*b
^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (a^3*b
- a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b^2)*d*co
sh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 13*a^3*b
+ 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d
*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b - a^2*b^2
)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sin
```

h(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^3*b - a^
2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c
)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b - a^2*b^
2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 35*(a^3*
b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*
x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3
+ (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d*cosh(d*
x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12*(8*a^4
- 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*sinh(d*x
 + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^9 -
20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d
*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 + 5*(
a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a
^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 34
50*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11
*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 -
3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b^3 + 2625
*a*b^4 - 625*b^5 - 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a
^5*b^4)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 +
 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a
^10*b^5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*
b^5)*cosh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^
5)*cosh(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4
- 625*b^5)*sinh(d*x + c)^2 + 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b
^3 + 5*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450
*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b
^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3
 - 50*a^3*b^4)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((230
4*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*
a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^
4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*
d^2))) + ((a^3*b - a^2*b^2)*d*cosh(d*x + c)^10 + 10*(a^3*b - a^2*b^2)*d*cos
h(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2*b^2)*d*sinh(d*x + c)^10 - 5*(a^3*
b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 -
 (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*
cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - (a^3*b - a^2*
b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x
 + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (8*a^4 - 13*a^3*b + 5*a^
2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c
)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 70*(a^3*b - a^2*b^2)*d*co
sh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 175*(a^3*b - a^2*b^2)
*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 +
(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^4 + 5*(a^3*b - a^2*b^2)*d*c
osh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 35*(a^3*b - a^
2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)
^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*
(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140*(a^3*b - a^2*b^2)*d*cosh(d*x + c)
^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 12*(8*a^4 - 13*a
^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(a^3*b - a^2*b^2)*d)*sinh(d*x + c)^
2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^9 - 20*(a^
3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(
d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 + 5*(a^3*b
- a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2
 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b
^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 -
 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*
b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4
 - 625*b^5 - 2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4

```
)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b
^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^
5 + a^9*b^6)*d^4)) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*c
osh(d*x + c)^2 - 2*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos
h(d*x + c)*sinh(d*x + c) - (1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*
b^5)*sinh(d*x + c)^2 - 2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5
*a^7*b^4)*d^3*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6
 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6
*a^10*b^5 + a^9*b^6)*d^4)) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3 - 50*
a^3*b^4)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*
b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b
 + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) +
36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)))
 + 32*a*b - 40*b^2 + 32*(2*(6*a*b - 5*b^2)*cosh(d*x + c)^7 - 3*(13*a*b - 10
*b^2)*cosh(d*x + c)^5 - 2*(32*a^2 - 47*a*b + 15*b^2)*cosh(d*x + c)^3 - (7*a
*b - 10*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^3*b - a^2*b^2)*d*cosh(d*x +
c)^10 + 10*(a^3*b - a^2*b^2)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^3*b - a^2
*b^2)*d*sinh(d*x + c)^10 - 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 + 5*(9*(a^
3*b - a^2*b^2)*d*cosh(d*x + c)^2 - (a^3*b - a^2*b^2)*d)*sinh(d*x + c)^8 - 2
*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^6 + 40*(3*(a^3*b - a^2*b^2)
*d*cosh(d*x + c)^3 - (a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2
*(105*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x
 + c)^2 - (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*x + c)^6 + 2*(8*a^4 - 13
*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^4 + 4*(63*(a^3*b - a^2*b^2)*d*cosh(d*x
+ c)^5 - 70*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^3 - 3*(8*a^4 - 13*a^3*b + 5*a
^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^3*b - a^2*b^2)*d*cosh(
d*x + c)^6 - 175*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^4 - 15*(8*a^4 - 13*a^3*b
 + 5*a^2*b^2)*d*cosh(d*x + c)^2 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d)*sinh(d*
x + c)^4 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^2 + 8*(15*(a^3*b - a^2*b^2)*
d*cosh(d*x + c)^7 - 35*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^5 - 5*(8*a^4 - 13*
a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^3 + (8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (45*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^8 - 140
*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^6 - 30*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*
cosh(d*x + c)^4 + 12*(8*a^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^2 + 5*(
a^3*b - a^2*b^2)*d)*sinh(d*x + c)^2 - (a^3*b - a^2*b^2)*d + 2*(5*(a^3*b - a
^2*b^2)*d*cosh(d*x + c)^9 - 20*(a^3*b - a^2*b^2)*d*cosh(d*x + c)^7 - 6*(8*a
^4 - 13*a^3*b + 5*a^2*b^2)*d*cosh(d*x + c)^5 + 4*(8*a^4 - 13*a^3*b + 5*a^2*
b^2)*d*cosh(d*x + c)^3 + 5*(a^3*b - a^2*b^2)*d*cosh(d*x + c))*sinh(d*x + c)
)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**2,x)

[Out] Timed out
```

Giac [A] time = 5.79865, size = 321, normalized size = 1.35

$$-\frac{6\,abe^{(8\,dx+8\,c)} - 5\,b^2e^{(8\,dx+8\,c)} - 26\,abe^{(6\,dx+6\,c)} + 20\,b^2e^{(6\,dx+6\,c)} - 64\,a^2e^{(4\,dx+4\,c)} + 94\,abe^{(4\,dx+4\,c)} - 30\,b^2e^{(4\,dx+4\,c)} - 14\,abe^{(2}}{2\left(a^3d - a^2bd\right)\left(be^{(10\,dx+10\,c)} - 5\,be^{(8\,dx+8\,c)} - 16\,ae^{(6\,dx+6\,c)} + 10\,be^{(6\,dx+6\,c)} + 16\,ae^{(4\,dx+4\,c)} - 10\,be^{(4\,dx+4\,c)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/2*(6*a*b*e^(8*d*x + 8*c) - 5*b^2*e^(8*d*x + 8*c) - 26*a*b*e^(6*d*x + 6*c
) + 20*b^2*e^(6*d*x + 6*c) - 64*a^2*e^(4*d*x + 4*c) + 94*a*b*e^(4*d*x + 4*c
) - 30*b^2*e^(4*d*x + 4*c) - 14*a*b*e^(2*d*x + 2*c) + 20*b^2*e^(2*d*x + 2*c
) + 4*a*b - 5*b^2)/((a^3*d - a^2*b*d)*(b*e^(10*d*x + 10*c) - 5*b*e^(8*d*x +
 8*c) - 16*a*e^(6*d*x + 6*c) + 10*b*e^(6*d*x + 6*c) + 16*a*e^(4*d*x + 4*c)
- 10*b*e^(4*d*x + 4*c) + 5*b*e^(2*d*x + 2*c) - b))
```

3.253 $\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=315

$$-\frac{\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-\ldots\right.}$$

```
[Out] ((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) + ((5*a +
 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) + (a*Cosh[c + d
*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cosh[c + d*x
]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(
2*a - 5*b)*b*Cosh[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cosh[c + d*
x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.577358, antiderivative size = 315, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1205, 1678, 1166, 205, 208}

$$-\frac{\cosh(c+dx)\left(9a^2-2b(2a-5b)\cosh^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{a\cosh(c+dx)\left(a-b\cosh^2(c+dx)+b\right)}{8b^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) + ((5*a +
 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) + (a*Cosh[c + d
*x]*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cosh[c + d*x
]^2 - b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(
2*a - 5*b)*b*Cosh[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cosh[c + d*
x]^2 - b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^9(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a\left(a^2+ab-8b^2\right)}{b}-2a(11a-1}{\left(a-b+2bx^2-\right.}\right.}{16a}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(9a^2-11ab-1\right.}{32(a-b)^2b^2d\left(a-b+2b\,\mathrm{co}\right.}$$

$$= \frac{a\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(9a^2-11ab-1\right.}{32(a-b)^2b^2d\left(a-b+2b\,\mathrm{co}\right.}$$

$$= \frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d} + \frac{\left(5a+14\sqrt{a}\sqrt{b}+12b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right.}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{9/4}d}$$

Mathematica [C] time = 1.68906, size = 1021, normalized size = 3.24

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^9/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cosh[2*(c + d*x
)]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a
- b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[
2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 - RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + 5*b^2*c - 2*a*b*d*x + 5*b^2*d*
x - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
 - Sinh[(c + d*x)/2]*#1] + 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2
] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 10*a^2*c*#1^2 + 28*a*b*c
*#1^2 - 39*b^2*c*#1^2 - 10*a^2*d*x*#1^2 + 28*a*b*d*x*#1^2 - 39*b^2*d*x*#1^2
 - 20*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1
 - Sinh[(c + d*x)/2]*#1]*#1^2 + 56*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d
*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 78*b^2*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^2 + 10*a^2*c*#1^4 - 28*a*b*c*#1^4 + 39*b^2*c*#1^4 + 10*a^2*d*x*#
1^4 - 28*a*b*d*x*#1^4 + 39*b^2*d*x*#1^4 + 20*a^2*Log[-Cosh[(c + d*x)/2] - S
inh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 56*a
*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^4 + 78*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
+ Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 2*a*b*c*#1^6 - 5*b^2*
c*#1^6 + 2*a*b*d*x*#1^6 - 5*b^2*d*x*#1^6 + 4*a*b*Log[-Cosh[(c + d*x)/2] - S
inh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 10*b
^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh
[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7)
 & ])/(128*(a - b)^2*b^2*d)
```

Maple [B] time = 0.099, size = 3542, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -1/16/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tan
h(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(
1/2)-77/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^
2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-
4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c
)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c
)^14-35/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^
3/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+85/16/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^12+105/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a^3-11/64/d/b/(a^2-2*a*b+b^2)*
a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^
(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-5/64/d/b^2/(a^2-2*a*b+b^2)*a^2/(-a*b
-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+
2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+11/64/d/b/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^(1
/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*
b-(a*b)^(1/2)*a)^(1/2))+35/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-175/16/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^8*a^3+5/32/d/b/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(
1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(
1/2)*a)^(1/2))*(a*b)^(1/2)+5/32/d/b/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1
/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(
1/2)*a)^(1/2))*(a*b)^(1/2)+5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+865/16/d/
(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*
a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^8*a^2+175/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*a^3-849/16
/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^6*a^2-105/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+383
/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-407/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh
(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*
tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a^2+5
/64/d/b^2/(a^2-2*a*b+b^2)*a^2/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh
(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))-1/16/d/b
^2/(a^2-2*a*b+b^2)*a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+
1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+88/d/
```

```
(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*
a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^8+20/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c
)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-5/16/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^3/b^2/(a^2-2*a*b+b^2)+11/1
6/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-
2*a*b+b^2)-106/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a+189/4/d/(tanh(1/2*d*x+1/2*c)^8*a-
4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c
)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*a-
31/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-3/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2+3/4/d/(tanh(
1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^14-13/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+163/4/d/(tanh(1/2*d*x+
1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/
2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^10*a-20/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-3/16/d/(a^2-2*a*b+b^2)/(-a*b-(a*
b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)
/(-a*b-(a*b)^(1/2)*a)^(1/2))+3/16/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1
/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1
/2)*a)^(1/2))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")

[Out] Timed out
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**9/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^9/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.254 $\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=290

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cosh(c+dx)\left(-3(a-3b)\cosh^2(c+dx)\right.}{32bd(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh\right.}$$

```
[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*S
qrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[
a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*
x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2)
+ (Cosh[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cosh[c + d*x]^2))/(32*(a - b)^2*
b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.467115, antiderivative size = 290, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cosh(c+dx)\left(-3(a-3b)\cosh^2(c+dx)\right.}{32bd(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]
```



```
[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*S
qrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[
a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cosh[c + d*x]*(2 - Cosh[c + d*
x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2)
+ (Cosh[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cosh[c + d*x]^2))/(32*(a - b)^2*
b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
```

```
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^7(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a(a-4b)-2a(3a-8b\cdots}{\left(a-b+2bx^2-bx^4\right)\cdots}\cdots\right)}{16a(a-\cdots}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\left(5a-17\cdots\right.}{32(a-b)^2bd\left(a-b+2b\cdots\right.}$$

$$= -\frac{a\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\left(5a-17\cdots\right.}{32(a-b)^2bd\left(a-b+2b\cdots\right.}$$

$$= \frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{\cdots}{8(a-b)b\cdots}$$

Mathematica [C] time = 1.29495, size = 802, normalized size = 2.77

$$-\frac{\frac{32\cosh(c+dx)(-7a+25b+3(a-3b)\cosh(2(c+dx)))}{8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx))} - 3\text{RootSum}\left[b\#1^8 - 4b\#1^6 - 16a\#1^4 + 6b\#1^4 - 4b\#1^2 + b\&, \frac{-ac\#1^6+3bc\#1^6-adx\#1^6}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^7/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cosh[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cosh[2*(c + d*x)]))/(8*a - 3
*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a - b)*(-5*Cosh
[c + d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cos
h[4*(c + d*x)])^2 - 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^
6 + b*#1^8 & , (a*c - 3*b*c + a*d*x - 3*b*d*x + 2*a*Log[-Cosh[(c + d*x)/2]
- Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 6*b*Lo
g[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c +
 d*x)/2]*#1] - 3*a*c*#1^2 + 17*b*c*#1^2 - 3*a*d*x*#1^2 + 17*b*d*x*#1^2 - 6*
a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[
(c + d*x)/2]*#1]*#1^2 + 34*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 3*a*c*#1^4 - 17*b*c*#1^4
 + 3*a*d*x*#1^4 - 17*b*d*x*#1^4 + 6*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*
x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 34*b*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^4 - a*c*#1^6 + 3*b*c*#1^6 - a*d*x*#1^6 + 3*b*d*x*#1^6 - 2*a*Log[-Co
sh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)
/2]*#1]*#1^6 + 6*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d
*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*
b*#1^5 + b*#1^7) & ])/(256*(a - b)^2*b*d)
```

Maple [B] time = 0.088, size = 2563, normalized size = 8.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 3/64/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh
(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1
/2)+1/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-32/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8-3/8/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-9/64/d/b/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*
a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*
b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-9/64/d/b/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a
)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*
b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-35/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a^2+5/d/(tan
h(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16
*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*t
```

```
anh(1/2*d*x+1/2*c)^6*a^2-25/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+2/d/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^10*a^2+3/64/d/b^2/(a^2-2*a*b+b^2)*a/(-a*b+(a*b)^(1/2)*a)^(1/2)
*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)
*a)^(1/2))*(a*b)^(1/2)+8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8-42/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^6-1/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^
2/b/(a^2-2*a*b+b^2)+13/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a+95/8/d/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^6*a-27/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+19/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*
tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^
4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-3/
8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^14+15/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-111/8/d/(tan
h(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16
*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^10*a+10/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10-1/4/d/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)+8/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^4+3/32/d/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/
4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2)
)-3/32/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*
d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(3*(a*b*e^(15*c) - 3*b^2*e^(15*c))*e^(15*d*x) - (23*a*b*e^(13*c) - 77*
b^2*e^(13*c))*e^(13*d*x) + (16*a^2*e^(11*c) + 131*a*b*e^(11*c) - 177*b^2*e^
(11*c))*e^(11*d*x) - (144*a^2*e^(9*c) + 367*a*b*e^(9*c) - 109*b^2*e^(9*c))*
e^(9*d*x) - (144*a^2*e^(7*c) + 367*a*b*e^(7*c) - 109*b^2*e^(7*c))*e^(7*d*x)
 + (16*a^2*e^(5*c) + 131*a*b*e^(5*c) - 177*b^2*e^(5*c))*e^(5*d*x) - (23*a*b
*e^(3*c) - 77*b^2*e^(3*c))*e^(3*d*x) + 3*(a*b*e^c - 3*b^2*e^c)*e^(d*x))/(a^
```

```
2*b^3*d - 2*a*b^4*d + b^5*d + (a^2*b^3*d*e^(16*c) - 2*a*b^4*d*e^(16*c) + b^
5*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^3*d*e^(14*c) - 2*a*b^4*d*e^(14*c) + b^5
*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b^2*d*e^(12*c) - 23*a^2*b^3*d*e^(12*c) +
 22*a*b^4*d*e^(12*c) - 7*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^3*b^2*d*e^(10
*c) - 39*a^2*b^3*d*e^(10*c) + 30*a*b^4*d*e^(10*c) - 7*b^5*d*e^(10*c))*e^(10
*d*x) + 2*(128*a^4*b*d*e^(8*c) - 352*a^3*b^2*d*e^(8*c) + 355*a^2*b^3*d*e^(8
*c) - 166*a*b^4*d*e^(8*c) + 35*b^5*d*e^(8*c))*e^(8*d*x) + 8*(16*a^3*b^2*d*e
^(6*c) - 39*a^2*b^3*d*e^(6*c) + 30*a*b^4*d*e^(6*c) - 7*b^5*d*e^(6*c))*e^(6*
d*x) - 4*(8*a^3*b^2*d*e^(4*c) - 23*a^2*b^3*d*e^(4*c) + 22*a*b^4*d*e^(4*c) -
 7*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^2*b^3*d*e^(2*c) - 2*a*b^4*d*e^(2*c) + b^
5*d*e^(2*c))*e^(2*d*x)) + 1/128*integrate(24*((a*e^(7*c) - 3*b*e^(7*c))*e^(
7*d*x) - (3*a*e^(5*c) - 17*b*e^(5*c))*e^(5*d*x) + (3*a*e^(3*c) - 17*b*e^(3*
c))*e^(3*d*x) - (a*e^c - 3*b*e^c)*e^(d*x))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*
b^2*e^(8*c) - 2*a*b^3*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) - 4*(a^2*b^2*e^(6*c)
 - 2*a*b^3*e^(6*c) + b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^3*b*e^(4*c) - 19*a^2*b
^2*e^(4*c) + 14*a*b^3*e^(4*c) - 3*b^4*e^(4*c))*e^(4*d*x) - 4*(a^2*b^2*e^(2*
c) - 2*a*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**7/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^7/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.255 $\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$-\frac{\cosh(c+dx)\left(a^2+2b(2a+b)\cosh^2(c+dx)-11ab-2b^2\right)}{32abd(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\right.}{}$$

```
[Out] -((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) - ((3*a +
 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) + (Cosh[c + d*x]
*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a +
b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*C
osh[c + d*x]^4))
```

Rubi [A] time = 0.498432, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$-\frac{\cosh(c+dx)\left(a^2+2b(2a+b)\cosh^2(c+dx)-11ab-2b^2\right)}{32abd(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] -((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) - ((3*a +
 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) + (Cosh[c + d*x]
*(a + b - b*Cosh[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4)^2) - (Cosh[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a +
b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*C
osh[c + d*x]^4))
```


Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^5(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{2a(a-7b)+10abx^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x,\right.}{16a(a-b)bd}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(a^2-11ab-2b^2\right.}{32a(a-b)^2bd\left(a-b+2b\cosh\right.}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(a^2-11ab-2b^2\right.}{32a(a-b)^2bd\left(a-b+2b\cosh\right.}$$

$$= -\frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d} - \frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right.}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{5/4}d}$$

Mathematica [C] time = 1.90983, size = 1019, normalized size = 3.26

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^5/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((32*Cosh[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cosh[2*(c + d*x)]))/(a
*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) - (512*(a - b)*
Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c
+ d*x)] + b*Cosh[4*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#
1^4 - 4*b*#1^6 + b*#1^8 & , (2*a*b*c + b^2*c + 2*a*b*d*x + b^2*d*x + 4*a*b*
Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
 + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c
 + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 6*a^2*c*#1^2 - 32*a*b*c*#1^2 + 5*b^
2*c*#1^2 + 6*a^2*d*x*#1^2 - 32*a*b*d*x*#1^2 + 5*b^2*d*x*#1^2 + 12*a^2*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*
x)/2]*#1]*#1^2 - 64*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b^2*Log[-Cosh[(c + d*x)/2]
 - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 -
6*a^2*c*#1^4 + 32*a*b*c*#1^4 - 5*b^2*c*#1^4 - 6*a^2*d*x*#1^4 + 32*a*b*d*x*#
1^4 - 5*b^2*d*x*#1^4 - 12*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 64*a*b*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^4 - 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]
*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 2*a*b*c*#1^6 - b^2*c*#1^6 - 2*a*b*d*x*#1
^6 - b^2*d*x*#1^6 - 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh
[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 2*b^2*Log[-Cosh[(c + d*x)/2
] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/
(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ]/a)/(128*(a - b)^2*b
*d)
```

Maple [B] time = 0.096, size = 3511, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 1/32/d/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2
*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+
4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+1/16/d*b/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/
2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b
-(a*b)^(1/2)*a)^(1/2))-1/16/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2
)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2
)*a)^(1/2))+1/32/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*
(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(
a*b)^(1/2)-4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*
b^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^10-21/16/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
```

```
2*c)^2+24/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d
*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b
^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8-3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*
tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^
4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
14+21/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*
x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/
b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-3/64/d/b/(a^2-2*a*b+b^2)*a/(-a*b+(
a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a
)/(-a*b+(a*b)^(1/2)*a)^(1/2))+3/64/d/b/(a^2-2*a*b+b^2)*a/(-a*b-(a*b)^(1/2)*
a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a
*b)^(1/2)*a)^(1/2))+1/16/d/b/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arc
tan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^
(1/2))*(a*b)^(1/2)+1/16/d/b/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arct
an(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^
(1/2))*(a*b)^(1/2)+105/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a^2-105/16/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^6*a^2+63/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^
6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2
*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-63/16/d/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^10*a^2+6/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+113/4/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^6+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2
*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a
^2/b/(a^2-2*a*b+b^2)-183/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c
)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a-9/16/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^6*a+87/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*ta
nh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+
a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2
-1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+3/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*
d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(
1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-37/16/d/(t
anh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-
16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^2+13/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14-99/16/d/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^12+225/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a
+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)
^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a-17/4/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
```

```
/2*c)^10+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
*a/(a^2-2*a*b+b^2)-15/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*
a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c
)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-13/64/d/(a^2-2*a*b+b^2)/
(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(
1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+13/64/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(
1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*
b+(a*b)^(1/2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((2*a*b^2*e^(15*c) + b^3*e^(15*c))*e^(15*d*x) + (2*a^2*b*e^(13*c) - 24*
a*b^2*e^(13*c) - 5*b^3*e^(13*c))*e^(13*d*x) - (70*a^2*b*e^(11*c) - 76*a*b^2
*e^(11*c) - 9*b^3*e^(11*c))*e^(11*d*x) + (96*a^3*e^(9*c) + 164*a^2*b*e^(9*c
) - 54*a*b^2*e^(9*c) - 5*b^3*e^(9*c))*e^(9*d*x) + (96*a^3*e^(7*c) + 164*a^2
*b*e^(7*c) - 54*a*b^2*e^(7*c) - 5*b^3*e^(7*c))*e^(7*d*x) - (70*a^2*b*e^(5*c
) - 76*a*b^2*e^(5*c) - 9*b^3*e^(5*c))*e^(5*d*x) + (2*a^2*b*e^(3*c) - 24*a*b
^2*e^(3*c) - 5*b^3*e^(3*c))*e^(3*d*x) + (2*a*b^2*e^c + b^3*e^c)*e^(d*x))/(a
^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*
c) + a*b^5*d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(
14*c) + a*b^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3
*d*e^(12*c) + 22*a^2*b^4*d*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(1
6*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*
b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c)
 + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8
*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*
c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e
^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d
*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*x)) + 1/32*integra
te(4*((2*a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (6*a^2*e^(5*c) - 32*a*b*e^(
5*c) + 5*b^2*e^(5*c))*e^(5*d*x) - (6*a^2*e^(3*c) - 32*a*b*e^(3*c) + 5*b^2*e
^(3*c))*e^(3*d*x) - (2*a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b^2 - 2*a^2*b^3 + a
*b^4 + (a^3*b^2*e^(8*c) - 2*a^2*b^3*e^(8*c) + a*b^4*e^(8*c))*e^(8*d*x) - 4*
(a^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^4*
b*e^(4*c) - 19*a^3*b^2*e^(4*c) + 14*a^2*b^3*e^(4*c) - 3*a*b^4*e^(4*c))*e^(4
*d*x) - 4*(a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**5/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^5/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.256 $\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cosh(c+dx)\left(-(5a+b)\cosh^2(c+\right.}{32ad(a-b)^2\left(a-b\cosh^4(c+dx)+2b\right.}$$

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqr
t[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2*
Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a^(3
/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cosh[c + d*x]*(2 - Cosh[c + d*x
]^2))/(8*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (
Cosh[c + d*x]*(11*a + b - (5*a + b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*d*(a
- b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.518835, antiderivative size = 288, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cosh(c+dx)\left(-(5a+b)\cosh^2(c+\right.}{32ad(a-b)^2\left(a-b\cosh^4(c+dx)+2b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqr
t[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2*
Sqrt[b])*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a^(3
/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cosh[c + d*x]*(2 - Cosh[c + d*x
]^2))/(8*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)^2) - (
Cosh[c + d*x]*(11*a + b - (5*a + b)*Cosh[c + d*x]^2))/(32*a*(a - b)^2*d*(a
- b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^3(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-12ab+10abx^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x \ldots\right.}{16a(a-b)bd}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(11a+b-\ldots\right.}{32a(a-b)^2d\left(a-b+2b\cosh\ldots\right.}$$

$$= -\frac{\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\cosh(c+dx)\left(11a+b-\ldots\right.}{32a(a-b)^2d\left(a-b+2b\cosh\ldots\right.}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d} + \frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/4}d} - \frac{\ldots}{8(a-b)d\left(\ldots\right.}$$

Mathematica [C] time = 1.30044, size = 802, normalized size = 2.78

$$\frac{\frac{32\cosh(c+dx)(-17a-b+(5a+b)\cosh(2(c+dx)))}{a(8a-3b+4b\cosh(2(c+dx))-b\cosh(4(c+dx)))} + \frac{\text{RootSum}\left[b\#1^8-4b\#1^6-16a\#1^4+6b\#1^4-4b\#1^2+b\&, \frac{-5ac\#1^6-bc\#1^6-5adx\#1^6-bdx\#1^6-10a\log\left(\#1\cosh\left(\frac{1}{2}(c+dx)\right)-\ldots\right.}{\ldots}\right.}{\ldots}}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^3/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(-17*a - b + (5*a + b)*Cosh[2*(c + d*x)]))/(a*(8*a - 3*b
 + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512*(a - b)*(-5*Cosh[c
+ d*x] + Cosh[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4
```

```
*(c + d*x)])^2 + RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b
*#1^8 & , (5*a*c + b*c + 5*a*d*x + b*d*x + 10*a*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 2*b*Log[-C
osh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x
)/2]*#1] - 47*a*c*#1^2 + 5*b*c*#1^2 - 47*a*d*x*#1^2 + 5*b*d*x*#1^2 - 94*a*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
+ d*x)/2]*#1]*#1^2 + 10*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh
[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 47*a*c*#1^4 - 5*b*c*#1^4 +
47*a*d*x*#1^4 - 5*b*d*x*#1^4 + 94*a*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)
/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 10*b*Log[-Cosh[(c
 + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#
1]*#1^4 - 5*a*c*#1^6 - b*c*#1^6 - 5*a*d*x*#1^6 - b*d*x*#1^6 - 10*a*Log[-Cos
h[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/
2]*#1]*#1^6 - 2*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*
x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b
*#1^5 + b*#1^7) & ]/a)/(256*(a - b)^2*d)
```

Maple [B] time = 0.082, size = 2916, normalized size = 10.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -1/64/d/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/
2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)
+14/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2
*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6-1/32/d*b/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(
1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a
*b-(a*b)^(1/2)*a)^(1/2))+1/32/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1
/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1
/2)*a)^(1/2))-1/64/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/
4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
*(a*b)^(1/2)+18/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh
(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)
^2*b^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^10-4/d/(tanh(1/2*d*x+1/2*c)^8*
a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2
*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
4*b^2-4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x
+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-
2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^12*b^2-104/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8-
5/64/d/b/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*
d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-5
/64/d/b/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*
d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+3
2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*b^3+275/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(
1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*t
anh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8-533/8/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^6-175/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
```

```
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a+55/2/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2
*d*x+1/2*c)^6*a-141/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4-17/8/d/(tanh(1/2*d*x+1/2*c
)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x
+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)^2+1/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d
*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+29/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tan
h(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4
*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+11/2
/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b
+b^2)*tanh(1/2*d*x+1/2*c)^2-1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+1/8/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^12+15/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a-219/8/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^10+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
(a^2-2*a*b+b^2)*b-5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)^2*a/(a^2-2*a*b+b^2)+79/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+1/8/d/(a^2-2*a*b
+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(
a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))-1/8/d/(a^2-2*a*b+b^2)/(-a*b+(a*
b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/
(-a*b+(a*b)^(1/2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((5*a*b*e^(15*c) + b^2*e^(15*c))*e^(15*d*x) - (49*a*b*e^(13*c) + 5*b^
2*e^(13*c))*e^(13*d*x) - 3*(48*a^2*e^(11*c) - 55*a*b*e^(11*c) - 3*b^2*e^(11
*c))*e^(11*d*x) + (784*a^2*e^(9*c) - 377*a*b*e^(9*c) - 5*b^2*e^(9*c))*e^(9*
d*x) + (784*a^2*e^(7*c) - 377*a*b*e^(7*c) - 5*b^2*e^(7*c))*e^(7*d*x) - 3*(4
8*a^2*e^(5*c) - 55*a*b*e^(5*c) - 3*b^2*e^(5*c))*e^(5*d*x) - (49*a*b*e^(3*c)
 + 5*b^2*e^(3*c))*e^(3*d*x) + (5*a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b^2*d - 2
*a^2*b^3*d + a*b^4*d + (a^3*b^2*d*e^(16*c) - 2*a^2*b^3*d*e^(16*c) + a*b^4*d
*e^(16*c))*e^(16*d*x) - 8*(a^3*b^2*d*e^(14*c) - 2*a^2*b^3*d*e^(14*c) + a*b^
4*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b*d*e^(12*c) - 23*a^3*b^2*d*e^(12*c) +
22*a^2*b^3*d*e^(12*c) - 7*a*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b*d*e^(1
0*c) - 39*a^3*b^2*d*e^(10*c) + 30*a^2*b^3*d*e^(10*c) - 7*a*b^4*d*e^(10*c))*
```

```
e^(10*d*x) + 2*(128*a^5*d*e^(8*c) - 352*a^4*b*d*e^(8*c) + 355*a^3*b^2*d*e^(
8*c) - 166*a^2*b^3*d*e^(8*c) + 35*a*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b*
d*e^(6*c) - 39*a^3*b^2*d*e^(6*c) + 30*a^2*b^3*d*e^(6*c) - 7*a*b^4*d*e^(6*c)
)*e^(6*d*x) - 4*(8*a^4*b*d*e^(4*c) - 23*a^3*b^2*d*e^(4*c) + 22*a^2*b^3*d*e^
(4*c) - 7*a*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^2*d*e^(2*c) - 2*a^2*b^3*d*e
^(2*c) + a*b^4*d*e^(2*c))*e^(2*d*x)) - 1/8*integrate(1/2*((5*a*e^(7*c) + b*
e^(7*c))*e^(7*d*x) - (47*a*e^(5*c) - 5*b*e^(5*c))*e^(5*d*x) + (47*a*e^(3*c)
 - 5*b*e^(3*c))*e^(3*d*x) - (5*a*e^c + b*e^c)*e^(d*x))/(a^3*b - 2*a^2*b^2 +
 a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2*e^(8*c) + a*b^3*e^(8*c))*e^(8*d*x) - 4*
(a^3*b*e^(6*c) - 2*a^2*b^2*e^(6*c) + a*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^4*e^
(4*c) - 19*a^3*b*e^(4*c) + 14*a^2*b^2*e^(4*c) - 3*a*b^3*e^(4*c))*e^(4*d*x)
- 4*(a^3*b*e^(2*c) - 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**3/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^3/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.257 $\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$\frac{\cosh(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cosh^2(c+dx)\right)}{32a^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{3\left(10\sqrt{a}\right.}{}$$

```
[Out] (3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) + (3*(7*
a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) + (Cosh[c + d
*x]*(a + b - b*Cosh[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^
2 - b*Cosh[c + d*x]^4)^2) + (Cosh[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a
- b)*b*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.463176, antiderivative size = 313, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3215, 1092, 1178, 1166, 205, 208}

$$\frac{\cosh(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cosh^2(c+dx)\right)}{32a^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}+\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{3\left(10\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) + (3*(7*
a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) + (Cosh[c + d
*x]*(a + b - b*Cosh[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^
2 - b*Cosh[c + d*x]^4)^2) + (Cosh[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a
- b)*b*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
```

```
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-4(4(a-\cdots}{\left(a-b+2bx^2\cdots\right.}\cdots\right.}{16\cdots}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\left((7a-3b)(a\cdots\right.}{32a^2(a-b)^2d\left(a-b+2b\cdots\right.}$$

$$= \frac{\cosh(c+dx)\left(a+b-b\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} + \frac{\cosh(c+dx)\left((7a-3b)(a\cdots\right.}{32a^2(a-b)^2d\left(a-b+2b\cdots\right.}$$

$$= \frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}d} + \frac{3\left(7a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(\cdots}{\sqrt{\sqrt{a}+\cdots}}\right.}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt[4]{b}d}$$

Mathematica [C] time = 1.36658, size = 1018, normalized size = 3.25

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cosh[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cosh[2*(c + d*x)
]))/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)]) + (512*a*(a -
 b)*Cosh[c + d*x]*(2*a + b - b*Cosh[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[2
*(c + d*x)] + b*Cosh[4*(c + d*x)])^2 + 3*RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*a*b*c + b^2*c - 2*a*b*d*x + b^2*d*x -
 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 -
Sinh[(c + d*x)/2]*#1] + 2*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 14*a^2*c*#1^2 - 12*a*b*c*#1^
2 + 5*b^2*c*#1^2 + 14*a^2*d*x*#1^2 - 12*a*b*d*x*#1^2 + 5*b^2*d*x*#1^2 + 28*
a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sin
h[(c + d*x)/2]*#1]*#1^2 - 24*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2]
 + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 + 10*b^2*Log[-Cosh[(c
+ d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1
]*#1^2 - 14*a^2*c*#1^4 + 12*a*b*c*#1^4 - 5*b^2*c*#1^4 - 14*a^2*d*x*#1^4 + 1
2*a*b*d*x*#1^4 - 5*b^2*d*x*#1^4 - 28*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 24*a*b*Log[-
Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*
x)/2]*#1]*#1^4 - 10*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 + 2*a*b*c*#1^6 - b^2*c*#1^6 + 2
*a*b*d*x*#1^6 - b^2*d*x*#1^6 + 4*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x
)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 2*b^2*Log[-Cosh[
(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]
*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(128*a^
2*(a - b)^2*d)
```

Maple [B] time = 0.106, size = 3512, normalized size = 11.2

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 12/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*b^3-12/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6*b^3+5/
4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^2*b^2-5/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*
d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(
1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14*b^2-3/16/d
/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x
+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*b^2+3/16/d/a^2/(
a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/2*c)
^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*b^2+3/16/d/(a^2-2*a*b+b
^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(
a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)+37/4/d/(tanh(1/2*d*
x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(
1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(
1/2*d*x+1/2*c)^6+27/64/d*b/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arc
tan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)
^(1/2))-27/64/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(
2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))+3/
```

16/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*
x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-197
/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/(a^2-2
*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^10+1/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4*b^2+19/2/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^
2)/a*tanh(1/2*d*x+1/2*c)^12*b^2+118/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8-3/32/d/a
^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/
2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)*b-3/32/
d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*
x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)*b-4
0/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*b^3-1231/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tan
h(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4
*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8+831/1
6/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^6+385/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*a-385/16/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^6*a+231/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+3/16/d/(tanh(1/2*d*
x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(
1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*
d*x+1/2*c)^2+37/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*t
anh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a
+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14-283/16/d/(tanh(1/2*d*x+1/2*c
)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x
+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2
*c)^12-77/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*
a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*
tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^
4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14+7
7/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-231/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*a+857/16/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^10-5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+
1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*
d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*b+11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)-209/16/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^4-21/64/d/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*t
anh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))+21/64

```
/d/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2*d*x+1/
2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*(2*a*b^2*e^(15*c) - b^3*e^(15*c))*e^(15*d*x) - (14*a^2*b*e^(13*c) +
28*a*b^2*e^(13*c) - 15*b^3*e^(13*c))*e^(13*d*x) - (86*a^2*b*e^(11*c) - 128*
a*b^2*e^(11*c) + 27*b^3*e^(11*c))*e^(11*d*x) + (352*a^3*e^(9*c) - 60*a^2*b*
e^(9*c) - 106*a*b^2*e^(9*c) + 15*b^3*e^(9*c))*e^(9*d*x) + (352*a^3*e^(7*c)
- 60*a^2*b*e^(7*c) - 106*a*b^2*e^(7*c) + 15*b^3*e^(7*c))*e^(7*d*x) - (86*a^
2*b*e^(5*c) - 128*a*b^2*e^(5*c) + 27*b^3*e^(5*c))*e^(5*d*x) - (14*a^2*b*e^(
3*c) + 28*a*b^2*e^(3*c) - 15*b^3*e^(3*c))*e^(3*d*x) + 3*(2*a*b^2*e^c - b^3*
e^c)*e^(d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(16*c) -
2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x) - 8*(a^4*b^2*d*e^(14*
c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14*d*x) - 4*(8*a^5*b*d*e
^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*c) - 7*a^2*b^4*d*e^(12
*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2*d*e^(10*c) + 30*a^3*b
^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a^6*d*e^(8*c) - 3
52*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*b^3*d*e^(8*c) + 35*a^2
*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 39*a^4*b^2*d*e^(6*c) +
30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c
) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4
*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(
2*d*x)) + 1/2*integrate(3/4*((2*a*b*e^(7*c) - b^2*e^(7*c))*e^(7*d*x) - (14*
a^2*e^(5*c) - 12*a*b*e^(5*c) + 5*b^2*e^(5*c))*e^(5*d*x) + (14*a^2*e^(3*c) -
 12*a*b*e^(3*c) + 5*b^2*e^(3*c))*e^(3*d*x) - (2*a*b*e^c - b^2*e^c)*e^(d*x))
/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^4*b*e^(8*c) - 2*a^3*b^2*e^(8*c) + a^2*b^
3*e^(8*c))*e^(8*d*x) - 4*(a^4*b*e^(6*c) - 2*a^3*b^2*e^(6*c) + a^2*b^3*e^(6*
c))*e^(6*d*x) - 2*(8*a^5*e^(4*c) - 19*a^4*b*e^(4*c) + 14*a^3*b^2*e^(4*c) -
3*a^2*b^3*e^(4*c))*e^(4*d*x) - 4*(a^4*b*e^(2*c) - 2*a^3*b^2*e^(2*c) + a^2*b
^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.258 $\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=617

$$-\frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4a^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{b\cosh(c+dx)\left(-(5a+b)\cosh^2(c+dx)+11a+b\right)}{32a^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[
a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (b^(1/4)*ArcTan[
(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - Sqr
t[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqr
t[b]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c + d*x]]/(a^3*d)
+ (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(
5/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x]
)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]*d) + ((5*Sqrt[a]
 + 2*Sqrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b*Cosh[c + d*x]*(2 - Cosh[
c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^
4)^2) - (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - b + 2
*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)) - (b*Cosh[c + d*x]*(11*a + b - (5*
a + b)*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4))
```

Rubi [A] time = 0.81855, antiderivative size = 617, normalized size of antiderivative = 1., number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$-\frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{4a^2d(a-b)\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}-\frac{b\cosh(c+dx)\left(-(5a+b)\cosh^2(c+dx)+11a+b\right)}{32a^2d(a-b)^2\left(a-b\cosh^4(c+dx)+2b\cosh^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[
a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (b^(1/4)*ArcTan[
(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - Sqr
t[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] - Sqr
t[b]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cosh[c + d*x]]/(a^3*d)
+ (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(
5/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x]
)/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]*d) + ((5*Sqrt[a]
 + 2*Sqrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cosh[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b
]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b*Cosh[c + d*x]*(2 - Cosh[
c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^
4)^2) - (b*Cosh[c + d*x]*(2 - Cosh[c + d*x]^2))/(4*a^2*(a - b)*d*(a - b + 2
*b*Cosh[c + d*x]^2 - b*Cosh[c + d*x]^4)) - (b*Cosh[c + d*x]*(11*a + b - (5*
a + b)*Cosh[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cosh[c + d*x]^2 -
 b*Cosh[c + d*x]^4))
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cosh(c+dx)\right)}{d}$$

$$= -\frac{\operatorname{Subst}\left(\int \left(-\frac{1}{a^3(-1+x^2)} + \frac{b-bx^2}{a\left(a-b+2bx^2-bx^4\right)^3} + \frac{b-bx^2}{a^2\left(a-b+2bx^2-bx^4\right)^2} + \frac{b-bx^2}{a^3\left(a-b+2bx^2-bx^4\right)}\right) dx, x, \cosh\right.}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cosh(c+dx)\right)}{a^3 d} - \frac{\operatorname{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cosh(c+dx)\right)}{a^3 d} - \frac{\text{Subs}}{}$$

$$= -\frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} - \frac{b\cosh(c+dx)\left(2-\cosh^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cosh^2(c+dx)-b\cosh^4(c+dx)\right)^2} - \frac{}{4a^2}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}+\sqrt{b}}d} - \frac{}{8a(a-}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}d} - \frac{\tanh^{-1}(\cosh(c+dx))}{a^3 d} + \frac{\sqrt[4]{b}\tanh}{8a^{5/2}}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cosh(c+}{\sqrt{\sqrt{a}-\sqrt{}}}\right.}{2a^3\sqrt{\sqrt{a}-\sqrt{b}d}}$$

Mathematica [C] time = 5.75688, size = 1189, normalized size = 1.93

result too large to display

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((32*a*b*Cosh[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cosh[2*(c + d*x)]))/((a
 - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (512*a
^2*b*(-5*Cosh[c + d*x] + Cosh[3*(c + d*x)]))/((a - b)*(-8*a + 3*b - 4*b*Cos
h[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2) + 256*Log[Tanh[(c + d*x)/2]] - (b*
RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-45*a^
2*c + 71*a*b*c - 32*b^2*c - 45*a^2*d*x + 71*a*b*d*x - 32*b^2*d*x - 90*a^2*L
og[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
+ d*x)/2]*#1] + 142*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(
c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] - 64*b^2*Log[-Cosh[(c + d*x)/2] - Si
nh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1] + 199*a^2*c*
#1^2 - 253*a*b*c*#1^2 + 96*b^2*c*#1^2 + 199*a^2*d*x*#1^2 - 253*a*b*d*x*#1^2
 + 96*b^2*d*x*#1^2 + 398*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + C
osh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 506*a*b*Log[-Cosh[(c + d
*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#
1^2 + 192*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2
]*#1 - Sinh[(c + d*x)/2]*#1]*#1^2 - 199*a^2*c*#1^4 + 253*a*b*c*#1^4 - 96*b^
2*c*#1^4 - 199*a^2*d*x*#1^4 + 253*a*b*d*x*#1^4 - 96*b^2*d*x*#1^4 - 398*a^2*
Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c
 + d*x)/2]*#1]*#1^4 + 506*a*b*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] +
Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^4 - 192*b^2*Log[-Cosh[(c +
```

```
d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*
#1^4 + 45*a^2*c*#1^6 - 71*a*b*c*#1^6 + 32*b^2*c*#1^6 + 45*a^2*d*x*#1^6 - 71
*a*b*d*x*#1^6 + 32*b^2*d*x*#1^6 + 90*a^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c +
 d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6 - 142*a*b*Log[
-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[(c + d*x)/2]*#1 - Sinh[(c + d
*x)/2]*#1]*#1^6 + 64*b^2*Log[-Cosh[(c + d*x)/2] - Sinh[(c + d*x)/2] + Cosh[
(c + d*x)/2]*#1 - Sinh[(c + d*x)/2]*#1]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^
3 - 3*b*#1^5 + b*#1^7) & ])/(a - b)^2)/(256*a^3*d)
```

Maple [B] time = 0.138, size = 3159, normalized size = 5.1

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -1/2/d*b^2/a^3/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^
(1/2)-1/2/d*b^2/a^3/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(
2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a
*b)^(1/2)+26/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^10*b^3+70/d/(tanh(1/2*d*x+1/2*c)^8*
a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2
*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c
)^6*b^3-53/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2*b^2+5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-
4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c
)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^14
*b^2-5/32/d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*t
anh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*b^2+5
/32/d/a^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*tanh(1/2
*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*b^2-45/64/d/
(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/
2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2))*(a*b)^(1/2)-1161/8/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+1/4/d*b/(a^2-2*a*b+b^2)/a/(-a*b-(a*b)^(1/2)*
a)^(1/2)*arctan(1/4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a
*b)^(1/2)*a)^(1/2))-1/4/d*b/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*ar
ctan(1/4*(2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)
^(1/2))-45/64/d/(a^2-2*a*b+b^2)/a/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2*
tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*b
)^(1/2)-327/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
*b^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^10+185/4/d/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c
)^4*b^2+43/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^12*b^2+537/4/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^8+71/64/d/a^2/(a^2-2*a*b+b^2)/(-a*b+(a*b)^(1/2)*a)^(1/2)*arctan(1/4*(2
*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)-2*a)/(-a*b+(a*b)^(1/2)*a)^(1/2))*(a*
```

```
b)^(1/2)*b+71/64/d/a^2/(a^2-2*a*b+b^2)/(-a*b-(a*b)^(1/2)*a)^(1/2)*arctan(1/
4*(-2*tanh(1/2*d*x+1/2*c)^2*a+4*(a*b)^(1/2)+2*a)/(-a*b-(a*b)^(1/2)*a)^(1/2)
)*(a*b)^(1/2)*b-216/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^8*b^3-315/8/d/(tanh(1/2*d*x+
1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/
2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*
x+1/2*c)^8+50/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^6+10/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^2-1/d/
(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*
a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^
2)*tanh(1/2*d*x+1/2*c)^14+5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12+3/4/d*b^2/a/(tanh
(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*
b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)+13/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^10-16/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4+96/d*b^4/a^3
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^8-8/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x
+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2
*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^12-9/8/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*b+1/d/a^
3*ln(tanh(1/2*d*x+1/2*c))-257/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*((13*a*b^2*e^(15*c) - 7*b^3*e^(15*c))*e^(15*d*x) - (121*a*b^2*e^(13*c
) - 67*b^3*e^(13*c))*e^(13*d*x) - (272*a^2*b*e^(11*c) - 461*a*b^2*e^(11*c)
+ 159*b^3*e^(11*c))*e^(11*d*x) + (1424*a^2*b*e^(9*c) - 1121*a*b^2*e^(9*c) +
 99*b^3*e^(9*c))*e^(9*d*x) + (1424*a^2*b*e^(7*c) - 1121*a*b^2*e^(7*c) + 99*
b^3*e^(7*c))*e^(7*d*x) - (272*a^2*b*e^(5*c) - 461*a*b^2*e^(5*c) + 159*b^3*e
^(5*c))*e^(5*d*x) - (121*a*b^2*e^(3*c) - 67*b^3*e^(3*c))*e^(3*d*x) + (13*a*
b^2*e^c - 7*b^3*e^c)*e^(d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b
^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x) - 8*(
a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14*d*x)
- 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*c) - 7
*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2*d*e^(
10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a
^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*b^3*d*
e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 39*a^4*
```

```
b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x) - 4*(
8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a^2*b^4
*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) + a^2*b^
4*d*e^(2*c))*e^(2*d*x)) - log((e^(d*x + c) + 1)*e^(-c))/(a^3*d) + log((e^(d
*x + c) - 1)*e^(-c))/(a^3*d) - 2*integrate(1/32*((45*a^2*b*e^(7*c) - 71*a*b
^2*e^(7*c) + 32*b^3*e^(7*c))*e^(7*d*x) - (199*a^2*b*e^(5*c) - 253*a*b^2*e^(
5*c) + 96*b^3*e^(5*c))*e^(5*d*x) + (199*a^2*b*e^(3*c) - 253*a*b^2*e^(3*c) +
 96*b^3*e^(3*c))*e^(3*d*x) - (45*a^2*b*e^c - 71*a*b^2*e^c + 32*b^3*e^c)*e^(
d*x))/(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*b*e^(8*c) - 2*a^4*b^2*e^(8*c) + a
^3*b^3*e^(8*c))*e^(8*d*x) - 4*(a^5*b*e^(6*c) - 2*a^4*b^2*e^(6*c) + a^3*b^3*
e^(6*c))*e^(6*d*x) - 2*(8*a^6*e^(4*c) - 19*a^5*b*e^(4*c) + 14*a^4*b^2*e^(4*
c) - 3*a^3*b^3*e^(4*c))*e^(4*d*x) - 4*(a^5*b*e^(2*c) - 2*a^4*b^2*e^(2*c) +
a^3*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError
```

3.259 $\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=319

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tanh^9(c+d}{8ad\left((a-b)\tanh^4(c+dx)-2a\right.}$$

```
[Out] -((2*Sqrt[a] - 5*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a
^(1/4)])/(64*a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5
*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(
3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tanh[c + d*x])/(32*a
*(a - b)^2*b*d) - Tanh[c + d*x]^3/(32*a*(a - b)*b*d) + Tanh[c + d*x]^9/(8*a
*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Sech[c + d*x]^
2*Tanh[c + d*x]^5)/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*
x]^4))
```

Rubi [A] time = 0.48526, antiderivative size = 319, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3217, 1275, 12, 1120, 1279, 1166, 208}

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tanh^9(c+d}{8ad\left((a-b)\tanh^4(c+dx)-2a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] -((2*Sqrt[a] - 5*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a
^(1/4)])/(64*a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5
*Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(
3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tanh[c + d*x])/(32*a
*(a - b)^2*b*d) - Tanh[c + d*x]^3/(32*a*(a - b)*b*d) + Tanh[c + d*x]^9/(8*a
*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) - (Sech[c + d*x]^
2*Tanh[c + d*x]^5)/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*
x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
```

```
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^8(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^8(1-x^2)}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int -\frac{2bx^8}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \text{t}\right.}{16abd}$$

$$= \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8}{\left(a-2ax^2+(a-b)x^4\right)^2}\,dx, x, \text{ta}\right.}{8ad}$$

$$= \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{sech}^2(c+dx)\tanh^5(\cdots}{32abd\left(a-2a\tanh^2(c+dx)+(a\cdots\right.}$$

$$= -\frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{se}\cdots}{32abd\left(a-2a\,\text{t}\cdots\right.}$$

$$= -\frac{(a+5b)\tanh(c+dx)}{32a(a-b)^2bd} - \frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+\cdots\right.}$$

$$= -\frac{(a+5b)\tanh(c+dx)}{32a(a-b)^2bd} - \frac{\tanh^3(c+dx)}{32a(a-b)bd} + \frac{\tanh^9(c+dx)}{8ad\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+\cdots\right.}$$
$$- \frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d} - \cdots$$

Mathematica [A] time = 3.95342, size = 331, normalized size = 1.04

$$\frac{\frac{\left(2a^{3/2}\sqrt{b}-8\sqrt{a}b^{3/2}+ab+5b^2\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\sqrt{b}\left(2\sqrt{a}-5\sqrt{b}\right)\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{8b\sinh(2(c+dx))((5b-2a)\cosh(2(c+dx))\cdots}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+d\cdots}}{64b^2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^8/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (((2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*Sqrt[b]*ArcTan[((Sqrt[a] -
Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqr
t[a]*Sqrt[b]]) + ((2*a^(3/2)*Sqrt[b] + a*b - 8*Sqrt[a]*b^(3/2) + 5*b^2)*Arc
Tanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[
a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cosh[2*(c +
 d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c
 + d*x)]) + (64*a*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(-8
*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*(a - b)^2*b^
2*d)
```

Maple [C] time = 0.092, size = 2236, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -1/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+5/16/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^3+49/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-9/16/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^5*a^2-165/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a+9/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^5+5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a^2+377/16/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7
*a-49/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*a^2+377/16
/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)
^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^9*a-49/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/
2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*
x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-9/16/d/(tanh(1/2*
d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tan
h(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/
2*d*x+1/2*c)^11*a^2-165/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11*a+9/d/(tanh(1/2*d*x+1/
2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*
d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^11+5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13+49/16/d/(tanh(1/2*d*x+1/2*c)
^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+
1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^13-1/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^
2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15-5/16/d/(tanh(1/2*d*x+1/2*c)^8*a-
4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c
)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15
-1/128/d/b/(a^2-2*a*b+b^2)*sum(((a+5*b)*_R^6+(5*a-47*b)*_R^4+(-5*a+47*b)*_R
^2-a-5*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_
R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(2*a*b^2 - 5*b^3 + (a*b^2*e^(14*c) - 4*b^3*e^(14*c))*e^(14*d*x) - (32*
a^2*b*e^(12*c) - 58*a*b^2*e^(12*c) - b^3*e^(12*c))*e^(12*d*x) + 3*(48*a^2*b
*e^(10*c) - 73*a*b^2*e^(10*c) + 20*b^3*e^(10*c))*e^(10*d*x) + (256*a^3*e^(8
*c) - 832*a^2*b*e^(8*c) + 550*a*b^2*e^(8*c) - 175*b^3*e^(8*c))*e^(8*d*x) +
(112*a^2*b*e^(6*c) - 533*a*b^2*e^(6*c) + 220*b^3*e^(6*c))*e^(6*d*x) - (32*a
^2*b*e^(4*c) - 158*a*b^2*e^(4*c) + 141*b^3*e^(4*c))*e^(4*d*x) - (17*a*b^2*e
^(2*c) - 44*b^3*e^(2*c))*e^(2*d*x))/(a^2*b^4*d - 2*a*b^5*d + b^6*d + (a^2*b
^4*d*e^(16*c) - 2*a*b^5*d*e^(16*c) + b^6*d*e^(16*c))*e^(16*d*x) - 8*(a^2*b^
4*d*e^(14*c) - 2*a*b^5*d*e^(14*c) + b^6*d*e^(14*c))*e^(14*d*x) - 4*(8*a^3*b
^3*d*e^(12*c) - 23*a^2*b^4*d*e^(12*c) + 22*a*b^5*d*e^(12*c) - 7*b^6*d*e^(12
*c))*e^(12*d*x) + 8*(16*a^3*b^3*d*e^(10*c) - 39*a^2*b^4*d*e^(10*c) + 30*a*b
^5*d*e^(10*c) - 7*b^6*d*e^(10*c))*e^(10*d*x) + 2*(128*a^4*b^2*d*e^(8*c) - 3
52*a^3*b^3*d*e^(8*c) + 355*a^2*b^4*d*e^(8*c) - 166*a*b^5*d*e^(8*c) + 35*b^6
*d*e^(8*c))*e^(8*d*x) + 8*(16*a^3*b^3*d*e^(6*c) - 39*a^2*b^4*d*e^(6*c) + 30
*a*b^5*d*e^(6*c) - 7*b^6*d*e^(6*c))*e^(6*d*x) - 4*(8*a^3*b^3*d*e^(4*c) - 23
*a^2*b^4*d*e^(4*c) + 22*a*b^5*d*e^(4*c) - 7*b^6*d*e^(4*c))*e^(4*d*x) - 8*(a
^2*b^4*d*e^(2*c) - 2*a*b^5*d*e^(2*c) + b^6*d*e^(2*c))*e^(2*d*x)) - 1/256*in
tegrate(64*((a*e^(6*c) - 4*b*e^(6*c))*e^(6*d*x) + (a*e^(2*c) - 4*b*e^(2*c))
*e^(2*d*x) + 18*b*e^(4*d*x + 4*c))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2*e^(8
*c) - 2*a*b^3*e^(8*c) + b^4*e^(8*c))*e^(8*d*x) - 4*(a^2*b^2*e^(6*c) - 2*a*b
^3*e^(6*c) + b^4*e^(6*c))*e^(6*d*x) - 2*(8*a^3*b*e^(4*c) - 19*a^2*b^2*e^(4*
c) + 14*a*b^3*e^(4*c) - 3*b^4*e^(4*c))*e^(4*d*x) - 4*(a^2*b^2*e^(2*c) - 2*a
*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**8/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 17.4165, size = 527, normalized size = 1.65

$$-\frac{ab^2e^{(14\,dx+14\,c)} - 4\,b^3e^{(14\,dx+14\,c)} - 32\,a^2be^{(12\,dx+12\,c)} + 58\,ab^2e^{(12\,dx+12\,c)} + b^3e^{(12\,dx+12\,c)} + 144\,a^2be^{(10\,dx+10\,c)} - 219\,ab^2e^{(}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^8/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/8*(a*b^2*e^(14*d*x + 14*c) - 4*b^3*e^(14*d*x + 14*c) - 32*a^2*b*e^(12*d*
x + 12*c) + 58*a*b^2*e^(12*d*x + 12*c) + b^3*e^(12*d*x + 12*c) + 144*a^2*b*
e^(10*d*x + 10*c) - 219*a*b^2*e^(10*d*x + 10*c) + 60*b^3*e^(10*d*x + 10*c)
+ 256*a^3*e^(8*d*x + 8*c) - 832*a^2*b*e^(8*d*x + 8*c) + 550*a*b^2*e^(8*d*x
+ 8*c) - 175*b^3*e^(8*d*x + 8*c) + 112*a^2*b*e^(6*d*x + 6*c) - 533*a*b^2*e^
(6*d*x + 6*c) + 220*b^3*e^(6*d*x + 6*c) - 32*a^2*b*e^(4*d*x + 4*c) + 158*a*
b^2*e^(4*d*x + 4*c) - 141*b^3*e^(4*d*x + 4*c) - 17*a*b^2*e^(2*d*x + 2*c) +
44*b^3*e^(2*d*x + 2*c) + 2*a*b^2 - 5*b^3)/((a^2*b^2*d - 2*a*b^3*d + b^4*d)*
(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*
d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2)
```

3.260 $\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=345

$$\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tanh(c+dx)\left(2a \dots\right.}{32abd\left((a-b)\,\mathrm{ta}\dots\right.}$$

```
[Out] ((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
 d*x])/a^(1/4)])/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) - ((4*a +
 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/
a^(1/4)])/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) + (Tanh[c + d*x]
*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2
*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((2*a*(a^
2 - a*b - 8*b^2))/(a - b)^3 - ((2*a^2 + 15*a*b + 3*b^2)*Tanh[c + d*x]^2)/(a
 - b)^2))/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.732552, antiderivative size = 345, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 208}

$$\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tanh(c+dx)\left(2a \dots\right.}{32abd\left((a-b)\,\mathrm{ta}\dots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c +
 d*x])/a^(1/4)])/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) - ((4*a +
 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/
a^(1/4)])/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) + (Tanh[c + d*x]
*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2
*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((2*a*(a^
2 - a*b - 8*b^2))/(a - b)^3 - ((2*a^2 + 15*a*b + 3*b^2)*Tanh[c + d*x]^2)/(a
 - b)^2))/(32*a*b*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^6(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(1-x^2\right)^2}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^3b(a+3b)}{(a-b)^3}+\frac{2a^2b(5a\cdots}{\cdots}}{(a-\cdots}\cdots\right.}{\cdots}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)\left(\frac{2a\left(a^2-ab-8\cdots\right.}{(a-b)^3}\cdots\right.}{32abd\left(a-2a\tanh^2(c+\cdots\right.}$$

$$= \frac{\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)\left(\frac{2a\left(a^2-ab-8\cdots\right.}{(a-b)^3}\cdots\right.}{32abd\left(a-2a\tanh^2(c+\cdots\right.}$$

$$= \frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} - \frac{\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\cdots}}{\cdots}\right.}{64a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}\cdots}$$

Mathematica [A] time = 3.30722, size = 351, normalized size = 1.02

$$-\frac{\frac{4\sinh(2(c+dx))\left(4a^2+3b(a+b)\cosh(2(c+dx))-19ab-3b^2\right)}{ab(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)}+\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{ab^{3/2}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tan}{ab^{3/2}\sqrt{\sqrt{a}\sqrt{b}\cdots}}}{64d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^6/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] -((((Sqrt[a] + Sqrt[b])^2*(4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((Sqrt[a]
- Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a*Sqrt[-a + Sqrt[a]
*Sqrt[b]]*b^(3/2)) + ((Sqrt[a] - Sqrt[b])^2*(4*a + 10*Sqrt[a]*Sqrt[b] + 3*b
)*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(
a*Sqrt[a + Sqrt[a]*Sqrt[b]]*b^(3/2)) + (4*(4*a^2 - 19*a*b - 3*b^2 + 3*b*(a
+ b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a*b*(8*a - 3*b + 4*b*Cosh[2*(c
+ d*x)] - b*Cosh[4*(c + d*x)])) - (128*(a - b)*(2*a + b - b*Cosh[2*(c + d*x
)])*Sinh[2*(c + d*x)])/(b*(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c
 + d*x)])^2))/(64*(a - b)^2*d)
```

Maple [C] time = 0.088, size = 2681, normalized size = 7.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] 12/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+12/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*
d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(
1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-5/8/d/(
tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a
-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^7*a^2-5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*a^2+9/8/d/(tanh(
1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^11*a^2-5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c
)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13+1/8/d/(tanh(1/2
*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*ta
nh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^15-5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+9/8/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^5*a^2+1/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)^2*a^2/b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+1/4/d/(tanh(1/2*d*x+1/2*c)
^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+
1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
```

```
c)^13+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d
*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(
a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+19/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh
(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*
tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11*a-35/4/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^11-3/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-3/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^3+19/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tan
h(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a
)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a-35/4/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5
-27/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*
a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a+41/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-27/4/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^9*a+41/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+1/4/d/(tanh(1/2*d*x+1
/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2
*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x
+1/2*c)^15+1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-1/64/d/a/b/(a^2-2*a*b+b^2)*sum((a*(
-a-2*b)*_R^6+(-5*a^2+32*a*b-6*b^2)*_R^4+(5*a^2-32*a*b+6*b^2)*_R^2+a^2+2*a*b
)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Ro
otOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(3*a*b^2 + 3*b^3 - (4*a^2*b*e^(14*c) - 13*a*b^2*e^(14*c) + 3*b^3*e^(1
4*c))*e^(14*d*x) + 3*(8*a^2*b*e^(12*c) - 33*a*b^2*e^(12*c) + 7*b^3*e^(12*c)
)*e^(12*d*x) - (64*a^3*e^(10*c) + 68*a^2*b*e^(10*c) - 225*a*b^2*e^(10*c) +
63*b^3*e^(10*c))*e^(10*d*x) + 3*(128*a^3*e^(8*c) + 32*a^2*b*e^(8*c) - 61*a*
b^2*e^(8*c) + 35*b^3*e^(8*c))*e^(8*d*x) + (64*a^3*e^(6*c) + 452*a^2*b*e^(6*
c) - 9*a*b^2*e^(6*c) - 105*b^3*e^(6*c))*e^(6*d*x) - 3*(40*a^2*b*e^(4*c) - 2
9*a*b^2*e^(4*c) - 21*b^3*e^(4*c))*e^(4*d*x) + (4*a^2*b*e^(2*c) - 37*a*b^2*e
^(2*c) - 21*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^3*d - 2*a^2*b^4*d + a*b^5*d + (a
^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) + a*b^5*d*e^(16*c))*e^(16*d*x) - 8
*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*c) + a*b^5*d*e^(14*c))*e^(14*d*x)
- 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d*e^(12*c) + 22*a^2*b^4*d*e^(12*c) -
 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b^2*d*e^(10*c) - 39*a^3*b^3*d*e
^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*b^5*d*e^(10*c))*e^(10*d*x) + 2*(128*a
```

```
^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*b^3*d*e^(8*c) - 166*a^2*b^
4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8*d*x) + 8*(16*a^4*b^2*d*e^(6*c) - 39*
a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^5*d*e^(6*c))*e^(6*d*x) - 4
*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22*a^2*b^4*d*e^(4*c) - 7*a*b
^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) - 2*a^2*b^4*d*e^(2*c) + a*b^
5*d*e^(2*c))*e^(2*d*x)) + 1/64*integrate(8*((4*a^2*e^(6*c) - 13*a*b*e^(6*c)
 + 3*b^2*e^(6*c))*e^(6*d*x) + 6*(7*a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + (
4*a^2*e^(2*c) - 13*a*b*e^(2*c) + 3*b^2*e^(2*c))*e^(2*d*x))/(a^3*b^2 - 2*a^2
*b^3 + a*b^4 + (a^3*b^2*e^(8*c) - 2*a^2*b^3*e^(8*c) + a*b^4*e^(8*c))*e^(8*d
*x) - 4*(a^3*b^2*e^(6*c) - 2*a^2*b^3*e^(6*c) + a*b^4*e^(6*c))*e^(6*d*x) - 2
*(8*a^4*b*e^(4*c) - 19*a^3*b^2*e^(4*c) + 14*a^2*b^3*e^(4*c) - 3*a*b^4*e^(4*
c))*e^(4*d*x) - 4*(a^3*b^2*e^(2*c) - 2*a^2*b^3*e^(2*c) + a*b^4*e^(2*c))*e^(
2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**6/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 18.2037, size = 609, normalized size = 1.77

$$\frac{4\,a^2be^{(14\,dx+14\,c)} - 13\,ab^2e^{(14\,dx+14\,c)} + 3\,b^3e^{(14\,dx+14\,c)} - 24\,a^2be^{(12\,dx+12\,c)} + 99\,ab^2e^{(12\,dx+12\,c)} - 21\,b^3e^{(12\,dx+12\,c)} + 64\,a^3e^{(10\,dx}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^6/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/16*(4*a^2*b*e^(14*d*x + 14*c) - 13*a*b^2*e^(14*d*x + 14*c) + 3*b^3*e^(14*
d*x + 14*c) - 24*a^2*b*e^(12*d*x + 12*c) + 99*a*b^2*e^(12*d*x + 12*c) - 21*
b^3*e^(12*d*x + 12*c) + 64*a^3*e^(10*d*x + 10*c) + 68*a^2*b*e^(10*d*x + 10*
c) - 225*a*b^2*e^(10*d*x + 10*c) + 63*b^3*e^(10*d*x + 10*c) - 384*a^3*e^(8*
d*x + 8*c) - 96*a^2*b*e^(8*d*x + 8*c) + 183*a*b^2*e^(8*d*x + 8*c) - 105*b^3
*e^(8*d*x + 8*c) - 64*a^3*e^(6*d*x + 6*c) - 452*a^2*b*e^(6*d*x + 6*c) + 9*a
*b^2*e^(6*d*x + 6*c) + 105*b^3*e^(6*d*x + 6*c) + 120*a^2*b*e^(4*d*x + 4*c)
- 87*a*b^2*e^(4*d*x + 4*c) - 63*b^3*e^(4*d*x + 4*c) - 4*a^2*b*e^(2*d*x + 2*
```

```
c) + 37*a*b^2*e^(2*d*x + 2*c) + 21*b^3*e^(2*d*x + 2*c) - 3*a*b^2 - 3*b^3)/(
(a^3*b*d - 2*a^2*b^2*d + a*b^3*d)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c)
- 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2)
```

3.261 $\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=314

$$-\frac{\tanh(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}-\frac{(17a+3b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tanh}{64a^{7/4}\sqrt{b}d}$$

```
[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] +
Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7
/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tanh[c + d*x]*(3*a + b - 4*(a
 + b)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*T
anh[c + d*x]^4)^2) - (Tanh[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 - ((1
7*a + 3*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a*d*(a - 2*a*Tanh[c + d*x]^2 +
(a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.64847, antiderivative size = 314, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 208}

$$-\frac{\tanh(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}-\frac{(17a+3b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32ad\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tanh}{64a^{7/4}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c + d*x])/a^
(1/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] +
Sqrt[b])*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(7
/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tanh[c + d*x]*(3*a + b - 4*(a
 + b)*Tanh[c + d*x]^2))/(8*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*T
anh[c + d*x]^4)^2) - (Tanh[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 - ((1
7*a + 3*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a*d*(a - 2*a*Tanh[c + d*x]^2 +
(a - b)*Tanh[c + d*x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^4(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4\left(1-x^2\right)^3}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3 d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b^2(3a+b)}{(a-b)^3}-\frac{8a^2 \cdots}{\cdots}}{(\cdots}\right.}{\cdots}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3 d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\tanh(c+dx)\left(\frac{9a^2-24\cdots}{(a-\cdots}\right.}{32ad\left(a-2a\tanh^2(c+\cdots\right.}$$

$$= -\frac{b\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8(a-b)^3 d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\tanh(c+dx)\left(\frac{9a^2-24\cdots}{(a-\cdots}\right.}{32ad\left(a-2a\tanh^2(c+\cdots\right.}$$

$$= \frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}d} - \frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}d}$$

Mathematica [A] time = 4.7506, size = 316, normalized size = 1.01

$$\frac{-\frac{3\left(2a^{3/2}-3a\sqrt{b}+b^{3/2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{3\left(2a^{3/2}+3a\sqrt{b}-b^{3/2}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}+\frac{8\sinh(2(c+dx))((2a+b)\cosh(2(c+dx))-7a-2b)}{a(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)}+\cdots}{64d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^4/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((-3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTan[((Sqrt[a] - Sqrt[b])*Tanh[c
 + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*S
qrt[b]) - (3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTanh[((Sqrt[a] + Sqrt[b
])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[a + Sqrt[a]*Sqr
t[b]]*Sqrt[b]) + (8*(-7*a - 2*b + (2*a + b)*Cosh[2*(c + d*x)])*Sinh[2*(c +
d*x)])/(a*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (64*
(a - b)*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))/(-8*a + 3*b - 4*b*Cosh[
2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*(a - b)^2*d)
```

Maple [C] time = 0.083, size = 2214, normalized size = 7.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -9/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1
/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+77/16/d/(tanh(
1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^3-23/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-177/16/d/(tanh(1/2*d*x+
1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/
2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^5*a+131/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*ta
nh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+
a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5+1/d/(tanh(1/2*d*x+1/2*c)^8*a-4
*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)
^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5*b
^2+109/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d
*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a-367/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-9/d/
(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*
a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^7+109/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d
*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1
/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*a-367/16/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
```

```
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^9-9/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*
a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c
)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9-177/16/d/(tanh(1/2*d
*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh
(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d
*x+1/2*c)^11*a+131/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+
6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^
2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+1/d/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c
)^11*b^2+77/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(
1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^
2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-23/16/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^1
3-9/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+
1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15+3/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15*b-3/128/d
/(a^2-2*a*b+b^2)/a*sum(((3*a-b)*_R^6+(-17*a+3*b)*_R^4+(17*a-3*b)*_R^2-3*a+b
)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=Ro
otOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(3*a*b^2*e^(14*d*x + 14*c) + 2*a*b^2 + b^3 - 3*(10*a*b^2*e^(12*c) - b^3
*e^(12*c))*e^(12*d*x) - (80*a^2*b*e^(10*c) - 111*a*b^2*e^(10*c) + 16*b^3*e^
(10*c))*e^(10*d*x) + (256*a^3*e^(8*c) - 64*a^2*b*e^(8*c) - 26*a*b^2*e^(8*c)
 + 35*b^3*e^(8*c))*e^(8*d*x) + (336*a^2*b*e^(6*c) - 95*a*b^2*e^(6*c) - 40*b
^3*e^(6*c))*e^(6*d*x) - (64*a^2*b*e^(4*c) - 54*a*b^2*e^(4*c) - 25*b^3*e^(4*
c))*e^(4*d*x) - (19*a*b^2*e^(2*c) + 8*b^3*e^(2*c))*e^(2*d*x))/(a^3*b^3*d -
2*a^2*b^4*d + a*b^5*d + (a^3*b^3*d*e^(16*c) - 2*a^2*b^4*d*e^(16*c) + a*b^5*
d*e^(16*c))*e^(16*d*x) - 8*(a^3*b^3*d*e^(14*c) - 2*a^2*b^4*d*e^(14*c) + a*b
^5*d*e^(14*c))*e^(14*d*x) - 4*(8*a^4*b^2*d*e^(12*c) - 23*a^3*b^3*d*e^(12*c)
 + 22*a^2*b^4*d*e^(12*c) - 7*a*b^5*d*e^(12*c))*e^(12*d*x) + 8*(16*a^4*b^2*d
*e^(10*c) - 39*a^3*b^3*d*e^(10*c) + 30*a^2*b^4*d*e^(10*c) - 7*a*b^5*d*e^(10
*c))*e^(10*d*x) + 2*(128*a^5*b*d*e^(8*c) - 352*a^4*b^2*d*e^(8*c) + 355*a^3*
b^3*d*e^(8*c) - 166*a^2*b^4*d*e^(8*c) + 35*a*b^5*d*e^(8*c))*e^(8*d*x) + 8*(
16*a^4*b^2*d*e^(6*c) - 39*a^3*b^3*d*e^(6*c) + 30*a^2*b^4*d*e^(6*c) - 7*a*b^
5*d*e^(6*c))*e^(6*d*x) - 4*(8*a^4*b^2*d*e^(4*c) - 23*a^3*b^3*d*e^(4*c) + 22
*a^2*b^4*d*e^(4*c) - 7*a*b^5*d*e^(4*c))*e^(4*d*x) - 8*(a^3*b^3*d*e^(2*c) -
2*a^2*b^4*d*e^(2*c) + a*b^5*d*e^(2*c))*e^(2*d*x)) + 1/16*integrate(-12*(2*(
4*a*e^(4*c) - b*e^(4*c))*e^(4*d*x) - a*e^(6*d*x + 6*c) - a*e^(2*d*x + 2*c))
/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(8*c) - 2*a^2*b^2*e^(8*c) + a*b^3*e^
(8*c))*e^(8*d*x) - 4*(a^3*b*e^(6*c) - 2*a^2*b^2*e^(6*c) + a*b^3*e^(6*c))*e^
(6*d*x) - 2*(8*a^4*e^(4*c) - 19*a^3*b*e^(4*c) + 14*a^2*b^2*e^(4*c) - 3*a*b^
3*e^(4*c))*e^(4*d*x) - 4*(a^3*b*e^(2*c) - 2*a^2*b^2*e^(2*c) + a*b^3*e^(2*c)
)*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**4/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 17.54, size = 489, normalized size = 1.56

$$\frac{3\,ab^2e^{(14\,dx+14\,c)} - 30\,ab^2e^{(12\,dx+12\,c)} + 3\,b^3e^{(12\,dx+12\,c)} - 80\,a^2be^{(10\,dx+10\,c)} + 111\,ab^2e^{(10\,dx+10\,c)} - 16\,b^3e^{(10\,dx+10\,c)} + 256\,a^3e^{(8}}{8\left(a^3bd\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^4/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*a*b^2*e^(14*d*x + 14*c) - 30*a*b^2*e^(12*d*x + 12*c) + 3*b^3*e^(12*d
*x + 12*c) - 80*a^2*b*e^(10*d*x + 10*c) + 111*a*b^2*e^(10*d*x + 10*c) - 16*
b^3*e^(10*d*x + 10*c) + 256*a^3*e^(8*d*x + 8*c) - 64*a^2*b*e^(8*d*x + 8*c)
- 26*a*b^2*e^(8*d*x + 8*c) + 35*b^3*e^(8*d*x + 8*c) + 336*a^2*b*e^(6*d*x +
6*c) - 95*a*b^2*e^(6*d*x + 6*c) - 40*b^3*e^(6*d*x + 6*c) - 64*a^2*b*e^(4*d*
x + 4*c) + 54*a*b^2*e^(4*d*x + 4*c) + 25*b^3*e^(4*d*x + 4*c) - 19*a*b^2*e^(
2*d*x + 2*c) - 8*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + b^3)/((a^3*b*d - 2*a^2*b^2
*d + a*b^3*d)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*
c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2)
```

3.262 $\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=348

$$\frac{\tanh(c+dx)\left(\frac{2a\left(5a^2-9ab-4b^2\right)}{(a-b)^3}-\frac{5\left(2a^2+3ab-b^2\right)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8ad(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}$$

```
[Out] -((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c
 + d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) + ((12*
a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x
])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) + (b*Tanh[c +
 d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d
*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((
2*a*(5*a^2 - 9*a*b - 4*b^2))/(a - b)^3 - (5*(2*a^2 + 3*a*b - b^2)*Tanh[c +
d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d
*x]^4))
```

Rubi [A] time = 0.664354, antiderivative size = 348, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 208}

$$\frac{\tanh(c+dx)\left(\frac{2a\left(5a^2-9ab-4b^2\right)}{(a-b)^3}-\frac{5\left(2a^2+3ab-b^2\right)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8ad(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] -((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c
 + d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) + ((12*
a + 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x
])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) + (b*Tanh[c +
 d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d
*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4)^2) + (Tanh[c + d*x]*((
2*a*(5*a^2 - 9*a*b - 4*b^2))/(a - b)^3 - (5*(2*a^2 + 3*a*b - b^2)*Tanh[c +
d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d
*x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
```

```
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\sinh^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1-x^2\right)^4}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{\frac{2a^2b^2(a+3b)}{(a-b)^3}-\frac{2ab\left(8a^3-2\right.}{}}{}\right.}{}$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)\left(\frac{2a\left(5a^2-9ab-4b\right.}{(a-b)^3}\right.}{32a^2d\left(a-2a\tanh^2(c+a\right.}$$

$$= \frac{b\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{\tanh(c+dx)\left(\frac{2a\left(5a^2-9ab-4b\right.}{(a-b)^3}\right.}{32a^2d\left(a-2a\tanh^2(c+a\right.}$$

$$= -\frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}d} + \frac{\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\tanh^{-1}\left(\sqrt{\sqrt{a}\right.}{64a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}}$$

Mathematica [A] time = 4.87274, size = 343, normalized size = 0.99

$$\frac{\frac{4\sinh(2(c+dx))\left(12a^2+b(5b-11a)\cosh(2(c+dx))+11ab-5b^2\right)}{8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b}+\frac{\left(14\sqrt{a}\sqrt{b}+12a+5b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-14\sqrt{a}\sqrt{b}+12a+\right.}{\sqrt{b}\sqrt{}}}{64a^2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sinh[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] (((Sqrt[a] + Sqrt[b])^2*(12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[((Sqrt[a]
- Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*S
qrt[b]]*Sqrt[b]) + ((Sqrt[a] - Sqrt[b])^2*(12*a + 14*Sqrt[a]*Sqrt[b] + 5*b)
*ArcTanh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(S
qrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + (4*(12*a^2 + 11*a*b - 5*b^2 + b*(-11*a
+ 5*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d
*x)] - b*Cosh[4*(c + d*x)]) + (128*a*(a - b)*(2*a + b - b*Cosh[2*(c + d*x)]
)*Sinh[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*
x)])^2)/(64*a^2*(a - b)^2*d)
```

Maple [C] time = 0.095, size = 2670, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -1/64/d/a^2/(a^2-2*a*b+b^2)*sum((a*(-5*a+2*b)*_R^6+(39*a^2-28*a*b+10*b^2)*_
R^4+(-39*a^2+28*a*b-10*b^2)*_R^2+5*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_
R^3*b-_R*a)*ln(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)
*_Z^4-4*a*_Z^2+a))+97/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*
a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c
)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+97/2/d/(tanh(1/2*d*x
+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1
/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1
/2*d*x+1/2*c)^7-20/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*t
anh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a
+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b^3+9/4/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^3*b^2+9/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh
(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)
^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13*b^2-20/d/(tanh(1/2*d*x+1/2*c)^8
*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/
2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^9*b^3-27/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5*b^2-27/4/d/(tanh(1/2*d*x+1/2*c)^8*
a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2
*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^
11*b^2-5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-5/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tan
h(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4
```

```
*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3-1/4/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2
)*tanh(1/2*d*x+1/2*c)^15*b-1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/
2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*
x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+45/8/d/(tanh(1/2*d*
x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(
1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*
x+1/2*c)^11*a+3/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh
(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)
^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-25/8/d/(tanh(1/2*d*x+1/2*c)^8*a
-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*
c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^1
3-25/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+
1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2
-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+45/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5*a+3/d/(tanh
(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*
b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^5-25/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6
*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*
c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*a-1/4/d/(tanh(1/2*d*x+1/2
*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d
*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1
/2*c)^7-25/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/
2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/
(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*a-1/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*ta
nh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-
4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+5/8/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+
b^2)*tanh(1/2*d*x+1/2*c)^15+5/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2*a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(11*a*b^2 - 5*b^3 + (12*a^2*b*e^(14*c) - 11*a*b^2*e^(14*c) + 5*b^3*e^
(14*c))*e^(14*d*x) - (104*a^2*b*e^(12*c) - 85*a*b^2*e^(12*c) + 35*b^3*e^(12
*c))*e^(12*d*x) - (320*a^3*e^(10*c) - 652*a^2*b*e^(10*c) + 407*a*b^2*e^(10*
c) - 105*b^3*e^(10*c))*e^(10*d*x) + (1408*a^3*e^(8*c) - 1696*a^2*b*e^(8*c)
+ 865*a*b^2*e^(8*c) - 175*b^3*e^(8*c))*e^(8*d*x) + (320*a^3*e^(6*c) + 756*a
^2*b*e^(6*c) - 849*a*b^2*e^(6*c) + 175*b^3*e^(6*c))*e^(6*d*x) - (248*a^2*b*
e^(4*c) - 383*a*b^2*e^(4*c) + 105*b^3*e^(4*c))*e^(4*d*x) - (12*a^2*b*e^(2*c
) + 77*a*b^2*e^(2*c) - 35*b^3*e^(2*c))*e^(2*d*x))/(a^4*b^2*d - 2*a^3*b^3*d
+ a^2*b^4*d + (a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*
c))*e^(16*d*x) - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e
^(14*c))*e^(14*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^
3*b^3*d*e^(12*c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c
) - 39*a^4*b^2*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e
```

```
^(10*d*x) + 2*(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8
*c) - 166*a^3*b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b
*d*e^(6*c) - 39*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6
*c))*e^(6*d*x) - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d
*e^(4*c) - 7*a^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^
3*d*e^(2*c) + a^2*b^4*d*e^(2*c))*e^(2*d*x)) - 1/4*integrate(1/2*((12*a^2*e^
(6*c) - 11*a*b*e^(6*c) + 5*b^2*e^(6*c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 19*
a*b*e^(4*c) + 5*b^2*e^(4*c))*e^(4*d*x) + (12*a^2*e^(2*c) - 11*a*b*e^(2*c) +
 5*b^2*e^(2*c))*e^(2*d*x))/(a^4*b - 2*a^3*b^2 + a^2*b^3 + (a^4*b*e^(8*c) -
2*a^3*b^2*e^(8*c) + a^2*b^3*e^(8*c))*e^(8*d*x) - 4*(a^4*b*e^(6*c) - 2*a^3*b
^2*e^(6*c) + a^2*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^5*e^(4*c) - 19*a^4*b*e^(4*
c) + 14*a^3*b^2*e^(4*c) - 3*a^2*b^3*e^(4*c))*e^(4*d*x) - 4*(a^4*b*e^(2*c) -
 2*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 17.2049, size = 608, normalized size = 1.75

$$-\frac{12\,a^2be^{(14\,dx+14\,c)} - 11\,ab^2e^{(14\,dx+14\,c)} + 5\,b^3e^{(14\,dx+14\,c)} - 104\,a^2be^{(12\,dx+12\,c)} + 85\,ab^2e^{(12\,dx+12\,c)} - 35\,b^3e^{(12\,dx+12\,c)} - 320}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sinh(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/16*(12*a^2*b*e^(14*d*x + 14*c) - 11*a*b^2*e^(14*d*x + 14*c) + 5*b^3*e^(1
4*d*x + 14*c) - 104*a^2*b*e^(12*d*x + 12*c) + 85*a*b^2*e^(12*d*x + 12*c) -
35*b^3*e^(12*d*x + 12*c) - 320*a^3*e^(10*d*x + 10*c) + 652*a^2*b*e^(10*d*x
+ 10*c) - 407*a*b^2*e^(10*d*x + 10*c) + 105*b^3*e^(10*d*x + 10*c) + 1408*a^
3*e^(8*d*x + 8*c) - 1696*a^2*b*e^(8*d*x + 8*c) + 865*a*b^2*e^(8*d*x + 8*c)
- 175*b^3*e^(8*d*x + 8*c) + 320*a^3*e^(6*d*x + 6*c) + 756*a^2*b*e^(6*d*x +
6*c) - 849*a*b^2*e^(6*d*x + 6*c) + 175*b^3*e^(6*d*x + 6*c) - 248*a^2*b*e^(4
*d*x + 4*c) + 383*a*b^2*e^(4*d*x + 4*c) - 105*b^3*e^(4*d*x + 4*c) - 12*a^2*
```

```
b*e^(2*d*x + 2*c) - 77*a*b^2*e^(2*d*x + 2*c) + 35*b^3*e^(2*d*x + 2*c) + 11*
a*b^2 - 5*b^3)/((a^4*d - 2*a^3*b*d + a^2*b^2*d)*(b*e^(8*d*x + 8*c) - 4*b*e^
(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d*x + 4*c) - 4*b*e^(2*d*x +
 2*c) + b)^2)
```

3.263 $\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=320

$$-\frac{b\tanh(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}-\frac{(33a-13b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{(50\cdots}{\cdots}$$

```
[Out] ((32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c
 + d*x])/a^(1/4)])/(64*a^(11/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + ((32*a + 50*
Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(
1/4)])/(64*a^(11/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b^2*Tanh[c + d*x]*(3*a
+ b - 4*(a + b)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2
 + (a - b)*Tanh[c + d*x]^4)^2) - (b*Tanh[c + d*x]*((17*a^2 - 40*a*b + 7*b^2
)/(a - b)^3 - ((33*a - 13*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*
a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 0.60877, antiderivative size = 320, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3209, 1205, 1678, 1166, 208}

$$-\frac{b\tanh(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}-\frac{(33a-13b)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}+\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{(50\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sinh[c + d*x]^4)^(-3),x]
```


```
[Out] ((32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt[b]]*Tanh[c
 + d*x])/a^(1/4)])/(64*a^(11/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + ((32*a + 50*
Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b]]*Tanh[c + d*x])/a^(
1/4)])/(64*a^(11/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b^2*Tanh[c + d*x]*(3*a
+ b - 4*(a + b)*Tanh[c + d*x]^2))/(8*a*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2
 + (a - b)*Tanh[c + d*x]^4)^2) - (b*Tanh[c + d*x]*((17*a^2 - 40*a*b + 7*b^2
)/(a - b)^3 - ((33*a - 13*b)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a - 2*
a*Tanh[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```


Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
```

```
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^5}{\left(a-2ax^2+(a-b)x^4\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=-\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-\frac{\text{Subst}\left(\int \frac{-\frac{2ab\left(8a^3-24a^2b+27ab\ldots\right.}{(a-b)^3}}{\ldots}\right.}{\ldots}$$

$$=-\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-\frac{b\tanh(c+dx)\left(\frac{17a^2-40a\ldots}{(a-b)\ldots}\right.}{32a^2d\left(a-2a\tanh^2(c+\ldots\right.}$$

$$=-\frac{b^2\tanh(c+dx)\left(3a+b-4(a+b)\tanh^2(c+dx)\right)}{8a(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2}-\frac{b\tanh(c+dx)\left(\frac{17a^2-40a\ldots}{(a-b)\ldots}\right.}{32a^2d\left(a-2a\tanh^2(c+\ldots\right.}$$

$$=\frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{\ldots}}}{\ldots}\right.}{64a^{11/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\ldots}$$

Mathematica [A] time = 2.98798, size = 333, normalized size = 1.04

$$\frac{\frac{64a^{3/2}b(a-b)(\sinh(4(c+dx))-6\sinh(2(c+dx)))}{(-8a-4b\cosh(2(c+dx))+b\cosh(4(c+dx))+3b)^2}+\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tan^{-1}\left(\right.}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{64a^{5/2}d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sinh[c + d*x]^4)^(-3),x]
```

```
[Out] (-(((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt[
a] - Sqrt[b])*Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]
*Sqrt[b]]) + ((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcT
anh[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a
+ Sqrt[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cosh[2*(c + d
*x)])*Sinh[2*(c + d*x)])/(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c +
 d*x)]) + (64*a^(3/2)*(a - b)*b*(-6*Sinh[2*(c + d*x)] + Sinh[4*(c + d*x)]))
/(-8*a + 3*b - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2)/(64*a^(5/2)*
(a - b)^2*d)
```

Maple [C] time = 0.103, size = 2290, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -17/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+
1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2
*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b+11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)+149/16/d
/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4
*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b
^2)*tanh(1/2*d*x+1/2*c)^3-95/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1
/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d
*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2-345/16/d/(ta
nh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-1
6*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*
tanh(1/2*d*x+1/2*c)^5+427/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*
c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+
1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^5*b^2-7/d/(tanh(1/2*d
*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh
(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^5+213/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)
^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/
2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-1111/16/d/(tanh(1/2*d
*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh
(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh
(1/2*d*x+1/2*c)^7+31/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7*b^3+213/16/d/(tanh(1/2*d*
x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(
1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*
```

```
d*x+1/2*c)^9-1111/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6
*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2
*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+31/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+
1/2*c)^9*b^3-345/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*
tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*
a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+427/16/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/
2*c)^11*b^2-7/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+149/16/d/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/
2*c)^13-95/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1
/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2
/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13*b^2-17/16/d/(tanh(1/2*d*x+1/2*c)^
8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1
/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^
15*b+11/16/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*
d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^
2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15-1/128/d/(a^2-2*a*b+b^2)/a^2*sum(
((32*a^2-47*a*b+21*b^2)*_R^6+(-96*a^2+85*a*b-31*b^2)*_R^4+(96*a^2-85*a*b+31
*b^2)*_R^2-32*a^2+47*a*b-21*b^2)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*l
n(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^
2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(6*a*b^2 - 3*b^3 + (7*a*b^2*e^(14*c) - 4*b^3*e^(14*c))*e^(14*d*x) - (32
*a^2*b*e^(12*c) + 2*a*b^2*e^(12*c) - 7*b^3*e^(12*c))*e^(12*d*x) - (16*a^2*b
*e^(10*c) - 3*a*b^2*e^(10*c) - 28*b^3*e^(10*c))*e^(10*d*x) + 3*(256*a^3*e^(
8*c) - 320*a^2*b*e^(8*c) + 166*a*b^2*e^(8*c) - 35*b^3*e^(8*c))*e^(8*d*x) +
(784*a^2*b*e^(6*c) - 723*a*b^2*e^(6*c) + 140*b^3*e^(6*c))*e^(6*d*x) - (160*
a^2*b*e^(4*c) - 266*a*b^2*e^(4*c) + 91*b^3*e^(4*c))*e^(4*d*x) - (55*a*b^2*e
^(2*c) - 28*b^3*e^(2*c))*e^(2*d*x))/(a^4*b^2*d - 2*a^3*b^3*d + a^2*b^4*d +
(a^4*b^2*d*e^(16*c) - 2*a^3*b^3*d*e^(16*c) + a^2*b^4*d*e^(16*c))*e^(16*d*x)
 - 8*(a^4*b^2*d*e^(14*c) - 2*a^3*b^3*d*e^(14*c) + a^2*b^4*d*e^(14*c))*e^(14
*d*x) - 4*(8*a^5*b*d*e^(12*c) - 23*a^4*b^2*d*e^(12*c) + 22*a^3*b^3*d*e^(12*
c) - 7*a^2*b^4*d*e^(12*c))*e^(12*d*x) + 8*(16*a^5*b*d*e^(10*c) - 39*a^4*b^2
*d*e^(10*c) + 30*a^3*b^3*d*e^(10*c) - 7*a^2*b^4*d*e^(10*c))*e^(10*d*x) + 2*
(128*a^6*d*e^(8*c) - 352*a^5*b*d*e^(8*c) + 355*a^4*b^2*d*e^(8*c) - 166*a^3*
b^3*d*e^(8*c) + 35*a^2*b^4*d*e^(8*c))*e^(8*d*x) + 8*(16*a^5*b*d*e^(6*c) - 3
9*a^4*b^2*d*e^(6*c) + 30*a^3*b^3*d*e^(6*c) - 7*a^2*b^4*d*e^(6*c))*e^(6*d*x)
 - 4*(8*a^5*b*d*e^(4*c) - 23*a^4*b^2*d*e^(4*c) + 22*a^3*b^3*d*e^(4*c) - 7*a
^2*b^4*d*e^(4*c))*e^(4*d*x) - 8*(a^4*b^2*d*e^(2*c) - 2*a^3*b^3*d*e^(2*c) +
a^2*b^4*d*e^(2*c))*e^(2*d*x)) + integrate(1/4*((7*a*b*e^(6*c) - 4*b^2*e^(6*
c))*e^(6*d*x) - 2*(32*a^2*e^(4*c) - 40*a*b*e^(4*c) + 17*b^2*e^(4*c))*e^(4*d
*x) + (7*a*b*e^(2*c) - 4*b^2*e^(2*c))*e^(2*d*x))/(a^4*b - 2*a^3*b^2 + a^2*b
^3 + (a^4*b*e^(8*c) - 2*a^3*b^2*e^(8*c) + a^2*b^3*e^(8*c))*e^(8*d*x) - 4*(a
```

```
^4*b*e^(6*c) - 2*a^3*b^2*e^(6*c) + a^2*b^3*e^(6*c))*e^(6*d*x) - 2*(8*a^5*e^
(4*c) - 19*a^4*b*e^(4*c) + 14*a^3*b^2*e^(4*c) - 3*a^2*b^3*e^(4*c))*e^(4*d*x
) - 4*(a^4*b*e^(2*c) - 2*a^3*b^2*e^(2*c) + a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

Giac [A] time = 17.8292, size = 529, normalized size = 1.65

$$7\, ab^2 e^{(14\, dx+14\, c)} - 4\, b^3 e^{(14\, dx+14\, c)} - 32\, a^2 b e^{(12\, dx+12\, c)} - 2\, ab^2 e^{(12\, dx+12\, c)} + 7\, b^3 e^{(12\, dx+12\, c)} - 16\, a^2 b e^{(10\, dx+10\, c)} + 3\, ab^2 e^{(10\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/8*(7*a*b^2*e^(14*d*x + 14*c) - 4*b^3*e^(14*d*x + 14*c) - 32*a^2*b*e^(12*d
*x + 12*c) - 2*a*b^2*e^(12*d*x + 12*c) + 7*b^3*e^(12*d*x + 12*c) - 16*a^2*b
*e^(10*d*x + 10*c) + 3*a*b^2*e^(10*d*x + 10*c) + 28*b^3*e^(10*d*x + 10*c) +
 768*a^3*e^(8*d*x + 8*c) - 960*a^2*b*e^(8*d*x + 8*c) + 498*a*b^2*e^(8*d*x +
 8*c) - 105*b^3*e^(8*d*x + 8*c) + 784*a^2*b*e^(6*d*x + 6*c) - 723*a*b^2*e^(
6*d*x + 6*c) + 140*b^3*e^(6*d*x + 6*c) - 160*a^2*b*e^(4*d*x + 4*c) + 266*a*
b^2*e^(4*d*x + 4*c) - 91*b^3*e^(4*d*x + 4*c) - 55*a*b^2*e^(2*d*x + 2*c) + 2
8*b^3*e^(2*d*x + 2*c) + 6*a*b^2 - 3*b^3)/((a^4*d - 2*a^3*b*d + a^2*b^2*d)*(
b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(4*d
*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2)
```

3.264 $\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=359

$$\frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2d(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}+\frac{b\tanh(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}-\frac{\left(18a^2+15ab-13b^2\right)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^3d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-$$

```
[Out] (-3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt
[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + (
3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot
h[c + d*x]/(a^3*d) + (b^2*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*
Tanh[c + d*x]^2))/(8*a^2*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tan
h[c + d*x]^4)^2) + (b*Tanh[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 - ((18*
a^2 + 15*a*b - 13*b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a - 2*a*Tanh
[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```

Rubi [A] time = 1.1577, antiderivative size = 359, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3217, 1334, 1669, 1664, 1166, 208}

$$\frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2d(a-b)^3\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)^2}+\frac{b\tanh(c+dx)\left(\frac{2a^2(9a-17b)}{(a-b)^3}-\frac{\left(18a^2+15ab-13b^2\right)\tanh^2(c+dx)}{(a-b)^2}\right)}{32a^3d\left((a-b)\tanh^4(c+dx)-2a\tanh^2(c+dx)+a\right)}-$$

Antiderivative was successfully verified.

```
[In] Int[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```


```
[Out] (-3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] - Sqrt
[b]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + (
3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh[(Sqrt[Sqrt[a] + Sqrt[b
]]*Tanh[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot
h[c + d*x]/(a^3*d) + (b^2*Tanh[c + d*x]*(a*(a + 3*b) - (a^2 + 6*a*b + b^2)*
Tanh[c + d*x]^2))/(8*a^2*(a - b)^3*d*(a - 2*a*Tanh[c + d*x]^2 + (a - b)*Tan
h[c + d*x]^4)^2) + (b*Tanh[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 - ((18*
a^2 + 15*a*b - 13*b^2)*Tanh[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a - 2*a*Tanh
[c + d*x]^2 + (a - b)*Tanh[c + d*x]^4))
```


Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rule 1334

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
```

```
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{csch}^2(c+dx)}{\left(a-b\sinh^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^6}{x^2\left(a-2ax^2+(a-b)x^4\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-16ab+\frac{2ab(32a^3-96a\cdots}{(\cdots}}{\cdots}\right)}{\cdots}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{b\tanh(c+dx)\left(\frac{2a^2(9a-17b\cdots}{(a-b)^3}\cdots\right)}{32a^3d\left(a-2a\tanh^2(c+\cdots\right)}$$

$$= \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{b\tanh(c+dx)\left(\frac{2a^2(9a-17b\cdots}{(a-b)^3}\cdots\right)}{32a^3d\left(a-2a\tanh^2(c+\cdots\right)}$$

$$= -\frac{\coth(c+dx)}{a^3d} + \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{b\tanh(\cdots}{32a^3d\,(\cdots}$$

$$= -\frac{\coth(c+dx)}{a^3d} + \frac{b^2\tanh(c+dx)\left(a(a+3b)-\left(a^2+6ab+b^2\right)\tanh^2(c+dx)\right)}{8a^2(a-b)^3d\left(a-2a\tanh^2(c+dx)+(a-b)\tanh^4(c+dx)\right)^2} + \frac{b\tanh(\cdots}{32a^3d\,(\cdots}$$

$$= -\frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tanh^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tanh(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} + \frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+15b\right)\cdots}{64a^{13/4}\left(\sqrt{a}\cdots\right.}$$

Mathematica [A] time = 3.45587, size = 357, normalized size = 0.99

$$\frac{\frac{4b\sinh(2(c+dx))\left(28a^2+b(13b-19a)\cosh(2(c+dx))+3ab-13b^2\right)}{(a-b)^2(8a+4b\cosh(2(c+dx))-b\cosh(4(c+dx))-3b)} + \frac{3\sqrt{b}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{3\sqrt{b}\left(-34\sqrt{a}\sqrt{b}+20a+15b\right)\tan^{-1}\left(\cdots\right)}{\left(\sqrt{a}-\sqrt{b}\right)^2\sqrt{\sqrt{a}\sqrt{b}\cdots}}\cdots}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csch[c + d*x]^2/(a - b*Sinh[c + d*x]^4)^3,x]
```

```
[Out] ((3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[((Sqrt[a] - Sqrt[b])*
Tanh[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])^2*Sqrt[-a
+ Sqrt[a]*Sqrt[b]]) + (3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTanh
[((Sqrt[a] + Sqrt[b])*Tanh[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a]
+ Sqrt[b])^2*Sqrt[a + Sqrt[a]*Sqrt[b]]) - 64*Coth[c + d*x] + (4*b*(28*a^2 +
 3*a*b - 13*b^2 + b*(-19*a + 13*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((
a - b)^2*(8*a - 3*b + 4*b*Cosh[2*(c + d*x)] - b*Cosh[4*(c + d*x)])) + (128*
a*b*(2*a + b - b*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/((a - b)*(-8*a + 3*b
 - 4*b*Cosh[2*(c + d*x)] + b*Cosh[4*(c + d*x)])^2))/(64*a^3*d)
```

Maple [C] time = 0.158, size = 2747, normalized size = 7.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x)
```

```
[Out] -3/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/
2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/a/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)-19/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/
2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tan
h(1/2*d*x+1/2*c)^2*a+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^5-19/
4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2*b^3/(a^
2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11-3/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1
/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*ta
nh(1/2*d*x+1/2*c)^2*a+a)^2*b^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^15+25/
4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c
)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-
2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+25/4/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2
*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh
(1/2*d*x+1/2*c)^2*a+a)^2/a*b^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7+153/2/
d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^
4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*
b+b^2)*tanh(1/2*d*x+1/2*c)^7*b^3-2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3*b^2-2/d/(tanh
(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*
b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/a/(a^2-2*a*b+b^2)*ta
nh(1/2*d*x+1/2*c)^13*b^2+153/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/
2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*
x+1/2*c)^2*a+a)^2/a^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9*b^3-7/2/d/(tanh
(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*
b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*ta
nh(1/2*d*x+1/2*c)^5*b^2-7/2/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c
)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1
/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)/a*tanh(1/2*d*x+1/2*c)^11*b^2-3/64/d*b/a^3/(a
^2-2*a*b+b^2)*sum((a*(-3*a+2*b)*_R^6+(49*a^2-72*a*b+30*b^2)*_R^4+(-49*a^2+7
2*a*b-30*b^2)*_R^2+3*a^2-2*a*b)/(_R^7*a-3*_R^5*a+3*_R^3*a-8*_R^3*b-_R*a)*ln
(tanh(1/2*d*x+1/2*c)-_R),_R=RootOf(a*_Z^8-4*a*_Z^6+(6*a-16*b)*_Z^4-4*a*_Z^2
+a))+17/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*ta
nh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+
a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-52/d*b^4/a^3/(tanh(1/2*d*x+1/2*
c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*
x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*
c)^7-52/d*b^4/a^3/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh
(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)
^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^9+17/4/d*b^3/a^2/(tanh(1/2*d*x+1/2*c
)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x
+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c
)^3-45/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*
x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a
^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^13-45/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh
(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*
tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^3+9/8/d/
(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*
a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)
*tanh(1/2*d*x+1/2*c)^15*b+9/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2
*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x
+1/2*c)^2*a+a)^2/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)*b-1/2/d/a^3*tanh(1/2*d
*x+1/2*c)-1/2/d/a^3/tanh(1/2*d*x+1/2*c)+81/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*t
anh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4
```

```
-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^11+81
/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*
c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a
*b+b^2)*tanh(1/2*d*x+1/2*c)^5-45/8/d/(tanh(1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*
x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/
2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tanh(1/2*d*x+1/2*c)^7-45/8/d/(tanh(
1/2*d*x+1/2*c)^8*a-4*tanh(1/2*d*x+1/2*c)^6*a+6*tanh(1/2*d*x+1/2*c)^4*a-16*b
*tanh(1/2*d*x+1/2*c)^4-4*tanh(1/2*d*x+1/2*c)^2*a+a)^2*b/(a^2-2*a*b+b^2)*tan
h(1/2*d*x+1/2*c)^9
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(32*a^2*b^2 - 83*a*b^3 + 45*b^4 + 3*(20*a^2*b^2*e^(16*c) - 33*a*b^3*e^
(16*c) + 15*b^4*e^(16*c))*e^(16*d*x) - 12*(43*a^2*b^2*e^(14*c) - 68*a*b^3*e
^(14*c) + 30*b^4*e^(14*c))*e^(14*d*x) - 4*(400*a^3*b*e^(12*c) - 1137*a^2*b^
2*e^(12*c) + 1031*a*b^3*e^(12*c) - 315*b^4*e^(12*c))*e^(12*d*x) + 12*(592*a
^3*b*e^(10*c) - 1237*a^2*b^2*e^(10*c) + 886*a*b^3*e^(10*c) - 210*b^4*e^(10*
c))*e^(10*d*x) + 2*(4096*a^4*e^(8*c) - 12192*a^3*b*e^(8*c) + 13634*a^2*b^2*
e^(8*c) - 7113*a*b^3*e^(8*c) + 1575*b^4*e^(8*c))*e^(8*d*x) + 4*(880*a^3*b*e
^(6*c) - 2855*a^2*b^2*e^(6*c) + 2512*a*b^3*e^(6*c) - 630*b^4*e^(6*c))*e^(6*
d*x) - 4*(256*a^3*b*e^(4*c) - 823*a^2*b^2*e^(4*c) + 903*a*b^3*e^(4*c) - 315
*b^4*e^(4*c))*e^(4*d*x) - 12*(19*a^2*b^2*e^(2*c) - 54*a*b^3*e^(2*c) + 30*b^
4*e^(2*c))*e^(2*d*x))/(a^5*b^2*d - 2*a^4*b^3*d + a^3*b^4*d - (a^5*b^2*d*e^(
18*c) - 2*a^4*b^3*d*e^(18*c) + a^3*b^4*d*e^(18*c))*e^(18*d*x) + 9*(a^5*b^2*
d*e^(16*c) - 2*a^4*b^3*d*e^(16*c) + a^3*b^4*d*e^(16*c))*e^(16*d*x) + 4*(8*a
^6*b*d*e^(14*c) - 25*a^5*b^2*d*e^(14*c) + 26*a^4*b^3*d*e^(14*c) - 9*a^3*b^4
*d*e^(14*c))*e^(14*d*x) - 4*(40*a^6*b*d*e^(12*c) - 101*a^5*b^2*d*e^(12*c) +
 82*a^4*b^3*d*e^(12*c) - 21*a^3*b^4*d*e^(12*c))*e^(12*d*x) - 2*(128*a^7*d*e
^(10*c) - 416*a^6*b*d*e^(10*c) + 511*a^5*b^2*d*e^(10*c) - 286*a^4*b^3*d*e^(
10*c) + 63*a^3*b^4*d*e^(10*c))*e^(10*d*x) + 2*(128*a^7*d*e^(8*c) - 416*a^6*
b*d*e^(8*c) + 511*a^5*b^2*d*e^(8*c) - 286*a^4*b^3*d*e^(8*c) + 63*a^3*b^4*d*
e^(8*c))*e^(8*d*x) + 4*(40*a^6*b*d*e^(6*c) - 101*a^5*b^2*d*e^(6*c) + 82*a^4
*b^3*d*e^(6*c) - 21*a^3*b^4*d*e^(6*c))*e^(6*d*x) - 4*(8*a^6*b*d*e^(4*c) - 2
5*a^5*b^2*d*e^(4*c) + 26*a^4*b^3*d*e^(4*c) - 9*a^3*b^4*d*e^(4*c))*e^(4*d*x)
 - 9*(a^5*b^2*d*e^(2*c) - 2*a^4*b^3*d*e^(2*c) + a^3*b^4*d*e^(2*c))*e^(2*d*x
)) - 4*integrate(3/32*((20*a^2*b*e^(6*c) - 33*a*b^2*e^(6*c) + 15*b^3*e^(6*c
))*e^(6*d*x) - 2*(32*a^2*b*e^(4*c) - 41*a*b^2*e^(4*c) + 15*b^3*e^(4*c))*e^(
4*d*x) + (20*a^2*b*e^(2*c) - 33*a*b^2*e^(2*c) + 15*b^3*e^(2*c))*e^(2*d*x))/
(a^5*b - 2*a^4*b^2 + a^3*b^3 + (a^5*b*e^(8*c) - 2*a^4*b^2*e^(8*c) + a^3*b^3
*e^(8*c))*e^(8*d*x) - 4*(a^5*b*e^(6*c) - 2*a^4*b^2*e^(6*c) + a^3*b^3*e^(6*c
))*e^(6*d*x) - 2*(8*a^6*e^(4*c) - 19*a^5*b*e^(4*c) + 14*a^4*b^2*e^(4*c) - 3
*a^3*b^3*e^(4*c))*e^(4*d*x) - 4*(a^5*b*e^(2*c) - 2*a^4*b^2*e^(2*c) + a^3*b^
3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="fricas")

[Out] Timed out

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)**2/(a-b*sinh(d*x+c)**4)**3,x)

[Out] Timed out

Giac [A] time = 23.5837, size = 660, normalized size = 1.84

$$-\frac{28\,a^2b^2e^{(14\,dx+14\,c)} - 35\,ab^3e^{(14\,dx+14\,c)} + 13\,b^4e^{(14\,dx+14\,c)} - 232\,a^2b^2e^{(12\,dx+12\,c)} + 269\,ab^3e^{(12\,dx+12\,c)} - 91\,b^4e^{(12\,dx+12\,c)} -}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csch(d*x+c)^2/(a-b*sinh(d*x+c)^4)^3,x, algorithm="giac")

```
[Out] -1/16*(28*a^2*b^2*e^(14*d*x + 14*c) - 35*a*b^3*e^(14*d*x + 14*c) + 13*b^4*e
^(14*d*x + 14*c) - 232*a^2*b^2*e^(12*d*x + 12*c) + 269*a*b^3*e^(12*d*x + 12
*c) - 91*b^4*e^(12*d*x + 12*c) - 576*a^3*b*e^(10*d*x + 10*c) + 1372*a^2*b^2
*e^(10*d*x + 10*c) - 1039*a*b^3*e^(10*d*x + 10*c) + 273*b^4*e^(10*d*x + 10*
c) + 2432*a^3*b*e^(8*d*x + 8*c) - 3488*a^2*b^2*e^(8*d*x + 8*c) + 1913*a*b^3
*e^(8*d*x + 8*c) - 455*b^4*e^(8*d*x + 8*c) + 576*a^3*b*e^(6*d*x + 6*c) + 10
60*a^2*b^2*e^(6*d*x + 6*c) - 1689*a*b^3*e^(6*d*x + 6*c) + 455*b^4*e^(6*d*x
+ 6*c) - 376*a^2*b^2*e^(4*d*x + 4*c) + 679*a*b^3*e^(4*d*x + 4*c) - 273*b^4*
e^(4*d*x + 4*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 117*a*b^3*e^(2*d*x + 2*c) +
91*b^4*e^(2*d*x + 2*c) + 19*a*b^3 - 13*b^4)/((a^5*d - 2*a^4*b*d + a^3*b^2*d
)*(b*e^(8*d*x + 8*c) - 4*b*e^(6*d*x + 6*c) - 16*a*e^(4*d*x + 4*c) + 6*b*e^(
4*d*x + 4*c) - 4*b*e^(2*d*x + 2*c) + b)^2) - 2/(a^3*d*(e^(2*d*x + 2*c) - 1)
)
```

3.265 $\int \frac{1}{1-\sinh^4(x)} dx$

Optimal. Leaf size=25

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rubi [A] time = 0.018092, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3209, 388, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(2*Sqrt[2]) + Tanh[x]/2
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^4(x)} dx &= \text{Subst}\left(\int \frac{1-x^2}{1-2x^2} dx, x, \tanh(x)\right)\\ &= \frac{\tanh(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1-2x^2} dx, x, \tanh(x)\right)\\ &= \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{2\sqrt{2}}+\frac{\tanh(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.0995156, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)+2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/4
```

Maple [B] time = 0.02, size = 55, normalized size = 2.2

$$\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-1}+\frac{\sqrt{2}}{4}\mathrm{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)-2)\right)+\frac{\sqrt{2}}{4}\mathrm{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)+2)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^4),x)
```

```
[Out] tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(
1/2))+1/4*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [B] time = 1.54143, size = 93, normalized size = 3.72

$$\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{8}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)+\frac{1}{e^{(-2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="maxima")
```

```
[Out] 1/8*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/8*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) + 1/(e^(-2*x) + 1)
```

Fricas [B] time = 2.04111, size = 382, normalized size = 15.28

$$\frac{\left(\sqrt{2}\cosh(x)^{2}+2\sqrt{2}\cosh(x)\sinh(x)+\sqrt{2}\sinh(x)^{2}+\sqrt{2}\right)\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^{2}-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh}{\cosh(x)^{2}+\sinh(x)^{2}-3}\right.}{8\left(\cosh(x)^{2}+2\cosh(x)\sinh(x)+\sinh(x)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(x) + sqrt(2)*sinh(x)^2 + s
qrt(2))*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*sinh(
x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^2 -
3)) - 8)/(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2 + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**4),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.14865, size = 65, normalized size = 2.6

$$-\frac{1}{8}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{1}{e^{(2x)}+1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^4),x, algorithm="giac")
```

```
[Out] -1/8*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - 1/(e^(2*x) + 1)
```

3.266 $\int \frac{1}{1+\sinh^4(x)}\,dx$

Optimal. Leaf size=176

$$-\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(2\tanh^2(x)-2\sqrt{1+\sqrt{2}}\tanh(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(\sqrt{2}\tanh^2(x)+\sqrt{2\left(1+\sqrt{2}\right)}\tanh(x)+1\right.$$

```
[Out] -ArcTan[(Sqrt[1 + Sqrt[2]] - 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[1 + Sqr
t[2]]) + ArcTan[(Sqrt[1 + Sqrt[2]] + 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt
[1 + Sqrt[2]]) - (Sqrt[1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[1 + Sqrt[2]]*Tanh[
x] + 2*Tanh[x]^2])/8 + (Sqrt[1 + Sqrt[2]]*Log[1 + Sqrt[2*(1 + Sqrt[2])]*Tan
h[x] + Sqrt[2]*Tanh[x]^2])/8
```

Rubi [A] time = 0.159089, antiderivative size = 176, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3209, 1169, 634, 618, 204, 628}

$$-\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(2\tanh^2(x)-2\sqrt{1+\sqrt{2}}\tanh(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(\sqrt{2}\tanh^2(x)+\sqrt{2\left(1+\sqrt{2}\right)}\tanh(x)+1\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^4)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[1 + Sqrt[2]] - 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt[1 + Sqr
t[2]]) + ArcTan[(Sqrt[1 + Sqrt[2]] + 2*Tanh[x])/Sqrt[-1 + Sqrt[2]]]/(4*Sqrt
[1 + Sqrt[2]]) - (Sqrt[1 + Sqrt[2]]*Log[Sqrt[2] - 2*Sqrt[1 + Sqrt[2]]*Tanh[
x] + 2*Tanh[x]^2])/8 + (Sqrt[1 + Sqrt[2]]*Log[1 + Sqrt[2*(1 + Sqrt[2])]*Tan
h[x] + Sqrt[2]*Tanh[x]^2])/8
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] && !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rubi steps

$$\int \frac{1}{1+\sinh^4(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{1-2x^2+2x^4}\,dx, x, \tanh(x)\right)$$

$$= \frac{\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{2}}-\left(1+\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{1+\sqrt{2}}x+x^2}\,dx, x, \tanh(x)\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{1+\sqrt{2}}+\left(1+\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}+\sqrt{1+\sqrt{2}}x+x^2}\,dx, x, \tanh(x)\right)}{2\sqrt{2\left(1+\sqrt{2}\right)}}$$

$$= \frac{1}{8}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{2}}-\sqrt{1+\sqrt{2}}x+x^2}\,dx, x, \tanh(x)\right) + \frac{1}{8}\sqrt{3-2\sqrt{2}}\,\text{Subst}\left(\int \frac{1}{\frac{1}{\sqrt{2}}+\sqrt{1+}\right.$$

$$= -\frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(\sqrt{2}-2\sqrt{1+\sqrt{2}}\tanh(x)+2\tanh^2(x)\right) + \frac{1}{8}\sqrt{1+\sqrt{2}}\log\left(1+\sqrt{2\left(1+\sqrt{2}\right)}\tanh\right.$$

$$= -\frac{1}{4}\sqrt{-1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1+\sqrt{2}}-2\tanh(x)}{\sqrt{-1+\sqrt{2}}}\right) + \frac{1}{4}\sqrt{-1+\sqrt{2}}\tan^{-1}\left(\frac{\sqrt{1+\sqrt{2}}+2\tanh(x)}{\sqrt{-1+\sqrt{2}}}\right) - \frac{1}{8}\sqrt{1}$$

Mathematica [C] time = 0.072179, size = 45, normalized size = 0.26

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{2\sqrt{1-i}} + \frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^4)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(2*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]
/(2*Sqrt[1 + I])
```

Maple [C] time = 0.024, size = 44, normalized size = 0.3

$$\frac{1}{4}\sum_{\_R=\text{RootOf}\left(2\_Z^4-2\_Z^2+1\right)} \_R\ \ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2 + \left(-4\_R^3+4\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^4),x)
```

```
[Out] 1/4*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^3+4*_R)*tanh(1/2*x)+1),_R=RootOf(2*_Z^4-
2*_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh{(x)}^4 + 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sinh(x)^4 + 1), x)
```

Fricas [B] time = 2.257, size = 1874, normalized size = 10.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="fricas")
```

```
[Out] 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log((2^(3/4)*e^(2*x) - 2^(1
/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x)
 + 5) - 1/16*2^(1/4)*(sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2*
x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) -
2*e^(2*x) + 5) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt(2)*(5*s
qrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) - (2
^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) +
 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*
sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 1/14*sqrt(2)*(3*sqrt(
2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2
*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2)
+ 4) - 1/7*sqrt(2) + 3/7) - 1/4*2^(1/4)*sqrt(-2*sqrt(2) + 4)*arctan(-1/14*(
sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) + 1/28*(2*sqrt(2)*(5*sqrt(
2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sq
rt(2) + 4) + 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4
))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) + 1/14*sqrt(
2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2)
 + 6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(
-2*sqrt(2) + 4) + 1/7*sqrt(2) - 3/7)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**4),x)
```

```
[Out] Timed out
```

Giac [C] time = 1.24228, size = 293, normalized size = 1.66

$$\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}-2}\left(-\frac{i}{\sqrt{2}-1}+1\right)\log\left(2\sqrt{10\sqrt{2}+14}\left(-\frac{i}{5\sqrt{2}+7}+1\right)+(4i+2)\,e^{(2x)}-10\right)-\left(\frac{1}{16}i+\frac{1}{16}\right)\sqrt{2\sqrt{2}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^4),x, algorithm="giac")
```

```
[Out] (1/16*I + 1/16)*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log(2*sqrt(10*sq
rt(2) + 14)*(-I/(5*sqrt(2) + 7) + 1) + (4*I + 2)*e^(2*x) - 10) - (1/16*I +
1/16)*sqrt(2*sqrt(2) - 2)*(-I/(sqrt(2) - 1) + 1)*log(-2*sqrt(10*sqrt(2) + 1
4)*(-I/(5*sqrt(2) + 7) + 1) + (4*I + 2)*e^(2*x) - 10) + (1/16*I + 1/16)*sqr
t(2*sqrt(2) + 2)*(-I/(sqrt(2) + 1) + 1)*log(2*sqrt(2*sqrt(2) - 2)*(I/(sqrt(
2) - 1) + 1) + 2*e^(2*x) - 4*I - 2) - (1/16*I + 1/16)*sqrt(2*sqrt(2) + 2)*(
-I/(sqrt(2) + 1) + 1)*log(-2*sqrt(2*sqrt(2) - 2)*(I/(sqrt(2) - 1) + 1) + 2*
e^(2*x) - 4*I - 2)
```

3.267 $\int \frac{1}{a+b\sinh^5(x)}\,dx$

Optimal. Leaf size=435

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}}+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{9/10}\left(\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}\right)}{\sqrt{\sqrt[5]{-1}b^{2/5}-(-1)^{4/5}a^{2/5}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{-1}b^{2/5}-(-1)^{4/5}a^{2/5}}}+\frac{2\sqrt[5]{-1}\tanh^{-1}\left(\frac{\sqrt[5]{-1}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}+\frac{2(-}{}$$

```
[Out] (-2*ArcTanh[(b^(1/5) - a^(1/5)*Tanh[x/2])/Sqrt[a^(2/5) + b^(2/5)]])/(5*a^(4
/5)*Sqrt[a^(2/5) + b^(2/5)]) + (2*(-1)^(9/10)*ArcTanh[((-1)^(9/10)*((-1)^(1
/5)*b^(1/5) + a^(1/5)*Tanh[x/2]))/Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(1/5)*b
^(2/5)]])/(5*a^(4/5)*Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(1/5)*b^(2/5)]) + (2
*(-1)^(1/5)*ArcTanh[(b^(1/5) + (-1)^(1/5)*a^(1/5)*Tanh[x/2])/Sqrt[(-1)^(2/5
)*a^(2/5) + b^(2/5)]])/(5*a^(4/5)*Sqrt[(-1)^(2/5)*a^(2/5) + b^(2/5)]) + (2*
(-1)^(9/10)*ArcTanh[((-1)^(3/10)*(b^(1/5) + (-1)^(3/5)*a^(1/5)*Tanh[x/2]))/
Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[-((-1)^(
4/5)*a^(2/5)) + (-1)^(3/5)*b^(2/5)]) - (2*(-1)^(9/10)*ArcTanh[(I*b^(1/5) -
(-1)^(9/10)*a^(1/5)*Tanh[x/2])/Sqrt[-((-1)^(4/5)*a^(2/5)) - b^(2/5)]])/(5*a
^(4/5)*Sqrt[-((-1)^(4/5)*a^(2/5)) - b^(2/5)])
```

Rubi [A] time = 0.974696, antiderivative size = 435, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.5, Rules used = {3213, 2660, 618, 206, 204}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}}+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{9/10}\left(\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)+\sqrt[5]{-1}\sqrt[5]{b}\right)}{\sqrt{\sqrt[5]{-1}b^{2/5}-(-1)^{4/5}a^{2/5}}}\right)}{5a^{4/5}\sqrt{\sqrt[5]{-1}b^{2/5}-(-1)^{4/5}a^{2/5}}}+\frac{2\sqrt[5]{-1}\tanh^{-1}\left(\frac{\sqrt[5]{-1}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}+\frac{2(-}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^5)^(-1),x]
```


```
[Out] (-2*ArcTanh[(b^(1/5) - a^(1/5)*Tanh[x/2])/Sqrt[a^(2/5) + b^(2/5)]])/(5*a^(4
/5)*Sqrt[a^(2/5) + b^(2/5)]) + (2*(-1)^(9/10)*ArcTanh[((-1)^(9/10)*((-1)^(1
/5)*b^(1/5) + a^(1/5)*Tanh[x/2]))/Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(1/5)*b
^(2/5)]])/(5*a^(4/5)*Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(1/5)*b^(2/5)]) + (2
*(-1)^(1/5)*ArcTanh[(b^(1/5) + (-1)^(1/5)*a^(1/5)*Tanh[x/2])/Sqrt[(-1)^(2/5
)*a^(2/5) + b^(2/5)]])/(5*a^(4/5)*Sqrt[(-1)^(2/5)*a^(2/5) + b^(2/5)]) + (2*
(-1)^(9/10)*ArcTanh[((-1)^(3/10)*(b^(1/5) + (-1)^(3/5)*a^(1/5)*Tanh[x/2]))/
Sqrt[-((-1)^(4/5)*a^(2/5)) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[-((-1)^(
4/5)*a^(2/5)) + (-1)^(3/5)*b^(2/5)]) - (2*(-1)^(9/10)*ArcTanh[(I*b^(1/5) -
(-1)^(9/10)*a^(1/5)*Tanh[x/2])/Sqrt[-((-1)^(4/5)*a^(2/5)) - b^(2/5)]])/(5*a
^(4/5)*Sqrt[-((-1)^(4/5)*a^(2/5)) - b^(2/5)])
```


Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
```

```
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sinh^5(x)}\,dx = \int \left(-\frac{(-1)^{9/10}}{5a^{4/5}\left(-(-1)^{9/10}\sqrt[5]{a}-i\sqrt[5]{b}\sinh(x)\right)} - \frac{(-1)^{9/10}}{5a^{4/5}\left(-(-1)^{9/10}\sqrt[5]{a}-\sqrt[10]{-1}\sqrt[5]{b}\sinh(x)\right)} - \frac{}{5a^{4/5}\left(-(-1)^{9}\right.}\right.$$

$$= -\frac{(-1)^{9/10}\int \frac{1}{-(-1)^{9/10}\sqrt[5]{a}-i\sqrt[5]{b}\sinh(x)}\,dx}{5a^{4/5}} - \frac{(-1)^{9/10}\int \frac{1}{-(-1)^{9/10}\sqrt[5]{a}-\sqrt[10]{-1}\sqrt[5]{b}\sinh(x)}\,dx}{5a^{4/5}} - \frac{(-1)^{9/10}\int \frac{}{-(-1)^{9/10}\sqrt[5]{}}}{5a}$$

$$= -\frac{\left(2(-1)^{9/10}\right)\operatorname{Subst}\left(\int \frac{1}{-(-1)^{9/10}\sqrt[5]{a}-2i\sqrt[5]{b}x+(-1)^{9/10}\sqrt[5]{a}x^2}\,dx, x, \tanh\left(\frac{x}{2}\right)\right)}{5a^{4/5}} - \frac{\left(2(-1)^{9/10}\right)\operatorname{Subst}\left(\int \frac{}{-(-1)^{9/}}\right.}{}$$

$$= \frac{\left(4(-1)^{9/10}\right)\operatorname{Subst}\left(\int \frac{1}{-4(-1)^{4/5}\left(a^{2/5}+b^{2/5}\right)-x^2}\,dx, x, -2(-1)^{9/10}\sqrt[5]{b}+2(-1)^{9/10}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)\right)}{5a^{4/5}} + \frac{\left(4(-1)^{9/}\right.}{}$$

$$= -\frac{2(-1)^{7/10}\tan^{-1}\left(\frac{i\sqrt[5]{b}+(-1)^{7/10}\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{(-1)^{2/5}a^{2/5}+b^{2/5}}} - \frac{2\tanh^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tanh\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+b^{2/5}}} - \frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{i\sqrt[5]{b}-(-1)^{}}{\sqrt{-(-1}}\right.}{5a^{4/5}\sqrt{-(-1)^{4/5}a^{2/5}}}$$

Mathematica [C] time = 0.325722, size = 141, normalized size = 0.32

$$\frac{8}{5}\operatorname{RootSum}\left[32\#1^5a+\#1^{10}b-5\#1^8b+10\#1^6b-10\#1^4b+5\#1^2b-b\&, \frac{\#1^3x+2\#1^3\log\left(-\#1\sinh\left(\frac{x}{2}\right)+\#1\cosh\left(\frac{x}{2}\right)-\right.}{16\#1^3a+\#1^8b-4\#1^6b+6\#1^4b-}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^5)^(-1),x]
```

```
[Out] (8*RootSum[-b + 5*b*#1^2 - 10*b*#1^4 + 32*a*#1^5 + 10*b*#1^6 - 5*b*#1^8 + b
*#1^10 & , (x*#1^3 + 2*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x/2]*#1 - Sinh[x/2
]*#1]*#1^3)/(b - 4*b*#1^2 + 16*a*#1^3 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8) & ])/
5
```

Maple [C] time = 0.032, size = 113, normalized size = 0.3

$$\frac{1}{5}\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{10}-5a\_Z^{8}+10a\_Z^{6}-32b\_Z^{5}-10a\_Z^{4}+5a\_Z^{2}-a\right)}\frac{-\_R^{8}+4\_R^{6}-6\_R^{4}+4\_R^{2}-1}{\_R^{9}a-4\_R^{7}a+6\_R^{5}a-16\_R^{4}b-4\_R^{3}a+\_Ra}\ln\left(\tanh\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^5),x)
```

```
[Out] 1/5*sum((-_R^8+4*_R^6-6*_R^4+4*_R^2-1)/(_R^9*a-4*_R^7*a+6*_R^5*a-16*_R^4*b-
4*_R^3*a+_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^10*a-5*_Z^8*a+10*_Z^6*a-32*_
Z^5*b-10*_Z^4*a+5*_Z^2*a-a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sinh(x)^5 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^5 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b\sinh^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**5),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**5), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sinh(x)^5 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sinh(x)^5 + a), x)
```

3.268 $\int \frac{1}{a+b\sinh^6(x)}\,dx$

Optimal. Leaf size=175

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3)
- b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tanh[x])/a^(1/6)]
/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) - (
-1)^(2/3)*b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b
^(1/3)])
```

Rubi [A] time = 0.279326, antiderivative size = 175, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^6)^(-1),x]
```


```
[Out] ArcTanh[(Sqrt[a^(1/3) - b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3)
- b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tanh[x])/a^(1/6)]
/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTanh[(Sqrt[a^(1/3) - (
-1)^(2/3)*b^(1/3)]*Tanh[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b
^(1/3)])
```


Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sinh^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\sinh^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tanh(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tanh(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\right.}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tanh(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.16913, size = 134, normalized size = 0.77

$$\frac{16}{3}\text{RootSum}\left[64\#1^3 a+\#1^6 b-6\#1^5 b+15\#1^4 b-20\#1^3 b+15\#1^2 b-6\#1 b+b\&, \frac{\#1^2 x+\#1^2\log(-\#1\sinh(x)+\#1\cosh}{32\#1^2 a+\#1^5 b-5\#1^4 b+10\#1^3 b}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^6)^(-1),x]
```

```
[Out] (16*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (x*#1^2 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]
*#1]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3 - 5*b*#1^4 + b*
#1^5) & ])/3
```

Maple [C] time = 0.03, size = 128, normalized size = 0.7

$$\frac{1}{6}\sum_{\_R=\text{RootOf}\left(a\_Z^{12}-6a\_Z^{10}+15a\_Z^8+(-20a+64b)\_Z^6+15a\_Z^4-6a\_Z^2+a\right)} \frac{-\_R^{10}+5\_R^8-10\_R^6+10\_R^4-5\_R^2+1}{\_R^{11}a-5\_R^9 a+10\_R^7 a-10\_R^5 a+32\_R^5 b+5\_R^3 a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^6),x)
```

```
[Out] 1/6*sum((-_R^10+5*_R^8-10*_R^6+10*_R^4-5*_R^2+1)/(_R^11*a-5*_R^9*a+10*_R^7*
a-10*_R^5*a+32*_R^5*b+5*_R^3*a-_R*a)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^12-6
*a*_Z^10+15*a*_Z^8+(-20*a+64*b)*_Z^6+15*a*_Z^4-6*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sinh(x)^6+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^6 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{a + b \sinh^{6}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**6),x)
```

```
[Out] Integral(1/(a + b*sinh(x)**6), x)
```

Giac [A] time = 1.37914, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^6),x, algorithm="giac")
```

```
[Out] 0
```

3.269 $\int \frac{1}{a+b\sinh^8(x)}\,dx$

Optimal. Leaf size=245

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

```
[Out] -ArcTanh[(Sqrt[(-a)^(1/4) - b^(1/4)]*Tanh[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqr
t[(-a)^(1/4) - b^(1/4)]) - ArcTanh[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tanh[x])/(
-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) - I*b^(1/4)]) - ArcTanh[(Sqrt[(-a)
^(1/4) + I*b^(1/4)]*Tanh[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*
b^(1/4)]) - ArcTanh[(Sqrt[(-a)^(1/4) + b^(1/4)]*Tanh[x])/(-a)^(1/8)]/(4*(-a
)^(7/8)*Sqrt[(-a)^(1/4) + b^(1/4)])
```

Rubi [A] time = 0.516823, antiderivative size = 245, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3211, 3181, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\tanh(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[x]^8)^(-1),x]
```

```
[Out] -ArcTanh[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tanh[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*S
qrt[(-a)^(1/4) - I*b^(1/4)]) - ArcTanh[(Sqrt[(-a)^(1/4) + I*b^(1/4)]*Tanh[x
])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ArcTanh[(Sqrt[
(-a)^(1/4) + b^(1/4)]*Tanh[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) +
b^(1/4)]) - ArcTanh[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Tanh[x])/(-a)^(5/8)]/(4*(
-a)^(3/8)*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{a+b\sinh^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sinh^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tanh(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1-\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tanh(x)\right)}{4a} + \text{Subst}\left(\int \right.$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} - \frac{\tanh^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\tanh(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}} - \frac{\tanh}{4(}$$

Mathematica [C] time = 0.245581, size = 160, normalized size = 0.65

$$16\text{RootSum}\left[256\#1^4a+\#1^8b-8\#1^7b+28\#1^6b-56\#1^5b+70\#1^4b-56\#1^3b+28\#1^2b-8\#1b+b\&, \frac{\#1^3x+}{128\#1^3a+\#1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[x]^8)^(-1),x]
```

```
[Out] 16*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56
*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x
] + Cosh[x]*#1 - Sinh[x]*#1]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2 + 128*a*#1^3 +
35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

Maple [C] time = 0.033, size = 162, normalized size = 0.7

$$\frac{1}{8}\sum_{\_R=\text{RootOf}\left(a\_Z^{16}-8a\_Z^{14}+28a\_Z^{12}-56a\_Z^{10}+(70a+256b)\_Z^8-56a\_Z^6+28a\_Z^4-8a\_Z^2+a\right)} \frac{-\_R^{14}+7\_R^{12}-21\_R^{10}+}{\_R^{15}a-7\_R^{13}a+21\_R^{11}a-35\_R^9a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(x)^8),x)
```

```
[Out] 1/8*sum((-_R^14+7*_R^12-21*_R^10+35*_R^8-35*_R^6+21*_R^4-7*_R^2+1)/(_R^15*a
-7*_R^13*a+21*_R^11*a-35*_R^9*a+35*_R^7*a+128*_R^7*b-21*_R^5*a+7*_R^3*a-_R*
a)*ln(tanh(1/2*x)-_R),_R=RootOf(a*_Z^16-8*a*_Z^14+28*a*_Z^12-56*a*_Z^10+(70
*a+256*b)*_Z^8-56*a*_Z^6+28*a*_Z^4-8*a*_Z^2+a))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{b\sinh(x)^8 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sinh(x)^8 + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)**8),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.90444, size = 1, normalized size = 0.

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(x)^8),x, algorithm="giac")
```

```
[Out] 0
```

3.270 $\int \frac{1}{1+\sinh^5(x)}\,dx$

Optimal. Leaf size=242

$$-\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{7/10}\left(\sqrt[5]{-1}\tanh\left(\frac{x}{2}\right)+1\right)}{\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}\right)}{5\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}-\frac{2(-1)^{4/5}\tanh^{-1}\left(\frac{1-(-1)^{4/5}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2(-1)}{}$$

```
[Out] (-2*(-1)^(3/5)*ArcTan[(1 + (-1)^(3/5)*Tanh[x/2])/Sqrt[-1 + (-1)^(1/5)]])/(5
*Sqrt[-1 + (-1)^(1/5)]) + (2*(-1)^(9/10)*ArcTan[(I - (-1)^(9/10)*Tanh[x/2])
/Sqrt[1 + (-1)^(4/5)]])/(5*Sqrt[1 + (-1)^(4/5)]) - (Sqrt[2]*ArcTanh[(1 - Ta
nh[x/2])/Sqrt[2]])/5 + (2*(-1)^(9/10)*ArcTanh[((-1)^(7/10)*(1 + (-1)^(1/5)*
Tanh[x/2]))/Sqrt[-((-1)^(2/5)*(1 + (-1)^(2/5)))]])/(5*Sqrt[-((-1)^(2/5)*(1
+ (-1)^(2/5)))]) - (2*(-1)^(4/5)*ArcTanh[(1 - (-1)^(4/5)*Tanh[x/2])/Sqrt[1
- (-1)^(3/5)]])/(5*Sqrt[1 - (-1)^(3/5)])
```

Rubi [A] time = 0.5185, antiderivative size = 242, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3213, 2660, 618, 204, 206, 617}

$$-\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\frac{2(-1)^{9/10}\tanh^{-1}\left(\frac{(-1)^{7/10}\left(\sqrt[5]{-1}\tanh\left(\frac{x}{2}\right)+1\right)}{\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}\right)}{5\sqrt{-(-1)^{2/5}\left(1+(-1)^{2/5}\right)}}-\frac{2(-1)^{4/5}\tanh^{-1}\left(\frac{1-(-1)^{4/5}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2(-1)}{}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^5)^(-1),x]
```

```
[Out] (-2*(-1)^(3/5)*ArcTan[(1 + (-1)^(3/5)*Tanh[x/2])/Sqrt[-1 + (-1)^(1/5)]])/(5
*Sqrt[-1 + (-1)^(1/5)]) + (2*(-1)^(9/10)*ArcTan[(I - (-1)^(9/10)*Tanh[x/2])
/Sqrt[1 + (-1)^(4/5)]])/(5*Sqrt[1 + (-1)^(4/5)]) - (Sqrt[2]*ArcTanh[(1 - Ta
nh[x/2])/Sqrt[2]])/5 + (2*(-1)^(9/10)*ArcTanh[((-1)^(7/10)*(1 + (-1)^(1/5)*
Tanh[x/2]))/Sqrt[-((-1)^(2/5)*(1 + (-1)^(2/5)))]])/(5*Sqrt[-((-1)^(2/5)*(1
+ (-1)^(2/5)))]) - (2*(-1)^(4/5)*ArcTanh[(1 - (-1)^(4/5)*Tanh[x/2])/Sqrt[1
- (-1)^(3/5)]])/(5*Sqrt[1 - (-1)^(3/5)])
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
```

```
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rubi steps

$$\int \frac{1}{1+\sinh^5(x)}\,dx = \int\left(-\frac{(-1)^{9/10}}{5\left(-(-1)^{9/10}-i\sinh(x)\right)}-\frac{(-1)^{9/10}}{5\left(-(-1)^{9/10}-\sqrt[10]{-1}\sinh(x)\right)}-\frac{(-1)^{9/10}}{5\left(-(-1)^{9/10}+(-1)^{3/10}\sinh(x)\right)}-$$

$$=-\left(\frac{1}{5}(-1)^{9/10}\int\frac{1}{-(-1)^{9/10}-i\sinh(x)}\,dx\right)-\frac{1}{5}(-1)^{9/10}\int\frac{1}{-(-1)^{9/10}-\sqrt[10]{-1}\sinh(x)}\,dx-\frac{1}{5}(-1)^{9/10}\int$$

$$=-\left(\frac{1}{5}\left(2(-1)^{9/10}\right)\text{Subst}\left(\int\frac{1}{-(-1)^{9/10}-2ix+(-1)^{9/10}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)\right)-\frac{1}{5}\left(2(-1)^{9/10}\right)\text{Subst}\left(\int\right.$$

$$=-\left(\frac{2}{5}\text{Subst}\left(\int\frac{1}{2-x^2}\,dx,x,1-\tanh\left(\frac{x}{2}\right)\right)\right)+\frac{1}{5}\left(4(-1)^{9/10}\right)\text{Subst}\left(\int\frac{1}{4(-1)^{3/5}\left(1-\sqrt[5]{-1}\right)-x^2}\,dx,\right.$$

$$=\frac{2(-1)^{9/10}\tan^{-1}\left(\frac{i-(-1)^{9/10}\tanh\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{4/5}}}\right)}{5\sqrt{1+(-1)^{4/5}}}-\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1-\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)-\frac{2\sqrt[10]{-1}\tanh^{-1}\left(\frac{i-\sqrt[10]{-1}\tanh\left(\frac{x}{2}\right)}{\sqrt{-1+\sqrt[5]{-1}}}\right)}{5\sqrt{-1+\sqrt[5]{-1}}}$$

Mathematica [C] time = 0.915842, size = 439, normalized size = 1.81

$$\frac{1}{10}\left(2\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)-1}{\sqrt{2}}\right)-\text{RootSum}\left[\#1^8-2\#1^7-2\#1^5+14\#1^4+2\#1^3+2\#1+1\&,\frac{\#1^6x-4\#1^5x+9\#1^4x-}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^5)^(-1),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(-1 + Tanh[x/2])/Sqrt[2]] - RootSum[1 + 2*#1 + 2*#1^3 +
14*#1^4 - 2*#1^5 - 2*#1^7 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + C
osh[x/2]*#1 - Sinh[x/2]*#1] - 4*x*#1 - 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[
x/2]*#1 - Sinh[x/2]*#1]*#1 - 9*x*#1^2 - 18*Log[-Cosh[x/2] - Sinh[x/2] + Cos
h[x/2]*#1 - Sinh[x/2]*#1]*#1^2 - 24*x*#1^3 - 48*Log[-Cosh[x/2] - Sinh[x/2]
+ Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^3 + 9*x*#1^4 + 18*Log[-Cosh[x/2] - Sinh[x
/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 - 4*x*#1^5 - 8*Log[-Cosh[x/2] - Sin
h[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Si
```

```
nh[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(1 + 3*#1^2 + 28*#1^3 - 5*#1^4
 - 7*#1^6 + 4*#1^7) & ])/10
```

Maple [C] time = 0.033, size = 124, normalized size = 0.5

$$\frac{2}{5}\sum_{\_R=\mathrm{RootOf}\left(\_Z^8+2\_Z^7+2\_Z^5+14\_Z^4-2\_Z^3-2\_Z+1\right)}\frac{-2\_R^6-3\_R^5+2\_R^4+2\_R^3-2\_R^2-3\_R+2}{4\_R^7+7\_R^6+5\_R^4+28\_R^3-3\_R^2-1}\ln\left(\tanh\left(\frac{x}{2}\right)-\_$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^5),x)
```

```
[Out] 2/5*sum((-2*_R^6-3*_R^5+2*_R^4+2*_R^3-2*_R^2-3*_R+2)/(4*_R^7+7*_R^6+5*_R^4+
28*_R^3-3*_R^2-1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^8+2*_Z^7+2*_Z^5+14*_Z^4-2
*_Z^3-2*_Z+1))+1/5*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{1}{10}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x-1}{\sqrt{2}+e^x+1}\right)-\int\frac{2\left(e^{(7x)}-4e^{(6x)}+9e^{(5x)}-24e^{(4x)}-9e^{(3x)}-4e^{(2x)}-e^x\right)}{5\left(e^{(8x)}-2e^{(7x)}-2e^{(5x)}+14e^{(4x)}+2e^{(3x)}+2e^x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="maxima")
```

```
[Out] 1/10*sqrt(2)*log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - integrate(2/5*
(e^(7*x) - 4*e^(6*x) + 9*e^(5*x) - 24*e^(4*x) - 9*e^(3*x) - 4*e^(2*x) - e^x
)/(e^(8*x) - 2*e^(7*x) - 2*e^(5*x) + 14*e^(4*x) + 2*e^(3*x) + 2*e^x + 1), x
)
```

Fricas [B] time = 3.48642, size = 11367, normalized size = 46.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="fricas")
```

```
[Out] -1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5)
 + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt
(5) + 3)*arctan(1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x - 7*sqrt(5) + 15)*sqrt(
2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 1/80*sqrt(2)*(sqrt(2)*((11*sqrt(5) - 25)
*e^x + 4*sqrt(5) - 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((3*sqrt(5
) - 5)*e^x + 7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(5) + 3) + 1/128
00*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 40*
sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2
*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) + sqrt(2*sqrt(2)*(2*s
qrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7*sqrt(5) - 15)*
sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
+ 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*sqrt(2*sqrt(5)
+ 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5
```

```
) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt(-20*sqrt(2)*sq
rt(sqrt(5) + 3)*(sqrt(5) - 3) + 40*(sqrt(5) - 1)*e^x - 2*(sqrt(2)*((2*sqrt(
5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^x - 3*sq
rt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 2
0)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*sqrt(2)*(2*sqrt
(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*sqrt(5) - 7)*e^x
 + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) -
11)*e^x + 2*sqrt(5) - 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(s
qrt(2)*((7*sqrt(5) - 17)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqr
t(5) + 3) + 2*((5*sqrt(5) - 11)*e^x + 5*sqrt(5) - 9)*sqrt(2*sqrt(5) + 5))*(
8*sqrt(5) + 24)^(1/4)) + 1/20*(2*(4*sqrt(5) - 5)*e^x - sqrt(5) + 5)*sqrt(2*
sqrt(5) + 5)) - 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) +
 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5)
 + 5)*sqrt(sqrt(5) + 3)*arctan(-1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x - 7*sqr
t(5) + 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) - 1/80*sqrt(2)*(sqrt(2)*((
11*sqrt(5) - 25)*e^x + 4*sqrt(5) - 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3
) + 2*((3*sqrt(5) - 5)*e^x + 7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt
(5) + 3) - 1/12800*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sq
rt(5) + 3) + 40*sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(
sqrt(5) + 3) + 2*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) - sqr
t(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(
7*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)
*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)
*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
 + 5))*(8*sqrt(5) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqr
t(-20*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) + 40*(sqrt(5) - 1)*e^x + 2*(s
qrt(2)*((2*sqrt(5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5)
 - 5)*e^x - 3*sqrt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3)
 - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2
*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*
sqrt(5) - 7)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) +
2*((5*sqrt(5) - 11)*e^x + 2*sqrt(5) - 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) +
24)^(3/4) + 4*(sqrt(2)*((7*sqrt(5) - 17)*e^x + 8*sqrt(5) - 18)*sqrt(2*sqrt(
5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 11)*e^x + 5*sqrt(5) - 9)*sqrt(2
*sqrt(5) + 5))*(8*sqrt(5) + 24)^(1/4)) - 1/20*(2*(4*sqrt(5) - 5)*e^x - sqrt
(5) + 5)*sqrt(2*sqrt(5) + 5)) + 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5)
 + 24) + 4*sqrt(5) + 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) +
 24)^(3/4)*arctan(-1/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x +
 8*sqrt(5) + 18)*sqrt(-8*sqrt(5) + 24) + 8*sqrt(5) + 16)*(-8*sqrt(5) + 24)^
(3/4) + 4*(4*(5*sqrt(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x + 8*sqrt(5) + 18)
*sqrt(-8*sqrt(5) + 24) + 20*sqrt(5) + 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2
*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5)
+ 1/25600*((((7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8
*sqrt(5) + 24)^(3/4) + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt
(5) + 100)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) +
 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) + 20*(((5*sqrt(5) + 11)*sqrt(-8
*sqrt(5) + 24) + 4*sqrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)
*sqrt(-8*sqrt(5) + 24) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(-40*(
sqrt(5) + 1)*e^x + (4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5)
- 5)*sqrt(-8*sqrt(5) + 24) - 6*sqrt(5) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*
sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*
sqrt(-8*sqrt(5) + 24) + 80*e^(2*x) + 80) - 1/320*(32*(4*sqrt(5) + 5)*e^x +
4*((11*sqrt(5) + 25)*e^x - 7*sqrt(5) - 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sq
rt(5) + 5)*e^x + ((11*sqrt(5) + 25)*e^x + 4*sqrt(5) + 10)*sqrt(-8*sqrt(5) +
 24) + 28*sqrt(5) + 60)*sqrt(-8*sqrt(5) + 24) - 16*sqrt(5) - 80)*sqrt(-2*sq
rt(5) + 5)) + 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5)
 + 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) + 24)^(3/4)*arctan(
-1/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x + 8*sqrt(5) + 18)*s
qrt(-8*sqrt(5) + 24) + 8*sqrt(5) + 16)*(-8*sqrt(5) + 24)^(3/4) + 4*(4*(5*sq
```

```
rt(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x + 8*sqrt(5) + 18)*sqrt(-8*sqrt(5) +
 24) + 20*sqrt(5) + 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt
(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) + 1/25600*((((7*sq
rt(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*sqrt(5) + 24)^(3/
4) + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*sqr
t(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) +
 20)*sqrt(-2*sqrt(5) + 5) - 20*(((5*sqrt(5) + 11)*sqrt(-8*sqrt(5) + 24) + 4
*sqrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)*sqrt(-8*sqrt(5) +
 24) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(-40*(sqrt(5) + 1)*e^x -
 (4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5) - 5)*sqrt(-8*sqrt(
5) + 24) - 6*sqrt(5) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*
sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*sqrt(-8*sqrt(5) +
24) + 80*e^(2*x) + 80) + 1/320*(32*(4*sqrt(5) + 5)*e^x + 4*((11*sqrt(5) + 2
5)*e^x - 7*sqrt(5) - 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sqrt(5) + 5)*e^x + (
(11*sqrt(5) + 25)*e^x + 4*sqrt(5) + 10)*sqrt(-8*sqrt(5) + 24) + 28*sqrt(5)
+ 60)*sqrt(-8*sqrt(5) + 24) - 16*sqrt(5) - 80)*sqrt(-2*sqrt(5) + 5)) + 1/80
0*(sqrt(2)*(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 8*sqrt(5))*sqrt(2*sqrt(2)*(2
*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*lo
g(-4*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) + 8*(sqrt(5) - 1)*e^x + 2/5*(s
qrt(2)*((2*sqrt(5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5)
 - 5)*e^x - 3*sqrt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3)
 - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) - 1/800*(sqrt(
2)*(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 8*sqrt(5))*sqrt(2*sqrt(2)*(2*sqrt(5)
 - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*log(-4*sqr
t(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) + 8*(sqrt(5) - 1)*e^x - 2/5*(sqrt(2)*(
(2*sqrt(5) - 5)*e^x - 3*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^
x - 3*sqrt(5) + 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqr
t(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) + 1/1600*((3*sqrt(5) +
 5)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(
5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4)*log(-8*(sqrt(5) + 1)*e^x
 + 1/5*(4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5) - 5)*sqrt(-8
*sqrt(5) + 24) - 6*sqrt(5) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24
) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(
5) + 24) + 16*e^(2*x) + 16) - 1/1600*((3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24)
 + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20
)*(-8*sqrt(5) + 24)^(1/4)*log(-8*(sqrt(5) + 1)*e^x - 1/5*(4*(sqrt(5) + 5)*e
^x + ((2*sqrt(5) + 5)*e^x - 3*sqrt(5) - 5)*sqrt(-8*sqrt(5) + 24) - 6*sqrt(5
) - 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*s
qrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24) + 16*e^(2*x) + 1
6) + 1/10*sqrt(2)*log(-(2*(sqrt(2) - 1)*e^x + 2*sqrt(2) - e^(2*x) - 3)/(e^(
2*x) + 2*e^x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**5),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(x)^5 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(sinh(x)^5 + 1), x)
```

3.271 $\int \frac{1}{1+\sinh^6(x)} dx$

Optimal. Leaf size=71

$$\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tanh(x)}{3}$$

```
[Out] ArcTanh[Sqrt[1 + (-1)^(1/3)]*Tanh[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTanh[Sq
rt[1 - (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tanh[x]/3
```

Rubi [A] time = 0.137859, antiderivative size = 71, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.75, Rules used = {3211, 3181, 206, 3175, 3767, 8}

$$\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tanh(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 + (-1)^(1/3)]*Tanh[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTanh[Sq
rt[1 - (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tanh[x]/3
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sinh^6(x)}\,dx &= \frac{1}{3}\int \frac{1}{1+\sinh^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sinh^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\sinh^2(x)}\,dx\\
&= \frac{1}{3}\int \operatorname{sech}^2(x)\,dx+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-\left(1+\sqrt[3]{-1}\right)x^2}\,dx,x,\tanh(x)\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1-\left(1-(-1)^{2/3}\right)x\right.\\
&= \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{1}{3}i\operatorname{Subst}\left(\int 1\,dx,x,-i\tanh(x)\right)\\
&= \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tanh(x)}{3}
\end{aligned}$$

Mathematica [C] time = 0.201319, size = 87, normalized size = 1.23

$$\frac{1}{18}\left(6\tanh(x)+\sqrt[4]{-3}\left(\left(-3-i\sqrt{3}\right)\tan^{-1}\left(\frac{1}{2}\sqrt[4]{-3}\left(1+i\sqrt{3}\right)\tanh(x)\right)-\left(\sqrt{3}+3i\right)\tan^{-1}\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}\left(3+i\sqrt{3}\right)\tanh(x)\right)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^6)^(-1),x]
```

```
[Out] ((-3)^(1/4)*((-3 - I*Sqrt[3])*ArcTan[((-3)^(1/4)*(1 + I*Sqrt[3])*Tanh[x])/2
] - (3*I + Sqrt[3])*ArcTan[((-1/3)^(1/4)*(3 + I*Sqrt[3])*Tanh[x])/2]) + 6*T
anh[x])/18
```

Maple [C] time = 0.033, size = 61, normalized size = 0.9

$$\frac{1}{6}\sum_{\_R=\operatorname{RootOf}\left(3\_Z^4-3\_Z^2+1\right)}\_R\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(-6\_R^3+6\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)+\frac{2}{3}\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^6),x)
```

```
[Out] 1/6*sum(_R*ln(tanh(1/2*x)^2+(-6*_R^3+6*_R)*tanh(1/2*x)+1),_R=RootOf(3*_Z^4-
3*_Z^2+1))+2/3*tanh(1/2*x)/(tanh(1/2*x)^2+1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{2}{3\left(e^{(2x)}+1\right)}-\int \frac{4\left(e^{(6x)}-10e^{(4x)}+e^{(2x)}\right)}{3\left(e^{(8x)}-8e^{(6x)}+30e^{(4x)}-8e^{(2x)}+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="maxima")
```

```
[Out] -2/3/(e^(2*x) + 1) - integrate(4/3*(e^(6*x) - 10*e^(4*x) + e^(2*x))/(e^(8*x
) - 8*e^(6*x) + 30*e^(4*x) - 8*e^(2*x) + 1), x)
```

Fricas [B] time = 1.84705, size = 2303, normalized size = 32.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="fricas")
```

```
[Out] -1/144*(4*(12^(1/4)*sqrt(6)*e^(2*x) + 12^(1/4)*sqrt(6))*sqrt(-4*sqrt(3) + 8
)*arctan((sqrt(3) + 2)*e^(2*x) + 1/216*sqrt(-6*(12^(1/4)*sqrt(6)*(sqrt(3) +
 3)*e^(2*x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*
sqrt(3) + 36*e^(4*x) - 144*e^(2*x) + 252)*((12^(3/4)*sqrt(6)*(sqrt(3) + 3)
+ 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8) - 36*sqrt(3) - 72)
 - 2/3*sqrt(3)*(2*sqrt(3) - 3) - 1/36*(12^(3/4)*sqrt(6)*(sqrt(3) - 3) + (12
^(3/4)*sqrt(6)*(sqrt(3) + 3) + 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*e^(2*x) +
3*12^(1/4)*sqrt(6)*(sqrt(3) - 3))*sqrt(-4*sqrt(3) + 8) - 2*sqrt(3) + 4) + 4
*(12^(1/4)*sqrt(6)*e^(2*x) + 12^(1/4)*sqrt(6))*sqrt(-4*sqrt(3) + 8)*arctan(
-(sqrt(3) + 2)*e^(2*x) + 1/216*sqrt(6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2*
x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3) +
 36*e^(4*x) - 144*e^(2*x) + 252)*((12^(3/4)*sqrt(6)*(sqrt(3) + 3) + 3*12^(1
/4)*sqrt(6)*(sqrt(3) + 3))*sqrt(-4*sqrt(3) + 8) + 36*sqrt(3) + 72) + 2/3*sq
rt(3)*(2*sqrt(3) - 3) - 1/36*(12^(3/4)*sqrt(6)*(sqrt(3) - 3) + (12^(3/4)*sq
rt(6)*(sqrt(3) + 3) + 3*12^(1/4)*sqrt(6)*(sqrt(3) + 3))*e^(2*x) + 3*12^(1/4
)*sqrt(6)*(sqrt(3) - 3))*sqrt(-4*sqrt(3) + 8) + 2*sqrt(3) - 4) - (12^(1/4)*
sqrt(6)*(sqrt(3) + 2)*e^(2*x) + 12^(1/4)*sqrt(6)*(sqrt(3) + 2))*sqrt(-4*sqr
t(3) + 8)*log(6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(2*x) - 12^(1/4)*sqrt(6)*
(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3) + 36*e^(4*x) - 144*e^(2
*x) + 252) + (12^(1/4)*sqrt(6)*(sqrt(3) + 2)*e^(2*x) + 12^(1/4)*sqrt(6)*(sq
rt(3) + 2))*sqrt(-4*sqrt(3) + 8)*log(-6*(12^(1/4)*sqrt(6)*(sqrt(3) + 3)*e^(
2*x) - 12^(1/4)*sqrt(6)*(5*sqrt(3) + 9))*sqrt(-4*sqrt(3) + 8) + 144*sqrt(3)
 + 36*e^(4*x) - 144*e^(2*x) + 252) + 96)/(e^(2*x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**6),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27431, size = 14, normalized size = 0.2

$$-\frac{2}{3\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^6),x, algorithm="giac")
```

```
[Out] -2/3/(e^(2*x) + 1)
```

3.272 $\int \frac{1}{1+\sinh^8(x)}\,dx$

Optimal. Leaf size=129

$$\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{3/4}}\tanh(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{3/4}}\tanh(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

```
[Out] ArcTanh[Sqrt[1 - (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTanh[Sq
rt[1 + (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTanh[Sqrt[1 - (-1
)^(3/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTanh[Sqrt[1 + (-1)^(3/4)]*T
anh[x]]/(4*Sqrt[1 + (-1)^(3/4)])
```

Rubi [A] time = 0.197778, antiderivative size = 129, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1-(-1)^{3/4}}\tanh(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{3/4}}\tanh(x)\right)}{4\sqrt{1+(-1)^{3/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sinh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTanh[Sq
rt[1 + (-1)^(1/4)]*Tanh[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTanh[Sqrt[1 - (-1
)^(3/4)]*Tanh[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTanh[Sqrt[1 + (-1)^(3/4)]*T
anh[x]]/(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1+\sinh^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sinh^2(x)}\,dx + \frac{1}{4}\int \frac{}{1+(-}$$
$$= \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \tanh(x)\right) + \frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-\left(1+\sqrt[4]{-1}\right)x^2}\,dx, x, \tanh(x)\right) + \frac{1}{4}$$
$$= \frac{\tanh^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\tanh(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\tanh(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1-(-1)^{3/4}}\tanh(x)\right)}{4\sqrt{1-(-1)^{3/4}}} +$$

Mathematica [C] time = 0.123296, size = 127, normalized size = 0.98

$$16\text{RootSum}\left[\#1^8-8\#1^7+28\#1^6-56\#1^5+326\#1^4-56\#1^3+28\#1^2-8\#1+1\&, \frac{\#1^3x+\#1^3\log(-\#1\sinh(x)+\#1\cos}{\#1^7-7\#1^6+21\#1^5-35\#1^4+16}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sinh[x]^8)^(-1),x]
```

```
[Out] 16*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*
#1^7 + #1^8 & , (x*#1^3 + Log[-Cosh[x] - Sinh[x] + Cosh[x]*#1 - Sinh[x]*#1]
*#1^3)/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5 - 7*#1^6 + #1^7)
 & ]
```

Maple [C] time = 0.036, size = 64, normalized size = 0.5

$$\frac{1}{8}\sum_{\_R=\text{RootOf}\left(2\_Z^8-4\_Z^6+6\_Z^4-4\_Z^2+1\right)}\_R\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(-4\_R^7+8\_R^5-12\_R^3+8\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sinh(x)^8),x)
```

```
[Out] 1/8*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^7+8*_R^5-12*_R^3+8*_R)*tanh(1/2*x)+1),_R
=RootOf(2*_Z^8-4*_Z^6+6*_Z^4-4*_Z^2+1))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sinh(x)^8+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(sinh(x)^8 + 1), x)
```

Fricas [B] time = 2.8748, size = 11979, normalized size = 92.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2)
 + 4)^(3/4)*sqrt(2*sqrt(2) + 3)*(sqrt(2) - 1)*arctan(1/31*(2*(13*sqrt(2) -
20)*e^(2*x) + 23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 1/
496*(32*(10*sqrt(2) - 13)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (((355*
sqrt(2) - 508)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 86
)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(2
*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3))
*(2*sqrt(2) + 4)^(1/4))*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt
(2) + 8) + 4*((76*sqrt(2) - 105)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) +
2*(53*sqrt(2) - 72)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt(
2) - 33)*sqrt(2*sqrt(2) + 3))*sqrt(4*(sqrt(2) - 1)*e^(2*x) - (2*(sqrt(2) -
1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) -
6*sqrt(2) + 6)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*
(2*sqrt(2) + 4)^(1/4) - 4*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2) - 4*sqrt(2) + 2
*e^(4*x) + 10) + 1/248*((((254*sqrt(2) - 355)*e^(2*x) - 102*sqrt(2) + 145)*
sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*(3*(43*sqrt(2) - 59)*e^(2*x) -
23*sqrt(2) + 33)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt
(2) - 164)*e^(2*x) - 39*sqrt(2) + 60)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) +
3) + 2*((63*sqrt(2) - 85)*e^(2*x) - 17*sqrt(2) + 19)*sqrt(2*sqrt(2) + 3))*(
2*sqrt(2) + 4)^(1/4))*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2
) + 8) + 1/124*(((105*sqrt(2) - 152)*e^(2*x) + 13*sqrt(2) - 20)*sqrt(2*sqrt
(2) + 4)*sqrt(2*sqrt(2) + 3) + 4*((36*sqrt(2) - 53)*e^(2*x) - 23*sqrt(2) +
33)*sqrt(2*sqrt(2) + 3))*sqrt(2*sqrt(2) + 4) + 1/31*((33*sqrt(2) - 46)*e^(2
*x) - 3*sqrt(2) + 7)*sqrt(2*sqrt(2) + 3)) + 1/16*sqrt(2*sqrt(2*sqrt(2) + 4)
*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(3/4)*sqrt(2*sqrt(2) + 3)
*(sqrt(2) - 1)*arctan(-1/31*(2*(13*sqrt(2) - 20)*e^(2*x) + 23*sqrt(2) - 33)
*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) - 1/496*(32*(10*sqrt(2) - 13)*sqrt
(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) - (((355*sqrt(2) - 508)*sqrt(2*sqrt(2)
+ 4)*sqrt(2*sqrt(2) + 3) + 6*(59*sqrt(2) - 86)*sqrt(2*sqrt(2) + 3))*(2*sqrt
(2) + 4)^(3/4) + 4*((82*sqrt(2) - 119)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) +
 3) + (85*sqrt(2) - 126)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(1/4))*sqrt(2
*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8) + 4*((76*sqrt(2) - 10
5)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*(53*sqrt(2) - 72)*sqrt(2*sqr
t(2) + 3))*sqrt(2*sqrt(2) + 4) + 16*(23*sqrt(2) - 33)*sqrt(2*sqrt(2) + 3))*
sqrt(4*(sqrt(2) - 1)*e^(2*x) + (2*(sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^
(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) - 6*sqrt(2) + 6)*sqrt(2*sqrt(2*s
qrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 4*sqrt
(2*sqrt(2) + 4)*(sqrt(2) - 2) - 4*sqrt(2) + 2*e^(4*x) + 10) + 1/248*((((254
*sqrt(2) - 355)*e^(2*x) - 102*sqrt(2) + 145)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqr
t(2) + 3) + 2*(3*(43*sqrt(2) - 59)*e^(2*x) - 23*sqrt(2) + 33)*sqrt(2*sqrt(2
) + 3))*(2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) - 164)*e^(2*x) - 39*sqrt(2
) + 60)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) + 2*((63*sqrt(2) - 85)*e^(2
*x) - 17*sqrt(2) + 19)*sqrt(2*sqrt(2) + 3))*(2*sqrt(2) + 4)^(1/4))*sqrt(2*s
qrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8) - 1/124*(((105*sqrt(2)
- 152)*e^(2*x) + 13*sqrt(2) - 20)*sqrt(2*sqrt(2) + 4)*sqrt(2*sqrt(2) + 3) +
 4*((36*sqrt(2) - 53)*e^(2*x) - 23*sqrt(2) + 33)*sqrt(2*sqrt(2) + 3))*sqrt(
2*sqrt(2) + 4) - 1/31*((33*sqrt(2) - 46)*e^(2*x) - 3*sqrt(2) + 7)*sqrt(2*sq
rt(2) + 3)) - 1/16*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2)
 + 8)*(sqrt(2) + 1)*(-2*sqrt(2) + 4)^(3/4)*sqrt(-2*sqrt(2) + 3)*arctan(1/31
*(2*(13*sqrt(2) + 20)*e^(2*x) + 23*sqrt(2) + 33)*sqrt(-2*sqrt(2) + 4)*sqrt(
-2*sqrt(2) + 3) - 1/496*(32*(10*sqrt(2) + 13)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*
sqrt(2) + 3) - (((355*sqrt(2) + 508)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) +
 3) + 6*(59*sqrt(2) + 86)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 4*
((82*sqrt(2) + 119)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + (85*sqrt(2)
 + 126)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sqrt(-2*(2*sqrt(2) +
```

```
3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 4*((76*sqrt(2) + 105)*sqrt(-2*sq
rt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*(53*sqrt(2) + 72)*sqrt(-2*sqrt(2) + 3))
*sqrt(-2*sqrt(2) + 4) + 16*(23*sqrt(2) + 33)*sqrt(-2*sqrt(2) + 3))*sqrt(-4*
(sqrt(2) + 1)*e^(2*x) - (2*(sqrt(2) + 1)*e^(2*x) + ((sqrt(2) + 2)*e^(2*x) -
 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) + 4) - 6*sqrt(2) - 6)*sqrt(-2*(2*sqrt(2) +
3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2) + 4)^(1/4) + 4*(sqrt(2
) + 2)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 2*e^(4*x) + 10) - 1/248*((((254*s
qrt(2) + 355)*e^(2*x) - 102*sqrt(2) - 145)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqr
t(2) + 3) + 2*(3*(43*sqrt(2) + 59)*e^(2*x) - 23*sqrt(2) - 33)*sqrt(-2*sqrt(
2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) + 164)*e^(2*x) - 39*sqrt
(2) - 60)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*((63*sqrt(2) + 85)*
e^(2*x) - 17*sqrt(2) - 19)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sq
rt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 1/124*(((105*
sqrt(2) + 152)*e^(2*x) + 13*sqrt(2) + 20)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt
(2) + 3) + 4*((36*sqrt(2) + 53)*e^(2*x) - 23*sqrt(2) - 33)*sqrt(-2*sqrt(2)
+ 3))*sqrt(-2*sqrt(2) + 4) + 1/31*((33*sqrt(2) + 46)*e^(2*x) - 3*sqrt(2) -
7)*sqrt(-2*sqrt(2) + 3)) - 1/16*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4
) + 4*sqrt(2) + 8)*(sqrt(2) + 1)*(-2*sqrt(2) + 4)^(3/4)*sqrt(-2*sqrt(2) + 3
)*arctan(-1/31*(2*(13*sqrt(2) + 20)*e^(2*x) + 23*sqrt(2) + 33)*sqrt(-2*sqrt
(2) + 4)*sqrt(-2*sqrt(2) + 3) + 1/496*(32*(10*sqrt(2) + 13)*sqrt(-2*sqrt(2)
 + 4)*sqrt(-2*sqrt(2) + 3) + (((355*sqrt(2) + 508)*sqrt(-2*sqrt(2) + 4)*sqr
t(-2*sqrt(2) + 3) + 6*(59*sqrt(2) + 86)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) +
 4)^(3/4) + 4*((82*sqrt(2) + 119)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3)
 + (85*sqrt(2) + 126)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(1/4))*sqrt(-2
*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) + 4*((76*sqrt(2) + 1
05)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*(53*sqrt(2) + 72)*sqrt(-2
*sqrt(2) + 3))*sqrt(-2*sqrt(2) + 4) + 16*(23*sqrt(2) + 33)*sqrt(-2*sqrt(2)
+ 3))*sqrt(-4*(sqrt(2) + 1)*e^(2*x) + (2*(sqrt(2) + 1)*e^(2*x) + ((sqrt(2)
+ 2)*e^(2*x) - 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) + 4) - 6*sqrt(2) - 6)*sqrt(-2
*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2) + 4)^(1/
4) + 4*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 2*e^(4*x) + 10) - 1
/248*((((254*sqrt(2) + 355)*e^(2*x) - 102*sqrt(2) - 145)*sqrt(-2*sqrt(2) +
4)*sqrt(-2*sqrt(2) + 3) + 2*(3*(43*sqrt(2) + 59)*e^(2*x) - 23*sqrt(2) - 33)
*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2) + 4)^(3/4) + 2*(((119*sqrt(2) + 164)*e^(
2*x) - 39*sqrt(2) - 60)*sqrt(-2*sqrt(2) + 4)*sqrt(-2*sqrt(2) + 3) + 2*((63*
sqrt(2) + 85)*e^(2*x) - 17*sqrt(2) - 19)*sqrt(-2*sqrt(2) + 3))*(-2*sqrt(2)
+ 4)^(1/4))*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8) -
 1/124*(((105*sqrt(2) + 152)*e^(2*x) + 13*sqrt(2) + 20)*sqrt(-2*sqrt(2) + 4
)*sqrt(-2*sqrt(2) + 3) + 4*((36*sqrt(2) + 53)*e^(2*x) - 23*sqrt(2) - 33)*sq
rt(-2*sqrt(2) + 3))*sqrt(-2*sqrt(2) + 4) - 1/31*((33*sqrt(2) + 46)*e^(2*x)
- 3*sqrt(2) - 7)*sqrt(-2*sqrt(2) + 3)) + 1/64*sqrt(-2*(2*sqrt(2) + 3)*sqrt(
-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*((sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + 2*sq
rt(2))*(-2*sqrt(2) + 4)^(1/4)*log(-2*(sqrt(2) + 1)*e^(2*x) + 1/2*(2*(sqrt(2
) + 1)*e^(2*x) + ((sqrt(2) + 2)*e^(2*x) - 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) +
4) - 6*sqrt(2) - 6)*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2
) + 8)*(-2*sqrt(2) + 4)^(1/4) + 2*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 2*sq
rt(2) + e^(4*x) + 5) - 1/64*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) +
4*sqrt(2) + 8)*((sqrt(2) + 1)*sqrt(-2*sqrt(2) + 4) + 2*sqrt(2))*(-2*sqrt(2)
 + 4)^(1/4)*log(-2*(sqrt(2) + 1)*e^(2*x) - 1/2*(2*(sqrt(2) + 1)*e^(2*x) + (
(sqrt(2) + 2)*e^(2*x) - 5*sqrt(2) - 6)*sqrt(-2*sqrt(2) + 4) - 6*sqrt(2) - 6
)*sqrt(-2*(2*sqrt(2) + 3)*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + 8)*(-2*sqrt(2)
 + 4)^(1/4) + 2*(sqrt(2) + 2)*sqrt(-2*sqrt(2) + 4) + 2*sqrt(2) + e^(4*x) +
5) + 1/64*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(sqrt
(2*sqrt(2) + 4)*(sqrt(2) - 1) + 2*sqrt(2))*(2*sqrt(2) + 4)^(1/4)*log(2*(sqr
t(2) - 1)*e^(2*x) + 1/2*(2*(sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) -
 5*sqrt(2) + 6)*sqrt(2*sqrt(2) + 4) - 6*sqrt(2) + 6)*sqrt(2*sqrt(2*sqrt(2)
+ 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 2*sqrt(2*sqrt
(2) + 4)*(sqrt(2) - 2) - 2*sqrt(2) + e^(4*x) + 5) - 1/64*sqrt(2*sqrt(2*sqrt
(2) + 4)*(2*sqrt(2) - 3) - 4*sqrt(2) + 8)*(sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1
```

```
) + 2*sqrt(2))*(2*sqrt(2) + 4)^(1/4)*log(2*(sqrt(2) - 1)*e^(2*x) - 1/2*(2*(
sqrt(2) - 1)*e^(2*x) + ((sqrt(2) - 2)*e^(2*x) - 5*sqrt(2) + 6)*sqrt(2*sqrt(
2) + 4) - 6*sqrt(2) + 6)*sqrt(2*sqrt(2*sqrt(2) + 4)*(2*sqrt(2) - 3) - 4*sqr
t(2) + 8)*(2*sqrt(2) + 4)^(1/4) - 2*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 2) - 2*s
qrt(2) + e^(4*x) + 5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)**8),x)

[Out] Timed out
```

Giac [A] time = 1.27901, size = 1, normalized size = 0.01

$$0$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sinh(x)^8),x, algorithm="giac")

[Out] 0
```

3.273 $\int \frac{1}{1-\sinh^5(x)}\,dx$

Optimal. Leaf size=228

$$-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2\sqrt[10]{-1}\tanh^{-1}\left(\frac{(-1)^{3/10}\left((-1)^{4/5}\tan\right.}{\sqrt{\sqrt[5]{-1}+(-1)}}\right.}{5\sqrt{\sqrt[5]{-1}+(-1)^{3/5}}}$$

```
[Out] (-2*(-1)^(1/10)*ArcTan[(I + (-1)^(1/10)*Tanh[x/2])/Sqrt[1 - (-1)^(1/5)]])/(
5*Sqrt[1 - (-1)^(1/5)]) - (2*ArcTanh[((-1)^(3/5) - Tanh[x/2])/Sqrt[1 - (-1)
^(1/5)]])/(5*Sqrt[1 - (-1)^(1/5)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[
2]])/5 + (2*ArcTanh[((-1)^(4/5) + Tanh[x/2])/Sqrt[1 - (-1)^(3/5)]])/(5*Sqrt
[1 - (-1)^(3/5)]) - (2*(-1)^(1/10)*ArcTanh[((-1)^(3/10)*(1 + (-1)^(4/5)*Tan
h[x/2]))/Sqrt[(-1)^(1/5) + (-1)^(3/5)]])/(5*Sqrt[(-1)^(1/5) + (-1)^(3/5)])
```

Rubi [A] time = 0.417044, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 17, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3213, 2660, 618, 204, 617, 206}

$$-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+1}{\sqrt{2}}\right)+\frac{2\tanh^{-1}\left(\frac{\tanh\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1-(-1)^{3/5}}}\right)}{5\sqrt{1-(-1)^{3/5}}}-\frac{2\sqrt[10]{-1}\tanh^{-1}\left(\frac{(-1)^{3/10}\left((-1)^{4/5}\tan\right.}{\sqrt{\sqrt[5]{-1}+(-1)}}\right.}{5\sqrt{\sqrt[5]{-1}+(-1)^{3/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^5)^(-1),x]
```

```
[Out] (-2*(-1)^(1/10)*ArcTan[(I + (-1)^(1/10)*Tanh[x/2])/Sqrt[1 - (-1)^(1/5)]])/(
5*Sqrt[1 - (-1)^(1/5)]) - (2*ArcTanh[((-1)^(3/5) - Tanh[x/2])/Sqrt[1 - (-1)
^(1/5)]])/(5*Sqrt[1 - (-1)^(1/5)]) + (Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[
2]])/5 + (2*ArcTanh[((-1)^(4/5) + Tanh[x/2])/Sqrt[1 - (-1)^(3/5)]])/(5*Sqrt
[1 - (-1)^(3/5)]) - (2*(-1)^(1/10)*ArcTanh[((-1)^(3/10)*(1 + (-1)^(4/5)*Tan
h[x/2]))/Sqrt[(-1)^(1/5) + (-1)^(3/5)]])/(5*Sqrt[(-1)^(1/5) + (-1)^(3/5)])
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-\sinh^5(x)}\,dx = \int \left(\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}-i\sinh(x)\right)}+\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}-\sqrt[10]{-1}\sinh(x)\right)}+\frac{\sqrt[10]{-1}}{5\left(\sqrt[10]{-1}+(-1)^{3/10}\sinh(x)\right)}+\frac{}{5\left(\sqrt[10]{-1}\right.}\right.$$

$$=\frac{1}{5}\sqrt[10]{-1}\int \frac{1}{\sqrt[10]{-1}-i\sinh(x)}\,dx+\frac{1}{5}\sqrt[10]{-1}\int \frac{1}{\sqrt[10]{-1}-\sqrt[10]{-1}\sinh(x)}\,dx+\frac{1}{5}\sqrt[10]{-1}\int \frac{1}{\sqrt[10]{-1}+(-1)^{3/}}$$

$$=\frac{1}{5}\left(2\sqrt[10]{-1}\right)\text{Subst}\left(\int \frac{1}{\sqrt[10]{-1}-2ix-\sqrt[10]{-1}x^2}\,dx,x,\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{5}\left(2\sqrt[10]{-1}\right)\text{Subst}\left(\int \frac{}{\sqrt[10]{-1}-2}\right.$$

$$=\frac{2}{5}\text{Subst}\left(\int \frac{1}{2-x^2}\,dx,x,1+\tanh\left(\frac{x}{2}\right)\right)-\frac{1}{5}\left(4\sqrt[10]{-1}\right)\text{Subst}\left(\int \frac{1}{-4\left(1-\sqrt[5]{-1}\right)-x^2}\,dx,x,-2i-\right.$$

$$=-\frac{2\sqrt[10]{-1}\tan^{-1}\left(\frac{i+\sqrt[10]{-1}\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{3/5}-\tanh\left(\frac{x}{2}\right)}{\sqrt{1-\sqrt[5]{-1}}}\right)}{5\sqrt{1-\sqrt[5]{-1}}}+\frac{1}{5}\sqrt{2}\tanh^{-1}\left(\frac{1+\tanh\left(\frac{x}{2}\right)}{\sqrt{2}}\right)+\frac{2}{}$$

Mathematica [C] time = 0.868295, size = 437, normalized size = 1.92

$$\frac{1}{10}\left(\text{RootSum}\left[\#1^8+2\#1^7+2\#1^5+14\#1^4-2\#1^3-2\#1+1\&,\frac{\#1^6x+4\#1^5x+9\#1^4x+24\#1^3x-9\#1^2x+2\#1^6\log}{}\right.\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^5)^(-1),x]
```

```
[Out] (2*Sqrt[2]*ArcTanh[(1 + Tanh[x/2])/Sqrt[2]] + RootSum[1 - 2*#1 - 2*#1^3 + 1
4*#1^4 + 2*#1^5 + 2*#1^7 + #1^8 & , (-x - 2*Log[-Cosh[x/2] - Sinh[x/2] + Co
sh[x/2]*#1 - Sinh[x/2]*#1] + 4*x*#1 + 8*Log[-Cosh[x/2] - Sinh[x/2] + Cosh[x
/2]*#1 - Sinh[x/2]*#1]*#1 - 9*x*#1^2 - 18*Log[-Cosh[x/2] - Sinh[x/2] + Cosh
[x/2]*#1 - Sinh[x/2]*#1]*#1^2 + 24*x*#1^3 + 48*Log[-Cosh[x/2] - Sinh[x/2] +
 Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^3 + 9*x*#1^4 + 18*Log[-Cosh[x/2] - Sinh[x/
2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^4 + 4*x*#1^5 + 8*Log[-Cosh[x/2] - Sinh
[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^5 + x*#1^6 + 2*Log[-Cosh[x/2] - Sin
h[x/2] + Cosh[x/2]*#1 - Sinh[x/2]*#1]*#1^6)/(-1 - 3*#1^2 + 28*#1^3 + 5*#1^4
 + 7*#1^6 + 4*#1^7) & ])/10
```

Maple [C] time = 0.036, size = 124, normalized size = 0.5

$$\frac{2}{5} \sum_{\_R=\mathrm{RootOf}\left(\_Z^8-2\_Z^7-2\_Z^5+14\_Z^4+2\_Z^3+2\_Z+1\right)} \frac{-2\_R^6+3\_R^5+2\_R^4-2\_R^3-2\_R^2+3\_R+2}{4\_R^7-7\_R^6-5\_R^4+28\_R^3+3\_R^2+1} \ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^5),x)
```

```
[Out] 2/5*sum((-2*_R^6+3*_R^5+2*_R^4-2*_R^3-2*_R^2+3*_R+2)/(4*_R^7-7*_R^6-5*_R^4+
28*_R^3+3*_R^2+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z^8-2*_Z^7-2*_Z^5+14*_Z^4+2
*_Z^3+2*_Z+1))+1/5*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{10}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x+1}{\sqrt{2}+e^x-1}\right)+\int \frac{2\left(e^{(7x)}+4e^{(6x)}+9e^{(5x)}+24e^{(4x)}-9e^{(3x)}+4e^{(2x)}-e^x\right)}{5\left(e^{(8x)}+2e^{(7x)}+2e^{(5x)}+14e^{(4x)}-2e^{(3x)}-2e^x+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="maxima")
```

```
[Out] -1/10*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + integrate(2/5
*(e^(7*x) + 4*e^(6*x) + 9*e^(5*x) + 24*e^(4*x) - 9*e^(3*x) + 4*e^(2*x) - e^
x)/(e^(8*x) + 2*e^(7*x) + 2*e^(5*x) + 14*e^(4*x) - 2*e^(3*x) - 2*e^x + 1),
x)
```

Fricas [B] time = 4.08645, size = 11359, normalized size = 49.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="fricas")
```

```
[Out] 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5)
+ 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(
5) + 3)*arctan(1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x + 7*sqrt(5) - 15)*sqrt(2
*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 1/80*sqrt(2)*(sqrt(2)*((11*sqrt(5) - 25)*
e^x - 4*sqrt(5) + 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((3*sqrt(5)
 - 5)*e^x - 7*sqrt(5) + 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(5) + 3) + 1/1280
0*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 40*s
qrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*
sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) + sqrt(2*sqrt(2)*(2*sq
rt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7*sqrt(5) - 15)*s
qrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) +
 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*sqrt(2*sqrt(5) +
 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5)
 + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt(-20*sqrt(2)*sqr
t(sqrt(5) + 3)*(sqrt(5) - 3) - 40*(sqrt(5) - 1)*e^x + 2*(sqrt(2)*((2*sqrt(5
) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^x + 3*sqr
```

```
t(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20
)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*sqrt(2)*(2*sqrt(
5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*sqrt(5) - 7)*e^x
- 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 1
1)*e^x - 2*sqrt(5) + 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sq
rt(2)*((7*sqrt(5) - 17)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt
(5) + 3) + 2*((5*sqrt(5) - 11)*e^x - 5*sqrt(5) + 9)*sqrt(2*sqrt(5) + 5))*(8
*sqrt(5) + 24)^(1/4)) + 1/20*(2*(4*sqrt(5) - 5)*e^x + sqrt(5) - 5)*sqrt(2*s
qrt(5) + 5)) + 1/200*sqrt(2)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) +
3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*(3*sqrt(5) - 5)*sqrt(2*sqrt(5)
+ 5)*sqrt(sqrt(5) + 3)*arctan(-1/40*sqrt(2)*((11*sqrt(5) - 25)*e^x + 7*sqrt
(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) - 1/80*sqrt(2)*(sqrt(2)*((1
1*sqrt(5) - 25)*e^x - 4*sqrt(5) + 10)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3)
 + 2*((3*sqrt(5) - 5)*e^x - 7*sqrt(5) + 15)*sqrt(2*sqrt(5) + 5))*sqrt(sqrt(
5) + 3) - 1/12800*(80*sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(sqr
t(5) + 3) + 40*sqrt(2)*(sqrt(2)*(5*sqrt(5) - 11)*sqrt(2*sqrt(5) + 5)*sqrt(s
qrt(5) + 3) + 2*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 3))*sqrt(sqrt(5) + 3) - sqrt
(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*(7
*sqrt(5) - 15)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*
sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 24)^(3/4) + 4*(sqrt(2)*(17*sqrt(5) - 35)*
sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2*(11*sqrt(5) - 25)*sqrt(2*sqrt(5)
+ 5))*(8*sqrt(5) + 24)^(1/4)) + 320*sqrt(2*sqrt(5) + 5)*(sqrt(5) - 4))*sqrt
(-20*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) - 40*(sqrt(5) - 1)*e^x - 2*(sq
rt(2)*((2*sqrt(5) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5)
- 5)*e^x + 3*sqrt(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3)
- 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 80*e^(2*x) + 80) + 1/640*sqrt(2*
sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*((sqrt(2)*((3*s
qrt(5) - 7)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5) + 5)*sqrt(sqrt(5) + 3) + 2
*((5*sqrt(5) - 11)*e^x - 2*sqrt(5) + 4)*sqrt(2*sqrt(5) + 5))*(8*sqrt(5) + 2
4)^(3/4) + 4*(sqrt(2)*((7*sqrt(5) - 17)*e^x - 8*sqrt(5) + 18)*sqrt(2*sqrt(5
) + 5)*sqrt(sqrt(5) + 3) + 2*((5*sqrt(5) - 11)*e^x - 5*sqrt(5) + 9)*sqrt(2*
sqrt(5) + 5))*(8*sqrt(5) + 24)^(1/4)) - 1/20*(2*(4*sqrt(5) - 5)*e^x + sqrt(
5) - 5)*sqrt(2*sqrt(5) + 5)) - 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5)
+ 24) + 4*sqrt(5) + 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) +
24)^(3/4)*arctan(-1/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x -
8*sqrt(5) - 18)*sqrt(-8*sqrt(5) + 24) - 8*sqrt(5) - 16)*(-8*sqrt(5) + 24)^(
3/4) + 4*(4*(5*sqrt(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x - 8*sqrt(5) - 18)*
sqrt(-8*sqrt(5) + 24) - 20*sqrt(5) - 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*
sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) +
 1/25600*((((7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*
sqrt(5) + 24)^(3/4) + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(
5) + 100)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) +
24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) - 20*(((5*sqrt(5) + 11)*sqrt(-8*
sqrt(5) + 24) + 4*sqrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)*
sqrt(-8*sqrt(5) + 24) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(40*(sq
rt(5) + 1)*e^x + (4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) +
5)*sqrt(-8*sqrt(5) + 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sq
rt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*sq
rt(-8*sqrt(5) + 24) + 80*e^(2*x) + 80) + 1/320*(32*(4*sqrt(5) + 5)*e^x + 4*
((11*sqrt(5) + 25)*e^x + 7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sqrt
(5) + 5)*e^x + ((11*sqrt(5) + 25)*e^x - 4*sqrt(5) - 10)*sqrt(-8*sqrt(5) + 2
4) - 28*sqrt(5) - 60)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5) + 80)*sqrt(-2*sqrt
(5) + 5)) - 1/400*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) +
 20)*(3*sqrt(5) + 5)*sqrt(-2*sqrt(5) + 5)*(-8*sqrt(5) + 24)^(3/4)*arctan(-1
/1280*((4*(5*sqrt(5) + 11)*e^x + ((3*sqrt(5) + 7)*e^x - 8*sqrt(5) - 18)*sqr
t(-8*sqrt(5) + 24) - 8*sqrt(5) - 16)*(-8*sqrt(5) + 24)^(3/4) + 4*(4*(5*sqrt
(5) + 11)*e^x + ((7*sqrt(5) + 17)*e^x - 8*sqrt(5) - 18)*sqrt(-8*sqrt(5) + 2
4) - 20*sqrt(5) - 36)*(-8*sqrt(5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-
8*sqrt(5) + 24) + 4*sqrt(5) + 20)*sqrt(-2*sqrt(5) + 5) + 1/25600*((((7*sqrt
```

```
(5) + 15)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*sqrt(5) + 24)^(3/4)
 + 4*((17*sqrt(5) + 35)*sqrt(-8*sqrt(5) + 24) + 44*sqrt(5) + 100)*(-8*sqrt(
5) + 24)^(1/4))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 2
0)*sqrt(-2*sqrt(5) + 5) + 20*(((5*sqrt(5) + 11)*sqrt(-8*sqrt(5) + 24) + 4*s
qrt(5) + 12)*sqrt(-8*sqrt(5) + 24) + 4*(5*sqrt(5) + 11)*sqrt(-8*sqrt(5) + 2
4) + 32*sqrt(5) + 128)*sqrt(-2*sqrt(5) + 5))*sqrt(40*(sqrt(5) + 1)*e^x - (4
*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) + 5)*sqrt(-8*sqrt(5)
+ 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqr
t(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 10*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24)
 + 80*e^(2*x) + 80) - 1/320*(32*(4*sqrt(5) + 5)*e^x + 4*((11*sqrt(5) + 25)*
e^x + 7*sqrt(5) + 15)*sqrt(-8*sqrt(5) + 24) + (4*(3*sqrt(5) + 5)*e^x + ((11
*sqrt(5) + 25)*e^x - 4*sqrt(5) - 10)*sqrt(-8*sqrt(5) + 24) - 28*sqrt(5) - 6
0)*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5) + 80)*sqrt(-2*sqrt(5) + 5)) - 1/800*(
sqrt(2)*(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 8*sqrt(5))*sqrt(2*sqrt(2)*(2*sq
rt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*log(-
4*sqrt(2)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) - 8*(sqrt(5) - 1)*e^x + 2/5*(sqrt
(2)*((2*sqrt(5) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) -
5)*e^x + 3*sqrt(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) -
4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) + 1/800*(sqrt(2)*
(3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 8*sqrt(5))*sqrt(2*sqrt(2)*(2*sqrt(5) -
5)*sqrt(sqrt(5) + 3) - 4*sqrt(5) + 20)*(8*sqrt(5) + 24)^(1/4)*log(-4*sqrt(2
)*sqrt(sqrt(5) + 3)*(sqrt(5) - 3) - 8*(sqrt(5) - 1)*e^x - 2/5*(sqrt(2)*((2*
sqrt(5) - 5)*e^x + 3*sqrt(5) - 5)*sqrt(sqrt(5) + 3) + 2*(sqrt(5) - 5)*e^x +
 3*sqrt(5) - 5)*sqrt(2*sqrt(2)*(2*sqrt(5) - 5)*sqrt(sqrt(5) + 3) - 4*sqrt(5
) + 20)*(8*sqrt(5) + 24)^(1/4) + 16*e^(2*x) + 16) - 1/1600*((3*sqrt(5) + 5)
*sqrt(-8*sqrt(5) + 24) + 16*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5)
+ 24) + 4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4)*log(8*(sqrt(5) + 1)*e^x + 1
/5*(4*(sqrt(5) + 5)*e^x + ((2*sqrt(5) + 5)*e^x + 3*sqrt(5) + 5)*sqrt(-8*sqr
t(5) + 24) + 6*sqrt(5) + 10)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) +
4*sqrt(5) + 20)*(-8*sqrt(5) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) +
 24) + 16*e^(2*x) + 16) + 1/1600*((3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 1
6*sqrt(5))*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-
8*sqrt(5) + 24)^(1/4)*log(8*(sqrt(5) + 1)*e^x - 1/5*(4*(sqrt(5) + 5)*e^x +
((2*sqrt(5) + 5)*e^x + 3*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 6*sqrt(5) + 1
0)*sqrt(-(2*sqrt(5) + 5)*sqrt(-8*sqrt(5) + 24) + 4*sqrt(5) + 20)*(-8*sqrt(5
) + 24)^(1/4) + 2*(sqrt(5) + 3)*sqrt(-8*sqrt(5) + 24) + 16*e^(2*x) + 16) +
1/10*sqrt(2)*log((2*(sqrt(2) - 1)*e^x - 2*sqrt(2) + e^(2*x) + 3)/(e^(2*x) -
 2*e^x - 1))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**5),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int -\frac{1}{\sinh(x)^5 - 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^5),x, algorithm="giac")
```

```
[Out] integrate(-1/(sinh(x)^5 - 1), x)
```

3.274 $\int \frac{1}{1-\sinh^6(x)} dx$

Optimal. Leaf size=83

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(3*Sqrt[2]) + ArcTanh[Sqrt[1 - (-1)^(1/3)]*Tanh[x]
]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTanh[Sqrt[1 + (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[
1 + (-1)^(2/3)])
```

Rubi [A] time = 0.0998656, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.3, Rules used = {3211, 3181, 206}

$$\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{3\sqrt{2}}+\frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^6)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[2]*Tanh[x]]/(3*Sqrt[2]) + ArcTanh[Sqrt[1 - (-1)^(1/3)]*Tanh[x]
]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTanh[Sqrt[1 + (-1)^(2/3)]*Tanh[x]]/(3*Sqrt[
1 + (-1)^(2/3)])
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{1}{1-\sinh^6(x)}dx = \frac{1}{3}\int \frac{1}{1-\sinh^2(x)}dx + \frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\sinh^2(x)}dx + \frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sinh^2(x)}dx$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-2x^2}dx, x, \tanh(x)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1-\left(1-\sqrt[3]{-1}\right)x^2}dx, x, \tanh(x)\right) + \frac{1}{3}\text{Subst}$$
$$= \frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{3\sqrt{2}} + \frac{\tanh^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\tanh(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}} + \frac{\tanh^{-1}\left(\sqrt{1+(-1)^{2/3}}\tanh(x)\right)}{3\sqrt{1+(-1)^{2/3}}}$$

Mathematica [C] time = 0.431108, size = 70, normalized size = 0.84

$$\frac{1}{6}\left(\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right) + i\sqrt{3}\left(\tan^{-1}\left(\frac{1-2i\tanh(x)}{\sqrt{3}}\right) - \tan^{-1}\left(\frac{1+2i\tanh(x)}{\sqrt{3}}\right)\right) - \tan^{-1}(\text{csch}(x)\text{sech}(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^6)^(-1),x]
```

```
[Out] (-ArcTan[Csch[x]*Sech[x]] + I*Sqrt[3]*(ArcTan[(1 - (2*I)*Tanh[x])/Sqrt[3]]
- ArcTan[(1 + (2*I)*Tanh[x])/Sqrt[3]]) + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]])/
6
```

Maple [C] time = 0.032, size = 160, normalized size = 1.9

$$\frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^4-2\_Z^3+2\_Z^2+2\_Z+1\right)} \frac{-\_R^2+\_R+1}{2\_R^3-3\_R^2+2\_R+1}\ln\left(\tanh\left(\frac{x}{2}\right)-\_R\right) + \frac{1}{3}\sum_{\_R=\text{RootOf}\left(\_Z^4+2\_Z^3+2\_Z^2-2\_Z+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^6),x)
```

```
[Out] 1/3*sum((-_R^2+_R+1)/(2*_R^3-3*_R^2+2*_R+1)*ln(tanh(1/2*x)-_R),_R=RootOf(_Z
^4-2*_Z^3+2*_Z^2+2*_Z+1))+1/3*sum((-_R^2-_R+1)/(2*_R^3+3*_R^2+2*_R-1)*ln(ta
nh(1/2*x)-_R),_R=RootOf(_Z^4+2*_Z^3+2*_Z^2-2*_Z+1))+1/6*2^(1/2)*arctanh(1/4
*(2*tanh(1/2*x)-2)*2^(1/2))+1/6*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/
2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x+1}{\sqrt{2}+e^x-1}\right) + \frac{1}{12}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x-1}{\sqrt{2}+e^x+1}\right) + \int \frac{e^{(3x)}+4e^{(2x)}-e^x}{3\left(e^{(4x)}+2e^{(3x)}+2e^{(2x)}-2e^x+1\right)}dx - \int \frac{e}{3\left(e^{(4x)}-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="maxima")
```

```
[Out] -1/12*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + 1/12*sqrt(2)*
log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) + integrate(1/3*(e^(3*x) + 4*
e^(2*x) - e^x)/(e^(4*x) + 2*e^(3*x) + 2*e^(2*x) - 2*e^x + 1), x) - integrat
```

```
e(1/3*(e^(3*x) - 4*e^(2*x) - e^x)/(e^(4*x) - 2*e^(3*x) + 2*e^(2*x) + 2*e^x
+ 1), x)
```

Fricas [B] time = 1.74409, size = 505, normalized size = 6.08

$$-\frac{1}{12}\sqrt{3}\log\left(16\sqrt{3}+4e^{(4x)}+28\right)+\frac{1}{12}\sqrt{3}\log\left(-16\sqrt{3}+4e^{(4x)}+28\right)+\frac{1}{12}\sqrt{2}\log\left(\frac{2\left(2\sqrt{2}-3\right)e^{(2x)}-12\sqrt{2}+e^{(4x)}+}{e^{(4x)}-6e^{(2x)}+1}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="fricas")
```

```
[Out] -1/12*sqrt(3)*log(16*sqrt(3) + 4*e^(4*x) + 28) + 1/12*sqrt(3)*log(-16*sqrt(
3) + 4*e^(4*x) + 28) + 1/12*sqrt(2)*log((2*(2*sqrt(2) - 3)*e^(2*x) - 12*sqr
t(2) + e^(4*x) + 17)/(e^(4*x) - 6*e^(2*x) + 1)) - 1/3*arctan(-(sqrt(3) + 2)
*e^(2*x) + 1/2*(sqrt(3) + 2)*sqrt(-16*sqrt(3) + 4*e^(4*x) + 28)) + 1/3*arct
an(-(sqrt(3) - 2)*e^(2*x) + sqrt(4*sqrt(3) + e^(4*x) + 7)*(sqrt(3) - 2))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**6),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17511, size = 193, normalized size = 2.33

$$-\frac{1}{36}\left(\left(2\sqrt{3}-3\right)e^{(4x)}+2\sqrt{3}-3\right)\arctan\left(\frac{e^{(2x)}}{\sqrt{3}+2}\right)+\frac{1}{36}\left(\left(2\sqrt{3}+3\right)e^{(4x)}+2\sqrt{3}+3\right)\arctan\left(-\frac{e^{(2x)}}{\sqrt{3}-2}\right)-\frac{1}{12}\sqrt{3}\log\left(\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^6),x, algorithm="giac")
```

```
[Out] -1/36*((2*sqrt(3) - 3)*e^(4*x) + 2*sqrt(3) - 3)*arctan(e^(2*x)/(sqrt(3) + 2
)) + 1/36*((2*sqrt(3) + 3)*e^(4*x) + 2*sqrt(3) + 3)*arctan(-e^(2*x)/(sqrt(3
) - 2)) - 1/12*sqrt(3)*log((sqrt(3) + 2)^2 + e^(4*x)) + 1/12*sqrt(3)*log((s
qrt(3) - 2)^2 + e^(4*x)) - 1/12*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)
/abs(4*sqrt(2) + 2*e^(2*x) - 6))
```

3.275 $\int \frac{1}{1-\sinh^8(x)}\,dx$

Optimal. Leaf size=69

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{4}$$

```
[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(4*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]
/(4*Sqrt[1 + I]) + ArcTanh[Sqrt[2]*Tanh[x]]/(4*Sqrt[2]) + Tanh[x]/4
```

Rubi [A] time = 0.0772041, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.6, Rules used = {3211, 3181, 206, 3175, 3767, 8}

$$\frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{4}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sinh[x]^8)^(-1),x]
```

```
[Out] ArcTanh[Sqrt[1 - I]*Tanh[x]]/(4*Sqrt[1 - I]) + ArcTanh[Sqrt[1 + I]*Tanh[x]]
/(4*Sqrt[1 + I]) + ArcTanh[Sqrt[2]*Tanh[x]]/(4*Sqrt[2]) + Tanh[x]/4
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sinh^8(x)}\,dx &= \frac{1}{4}\int \frac{1}{1-\sinh^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1-i\sinh^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+i\sinh^2(x)}\,dx+\frac{1}{4}\int \frac{1}{1+\sinh^2(x)}\,dx\\ &= \frac{1}{4}\int \operatorname{sech}^2(x)\,dx+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-2x^2}\,dx,x,\tanh(x)\right)+\frac{1}{4}\operatorname{Subst}\left(\int \frac{1}{1-(1+i)x^2}\,dx,x,\tanh(x)\right.\\ &= \frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{1}{4}i\operatorname{Subst}(\int 1\,dx,\\ &= \frac{\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{4\sqrt{1-i}}+\frac{\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{4\sqrt{1+i}}+\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{4\sqrt{2}}+\frac{\tanh(x)}{4}\end{aligned}$$

Mathematica [A] time = 0.482848, size = 64, normalized size = 0.93

$$\frac{1}{8}\left(\frac{2\tanh^{-1}\left(\sqrt{1-i}\tanh(x)\right)}{\sqrt{1-i}}+\frac{2\tanh^{-1}\left(\sqrt{1+i}\tanh(x)\right)}{\sqrt{1+i}}+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)+2\tanh(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sinh[x]^8)^(-1),x]
```

```
[Out] ((2*ArcTanh[Sqrt[1 - I]*Tanh[x]])/Sqrt[1 - I] + (2*ArcTanh[Sqrt[1 + I]*Tanh
[x]])/Sqrt[1 + I] + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + 2*Tanh[x])/8
```

Maple [C] time = 0.033, size = 99, normalized size = 1.4

$$\frac{1}{8}\sum_{\_R=\mathrm{RootOf}\left(2\_Z^4-2\_Z^2+1\right)}\_R\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+\left(-4\_R^3+4\_R\right)\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2}\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-1}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sinh(x)^8),x)
```

```
[Out] 1/8*sum(_R*ln(tanh(1/2*x)^2+(-4*_R^3+4*_R)*tanh(1/2*x)+1),_R=RootOf(2*_Z^4-
2*_Z^2+1))+1/2*tanh(1/2*x)/(tanh(1/2*x)^2+1)+1/8*2^(1/2)*arctanh(1/4*(2*tan
h(1/2*x)-2)*2^(1/2))+1/8*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x+1}{\sqrt{2}+e^x-1}\right)+\frac{1}{16}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^x-1}{\sqrt{2}+e^x+1}\right)-\frac{1}{2\left(e^{(2x)}+1\right)}+8\int \frac{e^{(4x)}}{e^{(8x)}-4e^{(6x)}+22e^{(4x)}-4e^{(2x)}+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="maxima")
```

```
[Out] -1/16*sqrt(2)*log(-(sqrt(2) - e^x + 1)/(sqrt(2) + e^x - 1)) + 1/16*sqrt(2)*
log(-(sqrt(2) - e^x - 1)/(sqrt(2) + e^x + 1)) - 1/2/(e^(2*x) + 1) + 8*integ
rate(e^(4*x)/(e^(8*x) - 4*e^(6*x) + 22*e^(4*x) - 4*e^(2*x) + 1), x)
```

Fricas [B] time = 1.85785, size = 2198, normalized size = 31.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="fricas")
```

```
[Out] -1/32*(4*(2^(1/4)*e^(2*x) + 2^(1/4))*sqrt(-2*sqrt(2) + 4)*arctan(1/14*(sqrt
(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x) - 1/28*(2*sqrt(2)*(5*sqrt(2) +
 6) - (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) + 6))*sqrt(-2*sqrt(2
) + 4) + 16*sqrt(2) + 8)*sqrt(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*
sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 1/14*sqrt(2)*
(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(5*sqrt(2) +
6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1/4)*(3*sqrt(2) - 2))*sqrt(-2*
sqrt(2) + 4) - 1/7*sqrt(2) + 3/7) + 4*(2^(1/4)*e^(2*x) + 2^(1/4))*sqrt(-2*s
qrt(2) + 4)*arctan(-1/14*(sqrt(2)*(5*sqrt(2) + 6) + 8*sqrt(2) + 4)*e^(2*x)
+ 1/28*(2*sqrt(2)*(5*sqrt(2) + 6) + (2^(3/4)*(8*sqrt(2) + 11) + 2*2^(1/4)*(
5*sqrt(2) + 6))*sqrt(-2*sqrt(2) + 4) + 16*sqrt(2) + 8)*sqrt((2^(3/4)*e^(2*x
) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2
*e^(2*x) + 5) + 1/14*sqrt(2)*(3*sqrt(2) - 2) - 1/28*((2^(3/4)*(8*sqrt(2) +
11) + 2*2^(1/4)*(5*sqrt(2) + 6))*e^(2*x) - 2^(3/4)*(2*sqrt(2) + 1) - 2*2^(1
/4)*(3*sqrt(2) - 2))*sqrt(-2*sqrt(2) + 4) + 1/7*sqrt(2) - 3/7) - (2^(1/4)*(
sqrt(2) + 1)*e^(2*x) + 2^(1/4)*(sqrt(2) + 1))*sqrt(-2*sqrt(2) + 4)*log((2^(
3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) +
e^(4*x) - 2*e^(2*x) + 5) + (2^(1/4)*(sqrt(2) + 1)*e^(2*x) + 2^(1/4)*(sqrt(2
) + 1))*sqrt(-2*sqrt(2) + 4)*log(-(2^(3/4)*e^(2*x) - 2^(1/4)*(3*sqrt(2) + 4
))*sqrt(-2*sqrt(2) + 4) + 4*sqrt(2) + e^(4*x) - 2*e^(2*x) + 5) - 2*(sqrt(2)
*e^(2*x) + sqrt(2))*log((2*(2*sqrt(2) - 3)*e^(2*x) - 12*sqrt(2) + e^(4*x) +
 17)/(e^(4*x) - 6*e^(2*x) + 1)) + 16)/(e^(2*x) + 1)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)**8),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.38856, size = 65, normalized size = 0.94

$$-\frac{1}{16}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-\frac{1}{2\left(e^{(2x)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sinh(x)^8),x, algorithm="giac")
```

```
[Out] -1/16*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
 - 6)) - 1/2/(e^(2*x) + 1)
```

3.276 $\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)}{a}$$

```
[Out] Sinh[x]/a + Sinh[x]^3/(3*a)
```

Rubi [A] time = 0.0516122, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3175, 2633}

$$\frac{\sinh^3(x)}{3a}+\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^5/(a + a*Sinh[x]^2),x]
```

```
[Out] Sinh[x]/a + Sinh[x]^3/(3*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^5(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \cosh^3(x)\,dx}{a}\\ &= \frac{i\,\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-i\sinh(x)\right)}{a}\\ &= \frac{\sinh(x)}{a}+\frac{\sinh^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.0034921, size = 19, normalized size = 1.06

$$\frac{\frac{3\sinh(x)}{4}+\frac{1}{12}\sinh(3x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^5/(a + a*Sinh[x]^2),x]
```

```
[Out] ((3*Sinh[x])/4 + Sinh[3*x]/12)/a
```

Maple [B] time = 0.025, size = 67, normalized size = 3.7

$$2\frac{1}{a}\left(-1/6\,(\tanh(x/2)+1)^{-3}+1/4\,(\tanh(x/2)+1)^{-2}-1/2\,(\tanh(x/2)+1)^{-1}-1/6\,(\tanh(x/2)-1)^{-3}-1/4\,(\tanh(x/2)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^5/(a+a*sinh(x)^2),x)
```

```
[Out] 2/a*(-1/6/(tanh(1/2*x)+1)^3+1/4/(tanh(1/2*x)+1)^2-1/2/(tanh(1/2*x)+1)-1/6/(
tanh(1/2*x)-1)^3-1/4/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)-1))
```

Maxima [B] time = 1.03111, size = 46, normalized size = 2.56

$$\frac{\left(9\,e^{(-2\,x)}+1\right)e^{(3\,x)}}{24\,a}-\frac{9\,e^{(-x)}+e^{(-3\,x)}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/24*(9*e^(-2*x) + 1)*e^(3*x)/a - 1/24*(9*e^(-x) + e^(-3*x))/a
```

Fricas [A] time = 1.47582, size = 65, normalized size = 3.61

$$\frac{\sinh(x)^3+3\left(\cosh(x)^2+3\right)\sinh(x)}{12\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(sinh(x)^3 + 3*(cosh(x)^2 + 3)*sinh(x))/a
```

Sympy [B] time = 10.0482, size = 124, normalized size = 6.89

$$-\frac{6\tanh^5\left(\frac{x}{2}\right)}{3a\tanh^6\left(\frac{x}{2}\right)-9a\tanh^4\left(\frac{x}{2}\right)+9a\tanh^2\left(\frac{x}{2}\right)-3a}+\frac{4\tanh^3\left(\frac{x}{2}\right)}{3a\tanh^6\left(\frac{x}{2}\right)-9a\tanh^4\left(\frac{x}{2}\right)+9a\tanh^2\left(\frac{x}{2}\right)-3a}-\frac{}{3a\tanh^6\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**5/(a+a*sinh(x)**2),x)
```

```
[Out] -6*tanh(x/2)**5/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**2 - 3
*a) + 4*tanh(x/2)**3/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)**
2 - 3*a) - 6*tanh(x/2)/(3*a*tanh(x/2)**6 - 9*a*tanh(x/2)**4 + 9*a*tanh(x/2)
```

```
**2 - 3*a)
```

Giac [A] time = 1.14123, size = 39, normalized size = 2.17

$$-\frac{\left(9\,e^{(2\,x)}+1\right)e^{(-3\,x)}-e^{(3\,x)}-9\,e^{x}}{24\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^5/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/24*((9*e^(2*x) + 1)*e^(-3*x) - e^(3*x) - 9*e^x)/a
```

3.277 $\int \frac{\cosh^4(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}+\frac{\sinh(x)\cosh(x)}{2a}$$

```
[Out] x/(2*a) + (Cosh[x]*Sinh[x])/(2*a)
```

Rubi [A] time = 0.0501591, antiderivative size = 20, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3175, 2635, 8}

$$\frac{x}{2a}+\frac{\sinh(x)\cosh(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(a + a*Sinh[x]^2),x]

[Out] x/(2*a) + (Cosh[x]*Sinh[x])/(2*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^4(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \cosh^2(x)\,dx}{a}\\ &= \frac{\cosh(x)\sinh(x)}{2a}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{\cosh(x)\sinh(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0024033, size = 18, normalized size = 0.9

$$\frac{\frac{x}{2}+\frac{1}{4}\sinh(2x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(a + a*Sinh[x]^2),x]
```

```
[Out] (x/2 + Sinh[2*x]/4)/a
```

Maple [B] time = 0.022, size = 78, normalized size = 3.9

$$-\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2}+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\frac{1}{2a}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-2}+\frac{1}{2a}\left(\tanh\left(\frac{x}{2}\right)-1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(a+a*sinh(x)^2),x)
```

```
[Out] -1/2/a/(tanh(1/2*x)+1)^2+1/2/a/(tanh(1/2*x)+1)+1/2/a*ln(tanh(1/2*x)+1)+1/2/
a/(tanh(1/2*x)-1)^2+1/2/a/(tanh(1/2*x)-1)-1/2/a*ln(tanh(1/2*x)-1)
```

Maxima [A] time = 1.05272, size = 34, normalized size = 1.7

$$\frac{x}{2a}+\frac{e^{(2x)}}{8a}-\frac{e^{(-2x)}}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x/a + 1/8*e^(2*x)/a - 1/8*e^(-2*x)/a
```

Fricas [A] time = 1.46211, size = 39, normalized size = 1.95

$$\frac{\cosh(x)\sinh(x)+x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(cosh(x)*sinh(x) + x)/a
```

Sympy [B] time = 6.11654, size = 153, normalized size = 7.65

$$\frac{x\tanh^4\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}-\frac{2x\tanh^2\left(\frac{x}{2}\right)}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{x}{2a\tanh^4\left(\frac{x}{2}\right)-4a\tanh^2\left(\frac{x}{2}\right)+2a}+\frac{}{2a\tan}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(a+a*sinh(x)**2),x)
```

```
[Out] x*tanh(x/2)**4/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) - 2*x*tanh(x/2)*
*2/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a) + x/(2*a*tanh(x/2)**4 - 4*a*
tanh(x/2)**2 + 2*a) + 2*tanh(x/2)**3/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 +
 2*a) + 2*tanh(x/2)/(2*a*tanh(x/2)**4 - 4*a*tanh(x/2)**2 + 2*a)
```

Giac [A] time = 1.13112, size = 38, normalized size = 1.9

$$-\frac{\left(2\,e^{(2\,x)}+1\right)e^{(-2\,x)}-4\,x-e^{(2\,x)}}{8\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/8*((2*e^(2*x) + 1)*e^(-2*x) - 4*x - e^(2*x))/a
```

3.278 $\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=6

$$\frac{\sinh(x)}{a}$$

```
[Out] Sinh[x]/a
```

Rubi [A] time = 0.0439533, antiderivative size = 6, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3175, 2637}

$$\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] Sinh[x]/a
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \cosh(x)\,dx}{a}\\ &= \frac{\sinh(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.0016884, size = 6, normalized size = 1.

$$\frac{\sinh(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] Sinh[x]/a
```

Maple [A] time = 0.012, size = 7, normalized size = 1.2

$$\frac{\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(a+a*sinh(x)^2),x)
```

```
[Out] sinh(x)/a
```

Maxima [B] time = 1.17094, size = 23, normalized size = 3.83

$$-\frac{e^{(-x)}}{2a} + \frac{e^x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*e^(-x)/a + 1/2*e^x/a
```

Fricas [A] time = 1.44102, size = 15, normalized size = 2.5

$$\frac{\sinh(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] sinh(x)/a
```

Sympy [B] time = 3.29542, size = 17, normalized size = 2.83

$$-\frac{2\tanh\left(\frac{x}{2}\right)}{a\tanh^2\left(\frac{x}{2}\right) - a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(a+a*sinh(x)**2),x)
```

```
[Out] -2*tanh(x/2)/(a*tanh(x/2)**2 - a)
```

Giac [B] time = 1.10998, size = 19, normalized size = 3.17

$$-\frac{e^{(-x)} - e^x}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(e^(-x) - e^x)/a
```

3.279 $\int \frac{\cosh^2(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a}$$

[Out] x/a

Rubi [A] time = 0.0415225, antiderivative size = 5, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3175, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Int[Cosh[x]^2/(a + a*Sinh[x]^2),x]

[Out] x/a

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^2(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int 1\,dx}{a}\\ &= \frac{x}{a}\end{aligned}$$

Mathematica [A] time = 0.0003036, size = 5, normalized size = 1.

$$\frac{x}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cosh[x]^2/(a + a*Sinh[x]^2),x]

[Out] x/a

Maple [C] time = 0.012, size = 11, normalized size = 2.2

$$2\,\frac{\operatorname{Artanh}\left(\tanh\left(x/2\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(a+a*sinh(x)^2),x)
```

```
[Out] 2/a*arctanh(tanh(1/2*x))
```

Maxima [A] time = 1.11412, size = 7, normalized size = 1.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] x/a
```

Fricas [A] time = 1.42275, size = 7, normalized size = 1.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] x/a
```

Sympy [A] time = 1.77234, size = 2, normalized size = 0.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(a+a*sinh(x)**2),x)
```

```
[Out] x/a
```

Giac [A] time = 1.15493, size = 7, normalized size = 1.4

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] x/a
```

3.280 $\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=7

$$\frac{\tan^{-1}(\sinh(x))}{a}$$

```
[Out] ArcTan[Sinh[x]]/a
```

Rubi [A] time = 0.0276392, antiderivative size = 7, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3175, 3770}

$$\frac{\tan^{-1}(\sinh(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] ArcTan[Sinh[x]]/a
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \operatorname{sech}(x)\,dx}{a}\\ &= \frac{\tan^{-1}(\sinh(x))}{a}\end{aligned}$$

Mathematica [A] time = 0.0027435, size = 12, normalized size = 1.71

$$\frac{2\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] (2*ArcTan[Tanh[x/2]])/a
```

Maple [A] time = 0.008, size = 8, normalized size = 1.1

$$\frac{\arctan(\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)/(a+a*sinh(x)^2),x)
```

```
[Out] arctan(sinh(x))/a
```

Maxima [A] time = 1.57621, size = 14, normalized size = 2.

$$-\frac{2\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -2*arctan(e^(-x))/a
```

Fricas [A] time = 1.44555, size = 42, normalized size = 6.

$$\frac{2\arctan(\cosh(x)+\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 2*arctan(cosh(x) + sinh(x))/a
```

Sympy [A] time = 0.250555, size = 5, normalized size = 0.71

$$\frac{\operatorname{atan}(\sinh(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)**2),x)
```

```
[Out] atan(sinh(x))/a
```

Giac [A] time = 1.10845, size = 11, normalized size = 1.57

$$\frac{2\arctan(e^x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 2*arctan(e^x)/a
```

3.281 $\int \frac{\operatorname{sech}(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

```
[Out] ArcTan[Sinh[x]]/(2*a) + (Sech[x]*Tanh[x])/(2*a)
```

Rubi [A] time = 0.0454984, antiderivative size = 22, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3175, 3768, 3770}

$$\frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\tanh(x)\operatorname{sech}(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] ArcTan[Sinh[x]]/(2*a) + (Sech[x]*Tanh[x])/(2*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\operatorname{sech}(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \operatorname{sech}^3(x)\,dx}{a}\\ &= \frac{\operatorname{sech}(x)\tanh(x)}{2a}+\frac{\int \operatorname{sech}(x)\,dx}{2a}\\ &= \frac{\tan^{-1}(\sinh(x))}{2a}+\frac{\operatorname{sech}(x)\tanh(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.0037607, size = 20, normalized size = 0.91

$$\frac{\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{2}\tanh(x)\operatorname{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]/(a + a*Sinh[x]^2),x]
```

```
[Out] (ArcTan[Tanh[x/2]] + (Sech[x]*Tanh[x])/2)/a
```

Maple [B] time = 0.021, size = 50, normalized size = 2.3

$$-\frac{1}{a}\left(\tanh\left(\frac{x}{2}\right)\right)^{3}\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-2}+\frac{1}{a}\tanh\left(\frac{x}{2}\right)\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+1\right)^{-2}+\frac{1}{a}\arctan\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)/(a+a*sinh(x)^2),x)
```

```
[Out] -1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)^3+1/a/(tanh(1/2*x)^2+1)^2*tanh(1/2*x)+
1/a*arctan(tanh(1/2*x))
```

Maxima [B] time = 1.52255, size = 54, normalized size = 2.45

$$\frac{e^{(-x)}-e^{(-3x)}}{2\,ae^{(-2x)}+ae^{(-4x)}+a}-\frac{\arctan\left(e^{(-x)}\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] (e^(-x) - e^(-3*x))/(2*a*e^(-2*x) + a*e^(-4*x) + a) - arctan(e^(-x))/a
```

Fricas [B] time = 1.48705, size = 522, normalized size = 23.73

$$\frac{\cosh(x)^{3}+3\,\cosh(x)\sinh(x)^{2}+\sinh(x)^{3}+\left(\cosh(x)^{4}+4\,\cosh(x)\sinh(x)^{3}+\sinh(x)^{4}+2\left(3\,\cosh(x)^{2}+1\right)\sinh(x\cdots}{a\cosh(x)^{4}+4\,a\cosh(x)\sinh(x)^{3}+a\sinh(x)^{4}+2\,a\cosh(x)^{2}+2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] (cosh(x)^3 + 3*cosh(x)*sinh(x)^2 + sinh(x)^3 + (cosh(x)^4 + 4*cosh(x)*sinh(
x)^3 + sinh(x)^4 + 2*(3*cosh(x)^2 + 1)*sinh(x)^2 + 2*cosh(x)^2 + 4*(cosh(x)
^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + sinh(x)) + (3*cosh(x)^2 - 1)*si
nh(x) - cosh(x))/(a*cosh(x)^4 + 4*a*cosh(x)*sinh(x)^3 + a*sinh(x)^4 + 2*a*c
osh(x)^2 + 2*(3*a*cosh(x)^2 + a)*sinh(x)^2 + 4*(a*cosh(x)^3 + a*cosh(x))*si
nh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}(x)}{\sinh^{2}(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)**2),x)
```

```
[Out] Integral(sech(x)/(sinh(x)**2 + 1), x)/a
```

Giac [B] time = 1.13744, size = 70, normalized size = 3.18

$$\frac{\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2x)} - 1\right)e^{(-x)}\right)}{4a} - \frac{e^{(-x)} - e^{x}}{\left(\left(e^{(-x)} - e^{x}\right)^{2} + 4\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - (e^(-x) - e^x)/(((e^(-x)
- e^x)^2 + 4)*a)
```

3.282 $\int \frac{\text{sech}^3(x)}{a+a\sinh^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{3\tan^{-1}(\sinh(x))}{8a}+\frac{\tanh(x)\text{sech}^3(x)}{4a}+\frac{3\tanh(x)\text{sech}(x)}{8a}$$

```
[Out] (3*ArcTan[Sinh[x]])/(8*a) + (3*Sech[x]*Tanh[x])/(8*a) + (Sech[x]^3*Tanh[x])
/(4*a)
```

Rubi [A] time = 0.0598158, antiderivative size = 35, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3175, 3768, 3770}

$$\frac{3\tan^{-1}(\sinh(x))}{8a}+\frac{\tanh(x)\text{sech}^3(x)}{4a}+\frac{3\tanh(x)\text{sech}(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] (3*ArcTan[Sinh[x]])/(8*a) + (3*Sech[x]*Tanh[x])/(8*a) + (Sech[x]^3*Tanh[x])
/(4*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\text{sech}^3(x)}{a+a\sinh^2(x)}\,dx &= \frac{\int \text{sech}^5(x)\,dx}{a}\\
&= \frac{\text{sech}^3(x)\tanh(x)}{4a}+\frac{3\int \text{sech}^3(x)\,dx}{4a}\\
&= \frac{3\text{sech}(x)\tanh(x)}{8a}+\frac{\text{sech}^3(x)\tanh(x)}{4a}+\frac{3\int \text{sech}(x)\,dx}{8a}\\
&= \frac{3\tan^{-1}(\sinh(x))}{8a}+\frac{3\text{sech}(x)\tanh(x)}{8a}+\frac{\text{sech}^3(x)\tanh(x)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.0044584, size = 34, normalized size = 0.97

$$\frac{\frac{3}{4}\tan^{-1}\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{4}\tanh(x)\mathrm{sech}^3(x)+\frac{3}{8}\tanh(x)\mathrm{sech}(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[x]^3/(a + a*Sinh[x]^2),x]
```

```
[Out] ((3*ArcTan[Tanh[x/2]])/4 + (3*Sech[x]*Tanh[x])/8 + (Sech[x]^3*Tanh[x])/4)/a
```

Maple [B] time = 0.026, size = 94, normalized size = 2.7

$$-\frac{5}{4a}\left(\tanh\left(\frac{x}{2}\right)\right)^7\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-4}+\frac{3}{4a}\left(\tanh\left(\frac{x}{2}\right)\right)^5\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right)^{-4}-\frac{3}{4a}\left(\tanh\left(\frac{x}{2}\right)\right)^3\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+1\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(x)^3/(a+a*sinh(x)^2),x)
```

```
[Out] -5/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^7+3/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2
*x)^5-3/4/a/(tanh(1/2*x)^2+1)^4*tanh(1/2*x)^3+5/4/a/(tanh(1/2*x)^2+1)^4*tan
h(1/2*x)+3/4/a*arctan(tanh(1/2*x))
```

Maxima [B] time = 1.51705, size = 93, normalized size = 2.66

$$\frac{3\,e^{(-x)}+11\,e^{(-3\,x)}-11\,e^{(-5\,x)}-3\,e^{(-7\,x)}}{4\left(4\,ae^{(-2\,x)}+6\,ae^{(-4\,x)}+4\,ae^{(-6\,x)}+ae^{(-8\,x)}+a\right)}-\frac{3\,\arctan\left(e^{(-x)}\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*e^(-x) + 11*e^(-3*x) - 11*e^(-5*x) - 3*e^(-7*x))/(4*a*e^(-2*x) + 6*a
*e^(-4*x) + 4*a*e^(-6*x) + a*e^(-8*x) + a) - 3/4*arctan(e^(-x))/a
```

Fricas [B] time = 1.49142, size = 1607, normalized size = 45.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(3*cosh(x)^7 + 21*cosh(x)*sinh(x)^6 + 3*sinh(x)^7 + (63*cosh(x)^2 + 11)
*sinh(x)^5 + 11*cosh(x)^5 + 5*(21*cosh(x)^3 + 11*cosh(x))*sinh(x)^4 + (105*
cosh(x)^4 + 110*cosh(x)^2 - 11)*sinh(x)^3 - 11*cosh(x)^3 + (63*cosh(x)^5 +
110*cosh(x)^3 - 33*cosh(x))*sinh(x)^2 + 3*(cosh(x)^8 + 8*cosh(x)*sinh(x)^7
+ sinh(x)^8 + 4*(7*cosh(x)^2 + 1)*sinh(x)^6 + 4*cosh(x)^6 + 8*(7*cosh(x)^3
+ 3*cosh(x))*sinh(x)^5 + 2*(35*cosh(x)^4 + 30*cosh(x)^2 + 3)*sinh(x)^4 + 6*
cosh(x)^4 + 8*(7*cosh(x)^5 + 10*cosh(x)^3 + 3*cosh(x))*sinh(x)^3 + 4*(7*cos
```

```
h(x)^6 + 15*cosh(x)^4 + 9*cosh(x)^2 + 1)*sinh(x)^2 + 4*cosh(x)^2 + 8*(cosh(
x)^7 + 3*cosh(x)^5 + 3*cosh(x)^3 + cosh(x))*sinh(x) + 1)*arctan(cosh(x) + s
inh(x)) + (21*cosh(x)^6 + 55*cosh(x)^4 - 33*cosh(x)^2 - 3)*sinh(x) - 3*cosh
(x))/(a*cosh(x)^8 + 8*a*cosh(x)*sinh(x)^7 + a*sinh(x)^8 + 4*a*cosh(x)^6 + 4
*(7*a*cosh(x)^2 + a)*sinh(x)^6 + 8*(7*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^5
+ 6*a*cosh(x)^4 + 2*(35*a*cosh(x)^4 + 30*a*cosh(x)^2 + 3*a)*sinh(x)^4 + 8*(
7*a*cosh(x)^5 + 10*a*cosh(x)^3 + 3*a*cosh(x))*sinh(x)^3 + 4*a*cosh(x)^2 + 4
*(7*a*cosh(x)^6 + 15*a*cosh(x)^4 + 9*a*cosh(x)^2 + a)*sinh(x)^2 + 8*(a*cosh
(x)^7 + 3*a*cosh(x)^5 + 3*a*cosh(x)^3 + a*cosh(x))*sinh(x) + a)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\frac{\int \frac{\operatorname{sech}^3(x)}{\sinh^2(x)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)**3/(a+a*sinh(x)**2),x)
```

```
[Out] Integral(sech(x)**3/(sinh(x)**2 + 1), x)/a
```

Giac [B] time = 1.15806, size = 90, normalized size = 2.57

$$\frac{3\left(\pi+2\arctan\left(\frac{1}{2}\left(e^{(2x)}-1\right)e^{(-x)}\right)\right)}{16\,a}-\frac{3\left(e^{(-x)}-e^{x}\right)^3+20\,e^{(-x)}-20\,e^{x}}{4\left(\left(e^{(-x)}-e^{x}\right)^2+4\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(x)^3/(a+a*sinh(x)^2),x, algorithm="giac")
```

```
[Out] 3/16*(pi + 2*arctan(1/2*(e^(2*x) - 1)*e^(-x)))/a - 1/4*(3*(e^(-x) - e^x)^3
+ 20*e^(-x) - 20*e^x)/(((e^(-x) - e^x)^2 + 4)^2*a)
```

3.283 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$

Optimal. Leaf size=89

$$\frac{(6a-b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{(6a-b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-b)+\frac{b\sinh(c+dx)\cosh^5(c+d}{6d}$$

```
[Out] ((6*a - b)*x)/16 + ((6*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) + ((6*a -
 b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*Cosh[c + d*x]^5*Sinh[c + d*x
])/(6*d)
```

Rubi [A] time = 0.0571499, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3191, 385, 199, 206}

$$\frac{(6a-b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{(6a-b)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x(6a-b)+\frac{b\sinh(c+dx)\cosh^5(c+d}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((6*a - b)*x)/16 + ((6*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(16*d) + ((6*a -
 b)*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*Cosh[c + d*x]^5*Sinh[c + d*x
])/(6*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^4}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d} + \frac{(6a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)^3}dx, x, \tanh(c+dx)\right)}{6d}$$
$$= \frac{(6a-b)\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(c+dx)}{6d} + \frac{(6a-}{}$$
$$= \frac{(6a-b)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(6a-b)\cosh^3(c+dx)\sinh(c+dx)}{24d} +$$
$$= \frac{1}{16}(6a-b)x + \frac{(6a-b)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(6a-b)\cosh^3(c+dx)\text{s}}{24d}$$

Mathematica [A] time = 0.168358, size = 63, normalized size = 0.71

$$\frac{(48a-3b)\sinh(2(c+dx))+3(2a+b)\sinh(4(c+dx))+72ac+72adx+b\sinh(6(c+dx))-12bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (72*a*c + 72*a*d*x - 12*b*d*x + (48*a - 3*b)*Sinh[2*(c + d*x)] + 3*(2*a + b
)*Sinh[4*(c + d*x)] + b*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.059, size = 95, normalized size = 1.1

$$\frac{1}{d}\left(b\left(\frac{\sinh(dx+c)(\cosh(dx+c))^5}{6} - \frac{\sinh(dx+c)}{6}\left(\frac{(\cosh(dx+c))^3}{4} + \frac{3\cosh(dx+c)}{8}\right) - \frac{dx}{16} - \frac{c}{16}\right) + a\left(\left(\frac{(\cosh(dx+}{4}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c
))*sinh(d*x+c)-1/16*d*x-1/16*c)+a*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh
(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.01327, size = 205, normalized size = 2.3

$$\frac{1}{64}a\left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) + \frac{1}{384}b\left(\frac{\left(3e^{(-2dx-2c)} - 3e^{(-4dx-4c)} + 1\right)e^{(6dx+6c)}}{d} - \frac{24(d}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/64*a*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*c)
/d - e^(-4*d*x - 4*c)/d) + 1/384*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c
```

```
) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d
*x - 4*c) - e^(-6*d*x - 6*c))/d)
```

Fricas [A] time = 1.48793, size = 306, normalized size = 3.44

$$\frac{3\,b\cosh\left(dx+c\right)\sinh\left(dx+c\right)^{5}+2\left(5\,b\cosh\left(dx+c\right)^{3}+3\left(2\,a+b\right)\cosh\left(dx+c\right)\right)\sinh\left(dx+c\right)^{3}+6\left(6\,a-b\right)dx+3\left(b\right.}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/96*(3*b*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b*cosh(d*x + c)^3 + 3*(2*a +
 b)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(6*a - b)*d*x + 3*(b*cosh(d*x + c)^5
 + 2*(2*a + b)*cosh(d*x + c)^3 + (16*a - b)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 4.28559, size = 250, normalized size = 2.81

$$\begin{cases}\frac{3ax\sinh^{4}(c+dx)}{8}-\frac{3ax\sinh^{2}(c+dx)\cosh^{2}(c+dx)}{4}+\frac{3ax\cosh^{4}(c+dx)}{8}-\frac{3a\sinh^{3}(c+dx)\cosh(c+dx)}{8d}+\frac{5a\sinh(c+dx)\cosh^{3}(c+dx)}{8d}+\frac{bx\sinh^{6}(c+}{16}\\ x\left(a+b\sinh^{2}(c)\right)\cosh^{4}(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sinh(c + d*x)**4/8 - 3*a*x*sinh(c + d*x)**2*cosh(c + d*x)*
*2/4 + 3*a*x*cosh(c + d*x)**4/8 - 3*a*sinh(c + d*x)**3*cosh(c + d*x)/(8*d)
+ 5*a*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + b*x*sinh(c + d*x)**6/16 - 3*b*
x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 3*b*x*sinh(c + d*x)**2*cosh(c + d*
x)**4/16 - b*x*cosh(c + d*x)**6/16 - b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d
) + b*sinh(c + d*x)**3*cosh(c + d*x)**3/(6*d) + b*sinh(c + d*x)*cosh(c + d*
x)**5/(16*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**4, True))
```

Giac [B] time = 1.20001, size = 221, normalized size = 2.48

$$\frac{24\left(dx+c\right)\left(6\,a-b\right)+be^{\left(6\,dx+6\,c\right)}+6\,ae^{\left(4\,dx+4\,c\right)}+3\,be^{\left(4\,dx+4\,c\right)}+48\,ae^{\left(2\,dx+2\,c\right)}-3\,be^{\left(2\,dx+2\,c\right)}-\left(132\,ae^{\left(6\,dx+6\,c\right)}-22\,be^{\left(6\,dx\right.}\right.}{384\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/384*(24*(d*x + c)*(6*a - b) + b*e^(6*d*x + 6*c) + 6*a*e^(4*d*x + 4*c) + 3
*b*e^(4*d*x + 4*c) + 48*a*e^(2*d*x + 2*c) - 3*b*e^(2*d*x + 2*c) - (132*a*e^
(6*d*x + 6*c) - 22*b*e^(6*d*x + 6*c) + 48*a*e^(4*d*x + 4*c) - 3*b*e^(4*d*x
+ 4*c) + 6*a*e^(2*d*x + 2*c) + 3*b*e^(2*d*x + 2*c) + b)*e^(-6*d*x - 6*c))/d
```

3.284 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=46

$$\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^5(c+dx)}{5d}$$

```
[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^5)
/(5*d)
```

Rubi [A] time = 0.0416845, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3190, 373}

$$\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Sinh[c + d*x])/d + ((a + b)*Sinh[c + d*x]^3)/(3*d) + (b*Sinh[c + d*x]^5)
/(5*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2+bx^4\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a\sinh(c+dx)}{d}+\frac{(a+b)\sinh^3(c+dx)}{3d}+\frac{b\sinh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.139109, size = 48, normalized size = 1.04

$$\frac{\sinh(c+dx)(4(5a+2b)\cosh(2(c+dx))+100a+3b\cosh(4(c+dx))-11b)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((100*a - 11*b + 4*(5*a + 2*b)*Cosh[2*(c + d*x)] + 3*b*Cosh[4*(c + d*x)])*S
inh[c + d*x])/(120*d)
```

Maple [A] time = 0.053, size = 65, normalized size = 1.4

$$\frac{1}{d}\left(b\left(\frac{\sinh(dx+c)(\cosh(dx+c))^4}{5}-\frac{\sinh(dx+c)}{5}\left(\frac{2}{3}+\frac{(\cosh(dx+c))^2}{3}\right)\right)+a\left(\frac{2}{3}+\frac{(\cosh(dx+c))^2}{3}\right)\sinh(dx+c)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/5*sinh(d*x+c)*cosh(d*x+c)^4-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+
c))+a*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))
```

Maxima [B] time = 1.01288, size = 184, normalized size = 4.

$$\frac{1}{480}b\left(\frac{\left(5e^{(-2dx-2c)}-30e^{(-4dx-4c)}+3\right)e^{(5dx+5c)}}{d}+\frac{30e^{(-dx-c)}-5e^{(-3dx-3c)}-3e^{(-5dx-5c)}}{d}\right)+\frac{1}{24}a\left(\frac{e^{(3dx+3c)}}{d}+\frac{9e^{(dx+c}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d +
 (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a*(e
^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Fricas [A] time = 1.48221, size = 220, normalized size = 4.78

$$\frac{3b\sinh(dx+c)^5+5\left(6b\cosh(dx+c)^2+4a+b\right)\sinh(dx+c)^3+15\left(b\cosh(dx+c)^4+(4a+b)\cosh(dx+c)^2+1\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/240*(3*b*sinh(d*x + c)^5 + 5*(6*b*cosh(d*x + c)^2 + 4*a + b)*sinh(d*x + c
)^3 + 15*(b*cosh(d*x + c)^4 + (4*a + b)*cosh(d*x + c)^2 + 12*a - 2*b)*sinh(
d*x + c))/d
```

Sympy [A] time = 2.04711, size = 85, normalized size = 1.85

$$\begin{cases} -\frac{2a\sinh^3(c+dx)}{3d}+\frac{a\sinh(c+dx)\cosh^2(c+dx)}{d}-\frac{2b\sinh^5(c+dx)}{15d}+\frac{b\sinh^3(c+dx)\cosh^2(c+dx)}{3d} & \text{for } d\neq 0 \\ x\left(a+b\sinh^2(c)\right)\cosh^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((-2*a*sinh(c + d*x)**3/(3*d) + a*sinh(c + d*x)*cosh(c + d*x)**2/d
 - 2*b*sinh(c + d*x)**5/(15*d) + b*sinh(c + d*x)**3*cosh(c + d*x)**2/(3*d),
 Ne(d, 0)), (x*(a + b*sinh(c)**2)*cosh(c)**3, True))
```

Giac [B] time = 1.17355, size = 166, normalized size = 3.61

$$\frac{3\,be^{(5\,dx+5\,c)} + 20\,ae^{(3\,dx+3\,c)} + 5\,be^{(3\,dx+3\,c)} + 180\,ae^{(dx+c)} - 30\,be^{(dx+c)} - \left(180\,ae^{(4\,dx+4\,c)} - 30\,be^{(4\,dx+4\,c)} + 20\,ae^{(2\,dx+2\,c)} + 5\,b}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/480*(3*b*e^(5*d*x + 5*c) + 20*a*e^(3*d*x + 3*c) + 5*b*e^(3*d*x + 3*c) + 1
80*a*e^(d*x + c) - 30*b*e^(d*x + c) - (180*a*e^(4*d*x + 4*c) - 30*b*e^(4*d*
x + 4*c) + 20*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 3*b)*e^(-5*d*x - 5*
c))/d
```

3.285 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$\frac{(4a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(4a-b)+\frac{b\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

```
[Out] ((4*a - b)*x)/8 + ((4*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c
 + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rubi [A] time = 0.0488858, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3191, 385, 199, 206}

$$\frac{(4a-b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{1}{8}x(4a-b)+\frac{b\sinh(c+dx)\cosh^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((4*a - b)*x)/8 + ((4*a - b)*Cosh[c + d*x]*Sinh[c + d*x])/(8*d) + (b*Cosh[c
 + d*x]^3*Sinh[c + d*x])/(4*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a-(a-b)x^2}{(1-x^2)^3}dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{b\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{(4a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)^2}dx, x, \tanh(c+dx)\right)}{4d}$$
$$= \frac{(4a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{(4a-b}{}$$
$$= \frac{1}{8}(4a-b)x + \frac{(4a-b)\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b\cosh^3(c+dx)\sinh(c+}{4d}$$

Mathematica [A] time = 0.0695163, size = 43, normalized size = 0.7

$$\frac{8a\sinh(2(c+dx)) + 16ac + 16adx + b\sinh(4(c+dx)) - 4bdx}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (16*a*c + 16*a*d*x - 4*b*d*x + 8*a*Sinh[2*(c + d*x)] + b*Sinh[4*(c + d*x)])
/(32*d)
```

Maple [A] time = 0.026, size = 70, normalized size = 1.2

$$\frac{1}{d}\left(b\left(\frac{\sinh(dx+c)(\cosh(dx+c))^3}{4} - \frac{\cosh(dx+c)\sinh(dx+c)}{8} - \frac{dx}{8} - \frac{c}{8}\right) + a\left(\frac{\cosh(dx+c)\sinh(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(1/4*sinh(d*x+c)*cosh(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1
/8*c)+a*(1/2*cosh(d*x+c)*sinh(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.00693, size = 103, normalized size = 1.69

$$\frac{1}{8}a\left(4x + \frac{e^{(2dx+2c)}}{d} - \frac{e^{(-2dx-2c)}}{d}\right) - \frac{1}{64}b\left(\frac{8(dx+c)}{d} - \frac{e^{(4dx+4c)}}{d} + \frac{e^{(-4dx-4c)}}{d}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*a*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/64*b*(8*(d*x + c)/
d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [A] time = 1.45926, size = 153, normalized size = 2.51

$$\frac{b\cosh(dx+c)\sinh(dx+c)^3+(4a-b)dx+\left(b\cosh(dx+c)^3+4a\cosh(dx+c)\right)\sinh(dx+c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*(b*cosh(d*x + c)*sinh(d*x + c)^3 + (4*a - b)*d*x + (b*cosh(d*x + c)^3 +
 4*a*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 1.14033, size = 150, normalized size = 2.46

$$\begin{cases}-\frac{ax\sinh^2(c+dx)}{2}+\frac{ax\cosh^2(c+dx)}{2}+\frac{a\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{bx\sinh^4(c+dx)}{8}+\frac{bx\sinh^2(c+dx)\cosh^2(c+dx)}{4}-\frac{bx\cosh^4(c+dx)}{8}+\frac{b\sinh^3}{} \\ x\left(a+b\sinh^2(c)\right)\cosh^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((-a*x*sinh(c + d*x)**2/2 + a*x*cosh(c + d*x)**2/2 + a*sinh(c + d*
x)*cosh(c + d*x)/(2*d) - b*x*sinh(c + d*x)**4/8 + b*x*sinh(c + d*x)**2*cosh
(c + d*x)**2/4 - b*x*cosh(c + d*x)**4/8 + b*sinh(c + d*x)**3*cosh(c + d*x)/
(8*d) + b*sinh(c + d*x)*cosh(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sinh(c
)**2)*cosh(c)**2, True))
```

Giac [A] time = 1.12573, size = 124, normalized size = 2.03

$$\frac{8(dx+c)(4a-b)+be^{(4dx+4c)}+8ae^{(2dx+2c)}-\left(24ae^{(4dx+4c)}-6be^{(4dx+4c)}+8ae^{(2dx+2c)}+b\right)e^{(-4dx-4c)}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*(8*(d*x + c)*(4*a - b) + b*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) - (24
*a*e^(4*d*x + 4*c) - 6*b*e^(4*d*x + 4*c) + 8*a*e^(2*d*x + 2*c) + b)*e^(-4*d
*x - 4*c))/d
```

3.286 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

```
[Out] (a*Sinh[c + d*x])/d + (b*Sinh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0216874, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3190}

$$\frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Sinh[c + d*x])/d + (b*Sinh[c + d*x]^3)/(3*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+bx^2\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a\sinh(c+dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0117645, size = 39, normalized size = 1.39

$$\frac{a\sinh(c)\cosh(dx)}{d}+\frac{a\cosh(c)\sinh(dx)}{d}+\frac{b\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Cosh[d*x]*Sinh[c])/d + (a*Cosh[c]*Sinh[d*x])/d + (b*Sinh[c + d*x]^3)/(3*
d)
```

Maple [A] time = 0.01, size = 25, normalized size = 0.9

$$\frac{1}{d}\left(\frac{b\left(\sinh(dx+c)\right)^3}{3}+a\sinh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(1/3*b*sinh(d*x+c)^3+a*sinh(d*x+c))
```

Maxima [A] time = 1.0335, size = 35, normalized size = 1.25

$$\frac{b\sinh(dx+c)^3}{3\,d} + \frac{a\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*b*sinh(d*x + c)^3/d + a*sinh(d*x + c)/d
```

Fricas [A] time = 1.4489, size = 103, normalized size = 3.68

$$\frac{b\sinh(dx+c)^3 + 3\left(b\cosh(dx+c)^2 + 4\,a - b\right)\sinh(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(b*sinh(d*x + c)^3 + 3*(b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))/d
```

Sympy [A] time = 0.487066, size = 36, normalized size = 1.29

$$\begin{cases} \frac{a\sinh(c+dx)}{d} + \frac{b\sinh^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x\left(a + b\sinh^2(c)\right)\cosh(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((a*sinh(c + d*x)/d + b*sinh(c + d*x)**3/(3*d), Ne(d, 0)), (x*(a +
 b*sinh(c)**2)*cosh(c), True))
```

Giac [B] time = 1.13263, size = 97, normalized size = 3.46

$$\frac{be^{(3\,dx+3\,c)} + 12\,ae^{(dx+c)} - 3\,be^{(dx+c)} - \left(12\,ae^{(2\,dx+2\,c)} - 3\,be^{(2\,dx+2\,c)} + b\right)e^{(-3\,dx-3\,c)}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/24*(b*e^(3*d*x + 3*c) + 12*a*e^(d*x + c) - 3*b*e^(d*x + c) - (12*a*e^(2*d
*x + 2*c) - 3*b*e^(2*d*x + 2*c) + b)*e^(-3*d*x - 3*c))/d
```

3.287 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=28

$$\frac{(a-b)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

```
[Out] ((a - b)*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x])/d
```

Rubi [A] time = 0.0319274, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.158, Rules used = {3190, 388, 203}

$$\frac{(a-b)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - b)*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x])/d
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a+bx^2}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{b\sinh(c+dx)}{d}+\frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{(a-b)\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.013777, size = 37, normalized size = 1.32

$$\frac{a\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\sinh(c+dx)}{d}-\frac{b\tan^{-1}(\sinh(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/d - (b*ArcTan[Sinh[c + d*x]])/d + (b*Sinh[c + d*x
])/d
```

Maple [A] time = 0.028, size = 39, normalized size = 1.4

$$\frac{b\sinh(dx+c)}{d}+2\,\frac{a\arctan\left(e^{dx+c}\right)}{d}-2\,\frac{b\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] b*sinh(d*x+c)/d+2/d*a*arctan(exp(d*x+c))-2/d*b*arctan(exp(d*x+c))
```

Maxima [A] time = 1.51922, size = 76, normalized size = 2.71

$$\frac{1}{2}\,b\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\frac{e^{(-dx-c)}}{d}\right)+\frac{a\arctan(\sinh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a*arcta
n(sinh(d*x + c))/d
```

Fricas [B] time = 1.51464, size = 282, normalized size = 10.07

$$\frac{b\cosh(dx+c)^2+2\,b\cosh(dx+c)\sinh(dx+c)+b\sinh(dx+c)^2+4\,((a-b)\cosh(dx+c)+(a-b)\sinh(dx+c))\,a}{2\,(d\cosh(dx+c)+d\sinh(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^
2 + 4*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*arctan(cosh(d*x + c)
+ sinh(d*x + c)) - b)/(d*cosh(d*x + c) + d*sinh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sinh^2(c+dx)\right)\operatorname{sech}(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x), x)
```

Giac [A] time = 1.12814, size = 61, normalized size = 2.18

$$\frac{2\,(a-b)\arctan\left(e^{(dx+c)}\right)}{d}+\frac{be^{(dx+c)}}{2\,d}-\frac{be^{(-dx-c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2*(a - b)*arctan(e^(d*x + c))/d + 1/2*b*e^(d*x + c)/d - 1/2*b*e^(-d*x - c)/
d
```

3.288 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=19

$$\frac{(a-b)\tanh(c+dx)}{d}+bx$$

```
[Out] b*x + ((a - b)*Tanh[c + d*x])/d
```

Rubi [A] time = 0.0340843, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3191, 388, 206}

$$\frac{(a-b)\tanh(c+dx)}{d}+bx$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] b*x + ((a - b)*Tanh[c + d*x])/d
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \frac{a-(a-b)x^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{(a-b)\tanh(c+dx)}{d}+\frac{b\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= bx+\frac{(a-b)\tanh(c+dx)}{d}
\end{aligned}$$

Mathematica [A] time = 0.0138384, size = 36, normalized size = 1.89

$$\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^{-1}(\tanh(c+dx))}{d}-\frac{b\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b*ArcTanh[Tanh[c + d*x]])/d + (a*Tanh[c + d*x])/d - (b*Tanh[c + d*x])/d
```

Maple [A] time = 0.056, size = 29, normalized size = 1.5

$$\frac{\tanh(dx+c)\,a+b\,(dx+c-\tanh(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(tanh(d*x+c)*a+b*(d*x+c-tanh(d*x+c)))
```

Maxima [B] time = 1.04978, size = 63, normalized size = 3.32

$$b\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+\frac{2\,a}{d\left(e^{(-2\,dx-2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 2*a/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 1.51005, size = 101, normalized size = 5.32

$$\frac{(bdx-a+b)\cosh(dx+c)+(a-b)\sinh(dx+c)}{d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] ((b*d*x - a + b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))/(d*cosh(d*x + c))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\left(a+b\sinh^2(c+dx)\right)\operatorname{sech}^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sinh(c + d*x)**2)*sech(c + d*x)**2, x)
```

Giac [A] time = 1.15137, size = 46, normalized size = 2.42

$$\frac{(dx+c)b}{d}-\frac{2\,(a-b)}{d\left(e^{(2\,dx+2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (d*x + c)*b/d - 2*(a - b)/(d*(e^(2*d*x + 2*c) + 1))
```

3.289 $\int \mathbf{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$

Optimal. Leaf size=42

$$\frac{(a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}$$

```
[Out] ((a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - b)*Sech[c + d*x]*Tanh[c + d*x
])/(2*d)
```

Rubi [A] time = 0.0395048, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 385, 203}

$$\frac{(a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)\tanh(c+dx)\mathbf{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a + b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((a - b)*Sech[c + d*x]*Tanh[c + d*x
])/(2*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \mathrm{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)dx &= \frac{\mathrm{Subst}\left(\int \frac{a+bx^2}{\left(1+x^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{(a-b)\mathbf{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{(a+b)\,\mathrm{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx\right.}{2d}\\ &= \frac{(a+b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)\mathbf{sech}(c+dx)\tanh(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.0229631, size = 71, normalized size = 1.69

$$\frac{a\tan^{-1}(\sinh(c+dx))}{2d}+\frac{a\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{b\tan^{-1}(\sinh(c+dx))}{2d}-\frac{b\tanh(c+dx)\text{sech}(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*ArcTan[Sinh[c + d*x]])/(2*d) + (b*ArcTan[Sinh[c + d*x]])/(2*d) + (a*Sech
[c + d*x]*Tanh[c + d*x])/(2*d) - (b*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Maple [B] time = 0.069, size = 82, normalized size = 2.

$$\frac{a\text{sech}\,(dx+c)\tanh\,(dx+c)}{2\,d}+\frac{a\arctan\left(e^{dx+c}\right)}{d}-\frac{b\sinh\,(dx+c)}{d\,(\cosh\,(dx+c))^2}+\frac{b\text{sech}\,(dx+c)\tanh\,(dx+c)}{2\,d}+\frac{b\arctan\left(e^{dx+c}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/2/d*a*sech(d*x+c)*tanh(d*x+c)+1/d*a*arctan(exp(d*x+c))-1/d*b*sinh(d*x+c)/
cosh(d*x+c)^2+1/2/d*b*sech(d*x+c)*tanh(d*x+c)+1/d*b*arctan(exp(d*x+c))
```

Maxima [B] time = 1.52533, size = 184, normalized size = 4.38

$$-b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)-a\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d\left(2\,e^{(-2\,dx-2\,c)}+e^{(-4\,dx-4\,c)}+1\right)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*
d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a*(arctan(e^(-d*x - c))/d - (e^(-d*x
 - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1)))
```

Fricas [B] time = 1.52328, size = 903, normalized size = 21.5

$$\frac{(a-b)\cosh\,(dx+c)^3+3\,(a-b)\cosh\,(dx+c)\sinh\,(dx+c)^2+(a-b)\sinh\,(dx+c)^3+\left((a+b)\cosh\,(dx+c)^4+4\,(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] ((a - b)*cosh(d*x + c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b
)*sinh(d*x + c)^3 + ((a + b)*cosh(d*x + c)^4 + 4*(a + b)*cosh(d*x + c)*sinh
(d*x + c)^3 + (a + b)*sinh(d*x + c)^4 + 2*(a + b)*cosh(d*x + c)^2 + 2*(3*(a
 + b)*cosh(d*x + c)^2 + a + b)*sinh(d*x + c)^2 + 4*((a + b)*cosh(d*x + c)^3
```

```
 + (a + b)*cosh(d*x + c))*sinh(d*x + c) + a + b)*arctan(cosh(d*x + c) + sin
h(d*x + c)) - (a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a + b)*s
inh(d*x + c))/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh(d*x + c)^3 + d*si
nh(d*x + c)^4 + 2*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c)^2 + d)*sinh(d*x
+ c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18395, size = 143, normalized size = 3.4

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)(a+b)}{4\,d} + \frac{a\left(e^{(dx+c)} - e^{(-dx-c)}\right) - b\left(e^{(dx+c)} - e^{(-dx-c)}\right)}{\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a + b)/d + (a*
(e^(d*x + c) - e^(-d*x - c)) - b*(e^(d*x + c) - e^(-d*x - c)))/(((e^(d*x +
c) - e^(-d*x - c))^2 + 4)*d)
```

3.290 $\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=32

$$\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}$$

```
[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0337702, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3191}

$$\frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((a - b)*Tanh[c + d*x]^3)/(3*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \left(a-(a-b)x^2\right)\,dx, x, \tanh(c+dx)\right)}{d}\\ &= \frac{a\tanh(c+dx)}{d}-\frac{(a-b)\tanh^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.0074434, size = 44, normalized size = 1.38

$$-\frac{a\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}+\frac{b\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - (a*Tanh[c + d*x]^3)/(3*d) + (b*Tanh[c + d*x]^3)/(3*d)
```

Maple [B] time = 0.062, size = 65, normalized size = 2.

$$\frac{1}{d}\left(a\left(\frac{2}{3}+\frac{(\operatorname{sech}(dx+c))^2}{3}\right)\tanh(dx+c)+b\left(-\frac{\sinh(dx+c)}{2(\cosh(dx+c))^3}+\frac{\tanh(dx+c)}{2}\left(\frac{2}{3}+\frac{(\operatorname{sech}(dx+c))^2}{3}\right)\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+b*(-1/2*sinh(d*x+c)/cosh(d*x+c)^
3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.0453, size = 250, normalized size = 7.81

$$\frac{4}{3}a\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}+\frac{1}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+\frac{2}{3}b\left(\frac{}{d\left(3e^{(-2dx-2c)}+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 4/3*a*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(
-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6
*d*x - 6*c) + 1))) + 2/3*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e
^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(
-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 1.38691, size = 427, normalized size = 13.34

$$-\frac{4\left((a+2b)\cosh(dx+c)^2-2(a-b)\cosh(dx+c)\sinh(dx+c)+(a\right.}{3\left(d\cosh(dx+c)^4+4d\cosh(dx+c)\sinh(dx+c)^3+d\sinh(dx+c)^4+4d\cosh(dx+c)^2+2\left(3d\cosh(dx+c)^2+2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -4/3*((a + 2*b)*cosh(d*x + c)^2 - 2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (
a + 2*b)*sinh(d*x + c)^2 + 3*a)/(d*cosh(d*x + c)^4 + 4*d*cosh(d*x + c)*sinh
(d*x + c)^3 + d*sinh(d*x + c)^4 + 4*d*cosh(d*x + c)^2 + 2*(3*d*cosh(d*x + c
)^2 + 2*d)*sinh(d*x + c)^2 + 4*(d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d
*x + c) + 3*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.18553, size = 63, normalized size = 1.97

$$-\frac{2\left(3\,be^{(4\,dx+4\,c)}+6\,ae^{(2\,dx+2\,c)}+2\,a+b\right)}{3\,d\left(e^{(2\,dx+2\,c)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2/3*(3*b*e^(4*d*x + 4*c) + 6*a*e^(2*d*x + 2*c) + 2*a + b)/(d*(e^(2*d*x + 2
*c) + 1)^3)
```

3.291 $\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)dx$

Optimal. Leaf size=70

$$\frac{(3a+b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}+\frac{(3a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}$$

```
[Out] ((3*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((3*a + b)*Sech[c + d*x]*Tanh[c +
 d*x])/(8*d) + ((a - b)*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.0470149, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3190, 385, 199, 203}

$$\frac{(3a+b)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d}+\frac{(3a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*ArcTan[Sinh[c + d*x]])/(8*d) + ((3*a + b)*Sech[c + d*x]*Tanh[c +
 d*x])/(8*d) + ((a - b)*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+bx^2}{(1+x^2)^3}dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d} + \frac{(3a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}dx, x, \sinh\right.}{4d}$$

$$= \frac{(3a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}$$

$$= \frac{(3a+b)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{(3a+b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a}{}$$

Mathematica [A] time = 0.149494, size = 60, normalized size = 0.86

$$\frac{(3a+b)\tan^{-1}(\sinh(c+dx)) + 2(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx) + (3a+b)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a + b)*ArcTan[Sinh[c + d*x]] + (3*a + b)*Sech[c + d*x]*Tanh[c + d*x] +
2*(a - b)*Sech[c + d*x]^3*Tanh[c + d*x])/(8*d)
```

Maple [A] time = 0.072, size = 124, normalized size = 1.8

$$\frac{\tanh(dx+c)\,a\,(\operatorname{sech}(dx+c))^3}{4\,d} + \frac{3\,a\operatorname{sech}(dx+c)\tanh(dx+c)}{8\,d} + \frac{3\,a\arctan\left(e^{dx+c}\right)}{4\,d} - \frac{b\sinh(dx+c)}{3\,d\,(\cosh(dx+c))^4} + \frac{b\tan}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/4/d*a*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a*sech(d*x+c)*tanh(d*x+c)+3/4/d*a*a
rctan(exp(d*x+c))-1/3/d*b*sinh(d*x+c)/cosh(d*x+c)^4+1/12/d*b*tanh(d*x+c)*se
ch(d*x+c)^3+1/8/d*b*sech(d*x+c)*tanh(d*x+c)+1/4/d*b*arctan(exp(d*x+c))
```

Maxima [B] time = 1.53705, size = 308, normalized size = 4.4

$$-\frac{1}{4}a\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{3\,e^{(-dx-c)} + 11\,e^{(-3\,dx-3\,c)} - 11\,e^{(-5\,dx-5\,c)} - 3\,e^{(-7\,dx-7\,c)}}{d\left(4\,e^{(-2\,dx-2\,c)} + 6\,e^{(-4\,dx-4\,c)} + 4\,e^{(-6\,dx-6\,c)} + e^{(-8\,dx-8\,c)} + 1\right)}\right) - \frac{1}{4}b\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d} - \frac{}{d(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*a*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) -
11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*b*(arctan(e
^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) -
```

```
e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x
 - 6*c) + e^(-8*d*x - 8*c) + 1)))
```

Fricas [B] time = 1.60049, size = 2838, normalized size = 40.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*((3*a + b)*cosh(d*x + c)^7 + 7*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)^6
+ (3*a + b)*sinh(d*x + c)^7 + (11*a - 7*b)*cosh(d*x + c)^5 + (21*(3*a + b)*
cosh(d*x + c)^2 + 11*a - 7*b)*sinh(d*x + c)^5 + 5*(7*(3*a + b)*cosh(d*x + c
)^3 + (11*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^4 - (11*a - 7*b)*cosh(d*x +
 c)^3 + (35*(3*a + b)*cosh(d*x + c)^4 + 10*(11*a - 7*b)*cosh(d*x + c)^2 - 1
1*a + 7*b)*sinh(d*x + c)^3 + (21*(3*a + b)*cosh(d*x + c)^5 + 10*(11*a - 7*b
)*cosh(d*x + c)^3 - 3*(11*a - 7*b)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a +
 b)*cosh(d*x + c)^8 + 8*(3*a + b)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a + b)
*sinh(d*x + c)^8 + 4*(3*a + b)*cosh(d*x + c)^6 + 4*(7*(3*a + b)*cosh(d*x +
c)^2 + 3*a + b)*sinh(d*x + c)^6 + 8*(7*(3*a + b)*cosh(d*x + c)^3 + 3*(3*a +
 b)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a + b)*cosh(d*x + c)^4 + 2*(35*(3
*a + b)*cosh(d*x + c)^4 + 30*(3*a + b)*cosh(d*x + c)^2 + 9*a + 3*b)*sinh(d*
x + c)^4 + 8*(7*(3*a + b)*cosh(d*x + c)^5 + 10*(3*a + b)*cosh(d*x + c)^3 +
3*(3*a + b)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a + b)*cosh(d*x + c)^2 +
4*(7*(3*a + b)*cosh(d*x + c)^6 + 15*(3*a + b)*cosh(d*x + c)^4 + 9*(3*a + b)
*cosh(d*x + c)^2 + 3*a + b)*sinh(d*x + c)^2 + 8*((3*a + b)*cosh(d*x + c)^7
+ 3*(3*a + b)*cosh(d*x + c)^5 + 3*(3*a + b)*cosh(d*x + c)^3 + (3*a + b)*cos
h(d*x + c))*sinh(d*x + c) + 3*a + b)*arctan(cosh(d*x + c) + sinh(d*x + c))
- (3*a + b)*cosh(d*x + c) + (7*(3*a + b)*cosh(d*x + c)^6 + 5*(11*a - 7*b)*c
osh(d*x + c)^4 - 3*(11*a - 7*b)*cosh(d*x + c)^2 - 3*a - b)*sinh(d*x + c))/(
d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x + c)^8 +
 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^6 + 8*(7*d
*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6*d*cosh(d*x + c)^4
 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*sinh(d*x + c)^4 +
8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 15*d*cosh(d*x + c)
^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh(d*x + c)^7 + 3*d*
cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15173, size = 209, normalized size = 2.99

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)(3a+b)}{16d} + \frac{3a\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + b\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 20a\left(e^{(dx+c)} - e^{(-d}\right.}{4\left(\left(e^{(dx+c)} - e^{(-dx-c)}\right)^2 + 4\right)^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a + b)/d +
1/4*(3*a*(e^(d*x + c) - e^(-d*x - c))^3 + b*(e^(d*x + c) - e^(-d*x - c))^3
+ 20*a*(e^(d*x + c) - e^(-d*x - c)) - 4*b*(e^(d*x + c) - e^(-d*x - c)))/(((
e^(d*x + c) - e^(-d*x - c))^2 + 4)^2*d)
```

3.292 $\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)\,dx$

Optimal. Leaf size=54

$$\frac{(a-b)\tanh^5(c+dx)}{5d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}$$

```
[Out] (a*Tanh[c + d*x])/d - ((2*a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)*Tanh[c +
 d*x]^5)/(5*d)
```

Rubi [A] time = 0.0474164, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3191, 373}

$$\frac{(a-b)\tanh^5(c+dx)}{5d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d - ((2*a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)*Tanh[c +
 d*x]^5)/(5*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)dx &= \frac{\operatorname{Subst}\left(\int \left(1-x^2\right)\left(a-(a-b)x^2\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \left(a-(2a-b)x^2+(a-b)x^4\right)dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a\tanh(c+dx)}{d}-\frac{(2a-b)\tanh^3(c+dx)}{3d}+\frac{(a-b)\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.050378, size = 102, normalized size = 1.89

$$\frac{a\tanh^5(c+dx)}{5d}-\frac{2a\tanh^3(c+dx)}{3d}+\frac{a\tanh(c+dx)}{d}+\frac{2b\tanh(c+dx)}{15d}-\frac{b\tanh(c+dx)\operatorname{sech}^4(c+dx)}{5d}+\frac{b\tanh(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (a*Tanh[c + d*x])/d + (2*b*Tanh[c + d*x])/(15*d) + (b*Sech[c + d*x]^2*Tanh[
c + d*x])/(15*d) - (b*Sech[c + d*x]^4*Tanh[c + d*x])/(5*d) - (2*a*Tanh[c +
d*x]^3)/(3*d) + (a*Tanh[c + d*x]^5)/(5*d)
```

Maple [A] time = 0.066, size = 85, normalized size = 1.6

$$\frac{1}{d}\left(a\left(\frac{8}{15}+\frac{(\operatorname{sech}(dx+c))^4}{5}+\frac{4(\operatorname{sech}(dx+c))^2}{15}\right)\tanh(dx+c)+b\left(-\frac{\sinh(dx+c)}{4(\cosh(dx+c))^5}+\frac{\tanh(dx+c)}{4}\left(\frac{8}{15}+\frac{(\operatorname{sec}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+b*(-1/4*sinh
(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(
d*x+c)))
```

Maxima [B] time = 1.07042, size = 656, normalized size = 12.15

$$\frac{16}{15}a\left(\frac{5e^{(-2dx-2c)}}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+10e^{(-6dx-6c)}+5e^{(-8dx-8c)}+e^{(-10dx-10c)}+1\right)}+\frac{1}{d\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}+1\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 16/15*a*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) +
10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x
- 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e
^(-10*d*x - 10*c) + 1))) + 4/15*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c
) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10
*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d
*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) +
 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(
5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*
x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [B] time = 1.45769, size = 930, normalized size = 17.22

$$-\frac{}{15\left(d\cosh(dx+c)^7+7d\cosh(dx+c)\sinh(dx+c)^6+d\sinh(dx+c)^7+5d\cosh(dx+c)^5+\left(21d\cosh(dx+c)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -8/15*(2*(a + 4*b)*cosh(d*x + c)^3 + 6*(a + 4*b)*cosh(d*x + c)*sinh(d*x + c
)^2 - (2*a - 7*b)*sinh(d*x + c)^3 + 30*a*cosh(d*x + c) - (3*(2*a - 7*b)*cos
h(d*x + c)^2 - 10*a + 5*b)*sinh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x
 + c)*sinh(d*x + c)^6 + d*sinh(d*x + c)^7 + 5*d*cosh(d*x + c)^5 + (21*d*cos
h(d*x + c)^2 + 5*d)*sinh(d*x + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 5*d*cosh(d*x
 + c))*sinh(d*x + c)^4 + 11*d*cosh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 50*
d*cosh(d*x + c)^2 + 9*d)*sinh(d*x + c)^3 + (21*d*cosh(d*x + c)^5 + 50*d*cos
h(d*x + c)^3 + 33*d*cosh(d*x + c))*sinh(d*x + c)^2 + 15*d*cosh(d*x + c) + (
7*d*cosh(d*x + c)^6 + 25*d*cosh(d*x + c)^4 + 27*d*cosh(d*x + c)^2 + 5*d)*si
nh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.15458, size = 112, normalized size = 2.07

$$-\frac{4\left(15\,be^{(6\,dx+6\,c)}+40\,ae^{(4\,dx+4\,c)}-5\,be^{(4\,dx+4\,c)}+20\,ae^{(2\,dx+2\,c)}+5\,be^{(2\,dx+2\,c)}+4\,a+b\right)}{15\,d\left(e^{(2\,dx+2\,c)}+1\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -4/15*(15*b*e^(6*d*x + 6*c) + 40*a*e^(4*d*x + 4*c) - 5*b*e^(4*d*x + 4*c) +
20*a*e^(2*d*x + 2*c) + 5*b*e^(2*d*x + 2*c) + 4*a + b)/(d*(e^(2*d*x + 2*c) +
 1)^5)
```

3.293 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=159

$$\frac{\left(48a^2-16ab+3b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(48a^2-16ab+3b^2\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(48a^2-16a\right.$$

```
[Out] ((48*a^2 - 16*a*b + 3*b^2)*x)/128 + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x
]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x]^3*Sinh[
c + d*x])/(192*d) + ((10*a - 3*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) +
 (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(8*d)
```

Rubi [A] time = 0.174047, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 413, 385, 199, 206}

$$\frac{\left(48a^2-16ab+3b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(48a^2-16ab+3b^2\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x\left(48a^2-16a\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```



```
[Out] ((48*a^2 - 16*a*b + 3*b^2)*x)/128 + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x
]*Sinh[c + d*x])/(128*d) + ((48*a^2 - 16*a*b + 3*b^2)*Cosh[c + d*x]^3*Sinh[
c + d*x])/(192*d) + ((10*a - 3*b)*b*Cosh[c + d*x]^5*Sinh[c + d*x])/(48*d) +
 (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(8*d)
```


Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{8d} - \frac{\text{Subst}\left(\int \frac{-a(8a-}{}\right.}{}$$

$$= \frac{(10a-3b)b\cosh^5(c+dx)\sinh(c+dx)}{48d} + \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-\right.}{8d}$$

$$= \frac{\left(48a^2-16ab+3b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{192d} + \frac{(10a-3b)b\cosh^5(c+d}{48d}$$

$$= \frac{\left(48a^2-16ab+3b^2\right)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{\left(48a^2-16ab+3b^2\right)\cosh}{192}$$

$$= \frac{1}{128}\left(48a^2-16ab+3b^2\right)x + \frac{\left(48a^2-16ab+3b^2\right)\cosh(c+dx)\sinh(c+dx}{128d}$$

Mathematica [A] time = 0.309086, size = 98, normalized size = 0.62

$$\frac{24\left(48a^2-16ab+3b^2\right)(c+dx)+24\left(4a^2+4ab-b^2\right)\sinh(4(c+dx))+32ab\sinh(6(c+dx))+96a(8a-b)\sinh(2(c+d}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (24*(48*a^2 - 16*a*b + 3*b^2)*(c + d*x) + 96*a*(8*a - b)*Sinh[2*(c + d*x)]
+ 24*(4*a^2 + 4*a*b - b^2)*Sinh[4*(c + d*x)] + 32*a*b*Sinh[6*(c + d*x)] + 3
*b^2*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A] time = 0.031, size = 172, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(\frac{(\sinh(dx+c))^3(\cosh(dx+c))^5}{8} - \frac{\sinh(dx+c)(\cosh(dx+c))^5}{16} + \frac{\sinh(dx+c)}{16}\left(\frac{(\cosh(dx+c))^3}{4} + \frac{3\cosh(dx}{8}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(1/8*sinh(d*x+c)^3*cosh(d*x+c)^5-1/16*sinh(d*x+c)*cosh(d*x+c)^5+1/
16*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/128*d*x+3/128*c)+2*a*b
*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sin
h(d*x+c)-1/16*d*x-1/16*c)+a^2*((1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x
+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.11009, size = 304, normalized size = 1.91

$$\frac{1}{64}a^2\left(24x + \frac{e^{(4dx+4c)}}{d} + \frac{8e^{(2dx+2c)}}{d} - \frac{8e^{(-2dx-2c)}}{d} - \frac{e^{(-4dx-4c)}}{d}\right) - \frac{1}{2048}b^2\left(\frac{\left(8e^{(-4dx-4c)}-1\right)e^{(8dx+8c)}}{d} - \frac{48(dx+c)}{d} - \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/64*a^2*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/2048*b^2*((8*e^(-4*d*x - 4*c) - 1)*e^(8*d*x
+ 8*c)/d - 48*(d*x + c)/d - (8*e^(-4*d*x - 4*c) - e^(-8*d*x - 8*c))/d) + 1/
192*a*b*((3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/d -
24*(d*x + c)/d + (3*e^(-2*d*x - 2*c) - 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c
))/d)
```

Fricas [A] time = 1.48994, size = 532, normalized size = 3.35

$$\frac{3b^2\cosh(dx+c)\sinh(dx+c)^7 + 3\left(7b^2\cosh(dx+c)^3 + 8ab\cosh(dx+c)\right)\sinh(dx+c)^5 + \left(21b^2\cosh(dx+c)^5 + \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^2*cosh(d*x + c)^3 + 8*a
*b*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^2*cosh(d*x + c)^5 + 80*a*b*cosh(d
*x + c)^3 + 12*(4*a^2 + 4*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(48
*a^2 - 16*a*b + 3*b^2)*d*x + 3*(b^2*cosh(d*x + c)^7 + 8*a*b*cosh(d*x + c)^5
 + 4*(4*a^2 + 4*a*b - b^2)*cosh(d*x + c)^3 + 8*(8*a^2 - a*b)*cosh(d*x + c))
*sinh(d*x + c))/d
```

Sympy [A] time = 12.8862, size = 481, normalized size = 3.03

$$\begin{cases} \frac{3a^2x\sinh^4(c+dx)}{8} - \frac{3a^2x\sinh^2(c+dx)\cosh^2(c+dx)}{4} + \frac{3a^2x\cosh^4(c+dx)}{8} - \frac{3a^2\sinh^3(c+dx)\cosh(c+dx)}{8d} + \frac{5a^2\sinh(c+dx)\cosh^3(c+dx)}{8d} + \frac{abx\sin}{} \\ x\left(a+b\sinh^2(c)\right)^2\cosh^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sinh(c + d*x)**4/8 - 3*a**2*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**2*x*cosh(c + d*x)**4/8 - 3*a**2*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) + 5*a**2*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + a*b*x*sinh(c +
d*x)**6/8 - 3*a*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/8 + 3*a*b*x*sinh(c +
```

```
d*x)**2*cosh(c + d*x)**4/8 - a*b*x*cosh(c + d*x)**6/8 - a*b*sinh(c + d*x)**
5*cosh(c + d*x)/(8*d) + a*b*sinh(c + d*x)**3*cosh(c + d*x)**3/(3*d) + a*b*s
inh(c + d*x)*cosh(c + d*x)**5/(8*d) + 3*b**2*x*sinh(c + d*x)**8/128 - 3*b**
2*x*sinh(c + d*x)**6*cosh(c + d*x)**2/32 + 9*b**2*x*sinh(c + d*x)**4*cosh(c
 + d*x)**4/64 - 3*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 + 3*b**2*x*co
sh(c + d*x)**8/128 - 3*b**2*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 11*b**
2*sinh(c + d*x)**5*cosh(c + d*x)**3/(128*d) + 11*b**2*sinh(c + d*x)**3*cosh
(c + d*x)**5/(128*d) - 3*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d,
 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**4, True))
```

Giac [A] time = 1.23409, size = 354, normalized size = 2.23

$$\frac{3\,b^2e^{(8\,dx+8\,c)} + 32\,abe^{(6\,dx+6\,c)} + 96\,a^2e^{(4\,dx+4\,c)} + 96\,abe^{(4\,dx+4\,c)} - 24\,b^2e^{(4\,dx+4\,c)} + 768\,a^2e^{(2\,dx+2\,c)} - 96\,abe^{(2\,dx+2\,c)} + 48\,(48}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6144*(3*b^2*e^(8*d*x + 8*c) + 32*a*b*e^(6*d*x + 6*c) + 96*a^2*e^(4*d*x +
4*c) + 96*a*b*e^(4*d*x + 4*c) - 24*b^2*e^(4*d*x + 4*c) + 768*a^2*e^(2*d*x +
 2*c) - 96*a*b*e^(2*d*x + 2*c) + 48*(48*a^2 - 16*a*b + 3*b^2)*(d*x + c) - (
2400*a^2*e^(8*d*x + 8*c) - 800*a*b*e^(8*d*x + 8*c) + 150*b^2*e^(8*d*x + 8*c
) + 768*a^2*e^(6*d*x + 6*c) - 96*a*b*e^(6*d*x + 6*c) + 96*a^2*e^(4*d*x + 4*
c) + 96*a*b*e^(4*d*x + 4*c) - 24*b^2*e^(4*d*x + 4*c) + 32*a*b*e^(2*d*x + 2*
c) + 3*b^2)*e^(-8*d*x - 8*c))/d
```

3.294 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=74

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(2a+b)\sinh^5(c+dx)}{5d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^7(c+dx)}{7d}$$

```
[Out] (a^2*Sinh[c + d*x])/d + (a*(a + 2*b)*Sinh[c + d*x]^3)/(3*d) + (b*(2*a + b)*
Sinh[c + d*x]^5)/(5*d) + (b^2*Sinh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0723943, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3190, 373}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{b(2a+b)\sinh^5(c+dx)}{5d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x])/d + (a*(a + 2*b)*Sinh[c + d*x]^3)/(3*d) + (b*(2*a + b)*
Sinh[c + d*x]^5)/(5*d) + (b^2*Sinh[c + d*x]^7)/(7*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx=\frac{\text{Subst}\left(\int \left(1+x^2\right)\left(a+bx^2\right)^2 dx,x,\sinh(c+dx)\right)}{d}$$
$$=\frac{\text{Subst}\left(\int \left(a^2+a(a+2b)x^2+b(2a+b)x^4+b^2x^6\right) dx,x,\sinh(c+dx)\right)}{d}$$
$$=\frac{a^2\sinh(c+dx)}{d}+\frac{a(a+2b)\sinh^3(c+dx)}{3d}+\frac{b(2a+b)\sinh^5(c+dx)}{5d}+$$

Mathematica [A] time = 0.114626, size = 64, normalized size = 0.86

$$\frac{105a^2\sinh(c+dx)+21b(2a+b)\sinh^5(c+dx)+35a(a+2b)\sinh^3(c+dx)+15b^2\sinh^7(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (105*a^2*Sinh[c + d*x] + 35*a*(a + 2*b)*Sinh[c + d*x]^3 + 21*b*(2*a + b)*Si
nh[c + d*x]^5 + 15*b^2*Sinh[c + d*x]^7)/(105*d)
```

Maple [A] time = 0.031, size = 128, normalized size = 1.7

$$\frac{1}{d}\left(b^2\left(\frac{(\sinh(dx+c))^3(\cosh(dx+c))^4}{7}-\frac{3\sinh(dx+c)(\cosh(dx+c))^4}{35}+\frac{3\sinh(dx+c)}{35}\left(\frac{2}{3}+\frac{(\cosh(dx+c))^2}{3}\right)\right)+2$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(1/7*sinh(d*x+c)^3*cosh(d*x+c)^4-3/35*sinh(d*x+c)*cosh(d*x+c)^4+3/
35*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+2*a*b*(1/5*sinh(d*x+c)*cosh(d*x+c)^
4-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+a^2*(2/3+1/3*cosh(d*x+c)^2)*sinh
(d*x+c))
```

Maxima [B] time = 1.08167, size = 327, normalized size = 4.42

$$-\frac{1}{4480}b^2\left(\frac{\left(7e^{(-2dx-2c)}+35e^{(-4dx-4c)}-105e^{(-6dx-6c)}-5\right)e^{(7dx+7c)}}{d}+\frac{105e^{(-dx-c)}-35e^{(-3dx-3c)}-7e^{(-5dx-5c)}+5e^{(-7dx}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4480*b^2*((7*e^(-2*d*x - 2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6*
c) - 5)*e^(7*d*x + 7*c)/d + (105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e^(
-5*d*x - 5*c) + 5*e^(-7*d*x - 7*c))/d) + 1/240*a*b*((5*e^(-2*d*x - 2*c) - 3
0*e^(-4*d*x - 4*c) + 3)*e^(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*d*x
- 3*c) - 3*e^(-5*d*x - 5*c))/d) + 1/24*a^2*(e^(3*d*x + 3*c)/d + 9*e^(d*x +
c)/d - 9*e^(-d*x - c)/d - e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.46437, size = 463, normalized size = 6.26

$$\frac{15b^2\sinh(dx+c)^7+21\left(15b^2\cosh(dx+c)^2+8ab-b^2\right)\sinh(dx+c)^5+35\left(15b^2\cosh(dx+c)^4+6\left(8ab-b^2\right)\cosh(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(15*b^2*sinh(d*x + c)^7 + 21*(15*b^2*cosh(d*x + c)^2 + 8*a*b - b^2)*
sinh(d*x + c)^5 + 35*(15*b^2*cosh(d*x + c)^4 + 6*(8*a*b - b^2)*cosh(d*x + c
)^2 + 16*a^2 + 8*a*b - 3*b^2)*sinh(d*x + c)^3 + 105*(b^2*cosh(d*x + c)^6 +
(8*a*b - b^2)*cosh(d*x + c)^4 + (16*a^2 + 8*a*b - 3*b^2)*cosh(d*x + c)^2 +
48*a^2 - 16*a*b + 3*b^2)*sinh(d*x + c))/d
```

Sympy [A] time = 6.89745, size = 136, normalized size = 1.84

$$\begin{cases} -\frac{2a^2\sinh^3(c+dx)}{3d} + \frac{a^2\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{4ab\sinh^5(c+dx)}{15d} + \frac{2ab\sinh^3(c+dx)\cosh^2(c+dx)}{3d} - \frac{2b^2\sinh^7(c+dx)}{35d} + \frac{b^2\sinh^5(c+dx)\cosh^2}{5d} \\ x\left(a+b\sinh^2(c)\right)^2\cosh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-2*a**2*sinh(c + d*x)**3/(3*d) + a**2*sinh(c + d*x)*cosh(c + d*x
)**2/d - 4*a*b*sinh(c + d*x)**5/(15*d) + 2*a*b*sinh(c + d*x)**3*cosh(c + d*
x)**2/(3*d) - 2*b**2*sinh(c + d*x)**7/(35*d) + b**2*sinh(c + d*x)**5*cosh(c
 + d*x)**2/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c)**3, True))
```

Giac [B] time = 1.22079, size = 332, normalized size = 4.49

$$\frac{15\,b^2e^{(7\,dx+7\,c)} + 168\,abe^{(5\,dx+5\,c)} - 21\,b^2e^{(5\,dx+5\,c)} + 560\,a^2e^{(3\,dx+3\,c)} + 280\,abe^{(3\,dx+3\,c)} - 105\,b^2e^{(3\,dx+3\,c)} + 5040\,a^2e^{(dx+c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/13440*(15*b^2*e^(7*d*x + 7*c) + 168*a*b*e^(5*d*x + 5*c) - 21*b^2*e^(5*d*x
 + 5*c) + 560*a^2*e^(3*d*x + 3*c) + 280*a*b*e^(3*d*x + 3*c) - 105*b^2*e^(3*
d*x + 3*c) + 5040*a^2*e^(d*x + c) - 1680*a*b*e^(d*x + c) + 315*b^2*e^(d*x +
 c) - (5040*a^2*e^(6*d*x + 6*c) - 1680*a*b*e^(6*d*x + 6*c) + 315*b^2*e^(6*d
*x + 6*c) + 560*a^2*e^(4*d*x + 4*c) + 280*a*b*e^(4*d*x + 4*c) - 105*b^2*e^(
4*d*x + 4*c) + 168*a*b*e^(2*d*x + 2*c) - 21*b^2*e^(2*d*x + 2*c) + 15*b^2)*e
^(-7*d*x - 7*c))/d
```

3.295 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=119

$$\frac{\left(8a^2-4ab+b^2\right)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x\left(8a^2-4ab+b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{b\sinh(c+\cdots}{\cdots}$$

```
[Out] ((8*a^2 - 4*a*b + b^2)*x)/16 + ((8*a^2 - 4*a*b + b^2)*Cosh[c + d*x]*Sinh[c
+ d*x])/(16*d) + ((8*a - 3*b)*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*
Cosh[c + d*x]^5*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(6*d)
```

Rubi [A] time = 0.14271, antiderivative size = 119, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 413, 385, 199, 206}

$$\frac{\left(8a^2-4ab+b^2\right)\sinh(c+dx)\cosh(c+dx)}{16d}+\frac{1}{16}x\left(8a^2-4ab+b^2\right)+\frac{b(8a-3b)\sinh(c+dx)\cosh^3(c+dx)}{24d}+\frac{b\sinh(c+\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + b^2)*x)/16 + ((8*a^2 - 4*a*b + b^2)*Cosh[c + d*x]*Sinh[c
+ d*x])/(16*d) + ((8*a - 3*b)*b*Cosh[c + d*x]^3*Sinh[c + d*x])/(24*d) + (b*
Cosh[c + d*x]^5*Sinh[c + d*x]*(a - (a - b)*Tanh[c + d*x]^2))/(6*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
```

```
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^4} dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^5(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)}{6d} - \frac{\text{Subst}\left(\int \frac{-a}{}\right.}{}$$

$$= \frac{(8a-3b)b\cosh^3(c+dx)\sinh(c+dx)}{24d} + \frac{b\cosh^5(c+dx)\sinh(c+dx)\left(a\right.}{6d}$$

$$= \frac{\left(8a^2-4ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(8a-3b)b\cosh^3(c+dx)}{24d}$$

$$= \frac{1}{16}\left(8a^2-4ab+b^2\right)x + \frac{\left(8a^2-4ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{16d} + \frac{(}{}$$

Mathematica [A] time = 0.294553, size = 79, normalized size = 0.66

$$\frac{12\left(8a^2-4ab+b^2\right)(c+dx)+3\left(16a^2-b^2\right)\sinh(2(c+dx))+3b(4a-b)\sinh(4(c+dx))+b^2\sinh(6(c+dx))}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (12*(8*a^2 - 4*a*b + b^2)*(c + d*x) + 3*(16*a^2 - b^2)*Sinh[2*(c + d*x)] +
3*(4*a - b)*b*Sinh[4*(c + d*x)] + b^2*Sinh[6*(c + d*x)])/(192*d)
```

Maple [A] time = 0.032, size = 134, normalized size = 1.1

$$\frac{1}{d}\left(b^2\left(\frac{(\sinh(dx+c))^3(\cosh(dx+c))^3}{6} - \frac{\sinh(dx+c)(\cosh(dx+c))^3}{8} + \frac{\cosh(dx+c)\sinh(dx+c)}{16} + \frac{dx}{16} + \frac{c}{16}\right) + \right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(b^2*(1/6*sinh(d*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3+1/1
6*cosh(d*x+c)*sinh(d*x+c)+1/16*d*x+1/16*c)+2*a*b*(1/4*sinh(d*x+c)*cosh(d*x+
c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a^2*(1/2*cosh(d*x+c)*sinh(d
*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.15212, size = 231, normalized size = 1.94

$$\frac{1}{8}a^2\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{384}b^2\left(\frac{\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}-1\right)e^{(6dx+6c)}}{d}-\frac{24(dx+c)}{d}-\frac{3e^{(-2dx-2c)}+3e^{(-4}}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/8*a^2*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/384*b^2*((3*e^(-
2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d -
 (3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 1/32*a*b
*(8*(d*x + c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [A] time = 1.4468, size = 347, normalized size = 2.92

$$\frac{3b^2\cosh(dx+c)\sinh(dx+c)^5+2\left(5b^2\cosh(dx+c)^3+3\left(4ab-b^2\right)\cosh(dx+c)\right)\sinh(dx+c)^3+6\left(8a^2-4ab+b^2\right.}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/96*(3*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(5*b^2*cosh(d*x + c)^3 + 3*(4
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(8*a^2 - 4*a*b + b^2)*d*x +
3*(b^2*cosh(d*x + c)^5 + 2*(4*a*b - b^2)*cosh(d*x + c)^3 + (16*a^2 - b^2)*c
osh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 4.5846, size = 314, normalized size = 2.64

$$\begin{cases}-\frac{a^2x\sinh^2(c+dx)}{2}+\frac{a^2x\cosh^2(c+dx)}{2}+\frac{a^2\sinh(c+dx)\cosh(c+dx)}{2d}-\frac{abx\sinh^4(c+dx)}{4}+\frac{abx\sinh^2(c+dx)\cosh^2(c+dx)}{2}-\frac{abx\cosh^4(c+dx)}{4}+\frac{ab\sin}{} \\ x\left(a+b\sinh^2(c)\right)^2\cosh^2(c)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-a**2*x*sinh(c + d*x)**2/2 + a**2*x*cosh(c + d*x)**2/2 + a**2*si
nh(c + d*x)*cosh(c + d*x)/(2*d) - a*b*x*sinh(c + d*x)**4/4 + a*b*x*sinh(c +
 d*x)**2*cosh(c + d*x)**2/2 - a*b*x*cosh(c + d*x)**4/4 + a*b*sinh(c + d*x)*
*3*cosh(c + d*x)/(4*d) + a*b*sinh(c + d*x)*cosh(c + d*x)**3/(4*d) - b**2*x*
sinh(c + d*x)**6/16 + 3*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 - 3*b**
2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 + b**2*x*cosh(c + d*x)**6/16 + b**
2*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + b**2*sinh(c + d*x)**3*cosh(c + d*
x)**3/(6*d) - b**2*sinh(c + d*x)*cosh(c + d*x)**5/(16*d), Ne(d, 0)), (x*(a
+ b*sinh(c)**2)**2*cosh(c)**2, True))
```

Giac [A] time = 1.18555, size = 277, normalized size = 2.33

$$\frac{b^2e^{(6dx+6c)}+12abe^{(4dx+4c)}-3b^2e^{(4dx+4c)}+48a^2e^{(2dx+2c)}-3b^2e^{(2dx+2c)}+24\left(8a^2-4ab+b^2\right)(dx+c)-\left(176a^2e^{(6dx+6}\right.}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/384*(b^2*e^(6*d*x + 6*c) + 12*a*b*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c)
 + 48*a^2*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + 24*(8*a^2 - 4*a*b + b^2
)*(d*x + c) - (176*a^2*e^(6*d*x + 6*c) - 88*a*b*e^(6*d*x + 6*c) + 22*b^2*e^
(6*d*x + 6*c) + 48*a^2*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 12*a*b*e^(
2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + b^2)*e^(-6*d*x - 6*c))/d
```

3.296 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=49

$$\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}$$

```
[Out] (a^2*Sinh[c + d*x])/d + (2*a*b*Sinh[c + d*x]^3)/(3*d) + (b^2*Sinh[c + d*x]^
5)/(5*d)
```

Rubi [A] time = 0.0370079, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3190, 194}

$$\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x])/d + (2*a*b*Sinh[c + d*x]^3)/(3*d) + (b^2*Sinh[c + d*x]^
5)/(5*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)^2\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^2+2abx^2+b^2x^4\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.0560869, size = 44, normalized size = 0.9

$$\frac{a^2\sinh(c+dx)+\frac{2}{3}ab\sinh^3(c+dx)+\frac{1}{5}b^2\sinh^5(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Sinh[c + d*x] + (2*a*b*Sinh[c + d*x]^3)/3 + (b^2*Sinh[c + d*x]^5)/5)/d
```

Maple [A] time = 0.011, size = 41, normalized size = 0.8

$$\frac{1}{d}\left(\frac{b^2(\sinh(dx+c))^5}{5}+\frac{2a(\sinh(dx+c))^3 b}{3}+a^2\sinh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/5*b^2*sinh(d*x+c)^5+2/3*a*sinh(d*x+c)^3*b+a^2*sinh(d*x+c))
```

Maxima [A] time = 1.16507, size = 61, normalized size = 1.24

$$\frac{b^2\sinh(dx+c)^5}{5d}+\frac{2ab\sinh(dx+c)^3}{3d}+\frac{a^2\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/5*b^2*sinh(d*x + c)^5/d + 2/3*a*b*sinh(d*x + c)^3/d + a^2*sinh(d*x + c)/d
```

Fricas [B] time = 1.47832, size = 261, normalized size = 5.33

$$\frac{3b^2\sinh(dx+c)^5+5\left(6b^2\cosh(dx+c)^2+8ab-3b^2\right)\sinh(dx+c)^3+15\left(b^2\cosh(dx+c)^4+\left(8ab-3b^2\right)\cosh(d\right.}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/240*(3*b^2*sinh(d*x + c)^5 + 5*(6*b^2*cosh(d*x + c)^2 + 8*a*b - 3*b^2)*si
nh(d*x + c)^3 + 15*(b^2*cosh(d*x + c)^4 + (8*a*b - 3*b^2)*cosh(d*x + c)^2 +
 16*a^2 - 8*a*b + 2*b^2)*sinh(d*x + c))/d
```

Sympy [A] time = 2.01711, size = 58, normalized size = 1.18

$$\begin{cases}\frac{a^2\sinh(c+dx)}{d}+\frac{2ab\sinh^3(c+dx)}{3d}+\frac{b^2\sinh^5(c+dx)}{5d} & \text{for } d\neq 0\\ x\left(a+b\sinh^2(c)\right)^2\cosh(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((a**2*sinh(c + d*x)/d + 2*a*b*sinh(c + d*x)**3/(3*d) + b**2*sinh(
c + d*x)**5/(5*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**2*cosh(c), True))
```

Giac [B] time = 1.17326, size = 221, normalized size = 4.51

$$\frac{3\,b^2e^{(5\,dx+5\,c)}+40\,abe^{(3\,dx+3\,c)}-15\,b^2e^{(3\,dx+3\,c)}+240\,a^2e^{(dx+c)}-120\,abe^{(dx+c)}+30\,b^2e^{(dx+c)}-\left(240\,a^2e^{(4\,dx+4\,c)}-120\,abe^{(4\,dx}\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/480*(3*b^2*e^(5*d*x + 5*c) + 40*a*b*e^(3*d*x + 3*c) - 15*b^2*e^(3*d*x + 3
*c) + 240*a^2*e^(d*x + c) - 120*a*b*e^(d*x + c) + 30*b^2*e^(d*x + c) - (240
*a^2*e^(4*d*x + 4*c) - 120*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 4
0*a*b*e^(2*d*x + 2*c) - 15*b^2*e^(2*d*x + 2*c) + 3*b^2)*e^(-5*d*x - 5*c))/d
```

3.297 $\int \mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=55

$$\frac{b(2a-b)\sinh(c+dx)}{d}+\frac{(a-b)^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\sinh^3(c+dx)}{3d}$$

```
[Out] ((a - b)^2*ArcTan[Sinh[c + d*x]])/d + ((2*a - b)*b*Sinh[c + d*x])/d + (b^2*
Sinh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.0577477, antiderivative size = 55, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 390, 203}

$$\frac{b(2a-b)\sinh(c+dx)}{d}+\frac{(a-b)^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^2\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)^2*ArcTan[Sinh[c + d*x]])/d + ((2*a - b)*b*Sinh[c + d*x])/d + (b^2*
Sinh[c + d*x]^3)/(3*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^2}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left((2a-b)b+b^2x^2+\frac{(a-b)^2}{1+x^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{(2a-b)b\sinh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)}{3d}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sin\right.}{d}$$
$$= \frac{(a-b)^2\tan^{-1}(\sinh(c+dx))}{d}+\frac{(2a-b)b\sinh(c+dx)}{d}+\frac{b^2\sinh^3(c+dx)}{3d}$$

Mathematica [A] time = 0.227902, size = 70, normalized size = 1.27

$$\frac{\sinh(c+dx)\left(b\left(6a+b\left(\sinh^2(c+dx)-3\right)\right)+\frac{3(a-b)^2\tanh^{-1}\left(\sqrt{-\sinh^2(c+dx)}\right)}{\sqrt{-\sinh^2(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Sinh[c + d*x]*((3*(a - b)^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c
+ d*x]^2] + b*(6*a + b*(-3 + Sinh[c + d*x]^2))))/(3*d)
```

Maple [A] time = 0.038, size = 89, normalized size = 1.6

$$2\,\frac{a^2\arctan\left(e^{dx+c}\right)}{d}+2\,\frac{ab\sinh(dx+c)}{d}-4\,\frac{ab\arctan\left(e^{dx+c}\right)}{d}+\frac{b^2(\sinh(dx+c))^3}{3d}-\frac{b^2\sinh(dx+c)}{d}+2\,\frac{b^2\arctan\left(e^{d}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 2/d*a^2*arctan(exp(d*x+c))+2/d*a*b*sinh(d*x+c)-4/d*a*b*arctan(exp(d*x+c))+1
/3*b^2*sinh(d*x+c)^3/d-b^2*sinh(d*x+c)/d+2/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.72436, size = 180, normalized size = 3.27

$$-\frac{1}{24}b^2\left(\frac{\left(15e^{(-2dx-2c)}-1\right)e^{(3dx+3c)}}{d}-\frac{15e^{(-dx-c)}-e^{(-3dx-3c)}}{d}+\frac{48\arctan\left(e^{(-dx-c)}\right)}{d}\right)+ab\left(\frac{4\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(dx+c)}}{d}-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x + 3*c)/d - (15*e^(-d*x - c) -
 e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c))/d) + a*b*(4*arctan(e^(-d*x -
 c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) + a^2*arctan(sinh(d*x + c))/d
```

Fricas [B] time = 1.60384, size = 1143, normalized size = 20.78

$$\frac{b^2\cosh(dx+c)^6 + 6\,b^2\cosh(dx+c)\sinh(dx+c)^5 + b^2\sinh(dx+c)^6 + 3\left(8\,ab - 5\,b^2\right)\cosh(dx+c)^4 + 3\left(5\,b^2\cosh\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(
d*x + c)^6 + 3*(8*a*b - 5*b^2)*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 +
 8*a*b - 5*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + 3*(8*a*b - 5*b
^2)*cosh(d*x + c))*sinh(d*x + c)^3 - 3*(8*a*b - 5*b^2)*cosh(d*x + c)^2 + 3*
(5*b^2*cosh(d*x + c)^4 + 6*(8*a*b - 5*b^2)*cosh(d*x + c)^2 - 8*a*b + 5*b^2)
*sinh(d*x + c)^2 - b^2 + 48*((a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 -
 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*
x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^3)*arctan(cosh(d
*x + c) + sinh(d*x + c)) + 6*(b^2*cosh(d*x + c)^5 + 2*(8*a*b - 5*b^2)*cosh(
d*x + c)^3 - (8*a*b - 5*b^2)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)
^3 + 3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*d*cosh(d*x + c)*sinh(d*x + c)^2
+ d*sinh(d*x + c)^3)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18227, size = 159, normalized size = 2.89

$$\frac{2\left(a^2 - 2\,ab + b^2\right)\arctan\left(e^{(dx+c)}\right)}{d} - \frac{\left(24\,abe^{(2\,dx+2\,c)} - 15\,b^2e^{(2\,dx+2\,c)} + b^2\right)e^{(-3\,dx-3\,c)}}{24\,d} + \frac{b^2d^2e^{(3\,dx+3\,c)} + 24\,abd^2e^{(dx+c)} -}{24\,d^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 2*(a^2 - 2*a*b + b^2)*arctan(e^(d*x + c))/d - 1/24*(24*a*b*e^(2*d*x + 2*c)
- 15*b^2*e^(2*d*x + 2*c) + b^2)*e^(-3*d*x - 3*c)/d + 1/24*(b^2*d^2*e^(3*d*x
 + 3*c) + 24*a*b*d^2*e^(d*x + c) - 15*b^2*d^2*e^(d*x + c))/d^3
```

3.298 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{(a-b)^2\tanh(c+dx)}{d}+\frac{1}{2}bx(4a-3b)+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] ((4*a - 3*b)*b*x)/2 + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + ((a - b)^2*
Tanh[c + d*x])/d
```

Rubi [A] time = 0.0949116, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 206}

$$\frac{(a-b)^2\tanh(c+dx)}{d}+\frac{1}{2}bx(4a-3b)+\frac{b^2\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((4*a - 3*b)*b*x)/2 + (b^2*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + ((a - b)^2*
Tanh[c + d*x])/d
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left((a-b)^2+\frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}\right)dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{(a-b)^2\tanh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(1-x^2\right)^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{(a-b)^2\tanh(c+dx)}{d}+\frac{((4a-3b)b)\,\text{Su}}{}$$
$$= \frac{1}{2}(4a-3b)bx+\frac{b^2\cosh(c+dx)\sinh(c+dx)}{2d}+\frac{(a-b)^2\tanh(c+dx)}{d}$$

Mathematica [A] time = 0.291894, size = 50, normalized size = 0.94

$$\frac{2b(4a-3b)(c+dx)+4(a-b)^2\tanh(c+dx)+b^2\sinh(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(4*a - 3*b)*b*(c + d*x) + b^2*Sinh[2*(c + d*x)] + 4*(a - b)^2*Tanh[c + d
*x])/(4*d)
```

Maple [A] time = 0.037, size = 71, normalized size = 1.3

$$\frac{1}{d}\left(a^2\tanh(dx+c)+2ab(dx+c-\tanh(dx+c))+b^2\left(\frac{(\sinh(dx+c))^3}{2\cosh(dx+c)}-\frac{3dx}{2}-\frac{3c}{2}+\frac{3\tanh(dx+c)}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*tanh(d*x+c)+2*a*b*(d*x+c-tanh(d*x+c))+b^2*(1/2*sinh(d*x+c)^3/cosh(
d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c)))
```

Maxima [B] time = 1.21566, size = 161, normalized size = 3.04

$$2ab\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2dx-2c)}+1\right)}\right)-\frac{1}{8}b^2\left(\frac{12(dx+c)}{d}+\frac{e^{(-2dx-2c)}}{d}-\frac{17e^{(-2dx-2c)}+1}{d\left(e^{(-2dx-2c)}+e^{(-4dx-4c)}\right)}\right)+\frac{2a^2}{d\left(e^{(-2dx-2c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 2*a*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) - 1/8*b^2*(12*(d*x + c)/d +
e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(-2*d*x - 2*c) + e^(-4
```

```
*d*x - 4*c)))) + 2*a^2/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [A] time = 1.53539, size = 235, normalized size = 4.43

$$\frac{b^2\sinh(dx+c)^3 + 4\left(\left(4\,ab-3\,b^2\right)dx - 2\,a^2 + 4\,ab - 2\,b^2\right)\cosh(dx+c) + \left(3\,b^2\cosh(dx+c)^2 + 8\,a^2 - 16\,ab + 9\,b^2\right)\sinh}{8\,d\cosh(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*sinh(d*x + c)^3 + 4*((4*a*b - 3*b^2)*d*x - 2*a^2 + 4*a*b - 2*b^2)*
cosh(d*x + c) + (3*b^2*cosh(d*x + c)^2 + 8*a^2 - 16*a*b + 9*b^2)*sinh(d*x +
 c))/(d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.18936, size = 184, normalized size = 3.47

$$\frac{b^2e^{(2\,dx+2\,c)}}{8\,d} + \frac{\left(4\,ab-3\,b^2\right)(dx+c)}{2\,d} - \frac{4\,abe^{(4\,dx+4\,c)} - 3\,b^2e^{(4\,dx+4\,c)} + 16\,a^2e^{(2\,dx+2\,c)} - 28\,abe^{(2\,dx+2\,c)} + 14\,b^2e^{(2\,dx+2\,c)} + b^2}{8\,d\left(e^{(4\,dx+4\,c)} + e^{(2\,dx+2\,c)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*b^2*e^(2*d*x + 2*c)/d + 1/2*(4*a*b - 3*b^2)*(d*x + c)/d - 1/8*(4*a*b*e^
(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 16*a^2*e^(2*d*x + 2*c) - 28*a*b*e^(
2*d*x + 2*c) + 14*b^2*e^(2*d*x + 2*c) + b^2)/(d*(e^(4*d*x + 4*c) + e^(2*d*x
 + 2*c)))
```

3.299 $\int \mathbf{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2 dx$

Optimal. Leaf size=64

$$\frac{(a+3b)(a-b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{b^2\sinh(c+dx)}{d}$$

```
[Out] ((a - b)*(a + 3*b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b^2*Sinh[c + d*x])/d + (
(a - b)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rubi [A] time = 0.079862, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 390, 385, 203}

$$\frac{(a+3b)(a-b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)^2\tanh(c+dx)\text{sech}(c+dx)}{2d}+\frac{b^2\sinh(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a - b)*(a + 3*b)*ArcTan[Sinh[c + d*x]])/(2*d) + (b^2*Sinh[c + d*x])/d + (
(a - b)^2*Sech[c + d*x]*Tanh[c + d*x])/(2*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^2}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(b^2+\frac{a^2-b^2+2(a-b)bx^2}{\left(1+x^2\right)^2}\right)dx,x,\sinh(c+dx)\right)}{d}$$

$$= \frac{b^2\sinh(c+dx)}{d}+\frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{\left(1+x^2\right)^2}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$= \frac{b^2\sinh(c+dx)}{d}+\frac{(a-b)^2\operatorname{sech}(c+dx)\tanh(c+dx)}{2d}+\frac{((a-b)(a+3b))\,\text{Su}}{}$$

$$= \frac{(a-b)(a+3b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{b^2\sinh(c+dx)}{d}+\frac{(a-b)^2\operatorname{sech}(c+d}{2d}$$

Mathematica [C] time = 7.36673, size = 253, normalized size = 3.95

$$\frac{\operatorname{csch}^3(c+dx)\left(-64\sinh^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2\right\},\left\{1,1,\frac{9}{2}\right\},-\sinh^2(c+dx)\right)\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c + d*x]^3*(-64*HypergeometricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 - 35*(a^2*(375 + 37*S
inh[c + d*x]^2) + b^2*Sinh[c + d*x]^4*(303 + 61*Sinh[c + d*x]^2) + 2*a*b*Si
nh[c + d*x]^2*(375 + 61*Sinh[c + d*x]^2)) + (105*ArcTanh[Sqrt[-Sinh[c + d*x
]^2]]*(b^2*Sinh[c + d*x]^4*(101 + 54*Sinh[c + d*x]^2 + Sinh[c + d*x]^4) + 2
*a*b*Sinh[c + d*x]^2*(125 + 62*Sinh[c + d*x]^2 + Sinh[c + d*x]^4) + a^2*(12
5 + 54*Sinh[c + d*x]^2 + 9*Sinh[c + d*x]^4)))/Sqrt[-Sinh[c + d*x]^2]))/(168
0*d)
```

Maple [B] time = 0.047, size = 169, normalized size = 2.6

$$\frac{a^2\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}+\frac{a^2\arctan\left(e^{dx+c}\right)}{d}-2\,\frac{ab\sinh(dx+c)}{d\left(\cosh(dx+c)\right)^2}+\frac{ab\operatorname{sech}(dx+c)\tanh(dx+c)}{d}+2\,\frac{ab\arctan}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*a^2*sech(d*x+c)*tanh(d*x+c)+1/d*a^2*arctan(exp(d*x+c))-2/d*a*b*sinh(d
*x+c)/cosh(d*x+c)^2+1/d*a*b*sech(d*x+c)*tanh(d*x+c)+2/d*a*b*arctan(exp(d*x+
c))+1/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^2+3/d*b^2*sinh(d*x+c)/cosh(d*x+c)^2-3
/2/d*b^2*sech(d*x+c)*tanh(d*x+c)-3/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.89297, size = 316, normalized size = 4.94

$$\frac{1}{2}\,b^2\left(\frac{6\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{e^{(-dx-c)}}{d}+\frac{4\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}+1}{d\left(e^{(-dx-c)}+2\,e^{(-3\,dx-3\,c)}+e^{(-5\,dx-5\,c)}\right)}\right)-2\,ab\left(\frac{\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{e^{(-dx-c)}-}{d\left(2\,e^{(-2\,dx-2\,c)}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) -
e^(-4*d*x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5
*c)))) - 2*a*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/
(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^2*(arctan(e^(-d*x - c)
)/d - (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x
- 4*c) + 1)))
```

Fricas [B] time = 1.62602, size = 1916, normalized size = 29.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d
*x + c)^6 + (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c)^4 + (15*b^2*cosh(d*x + c)
^2 + 2*a^2 - 4*a*b + 3*b^2)*sinh(d*x + c)^4 + 4*(5*b^2*cosh(d*x + c)^3 + (2
*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - (2*a^2 - 4*a*b + 3*b
^2)*cosh(d*x + c)^2 + (15*b^2*cosh(d*x + c)^4 + 6*(2*a^2 - 4*a*b + 3*b^2)*c
osh(d*x + c)^2 - 2*a^2 + 4*a*b - 3*b^2)*sinh(d*x + c)^2 - b^2 + 2*((a^2 + 2
*a*b - 3*b^2)*cosh(d*x + c)^5 + 5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)*sinh(
d*x + c)^4 + (a^2 + 2*a*b - 3*b^2)*sinh(d*x + c)^5 + 2*(a^2 + 2*a*b - 3*b^2
)*cosh(d*x + c)^3 + 2*(5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*
b - 3*b^2)*sinh(d*x + c)^3 + 2*(5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^3 + 3
*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^2 + 2*a*b - 3*b^
2)*cosh(d*x + c) + (5*(a^2 + 2*a*b - 3*b^2)*cosh(d*x + c)^4 + 6*(a^2 + 2*a*
b - 3*b^2)*cosh(d*x + c)^2 + a^2 + 2*a*b - 3*b^2)*sinh(d*x + c))*arctan(cos
h(d*x + c) + sinh(d*x + c)) + 2*(3*b^2*cosh(d*x + c)^5 + 2*(2*a^2 - 4*a*b +
 3*b^2)*cosh(d*x + c)^3 - (2*a^2 - 4*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x +
 c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)*sinh(d*x + c)^4 + d*sinh(d*x +
c)^5 + 2*d*cosh(d*x + c)^3 + 2*(5*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^3 +
2*(5*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x +
c) + (5*d*cosh(d*x + c)^4 + 6*d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2036, size = 225, normalized size = 3.52

$$\frac{b^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{2\,d}+\frac{\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^2+2\,ab-3\,b^2\right)}{4\,d}+\frac{a^2\left(e^{(dx+c)}-e^{(-dx-c)}\right)-2\,ab\left(e^{(dx+c)}-\right.}{\left(\left(e^{(dx+c)}-e^{(-dx}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*(e^(d*x + c) - e^(-d*x - c))/d + 1/4*(pi + 2*arctan(1/2*(e^(2*d*x +
 2*c) - 1)*e^(-d*x - c)))*(a^2 + 2*a*b - 3*b^2)/d + (a^2*(e^(d*x + c) - e^(
-d*x - c)) - 2*a*b*(e^(d*x + c) - e^(-d*x - c)) + b^2*(e^(d*x + c) - e^(-d*
x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)*d)
```

3.300 $\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=47

$$\frac{(a^2-b^2)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}+b^2x$$

```
[Out] b^2*x + ((a^2 - b^2)*Tanh[c + d*x])/d - ((a - b)^2*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.062746, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3191, 390, 206}

$$\frac{(a^2-b^2)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}+b^2x$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] b^2*x + ((a^2 - b^2)*Tanh[c + d*x])/d - ((a - b)^2*Tanh[c + d*x]^3)/(3*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^2}{1-x^2}\,dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{\operatorname{Subst}\left(\int \left(a^2-b^2-(a-b)^2x^2+\frac{b^2}{1-x^2}\right)dx,x,\tanh(c+dx)\right)}{d}\\
&= \frac{(a^2-b^2)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}+\frac{b^2\operatorname{Subst}\left(\int \frac{1}{1-x^2}\,dx,\right.}{d}\\
&= b^2x+\frac{(a^2-b^2)\tanh(c+dx)}{d}-\frac{(a-b)^2\tanh^3(c+dx)}{3d}
\end{aligned}$$

Mathematica [A] time = 0.331525, size = 57, normalized size = 1.21

$$\frac{(a-b)\tanh(c+dx)\text{sech}^2(c+dx)((a+2b)\cosh(2(c+dx))+2a+b)+3b^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (3*b^2*(c + d*x) + (a - b)*(2*a + b + (a + 2*b)*Cosh[2*(c + d*x)])*Sech[c +
 d*x]^2*Tanh[c + d*x])/(3*d)
```

Maple [B] time = 0.046, size = 96, normalized size = 2.

$$\frac{1}{d}\left(a^2\left(\frac{2}{3}+\frac{(\text{sech}(dx+c))^2}{3}\right)\tanh(dx+c)+2ab\left(-1/2\frac{\sinh(dx+c)}{(\cosh(dx+c))^3}+1/2\left(2/3+1/3(\text{sech}(dx+c))^2\right)\tanh(dx+c)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/2*sinh(d*x+c)/cosh(d
*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+b^2*(d*x+c-tanh(d*x+c)-1/3
*tanh(d*x+c)^3))
```

Maxima [B] time = 1.11575, size = 360, normalized size = 7.66

$$\frac{1}{3}b^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)+\frac{4}{3}a^2\left(\frac{3e^{(-2dx-2c)}}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}+\frac{}{d(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(
3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 4/3*a^2
*(3*e^(-2*d*x - 2*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*
x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x
- 6*c) + 1))) + 4/3*a*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-
4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*
d*x - 4*c) + e^(-6*d*x - 6*c) + 1)))
```

Fricas [B] time = 1.48203, size = 495, normalized size = 10.53

$$\frac{\left(3b^2dx-2a^2-2ab+4b^2\right)\cosh(dx+c)^3+3\left(3b^2dx-2a^2-2ab+4b^2\right)\cosh(dx+c)\sinh(dx+c)^2+2\left(a^2+ab-2\right.}{3\left(d\cosh(dx+c)^3+3d\cosh(dx-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*((3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)^3 + 3*(3*b^2*d*x - 2
*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2 + a*b - 2*b^2)
*sinh(d*x + c)^3 + 3*(3*b^2*d*x - 2*a^2 - 2*a*b + 4*b^2)*cosh(d*x + c) + 6*
((a^2 + a*b - 2*b^2)*cosh(d*x + c)^2 + a^2 - a*b)*sinh(d*x + c))/(d*cosh(d*
x + c)^3 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1663, size = 132, normalized size = 2.81

$$\frac{(dx+c)b^2}{d} - \frac{4\left(3\,abe^{(4\,dx+4\,c)} - 3\,b^2e^{(4\,dx+4\,c)} + 3\,a^2e^{(2\,dx+2\,c)} - 3\,b^2e^{(2\,dx+2\,c)} + a^2 + ab - 2\,b^2\right)}{3\,d\left(e^{(2\,dx+2\,c)} + 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^2/d - 4/3*(3*a*b*e^(4*d*x + 4*c) - 3*b^2*e^(4*d*x + 4*c) + 3*a^
2*e^(2*d*x + 2*c) - 3*b^2*e^(2*d*x + 2*c) + a^2 + a*b - 2*b^2)/(d*(e^(2*d*x
 + 2*c) + 1)^3)
```

3.301 $\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=96

$$\frac{(3a^2+2ab+3b^2)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3(a^2-b^2)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx)\left(a+\right.}{4d}$$

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (3*(a^2 - b^2)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) + ((a - b)*Sech[c + d*x]^3*(a + b*Sinh[c + d
*x]^2)*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.0927501, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 413, 385, 203}

$$\frac{(3a^2+2ab+3b^2)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{3(a^2-b^2)\tanh(c+dx)\operatorname{sech}(c+dx)}{8d}+\frac{(a-b)\tanh(c+dx)\operatorname{sech}^3(c+dx)\left(a+\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (3*(a^2 - b^2)*Sech
[c + d*x]*Tanh[c + d*x])/(8*d) + ((a - b)*Sech[c + d*x]^3*(a + b*Sinh[c + d
*x]^2)*Tanh[c + d*x])/(4*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{4d} + \frac{\operatorname{Subst}\left(\int \frac{a(3}{}\right.}{}$$

$$= \frac{3\left(a^2-b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d} + \frac{(a-b)\operatorname{sech}^3(c+dx)\left(a+b\sin\right.}{4d}$$

$$= \frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}(\sinh(c+dx))}{8d} + \frac{3\left(a^2-b^2\right)\operatorname{sech}(c+dx)\tanh}{8d}$$

Mathematica [C] time = 5.94609, size = 303, normalized size = 3.16

$$-\frac{\operatorname{csch}^3(c+dx)\left(128\sinh^6(c+dx)\left(7a^2+12ab\sinh^2(c+dx)+5b^2\sinh^4(c+dx)\right)\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -(Csch[c + d*x]^3*(128*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 9/2},
 -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2 + 128*Hypergeo
metricPFQ[{3/2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(7
*a^2 + 12*a*b*Sinh[c + d*x]^2 + 5*b^2*Sinh[c + d*x]^4) + 35*(3375*a^2 + a*(
657*a + 4643*b + 607*b*Cosh[2*(c + d*x)])*Sinh[c + d*x]^2 + 1947*b^2*Sinh[c
 + d*x]^4 + 485*b^2*Sinh[c + d*x]^6) - (105*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]
*(1125*a^2 + 2*a*(297*a + 875*b)*Sinh[c + d*x]^2 + (37*a^2 + 988*a*b + 649*
b^2)*Sinh[c + d*x]^4 + 2*b*(11*a + 189*b)*Sinh[c + d*x]^6 + 9*b^2*Sinh[c +
d*x]^8))/Sqrt[-Sinh[c + d*x]^2]))/(6720*d)
```

Maple [B] time = 0.051, size = 237, normalized size = 2.5

$$\frac{a^2\tanh(dx+c)(\operatorname{sech}(dx+c))^3}{4d} + \frac{3a^2\operatorname{sech}(dx+c)\tanh(dx+c)}{8d} + \frac{3a^2\arctan\left(e^{dx+c}\right)}{4d} - \frac{2ab\sinh(dx+c)}{3d(\cosh(dx+c))^4} + \frac{ab}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/4/d*a^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a^2*sech(d*x+c)*tanh(d*x+c)+3/4/d
*a^2*arctan(exp(d*x+c))-2/3/d*a*b*sinh(d*x+c)/cosh(d*x+c)^4+1/6/d*a*b*tanh(
d*x+c)*sech(d*x+c)^3+1/4/d*a*b*sech(d*x+c)*tanh(d*x+c)+1/2/d*a*b*arctan(exp
(d*x+c))-1/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^4-1/d*b^2*sinh(d*x+c)/cosh(d*x+c
)^4+1/4/d*b^2*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*b^2*sech(d*x+c)*tanh(d*x+c)+3
/4/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.72498, size = 468, normalized size = 4.88

$$-\frac{1}{4}b^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}+\frac{5e^{(-dx-c)}-3e^{(-3dx-3c)}+3e^{(-5dx-5c)}-5e^{(-7dx-7c)}}{d\left(4e^{(-2dx-2c)}+6e^{(-4dx-4c)}+4e^{(-6dx-6c)}+e^{(-8dx-8c)}+1\right)}\right)-\frac{1}{4}a^2\left(\frac{3\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{}{d(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x - c) - 3*e^(-3*d*x - 3*c) +
 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*
d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/4*a^2*(3*arct
an(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d*x - 3*c) - 11*e^(-5*d*x -
 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4
*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 1/2*a*b*(arctan(e^(-d*x - c))
/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(-5*d*x - 5*c) - e^(-7*d*x -
7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^
(-8*d*x - 8*c) + 1)))
```

Fricas [B] time = 1.5918, size = 3672, normalized size = 38.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*((3*a^2 + 2*a*b - 5*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 + 2*a*b - 5*b^2)*co
sh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 + 2*a*b - 5*b^2)*sinh(d*x + c)^7 + (11
*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 + 2*a*b - 5*b^2)*cosh(d
*x + c)^2 + 11*a^2 - 14*a*b + 3*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 + 2*a*b
- 5*b^2)*cosh(d*x + c)^3 + (11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*
x + c)^4 - (11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 + 2*a*b -
 5*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^2 - 11
*a^2 + 14*a*b - 3*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 + 2*a*b - 5*b^2)*cosh(d
*x + c)^5 + 10*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 - 14*a
*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^2 + 2*a*b + 3*b^2)*cosh(
d*x + c)^8 + 8*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a
^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x +
c)^6 + 4*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2
)*sinh(d*x + c)^6 + 8*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^3 + 3*(3*a^2
 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2 + 2*a*b + 3*b^2
)*cosh(d*x + c)^4 + 2*(35*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 30*(3*a
^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 9*a^2 + 6*a*b + 9*b^2)*sinh(d*x + c)^
4 + 8*(7*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 10*(3*a^2 + 2*a*b + 3*b^
2)*cosh(d*x + c)^3 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)
^3 + 4*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 + 2*a*b + 3*b^
2)*cosh(d*x + c)^6 + 15*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^4 + 9*(3*a^2
+ 2*a*b + 3*b^2)*cosh(d*x + c)^2 + 3*a^2 + 2*a*b + 3*b^2)*sinh(d*x + c)^2 +
 3*a^2 + 2*a*b + 3*b^2 + 8*((3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^7 + 3*(3*
a^2 + 2*a*b + 3*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 + 2*a*b + 3*b^2)*cosh(d*x +
 c)^3 + (3*a^2 + 2*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d
*x + c) + sinh(d*x + c)) - (3*a^2 + 2*a*b - 5*b^2)*cosh(d*x + c) + (7*(3*a^
2 + 2*a*b - 5*b^2)*cosh(d*x + c)^6 + 5*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x +
 c)^4 - 3*(11*a^2 - 14*a*b + 3*b^2)*cosh(d*x + c)^2 - 3*a^2 - 2*a*b + 5*b^2
)*sinh(d*x + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d
```

```
*sinh(d*x + c)^8 + 4*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + d)*sinh(d
*x + c)^6 + 8*(7*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^5 + 6
*d*cosh(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 30*d*cosh(d*x + c)^2 + 3*d)*
sinh(d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 10*d*cosh(d*x + c)^3 + 3*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 +
15*d*cosh(d*x + c)^4 + 9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 8*(d*cosh
(d*x + c)^7 + 3*d*cosh(d*x + c)^5 + 3*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*
sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.16042, size = 297, normalized size = 3.09

$$\frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(3\,a^2 + 2\,ab + 3\,b^2\right)}{16\,d} + \frac{3\,a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 + 2\,ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^3 - 5\,b}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(3*a^2 + 2*a*b
 + 3*b^2)/d + 1/4*(3*a^2*(e^(d*x + c) - e^(-d*x - c))^3 + 2*a*b*(e^(d*x + c
) - e^(-d*x - c))^3 - 5*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 20*a^2*(e^(d*x
 + c) - e^(-d*x - c)) - 8*a*b*(e^(d*x + c) - e^(-d*x - c)) - 12*b^2*(e^(d*x
 + c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2*d)
```

3.302 $\int \text{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=57

$$\frac{a^2\tanh(c+dx)}{d}+\frac{(a-b)^2\tanh^5(c+dx)}{5d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}$$

```
[Out] (a^2*Tanh[c + d*x])/d - (2*a*(a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)^2*Ta
nh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0601492, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 194}

$$\frac{a^2\tanh(c+dx)}{d}+\frac{(a-b)^2\tanh^5(c+dx)}{5d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (a^2*Tanh[c + d*x])/d - (2*a*(a - b)*Tanh[c + d*x]^3)/(3*d) + ((a - b)^2*Ta
nh[c + d*x]^5)/(5*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \text{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(a-(a-b)x^2\right)^2\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int\left(a^2-2a(a-b)x^2+(a-b)^2x^4\right)\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{a^2\tanh(c+dx)}{d}-\frac{2a(a-b)\tanh^3(c+dx)}{3d}+\frac{(a-b)^2\tanh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.373003, size = 69, normalized size = 1.21

$$\frac{\tanh(c+dx)\left(2\left(2a^2+ab-3b^2\right)\text{sech}^2(c+dx)+8a^2+3(a-b)^2\text{sech}^4(c+dx)+4ab+3b^2\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + 3*b^2 + 2*(2*a^2 + a*b - 3*b^2)*Sech[c + d*x]^2 + 3*(a -
b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)
```

Maple [B] time = 0.048, size = 158, normalized size = 2.8

$$\frac{1}{d}\left(a^2\left(\frac{8}{15}+\frac{(\operatorname{sech}(dx+c))^4}{5}+\frac{4\,(\operatorname{sech}(dx+c))^2}{15}\right)\tanh(dx+c)+2\,ab\left(-1/4\,\frac{\sinh(dx+c)}{(\cosh(dx+c))^5}+1/4\left(\frac{8}{15}+1/5\,(\operatorname{sech}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(a^2*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+2*a*b*(-1/
4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)
*tanh(d*x+c))+b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/cosh(d*
x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.12718, size = 942, normalized size = 16.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 16/15*a^2*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
+ 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 10*
e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*
x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*
x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
 e^(-10*d*x - 10*c) + 1))) + 8/15*a*b*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(
-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*
c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c)
 + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/
(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-
8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 2/5*b^2*(10*e^(-4*d*x - 4*c)/(d*
(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d
*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)/(d*(5*e^(-2*d*x -
 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^
(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 1
0*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)))
```

Fricas [B] time = 1.45971, size = 1040, normalized size = 18.25

$$-\frac{4\left(\left(4\,a^2+2\,ab+9\,b^2\right)\cosh(dx+c)^4-8\left(2\,a^2+ab-3\,b^2\right)\cosh(dx+c)\sinh(dx+c)^3+\left(4\,a^2+\right.\right.}{15\left(d\cosh(dx+c)^6+6\,d\cosh(dx+c)\sinh(dx+c)^5+d\sinh(dx+c)^6+6\,d\cosh(dx+c)^4+3\left(5\,d\cosh(dx+c)^2\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -4/15*((4*a^2 + 2*a*b + 9*b^2)*cosh(d*x + c)^4 - 8*(2*a^2 + a*b - 3*b^2)*co
sh(d*x + c)*sinh(d*x + c)^3 + (4*a^2 + 2*a*b + 9*b^2)*sinh(d*x + c)^4 + 20*
(a^2 + 2*a*b)*cosh(d*x + c)^2 + 2*(3*(4*a^2 + 2*a*b + 9*b^2)*cosh(d*x + c)^
2 + 10*a^2 + 20*a*b)*sinh(d*x + c)^2 + 40*a^2 - 10*a*b + 15*b^2 - 8*((2*a^2
 + a*b - 3*b^2)*cosh(d*x + c)^3 + 5*(a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c
))/(d*cosh(d*x + c)^6 + 6*d*cosh(d*x + c)*sinh(d*x + c)^5 + d*sinh(d*x + c)
^6 + 6*d*cosh(d*x + c)^4 + 3*(5*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^4 +
4*(5*d*cosh(d*x + c)^3 + 4*d*cosh(d*x + c))*sinh(d*x + c)^3 + 15*d*cosh(d*x
 + c)^2 + 3*(5*d*cosh(d*x + c)^4 + 12*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c
)^2 + 2*(3*d*cosh(d*x + c)^5 + 8*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sin
h(d*x + c) + 10*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.22235, size = 173, normalized size = 3.04

$$-\frac{2\left(15\,b^2e^{(8\,dx+8\,c)} + 60\,abe^{(6\,dx+6\,c)} + 80\,a^2e^{(4\,dx+4\,c)} - 20\,abe^{(4\,dx+4\,c)} + 30\,b^2e^{(4\,dx+4\,c)} + 40\,a^2e^{(2\,dx+2\,c)} + 20\,abe^{(2\,dx+2\,c)} + 8\,\ldots}{15\,d\left(e^{(2\,dx+2\,c)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -2/15*(15*b^2*e^(8*d*x + 8*c) + 60*a*b*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x +
4*c) - 20*a*b*e^(4*d*x + 4*c) + 30*b^2*e^(4*d*x + 4*c) + 40*a^2*e^(2*d*x +
2*c) + 20*a*b*e^(2*d*x + 2*c) + 8*a^2 + 4*a*b + 3*b^2)/(d*(e^(2*d*x + 2*c)
+ 1)^5)
```

3.303 $\int \mathrm{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx$

Optimal. Leaf size=131

$$\frac{\left(5a^2+2ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{\left(5a^2+2ab+b^2\right)\tanh(c+dx)\mathrm{sech}(c+dx)}{16d}+\frac{(a-b)(5a+3b)\tanh(c+dx)\mathrm{s}}{24d}$$

```
[Out] ((5*a^2 + 2*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((5*a^2 + 2*a*b + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Sech[c + d*x]
^3*Tanh[c + d*x])/(24*d) + ((a - b)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)
*Tanh[c + d*x])/(6*d)
```

Rubi [A] time = 0.132593, antiderivative size = 131, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 413, 385, 199, 203}

$$\frac{\left(5a^2+2ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{\left(5a^2+2ab+b^2\right)\tanh(c+dx)\mathrm{sech}(c+dx)}{16d}+\frac{(a-b)(5a+3b)\tanh(c+dx)\mathrm{s}}{24d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((5*a^2 + 2*a*b + b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((5*a^2 + 2*a*b + b^
2)*Sech[c + d*x]*Tanh[c + d*x])/(16*d) + ((a - b)*(5*a + 3*b)*Sech[c + d*x]
^3*Tanh[c + d*x])/(24*d) + ((a - b)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)
*Tanh[c + d*x])/(6*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
```

```
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{a(5a+b}{(1}\right.}{}$$

$$= \frac{(a-b)(5a+3b)\operatorname{sech}^3(c+dx)\tanh(c+dx)}{24d} + \frac{(a-b)\operatorname{sech}^5(c+dx)\left(a+b\,\mathrm{s}\right.}{6}$$

$$= \frac{\left(5a^2+2ab+b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{16d} + \frac{(a-b)(5a+3b)\operatorname{sech}^3(c+d}{24d}$$

$$= \frac{\left(5a^2+2ab+b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{\left(5a^2+2ab+b^2\right)\operatorname{sech}(c+dx)\tan}{16d}$$

Mathematica [C] time = 10.1684, size = 715, normalized size = 5.46

$$\operatorname{csch}^3(c+dx)\left(32\left(-\sinh^2(c+dx)\right)^{3/2}\sinh^4(c+dx)\left(5a^2+9ab\sinh^2(c+dx)+4b^2\sinh^4(c+dx)\right)\text{HypergeometricPF}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (Csch[c + d*x]^3*(65625*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] + 36855*a^2*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 + 91875*a*b*ArcTanh[Sqrt[-Sinh
[c + d*x]^2]]*Sinh[c + d*x]^2 + 1680*a^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Si
nh[c + d*x]^4 + 54180*a*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 +
 32970*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^4 + 1365*a*b*ArcTa
nh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 + 19845*b^2*ArcTanh[Sqrt[-Sinh[c
 + d*x]^2]]*Sinh[c + d*x]^6 + 525*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[
c + d*x]^8 - 65625*a^2*Sqrt[-Sinh[c + d*x]^2] + 14980*a^2*(-Sinh[c + d*x]^2
)^(3/2) + 91875*a*b*(-Sinh[c + d*x]^2)^(3/2) + 8855*b^2*Sinh[c + d*x]^4*(-S
inh[c + d*x]^2)^(3/2) + 16*a^2*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1,
1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2) +
 32*a*b*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c
+ d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 16*b^2*HypergeometricP
FQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]
^8*(-Sinh[c + d*x]^2)^(3/2) - 23555*a*b*(-Sinh[c + d*x]^2)^(5/2) - 32970*b^
2*(-Sinh[c + d*x]^2)^(5/2) + 32*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1,
 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]^2)^(3/2)*(5*a^2
 + 9*a*b*Sinh[c + d*x]^2 + 4*b^2*Sinh[c + d*x]^4) + 4*HypergeometricPFQ[{3/
```

```
2, 2, 2, 2}, {1, 1, 9/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^4*(-Sinh[c + d*x]
^2)^(3/2)*(155*a^2 + 242*a*b*Sinh[c + d*x]^2 + 95*b^2*Sinh[c + d*x]^4)))/(2
520*d*Sqrt[-Sinh[c + d*x]^2])
```

Maple [B] time = 0.082, size = 302, normalized size = 2.3

$$\frac{a^2\tanh(dx+c)(\operatorname{sech}(dx+c))^5}{6d}+\frac{5a^2\tanh(dx+c)(\operatorname{sech}(dx+c))^3}{24d}+\frac{5a^2\operatorname{sech}(dx+c)\tanh(dx+c)}{16d}+\frac{5a^2\operatorname{arcta}}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/6/d*a^2*tanh(d*x+c)*sech(d*x+c)^5+5/24/d*a^2*tanh(d*x+c)*sech(d*x+c)^3+5/
16/d*a^2*sech(d*x+c)*tanh(d*x+c)+5/8/d*a^2*arctan(exp(d*x+c))-2/5/d*a*b*sin
h(d*x+c)/cosh(d*x+c)^6+1/15/d*a*b*tanh(d*x+c)*sech(d*x+c)^5+1/12/d*a*b*tanh
(d*x+c)*sech(d*x+c)^3+1/8/d*a*b*sech(d*x+c)*tanh(d*x+c)+1/4/d*a*b*arctan(ex
p(d*x+c))-1/3/d*b^2*sinh(d*x+c)^3/cosh(d*x+c)^6-1/5/d*b^2*sinh(d*x+c)/cosh(
d*x+c)^6+1/30/d*b^2*tanh(d*x+c)*sech(d*x+c)^5+1/24/d*b^2*tanh(d*x+c)*sech(d
*x+c)^3+1/16/d*b^2*sech(d*x+c)*tanh(d*x+c)+1/8/d*b^2*arctan(exp(d*x+c))
```

Maxima [B] time = 1.79108, size = 652, normalized size = 4.98

$$-\frac{1}{24}a^2\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{15e^{(-dx-c)}+85e^{(-3dx-3c)}+198e^{(-5dx-5c)}-198e^{(-7dx-7c)}-85e^{(-9dx-9c)}-15e^{(-11dx}}{d\left(6e^{(-2dx-2c)}+15e^{(-4dx-4c)}+20e^{(-6dx-6c)}+15e^{(-8dx-8c)}+6e^{(-10dx-10c)}+e^{(-12dx-12}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/24*a^2*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*d*x - 3*
c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9*c) - 15
*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-
6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12
*c) + 1))) - 1/12*a*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 17*e^(-
3*d*x - 3*c) - 114*e^(-5*d*x - 5*c) + 114*e^(-7*d*x - 7*c) - 17*e^(-9*d*x -
 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c)
+ 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12
*d*x - 12*c) + 1))) - 1/24*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c)
- 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c) + 47*e^(-
9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x
- 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) +
 e^(-12*d*x - 12*c) + 1)))
```

Fricas [B] time = 1.71361, size = 7356, normalized size = 56.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 33*(5*a^2 + 2*a*b + b^2)*c
osh(d*x + c)*sinh(d*x + c)^10 + 3*(5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^11 +
(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^9 + (165*(5*a^2 + 2*a*b + b^2)*cos
h(d*x + c)^2 + 85*a^2 + 34*a*b - 47*b^2)*sinh(d*x + c)^9 + 9*(55*(5*a^2 + 2
*a*b + b^2)*cosh(d*x + c)^3 + (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c))*sin
h(d*x + c)^8 + 6*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^7 + 6*(165*(5*a^2
 + 2*a*b + b^2)*cosh(d*x + c)^4 + 6*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c
)^2 + 33*a^2 - 38*a*b + 13*b^2)*sinh(d*x + c)^7 + 42*(33*(5*a^2 + 2*a*b + b
^2)*cosh(d*x + c)^5 + 2*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^3 + (33*a^
2 - 38*a*b + 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^6 - 6*(33*a^2 - 38*a*b +
13*b^2)*cosh(d*x + c)^5 + 6*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 21
*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^4 + 21*(33*a^2 - 38*a*b + 13*b^2)
*cosh(d*x + c)^2 - 33*a^2 + 38*a*b - 13*b^2)*sinh(d*x + c)^5 + 6*(165*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 21*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x +
 c)^5 + 35*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - 5*(33*a^2 - 38*a*b
+ 13*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (85*a^2 + 34*a*b - 47*b^2)*cosh(
d*x + c)^3 + (495*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(85*a^2 + 34*a
*b - 47*b^2)*cosh(d*x + c)^6 + 210*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)
^4 - 60*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^2 - 85*a^2 - 34*a*b + 47*b
^2)*sinh(d*x + c)^3 + 3*(55*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^9 + 12*(85*
a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^7 + 42*(33*a^2 - 38*a*b + 13*b^2)*cosh
(d*x + c)^5 - 20*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^3 - (85*a^2 + 34*
a*b - 47*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((5*a^2 + 2*a*b + b^2)*cos
h(d*x + c)^12 + 12*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^11 + (
5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^12 + 6*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^10 + 6*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*
sinh(d*x + c)^10 + 20*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(5*a^2
+ 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^2 + 2*a*b + b^2)*co
sh(d*x + c)^8 + 15*(33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 18*(5*a^2 +
2*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^8 + 24*(3
3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 30*(5*a^2 + 2*a*b + b^2)*cosh(d*x
 + c)^3 + 5*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^6 + 4*(231*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^6 + 315*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 105*(5*a^2 + 2*a*b + b^2
)*cosh(d*x + c)^2 + 25*a^2 + 10*a*b + 5*b^2)*sinh(d*x + c)^6 + 24*(33*(5*a^
2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 63*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5
 + 35*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^3 + 5*(5*a^2 + 2*a*b + b^2)*cosh(
d*x + c))*sinh(d*x + c)^5 + 15*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(
33*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 84*(5*a^2 + 2*a*b + b^2)*cosh(d*
x + c)^6 + 70*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 20*(5*a^2 + 2*a*b + b
^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^4 + 20*(11*(5*a^2
+ 2*a*b + b^2)*cosh(d*x + c)^9 + 36*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 +
 42*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 20*(5*a^2 + 2*a*b + b^2)*cosh(d
*x + c)^3 + 3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 6*(5*a
^2 + 2*a*b + b^2)*cosh(d*x + c)^2 + 6*(11*(5*a^2 + 2*a*b + b^2)*cosh(d*x +
c)^10 + 45*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^8 + 70*(5*a^2 + 2*a*b + b^2)
*cosh(d*x + c)^6 + 50*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^4 + 15*(5*a^2 + 2
*a*b + b^2)*cosh(d*x + c)^2 + 5*a^2 + 2*a*b + b^2)*sinh(d*x + c)^2 + 5*a^2
+ 2*a*b + b^2 + 12*((5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^11 + 5*(5*a^2 + 2*a
*b + b^2)*cosh(d*x + c)^9 + 10*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^7 + 10*(
5*a^2 + 2*a*b + b^2)*cosh(d*x + c)^5 + 5*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c
)^3 + (5*a^2 + 2*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x +
 c) + sinh(d*x + c)) - 3*(5*a^2 + 2*a*b + b^2)*cosh(d*x + c) + 3*(11*(5*a^2
 + 2*a*b + b^2)*cosh(d*x + c)^10 + 3*(85*a^2 + 34*a*b - 47*b^2)*cosh(d*x +
c)^8 + 14*(33*a^2 - 38*a*b + 13*b^2)*cosh(d*x + c)^6 - 10*(33*a^2 - 38*a*b
+ 13*b^2)*cosh(d*x + c)^4 - (85*a^2 + 34*a*b - 47*b^2)*cosh(d*x + c)^2 - 5*
a^2 - 2*a*b - b^2)*sinh(d*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*
sinh(d*x + c)^11 + d*sinh(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh
(d*x + c)^2 + d)*sinh(d*x + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x
```

```
 + c))*sinh(d*x + c)^9 + 15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 +
18*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d
*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^
6 + 4*(231*d*cosh(d*x + c)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^
2 + 5*d)*sinh(d*x + c)^6 + 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5
+ 35*d*cosh(d*x + c)^3 + 5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x
 + c)^4 + 15*(33*d*cosh(d*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x +
 c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^
9 + 36*d*cosh(d*x + c)^7 + 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*
d*cosh(d*x + c))*sinh(d*x + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x +
 c)^10 + 45*d*cosh(d*x + c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4
 + 15*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d
*cosh(d*x + c)^9 + 10*d*cosh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d
*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.1794, size = 394, normalized size = 3.01

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(5\,a^2 + 2\,ab + b^2\right)}{32\,d} + \frac{15\,a^2\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 6\,ab\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 3\,b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/32*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^2 + 2*a*b
 + b^2)/d + 1/24*(15*a^2*(e^(d*x + c) - e^(-d*x - c))^5 + 6*a*b*(e^(d*x + c
) - e^(-d*x - c))^5 + 3*b^2*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^2*(e^(d*
x + c) - e^(-d*x - c))^3 + 64*a*b*(e^(d*x + c) - e^(-d*x - c))^3 - 32*b^2*(
e^(d*x + c) - e^(-d*x - c))^3 + 528*a^2*(e^(d*x + c) - e^(-d*x - c)) - 96*a
*b*(e^(d*x + c) - e^(-d*x - c)) - 48*b^2*(e^(d*x + c) - e^(-d*x - c)))/(((e
^(d*x + c) - e^(-d*x - c))^2 + 4)^3*d)
```

3.304 $\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=238

$$\frac{b\left(44a^2-28ab+5b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{160d}+\frac{(4a-b)\left(8a^2-2ab+b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{128d}+\frac{3(4a-b)\left(8a\right.}{}$$

```
[Out] (3*(4*a - b)*(8*a^2 - 2*a*b + b^2)*x)/256 + (3*(4*a - b)*(8*a^2 - 2*a*b + b
^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + ((4*a - b)*(8*a^2 - 2*a*b + b^2)
*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b*(44*a^2 - 28*a*b + 5*b^2)*Cosh
[c + d*x]^5*Sinh[c + d*x])/(160*d) + (b*Cosh[c + d*x]^9*Sinh[c + d*x]*(a -
(a - b)*Tanh[c + d*x]^2)^2)/(10*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a*(1
0*a - b) - 5*(a - b)*(2*a - b)*Tanh[c + d*x]^2))/(80*d)
```

Rubi [A] time = 0.331757, antiderivative size = 238, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3191, 413, 526, 385, 199, 206}

$$\frac{b\left(44a^2-28ab+5b^2\right)\sinh(c+dx)\cosh^5(c+dx)}{160d}+\frac{(4a-b)\left(8a^2-2ab+b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{128d}+\frac{3(4a-b)\left(8a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(4*a - b)*(8*a^2 - 2*a*b + b^2)*x)/256 + (3*(4*a - b)*(8*a^2 - 2*a*b + b
^2)*Cosh[c + d*x]*Sinh[c + d*x])/(256*d) + ((4*a - b)*(8*a^2 - 2*a*b + b^2)
*Cosh[c + d*x]^3*Sinh[c + d*x])/(128*d) + (b*(44*a^2 - 28*a*b + 5*b^2)*Cosh
[c + d*x]^5*Sinh[c + d*x])/(160*d) + (b*Cosh[c + d*x]^9*Sinh[c + d*x]*(a -
(a - b)*Tanh[c + d*x]^2)^2)/(10*d) + (b*Cosh[c + d*x]^7*Sinh[c + d*x]*(a*(1
0*a - b) - 5*(a - b)*(2*a - b)*Tanh[c + d*x]^2))/(80*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
```

```
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^6}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^9(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10d} - \frac{\text{Subst}\left(\int \frac{(\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{b\cosh^9(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{10d} + \frac{b\cosh^7(c+\cdots}{\cdots}$$

$$= \frac{b\left(44a^2-28ab+5b^2\right)\cosh^5(c+dx)\sinh(c+dx)}{160d} + \frac{b\cosh^9(c+dx)\sin\cdots}{\cdots}$$

$$= \frac{(4a-b)\left(8a^2-2ab+b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{128d} + \frac{b\left(44a^2-28ab+\cdots\right.}{\cdots}$$

$$= \frac{3(4a-b)\left(8a^2-2ab+b^2\right)\cosh(c+dx)\sinh(c+dx)}{256d} + \frac{(4a-b)\left(8a^2-\cdots\right.}{\cdots}$$

$$= \frac{3}{256}(4a-b)\left(8a^2-2ab+b^2\right)x + \frac{3(4a-b)\left(8a^2-2ab+b^2\right)\cosh(c+d\cdots}{256d}$$

Mathematica [A] time = 0.520767, size = 144, normalized size = 0.61

$$\frac{120(4a-b)\left(8a^2-2ab+b^2\right)(c+dx)-10b\left(b^2-16a^2\right)\sinh(6(c+dx))+20\left(-24a^2b+128a^3+b^3\right)\sinh(2(c+dx))+\cdots}{10240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (120*(4*a - b)*(8*a^2 - 2*a*b + b^2)*(c + d*x) + 20*(128*a^3 - 24*a^2*b + b
^3)*Sinh[2*(c + d*x)] + 40*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*Sinh[4*(c + d
*x)] - 10*b*(-16*a^2 + b^2)*Sinh[6*(c + d*x)] + 5*(6*a - b)*b^2*Sinh[8*(c +
 d*x)] + 2*b^3*Sinh[10*(c + d*x)])/(10240*d)
```

Maple [A] time = 0.032, size = 267, normalized size = 1.1

$$\frac{1}{d}\left(b^3\left(\frac{(\sinh(dx+c))^5(\cosh(dx+c))^5}{10}-\frac{(\sinh(dx+c))^3(\cosh(dx+c))^5}{16}+\frac{\sinh(dx+c)(\cosh(dx+c))^5}{32}-\frac{\sinh(dx+}{32}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(1/10*sinh(d*x+c)^5*cosh(d*x+c)^5-1/16*sinh(d*x+c)^3*cosh(d*x+c)^5
+1/32*sinh(d*x+c)*cosh(d*x+c)^5-1/32*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*si
nh(d*x+c)-3/256*d*x-3/256*c)+3*a*b^2*(1/8*sinh(d*x+c)^3*cosh(d*x+c)^5-1/16*
sinh(d*x+c)*cosh(d*x+c)^5+1/16*(1/4*cosh(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x
+c)+3/128*d*x+3/128*c)+3*a^2*b*(1/6*sinh(d*x+c)*cosh(d*x+c)^5-1/6*(1/4*cosh
(d*x+c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)-1/16*d*x-1/16*c)+a^3*((1/4*cosh(d*x+
c)^3+3/8*cosh(d*x+c))*sinh(d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A] time = 1.10372, size = 490, normalized size = 2.06

$$\frac{1}{64}a^3\left(24x+\frac{e^{(4dx+4c)}}{d}+\frac{8e^{(2dx+2c)}}{d}-\frac{8e^{(-2dx-2c)}}{d}-\frac{e^{(-4dx-4c)}}{d}\right)-\frac{1}{20480}b^3\left(\frac{\left(5e^{(-2dx-2c)}+10e^{(-4dx-4c)}-40e^{(-6dx-6c)}-\right.}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/64*a^3*(24*x + e^(4*d*x + 4*c)/d + 8*e^(2*d*x + 2*c)/d - 8*e^(-2*d*x - 2*
c)/d - e^(-4*d*x - 4*c)/d) - 1/20480*b^3*((5*e^(-2*d*x - 2*c) + 10*e^(-4*d*
x - 4*c) - 40*e^(-6*d*x - 6*c) - 20*e^(-8*d*x - 8*c) - 2)*e^(10*d*x + 10*c)
/d + 240*(d*x + c)/d + (20*e^(-2*d*x - 2*c) + 40*e^(-4*d*x - 4*c) - 10*e^(-
6*d*x - 6*c) - 5*e^(-8*d*x - 8*c) + 2*e^(-10*d*x - 10*c))/d) - 3/2048*a*b^2
*((8*e^(-4*d*x - 4*c) - 1)*e^(8*d*x + 8*c)/d - 48*(d*x + c)/d - (8*e^(-4*d*
x - 4*c) - e^(-8*d*x - 8*c))/d) + 1/128*a^2*b*((3*e^(-2*d*x - 2*c) - 3*e^(-
4*d*x - 4*c) + 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d + (3*e^(-2*d*x - 2*c)
- 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d)
```

Fricas [A] time = 1.49742, size = 906, normalized size = 3.81

$$\frac{5b^3\cosh(dx+c)\sinh(dx+c)^9+10\left(6b^3\cosh(dx+c)^3+\left(6ab^2-b^3\right)\cosh(dx+c)\right)\sinh(dx+c)^7+\left(126b^3\cosh(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2560*(5*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 10*(6*b^3*cosh(d*x + c)^3 + (
6*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + (126*b^3*cosh(d*x + c)^5 +
```

```
70*(6*a*b^2 - b^3)*cosh(d*x + c)^3 + 15*(16*a^2*b - b^3)*cosh(d*x + c))*sin
h(d*x + c)^5 + 10*(6*b^3*cosh(d*x + c)^7 + 7*(6*a*b^2 - b^3)*cosh(d*x + c)^
5 + 5*(16*a^2*b - b^3)*cosh(d*x + c)^3 + 4*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^
3)*cosh(d*x + c))*sinh(d*x + c)^3 + 30*(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*
d*x + 5*(b^3*cosh(d*x + c)^9 + 2*(6*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(16*a^
2*b - b^3)*cosh(d*x + c)^5 + 8*(8*a^3 + 12*a^2*b - 6*a*b^2 + b^3)*cosh(d*x
+ c)^3 + 2*(128*a^3 - 24*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c))/d
```

Sympy [A] time = 34.7673, size = 774, normalized size = 3.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((3*a**3*x*sinh(c + d*x)**4/8 - 3*a**3*x*sinh(c + d*x)**2*cosh(c +
 d*x)**2/4 + 3*a**3*x*cosh(c + d*x)**4/8 - 3*a**3*sinh(c + d*x)**3*cosh(c +
 d*x)/(8*d) + 5*a**3*sinh(c + d*x)*cosh(c + d*x)**3/(8*d) + 3*a**2*b*x*sinh
(c + d*x)**6/16 - 9*a**2*b*x*sinh(c + d*x)**4*cosh(c + d*x)**2/16 + 9*a**2*
b*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 - 3*a**2*b*x*cosh(c + d*x)**6/16 -
 3*a**2*b*sinh(c + d*x)**5*cosh(c + d*x)/(16*d) + a**2*b*sinh(c + d*x)**3*c
osh(c + d*x)**3/(2*d) + 3*a**2*b*sinh(c + d*x)*cosh(c + d*x)**5/(16*d) + 9*
a*b**2*x*sinh(c + d*x)**8/128 - 9*a*b**2*x*sinh(c + d*x)**6*cosh(c + d*x)**
2/32 + 27*a*b**2*x*sinh(c + d*x)**4*cosh(c + d*x)**4/64 - 9*a*b**2*x*sinh(c
 + d*x)**2*cosh(c + d*x)**6/32 + 9*a*b**2*x*cosh(c + d*x)**8/128 - 9*a*b**2
*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 33*a*b**2*sinh(c + d*x)**5*cosh(c
 + d*x)**3/(128*d) + 33*a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**5/(128*d) -
9*a*b**2*sinh(c + d*x)*cosh(c + d*x)**7/(128*d) + 3*b**3*x*sinh(c + d*x)**1
0/256 - 15*b**3*x*sinh(c + d*x)**8*cosh(c + d*x)**2/256 + 15*b**3*x*sinh(c
+ d*x)**6*cosh(c + d*x)**4/128 - 15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x)**
6/128 + 15*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**8/256 - 3*b**3*x*cosh(c +
 d*x)**10/256 - 3*b**3*sinh(c + d*x)**9*cosh(c + d*x)/(256*d) + 7*b**3*sinh
(c + d*x)**7*cosh(c + d*x)**3/(128*d) + b**3*sinh(c + d*x)**5*cosh(c + d*x)
**5/(10*d) - 7*b**3*sinh(c + d*x)**3*cosh(c + d*x)**7/(128*d) + 3*b**3*sinh
(c + d*x)*cosh(c + d*x)**9/(256*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cos
h(c)**4, True))
```

Giac [A] time = 1.26231, size = 598, normalized size = 2.51

$$\frac{2\,b^3e^{(10\,dx+10\,c)} + 30\,ab^2e^{(8\,dx+8\,c)} - 5\,b^3e^{(8\,dx+8\,c)} + 160\,a^2be^{(6\,dx+6\,c)} - 10\,b^3e^{(6\,dx+6\,c)} + 320\,a^3e^{(4\,dx+4\,c)} + 480\,a^2be^{(4\,dx+4\,c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/20480*(2*b^3*e^(10*d*x + 10*c) + 30*a*b^2*e^(8*d*x + 8*c) - 5*b^3*e^(8*d*
x + 8*c) + 160*a^2*b*e^(6*d*x + 6*c) - 10*b^3*e^(6*d*x + 6*c) + 320*a^3*e^(
4*d*x + 4*c) + 480*a^2*b*e^(4*d*x + 4*c) - 240*a*b^2*e^(4*d*x + 4*c) + 40*b
^3*e^(4*d*x + 4*c) + 2560*a^3*e^(2*d*x + 2*c) - 480*a^2*b*e^(2*d*x + 2*c) +
 20*b^3*e^(2*d*x + 2*c) + 240*(32*a^3 - 16*a^2*b + 6*a*b^2 - b^3)*(d*x + c)
 - (8768*a^3*e^(10*d*x + 10*c) - 4384*a^2*b*e^(10*d*x + 10*c) + 1644*a*b^2*
e^(10*d*x + 10*c) - 274*b^3*e^(10*d*x + 10*c) + 2560*a^3*e^(8*d*x + 8*c) -
```

```
480*a^2*b*e^(8*d*x + 8*c) + 20*b^3*e^(8*d*x + 8*c) + 320*a^3*e^(6*d*x + 6*c
) + 480*a^2*b*e^(6*d*x + 6*c) - 240*a*b^2*e^(6*d*x + 6*c) + 40*b^3*e^(6*d*x
 + 6*c) + 160*a^2*b*e^(4*d*x + 4*c) - 10*b^3*e^(4*d*x + 4*c) + 30*a*b^2*e^(
2*d*x + 2*c) - 5*b^3*e^(2*d*x + 2*c) + 2*b^3)*e^(-10*d*x - 10*c))/d
```

3.305 $\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=98

$$\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{a^3\sinh(c+dx)}{d}+\frac{b^2(3a+b)\sinh^7(c+dx)}{7d}+\frac{3ab(a+b)\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^9(c+dx)}{9d}$$

```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*(a + 3*b)*Sinh[c + d*x]^3)/(3*d) + (3*a*b*(a +
 b)*Sinh[c + d*x]^5)/(5*d) + (b^2*(3*a + b)*Sinh[c + d*x]^7)/(7*d) + (b^3*S
inh[c + d*x]^9)/(9*d)
```

Rubi [A] time = 0.0902005, antiderivative size = 98, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3190, 373}

$$\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{a^3\sinh(c+dx)}{d}+\frac{b^2(3a+b)\sinh^7(c+dx)}{7d}+\frac{3ab(a+b)\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^9(c+dx)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*(a + 3*b)*Sinh[c + d*x]^3)/(3*d) + (3*a*b*(a +
 b)*Sinh[c + d*x]^5)/(5*d) + (b^2*(3*a + b)*Sinh[c + d*x]^7)/(7*d) + (b^3*S
inh[c + d*x]^9)/(9*d)
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^3\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int\left(a^3+a^2(a+3b)x^2+3ab(a+b)x^4+b^2(3a+b)x^6+b^3x^8\right)dx,\right.}{d}\\ &= \frac{a^3\sinh(c+dx)}{d}+\frac{a^2(a+3b)\sinh^3(c+dx)}{3d}+\frac{3ab(a+b)\sinh^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.233608, size = 83, normalized size = 0.85

$$\frac{105a^2(a+3b)\sinh^3(c+dx)+315a^3\sinh(c+dx)+45b^2(3a+b)\sinh^7(c+dx)+189ab(a+b)\sinh^5(c+dx)+35b^3\text{s}}{315d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (315*a^3*Sinh[c + d*x] + 105*a^2*(a + 3*b)*Sinh[c + d*x]^3 + 189*a*b*(a + b
)*Sinh[c + d*x]^5 + 45*b^2*(3*a + b)*Sinh[c + d*x]^7 + 35*b^3*Sinh[c + d*x]
^9)/(315*d)
```

Maple [B] time = 0.035, size = 209, normalized size = 2.1

$$\frac{1}{d}\left(b^3\left(\frac{(\sinh(dx+c))^5(\cosh(dx+c))^4}{9}-\frac{5(\sinh(dx+c))^3(\cosh(dx+c))^4}{63}+\frac{\sinh(dx+c)(\cosh(dx+c))^4}{21}-\frac{\sinh(dx}{21}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(1/9*sinh(d*x+c)^5*cosh(d*x+c)^4-5/63*sinh(d*x+c)^3*cosh(d*x+c)^4+
1/21*sinh(d*x+c)*cosh(d*x+c)^4-1/21*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+3*
a*b^2*(1/7*sinh(d*x+c)^3*cosh(d*x+c)^4-3/35*sinh(d*x+c)*cosh(d*x+c)^4+3/35*
(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+3*a^2*b*(1/5*sinh(d*x+c)*cosh(d*x+c)^4
-1/5*(2/3+1/3*cosh(d*x+c)^2)*sinh(d*x+c))+a^3*(2/3+1/3*cosh(d*x+c)^2)*sinh(
d*x+c))
```

Maxima [B] time = 1.09882, size = 471, normalized size = 4.81

$$-\frac{1}{32256}b^3\left(\frac{\left(27e^{(-2dx-2c)}-168e^{(-6dx-6c)}+378e^{(-8dx-8c)}-7\right)e^{(9dx+9c)}}{d}-\frac{378e^{(-dx-c)}-168e^{(-3dx-3c)}+27e^{(-7dx-7c)}-7}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/32256*b^3*((27*e^(-2*d*x - 2*c) - 168*e^(-6*d*x - 6*c) + 378*e^(-8*d*x -
 8*c) - 7)*e^(9*d*x + 9*c)/d - (378*e^(-d*x - c) - 168*e^(-3*d*x - 3*c) + 2
7*e^(-7*d*x - 7*c) - 7*e^(-9*d*x - 9*c))/d) - 3/4480*a*b^2*((7*e^(-2*d*x -
2*c) + 35*e^(-4*d*x - 4*c) - 105*e^(-6*d*x - 6*c) - 5)*e^(7*d*x + 7*c)/d +
(105*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) - 7*e^(-5*d*x - 5*c) + 5*e^(-7*d*x
- 7*c))/d) + 1/160*a^2*b*((5*e^(-2*d*x - 2*c) - 30*e^(-4*d*x - 4*c) + 3)*e^
(5*d*x + 5*c)/d + (30*e^(-d*x - c) - 5*e^(-3*d*x - 3*c) - 3*e^(-5*d*x - 5*c
))/d) + 1/24*a^3*(e^(3*d*x + 3*c)/d + 9*e^(d*x + c)/d - 9*e^(-d*x - c)/d -
e^(-3*d*x - 3*c)/d)
```

Fricas [B] time = 1.52027, size = 787, normalized size = 8.03

$$\frac{35b^3\sinh(dx+c)^9+45\left(28b^3\cosh(dx+c)^2+12ab^2-3b^3\right)\sinh(dx+c)^7+63\left(70b^3\cosh(dx+c)^4+48a^2b-12ab^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/80640*(35*b^3*sinh(d*x + c)^9 + 45*(28*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 3
*b^3)*sinh(d*x + c)^7 + 63*(70*b^3*cosh(d*x + c)^4 + 48*a^2*b - 12*a*b^2 +
45*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 105*(28*b^3*cosh(d*x
+ c)^6 + 45*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 64*a^3 + 48*a^2*b - 36*a*b^2
+ 8*b^3 + 72*(4*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 315*(b^3*
cosh(d*x + c)^8 + 3*(4*a*b^2 - b^3)*cosh(d*x + c)^6 + 12*(4*a^2*b - a*b^2)*
cosh(d*x + c)^4 + 192*a^3 - 96*a^2*b + 36*a*b^2 - 6*b^3 + 4*(16*a^3 + 12*a^
2*b - 9*a*b^2 + 2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/d
```

Sympy [A] time = 19.4941, size = 182, normalized size = 1.86

$$\begin{cases} -\frac{2a^3\sinh^3(c+dx)}{3d} + \frac{a^3\sinh(c+dx)\cosh^2(c+dx)}{d} - \frac{2a^2b\sinh^5(c+dx)}{5d} + \frac{a^2b\sinh^3(c+dx)\cosh^2(c+dx)}{d} - \frac{6ab^2\sinh^7(c+dx)}{35d} + \frac{3ab^2\sinh^5(c+dx)\cosh \\ x\left(a+b\sinh^2(c)\right)^3\cosh^3(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((-2*a**3*sinh(c + d*x)**3/(3*d) + a**3*sinh(c + d*x)*cosh(c + d*x
)**2/d - 2*a**2*b*sinh(c + d*x)**5/(5*d) + a**2*b*sinh(c + d*x)**3*cosh(c +
 d*x)**2/d - 6*a*b**2*sinh(c + d*x)**7/(35*d) + 3*a*b**2*sinh(c + d*x)**5*c
osh(c + d*x)**2/(5*d) - 2*b**3*sinh(c + d*x)**9/(63*d) + b**3*sinh(c + d*x)
**7*cosh(c + d*x)**2/(7*d), Ne(d, 0)), (x*(a + b*sinh(c)**2)**3*cosh(c)**3,
 True))
```

Giac [B] time = 1.26016, size = 506, normalized size = 5.16

$$\frac{35\,b^3e^{(9\,dx+9\,c)} + 540\,ab^2e^{(7\,dx+7\,c)} - 135\,b^3e^{(7\,dx+7\,c)} + 3024\,a^2be^{(5\,dx+5\,c)} - 756\,ab^2e^{(5\,dx+5\,c)} + 6720\,a^3e^{(3\,dx+3\,c)} + 5040\,a^2}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/161280*(35*b^3*e^(9*d*x + 9*c) + 540*a*b^2*e^(7*d*x + 7*c) - 135*b^3*e^(7
*d*x + 7*c) + 3024*a^2*b*e^(5*d*x + 5*c) - 756*a*b^2*e^(5*d*x + 5*c) + 6720
*a^3*e^(3*d*x + 3*c) + 5040*a^2*b*e^(3*d*x + 3*c) - 3780*a*b^2*e^(3*d*x + 3
*c) + 840*b^3*e^(3*d*x + 3*c) + 60480*a^3*e^(d*x + c) - 30240*a^2*b*e^(d*x
+ c) + 11340*a*b^2*e^(d*x + c) - 1890*b^3*e^(d*x + c) - (60480*a^3*e^(8*d*x
 + 8*c) - 30240*a^2*b*e^(8*d*x + 8*c) + 11340*a*b^2*e^(8*d*x + 8*c) - 1890*
b^3*e^(8*d*x + 8*c) + 6720*a^3*e^(6*d*x + 6*c) + 5040*a^2*b*e^(6*d*x + 6*c)
 - 3780*a*b^2*e^(6*d*x + 6*c) + 840*b^3*e^(6*d*x + 6*c) + 3024*a^2*b*e^(4*d
*x + 4*c) - 756*a*b^2*e^(4*d*x + 4*c) + 540*a*b^2*e^(2*d*x + 2*c) - 135*b^3
*e^(2*d*x + 2*c) + 35*b^3)*e^(-9*d*x - 9*c))/d
```

3.306 $\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=203

$$\frac{b\left(88a^2-68ab+15b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(-48a^2b+64a^3+24ab^2-5b^3\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x$$

```
[Out] ((64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x)/128 + ((64*a^3 - 48*a^2*b + 24*a
*b^2 - 5*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (b*(88*a^2 - 68*a*b +
15*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + (b*Cosh[c + d*x]^7*Sinh[c
+ d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(8*d) + (b*Cosh[c + d*x]^5*Sinh[c +
 d*x]*(a*(8*a - b) - (8*a - 5*b)*(a - b)*Tanh[c + d*x]^2))/(48*d)
```

Rubi [A] time = 0.268032, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3191, 413, 526, 385, 199, 206}

$$\frac{b\left(88a^2-68ab+15b^2\right)\sinh(c+dx)\cosh^3(c+dx)}{192d}+\frac{\left(-48a^2b+64a^3+24ab^2-5b^3\right)\sinh(c+dx)\cosh(c+dx)}{128d}+\frac{1}{128}x$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*x)/128 + ((64*a^3 - 48*a^2*b + 24*a
*b^2 - 5*b^3)*Cosh[c + d*x]*Sinh[c + d*x])/(128*d) + (b*(88*a^2 - 68*a*b +
15*b^2)*Cosh[c + d*x]^3*Sinh[c + d*x])/(192*d) + (b*Cosh[c + d*x]^7*Sinh[c
+ d*x]*(a - (a - b)*Tanh[c + d*x]^2)^2)/(8*d) + (b*Cosh[c + d*x]^5*Sinh[c +
 d*x]*(a*(8*a - b) - (8*a - 5*b)*(a - b)*Tanh[c + d*x]^2))/(48*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^5}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8d} - \frac{\text{Subst}\left(\int \frac{(\cdots}{\cdots}\right.}{\cdots}$$

$$= \frac{b\cosh^7(c+dx)\sinh(c+dx)\left(a-(a-b)\tanh^2(c+dx)\right)^2}{8d} + \frac{b\cosh^5(c+\cdots}{\cdots}$$

$$= \frac{b\left(88a^2-68ab+15b^2\right)\cosh^3(c+dx)\sinh(c+dx)}{192d} + \frac{b\cosh^7(c+dx)\sinh\cdots}{\cdots}$$

$$= \frac{\left(64a^3-48a^2b+24ab^2-5b^3\right)\cosh(c+dx)\sinh(c+dx)}{128d} + \frac{b\left(88a^2-68\cdots\right.}{\cdots}$$

$$= \frac{1}{128}\left(64a^3-48a^2b+24ab^2-5b^3\right)x + \frac{\left(64a^3-48a^2b+24ab^2-5b^3\right)\cosh\cdots}{128d}$$

Mathematica [A] time = 0.302241, size = 120, normalized size = 0.59

$$\frac{24\left(-48a^2b+64a^3+24ab^2-5b^3\right)(c+dx)+24b\left(12a^2-6ab+b^2\right)\sinh(4(c+dx))+48\left(16a^3-3ab^2+b^3\right)\sinh(2(c\cdots}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (24*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*(c + d*x) + 48*(16*a^3 - 3*a*b^2
 + b^3)*Sinh[2*(c + d*x)] + 24*b*(12*a^2 - 6*a*b + b^2)*Sinh[4*(c + d*x)] +
 16*(3*a - b)*b^2*Sinh[6*(c + d*x)] + 3*b^3*Sinh[8*(c + d*x)])/(3072*d)
```

Maple [A] time = 0.032, size = 216, normalized size = 1.1

$$\frac{1}{d}\left(b^3\left(\frac{(\sinh(dx+c))^5(\cosh(dx+c))^3}{8}-\frac{5(\sinh(dx+c))^3(\cosh(dx+c))^3}{48}+\frac{5\sinh(dx+c)(\cosh(dx+c))^3}{64}-\frac{5\cosh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(b^3*(1/8*sinh(d*x+c)^5*cosh(d*x+c)^3-5/48*sinh(d*x+c)^3*cosh(d*x+c)^3+
5/64*sinh(d*x+c)*cosh(d*x+c)^3-5/128*cosh(d*x+c)*sinh(d*x+c)-5/128*d*x-5/12
8*c)+3*a*b^2*(1/6*sinh(d*x+c)^3*cosh(d*x+c)^3-1/8*sinh(d*x+c)*cosh(d*x+c)^3
+1/16*cosh(d*x+c)*sinh(d*x+c)+1/16*d*x+1/16*c)+3*a^2*b*(1/4*sinh(d*x+c)*cos
h(d*x+c)^3-1/8*cosh(d*x+c)*sinh(d*x+c)-1/8*d*x-1/8*c)+a^3*(1/2*cosh(d*x+c)*
sinh(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A] time = 1.11971, size = 387, normalized size = 1.91

$$\frac{1}{8}a^3\left(4x+\frac{e^{(2dx+2c)}}{d}-\frac{e^{(-2dx-2c)}}{d}\right)-\frac{1}{6144}b^3\left(\frac{\left(16e^{(-2dx-2c)}-24e^{(-4dx-4c)}-48e^{(-6dx-6c)}-3\right)e^{(8dx+8c)}}{d}+\frac{240(dx+c)}{d}+\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*a^3*(4*x + e^(2*d*x + 2*c)/d - e^(-2*d*x - 2*c)/d) - 1/6144*b^3*((16*e^
(-2*d*x - 2*c) - 24*e^(-4*d*x - 4*c) - 48*e^(-6*d*x - 6*c) - 3)*e^(8*d*x +
8*c)/d + 240*(d*x + c)/d + (48*e^(-2*d*x - 2*c) + 24*e^(-4*d*x - 4*c) - 16*
e^(-6*d*x - 6*c) + 3*e^(-8*d*x - 8*c))/d) - 1/128*a*b^2*((3*e^(-2*d*x - 2*c
) + 3*e^(-4*d*x - 4*c) - 1)*e^(6*d*x + 6*c)/d - 24*(d*x + c)/d - (3*e^(-2*d
*x - 2*c) + 3*e^(-4*d*x - 4*c) - e^(-6*d*x - 6*c))/d) - 3/64*a^2*b*(8*(d*x
+ c)/d - e^(4*d*x + 4*c)/d + e^(-4*d*x - 4*c)/d)
```

Fricas [A] time = 1.48108, size = 626, normalized size = 3.08

$$\frac{3b^3\cosh(dx+c)\sinh(dx+c)^7+3\left(7b^3\cosh(dx+c)^3+4\left(3ab^2-b^3\right)\cosh(dx+c)\right)\sinh(dx+c)^5+\left(21b^3\cosh(dx+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/384*(3*b^3*cosh(d*x + c)*sinh(d*x + c)^7 + 3*(7*b^3*cosh(d*x + c)^3 + 4*(
3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + (21*b^3*cosh(d*x + c)^5 + 4
0*(3*a*b^2 - b^3)*cosh(d*x + c)^3 + 12*(12*a^2*b - 6*a*b^2 + b^3)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 3*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*d*x + 3*(b
^3*cosh(d*x + c)^7 + 4*(3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(12*a^2*b - 6*a*
b^2 + b^3)*cosh(d*x + c)^3 + 4*(16*a^3 - 3*a*b^2 + b^3)*cosh(d*x + c))*sinh
(d*x + c))/d
```

Sympy [A] time = 13.7037, size = 559, normalized size = 2.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((-a**3*x*sinh(c + d*x)**2/2 + a**3*x*cosh(c + d*x)**2/2 + a**3*si
nh(c + d*x)*cosh(c + d*x)/(2*d) - 3*a**2*b*x*sinh(c + d*x)**4/8 + 3*a**2*b*
x*sinh(c + d*x)**2*cosh(c + d*x)**2/4 - 3*a**2*b*x*cosh(c + d*x)**4/8 + 3*a
**2*b*sinh(c + d*x)**3*cosh(c + d*x)/(8*d) + 3*a**2*b*sinh(c + d*x)*cosh(c
+ d*x)**3/(8*d) - 3*a*b**2*x*sinh(c + d*x)**6/16 + 9*a*b**2*x*sinh(c + d*x)
**4*cosh(c + d*x)**2/16 - 9*a*b**2*x*sinh(c + d*x)**2*cosh(c + d*x)**4/16 +
 3*a*b**2*x*cosh(c + d*x)**6/16 + 3*a*b**2*sinh(c + d*x)**5*cosh(c + d*x)/(
16*d) + a*b**2*sinh(c + d*x)**3*cosh(c + d*x)**3/(2*d) - 3*a*b**2*sinh(c +
d*x)*cosh(c + d*x)**5/(16*d) - 5*b**3*x*sinh(c + d*x)**8/128 + 5*b**3*x*sin
h(c + d*x)**6*cosh(c + d*x)**2/32 - 15*b**3*x*sinh(c + d*x)**4*cosh(c + d*x
)**4/64 + 5*b**3*x*sinh(c + d*x)**2*cosh(c + d*x)**6/32 - 5*b**3*x*cosh(c +
 d*x)**8/128 + 5*b**3*sinh(c + d*x)**7*cosh(c + d*x)/(128*d) + 73*b**3*sinh
(c + d*x)**5*cosh(c + d*x)**3/(384*d) - 55*b**3*sinh(c + d*x)**3*cosh(c + d
*x)**5/(384*d) + 5*b**3*sinh(c + d*x)*cosh(c + d*x)**7/(128*d), Ne(d, 0)),
(x*(a + b*sinh(c)**2)**3*cosh(c)**2, True))
```

Giac [A] time = 1.27872, size = 482, normalized size = 2.37

$$\frac{3\,b^3e^{(8\,dx+8\,c)} + 48\,ab^2e^{(6\,dx+6\,c)} - 16\,b^3e^{(6\,dx+6\,c)} + 288\,a^2be^{(4\,dx+4\,c)} - 144\,ab^2e^{(4\,dx+4\,c)} + 24\,b^3e^{(4\,dx+4\,c)} + 768\,a^3e^{(2\,dx+2\,c)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/6144*(3*b^3*e^(8*d*x + 8*c) + 48*a*b^2*e^(6*d*x + 6*c) - 16*b^3*e^(6*d*x
+ 6*c) + 288*a^2*b*e^(4*d*x + 4*c) - 144*a*b^2*e^(4*d*x + 4*c) + 24*b^3*e^(
4*d*x + 4*c) + 768*a^3*e^(2*d*x + 2*c) - 144*a*b^2*e^(2*d*x + 2*c) + 48*b^3
*e^(2*d*x + 2*c) + 48*(64*a^3 - 48*a^2*b + 24*a*b^2 - 5*b^3)*(d*x + c) - (3
200*a^3*e^(8*d*x + 8*c) - 2400*a^2*b*e^(8*d*x + 8*c) + 1200*a*b^2*e^(8*d*x
+ 8*c) - 250*b^3*e^(8*d*x + 8*c) + 768*a^3*e^(6*d*x + 6*c) - 144*a*b^2*e^(6
*d*x + 6*c) + 48*b^3*e^(6*d*x + 6*c) + 288*a^2*b*e^(4*d*x + 4*c) - 144*a*b^
2*e^(4*d*x + 4*c) + 24*b^3*e^(4*d*x + 4*c) + 48*a*b^2*e^(2*d*x + 2*c) - 16*
b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-8*d*x - 8*c))/d
```

3.307 $\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=67

$$\frac{a^2b\sinh^3(c+dx)}{d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}$$

```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*b*Sinh[c + d*x]^3)/d + (3*a*b^2*Sinh[c + d*x]^
5)/(5*d) + (b^3*Sinh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0454635, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3190, 194}

$$\frac{a^2b\sinh^3(c+dx)}{d}+\frac{a^3\sinh(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x])/d + (a^2*b*Sinh[c + d*x]^3)/d + (3*a*b^2*Sinh[c + d*x]^
5)/(5*d) + (b^3*Sinh[c + d*x]^7)/(7*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \cosh(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\text{Subst}\left(\int \left(a+bx^2\right)^3\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{\text{Subst}\left(\int \left(a^3+3a^2bx^2+3ab^2x^4+b^3x^6\right)\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d}\end{aligned}$$

Mathematica [A] time = 0.11578, size = 59, normalized size = 0.88

$$\frac{a^2b\sinh^3(c+dx)+a^3\sinh(c+dx)+\frac{3}{5}ab^2\sinh^5(c+dx)+\frac{1}{7}b^3\sinh^7(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Sinh[c + d*x] + a^2*b*Sinh[c + d*x]^3 + (3*a*b^2*Sinh[c + d*x]^5)/5 +
(b^3*Sinh[c + d*x]^7)/7)/d
```

Maple [A] time = 0.011, size = 56, normalized size = 0.8

$$\frac{1}{d}\left(\frac{b^3(\sinh(dx+c))^7}{7}+\frac{3ab^2(\sinh(dx+c))^5}{5}+a^2b(\sinh(dx+c))^3+a^3\sinh(dx+c)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(1/7*b^3*sinh(d*x+c)^7+3/5*a*b^2*sinh(d*x+c)^5+a^2*b*sinh(d*x+c)^3+a^3*
sinh(d*x+c))
```

Maxima [A] time = 1.05374, size = 85, normalized size = 1.27

$$\frac{b^3\sinh(dx+c)^7}{7d}+\frac{3ab^2\sinh(dx+c)^5}{5d}+\frac{a^2b\sinh(dx+c)^3}{d}+\frac{a^3\sinh(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/7*b^3*sinh(d*x + c)^7/d + 3/5*a*b^2*sinh(d*x + c)^5/d + a^2*b*sinh(d*x +
c)^3/d + a^3*sinh(d*x + c)/d
```

Fricas [B] time = 1.54785, size = 512, normalized size = 7.64

$$5b^3\sinh(dx+c)^7+7\left(15b^3\cosh(dx+c)^2+12ab^2-5b^3\right)\sinh(dx+c)^5+35\left(5b^3\cosh(dx+c)^4+16a^2b-12ab^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/2240*(5*b^3*sinh(d*x + c)^7 + 7*(15*b^3*cosh(d*x + c)^2 + 12*a*b^2 - 5*b^
3)*sinh(d*x + c)^5 + 35*(5*b^3*cosh(d*x + c)^4 + 16*a^2*b - 12*a*b^2 + 3*b^
3 + 2*(12*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 35*(b^3*cosh(d*
x + c)^6 + (12*a*b^2 - 5*b^3)*cosh(d*x + c)^4 + 64*a^3 - 48*a^2*b + 24*a*b^
2 - 5*b^3 + 3*(16*a^2*b - 12*a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))
/d
```

Sympy [A] time = 6.6694, size = 75, normalized size = 1.12

$$\begin{cases}\frac{a^3\sinh(c+dx)}{d}+\frac{a^2b\sinh^3(c+dx)}{d}+\frac{3ab^2\sinh^5(c+dx)}{5d}+\frac{b^3\sinh^7(c+dx)}{7d} & \text{for } d\neq 0\\ x\left(a+b\sinh^2(c)\right)^3\cosh(c) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((a**3*sinh(c + d*x)/d + a**2*b*sinh(c + d*x)**3/d + 3*a*b**2*sinh
(c + d*x)**5/(5*d) + b**3*sinh(c + d*x)**7/(7*d), Ne(d, 0)), (x*(a + b*sinh
(c)**2)**3*cosh(c), True))
```

Giac [B] time = 1.2839, size = 387, normalized size = 5.78

$$\frac{5\,b^3e^{(7\,dx+7\,c)} + 84\,ab^2e^{(5\,dx+5\,c)} - 35\,b^3e^{(5\,dx+5\,c)} + 560\,a^2be^{(3\,dx+3\,c)} - 420\,ab^2e^{(3\,dx+3\,c)} + 105\,b^3e^{(3\,dx+3\,c)} + 2240\,a^3e^{(dx+c)} - 1}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4480*(5*b^3*e^(7*d*x + 7*c) + 84*a*b^2*e^(5*d*x + 5*c) - 35*b^3*e^(5*d*x
+ 5*c) + 560*a^2*b*e^(3*d*x + 3*c) - 420*a*b^2*e^(3*d*x + 3*c) + 105*b^3*e^
(3*d*x + 3*c) + 2240*a^3*e^(d*x + c) - 1680*a^2*b*e^(d*x + c) + 840*a*b^2*e
^(d*x + c) - 175*b^3*e^(d*x + c) - (2240*a^3*e^(6*d*x + 6*c) - 1680*a^2*b*e
^(6*d*x + 6*c) + 840*a*b^2*e^(6*d*x + 6*c) - 175*b^3*e^(6*d*x + 6*c) + 560*
a^2*b*e^(4*d*x + 4*c) - 420*a*b^2*e^(4*d*x + 4*c) + 105*b^3*e^(4*d*x + 4*c)
 + 84*a*b^2*e^(2*d*x + 2*c) - 35*b^3*e^(2*d*x + 2*c) + 5*b^3)*e^(-7*d*x - 7
*c))/d
```

3.308 $\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=86

$$\frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)}{d}+\frac{b^2(3a-b)\sinh^3(c+dx)}{3d}+\frac{(a-b)^3\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^3\sinh^5(c+dx)}{5d}$$

```
[Out] ((a - b)^3*ArcTan[Sinh[c + d*x]])/d + (b*(3*a^2 - 3*a*b + b^2)*Sinh[c + d*x
])/d + ((3*a - b)*b^2*Sinh[c + d*x]^3)/(3*d) + (b^3*Sinh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0755837, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 390, 203}

$$\frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)}{d}+\frac{b^2(3a-b)\sinh^3(c+dx)}{3d}+\frac{(a-b)^3\tan^{-1}(\sinh(c+dx))}{d}+\frac{b^3\sinh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^3*ArcTan[Sinh[c + d*x]])/d + (b*(3*a^2 - 3*a*b + b^2)*Sinh[c + d*x
])/d + ((3*a - b)*b^2*Sinh[c + d*x]^3)/(3*d) + (b^3*Sinh[c + d*x]^5)/(5*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^3}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(b\left(3a^2-3ab+b^2\right)+(3a-b)b^2x^2+b^3x^4+\frac{(a-b)^3}{1+x^2}\right)dx, x, \sinh(c+\right.}{d}$$

$$= \frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)}{d}+\frac{(3a-b)b^2\sinh^3(c+dx)}{3d}+\frac{b^3\sinh^5(c+d}{5d}$$

$$= \frac{(a-b)^3\tan^{-1}(\sinh(c+dx))}{d}+\frac{b\left(3a^2-3ab+b^2\right)\sinh(c+dx)}{d}+\frac{(3a-b)b^2}{}$$

Mathematica [A] time = 0.530513, size = 100, normalized size = 1.16

$$\frac{\sinh(c+dx)\left(b\left(45a^2+15ab\left(\sinh^2(c+dx)-3\right)+b^2\left(3\sinh^4(c+dx)-5\sinh^2(c+dx)+15\right)\right)+\frac{15(a-b)^3\tanh^{-1}\left(\sqrt{-\sinh^2(c}\right.}{\sqrt{-\sinh^2(c+dx)}}\right.}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Sinh[c + d*x]*((15*(a - b)^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]])/Sqrt[-Sinh[c
 + d*x]^2] + b*(45*a^2 + 15*a*b*(-3 + Sinh[c + d*x]^2) + b^2*(15 - 5*Sinh[c
 + d*x]^2 + 3*Sinh[c + d*x]^4))))/(15*d)
```

Maple [A] time = 0.056, size = 155, normalized size = 1.8

$$2\,\frac{a^3\arctan\left(e^{dx+c}\right)}{d}+3\,\frac{a^2b\sinh(dx+c)}{d}-6\,\frac{a^2b\arctan\left(e^{dx+c}\right)}{d}+\frac{ab^2\left(\sinh(dx+c)\right)^3}{d}-3\,\frac{ab^2\sinh(dx+c)}{d}+6\,\frac{ab^2\,\mathrm{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 2/d*a^3*arctan(exp(d*x+c))+3/d*a^2*b*sinh(d*x+c)-6/d*a^2*b*arctan(exp(d*x+c
))+1/d*a*b^2*sinh(d*x+c)^3-3/d*a*b^2*sinh(d*x+c)+6/d*a*b^2*arctan(exp(d*x+c
))+1/5*b^3*sinh(d*x+c)^5/d-1/3*b^3*sinh(d*x+c)^3/d+b^3*sinh(d*x+c)/d-2/d*b^
3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.69462, size = 315, normalized size = 3.66

$$-\frac{1}{480}\,b^3\left(\frac{\left(35\,e^{(-2\,dx-2\,c)}-330\,e^{(-4\,dx-4\,c)}-3\right)e^{(5\,dx+5\,c)}}{d}+\frac{330\,e^{(-dx-c)}-35\,e^{(-3\,dx-3\,c)}+3\,e^{(-5\,dx-5\,c)}}{d}-\frac{960\,\arctan\left(e^{(-dx-c)}\right)}{d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/480*b^3*((35*e^(-2*d*x - 2*c) - 330*e^(-4*d*x - 4*c) - 3)*e^(5*d*x + 5*c
)/d + (330*e^(-d*x - c) - 35*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c))/d - 960
*arctan(e^(-d*x - c))/d) - 1/8*a*b^2*((15*e^(-2*d*x - 2*c) - 1)*e^(3*d*x +
3*c)/d - (15*e^(-d*x - c) - e^(-3*d*x - 3*c))/d + 48*arctan(e^(-d*x - c))/d
) + 3/2*a^2*b*(4*arctan(e^(-d*x - c))/d + e^(d*x + c)/d - e^(-d*x - c)/d) +
 a^3*arctan(sinh(d*x + c))/d
```

Fricas [B] time = 1.62245, size = 2795, normalized size = 32.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/480*(3*b^3*cosh(d*x + c)^10 + 30*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 3*b^
3*sinh(d*x + c)^10 + 5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^8 + 5*(27*b^3*cosh(
d*x + c)^2 + 12*a*b^2 - 7*b^3)*sinh(d*x + c)^8 + 40*(9*b^3*cosh(d*x + c)^3
+ (12*a*b^2 - 7*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 30*(24*a^2*b - 30*a*b
^2 + 11*b^3)*cosh(d*x + c)^6 + 10*(63*b^3*cosh(d*x + c)^4 + 72*a^2*b - 90*a
*b^2 + 33*b^3 + 14*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*
(189*b^3*cosh(d*x + c)^5 + 70*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^3 + 45*(24*a
^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - 30*(24*a^2*b - 3
0*a*b^2 + 11*b^3)*cosh(d*x + c)^4 + 10*(63*b^3*cosh(d*x + c)^6 + 35*(12*a*b
^2 - 7*b^3)*cosh(d*x + c)^4 - 72*a^2*b + 90*a*b^2 - 33*b^3 + 45*(24*a^2*b -
 30*a*b^2 + 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 40*(9*b^3*cosh(d*x +
 c)^7 + 7*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^5 + 15*(24*a^2*b - 30*a*b^2 + 11
*b^3)*cosh(d*x + c)^3 - 3*(24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c))*sin
h(d*x + c)^3 - 3*b^3 - 5*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^2 + 5*(27*b^3*cos
h(d*x + c)^8 + 28*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^6 + 90*(24*a^2*b - 30*a*
b^2 + 11*b^3)*cosh(d*x + c)^4 - 12*a*b^2 + 7*b^3 - 36*(24*a^2*b - 30*a*b^2
+ 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 960*((a^3 - 3*a^2*b + 3*a*b^2
- b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^4*
sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^3*sinh(d*x
 + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh(d*x + c)^
3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^3
- 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*arctan(cosh(d*x + c) + sinh(d*x
 + c)) + 10*(3*b^3*cosh(d*x + c)^9 + 4*(12*a*b^2 - 7*b^3)*cosh(d*x + c)^7 +
 18*(24*a^2*b - 30*a*b^2 + 11*b^3)*cosh(d*x + c)^5 - 12*(24*a^2*b - 30*a*b^
2 + 11*b^3)*cosh(d*x + c)^3 - (12*a*b^2 - 7*b^3)*cosh(d*x + c))*sinh(d*x +
c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*d*cosh(d*x
+ c)^3*sinh(d*x + c)^2 + 10*d*cosh(d*x + c)^2*sinh(d*x + c)^3 + 5*d*cosh(d*
x + c)*sinh(d*x + c)^4 + d*sinh(d*x + c)^5)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.279, size = 309, normalized size = 3.59

$$\frac{2\left(a^3-3\,a^2b+3\,ab^2-b^3\right)\arctan\left(e^{(dx+c)}\right)}{d}-\frac{\left(720\,a^2be^{(4\,dx+4\,c)}-900\,ab^2e^{(4\,dx+4\,c)}+330\,b^3e^{(4\,dx+4\,c)}+60\,ab^2e^{(2\,dx+2\,c)}-35\right.}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*arctan(e^(d*x + c))/d - 1/480*(720*a^2*b*
e^(4*d*x + 4*c) - 900*a*b^2*e^(4*d*x + 4*c) + 330*b^3*e^(4*d*x + 4*c) + 60*
a*b^2*e^(2*d*x + 2*c) - 35*b^3*e^(2*d*x + 2*c) + 3*b^3)*e^(-5*d*x - 5*c)/d
+ 1/480*(3*b^3*d^4*e^(5*d*x + 5*c) + 60*a*b^2*d^4*e^(3*d*x + 3*c) - 35*b^3*
d^4*e^(3*d*x + 3*c) + 720*a^2*b*d^4*e^(d*x + c) - 900*a*b^2*d^4*e^(d*x + c)
 + 330*b^3*d^4*e^(d*x + c))/d^5
```

3.309 $\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3 dx$

Optimal. Leaf size=92

$$\frac{3}{8}bx\left(8a^2-12ab+5b^2\right)+\frac{3b^2(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{(a-b)^3\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh^3(c\cdot}{4d}$$

```
[Out] (3*b*(8*a^2 - 12*a*b + 5*b^2)*x)/8 + (3*(4*a - 3*b)*b^2*Cosh[c + d*x]*Sinh[
c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + ((a - b)^3*Ta
nh[c + d*x])/d
```

Rubi [A] time = 0.1291, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 390, 1157, 385, 206}

$$\frac{3}{8}bx\left(8a^2-12ab+5b^2\right)+\frac{3b^2(4a-3b)\sinh(c+dx)\cosh(c+dx)}{8d}+\frac{(a-b)^3\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh^3(c\cdot}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*b*(8*a^2 - 12*a*b + 5*b^2)*x)/8 + (3*(4*a - 3*b)*b^2*Cosh[c + d*x]*Sinh[
c + d*x])/(8*d) + (b^3*Cosh[c + d*x]^3*Sinh[c + d*x])/(4*d) + ((a - b)^3*Ta
nh[c + d*x])/d
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left((a-b)^3 + \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{\left(1-x^2\right)^3}\right) dx, x, \tanh(c+ \ldots}{d}$$

$$= \frac{(a-b)^3\tanh(c+dx)}{d} + \frac{\operatorname{Subst}\left(\int \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{\left(1-x^2\right)^3}\,dx, x \ldots}{d}$$

$$= \frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d} + \frac{(a-b)^3\tanh(c+dx)}{d} - \frac{\operatorname{Subst}\left(\int \frac{-3(2a-b)\ldots}{(\ldots}\right.}{}$$

$$= \frac{3(4a-3b)b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^3\cosh^3(c+dx)\sinh(c+dx)}{4d} + \ldots$$

$$= \frac{3}{8}b\left(8a^2-12ab+5b^2\right)x + \frac{3(4a-3b)b^2\cosh(c+dx)\sinh(c+dx)}{8d} + \frac{b^3\cos\ldots}{}$$

Mathematica [A] time = 0.510685, size = 78, normalized size = 0.85

$$\frac{12b\left(8a^2-12ab+5b^2\right)(c+dx)+8b^2(3a-2b)\sinh(2(c+dx))+32(a-b)^3\tanh(c+dx)+b^3\sinh(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (12*b*(8*a^2 - 12*a*b + 5*b^2)*(c + d*x) + 8*(3*a - 2*b)*b^2*Sinh[2*(c + d*
x)] + b^3*Sinh[4*(c + d*x)] + 32*(a - b)^3*Tanh[c + d*x])/(32*d)
```

Maple [A] time = 0.039, size = 131, normalized size = 1.4

$$\frac{1}{d}\left(a^3\tanh(dx+c)+3a^2b\,(dx+c-\tanh(dx+c))+3ab^2\left(1/2\,\frac{(\sinh(dx+c))^3}{\cosh(dx+c)}-3/2\,dx-3/2\,c+3/2\,\tanh(dx+c)\right)+ \ldots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*tanh(d*x+c)+3*a^2*b*(d*x+c-tanh(d*x+c))+3*a*b^2*(1/2*sinh(d*x+c)^3
/cosh(d*x+c)-3/2*d*x-3/2*c+3/2*tanh(d*x+c))+b^3*(1/4*sinh(d*x+c)^5/cosh(d*x
+c)-5/8*sinh(d*x+c)^3/cosh(d*x+c)+15/8*d*x+15/8*c-15/8*tanh(d*x+c)))
```

Maxima [B] time = 1.09097, size = 290, normalized size = 3.15

$$3\,a^2b\left(x+\frac{c}{d}-\frac{2}{d\left(e^{(-2\,dx-2\,c)}+1\right)}\right)+\frac{1}{64}\,b^3\left(\frac{120\,(dx+c)}{d}+\frac{16\,e^{(-2\,dx-2\,c)}-e^{(-4\,dx-4\,c)}}{d}-\frac{15\,e^{(-2\,dx-2\,c)}+144\,e^{(-4\,dx-4\,c)}-1}{d\left(e^{(-4\,dx-4\,c)}+e^{(-6\,dx-6\,c)}\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 3*a^2*b*(x + c/d - 2/(d*(e^(-2*d*x - 2*c) + 1))) + 1/64*b^3*(120*(d*x + c)/
d + (16*e^(-2*d*x - 2*c) - e^(-4*d*x - 4*c))/d - (15*e^(-2*d*x - 2*c) + 144
*e^(-4*d*x - 4*c) - 1)/(d*(e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c)))) - 3/8*a*b
^2*(12*(d*x + c)/d + e^(-2*d*x - 2*c)/d - (17*e^(-2*d*x - 2*c) + 1)/(d*(e^(
-2*d*x - 2*c) + e^(-4*d*x - 4*c)))) + 2*a^3/(d*(e^(-2*d*x - 2*c) + 1))
```

Fricas [B] time = 1.55093, size = 435, normalized size = 4.73

$$\frac{b^3\sinh(dx+c)^5+\left(10\,b^3\cosh(dx+c)^2+24\,ab^2-15\,b^3\right)\sinh(dx+c)^3-8\left(8\,a^3-24\,a^2b+24\,ab^2-8\,b^3-3\left(8\,a^2b-\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/64*(b^3*sinh(d*x + c)^5 + (10*b^3*cosh(d*x + c)^2 + 24*a*b^2 - 15*b^3)*si
nh(d*x + c)^3 - 8*(8*a^3 - 24*a^2*b + 24*a*b^2 - 8*b^3 - 3*(8*a^2*b - 12*a*
b^2 + 5*b^3)*d*x)*cosh(d*x + c) + (5*b^3*cosh(d*x + c)^4 + 64*a^3 - 192*a^2
*b + 216*a*b^2 - 80*b^3 + 9*(8*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
))/(d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30828, size = 284, normalized size = 3.09

$$\frac{3\left(8\,a^2b-12\,ab^2+5\,b^3\right)(dx+c)}{8\,d}-\frac{\left(144\,a^2be^{(4\,dx+4\,c)}-216\,ab^2e^{(4\,dx+4\,c)}+90\,b^3e^{(4\,dx+4\,c)}+24\,ab^2e^{(2\,dx+2\,c)}-16\,b^3e^{(2\,dx+\right.}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*(8*a^2*b - 12*a*b^2 + 5*b^3)*(d*x + c)/d - 1/64*(144*a^2*b*e^(4*d*x + 4
*c) - 216*a*b^2*e^(4*d*x + 4*c) + 90*b^3*e^(4*d*x + 4*c) + 24*a*b^2*e^(2*d*
x + 2*c) - 16*b^3*e^(2*d*x + 2*c) + b^3)*e^(-4*d*x - 4*c)/d + 1/64*(b^3*d*e
^(4*d*x + 4*c) + 24*a*b^2*d*e^(2*d*x + 2*c) - 16*b^3*d*e^(2*d*x + 2*c))/d^2
 - 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)/(d*(e^(2*d*x + 2*c) + 1))
```

3.310 $\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=91

$$\frac{b^2(3a-2b)\sinh(c+dx)}{d}+\frac{(a+5b)(a-b)^2\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)^3\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}+\frac{b^3\sinh^3(c+dx)}{3d}$$

```
[Out] ((a - b)^2*(a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((3*a - 2*b)*b^2*Sinh[c
 + d*x])/d + (b^3*Sinh[c + d*x]^3)/(3*d) + ((a - b)^3*Sech[c + d*x]*Tanh[c
+ d*x])/(2*d)
```

Rubi [A] time = 0.0957924, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 390, 385, 203}

$$\frac{b^2(3a-2b)\sinh(c+dx)}{d}+\frac{(a+5b)(a-b)^2\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(a-b)^3\tanh(c+dx)\operatorname{sech}(c+dx)}{2d}+\frac{b^3\sinh^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - b)^2*(a + 5*b)*ArcTan[Sinh[c + d*x]])/(2*d) + ((3*a - 2*b)*b^2*Sinh[c
 + d*x])/d + (b^3*Sinh[c + d*x]^3)/(3*d) + ((a - b)^3*Sech[c + d*x]*Tanh[c
+ d*x])/(2*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{\left(1+x^2\right)^2}dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left((3a-2b)b^2+b^3x^2+\frac{(a-b)^2(a+2b)+3(a-b)^2bx^2}{\left(1+x^2\right)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(3a-2b)b^2\sinh(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)}{3d}+\frac{\operatorname{Subst}\left(\int \frac{(a-b)^2(a+2b)+3(a-b)^2b}{\left(1+x^2\right)^2}\right.}{d}$$

$$= \frac{(3a-2b)b^2\sinh(c+dx)}{d}+\frac{b^3\sinh^3(c+dx)}{3d}+\frac{(a-b)^3\operatorname{sech}(c+dx)\tanh(c}{2d}$$

$$= \frac{(a-b)^2(a+5b)\tan^{-1}(\sinh(c+dx))}{2d}+\frac{(3a-2b)b^2\sinh(c+dx)}{d}+\frac{b^3\sinh^3}{3}$$

Mathematica [C] time = 7.32553, size = 347, normalized size = 3.81

$$\frac{\operatorname{csch}^5(c+dx)\left(-256\sinh^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2,2\right\},\left\{1,1,1,\frac{11}{2}\right\},-\sinh^2(c+\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-256*HypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}
, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3 + 21*(36015*a
^3 + 5*a^2*(3224*a + 21609*b)*Sinh[c + d*x]^2 + 3*a*(491*a^2 + 16120*a*b +
36015*b^2)*Sinh[c + d*x]^4 + 3*b*(753*a^2 + 18280*a*b + 10805*b^2)*Sinh[c +
 d*x]^6 + b^2*(2259*a + 17320*b)*Sinh[c + d*x]^8 + 753*b^3*Sinh[c + d*x]^10
) - (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(2401*a^3 + 3*a^2*(625*a + 2401*b)
*Sinh[c + d*x]^2 + 3*a*(81*a^2 + 1875*a*b + 2401*b^2)*Sinh[c + d*x]^4 + (-4
7*a^3 + 585*a^2*b + 6057*a*b^2 + 2161*b^3)*Sinh[c + d*x]^6 + 3*b*(a^2 + 243
*a*b + 625*b^2)*Sinh[c + d*x]^8 + 3*b^2*(a + 81*b)*Sinh[c + d*x]^10 + b^3*S
inh[c + d*x]^12))/Sqrt[-Sinh[c + d*x]^2]))/(30240*d)
```

Maple [B] time = 0.055, size = 286, normalized size = 3.1

$$\frac{a^3\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}+\frac{a^3\arctan\left(e^{dx+c}\right)}{d}-3\frac{a^2b\sinh(dx+c)}{d(\cosh(dx+c))^2}+\frac{3a^2b\operatorname{sech}(dx+c)\tanh(dx+c)}{2d}+3\frac{a^2b\operatorname{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d*a^3*sech(d*x+c)*tanh(d*x+c)+1/d*a^3*arctan(exp(d*x+c))-3/d*a^2*b*sinh
(d*x+c)/cosh(d*x+c)^2+3/2/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/d*a^2*b*arctan(
exp(d*x+c))+3/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^2+9/d*a*b^2*sinh(d*x+c)/cos
h(d*x+c)^2-9/2/d*a*b^2*sech(d*x+c)*tanh(d*x+c)-9/d*a*b^2*arctan(exp(d*x+c))
+1/3/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^2-5/3/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^
2-5/d*b^3*sinh(d*x+c)/cosh(d*x+c)^2+5/2/d*b^3*sech(d*x+c)*tanh(d*x+c)+5/d*b
```

```
^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.59216, size = 482, normalized size = 5.3

$$\frac{1}{24}b^3\left(\frac{27\,e^{(-dx-c)}-e^{(-3\,dx-3\,c)}}{d}-\frac{120\,\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{25\,e^{(-2\,dx-2\,c)}+77\,e^{(-4\,dx-4\,c)}+3\,e^{(-6\,dx-6\,c)}-1}{d\left(e^{(-3\,dx-3\,c)}+2\,e^{(-5\,dx-5\,c)}+e^{(-7\,dx-7\,c)}\right)}\right)+\frac{3}{2}ab^2\left(\frac{6\,\mathrm{arc}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/24*b^3*((27*e^(-d*x - c) - e^(-3*d*x - 3*c))/d - 120*arctan(e^(-d*x - c))
/d - (25*e^(-2*d*x - 2*c) + 77*e^(-4*d*x - 4*c) + 3*e^(-6*d*x - 6*c) - 1)/(
d*(e^(-3*d*x - 3*c) + 2*e^(-5*d*x - 5*c) + e^(-7*d*x - 7*c)))) + 3/2*a*b^2*
(6*arctan(e^(-d*x - c))/d - e^(-d*x - c)/d + (4*e^(-2*d*x - 2*c) - e^(-4*d*
x - 4*c) + 1)/(d*(e^(-d*x - c) + 2*e^(-3*d*x - 3*c) + e^(-5*d*x - 5*c)))) -
 3*a^2*b*(arctan(e^(-d*x - c))/d + (e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*
e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c) + 1))) - a^3*(arctan(e^(-d*x - c))/d -
(e^(-d*x - c) - e^(-3*d*x - 3*c))/(d*(2*e^(-2*d*x - 2*c) + e^(-4*d*x - 4*c)
 + 1)))
```

Fricas [B] time = 1.81228, size = 4188, normalized size = 46.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sin
h(d*x + c)^10 + (36*a*b^2 - 25*b^3)*cosh(d*x + c)^8 + (45*b^3*cosh(d*x + c)
^2 + 36*a*b^2 - 25*b^3)*sinh(d*x + c)^8 + 8*(15*b^3*cosh(d*x + c)^3 + (36*a
*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(12*a^3 - 36*a^2*b + 54*a
*b^2 - 25*b^3)*cosh(d*x + c)^6 + 2*(105*b^3*cosh(d*x + c)^4 + 12*a^3 - 36*a
^2*b + 54*a*b^2 - 25*b^3 + 14*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x
 + c)^6 + 4*(63*b^3*cosh(d*x + c)^5 + 14*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^
3 + 3*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^
5 - 2*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^4 + 2*(105*b^3*
cosh(d*x + c)^6 + 35*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^4 - 12*a^3 + 36*a^2*
b - 54*a*b^2 + 25*b^3 + 15*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^4 + 8*(15*b^3*cosh(d*x + c)^7 + 7*(36*a*b^2 - 25*b^3
)*cosh(d*x + c)^5 + 5*(12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)
^3 - (12*a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c))*sinh(d*x + c)^3
 - b^3 - (36*a*b^2 - 25*b^3)*cosh(d*x + c)^2 + (45*b^3*cosh(d*x + c)^8 + 28
*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^6 + 30*(12*a^3 - 36*a^2*b + 54*a*b^2 - 2
5*b^3)*cosh(d*x + c)^4 - 36*a*b^2 + 25*b^3 - 12*(12*a^3 - 36*a^2*b + 54*a*b
^2 - 25*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*((a^3 + 3*a^2*b - 9*a*b^
2 + 5*b^3)*cosh(d*x + c)^7 + 7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x +
 c)*sinh(d*x + c)^6 + (a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*sinh(d*x + c)^7 + 2
*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + (2*a^3 + 6*a^2*b - 18*
a*b^2 + 10*b^3 + 21*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh
(d*x + c)^5 + 5*(7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 2*(a
^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + (a^3 + 3*a
^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (35*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
```

```
b^3)*cosh(d*x + c)^4 + a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3 + 20*(a^3 + 3*a^2*b
- 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + (21*(a^3 + 3*a^2*b -
9*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 20*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cos
h(d*x + c)^3 + 3*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x
+ c)^2 + (7*(a^3 + 3*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 10*(a^3 + 3
*a^2*b - 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 3*(a^3 + 3*a^2*b - 9*a*b^2 + 5*
b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c))
+ 2*(5*b^3*cosh(d*x + c)^9 + 4*(36*a*b^2 - 25*b^3)*cosh(d*x + c)^7 + 6*(12*
a^3 - 36*a^2*b + 54*a*b^2 - 25*b^3)*cosh(d*x + c)^5 - 4*(12*a^3 - 36*a^2*b
+ 54*a*b^2 - 25*b^3)*cosh(d*x + c)^3 - (36*a*b^2 - 25*b^3)*cosh(d*x + c))*s
inh(d*x + c))/(d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)*sinh(d*x + c)^6 + d*si
nh(d*x + c)^7 + 2*d*cosh(d*x + c)^5 + (21*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x
 + c)^5 + 5*(7*d*cosh(d*x + c)^3 + 2*d*cosh(d*x + c))*sinh(d*x + c)^4 + d*c
osh(d*x + c)^3 + (35*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^3 + (21*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))
*sinh(d*x + c)^2 + (7*d*cosh(d*x + c)^6 + 10*d*cosh(d*x + c)^4 + 3*d*cosh(d
*x + c)^2)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.43009, size = 354, normalized size = 3.89

$$\frac{\left(\pi + 2\arctan\left(\frac{1}{2}\left(e^{(2dx+2c)} - 1\right)e^{(-dx-c)}\right)\right)\left(a^3 + 3a^2b - 9ab^2 + 5b^3\right)}{4d} + \frac{a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right) - 3a^2b\left(e^{(dx+c)} - e^{(-dx-c)}\right) + 3}{\left(\left(e^{(dx+c)} - e^{(-dx-\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/4*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(a^3 + 3*a^2*b
- 9*a*b^2 + 5*b^3)/d + (a^3*(e^(d*x + c) - e^(-d*x - c)) - 3*a^2*b*(e^(d*x
+ c) - e^(-d*x - c)) + 3*a*b^2*(e^(d*x + c) - e^(-d*x - c)) - b^3*(e^(d*x +
 c) - e^(-d*x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)*d) + 1/24*(b^3*d
^2*(e^(d*x + c) - e^(-d*x - c))^3 + 36*a*b^2*d^2*(e^(d*x + c) - e^(-d*x - c
)) - 24*b^3*d^2*(e^(d*x + c) - e^(-d*x - c)))/d^3
```

3.311 $\int \mathrm{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=82

$$\frac{1}{2}b^2x(6a-5b)-\frac{(a-b)^3\tanh^3(c+dx)}{3d}+\frac{(a-b)^2(a+2b)\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh(c+dx)}{2d}$$

```
[Out] ((6*a - 5*b)*b^2*x)/2 + (b^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + ((a - b)^
2*(a + 2*b)*Tanh[c + d*x])/d - ((a - b)^3*Tanh[c + d*x]^3)/(3*d)
```

Rubi [A] time = 0.108489, antiderivative size = 82, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 206}

$$\frac{1}{2}b^2x(6a-5b)-\frac{(a-b)^3\tanh^3(c+dx)}{3d}+\frac{(a-b)^2(a+2b)\tanh(c+dx)}{d}+\frac{b^3\sinh(c+dx)\cosh(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] ((6*a - 5*b)*b^2*x)/2 + (b^3*Cosh[c + d*x]*Sinh[c + d*x])/(2*d) + ((a - b)^
2*(a + 2*b)*Tanh[c + d*x])/d - ((a - b)^3*Tanh[c + d*x]^3)/(3*d)
```


Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\text{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{\left(1-x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left((a-b)^2(a+2b)-(a-b)^3x^2+\frac{(3a-2b)b^2-3(a-b)b^2x^2}{\left(1-x^2\right)^2}\right)dx, x, \tanh(c$$

$$= \frac{(a-b)^2(a+2b)\tanh(c+dx)}{d} - \frac{(a-b)^3\tanh^3(c+dx)}{3d} + \frac{\text{Subst}\left(\int \frac{(3a-2b)b}{(}\right.}{}$$

$$= \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2(a+2b)\tanh(c+dx)}{d} - \frac{(a-b)^3\operatorname{ta}}{3}$$

$$= \frac{1}{2}(6a-5b)b^2x + \frac{b^3\cosh(c+dx)\sinh(c+dx)}{2d} + \frac{(a-b)^2(a+2b)\tanh(c+}{d}$$

Mathematica [A] time = 0.738259, size = 84, normalized size = 1.02

$$\frac{6b^2(6a-5b)(c+dx)+2(a-b)^2\tanh(c+dx)\operatorname{sech}^2(c+dx)((2a+7b)\cosh(2(c+dx))+4a+5b)+3b^3\sinh(2(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (6*(6*a - 5*b)*b^2*(c + d*x) + 3*b^3*Sinh[2*(c + d*x)] + 2*(a - b)^2*(4*a +
 5*b + (2*a + 7*b)*Cosh[2*(c + d*x)])*Sech[c + d*x]^2*Tanh[c + d*x])/(12*d)
```

Maple [A] time = 0.054, size = 148, normalized size = 1.8

$$\frac{1}{d}\left(a^3\left(\frac{2}{3}+\frac{(\operatorname{sech}(dx+c))^2}{3}\right)\tanh(dx+c)+3a^2b\left(-1/2\frac{\sinh(dx+c)}{(\cosh(dx+c))^3}+1/2\left(2/3+1/3(\operatorname{sech}(dx+c))^2\right)\tanh(dx+c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-1/2*sinh(d*x+c)/cosh
(d*x+c)^3+1/2*(2/3+1/3*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(d*x+c-tanh(d*x+
c)-1/3*tanh(d*x+c)^3)+b^3*(1/2*sinh(d*x+c)^5/cosh(d*x+c)^3-5/2*d*x-5/2*c+5/
2*tanh(d*x+c)+5/6*tanh(d*x+c)^3))
```

Maxima [B] time = 1.14396, size = 516, normalized size = 6.29

$$ab^2\left(3x+\frac{3c}{d}-\frac{4\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+2\right)}{d\left(3e^{(-2dx-2c)}+3e^{(-4dx-4c)}+e^{(-6dx-6c)}+1\right)}\right)-\frac{1}{24}b^3\left(\frac{60(dx+c)}{d}+\frac{3e^{(-2dx-2c)}}{d}-\frac{121e^{(-2dx-2c)}+201}{d\left(e^{(-2dx-2c)}+3e^{(-4d}\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] a*b^2*(3*x + 3*c/d - 4*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + 2)/(d*(3*
e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) - 1/24*b^3*
(60*(d*x + c)/d + 3*e^(-2*d*x - 2*c)/d - (121*e^(-2*d*x - 2*c) + 201*e^(-4*
d*x - 4*c) + 147*e^(-6*d*x - 6*c) + 3)/(d*(e^(-2*d*x - 2*c) + 3*e^(-4*d*x -
 4*c) + 3*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c)))) + 4/3*a^3*(3*e^(-2*d*x - 2
*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1)) +
1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-6*d*x - 6*c) + 1))) + 2
*a^2*b*(3*e^(-4*d*x - 4*c)/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^
(-6*d*x - 6*c) + 1)) + 1/(d*(3*e^(-2*d*x - 2*c) + 3*e^(-4*d*x - 4*c) + e^(-
6*d*x - 6*c) + 1)))
```

Fricas [B] time = 1.5977, size = 788, normalized size = 9.61

$$\frac{3\,b^3\sinh\left(dx+c\right)^5-4\left(4\,a^3+6\,a^2b-24\,ab^2+14\,b^3-3\left(6\,ab^2-5\,b^3\right)dx\right)\cosh\left(dx+c\right)^3-12\left(4\,a^3+6\,a^2b-24\,ab^2+\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^3*sinh(d*x + c)^5 - 4*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6
*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)^3 - 12*(4*a^3 + 6*a^2*b - 24*a*b^2 + 14*
b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c)*sinh(d*x + c)^2 + (30*b^3*cosh
(d*x + c)^2 + 16*a^3 + 24*a^2*b - 96*a*b^2 + 65*b^3)*sinh(d*x + c)^3 - 12*(
4*a^3 + 6*a^2*b - 24*a*b^2 + 14*b^3 - 3*(6*a*b^2 - 5*b^3)*d*x)*cosh(d*x + c
) + 3*(5*b^3*cosh(d*x + c)^4 + 16*a^3 - 24*a^2*b + 10*b^3 + (16*a^3 + 24*a^
2*b - 96*a*b^2 + 65*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^3
 + 3*d*cosh(d*x + c)*sinh(d*x + c)^2 + 3*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.37152, size = 290, normalized size = 3.54

$$\frac{b^3e^{(2\,dx+2\,c)}}{8\,d}+\frac{\left(6\,ab^2-5\,b^3\right)\left(dx+c\right)}{2\,d}-\frac{\left(12\,ab^2e^{(2\,dx+2\,c)}-10\,b^3e^{(2\,dx+2\,c)}+b^3\right)e^{(-2\,dx-2\,c)}}{8\,d}-\frac{2\left(9\,a^2be^{(4\,dx+4\,c)}-18\,ab^2e^{(4\,d\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*b^3*e^(2*d*x + 2*c)/d + 1/2*(6*a*b^2 - 5*b^3)*(d*x + c)/d - 1/8*(12*a*b
^2*e^(2*d*x + 2*c) - 10*b^3*e^(2*d*x + 2*c) + b^3)*e^(-2*d*x - 2*c)/d - 2/3
*(9*a^2*b*e^(4*d*x + 4*c) - 18*a*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c
```

```
) + 6*a^3*e^(2*d*x + 2*c) - 18*a*b^2*e^(2*d*x + 2*c) + 12*b^3*e^(2*d*x + 2*
c) + 2*a^3 + 3*a^2*b - 12*a*b^2 + 7*b^3)/(d*(e^(2*d*x + 2*c) + 1)^3)
```

3.312 $\int \mathbf{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=103

$$\frac{3(a-b)\left((a+b)^2+4b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(a-b)^3\tanh(c+dx)\text{sech}^3(c+dx)}{4d}+\frac{3(a-b)^2(a+3b)\tanh(c+dx)\text{s}}{8d}$$

```
[Out] (3*(a - b)*(4*b^2 + (a + b)^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (b^3*Sinh[c +
 d*x])/d + (3*(a - b)^2*(a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + ((a
- b)^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rubi [A] time = 0.132009, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 390, 1157, 385, 203}

$$\frac{3(a-b)\left((a+b)^2+4b^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{(a-b)^3\tanh(c+dx)\text{sech}^3(c+dx)}{4d}+\frac{3(a-b)^2(a+3b)\tanh(c+dx)\text{s}}{8d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*(a - b)*(4*b^2 + (a + b)^2)*ArcTan[Sinh[c + d*x]])/(8*d) + (b^3*Sinh[c +
 d*x])/d + (3*(a - b)^2*(a + 3*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*d) + ((a
- b)^3*Sech[c + d*x]^3*Tanh[c + d*x])/(4*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(b^3+\frac{a^3-b^3+3b\left(a^2-b^2\right)x^2+3(a-b)b^2x^4}{\left(1+x^2\right)^3}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{b^3\sinh(c+dx)}{d}+\frac{\operatorname{Subst}\left(\int \frac{a^3-b^3+3b\left(a^2-b^2\right)x^2+3(a-b)b^2x^4}{\left(1+x^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{b^3\sinh(c+dx)}{d}+\frac{(a-b)^3\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4d}-\frac{\operatorname{Subst}\left(\int \frac{-3(a-b)(a\cdots}{\cdots}\right.}{\cdots}$$
$$= \frac{b^3\sinh(c+dx)}{d}+\frac{3(a-b)^2(a+3b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8d}+\frac{(a-b)^3\operatorname{se}\cdots}{\cdots}$$
$$= \frac{3(a-b)\left(4b^2+(a+b)^2\right)\tan^{-1}(\sinh(c+dx))}{8d}+\frac{b^3\sinh(c+dx)}{d}+\frac{3(a-b)^2\cdots}{\cdots}$$

Mathematica [C] time = 9.93774, size = 472, normalized size = 4.58

$$-\frac{\operatorname{csch}^5(c+dx)\left(256\sinh^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\text{HypergeometricPFQ}\left(\left\{\frac{3}{2},2,2,2,2,2\right\},\left\{1,1,1,1,\frac{11}{2}\right\},-\sinh\cdots\right.\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] -(Csch[c + d*x]^5*(256*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1,
 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3 + 384*H
ypergeometricPFQ[{3/2, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh
[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^2*(7*a + 5*b*Sinh[c + d*x]^2) - 21*(15*
a*b^2*Sinh[c + d*x]^4*(36015 + 21529*Sinh[c + d*x]^2 + 1128*Sinh[c + d*x]^4
) + 9*a^2*b*Sinh[c + d*x]^2*(72030 + 41615*Sinh[c + d*x]^2 + 2131*Sinh[c +
d*x]^4) + b^3*Sinh[c + d*x]^6*(149460 + 90805*Sinh[c + d*x]^2 + 4887*Sinh[c
 + d*x]^4) + a^3*(252105 + 140965*Sinh[c + d*x]^2 + 8226*Sinh[c + d*x]^4))
+ (315*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*(a^3*(16807 + 15000*Sinh[c + d*x]^2
+ 2187*Sinh[c + d*x]^4 - 62*Sinh[c + d*x]^6) + 9*a^2*b*Sinh[c + d*x]^2*(480
2 + 4375*Sinh[c + d*x]^2 + 640*Sinh[c + d*x]^4 + 3*Sinh[c + d*x]^6) + b^3*S
inh[c + d*x]^6*(9964 + 9375*Sinh[c + d*x]^2 + 1458*Sinh[c + d*x]^4 + 7*Sinh
[c + d*x]^6) + 3*a*b^2*Sinh[c + d*x]^4*(12005 + 11178*Sinh[c + d*x]^2 + 170
1*Sinh[c + d*x]^4 + 8*Sinh[c + d*x]^6)))/Sqrt[-Sinh[c + d*x]^2]))/(60480*d)
```

Maple [B] time = 0.062, size = 376, normalized size = 3.7

$$\frac{a^3\tanh(dx+c)(\operatorname{sech}(dx+c))^3}{4d}+\frac{3a^3\operatorname{sech}(dx+c)\tanh(dx+c)}{8d}+\frac{3a^3\arctan\left(e^{dx+c}\right)}{4d}-\frac{a^2b\sinh(dx+c)}{d(\cosh(dx+c))^4}+\frac{a^2b\,}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d*a^3*tanh(d*x+c)*sech(d*x+c)^3+3/8/d*a^3*sech(d*x+c)*tanh(d*x+c)+3/4/d
*a^3*arctan(exp(d*x+c))-1/d*a^2*b*sinh(d*x+c)/cosh(d*x+c)^4+1/4/d*a^2*b*tan
h(d*x+c)*sech(d*x+c)^3+3/8/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/4/d*a^2*b*arct
an(exp(d*x+c))-3/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^4-3/d*a*b^2*sinh(d*x+c)/
cosh(d*x+c)^4+3/4/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^3+9/8/d*a*b^2*sech(d*x+c)
*tanh(d*x+c)+9/4/d*a*b^2*arctan(exp(d*x+c))+1/d*b^3*sinh(d*x+c)^5/cosh(d*x+
c)^4+5/d*b^3*sinh(d*x+c)^3/cosh(d*x+c)^4+5/d*b^3*sinh(d*x+c)/cosh(d*x+c)^4-
5/4/d*b^3*tanh(d*x+c)*sech(d*x+c)^3-15/8/d*b^3*sech(d*x+c)*tanh(d*x+c)-15/4
/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.71048, size = 660, normalized size = 6.41

$$\frac{1}{4}b^3\left(\frac{15\arctan\left(e^{(-dx-c)}\right)}{d}-\frac{2e^{(-dx-c)}}{d}+\frac{17e^{(-2dx-2c)}+13e^{(-4dx-4c)}+7e^{(-6dx-6c)}-7e^{(-8dx-8c)}+2}{d\left(e^{(-dx-c)}+4e^{(-3dx-3c)}+6e^{(-5dx-5c)}+4e^{(-7dx-7c)}+e^{(-9dx-9c)}\right)}\right)-\frac{3}{4}ab^2\left(\frac{3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*b^3*(15*arctan(e^(-d*x - c))/d - 2*e^(-d*x - c)/d + (17*e^(-2*d*x - 2*c
) + 13*e^(-4*d*x - 4*c) + 7*e^(-6*d*x - 6*c) - 7*e^(-8*d*x - 8*c) + 2)/(d*(
e^(-d*x - c) + 4*e^(-3*d*x - 3*c) + 6*e^(-5*d*x - 5*c) + 4*e^(-7*d*x - 7*c)
 + e^(-9*d*x - 9*c)))) - 3/4*a*b^2*(3*arctan(e^(-d*x - c))/d + (5*e^(-d*x -
 c) - 3*e^(-3*d*x - 3*c) + 3*e^(-5*d*x - 5*c) - 5*e^(-7*d*x - 7*c))/(d*(4*e
^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c
) + 1))) - 1/4*a^3*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c) + 11*e^(-3*d
*x - 3*c) - 11*e^(-5*d*x - 5*c) - 3*e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c
) + 6*e^(-4*d*x - 4*c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1))) - 3/4
*a^2*b*(arctan(e^(-d*x - c))/d - (e^(-d*x - c) - 7*e^(-3*d*x - 3*c) + 7*e^(
-5*d*x - 5*c) - e^(-7*d*x - 7*c))/(d*(4*e^(-2*d*x - 2*c) + 6*e^(-4*d*x - 4*
c) + 4*e^(-6*d*x - 6*c) + e^(-8*d*x - 8*c) + 1)))
```

Fricas [B] time = 1.76507, size = 5520, normalized size = 53.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/4*(2*b^3*cosh(d*x + c)^10 + 20*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + 2*b^3*
sinh(d*x + c)^10 + 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 3*(3
0*b^3*cosh(d*x + c)^2 + a^3 + a^2*b - 5*a*b^2 + 5*b^3)*sinh(d*x + c)^8 + 24
*(10*b^3*cosh(d*x + c)^3 + (a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c))*s
```

```
inh(d*x + c)^7 + (11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + (4
20*b^3*cosh(d*x + c)^4 + 11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3 + 84*(a^3 + a^
2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 6*(84*b^3*cosh(d*
x + c)^5 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (11*a^3 - 2
1*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 - (11*a^3 - 21*a^
2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + (420*b^3*cosh(d*x + c)^6 + 210*(a^
3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - 11*a^3 + 21*a^2*b - 9*a*b^2
- 5*b^3 + 15*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 4*(60*b^3*cosh(d*x + c)^7 + 42*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*c
osh(d*x + c)^5 + 5*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 -
(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*b^
3 - 3*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 3*(30*b^3*cosh(d*x
+ c)^8 + 28*(a^3 + a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 5*(11*a^3 - 2
1*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - a^3 - a^2*b + 5*a*b^2 - 5*b^3
- 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 3*((a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^9 + 9*(a^3 + a^2*b + 3*a
*b^2 - 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^8 + (a^3 + a^2*b + 3*a*b^2 - 5*b^
3)*sinh(d*x + c)^9 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 4*
(a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x
 + c)^2)*sinh(d*x + c)^7 + 28*(3*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x +
 c)^3 + (a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^6 + 6*
(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 6*(21*(a^3 + a^2*b + 3*a*
b^2 - 5*b^3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 14*(a^3 + a^
2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 2*(63*(a^3 + a^2*
b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^5 + 70*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*c
osh(d*x + c)^3 + 15*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^4 + 4*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + 4*(21*(a^3 +
a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 35*(a^3 + a^2*b + 3*a*b^2 - 5*b^
3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 15*(a^3 + a^2*b + 3*a*
b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 12*(3*(a^3 + a^2*b + 3*a*b^
2 - 5*b^3)*cosh(d*x + c)^7 + 7*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c
)^5 + 5*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^3 + (a^3 + a^2*b + 3*
a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (a^3 + a^2*b + 3*a*b^2 - 5*
b^3)*cosh(d*x + c) + (9*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^8 + 2
8*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^6 + 30*(a^3 + a^2*b + 3*a*b
^2 - 5*b^3)*cosh(d*x + c)^4 + a^3 + a^2*b + 3*a*b^2 - 5*b^3 + 12*(a^3 + a^2
*b + 3*a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*arctan(cosh(d*x + c)
+ sinh(d*x + c)) + 2*(10*b^3*cosh(d*x + c)^9 + 12*(a^3 + a^2*b - 5*a*b^2 +
5*b^3)*cosh(d*x + c)^7 + 3*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x +
 c)^5 - 2*(11*a^3 - 21*a^2*b + 9*a*b^2 + 5*b^3)*cosh(d*x + c)^3 - 3*(a^3 +
a^2*b - 5*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c))/(d*cosh(d*x + c)^9 +
 9*d*cosh(d*x + c)*sinh(d*x + c)^8 + d*sinh(d*x + c)^9 + 4*d*cosh(d*x + c)^
7 + 4*(9*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^7 + 28*(3*d*cosh(d*x + c)^3 +
 d*cosh(d*x + c))*sinh(d*x + c)^6 + 6*d*cosh(d*x + c)^5 + 6*(21*d*cosh(d*x
+ c)^4 + 14*d*cosh(d*x + c)^2 + d)*sinh(d*x + c)^5 + 2*(63*d*cosh(d*x + c)^
5 + 70*d*cosh(d*x + c)^3 + 15*d*cosh(d*x + c))*sinh(d*x + c)^4 + 4*d*cosh(d
*x + c)^3 + 4*(21*d*cosh(d*x + c)^6 + 35*d*cosh(d*x + c)^4 + 15*d*cosh(d*x
+ c)^2 + d)*sinh(d*x + c)^3 + 12*(3*d*cosh(d*x + c)^7 + 7*d*cosh(d*x + c)^5
 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x + c))*sinh(d*x + c)^2 + d*cosh(d*x + c)
 + (9*d*cosh(d*x + c)^8 + 28*d*cosh(d*x + c)^6 + 30*d*cosh(d*x + c)^4 + 12*
d*cosh(d*x + c)^2 + d)*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.35793, size = 412, normalized size = 4.

$$\frac{b^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{2\,d}+\frac{3\left(\pi+2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}\right)\right)\left(a^3+a^2b+3\,ab^2-5\,b^3\right)}{16\,d}+\frac{3\,a^3\left(e^{(dx+c)}-e^{(-dx-c)}\right)^3}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/2*b^3*(e^(d*x + c) - e^(-d*x - c))/d + 3/16*(pi + 2*arctan(1/2*(e^(2*d*x
+ 2*c) - 1)*e^(-d*x - c)))*(a^3 + a^2*b + 3*a*b^2 - 5*b^3)/d + 1/4*(3*a^3*(
e^(d*x + c) - e^(-d*x - c))^3 + 3*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 - 15
*a*b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 9*b^3*(e^(d*x + c) - e^(-d*x - c))^
3 + 20*a^3*(e^(d*x + c) - e^(-d*x - c)) - 12*a^2*b*(e^(d*x + c) - e^(-d*x -
 c)) - 36*a*b^2*(e^(d*x + c) - e^(-d*x - c)) + 28*b^3*(e^(d*x + c) - e^(-d*
x - c)))/(((e^(d*x + c) - e^(-d*x - c))^2 + 4)^2*d)
```

3.313 $\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=74

$$\frac{(a^3-b^3)\tanh(c+dx)}{d}+\frac{(a-b)^3\tanh^5(c+dx)}{5d}-\frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}+b^3x$$

```
[Out] b^3*x + ((a^3 - b^3)*Tanh[c + d*x])/d - ((a - b)^2*(2*a + b)*Tanh[c + d*x]^
3)/(3*d) + ((a - b)^3*Tanh[c + d*x]^5)/(5*d)
```

Rubi [A] time = 0.0798852, antiderivative size = 74, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3191, 390, 206}

$$\frac{(a^3-b^3)\tanh(c+dx)}{d}+\frac{(a-b)^3\tanh^5(c+dx)}{5d}-\frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d}+b^3x$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] b^3*x + ((a^3 - b^3)*Tanh[c + d*x])/d - ((a - b)^2*(2*a + b)*Tanh[c + d*x]^
3)/(3*d) + ((a - b)^3*Tanh[c + d*x]^5)/(5*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^6(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a-(a-b)x^2\right)^3}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \left(a^3-b^3-(a-b)^2(2a+b)x^2+(a-b)^3x^4+\frac{b^3}{1-x^2}\right)dx, x, \tanh\right.}{d}$$
$$= \frac{\left(a^3-b^3\right)\tanh(c+dx)}{d} - \frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d} + \frac{(a-b)^3\tanh}{5d}$$
$$= b^3x + \frac{\left(a^3-b^3\right)\tanh(c+dx)}{d} - \frac{(a-b)^2(2a+b)\tanh^3(c+dx)}{3d} + \frac{(a-b}{}$$

Mathematica [A] time = 0.731567, size = 86, normalized size = 1.16

$$\frac{(a-b)\tanh(c+dx)\left(\left(4a^2+7ab-11b^2\right)\operatorname{sech}^2(c+dx)+8a^2+3(a-b)^2\operatorname{sech}^4(c+dx)+14ab+23b^2\right)+15b^3(c+dx)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (15*b^3*(c + d*x) + (a - b)*(8*a^2 + 14*a*b + 23*b^2 + (4*a^2 + 7*a*b - 11*
b^2)*Sech[c + d*x]^2 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(15*d)
```

Maple [B] time = 0.058, size = 199, normalized size = 2.7

$$\frac{1}{d}\left(a^3\left(\frac{8}{15}+\frac{(\operatorname{sech}(dx+c))^4}{5}+\frac{4(\operatorname{sech}(dx+c))^2}{15}\right)\tanh(dx+c)+3a^2b\left(-1/4\frac{\sinh(dx+c)}{(\cosh(dx+c))^5}+1/4\left(\frac{8}{15}+1/5(\operatorname{sec}\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c)+3*a^2*b*(-
1/4*sinh(d*x+c)/cosh(d*x+c)^5+1/4*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^
2)*tanh(d*x+c))+3*a*b^2*(-1/2*sinh(d*x+c)^3/cosh(d*x+c)^5-3/8*sinh(d*x+c)/c
osh(d*x+c)^5+3/8*(8/15+1/5*sech(d*x+c)^4+4/15*sech(d*x+c)^2)*tanh(d*x+c))+b
^3*(d*x+c-tanh(d*x+c)-1/3*tanh(d*x+c)^3-1/5*tanh(d*x+c)^5))
```

Maxima [B] time = 1.24156, size = 1112, normalized size = 15.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*b^3*(15*x + 15*c/d - 2*(70*e^(-2*d*x - 2*c) + 140*e^(-4*d*x - 4*c) + 9
0*e^(-6*d*x - 6*c) + 45*e^(-8*d*x - 8*c) + 23)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1))) + 16/15*a^3*(5*e^(-2*d*x - 2*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^
```

```
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)) + 10*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c
) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 4/5*a^2*b*(5*e^(-2*d*x - 2*c)/(
d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8
*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) - 5*e^(-4*d*x - 4*c)/(d*(5*e^(-2*d*x
 - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) +
e^(-10*d*x - 10*c) + 1)) + 15*e^(-6*d*x - 6*c)/(d*(5*e^(-2*d*x - 2*c) + 10*
e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x -
10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
 - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1))) + 6/5*a*b^2*(10*e^
(-4*d*x - 4*c)/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x
- 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 5*e^(-8*d*x - 8*c)
/(d*(5*e^(-2*d*x - 2*c) + 10*e^(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(
-8*d*x - 8*c) + e^(-10*d*x - 10*c) + 1)) + 1/(d*(5*e^(-2*d*x - 2*c) + 10*e^
(-4*d*x - 4*c) + 10*e^(-6*d*x - 6*c) + 5*e^(-8*d*x - 8*c) + e^(-10*d*x - 10
*c) + 1)))
```

Fricas [B] time = 1.56797, size = 1287, normalized size = 17.39

$$\frac{\left(15\,b^3dx-8\,a^3-6\,a^2b-9\,ab^2+23\,b^3\right)\cosh\left(dx+c\right)^5+5\left(15\,b^3dx-8\,a^3-6\,a^2b-9\,ab^2+23\,b^3\right)\cosh\left(dx+c\right)\sinh\left(dx\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*((15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)^5 + 5
*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)*sinh(d*x +
 c)^4 + (8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*sinh(d*x + c)^5 + 5*(15*b^3*d*
x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh(d*x + c)^3 + 5*(8*a^3 + 6*a^2*
b - 9*a*b^2 - 5*b^3 + 2*(8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^3 + 5*(2*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)
*cosh(d*x + c)^3 + 3*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 23*b^3)*cosh
(d*x + c))*sinh(d*x + c)^2 + 10*(15*b^3*d*x - 8*a^3 - 6*a^2*b - 9*a*b^2 + 2
3*b^3)*cosh(d*x + c) + 5*((8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)*cosh(d*x + c
)^4 + 16*a^3 - 24*a^2*b + 18*a*b^2 - 10*b^3 + 3*(8*a^3 + 6*a^2*b - 9*a*b^2
- 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(d*cosh(d*x + c)^5 + 5*d*cosh(d*x
+ c)*sinh(d*x + c)^4 + 5*d*cosh(d*x + c)^3 + 5*(2*d*cosh(d*x + c)^3 + 3*d*c
osh(d*x + c))*sinh(d*x + c)^2 + 10*d*cosh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.38027, size = 288, normalized size = 3.89

$$\frac{(dx+c)b^3}{d} - \frac{2\left(45\,ab^2e^{(8\,dx+8\,c)} - 45\,b^3e^{(8\,dx+8\,c)} + 90\,a^2be^{(6\,dx+6\,c)} - 90\,b^3e^{(6\,dx+6\,c)} + 80\,a^3e^{(4\,dx+4\,c)} - 30\,a^2be^{(4\,dx+4\,c)} + 9}{15\,d\big(e}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (d*x + c)*b^3/d - 2/15*(45*a*b^2*e^(8*d*x + 8*c) - 45*b^3*e^(8*d*x + 8*c) +
 90*a^2*b*e^(6*d*x + 6*c) - 90*b^3*e^(6*d*x + 6*c) + 80*a^3*e^(4*d*x + 4*c)
 - 30*a^2*b*e^(4*d*x + 4*c) + 90*a*b^2*e^(4*d*x + 4*c) - 140*b^3*e^(4*d*x +
 4*c) + 40*a^3*e^(2*d*x + 2*c) + 30*a^2*b*e^(2*d*x + 2*c) - 70*b^3*e^(2*d*x
 + 2*c) + 8*a^3 + 6*a^2*b + 9*a*b^2 - 23*b^3)/(d*(e^(2*d*x + 2*c) + 1)^5)
```

3.314 $\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=154

$$\frac{(a+b)\left(5a^2-2ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{(a-b)\left(15a^2+14ab+15b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d}+\frac{5\left(a^2-b^2\right)\operatorname{ta}}{}$$

```
[Out] ((a + b)*(5*a^2 - 2*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((a - b)*(
15*a^2 + 14*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*(a^2 - b
^2)*Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + ((a - b
)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```

Rubi [A] time = 0.152918, antiderivative size = 154, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 413, 526, 385, 203}

$$\frac{(a+b)\left(5a^2-2ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d}+\frac{(a-b)\left(15a^2+14ab+15b^2\right)\tanh(c+dx)\operatorname{sech}(c+dx)}{48d}+\frac{5\left(a^2-b^2\right)\operatorname{ta}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] ((a + b)*(5*a^2 - 2*a*b + 5*b^2)*ArcTan[Sinh[c + d*x]])/(16*d) + ((a - b)*(
15*a^2 + 14*a*b + 15*b^2)*Sech[c + d*x]*Tanh[c + d*x])/(48*d) + (5*(a^2 - b
^2)*Sech[c + d*x]^3*(a + b*Sinh[c + d*x]^2)*Tanh[c + d*x])/(24*d) + ((a - b
)*Sech[c + d*x]^5*(a + b*Sinh[c + d*x]^2)^2*Tanh[c + d*x])/(6*d)
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^7(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^3}{\left(1+x^2\right)^4}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a-b)\operatorname{sech}^5(c+dx)\left(a+b\sinh^2(c+dx)\right)^2\tanh(c+dx)}{6d} + \frac{\operatorname{Subst}\left(\int \frac{(a}{}\right.}{}$$

$$= \frac{5\left(a^2-b^2\right)\operatorname{sech}^3(c+dx)\left(a+b\sinh^2(c+dx)\right)\tanh(c+dx)}{24d} + \frac{(a-b)\mathrm{se}}{}$$

$$= \frac{(a-b)\left(15a^2+14ab+15b^2\right)\operatorname{sech}(c+dx)\tanh(c+dx)}{48d} + \frac{5\left(a^2-b^2\right)\mathrm{se}}{}$$

$$= \frac{(a+b)\left(5a^2-2ab+5b^2\right)\tan^{-1}(\sinh(c+dx))}{16d} + \frac{(a-b)\left(15a^2+14ab+\right.}{}$$

Mathematica [C] time = 14.2621, size = 1192, normalized size = 7.74

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[c + d*x]^7*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (Csch[c + d*x]^5*(-117228825*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]] - 10926562
5*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 274542345*a^2*b*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^2 - 17069535*a^3*ArcTanh[Sqrt[-S
inh[c + d*x]^2]]*Sinh[c + d*x]^4 - 260465625*a^2*b*ArcTanh[Sqrt[-Sinh[c + d
*x]^2]]*Sinh[c + d*x]^4 - 215549775*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*S
inh[c + d*x]^4 + 142065*a^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6
 - 41427855*a^2*b*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 2071732
95*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 58009455*b^3*Arc
Tanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^6 - 210735*a^2*b*ArcTanh[Sqrt[-S
inh[c + d*x]^2]]*Sinh[c + d*x]^8 - 33756345*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*
x]^2]]*Sinh[c + d*x]^8 - 56109375*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[
c + d*x]^8 - 174825*a*b^2*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10
- 9261945*b^3*ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^10 - 48825*b^3*
ArcTanh[Sqrt[-Sinh[c + d*x]^2]]*Sinh[c + d*x]^12 + 117228825*a^3*Sqrt[-Sinh
[c + d*x]^2] + 4093425*a^3*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 1689515
10*a^2*b*Sinh[c + d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 215549775*a*b^2*Sinh[c +
d*x]^4*Sqrt[-Sinh[c + d*x]^2] + 9514449*a^2*b*Sinh[c + d*x]^6*Sqrt[-Sinh[c
+ d*x]^2] + 135323370*a*b^2*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 580094
```

```
55*b^3*Sinh[c + d*x]^6*Sqrt[-Sinh[c + d*x]^2] + 7808535*a*b^2*Sinh[c + d*x]
^8*Sqrt[-Sinh[c + d*x]^2] + 36772890*b^3*Sinh[c + d*x]^8*Sqrt[-Sinh[c + d*x
]^2] + 2160711*b^3*Sinh[c + d*x]^10*Sqrt[-Sinh[c + d*x]^2] - 70189350*a^3*(
-Sinh[c + d*x]^2)^(3/2) - 274542345*a^2*b*(-Sinh[c + d*x]^2)^(3/2) + 1024*a
^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[
c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2) + 3072*a^2*b*Hypergeom
etricPFQ[{3/2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*
Sinh[c + d*x]^8*(-Sinh[c + d*x]^2)^(3/2) + 3072*a*b^2*HypergeometricPFQ[{3/
2, 2, 2, 2, 2, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x
]^10*(-Sinh[c + d*x]^2)^(3/2) + 1024*b^3*HypergeometricPFQ[{3/2, 2, 2, 2, 2
, 2, 2}, {1, 1, 1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^12*(-Sinh[c
 + d*x]^2)^(3/2) + 1536*HypergeometricPFQ[{3/2, 2, 2, 2, 2, 2}, {1, 1, 1, 1
, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c + d*x]^2)^(3/2)*(a + b*
Sinh[c + d*x]^2)^2*(9*a + 7*b*Sinh[c + d*x]^2) + 256*HypergeometricPFQ[{3/2
, 2, 2, 2, 2}, {1, 1, 1, 11/2}, -Sinh[c + d*x]^2]*Sinh[c + d*x]^6*(-Sinh[c
+ d*x]^2)^(3/2)*(295*a^3 + 741*a^2*b*Sinh[c + d*x]^2 + 621*a*b^2*Sinh[c + d
*x]^4 + 175*b^3*Sinh[c + d*x]^6)))/(725760*d*Sqrt[-Sinh[c + d*x]^2])
```

Maple [B] time = 0.062, size = 467, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/6/d*a^3*tanh(d*x+c)*sech(d*x+c)^5+5/24/d*a^3*tanh(d*x+c)*sech(d*x+c)^3+5/
16/d*a^3*sech(d*x+c)*tanh(d*x+c)+5/8/d*a^3*arctan(exp(d*x+c))-3/5/d*a^2*b*s
inh(d*x+c)/cosh(d*x+c)^6+1/10/d*a^2*b*tanh(d*x+c)*sech(d*x+c)^5+1/8/d*a^2*b
*tanh(d*x+c)*sech(d*x+c)^3+3/16/d*a^2*b*sech(d*x+c)*tanh(d*x+c)+3/8/d*a^2*b
*arctan(exp(d*x+c))-1/d*a*b^2*sinh(d*x+c)^3/cosh(d*x+c)^6-3/5/d*a*b^2*sinh(
d*x+c)/cosh(d*x+c)^6+1/10/d*a*b^2*tanh(d*x+c)*sech(d*x+c)^5+1/8/d*a*b^2*tan
h(d*x+c)*sech(d*x+c)^3+3/16/d*a*b^2*sech(d*x+c)*tanh(d*x+c)+3/8/d*a*b^2*arc
tan(exp(d*x+c))-1/d*b^3*sinh(d*x+c)^5/cosh(d*x+c)^6-5/3/d*b^3*sinh(d*x+c)^3
/cosh(d*x+c)^6-1/d*b^3*sinh(d*x+c)/cosh(d*x+c)^6+1/6/d*b^3*tanh(d*x+c)*sech
(d*x+c)^5+5/24/d*b^3*tanh(d*x+c)*sech(d*x+c)^3+5/16/d*b^3*sech(d*x+c)*tanh(
d*x+c)+5/8/d*b^3*arctan(exp(d*x+c))
```

Maxima [B] time = 1.83213, size = 872, normalized size = 5.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/24*b^3*(15*arctan(e^(-d*x - c))/d + (33*e^(-d*x - c) - 5*e^(-3*d*x - 3*c
) + 90*e^(-5*d*x - 5*c) - 90*e^(-7*d*x - 7*c) + 5*e^(-9*d*x - 9*c) - 33*e^(
-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) + 20*e^(-6*d*
x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*d*x - 12*c)
+ 1))) - 1/24*a^3*(15*arctan(e^(-d*x - c))/d - (15*e^(-d*x - c) + 85*e^(-3*
d*x - 3*c) + 198*e^(-5*d*x - 5*c) - 198*e^(-7*d*x - 7*c) - 85*e^(-9*d*x - 9
*c) - 15*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*x - 4*c) +
 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c) + e^(-12*
```

```
d*x - 12*c) + 1))) - 1/8*a^2*b*(3*arctan(e^(-d*x - c))/d - (3*e^(-d*x - c)
+ 17*e^(-3*d*x - 3*c) - 114*e^(-5*d*x - 5*c) + 114*e^(-7*d*x - 7*c) - 17*e^
(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*e^(-4*d*
x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x - 10*c)
 + e^(-12*d*x - 12*c) + 1))) - 1/8*a*b^2*(3*arctan(e^(-d*x - c))/d - (3*e^(
-d*x - c) - 47*e^(-3*d*x - 3*c) + 78*e^(-5*d*x - 5*c) - 78*e^(-7*d*x - 7*c)
 + 47*e^(-9*d*x - 9*c) - 3*e^(-11*d*x - 11*c))/(d*(6*e^(-2*d*x - 2*c) + 15*
e^(-4*d*x - 4*c) + 20*e^(-6*d*x - 6*c) + 15*e^(-8*d*x - 8*c) + 6*e^(-10*d*x
 - 10*c) + e^(-12*d*x - 12*c) + 1)))
```

Fricas [B] time = 1.81393, size = 9044, normalized size = 58.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/24*(3*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^11 + 33*(5*a^3 +
 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)*sinh(d*x + c)^10 + 3*(5*a^3 + 3*
a^2*b + 3*a*b^2 - 11*b^3)*sinh(d*x + c)^11 + (85*a^3 + 51*a^2*b - 141*a*b^2
 + 5*b^3)*cosh(d*x + c)^9 + (85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3 + 165*(5
*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^9 + 9*(55
*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^3 + (85*a^3 + 51*a^2*b
- 141*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^8 + 18*(11*a^3 - 19*a^2*b
 + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^7 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 -
11*b^3)*cosh(d*x + c)^4 + 11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3 + 2*(85*a^3
+ 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^7 + 42*(33*(
5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x + c)^5 + 2*(85*a^3 + 51*a^2*b
- 141*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*
b^3)*cosh(d*x + c))*sinh(d*x + c)^6 - 18*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*
b^3)*cosh(d*x + c)^5 + 18*(77*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)*cosh(d*x
 + c)^6 + 7*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^4 - 11*a^
3 + 19*a^2*b - 13*a*b^2 + 5*b^3 + 21*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 18*(55*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*
b^3)*cosh(d*x + c)^7 + 7*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x +
 c)^5 + 35*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - 5*(11*a
^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - (85*a^3
+ 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + (495*(5*a^3 + 3*a^2*b + 3
*a*b^2 - 11*b^3)*cosh(d*x + c)^8 + 84*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^
3)*cosh(d*x + c)^6 + 630*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x +
c)^4 - 85*a^3 - 51*a^2*b + 141*a*b^2 - 5*b^3 - 180*(11*a^3 - 19*a^2*b + 13*
a*b^2 - 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 3*(55*(5*a^3 + 3*a^2*b +
3*a*b^2 - 11*b^3)*cosh(d*x + c)^9 + 12*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b
^3)*cosh(d*x + c)^7 + 126*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x +
 c)^5 - 60*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^3 - (85*a^3
 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + 3*((5*a^3
 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^12 + 12*(5*a^3 + 3*a^2*b + 3*a*
b^2 + 5*b^3)*cosh(d*x + c)*sinh(d*x + c)^11 + (5*a^3 + 3*a^2*b + 3*a*b^2 +
5*b^3)*sinh(d*x + c)^12 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x +
c)^10 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 11*(5*a^3 + 3*a^2*b + 3*a*b^
2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^10 + 20*(11*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh
(d*x + c))*sinh(d*x + c)^9 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*
x + c)^8 + 15*(33*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 5*a
^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 18*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cos
h(d*x + c)^2)*sinh(d*x + c)^8 + 24*(33*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*
```

```
cosh(d*x + c)^5 + 30*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^3 +
5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 20*(
5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 4*(231*(5*a^3 + 3*a^2*
b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 315*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b
^3)*cosh(d*x + c)^4 + 25*a^3 + 15*a^2*b + 15*a*b^2 + 25*b^3 + 105*(5*a^3 +
3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 24*(33*(5*a^3
 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 63*(5*a^3 + 3*a^2*b + 3*a*b
^2 + 5*b^3)*cosh(d*x + c)^5 + 35*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d
*x + c)^3 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x +
 c)^5 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4 + 15*(33*(5*
a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 84*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^6 + 70*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cos
h(d*x + c)^4 + 5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 20*(5*a^3 + 3*a^2*b + 3*
a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 20*(11*(5*a^3 + 3*a^2*b +
 3*a*b^2 + 5*b^3)*cosh(d*x + c)^9 + 36*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*
cosh(d*x + c)^7 + 42*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^5 +
20*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^3 + 3*(5*a^3 + 3*a^2*b
 + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a^3 + 3*a^2*b + 3*a*
b^2 + 5*b^3 + 6*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^2 + 6*(11
*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^10 + 45*(5*a^3 + 3*a^2*b
 + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^8 + 70*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^6 + 50*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^4
+ 5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3 + 15*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 12*((5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3
)*cosh(d*x + c)^11 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^9
+ 10*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^7 + 10*(5*a^3 + 3*a^
2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c)^5 + 5*(5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b
^3)*cosh(d*x + c)^3 + (5*a^3 + 3*a^2*b + 3*a*b^2 + 5*b^3)*cosh(d*x + c))*si
nh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 3*(5*a^3 + 3*a^2*b + 3
*a*b^2 - 11*b^3)*cosh(d*x + c) + 3*(11*(5*a^3 + 3*a^2*b + 3*a*b^2 - 11*b^3)
*cosh(d*x + c)^10 + 3*(85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)
^8 + 42*(11*a^3 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^6 - 30*(11*a^3
 - 19*a^2*b + 13*a*b^2 - 5*b^3)*cosh(d*x + c)^4 - 5*a^3 - 3*a^2*b - 3*a*b^2
 + 11*b^3 - (85*a^3 + 51*a^2*b - 141*a*b^2 + 5*b^3)*cosh(d*x + c)^2)*sinh(d
*x + c))/(d*cosh(d*x + c)^12 + 12*d*cosh(d*x + c)*sinh(d*x + c)^11 + d*sinh
(d*x + c)^12 + 6*d*cosh(d*x + c)^10 + 6*(11*d*cosh(d*x + c)^2 + d)*sinh(d*x
 + c)^10 + 20*(11*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x + c)^9 +
15*d*cosh(d*x + c)^8 + 15*(33*d*cosh(d*x + c)^4 + 18*d*cosh(d*x + c)^2 + d)
*sinh(d*x + c)^8 + 24*(33*d*cosh(d*x + c)^5 + 30*d*cosh(d*x + c)^3 + 5*d*co
sh(d*x + c))*sinh(d*x + c)^7 + 20*d*cosh(d*x + c)^6 + 4*(231*d*cosh(d*x + c
)^6 + 315*d*cosh(d*x + c)^4 + 105*d*cosh(d*x + c)^2 + 5*d)*sinh(d*x + c)^6
+ 24*(33*d*cosh(d*x + c)^7 + 63*d*cosh(d*x + c)^5 + 35*d*cosh(d*x + c)^3 +
5*d*cosh(d*x + c))*sinh(d*x + c)^5 + 15*d*cosh(d*x + c)^4 + 15*(33*d*cosh(d
*x + c)^8 + 84*d*cosh(d*x + c)^6 + 70*d*cosh(d*x + c)^4 + 20*d*cosh(d*x + c
)^2 + d)*sinh(d*x + c)^4 + 20*(11*d*cosh(d*x + c)^9 + 36*d*cosh(d*x + c)^7
+ 42*d*cosh(d*x + c)^5 + 20*d*cosh(d*x + c)^3 + 3*d*cosh(d*x + c))*sinh(d*x
 + c)^3 + 6*d*cosh(d*x + c)^2 + 6*(11*d*cosh(d*x + c)^10 + 45*d*cosh(d*x +
c)^8 + 70*d*cosh(d*x + c)^6 + 50*d*cosh(d*x + c)^4 + 15*d*cosh(d*x + c)^2 +
 d)*sinh(d*x + c)^2 + 12*(d*cosh(d*x + c)^11 + 5*d*cosh(d*x + c)^9 + 10*d*c
osh(d*x + c)^7 + 10*d*cosh(d*x + c)^5 + 5*d*cosh(d*x + c)^3 + d*cosh(d*x +
c))*sinh(d*x + c) + d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**7*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.29618, size = 518, normalized size = 3.36

$$\frac{\left(\pi + 2\,\arctan\left(\frac{1}{2}\left(e^{(2\,dx+2\,c)} - 1\right)e^{(-dx-c)}\right)\right)\left(5\,a^3 + 3\,a^2b + 3\,ab^2 + 5\,b^3\right)}{32\,d} + \frac{15\,a^3\left(e^{(dx+c)} - e^{(-dx-c)}\right)^5 + 9\,a^2b\left(e^{(dx+c)} - e^{(-d}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^7*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/32*(pi + 2*arctan(1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)))*(5*a^3 + 3*a^2
*b + 3*a*b^2 + 5*b^3)/d + 1/24*(15*a^3*(e^(d*x + c) - e^(-d*x - c))^5 + 9*a
^2*b*(e^(d*x + c) - e^(-d*x - c))^5 + 9*a*b^2*(e^(d*x + c) - e^(-d*x - c))^
5 - 33*b^3*(e^(d*x + c) - e^(-d*x - c))^5 + 160*a^3*(e^(d*x + c) - e^(-d*x
- c))^3 + 96*a^2*b*(e^(d*x + c) - e^(-d*x - c))^3 - 96*a*b^2*(e^(d*x + c) -
 e^(-d*x - c))^3 - 160*b^3*(e^(d*x + c) - e^(-d*x - c))^3 + 528*a^3*(e^(d*x
 + c) - e^(-d*x - c)) - 144*a^2*b*(e^(d*x + c) - e^(-d*x - c)) - 144*a*b^2*
(e^(d*x + c) - e^(-d*x - c)) - 240*b^3*(e^(d*x + c) - e^(-d*x - c)))/(((e^(
d*x + c) - e^(-d*x - c))^2 + 4)^3*d)
```

3.315 $\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx$

Optimal. Leaf size=80

$$-\frac{a^2(a-b)\tanh^3(c+dx)}{d}+\frac{a^3\tanh(c+dx)}{d}-\frac{(a-b)^3\tanh^7(c+dx)}{7d}+\frac{3a(a-b)^2\tanh^5(c+dx)}{5d}$$

```
[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - b)*Tanh[c + d*x]^3)/d + (3*a*(a - b)^2*Ta
nh[c + d*x]^5)/(5*d) - ((a - b)^3*Tanh[c + d*x]^7)/(7*d)
```

Rubi [A] time = 0.0738987, antiderivative size = 80, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 194}

$$-\frac{a^2(a-b)\tanh^3(c+dx)}{d}+\frac{a^3\tanh(c+dx)}{d}-\frac{(a-b)^3\tanh^7(c+dx)}{7d}+\frac{3a(a-b)^2\tanh^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (a^3*Tanh[c + d*x])/d - (a^2*(a - b)*Tanh[c + d*x]^3)/d + (3*a*(a - b)^2*Ta
nh[c + d*x]^5)/(5*d) - ((a - b)^3*Tanh[c + d*x]^7)/(7*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^8(c+dx)\left(a+b\sinh^2(c+dx)\right)^3\,dx &= \frac{\operatorname{Subst}\left(\int\left(a-(a-b)x^2\right)^3\,dx,x,\tanh(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int\left(a^3-3a^2(a-b)x^2+3a(a-b)^2x^4-(a-b)^3x^6\right)dx,x,\tanh(c+d\right.}{d}\\ &= \frac{a^3\tanh(c+dx)}{d}-\frac{a^2(a-b)\tanh^3(c+dx)}{d}+\frac{3a(a-b)^2\tanh^5(c+dx)}{5d}-\end{aligned}$$

Mathematica [B] time = 0.569323, size = 163, normalized size = 2.04

$$\frac{\tanh(c+dx)\operatorname{sech}^6(c+dx)\left(\left(232a^2b+464a^3-246ab^2+75b^3\right)\cosh(2(c+dx))+2\left(32a^2b+64a^3+24ab^2-15b^3\right)\cosh\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^8*(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((512*a^3 - 304*a^2*b + 192*a*b^2 - 50*b^3 + (464*a^3 + 232*a^2*b - 246*a*b
^2 + 75*b^3)*Cosh[2*(c + d*x)] + 2*(64*a^3 + 32*a^2*b + 24*a*b^2 - 15*b^3)*
Cosh[4*(c + d*x)] + 16*a^3*Cosh[6*(c + d*x)] + 8*a^2*b*Cosh[6*(c + d*x)] +
6*a*b^2*Cosh[6*(c + d*x)] + 5*b^3*Cosh[6*(c + d*x)])*Sech[c + d*x]^6*Tanh[c
 + d*x])/(1120*d)
```

Maple [B] time = 0.089, size = 289, normalized size = 3.6

$$\frac{1}{d}\left(a^3\left(\frac{16}{35}+\frac{(\operatorname{sech}(dx+c))^6}{7}+\frac{6\,(\operatorname{sech}(dx+c))^4}{35}+\frac{8\,(\operatorname{sech}(dx+c))^2}{35}\right)\tanh(dx+c)+3\,a^2b\left(-1/6\,\frac{\sinh(dx+c)}{(\cosh(dx+c))^7}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d*(a^3*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*ta
nh(d*x+c)+3*a^2*b*(-1/6*sinh(d*x+c)/cosh(d*x+c)^7+1/6*(16/35+1/7*sech(d*x+c
)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+3*a*b^2*(-1/4*sinh(
d*x+c)^3/cosh(d*x+c)^7-1/8*sinh(d*x+c)/cosh(d*x+c)^7+1/8*(16/35+1/7*sech(d*
x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)^2)*tanh(d*x+c))+b^3*(-1/2*sinh(d
*x+c)^5/cosh(d*x+c)^7-5/8*sinh(d*x+c)^3/cosh(d*x+c)^7-5/16*sinh(d*x+c)/cosh
(d*x+c)^7+5/16*(16/35+1/7*sech(d*x+c)^6+6/35*sech(d*x+c)^4+8/35*sech(d*x+c)
^2)*tanh(d*x+c)))
```

Maxima [B] time = 1.28064, size = 2368, normalized size = 29.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 32/35*a^3*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*
e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d
*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4
*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 1
0*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 16/35*a^2*b*(7*e^
(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x
- 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
 + e^(-14*d*x - 14*c) + 1)) + 21*e^(-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) +
21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10
*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-6*d
*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 70*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^
(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x
- 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d
```

```
*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c)
 + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
 + 12/35*a*b^2*(7*e^(-2*d*x - 2*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x -
4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) +
7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 14*e^(-4*d*x - 4*c)/(d*(7
*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*
x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c
) + 1)) + 70*e^(-6*d*x - 6*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c)
+ 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(
-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) - 35*e^(-8*d*x - 8*c)/(d*(7*e^(-
2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8
*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1
)) + 35*e^(-10*d*x - 10*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 3
5*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12
*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(
-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x -
 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1))) + 2/7*b^3*(21*e^(
-4*d*x - 4*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x -
 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c)
+ e^(-14*d*x - 14*c) + 1)) + 35*e^(-8*d*x - 8*c)/(d*(7*e^(-2*d*x - 2*c) + 2
1*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*
d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)) + 7*e^(-12*d*
x - 12*c)/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 35*e^(-6*d*x - 6*c
) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12*d*x - 12*c) + e^
(-14*d*x - 14*c) + 1)) + 1/(d*(7*e^(-2*d*x - 2*c) + 21*e^(-4*d*x - 4*c) + 3
5*e^(-6*d*x - 6*c) + 35*e^(-8*d*x - 8*c) + 21*e^(-10*d*x - 10*c) + 7*e^(-12
*d*x - 12*c) + e^(-14*d*x - 14*c) + 1)))
```

Fricas [B] time = 1.53974, size = 2063, normalized size = 25.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] -4/35*((8*a^3 + 4*a^2*b + 3*a*b^2 + 20*b^3)*cosh(d*x + c)^6 - 6*(8*a^3 + 4*
a^2*b + 3*a*b^2 - 15*b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (8*a^3 + 4*a^2*b
+ 3*a*b^2 + 20*b^3)*sinh(d*x + c)^6 + 14*(4*a^3 + 2*a^2*b + 9*a*b^2)*cosh(d
*x + c)^4 + (56*a^3 + 28*a^2*b + 126*a*b^2 + 15*(8*a^3 + 4*a^2*b + 3*a*b^2
+ 20*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 - 4*(5*(8*a^3 + 4*a^2*b + 3*a*b^
2 - 15*b^3)*cosh(d*x + c)^3 + 28*(2*a^3 + a^2*b - 3*a*b^2)*cosh(d*x + c))*s
inh(d*x + c)^3 + 280*a^3 - 140*a^2*b + 210*a*b^2 + 7*(24*a^3 + 52*a^2*b - 2
1*a*b^2 + 20*b^3)*cosh(d*x + c)^2 + (15*(8*a^3 + 4*a^2*b + 3*a*b^2 + 20*b^3
)*cosh(d*x + c)^4 + 168*a^3 + 364*a^2*b - 147*a*b^2 + 140*b^3 + 84*(4*a^3 +
 2*a^2*b + 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*(3*(8*a^3 + 4*a^2*
b + 3*a*b^2 - 15*b^3)*cosh(d*x + c)^5 + 56*(2*a^3 + a^2*b - 3*a*b^2)*cosh(d
*x + c)^3 + 7*(24*a^3 - 28*a^2*b + 9*a*b^2 - 5*b^3)*cosh(d*x + c))*sinh(d*x
 + c))/(d*cosh(d*x + c)^8 + 8*d*cosh(d*x + c)*sinh(d*x + c)^7 + d*sinh(d*x
+ c)^8 + 8*d*cosh(d*x + c)^6 + 4*(7*d*cosh(d*x + c)^2 + 2*d)*sinh(d*x + c)^
6 + 4*(14*d*cosh(d*x + c)^3 + 9*d*cosh(d*x + c))*sinh(d*x + c)^5 + 28*d*cos
h(d*x + c)^4 + 2*(35*d*cosh(d*x + c)^4 + 60*d*cosh(d*x + c)^2 + 14*d)*sinh(
d*x + c)^4 + 8*(7*d*cosh(d*x + c)^5 + 15*d*cosh(d*x + c)^3 + 7*d*cosh(d*x +
 c))*sinh(d*x + c)^3 + 56*d*cosh(d*x + c)^2 + 4*(7*d*cosh(d*x + c)^6 + 30*d
*cosh(d*x + c)^4 + 42*d*cosh(d*x + c)^2 + 14*d)*sinh(d*x + c)^2 + 4*(2*d*co
sh(d*x + c)^7 + 9*d*cosh(d*x + c)^5 + 14*d*cosh(d*x + c)^3 + 7*d*cosh(d*x +
 c))*sinh(d*x + c) + 35*d)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**8*(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.26696, size = 351, normalized size = 4.39

$$-\frac{2\left(35\,b^3e^{(12\,dx+12\,c)}+210\,ab^2e^{(10\,dx+10\,c)}+560\,a^2be^{(8\,dx+8\,c)}-210\,ab^2e^{(8\,dx+8\,c)}+175\,b^3e^{(8\,dx+8\,c)}+560\,a^3e^{(6\,dx+6\,c)}-28\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^8*(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -2/35*(35*b^3*e^(12*d*x + 12*c) + 210*a*b^2*e^(10*d*x + 10*c) + 560*a^2*b*e
^(8*d*x + 8*c) - 210*a*b^2*e^(8*d*x + 8*c) + 175*b^3*e^(8*d*x + 8*c) + 560*
a^3*e^(6*d*x + 6*c) - 280*a^2*b*e^(6*d*x + 6*c) + 420*a*b^2*e^(6*d*x + 6*c)
 + 336*a^3*e^(4*d*x + 4*c) + 168*a^2*b*e^(4*d*x + 4*c) - 84*a*b^2*e^(4*d*x
+ 4*c) + 105*b^3*e^(4*d*x + 4*c) + 112*a^3*e^(2*d*x + 2*c) + 56*a^2*b*e^(2*
d*x + 2*c) + 42*a*b^2*e^(2*d*x + 2*c) + 16*a^3 + 8*a^2*b + 6*a*b^2 + 5*b^3)
/(d*(e^(2*d*x + 2*c) + 1)^7)
```

3.316 $\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=108

$$\frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d}-\frac{(a-3b)\sinh^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}d}+\frac{\sinh^5(c+dx)}{5bd}$$

```
[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d))
+ ((a^2 - 3*a*b + 3*b^2)*Sinh[c + d*x])/(b^3*d) - ((a - 3*b)*Sinh[c + d*x]^
3)/(3*b^2*d) + Sinh[c + d*x]^5/(5*b*d)
```

Rubi [A] time = 0.112928, antiderivative size = 108, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 390, 205}

$$\frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d}-\frac{(a-3b)\sinh^3(c+dx)}{3b^2d}-\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}d}+\frac{\sinh^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)^3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)*d))
+ ((a^2 - 3*a*b + 3*b^2)*Sinh[c + d*x])/(b^3*d) - ((a - 3*b)*Sinh[c + d*x]^
3)/(3*b^2*d) + Sinh[c + d*x]^5/(5*b*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^7(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{b^3} - \frac{(a-3b)x^2}{b^2} + \frac{x^4}{b} + \frac{-a^3+3a^2b-3ab^2+b^3}{b^3(a+bx^2)}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d} - \frac{(a-3b)\sinh^3(c+dx)}{3b^2d} + \frac{\sinh^5(c+dx)}{5bd} - \frac{(a-b)^3\,\text{Subst}}{}$$

$$= -\frac{(a-b)^3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{7/2}d} + \frac{(a^2-3ab+3b^2)\sinh(c+dx)}{b^3d} - \frac{(a-3b)\sinh^3(c+dx)}{3b^2d}$$

Mathematica [A] time = 0.476694, size = 117, normalized size = 1.08

$$\frac{30\sqrt{b}(8a^2-22ab+19b^2)\sinh(c+dx)+5b^{3/2}(9b-4a)\sinh(3(c+dx))+\frac{3\left(\sqrt{a}b^{5/2}\sinh(5(c+dx))+80(a-b)^3\tan^{-1}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)\right)}{\sqrt{a}}}{240b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^7/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (30*Sqrt[b]*(8*a^2 - 22*a*b + 19*b^2)*Sinh[c + d*x] + 5*b^(3/2)*(-4*a + 9*b
)*Sinh[3*(c + d*x)] + (3*(80*(a - b)^3*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[
b]] + Sqrt[a]*b^(5/2)*Sinh[5*(c + d*x)]))/Sqrt[a])/(240*b^(7/2)*d)
```

Maple [B] time = 0.081, size = 1656, normalized size = 15.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 11/8/d/b/(tanh(1/2*d*x+1/2*c)+1)^2-11/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2-4/d*a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*a/b/((2*(-b*(a-b))^(1/2
)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2))-1/5/d/b/(tanh(1/2*d*x+1/2*c)-1)^5-1/d/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/
2))+1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/(
(2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/b^3*a^4/(-b*(a-b))^(1/2)/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2))+1/d/b^3*a^4/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/
2))-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)
^2*a+3/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a+3/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-4/
d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-4/d*a^3/b^2/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+6/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
```

```
1/2)+a-2*b)*a)^(1/2))+6/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
))+1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/3/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^3*a-1/d/
b^3/(tanh(1/2*d*x+1/2*c)+1)*a^2+1/3/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3*a-1/d/b
^3/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^4+1/2/d/b/(t
anh(1/2*d*x+1/2*c)+1)^4+3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*ar
ctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^3/b^
3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5
/4/d/b/(tanh(1/2*d*x+1/2*c)+1)^3-5/4/d/b/(tanh(1/2*d*x+1/2*c)-1)^3-3/d/b/(t
anh(1/2*d*x+1/2*c)+1)-3/d/b/(tanh(1/2*d*x+1/2*c)-1)-3/d*a/b/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2))-4/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/5/d/b/(
tanh(1/2*d*x+1/2*c)+1)^5-1/d*a^3/b^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,b^2e^{(10\,dx+10\,c)}-3\,b^2-5\left(4\,abe^{(8\,c)}-9\,b^2e^{(8\,c)}\right)e^{(8\,dx)}+30\left(8\,a^2e^{(6\,c)}-22\,abe^{(6\,c)}+19\,b^2e^{(6\,c)}\right)e^{(6\,dx)}-30\left(8\,a^2e^{(4\,c)}-22\,abe\right.}{480\,b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/480*(3*b^2*e^(10*d*x + 10*c) - 3*b^2 - 5*(4*a*b*e^(8*c) - 9*b^2*e^(8*c))*
e^(8*d*x) + 30*(8*a^2*e^(6*c) - 22*a*b*e^(6*c) + 19*b^2*e^(6*c))*e^(6*d*x)
- 30*(8*a^2*e^(4*c) - 22*a*b*e^(4*c) + 19*b^2*e^(4*c))*e^(4*d*x) + 5*(4*a*b
*e^(2*c) - 9*b^2*e^(2*c))*e^(2*d*x))*e^(-5*d*x - 5*c)/(b^3*d) - 1/128*integ
rate(256*((a^3*e^(3*c) - 3*a^2*b*e^(3*c) + 3*a*b^2*e^(3*c) - b^3*e^(3*c))*e
^(3*d*x) + (a^3*e^c - 3*a^2*b*e^c + 3*a*b^2*e^c - b^3*e^c)*e^(d*x))/(b^4*e^
(4*d*x + 4*c) + b^4 + 2*(2*a*b^3*e^(2*c) - b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 1.85931, size = 7526, normalized size = 69.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/480*(3*a*b^3*cosh(d*x + c)^10 + 30*a*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
 3*a*b^3*sinh(d*x + c)^10 - 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^8 + 5*(27
*a*b^3*cosh(d*x + c)^2 - 4*a^2*b^2 + 9*a*b^3)*sinh(d*x + c)^8 + 40*(9*a*b^3
*cosh(d*x + c)^3 - (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 3
0*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^6 + 10*(63*a*b^3*cosh(d*x
 + c)^4 + 24*a^3*b - 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 - 9*a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*a*b^3*cosh(d*x + c)^5 - 70*(4*a^2*b^2
- 9*a*b^3)*cosh(d*x + c)^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x
```

```
+ c))*sinh(d*x + c)^5 - 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^
4 + 10*(63*a*b^3*cosh(d*x + c)^6 - 35*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4
 - 24*a^3*b + 66*a^2*b^2 - 57*a*b^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 - 3*a*b^3 + 40*(9*a*b^3*cosh(d*x + c)^7 -
7*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 15*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 - 3*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 + 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2 + 5*(27*a*b^3*cosh(
d*x + c)^8 - 28*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^6 + 90*(8*a^3*b - 22*a^
2*b^2 + 19*a*b^3)*cosh(d*x + c)^4 + 4*a^2*b^2 - 9*a*b^3 - 36*(8*a^3*b - 22*
a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 240*((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d
*x + c)^4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh
(d*x + c)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)
^4 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(-a*b)*log((b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2
*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2
 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(
d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*
x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x +
c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*
cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*
cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 10*(3*a*b^
3*cosh(d*x + c)^9 - 4*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^7 + 18*(8*a^3*b -
 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^5 - 12*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 + (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(
a*b^4*d*cosh(d*x + c)^5 + 5*a*b^4*d*cosh(d*x + c)^4*sinh(d*x + c) + 10*a*b^
4*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a*b^4*d*cosh(d*x + c)^2*sinh(d*x +
 c)^3 + 5*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a*b^4*d*sinh(d*x + c)^5),
 1/480*(3*a*b^3*cosh(d*x + c)^10 + 30*a*b^3*cosh(d*x + c)*sinh(d*x + c)^9 +
 3*a*b^3*sinh(d*x + c)^10 - 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^8 + 5*(27
*a*b^3*cosh(d*x + c)^2 - 4*a^2*b^2 + 9*a*b^3)*sinh(d*x + c)^8 + 40*(9*a*b^3
*cosh(d*x + c)^3 - (4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^7 + 3
0*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^6 + 10*(63*a*b^3*cosh(d*x
 + c)^4 + 24*a^3*b - 66*a^2*b^2 + 57*a*b^3 - 14*(4*a^2*b^2 - 9*a*b^3)*cosh(
d*x + c)^2)*sinh(d*x + c)^6 + 4*(189*a*b^3*cosh(d*x + c)^5 - 70*(4*a^2*b^2
- 9*a*b^3)*cosh(d*x + c)^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x
+ c))*sinh(d*x + c)^5 - 30*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^
4 + 10*(63*a*b^3*cosh(d*x + c)^6 - 35*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^4
 - 24*a^3*b + 66*a^2*b^2 - 57*a*b^3 + 45*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*
cosh(d*x + c)^2)*sinh(d*x + c)^4 - 3*a*b^3 + 40*(9*a*b^3*cosh(d*x + c)^7 -
7*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^5 + 15*(8*a^3*b - 22*a^2*b^2 + 19*a*b
^3)*cosh(d*x + c)^3 - 3*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c))*si
nh(d*x + c)^3 + 5*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^2 + 5*(27*a*b^3*cosh(
d*x + c)^8 - 28*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x + c)^6 + 90*(8*a^3*b - 22*a^
2*b^2 + 19*a*b^3)*cosh(d*x + c)^4 + 4*a^2*b^2 - 9*a*b^3 - 36*(8*a^3*b - 22*
a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 480*((a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d
*x + c)^4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
3*sinh(d*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh
(d*x + c)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)
^4 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(a*b)*arctan(1/2*
sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - 480*((a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*cosh(d*x + c)^5 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^
4*sinh(d*x + c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^3*sinh(d
*x + c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)^2*sinh(d*x + c
)^3 + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*sinh(d*x + c)^5)*sqrt(a*b)*arctan(1/2*(b*cosh(
d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a -
```

```
 b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b
)/(a*b)) + 10*(3*a*b^3*cosh(d*x + c)^9 - 4*(4*a^2*b^2 - 9*a*b^3)*cosh(d*x +
 c)^7 + 18*(8*a^3*b - 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^5 - 12*(8*a^3*b
- 22*a^2*b^2 + 19*a*b^3)*cosh(d*x + c)^3 + (4*a^2*b^2 - 9*a*b^3)*cosh(d*x +
 c))*sinh(d*x + c))/(a*b^4*d*cosh(d*x + c)^5 + 5*a*b^4*d*cosh(d*x + c)^4*si
nh(d*x + c) + 10*a*b^4*d*cosh(d*x + c)^3*sinh(d*x + c)^2 + 10*a*b^4*d*cosh(
d*x + c)^2*sinh(d*x + c)^3 + 5*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a*b^
4*d*sinh(d*x + c)^5)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**7/(a+b*sinh(d*x+c)**2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^7/(a+b*sinh(d*x+c)^2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.317 $\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{x\left(8a^2-20ab+15b^2\right)}{8b^3}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^3d}-\frac{(4a-7b)\sinh(c+dx)\cosh(c+dx)}{8b^2d}+\frac{\sinh(c+dx)\cosh}{4bd}$$

```
[Out] ((8*a^2 - 20*a*b + 15*b^2)*x)/(8*b^3) - ((a - b)^(5/2)*ArcTanh[(Sqrt[a - b]
*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^3*d) - ((4*a - 7*b)*Cosh[c + d*x]*Sinh
[c + d*x])/(8*b^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*b*d)
```

Rubi [A] time = 0.220512, antiderivative size = 121, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3191, 414, 527, 522, 206, 208}

$$\frac{x\left(8a^2-20ab+15b^2\right)}{8b^3}-\frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^3d}-\frac{(4a-7b)\sinh(c+dx)\cosh(c+dx)}{8b^2d}+\frac{\sinh(c+dx)\cosh}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((8*a^2 - 20*a*b + 15*b^2)*x)/(8*b^3) - ((a - b)^(5/2)*ArcTanh[(Sqrt[a - b]
*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^3*d) - ((4*a - 7*b)*Cosh[c + d*x]*Sinh
[c + d*x])/(8*b^2*d) + (Cosh[c + d*x]^3*Sinh[c + d*x])/(4*b*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^3(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{-a+4b-3(a-b)x^2}{(1-x^2)^2(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{4bd}$$

$$= -\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{4a^2-9ab+8b^2+}{(1-x^2)(a+}\right.}{}$$

$$= -\frac{(4a-7b)\cosh(c+dx)\sinh(c+dx)}{8b^2d} + \frac{\cosh^3(c+dx)\sinh(c+dx)}{4bd} - \frac{(a-b)^3\,\text{Subst}\left(\int \frac{}{a+(-}\right.}{}$$

$$= \frac{(8a^2-20ab+15b^2)x}{8b^3} - \frac{(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^3d} - \frac{(4a-7b)\cosh(c+dx)\sinh(c-}{8b^2d}$$

Mathematica [A] time = 0.316766, size = 106, normalized size = 0.88

$$\frac{\sqrt{a}\left(4\left(8a^2-20ab+15b^2\right)(c+dx)-8b(a-2b)\sinh(2(c+dx))+b^2\sinh(4(c+dx))\right)-32(a-b)^{5/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+d}{\sqrt{a}}\right.}{32\sqrt{a}b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-32*(a - b)^(5/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(
4*(8*a^2 - 20*a*b + 15*b^2)*(c + d*x) - 8*(a - 2*b)*b*Sinh[2*(c + d*x)] + b
^2*Sinh[4*(c + d*x)]))/(32*Sqrt[a]*b^3*d)
```

Maple [B] time = 0.063, size = 1497, normalized size = 12.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 15/8/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-15/8/d/b*ln(tanh(1/2*d*x+1/2*c)-1)-11/8/
d/b/(tanh(1/2*d*x+1/2*c)+1)^2+11/8/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+3/d*a/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*a/b/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2))+1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/
2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a-1
/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a+1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a^2-1/
2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)*a^2-5/2
/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+5/2/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/
d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^3/b^2/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2))-3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
))-1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/4/d/b/(tanh(1/2*d*x+1/2*c)-1)^4-1/4/d/b/
(tanh(1/2*d*x+1/2*c)+1)^4-3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*a^3/
b^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^3+9/8/d/
b/(tanh(1/2*d*x+1/2*c)+1)+9/8/d/b/(tanh(1/2*d*x+1/2*c)-1)+3/d*a/b/((2*(-b*(
a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)-a+2*b)*a)^(1/2))+3/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(
1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d
*a^3/b^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75907, size = 4566, normalized size = 37.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/64*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh
(d*x + c)^8 + 8*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b - 2*
b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b + 4*b^2)*sinh(d*x +
 c)^6 + 8*(7*b^2*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x
+ c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x - 60*(
a*b - 2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 +
4*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c) - 20*(a*b - 2*b^2)*cosh(d*x +
 c)^3)*sinh(d*x + c)^3 + 8*(a*b - 2*b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*
x + c)^6 + 12*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b - 2*b
^2)*cosh(d*x + c)^4 + 2*a*b - 4*b^2)*sinh(d*x + c)^2 + 32*((a^2 - 2*a*b + b
^2)*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3*sinh(d*x + c) +
 6*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^2 - 2*a*b + b
^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^4)*sq
rt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 -
 a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x +
 c))*sinh(d*x + c) + b)) - b^2 + 8*(b^2*cosh(d*x + c)^7 + 4*(8*a^2 - 20*a*b
 + 15*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c)^5 + 2*(a*b -
 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x + c)^4 + 4*b^3*d*cosh
(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^3
*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^4), 1/64*(b^2*cosh(d
*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 8*(
8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^4 - 8*(a*b - 2*b^2)*cosh(d*x + c
)^6 + 4*(7*b^2*cosh(d*x + c)^2 - 2*a*b + 4*b^2)*sinh(d*x + c)^6 + 8*(7*b^2*
cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^
2*cosh(d*x + c)^4 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x - 60*(a*b - 2*b^2)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 4*(8*a^2 - 20*a*b
 + 15*b^2)*d*x*cosh(d*x + c) - 20*(a*b - 2*b^2)*cosh(d*x + c)^3)*sinh(d*x +
 c)^3 + 8*(a*b - 2*b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 12*(8*
a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^2 - 30*(a*b - 2*b^2)*cosh(d*x + c)
^4 + 2*a*b - 4*b^2)*sinh(d*x + c)^2 + 64*((a^2 - 2*a*b + b^2)*cosh(d*x + c)
^4 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^2 - 2*a*b +
 b^2)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^2 - 2*a*b + b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^4)*sqrt(-(a - b)/a)*ar
ctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) - b^2 + 8*(b^2*cosh(d*x + c)^7
 + 4*(8*a^2 - 20*a*b + 15*b^2)*d*x*cosh(d*x + c)^3 - 6*(a*b - 2*b^2)*cosh(d
*x + c)^5 + 2*(a*b - 2*b^2)*cosh(d*x + c))*sinh(d*x + c))/(b^3*d*cosh(d*x +
 c)^4 + 4*b^3*d*cosh(d*x + c)^3*sinh(d*x + c) + 6*b^3*d*cosh(d*x + c)^2*sin
h(d*x + c)^2 + 4*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + b^3*d*sinh(d*x + c)^
4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.30117, size = 320, normalized size = 2.64

$$\frac{\left(8\,a^{2}-20\,ab+15\,b^{2}\right)(dx+c)}{8\,b^{3}d}-\frac{\left(48\,a^{2}e^{(4\,dx+4\,c)}-120\,abe^{(4\,dx+4\,c)}+90\,b^{2}e^{(4\,dx+4\,c)}-8\,abe^{(2\,dx+2\,c)}+16\,b^{2}e^{(2\,dx+2\,c)}+b^{2}\right.}{64\,b^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 - 20*a*b + 15*b^2)*(d*x + c)/(b^3*d) - 1/64*(48*a^2*e^(4*d*x + 4
*c) - 120*a*b*e^(4*d*x + 4*c) + 90*b^2*e^(4*d*x + 4*c) - 8*a*b*e^(2*d*x + 2
*c) + 16*b^2*e^(2*d*x + 2*c) + b^2)*e^(-4*d*x - 4*c)/(b^3*d) - (a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*
b))/(sqrt(-a^2 + a*b)*b^3*d) + 1/64*(b*d*e^(4*d*x + 4*c) - 8*a*d*e^(2*d*x +
 2*c) + 16*b*d*e^(2*d*x + 2*c))/(b^2*d^2)
```

3.318 $\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}+\frac{\sinh^3(c+dx)}{3bd}$$

```
[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(a - 2*b)*Sinh[c + d*x])/(b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```

Rubi [A] time = 0.0933289, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 390, 205}

$$-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}+\frac{\sinh^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```


```
[Out] ((a - b)^2*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)*d) - (
(a - 2*b)*Sinh[c + d*x])/(b^2*d) + Sinh[c + d*x]^3/(3*b*d)
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{a-2b}{b^2}+\frac{x^2}{b}+\frac{a^2-2ab+b^2}{b^2(a+bx^2)}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= -\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}+\frac{(a-b)^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{b^2d}$$
$$= \frac{(a-b)^2\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{5/2}d}-\frac{(a-2b)\sinh(c+dx)}{b^2d}+\frac{\sinh^3(c+dx)}{3bd}$$

Mathematica [A] time = 0.244737, size = 79, normalized size = 1.03

$$\frac{3\sqrt{b}(7b-4a)\sinh(c+dx)-\frac{12(a-b)^2\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}}+b^{3/2}\sinh(3(c+dx))}{12b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-12*(a - b)^2*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 3*Sqrt[b
]*(-4*a + 7*b)*Sinh[c + d*x] + b^(3/2)*Sinh[3*(c + d*x)])/(12*b^(5/2)*d)
```

Maple [B] time = 0.059, size = 1148, normalized size = 14.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/3/d/b/(tanh(1/2*d*x+1/2*c)+1)^3+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/d/b^
2/(tanh(1/2*d*x+1/2*c)+1)*a-2/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d*a^3/b^2/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))-3/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*
a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
)-1/d*a^2/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1
/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d*a/b/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2))-1/d/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)+a-2*b)*a)^(1/2))-1/d*a^3/b^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)
*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
)+3/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/d*a/(-b*(a-b))^
(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*
```

```
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
-2/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*
b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/3/d/b/(tanh(1/2*d*x+1/2*c
)-1)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a-
2/d/b/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(3\left(4\,ae^{(4\,c)}-7\,be^{(4\,c)}\right)e^{(4\,dx)}-3\left(4\,ae^{(2\,c)}-7\,be^{(2\,c)}\right)e^{(2\,dx)}-be^{(6\,dx+6\,c)}+b\right)e^{(-3\,dx-3\,c)}}{24\,b^2d}+\frac{1}{32}\int\frac{64\left(\left(a^2e^{(3\,c)}-2\,abe^{(3\,c)}+b^2\right.\right.}{b^3e^{(4\,dx+4\,c)}+b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/24*(3*(4*a*e^(4*c) - 7*b*e^(4*c))*e^(4*d*x) - 3*(4*a*e^(2*c) - 7*b*e^(2*
c))*e^(2*d*x) - b*e^(6*d*x + 6*c) + b)*e^(-3*d*x - 3*c)/(b^2*d) + 1/32*inte
grate(64*((a^2*e^(3*c) - 2*a*b*e^(3*c) + b^2*e^(3*c))*e^(3*d*x) + (a^2*e^c
- 2*a*b*e^c + b^2*e^c)*e^(d*x))/(b^3*e^(4*d*x + 4*c) + b^3 + 2*(2*a*b^2*e^(
2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 1.71961, size = 3804, normalized size = 49.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/24*(a*b^2*cosh(d*x + c)^6 + 6*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^
2*sinh(d*x + c)^6 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^4 + 3*(5*a*b^2*cosh
(d*x + c)^2 - 4*a^2*b + 7*a*b^2)*sinh(d*x + c)^4 + 4*(5*a*b^2*cosh(d*x + c)
^3 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a*b^2 + 3*(4*a^
2*b - 7*a*b^2)*cosh(d*x + c)^2 + 3*(5*a*b^2*cosh(d*x + c)^4 + 4*a^2*b - 7*a
*b^2 - 6*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 12*((a^2 -
2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d
*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*
b + b^2)*sinh(d*x + c)^3)*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x
+ c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*
(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2
*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)
*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c)
- cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sin
h(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cos
h(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*
cosh(d*x + c))*sinh(d*x + c) + b)) + 6*(a*b^2*cosh(d*x + c)^5 - 2*(4*a^2*b
- 7*a*b^2)*cosh(d*x + c)^3 + (4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x +
c))/(a*b^3*d*cosh(d*x + c)^3 + 3*a*b^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*
a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^2 + a*b^3*d*sinh(d*x + c)^3), 1/24*(a*b
^2*cosh(d*x + c)^6 + 6*a*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^2*sinh(d*x
 + c)^6 - 3*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^4 + 3*(5*a*b^2*cosh(d*x + c)^
```

```
2 - 4*a^2*b + 7*a*b^2)*sinh(d*x + c)^4 + 4*(5*a*b^2*cosh(d*x + c)^3 - 3*(4*
a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 - a*b^2 + 3*(4*a^2*b - 7*a*
b^2)*cosh(d*x + c)^2 + 3*(5*a*b^2*cosh(d*x + c)^4 + 4*a^2*b - 7*a*b^2 - 6*(
4*a^2*b - 7*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 24*((a^2 - 2*a*b + b^
2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)^2*sinh(d*x + c) +
3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*s
inh(d*x + c)^3)*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x +
c))/a) + 24*((a^2 - 2*a*b + b^2)*cosh(d*x + c)^3 + 3*(a^2 - 2*a*b + b^2)*co
sh(d*x + c)^2*sinh(d*x + c) + 3*(a^2 - 2*a*b + b^2)*cosh(d*x + c)*sinh(d*x
+ c)^2 + (a^2 - 2*a*b + b^2)*sinh(d*x + c)^3)*sqrt(a*b)*arctan(1/2*(b*cosh(
d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a -
 b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b
)/(a*b)) + 6*(a*b^2*cosh(d*x + c)^5 - 2*(4*a^2*b - 7*a*b^2)*cosh(d*x + c)^3
 + (4*a^2*b - 7*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x + c)
^3 + 3*a*b^3*d*cosh(d*x + c)^2*sinh(d*x + c) + 3*a*b^3*d*cosh(d*x + c)*sinh
(d*x + c)^2 + a*b^3*d*sinh(d*x + c)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.319 $\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=81

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d}-\frac{x(2a-3b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

```
[Out] -((2*a - 3*b)*x)/(2*b^2) + ((a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rubi [A] time = 0.132927, antiderivative size = 81, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 414, 522, 206, 208}

$$\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d}-\frac{x(2a-3b)}{2b^2}+\frac{\sinh(c+dx)\cosh(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((2*a - 3*b)*x)/(2*b^2) + ((a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x
])/Sqrt[a]])/(Sqrt[a]*b^2*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2*b*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{-a+2b+(-a+b)x^2}{(1-x^2)(a+(-a+b)x^2)}\,dx, x, \tanh(c+dx)\right)}{2bd}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{2b^2d} + \frac{(a-b)^2\,\text{Sub}}{}$$

$$= -\frac{(2a-3b)x}{2b^2} + \frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^2d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd}$$

Mathematica [A] time = 0.14953, size = 80, normalized size = 0.99

$$\frac{\sqrt{a}(b\sinh(2(c+dx))-2(2a-3b)(c+dx))+4(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{4\sqrt{a}b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (4*(a - b)^(3/2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]] + Sqrt[a]*(-2
*(2*a - 3*b)*(c + d*x) + b*Sinh[2*(c + d*x)]))/(4*Sqrt[a]*b^2*d)
```

Maple [B] time = 0.054, size = 993, normalized size = 12.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)+1)+3/2/d/b*
ln(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*a^2/b^2/(
(2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2))-2/d*a/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2))-1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))+2/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*a^2/b^2/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
```

```
/2))+2/d*a/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-
1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*ta
nh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d*a/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2))+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)^2+1/2/d/b/(tanh(1/2*d*x+1/2*c)-1)
+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-3/2/d/b*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.70902, size = 2222, normalized size = 27.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4 - 4*b*cosh(d*x
 + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)*d*x - 3*b*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 4*((a - b)*cosh(d*x + c)^2 + 2*(a - b)*cosh(
d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt((a - b)/a)*log((b^2*
cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4
 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2
)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b -
 b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*
x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
) + 4*(2*(2*a - 3*b)*d*x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x + c) +
 b)/(b^2*d*cosh(d*x + c)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*si
nh(d*x + c)^2), -1/8*(4*(2*a - 3*b)*d*x*cosh(d*x + c)^2 - b*cosh(d*x + c)^4
 - 4*b*cosh(d*x + c)*sinh(d*x + c)^3 - b*sinh(d*x + c)^4 + 2*(2*(2*a - 3*b)
*d*x - 3*b*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a - b)*cosh(d*x + c)^2 +
2*(a - b)*cosh(d*x + c)*sinh(d*x + c) + (a - b)*sinh(d*x + c)^2)*sqrt(-(a -
 b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) + 4*(2*(2*a - 3*b)*d*
x*cosh(d*x + c) - b*cosh(d*x + c)^3)*sinh(d*x + c) + b)/(b^2*d*cosh(d*x + c
)^2 + 2*b^2*d*cosh(d*x + c)*sinh(d*x + c) + b^2*d*sinh(d*x + c)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.32998, size = 194, normalized size = 2.4

$$-\frac{(dx+c)(2\,a-3\,b)}{2\,b^2d}+\frac{e^{(2\,dx+2\,c)}}{8\,bd}+\frac{\left(4\,ae^{(2\,dx+2\,c)}-6\,be^{(2\,dx+2\,c)}-b\right)e^{(-2\,dx-2\,c)}}{8\,b^2d}+\frac{\left(a^2-2\,ab+b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*(2*a - 3*b)/(b^2*d) + 1/8*e^(2*d*x + 2*c)/(b*d) + 1/8*(4*a*e
^(2*d*x + 2*c) - 6*b*e^(2*d*x + 2*c) - b)*e^(-2*d*x - 2*c)/(b^2*d) + (a^2 -
 2*a*b + b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(s
qrt(-a^2 + a*b)*b^2*d)
```

3.320 $\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{\sinh(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}$$

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) +
Sinh[c + d*x]/(b*d)
```

Rubi [A] time = 0.0704174, antiderivative size = 52, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 388, 205}

$$\frac{\sinh(c+dx)}{bd}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)*d)) +
Sinh[c + d*x]/(b*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\sinh(c+dx)}{bd} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{bd}$$
$$= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}d} + \frac{\sinh(c+dx)}{bd}$$

Mathematica [A] time = 0.0434289, size = 50, normalized size = 0.96

$$\frac{\frac{\sinh(c+dx)}{b} - \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}b^{3/2}}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-(((a - b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b^(3/2))) + S
inh[c + d*x]/b)/d
```

Maple [B] time = 0.049, size = 732, normalized size = 14.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a/b/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/
2)+a-2*b)*a)^(1/2))-2/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/
d*a^2/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*a/b/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))-2/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)
*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2))+1/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2))-1/d/b/(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(e^{(2\,dx+2\,c)}-1\right)e^{(-dx-c)}}{2\,bd} - \frac{1}{8}\int \frac{16\left(\left(ae^{(3\,c)}-be^{(3\,c)}\right)e^{(3\,dx)}+\left(ae^{c}-be^{c}\right)e^{(dx)}\right)}{b^2e^{(4\,dx+4\,c)}+b^2+2\left(2\,abe^{(2\,c)}-b^2e^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(e^(2*d*x + 2*c) - 1)*e^(-d*x - c)/(b*d) - 1/8*integrate(16*((a*e^(3*c)
 - b*e^(3*c))*e^(3*d*x) + (a*e^c - b*e^c)*e^(d*x))/(b^2*e^(4*d*x + 4*c) + b
^2 + 2*(2*a*b*e^(2*c) - b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 1.6135, size = 1754, normalized size = 33.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*
x + c)^2 + sqrt(-a*b)*((a - b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*log((
b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 -
 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x +
 c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(
cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*co
sh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d
*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
- b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - a*b)/
(a*b^2*d*cosh(d*x + c) + a*b^2*d*sinh(d*x + c)), 1/2*(a*b*cosh(d*x + c)^2 +
 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 - 2*sqrt(a*b)*((a
- b)*cosh(d*x + c) + (a - b)*sinh(d*x + c))*arctan(1/2*sqrt(a*b)*(cosh(d*x
+ c) + sinh(d*x + c))/a) - 2*sqrt(a*b)*((a - b)*cosh(d*x + c) + (a - b)*sin
h(d*x + c))*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)
^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4
*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) - a*b)/(a*b^2*d*cosh(d*x + c) + a*b
^2*d*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.321 $\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=50

$$\frac{x}{b} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

```
[Out] x/b - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b
*d)
```

Rubi [A] time = 0.0817953, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 391, 206, 208}

$$\frac{x}{b} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*b
*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-(a-b)x^2)}\,dx, x, \tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{bd} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{bd}$$
$$= \frac{x}{b} - \frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}bd}$$

Mathematica [A] time = 0.0816948, size = 50, normalized size = 1.

$$\frac{-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}}+c+dx}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a - b]*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[
a])/(b*d)
```

Maple [B] time = 0.04, size = 577, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d/b*ln(tanh(1/2*d*x+1/2*c)+1)-1/d*a/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*a/b/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2))+1/d*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arcta
n(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2))-1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1
/d/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)
-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2))-1/d/b*ln(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.62478, size = 1088, normalized size = 21.76

$$\left[\frac{2\,dx + \sqrt{\frac{a-b}{a}}\log\left(\frac{b^2\cosh(dx+c)^4+4\,b^2\cosh(dx+c)\sinh(dx+c)^3+b^2\sinh(dx+c)^4+2\left(2\,ab-b^2\right)\cosh(dx+c)^2+2\left(3\,b^2\cosh(dx+c)^2+2\,ab-b^2\right)\sinh(dx+c)^2+8}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\cosh(dx+c)^2}\right.}{2\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*d*x + sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c
)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 +
 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b +
b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) +
 4*(a*b*cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x
+ c)^2 + 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a
 - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/(b*d), (d*x + sqrt(-(a - b)/a)*ar
ctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x
 + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)))/(b*d)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.28074, size = 93, normalized size = 1.86

$$-\frac{(a-b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}bd}+\frac{dx+c}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -(a - b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-
a^2 + a*b)*b*d) + (d*x + c)/(b*d)
```

3.322 $\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rubi [A] time = 0.0365203, antiderivative size = 32, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3190, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sinh^2(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}\end{aligned}$$

Mathematica [A] time = 0.0099342, size = 32, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b]*d)
```

Maple [A] time = 0.01, size = 24, normalized size = 0.8

$$\frac{1}{d}\arctan\left(b\sinh\left(dx+c\right)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d/(a*b)^(1/2)*arctan(sinh(d*x+c)*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(dx+c\right)}{b\sinh\left(dx+c\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^2 + a), x)
```

Fricas [B] time = 1.57284, size = 1207, normalized size = 37.72

$$\left[-\frac{\sqrt{-ab}\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a+b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2-2\,a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3 \cdots\right.}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\cosh(dx+c)^2+2\,(\cdots}\right.}{2\,abd} \cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3
 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c
))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + s
inh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqr
t(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sin
h(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(
d*x + c) + b))/(a*b*d), (sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + si
nh(d*x + c))/a) + sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x +
c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cos
h(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)))/(a*b*d)]
```

Sympy [A] time = 5.01131, size = 128, normalized size = 4.

$$\begin{cases} \frac{\tilde{\infty} x \cosh(c)}{\sinh^2(c)} & \text{for } a = 0 \wedge b = 0 \wedge d = 0 \\ \frac{\sinh(c+dx)}{ad} & \text{for } b = 0 \\ -\frac{1}{bd\sinh(c+dx)} & \text{for } a = 0 \\ \frac{x\cosh(c)}{a+b\sinh^2(c)} & \text{for } d = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right)}{2\sqrt{a}bd\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right)}{2\sqrt{a}bd\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**2, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh
(c + d*x)/(a*d), Eq(b, 0)), (-1/(b*d*sinh(c + d*x)), Eq(a, 0)), (x*cosh(c)/
(a + b*sinh(c)**2), Eq(d, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x)
)/(2*sqrt(a)*b*d*sqrt(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))/(2
*sqrt(a)*b*d*sqrt(1/b)), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.323 $\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)}$$

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)*d) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])
/Sqrt[a]])/(Sqrt[a]*(a - b)*d)
```

Rubi [A] time = 0.0667097, antiderivative size = 59, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.19, Rules used = {3190, 391, 203, 205}

$$\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)*d) - (Sqrt[b]*ArcTan[(Sqrt[b]*Sinh[c + d*x])
/Sqrt[a]])/(Sqrt[a]*(a - b)*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)d} - \frac{b\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)d}$$
$$= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)d} - \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)d}$$

Mathematica [A] time = 0.126033, size = 54, normalized size = 0.92

$$\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} + 2\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)}{ad-bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((Sqrt[b]*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + 2*ArcTan[Tanh[
(c + d*x)/2]])/(a*d - b*d)
```

Maple [B] time = 0.058, size = 481, normalized size = 8.2

$$\frac{ab}{d(a-b)}\operatorname{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)\frac{1}{\sqrt{-b(a-b)}}\frac{1}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{b}{d(a-b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*b/(a-b)*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b/(a-b)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^2/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2))+1/d*b/(a-b)*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1
/d*b/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b^2/(a-b)/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))+2/d/(a-b)*arctan(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{2\arctan\left(e^{(dx+c)}\right)}{ad-bd} - 2\int \frac{be^{(3dx+3c)}+be^{(dx+c)}}{ab-b^2+\left(abe^{(4c)}-b^2e^{(4c)}\right)e^{(4dx)}+2\left(2a^2e^{(2c)}-3abe^{(2c)}+b^2e^{(2c)}\right)e^{(2dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 2*arctan(e^(d*x + c))/(a*d - b*d) - 2*integrate((b*e^(3*d*x + 3*c) + b*e^(d
*x + c))/(a*b - b^2 + (a*b*e^(4*c) - b^2*e^(4*c))*e^(4*d*x) + 2*(2*a^2*e^(2
*c) - 3*a*b*e^(2*c) + b^2*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 1.66076, size = 1341, normalized size = 22.73

$$\left[-\frac{\sqrt{-\frac{b}{a}}\log\left(\frac{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4-2\,(2\,a+b)\cosh(dx+c)^2+2\left(3\,b\cosh(dx+c)^2-2\,a-b\right)\sinh(dx+c)^2+4\left(b\cosh(dx+c)^3-\right.}{b\cosh(dx+c)^4+4\,b\cosh(dx+c)\sinh(dx+c)^3+b\sinh(dx+c)^4+2\,(2\,a-b)\cosh(dx+c)^2+}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^
3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^
2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x +
c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^
2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*
x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c
)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c
)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
+ c))*sinh(d*x + c) + b)) - 4*arctan(cosh(d*x + c) + sinh(d*x + c)))/((a -
b)*d), -(sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) +
sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2
 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a
 - b)*sinh(d*x + c))*sqrt(b/a)/b) - 2*arctan(cosh(d*x + c) + sinh(d*x + c))
)/((a - b)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sinh(c + d*x)**2), x)
```

Giac [B] time = 1.43074, size = 289, normalized size = 4.9

$$-\frac{2\sqrt{abd}|b|\arctan\left(\frac{2\sqrt{\frac{1}{2}}\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\sqrt{\frac{4\,ad+4\,bd-\sqrt{-64\,abd^2+16\,(ad+bd)^2}}{bd}}}\right)}{(ad-bd)^2b-\left(ab+b^2\right)d|-ad+bd|}-\frac{2\,bd\arctan\left(\frac{2\sqrt{\frac{1}{2}}\left(e^{(dx+c)}-e^{(-dx-c)}\right)}{\sqrt{\frac{4\,ad+4\,bd+\sqrt{-64\,abd^2+16\,(ad+bd)^2}}{bd}}}\right)}{ad|-ad+bd|+bd|-ad+bd|+(ad-bd)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -2*sqrt(a*b)*d*abs(b)*arctan(2*sqrt(1/2)*(e^(d*x + c) - e^(-d*x - c))/sqrt(
(4*a*d + 4*b*d - sqrt(-64*a*b*d^2 + 16*(a*d + b*d)^2))/(b*d)))/((a*d - b*d)
^2*b - (a*b + b^2)*d*abs(-a*d + b*d)) - 2*b*d*arctan(2*sqrt(1/2)*(e^(d*x +
c) - e^(-d*x - c))/sqrt((4*a*d + 4*b*d + sqrt(-64*a*b*d^2 + 16*(a*d + b*d)^
2))/(b*d)))/(a*d*abs(-a*d + b*d) + b*d*abs(-a*d + b*d) + (a*d - b*d)^2)
```

3.324 $\int \frac{\text{sech}^2(c+dx)}{a+b\sinh^2(c+dx)} \, dx$

Optimal. Leaf size=60

$$\frac{\tanh(c+dx)}{d(a-b)} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{3/2}}$$

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d
)) + Tanh[c + d*x]/((a - b)*d)
```

Rubi [A] time = 0.0788262, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3191, 388, 208}

$$\frac{\tanh(c+dx)}{d(a-b)} - \frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d
)) + Tanh[c + d*x]/((a - b)*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx=\frac{\operatorname{Subst}\left(\int \frac{1-x^2}{a-(a-b)x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$=\frac{\tanh(c+dx)}{(a-b)d}-\frac{b\operatorname{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx,x,\tanh(c+dx)\right)}{(a-b)d}$$
$$=-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{3/2}d}+\frac{\tanh(c+dx)}{(a-b)d}$$

Mathematica [A] time = 0.154095, size = 60, normalized size = 1.

$$\frac{\tanh(c+dx)}{d(a-b)}-\frac{b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] -((b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(3/2)*d
)) + Tanh[c + d*x]/((a - b)*d)
```

Maple [B] time = 0.06, size = 335, normalized size = 5.6

$$-\frac{b}{d(a-b)}\operatorname{Artanh}\left(a\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\frac{1}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}\right)\frac{1}{\sqrt{\left(2\sqrt{-b(a-b)}+a-2b\right)a}}+\frac{b^2}{d(a-b)}\operatorname{Artanh}\Bigg($$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d*b/(a-b)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^2/(a-b)/(-b*(a-b))^(1/2)/(
(2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(
a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b
^2/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tan
h(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d/(a-b)*tanh(1/2*d
*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.63066, size = 1770, normalized size = 29.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x +
c)^2 + b)*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*si
nh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(
3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2
+ 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(
b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2
*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 4*a^2 - 4*a*b)/((a^3 - 2*a^2*b + a*b^2)*d*cosh
(d*x + c)^2 + 2*(a^3 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^
3 - 2*a^2*b + a*b^2)*d*sinh(d*x + c)^2 + (a^3 - 2*a^2*b + a*b^2)*d), ((b*co
sh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + b)*sq
rt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x
+ c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) - 2*a^2 +
 2*a*b)/((a^3 - 2*a^2*b + a*b^2)*d*cosh(d*x + c)^2 + 2*(a^3 - 2*a^2*b + a*b
^2)*d*cosh(d*x + c)*sinh(d*x + c) + (a^3 - 2*a^2*b + a*b^2)*d*sinh(d*x + c)
^2 + (a^3 - 2*a^2*b + a*b^2)*d)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**2/(a + b*sinh(c + d*x)**2), x)
```

Giac [A] time = 1.22527, size = 111, normalized size = 1.85

$$-\frac{b\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\sqrt{-a^2+ab}(ad-bd)}-\frac{2}{(ad-bd)\left(e^{(2\,dx+2\,c)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -b*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
a*b)*(a*d - b*d)) - 2/((a*d - b*d)*(e^(2*d*x + 2*c) + 1))
```

3.325 $\int \frac{\mathbf{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=92

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^2}+\frac{(a-3b)\tan^{-1}(\sinh(c+dx))}{2d(a-b)^2}+\frac{\tanh(c+dx)\mathrm{sech}(c+dx)}{2d(a-b)}$$

```
[Out] ((a - 3*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^2*d) + (b^(3/2)*ArcTan[(Sqrt[b
]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^2*d) + (Sech[c + d*x]*Tanh[c +
d*x])/(2*(a - b)*d)
```

Rubi [A] time = 0.10543, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 414, 522, 203, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^2}+\frac{(a-3b)\tan^{-1}(\sinh(c+dx))}{2d(a-b)^2}+\frac{\tanh(c+dx)\mathrm{sech}(c+dx)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((a - 3*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^2*d) + (b^(3/2)*ArcTan[(Sqrt[b
]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^2*d) + (Sech[c + d*x]*Tanh[c +
d*x])/(2*(a - b)*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{2(a-b)d}$$
$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d} + \frac{(a-3b)\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{2(a-b)^2d} + \frac{b^2\operatorname{Subst}\left(\int \frac{1}{a+bx}\right.}{(a}$$
$$= \frac{(a-3b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^2d} + \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^2d} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d}$$

Mathematica [A] time = 0.222276, size = 91, normalized size = 0.99

$$\frac{-2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right) + 2\sqrt{a}(a-3b)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \sqrt{a}(a-b)\tanh(c+dx)\operatorname{sech}(c+dx)}{2\sqrt{a}d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (-2*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(a - 3*b)*A
rcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(2*
Sqrt[a]*(a - b)^2*d)
```

Maple [B] time = 0.068, size = 662, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d*b^2/(a-b)^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*ar
ctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^2/(
a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/(
(2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^3/(a-b)^2/(-b*(a-b))^(1/2)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2))-1/d*b^2/(a-b)^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2))+1/d*b^2/(a-b)^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d*b^3/(a-b)^2/
```

```
(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/(a-b)^2/(tanh(1/2*d*x+1/2
*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*a+1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*
tanh(1/2*d*x+1/2*c)^3*b+1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*
x+1/2*c)*a-1/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*b+1/
d/(a-b)^2*arctan(tanh(1/2*d*x+1/2*c))*a-3/d/(a-b)^2*arctan(tanh(1/2*d*x+1/2
*c))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(ae^{c} - 3\,be^{c})\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{a^2d - 2\,abd + b^2d} + \frac{e^{(3\,dx+3\,c)} - e^{(dx+c)}}{ad - bd + \left(ade^{(4\,c)} - bde^{(4\,c)}\right)e^{(4\,dx)} + 2\left(ade^{(2\,c)} - bde^{(2\,c)}\right)e^{(2\,dx)}} + 8\int \frac{}{4\left(a^2b - 2\,ab\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (a*e^c - 3*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^2*d - 2*a*b*d + b^2*d) + (e
^(3*d*x + 3*c) - e^(d*x + c))/(a*d - b*d + (a*d*e^(4*c) - b*d*e^(4*c))*e^(4
*d*x) + 2*(a*d*e^(2*c) - b*d*e^(2*c))*e^(2*d*x)) + 8*integrate(1/4*(b^2*e^(
3*d*x + 3*c) + b^2*e^(d*x + c))/(a^2*b - 2*a*b^2 + b^3 + (a^2*b*e^(4*c) - 2
*a*b^2*e^(4*c) + b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^3*e^(2*c) - 5*a^2*b*e^(2*c
) + 4*a*b^2*e^(2*c) - b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 1.92306, size = 4370, normalized size = 47.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(a - b)*cosh(d*x + c)^3 + 6*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 +
 2*(a - b)*sinh(d*x + c)^3 + (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*
x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2
 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x +
c) + b)*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x +
 c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)
^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d
*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 2*((a - 3*b)*cosh(d*x + c)^4 + 4*(a - 3*b)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a - 3*b)*sinh(d*x + c)^4 + 2*(a - 3*b)*cosh(d
*x + c)^2 + 2*(3*(a - 3*b)*cosh(d*x + c)^2 + a - 3*b)*sinh(d*x + c)^2 + 4*(
(a - 3*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d*x + c))*sinh(d*x + c) + a - 3*
b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a - b)*cosh(d*x + c) + 2*(3*(
a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c))/((a^2 - 2*a*b + b^2)*d*cosh(
d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^2 -
 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 +
 2*(3*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*d)*sinh(d
*x + c)^2 + (a^2 - 2*a*b + b^2)*d + 4*((a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^
```

```
3 + (a^2 - 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*x + c)), ((a - b)*cosh(d*x
+ c)^3 + 3*(a - b)*cosh(d*x + c)*sinh(d*x + c)^2 + (a - b)*sinh(d*x + c)^3
+ (b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^
4 + 2*b*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(
b*cosh(d*x + c)^3 + b*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(b/a)*arctan(1/
2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + (b*cosh(d*x + c)^4 + 4*b*cos
h(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*b*cosh(d*x + c)^2 + 2*(3
*b*cosh(d*x + c)^2 + b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + b*cosh(d*x
 + c))*sinh(d*x + c) + b)*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cos
h(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) +
(3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) + ((a - 3*b)*co
sh(d*x + c)^4 + 4*(a - 3*b)*cosh(d*x + c)*sinh(d*x + c)^3 + (a - 3*b)*sinh(
d*x + c)^4 + 2*(a - 3*b)*cosh(d*x + c)^2 + 2*(3*(a - 3*b)*cosh(d*x + c)^2 +
 a - 3*b)*sinh(d*x + c)^2 + 4*((a - 3*b)*cosh(d*x + c)^3 + (a - 3*b)*cosh(d
*x + c))*sinh(d*x + c) + a - 3*b)*arctan(cosh(d*x + c) + sinh(d*x + c)) - (
a - b)*cosh(d*x + c) + (3*(a - b)*cosh(d*x + c)^2 - a + b)*sinh(d*x + c))/(
(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^4 + 4*(a^2 - 2*a*b + b^2)*d*cosh(d*x +
c)*sinh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*sinh(d*x + c)^4 + 2*(a^2 - 2*a*b
 + b^2)*d*cosh(d*x + c)^2 + 2*(3*(a^2 - 2*a*b + b^2)*d*cosh(d*x + c)^2 + (a
^2 - 2*a*b + b^2)*d)*sinh(d*x + c)^2 + (a^2 - 2*a*b + b^2)*d + 4*((a^2 - 2*
a*b + b^2)*d*cosh(d*x + c)^3 + (a^2 - 2*a*b + b^2)*d*cosh(d*x + c))*sinh(d*
x + c))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(c+dx)}{a+b\sinh^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Integral(sech(c + d*x)**3/(a + b*sinh(c + d*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.326 $\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{5/2}}-\frac{\tanh^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\tanh(c+dx)}{d(a-b)^2}$$

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d
) + ((a - 2*b)*Tanh[c + d*x])/((a - b)^2*d) - Tanh[c + d*x]^3/(3*(a - b)*d)
```

Rubi [A] time = 0.10538, antiderivative size = 88, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3191, 390, 208}

$$\frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{5/2}}-\frac{\tanh^3(c+dx)}{3d(a-b)}+\frac{(a-2b)\tanh(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2)*d
) + ((a - 2*b)*Tanh[c + d*x])/((a - b)^2*d) - Tanh[c + d*x]^3/(3*(a - b)*d)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a-(a-b)x^2}\,dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-2b}{(a-b)^2}-\frac{x^2}{a-b}+\frac{b^2}{(a-b)^2(a-(a-b)x^2)}\right)dx,x,\tanh(c+dx)\right)}{d}$$
$$= \frac{(a-2b)\tanh(c+dx)}{(a-b)^2d}-\frac{\tanh^3(c+dx)}{3(a-b)d}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a-(a-b)x^2}\,dx,x,\tanh(c+dx)\right)}{(a-b)^2d}$$
$$= \frac{b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}d}+\frac{(a-2b)\tanh(c+dx)}{(a-b)^2d}-\frac{\tanh^3(c+dx)}{3(a-b)d}$$

Mathematica [A] time = 0.491389, size = 84, normalized size = 0.95

$$\frac{\frac{3b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{5/2}}+\frac{\tanh(c+dx)\left((a-b)\operatorname{sech}^2(c+dx)+2a-5b\right)}{(a-b)^2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(5/2
)) + ((2*a - 5*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^2)/(3*d)
```

Maple [B] time = 0.07, size = 535, normalized size = 6.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*b^2/(a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^3/(a-b)^2/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^2/(a-b)^2/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))-1/d*b^3/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d/(a-b)
^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a-4/d/(a-b)^2/(tanh(1/
2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*b+4/3/d/(a-b)^2/(tanh(1/2*d*x+1/2
*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*a-16/3/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)
^3*tanh(1/2*d*x+1/2*c)^3*b+2/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2
*d*x+1/2*c)*a-4/d/(a-b)^2/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError

Fricas [B] time = 1.81998, size = 5762, normalized size = 65.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

```
[Out] [1/6*(12*(a^2*b - a*b^2)*cosh(d*x + c)^4 + 48*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^3 + 12*(a^2*b - a*b^2)*sinh(d*x + c)^4 - 8*a^3 + 28*a^2*b -
20*a*b^2 - 24*(a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c)^2 - 24*(a^3 - 3*a^2*b
 + 2*a*b^2 - 3*(a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*(b^2*co
sh(d*x + c)^6 + 6*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 +
 3*b^2*cosh(d*x + c)^4 + 3*(5*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^4 +
3*b^2*cosh(d*x + c)^2 + 4*(5*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sin
h(d*x + c)^3 + 3*(5*b^2*cosh(d*x + c)^4 + 6*b^2*cosh(d*x + c)^2 + b^2)*sinh
(d*x + c)^2 + b^2 + 6*(b^2*cosh(d*x + c)^5 + 2*b^2*cosh(d*x + c)^3 + b^2*co
sh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^
2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cos
h(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*
a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*s
inh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*s
inh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*
x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 +
2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 +
(2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 48*((a^2*b - a*b^2)*cosh(d*x
 + c)^3 - (a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^6 + 6*(a^4 - 3*a^3*b + 3*a^2*b^
2 - a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a
*b^3)*d*sinh(d*x + c)^6 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x
+ c)^4 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + (a^4
- 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^4 + 3*(a^4 - 3*a^3*b + 3*a^
2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)
*d*cosh(d*x + c)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c))
*sinh(d*x + c)^3 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)
^4 + 6*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + (a^4 - 3*a^3
*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a
*b^3)*d + 6*((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^5 + 2*(a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + (a^4 - 3*a^3*b + 3*a^2*
b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/3*(6*(a^2*b - a*b^2)*cosh(d
*x + c)^4 + 24*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + 6*(a^2*b - a
*b^2)*sinh(d*x + c)^4 - 4*a^3 + 14*a^2*b - 10*a*b^2 - 12*(a^3 - 3*a^2*b + 2
*a*b^2)*cosh(d*x + c)^2 - 12*(a^3 - 3*a^2*b + 2*a*b^2 - 3*(a^2*b - a*b^2)*c
osh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^2*cosh(d*x + c)^6 + 6*b^2*cosh(d*x +
 c)*sinh(d*x + c)^5 + b^2*sinh(d*x + c)^6 + 3*b^2*cosh(d*x + c)^4 + 3*(5*b^
2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^4 + 3*b^2*cosh(d*x + c)^2 + 4*(5*b^2
*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*b^2*cosh(d*x
 + c)^4 + 6*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 6*(b^2*cosh(
d*x + c)^5 + 2*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt
(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 24*((a^2*
```

```
b - a*b^2)*cosh(d*x + c)^3 - (a^3 - 3*a^2*b + 2*a*b^2)*cosh(d*x + c))*sinh(
d*x + c))/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^6 + 6*(a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^4 - 3*a^
3*b + 3*a^2*b^2 - a*b^3)*d*sinh(d*x + c)^6 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 -
 a*b^3)*d*cosh(d*x + c)^4 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh
(d*x + c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^4 + 3*(a
^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 4*(5*(a^4 - 3*a^3*b +
 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^
3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(5*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b
^3)*d*cosh(d*x + c)^4 + 6*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x +
c)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^4 - 3*a^
3*b + 3*a^2*b^2 - a*b^3)*d + 6*((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(
d*x + c)^5 + 2*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^3 + (a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.41082, size = 192, normalized size = 2.18

$$\frac{b^2 \arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^2d-2\,abd+b^2d\right)\sqrt{-a^2+ab}}+\frac{2\left(3\,be^{(4\,dx+4\,c)}-6\,ae^{(2\,dx+2\,c)}+12\,be^{(2\,dx+2\,c)}-2\,a+5\,b\right)}{3\left(a^2d-2\,abd+b^2d\right)\left(e^{(2\,dx+2\,c)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^2*d - 2*
a*b*d + b^2*d)*sqrt(-a^2 + a*b)) + 2/3*(3*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x
+ 2*c) + 12*b*e^(2*d*x + 2*c) - 2*a + 5*b)/((a^2*d - 2*a*b*d + b^2*d)*(e^(2
*d*x + 2*c) + 1)^3)
```

3.327 $\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=138

$$\frac{(3a^2-10ab+15b^2)\tan^{-1}(\sinh(c+dx))}{8d(a-b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^3}+\frac{\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\tanh(\cdot}{8d(\cdot}$$

```
[Out] ((3*a^2 - 10*a*b + 15*b^2)*ArcTan[Sinh[c + d*x]])/(8*(a - b)^3*d) - (b^(5/2
)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^3*d) + ((3*a -
7*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*(a - b)^2*d) + (Sech[c + d*x]^3*Tanh[c
 + d*x])/(4*(a - b)*d)
```

Rubi [A] time = 0.161852, antiderivative size = 138, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{(3a^2-10ab+15b^2)\tan^{-1}(\sinh(c+dx))}{8d(a-b)^3}-\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^3}+\frac{\tanh(c+dx)\operatorname{sech}^3(c+dx)}{4d(a-b)}+\frac{(3a-7b)\tanh(\cdot}{8d(\cdot}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((3*a^2 - 10*a*b + 15*b^2)*ArcTan[Sinh[c + d*x]])/(8*(a - b)^3*d) - (b^(5/2
)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^3*d) + ((3*a -
7*b)*Sech[c + d*x]*Tanh[c + d*x])/(8*(a - b)^2*d) + (Sech[c + d*x]^3*Tanh[c
 + d*x])/(4*(a - b)*d)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^5(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^3(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} - \frac{\operatorname{Subst}\left(\int \frac{-3a+4b-3bx^2}{(1+x^2)^2(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{4(a-b)d}$$

$$= \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} + \frac{\operatorname{Subst}\left(\int \frac{3a^2-7ab+8b^2+(}{(1+x^2)(a+}\right.}{8(}$$

$$= \frac{(3a-7b)\operatorname{sech}(c+dx)\tanh(c+dx)}{8(a-b)^2d} + \frac{\operatorname{sech}^3(c+dx)\tanh(c+dx)}{4(a-b)d} - \frac{b^3\operatorname{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x\right.}{(a-b)^3}$$

$$= \frac{(3a^2-10ab+15b^2)\tan^{-1}(\sinh(c+dx))}{8(a-b)^3d} - \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^3d} + \frac{(3a-7b)\operatorname{sech}(c+dx}{8(a-b)}$$

Mathematica [A] time = 0.50705, size = 139, normalized size = 1.01

$$\frac{2\sqrt{a}(3a^2-10ab+15b^2)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) + \sqrt{a}(3a^2-10ab+7b^2)\tanh(c+dx)\operatorname{sech}(c+dx) + 8b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}}{}\right.}{8\sqrt{a}d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^5/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] (8*b^(5/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*Sqrt[a]*(3*a^2 - 10*
a*b + 15*b^2)*ArcTan[Tanh[(c + d*x)/2]] + Sqrt[a]*(3*a^2 - 10*a*b + 7*b^2)*
Sech[c + d*x]*Tanh[c + d*x] + 2*Sqrt[a]*(a - b)^2*Sech[c + d*x]^3*Tanh[c +
d*x])/(8*Sqrt[a]*(a - b)^3*d)
```

Maple [B] time = 0.073, size = 1023, normalized size = 7.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] 1/d*b^3/(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^3/(a
-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d*b^4/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)+a-2*b)*a)^(1/2))+1/d*b^3/(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2))-1/d*b^3/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*
tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d*b^4/(a-b)^3/(
-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/4/d/(a-b)^3/(tanh(1/2*d*x+1/
2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^7*a^2+7/2/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+
1)^4*tanh(1/2*d*x+1/2*c)^7*a*b-9/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*ta
nh(1/2*d*x+1/2*c)^7*b^2+3/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*
d*x+1/2*c)^5*a^2-1/2/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2
*c)^5*a*b-1/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^5*b
^2-3/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^3*a^2+1/2/
d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^3*a*b+1/4/d/(a-b)
^3/(tanh(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)^3*b^2+5/4/d/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)*a^2-7/2/d/(a-b)^3/(tanh(1/2*d*x
+1/2*c)^2+1)^4*tanh(1/2*d*x+1/2*c)*a*b+9/4/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2
+1)^4*tanh(1/2*d*x+1/2*c)*b^2+3/4/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))*a^2
-5/2/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))*a*b+15/4/d/(a-b)^3*arctan(tanh(1
/2*d*x+1/2*c))*b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(3\,a^{2}e^{c}-10\,abe^{c}+15\,b^{2}e^{c}\right)\arctan\left(e^{(dx+c)}\right)e^{(-c)}}{4\left(a^{3}d-3\,a^{2}bd+3\,ab^{2}d-b^{3}d\right)}+\frac{\left(3\,ae^{(7\,c)}-7\,be^{(7\,c)}\right)e^{(7\,dx)}}{4\left(a^{2}d-2\,abd+b^{2}d+\left(a^{2}de^{(8\,c)}-2\,abde^{(8\,c)}+b^{2}de^{(8\,c)}\right)e^{(8\,dx)}+4\,(a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(3*a^2*e^c - 10*a*b*e^c + 15*b^2*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*d
 - 3*a^2*b*d + 3*a*b^2*d - b^3*d) + 1/4*((3*a*e^(7*c) - 7*b*e^(7*c))*e^(7*d
*x) + (11*a*e^(5*c) - 15*b*e^(5*c))*e^(5*d*x) - (11*a*e^(3*c) - 15*b*e^(3*c
))*e^(3*d*x) - (3*a*e^c - 7*b*e^c)*e^(d*x))/(a^2*d - 2*a*b*d + b^2*d + (a^2
*d*e^(8*c) - 2*a*b*d*e^(8*c) + b^2*d*e^(8*c))*e^(8*d*x) + 4*(a^2*d*e^(6*c)
- 2*a*b*d*e^(6*c) + b^2*d*e^(6*c))*e^(6*d*x) + 6*(a^2*d*e^(4*c) - 2*a*b*d*e
^(4*c) + b^2*d*e^(4*c))*e^(4*d*x) + 4*(a^2*d*e^(2*c) - 2*a*b*d*e^(2*c) + b^
2*d*e^(2*c))*e^(2*d*x)) - 32*integrate(1/16*(b^3*e^(3*d*x + 3*c) + b^3*e^(d
*x + c))/(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4 + (a^3*b*e^(4*c) - 3*a^2*b^2*e^
(4*c) + 3*a*b^3*e^(4*c) - b^4*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 7*a^3
*b*e^(2*c) + 9*a^2*b^2*e^(2*c) - 5*a*b^3*e^(2*c) + b^4*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [B] time = 2.41039, size = 13526, normalized size = 98.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] [1/4*((3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 - 10*a*b + 7*b^2)*sinh(d*x + c)^7 + (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^2 + 11*a^2 - 26*a*b + 15*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^3 + (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^2 - 11*a^2 + 26*a*b - 15*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^5 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 2*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 30*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^5 + 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^2*cosh(d*x + c)^6 + 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*b^2*cosh(d*x + c)^5 + 3*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + ((3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 30*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 9*a^2 - 30*a*b + 45*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^5 + 10*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^6 + 15*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2 + 8*((3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^7 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + (3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c) + (7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^6 + 5*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^4 - 3*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^2 - 3*a^2 + 10*a*b - 7*b^2)*sinh(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sinh(d*x + c)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*sinh(d*x + c)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3

)*d)*sinh(d*x + c)^4 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^6 + 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*sinh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^5 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/4*((3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^7 + 7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)*sinh(d*x + c)^6 + (3*a^2 - 10*a*b + 7*b^2)*sinh(d*x + c)^7 + (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^5 + (21*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^2 + 11*a^2 - 26*a*b + 15*b^2)*sinh(d*x + c)^5 + 5*(7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^3 + (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - (11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 + (35*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^4 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^2 - 11*a^2 + 26*a*b - 15*b^2)*sinh(d*x + c)^3 + (21*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^5 + 10*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^3 - 3*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 - 4*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 30*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^5 + 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^2*cosh(d*x + c)^6 + 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*b^2*cosh(d*x + c)^5 + 3*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) - 4*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*b^2*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^6 + 6*b^2*cosh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*b^2*cosh(d*x + c)^4 + 30*b^2*cosh(d*x + c)^2 + 3*b^2)*sinh(d*x + c)^4 + 4*b^2*cosh(d*x + c)^2 + 8*(7*b^2*cosh(d*x + c)^5 + 10*b^2*cosh(d*x + c)^3 + 3*b^2*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*b^2*cosh(d*x + c)^6 + 15*b^2*cosh(d*x + c)^4 + 9*b^2*cosh(d*x + c)^2 + b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*b^2*cosh(d*x + c)^5 + 3*b^2*cosh(d*x + c)^3 + b^2*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) + ((3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^8 + 8*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^8 + 4*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^6 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 6*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 2*(35*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 30*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 9*a^2 - 30*a*b + 45*b^2)*sinh(d*x + c)^4 + 8*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^5 + 10*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 4*(7*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^6 + 15*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^4 + 9*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2)*sinh(d*x + c)^2 + 3*a^2 - 10*a*b + 15*b^2 + 8*((3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^7 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^5 + 3*(3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c)^3 + (3*a^2 - 10*a*b + 15*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - (3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c) + (7*(3*a^2 - 10*a*b + 7*b^2)*cosh(d*x + c)^6 + 5*(11*a^2 - 26*a*b + 15*b^2)*cosh(d*x + c)^4 - 3*(11*a^2 - 26*a*b + 15

```
*b^2)*cosh(d*x + c)^2 - 3*a^2 + 10*a*b - 7*b^2)*sinh(d*x + c))/((a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sinh(d*
x + c)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^3
- 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d)*sinh(d*x + c)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^
4 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + 3*(a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*
cosh(d*x + c)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*sinh(d*x + c)^4 + 4*
(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*cosh(d*x + c)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh
(d*x + c)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c))*sinh(d*x +
 c)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^6 + 15*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d*cosh(d*x + c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*sinh(d*x + c)^2
+ (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*
cosh(d*x + c)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^5 + 3*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cosh(d*x + c)^3 + (a^3 - 3*a^2*b + 3*a*b^2
 - b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**5/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^5/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.328 $\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx$

Optimal. Leaf size=126

$$\frac{(a^2-3ab+3b^2)\tanh(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{7/2}}+\frac{\tanh^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\tanh^3(c+dx)}{3d(a-b)^2}$$

```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)
*d)) + ((a^2 - 3*a*b + 3*b^2)*Tanh[c + d*x])/((a - b)^3*d) - ((2*a - 3*b)*T
anh[c + d*x]^3)/(3*(a - b)^2*d) + Tanh[c + d*x]^5/(5*(a - b)*d)
```

Rubi [A] time = 0.147213, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3191, 390, 208}

$$\frac{(a^2-3ab+3b^2)\tanh(c+dx)}{d(a-b)^3}-\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}d(a-b)^{7/2}}+\frac{\tanh^5(c+dx)}{5d(a-b)}-\frac{(2a-3b)\tanh^3(c+dx)}{3d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```


```
[Out] -((b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7/2)
*d)) + ((a^2 - 3*a*b + 3*b^2)*Tanh[c + d*x])/((a - b)^3*d) - ((2*a - 3*b)*T
anh[c + d*x]^3)/(3*(a - b)^2*d) + Tanh[c + d*x]^5/(5*(a - b)*d)
```


Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^6(c+dx)}{a+b\sinh^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a-(a-b)x^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+3b^2}{(a-b)^3} - \frac{(2a-3b)x^2}{(a-b)^2} + \frac{x^4}{a-b} - \frac{b^3}{(a-b)^3(a-(a-b)x^2)}\right) dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2 d} + \frac{\tanh^5(c+dx)}{5(a-b)d} - \frac{b^3\,\text{Subst}\left(\int \frac{}{a-}\right.}{}$$

$$= -\frac{b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}d} + \frac{(a^2-3ab+3b^2)\tanh(c+dx)}{(a-b)^3 d} - \frac{(2a-3b)\tanh^3(c+dx)}{3(a-b)^2 d} + \cdots$$

Mathematica [A] time = 0.893982, size = 119, normalized size = 0.94

$$\frac{\frac{\tanh(c+dx)\left((4a^2-13ab+9b^2)\operatorname{sech}^2(c+dx)+8a^2+3(a-b)^2\operatorname{sech}^4(c+dx)-26ab+33b^2\right)}{(a-b)^3} - \frac{15b^3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}(a-b)^{7/2}}}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^6/(a + b*Sinh[c + d*x]^2),x]
```

```
[Out] ((-15*b^3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a - b)^(7
/2)) + ((8*a^2 - 26*a*b + 33*b^2 + (4*a^2 - 13*a*b + 9*b^2)*Sech[c + d*x]^2
 + 3*(a - b)^2*Sech[c + d*x]^4)*Tanh[c + d*x])/(a - b)^3)/(15*d)
```

Maple [B] time = 0.075, size = 907, normalized size = 7.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x)
```

```
[Out] -1/d*b^3/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^4/(a-b)^3/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d*b^3/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))+1/d*b^4/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d/(a-b
)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^9*a^2-6/d/(a-b)^3/(tanh
(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^9*a*b+6/d/(a-b)^3/(tanh(1/2*d*x+
1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^9*b^2+8/3/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^
2+1)^5*tanh(1/2*d*x+1/2*c)^7*a^2-32/3/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5
*tanh(1/2*d*x+1/2*c)^7*a*b+16/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/
2*d*x+1/2*c)^7*b^2+116/15/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*
x+1/2*c)^5*a^2-332/15/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/
2*c)^5*a*b+132/5/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^
5*b^2+8/3/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^3*a^2-3
2/3/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^3*a*b+16/d/(a
-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)^3*b^2+2/d/(a-b)^3/(ta
```

```
nh(1/2*d*x+1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)*a^2-6/d/(a-b)^3/(tanh(1/2*d*x+
1/2*c)^2+1)^5*tanh(1/2*d*x+1/2*c)*a*b+6/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)
^5*tanh(1/2*d*x+1/2*c)*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 2.13266, size = 14236, normalized size = 112.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="fricas")

[Out] [-1/30*(60*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 480*(a^2*b^2 - a*b^3)*cosh(d
*x + c)*sinh(d*x + c)^7 + 60*(a^2*b^2 - a*b^3)*sinh(d*x + c)^8 - 120*(a^3*b
 - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 - 120*(a^3*b - 4*a^2*b^2 + 3*a*b^3
- 14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 240*(14*(a^2*b^2
- a*b^3)*cosh(d*x + c)^3 - 3*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c))*s
inh(d*x + c)^5 + 40*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c
)^4 + 40*(105*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - 31*a^3*b + 47*a^2
*b^2 - 24*a*b^3 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*
x + c)^4 + 32*a^4 - 136*a^3*b + 236*a^2*b^2 - 132*a*b^3 + 160*(21*(a^2*b^2
- a*b^3)*cosh(d*x + c)^5 - 15*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3
 + (8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^
3 + 40*(4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c)^2 + 40*(42*
(a^2*b^2 - a*b^3)*cosh(d*x + c)^6 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d
*x + c)^4 + 4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3 + 6*(8*a^4 - 31*a^3*b
+ 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 15*(b^3*cosh(d*
x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sinh(d*x + c)^10 + 5
*b^3*cosh(d*x + c)^8 + 10*b^3*cosh(d*x + c)^6 + 5*(9*b^3*cosh(d*x + c)^2 +
b^3)*sinh(d*x + c)^8 + 40*(3*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(
d*x + c)^7 + 10*b^3*cosh(d*x + c)^4 + 10*(21*b^3*cosh(d*x + c)^4 + 14*b^3*c
osh(d*x + c)^2 + b^3)*sinh(d*x + c)^6 + 4*(63*b^3*cosh(d*x + c)^5 + 70*b^3*
cosh(d*x + c)^3 + 15*b^3*cosh(d*x + c))*sinh(d*x + c)^5 + 5*b^3*cosh(d*x +
c)^2 + 10*(21*b^3*cosh(d*x + c)^6 + 35*b^3*cosh(d*x + c)^4 + 15*b^3*cosh(d*
x + c)^2 + b^3)*sinh(d*x + c)^4 + 40*(3*b^3*cosh(d*x + c)^7 + 7*b^3*cosh(d*
x + c)^5 + 5*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c)^3 + b^3
 + 5*(9*b^3*cosh(d*x + c)^8 + 28*b^3*cosh(d*x + c)^6 + 30*b^3*cosh(d*x + c)
^4 + 12*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^2 + 10*(b^3*cosh(d*x + c)^
9 + 4*b^3*cosh(d*x + c)^7 + 6*b^3*cosh(d*x + c)^5 + 4*b^3*cosh(d*x + c)^3 +
 b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4
 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b
^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)
^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x
+ c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c
) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*
```

cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 80*(6*(a^2*b^2 - a*b^3)*cosh(d*x + c)^7 - 9*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 + 2*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^10 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^10 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 5*(9*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^8 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 14*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^6 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 70*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 35*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^4 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + 7*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 5*(9*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 28*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 30*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 12*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d + 10*((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^9 + 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + 6*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/15*(30*(a^2*b^2 - a*b^3)*cosh(d*x + c)^8 + 240*(a^2*b^2 - a*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + 30*(a^2*b^2 - a*b^3)*sinh(d*x + c)^8 - 60*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 - 60*(a^3*b - 4*a^2*b^2 + 3*a*b^3 - 14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 120*(14*(a^2*b^2 - a*b^3)*cosh(d*x + c)^3 - 3*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 20*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^4 + 20*(105*(a^2*b^2 - a*b^3)*cosh(d*x + c)^4 + 8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*a^4 - 68*a^3*b + 118*a^2*b^2 - 66*a*b^3 + 80*(21*(a^2*b^2 - a*b^3)*cosh(d*x + c)^5 - 15*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 + (8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 20*(4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c)^2 + 20*(42*(a^2*b^2 - a*b^3)*cosh(d*x + c)^6 - 45*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^4 - 17*a^3*b + 28*a^2*b^2 - 15*a*b^3 + 6*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 15*(b^3*cosh(d*x + c)^10 + 10*b^3*cosh(d*x + c)*sinh(d*x + c)^9 + b^3*sinh(d*x + c)^10 + 5*b^3*cosh(d*x + c)^8 + 10*b^3*cosh(d*x + c)^6 + 5*(9*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^8 + 40*(3*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c)^7 + 10*b^3*cosh(d*x + c)^4 + 10*(21*

```
b^3*cosh(d*x + c)^4 + 14*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^6 + 4*(63
*b^3*cosh(d*x + c)^5 + 70*b^3*cosh(d*x + c)^3 + 15*b^3*cosh(d*x + c))*sinh(
d*x + c)^5 + 5*b^3*cosh(d*x + c)^2 + 10*(21*b^3*cosh(d*x + c)^6 + 35*b^3*co
sh(d*x + c)^4 + 15*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x + c)^4 + 40*(3*b^3*c
osh(d*x + c)^7 + 7*b^3*cosh(d*x + c)^5 + 5*b^3*cosh(d*x + c)^3 + b^3*cosh(d
*x + c))*sinh(d*x + c)^3 + b^3 + 5*(9*b^3*cosh(d*x + c)^8 + 28*b^3*cosh(d*x
 + c)^6 + 30*b^3*cosh(d*x + c)^4 + 12*b^3*cosh(d*x + c)^2 + b^3)*sinh(d*x +
 c)^2 + 10*(b^3*cosh(d*x + c)^9 + 4*b^3*cosh(d*x + c)^7 + 6*b^3*cosh(d*x +
c)^5 + 4*b^3*cosh(d*x + c)^3 + b^3*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2
+ a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b
*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 40*(6*(a^2*b^2
- a*b^3)*cosh(d*x + c)^7 - 9*(a^3*b - 4*a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5
+ 2*(8*a^4 - 31*a^3*b + 47*a^2*b^2 - 24*a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 1
7*a^3*b + 28*a^2*b^2 - 15*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/((a^5 - 4*a^
4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^10 + 10*(a^5 - 4*a^4*b
 + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^5 -
4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*sinh(d*x + c)^10 + 5*(a^5 - 4*a^
4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^8 + 5*(9*(a^5 - 4*a^4*
b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a
^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^8 + 10*(a^5 - 4*a^4*b + 6*a^3*
b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b
^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*
a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 10*(21*(a^5 - 4*a^4*b +
 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 14*(a^5 - 4*a^4*b + 6*a
^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2
- 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^6 + 10*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4
*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 4*(63*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*
a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 70*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*
b^3 + a*b^4)*d*cosh(d*x + c)^3 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
+ a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 10*(21*(a^5 - 4*a^4*b + 6*a^3*b
^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^6 + 35*(a^5 - 4*a^4*b + 6*a^3*b^2 -
 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 15*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a
^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
+ a*b^4)*d)*sinh(d*x + c)^4 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*
b^4)*d*cosh(d*x + c)^2 + 40*(3*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b
^4)*d*cosh(d*x + c)^7 + 7*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d
*cosh(d*x + c)^5 + 5*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh
(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x +
c))*sinh(d*x + c)^3 + 5*(9*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*
d*cosh(d*x + c)^8 + 28*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*co
sh(d*x + c)^6 + 30*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d
*x + c)^4 + 12*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x +
 c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^2
+ (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d + 10*((a^5 - 4*a^4*b +
6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^9 + 4*(a^5 - 4*a^4*b + 6*a^3
*b^2 - 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^7 + 6*(a^5 - 4*a^4*b + 6*a^3*b^2
- 4*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^5 + 4*(a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a
^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
+ a*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**6/(a+b*sinh(d*x+c)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.42036, size = 348, normalized size = 2.76

$$-\frac{b^3 \arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{\left(a^3d-3\,a^2bd+3\,ab^2d-b^3d\right)\sqrt{-a^2+ab}} - \frac{2\left(15\,b^2e^{(8\,dx+8\,c)}-30\,abe^{(6\,dx+6\,c)}+90\,b^2e^{(6\,dx+6\,c)}+80\,a^2e^{(4\,dx+4\,c)}-230\,abe^{(}\right.}{15\left(a^3d-3\,\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^6/(a+b*sinh(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -b^3*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/((a^3*d - 3
*a^2*b*d + 3*a*b^2*d - b^3*d)*sqrt(-a^2 + a*b)) - 2/15*(15*b^2*e^(8*d*x + 8
*c) - 30*a*b*e^(6*d*x + 6*c) + 90*b^2*e^(6*d*x + 6*c) + 80*a^2*e^(4*d*x + 4
*c) - 230*a*b*e^(4*d*x + 4*c) + 240*b^2*e^(4*d*x + 4*c) + 40*a^2*e^(2*d*x +
 2*c) - 130*a*b*e^(2*d*x + 2*c) + 150*b^2*e^(2*d*x + 2*c) + 8*a^2 - 26*a*b
+ 33*b^2)/((a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d)*(e^(2*d*x + 2*c) + 1)^5)
```

3.329 $\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=158

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(2a-b)(a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{x(4a-5b)}{2b^3}+\frac{\sinh(c+dx)\cosh(c}{2bd\left(a-(a-b)\tanh^2(\right.}$$

```
[Out] -((4*a - 5*b)*x)/(2*b^3) + ((a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]*Ta
nh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2
*b*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((a - b)*(2*a - b)*Tanh[c + d*x])/(2*
a*b^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.25785, antiderivative size = 158, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3191, 414, 527, 522, 206, 208}

$$\frac{(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d}+\frac{(2a-b)(a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{x(4a-5b)}{2b^3}+\frac{\sinh(c+dx)\cosh(c}{2bd\left(a-(a-b)\tanh^2(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]
```


```
[Out] -((4*a - 5*b)*x)/(2*b^3) + ((a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]*Ta
nh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^3*d) + (Cosh[c + d*x]*Sinh[c + d*x])/(2
*b*d*(a - (a - b)*Tanh[c + d*x]^2)) + ((a - b)*(2*a - b)*Tanh[c + d*x])/(2*
a*b^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```


Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-a+2b-3(a-b)x^2}{\left(1-x^2\right)\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{2bd}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{2\left(2a^2-2ab\right.}{(1-x}\right.}{}$$

$$= \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{(a-b)(2a-b)\tanh(c+dx)}{2ab^2d\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{(4a-5b)\,\text{Subst}\left(\right.}{}$$

$$= -\frac{(4a-5b)x}{2b^3} + \frac{(a-b)^{3/2}(4a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^3d} + \frac{\cosh(c+dx)\sinh(c+dx)}{2bd\left(a-(a-b)\tanh^2(c+dx\right.}$$

Mathematica [A] time = 0.576941, size = 118, normalized size = 0.75

$$\frac{\frac{2(4a+b)(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}} + 2(5b-4a)(c+dx) + \frac{2b(a-b)^2\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)} + b\sinh(2(c+dx))}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(-4*a + 5*b)*(c + d*x) + (2*(a - b)^(3/2)*(4*a + b)*ArcTanh[(Sqrt[a - b]
*Tanh[c + d*x])/Sqrt[a]])/a^(3/2) + b*Sinh[2*(c + d*x)] + (2*(a - b)^2*b*Si
nh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])))/(4*b^3*d)
```

Maple [B] time = 0.073, size = 1659, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -2/d/b^3*a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-7/2/d/b^2*a/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2))+7/2/d/b^2*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/b^2/(tanh(1/2*d*x+1/2
*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d
*x+1/2*c)^3+1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)-1/2/d/a/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))*b-2/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))-2/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+7/2/
d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+7/2/d/b*a/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2))*b-2/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)*a+2/d/b^3*ln(tanh(1/2*d*x+
1/2*c)-1)*a+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2
*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)
-1/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/(-b*(a-b))^(1/2)/((
2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/((
2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2))-2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/
2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3-2/d/b/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh
(1/2*d*x+1/2*c)+1/d/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/d/b/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2-1/2/d/b^2/(tanh(1/2*d*x+1/2
*c)+1)^2+5/2/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-5/2/d/b^2*ln(tanh(1/2*d*x+1/2*
c)-1)+2/d/b^3*a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/b^2/(tanh(1/2*d*x+1/2
*c)+1)+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.90558, size = 8500, normalized size = 53.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(a*b^2*cosh(d*x + c)^8 + 8*a*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + a*b^2
*sinh(d*x + c)^8 + 2*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x
 + c)^6 + 2*(14*a*b^2*cosh(d*x + c)^2 + 2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*
b^2)*d*x)*sinh(d*x + c)^6 + 4*(14*a*b^2*cosh(d*x + c)^3 + 3*(2*a^2*b - a*b^
2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 - 8*(2*a^3 -
5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^4
 + 2*(35*a*b^2*cosh(d*x + c)^4 - 8*a^3 + 20*a^2*b - 16*a*b^2 + 4*b^3 - 4*(8
*a^3 - 14*a^2*b + 5*a*b^2)*d*x + 15*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a*b^2*cosh(d*x + c)^5 + 5*(2
*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3 - 4*(2*a^3 - 5*
a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c))*si
nh(d*x + c)^3 - a*b^2 - 2*(6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2
)*d*x)*cosh(d*x + c)^2 + 2*(14*a*b^2*cosh(d*x + c)^6 + 15*(2*a^2*b - a*b^2
- 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 - 6*a^2*b + 9*a*b^2 - 4*b^3 -
2*(4*a^2*b - 5*a*b^2)*d*x - 24*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 -
14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 2*((4*a^2*b - 3
*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)*s
inh(d*x + c)^5 + (4*a^2*b - 3*a*b^2 - b^3)*sinh(d*x + c)^6 + 2*(8*a^3 - 10*
a^2*b + a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^3 - 20*a^2*b + 2*a*b^2 + 2*b^3
 + 15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*
a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(8*a^3 - 10*a^2*b + a*b^2 + b^3)
*cosh(d*x + c))*sinh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2
 + (15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^4 + 4*a^2*b - 3*a*b^2 - b^3
+ 12*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*
(3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(8*a^3 - 10*a^2*b + a*b^2
+ b^3)*cosh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x
+ c))*sqrt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d
*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^
2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*
(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*
cosh(d*x + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 +
 2*a^2 - a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(
d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(
d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*co
sh(d*x + c))*sinh(d*x + c) + b)) + 4*(2*a*b^2*cosh(d*x + c)^7 + 3*(2*a^2*b
- a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(2*a^3 - 5*a^2*b +
 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^2
*b - 9*a*b^2 + 4*b^3 + 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x +
 c))/(a*b^4*d*cosh(d*x + c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a
*b^4*d*sinh(d*x + c)^6 + a*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d*
cosh(d*x + c)^4 + (15*a*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d)*si
nh(d*x + c)^4 + 4*(5*a*b^4*d*cosh(d*x + c)^3 + 2*(2*a^2*b^3 - a*b^4)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + (15*a*b^4*d*cosh(d*x + c)^4 + a*b^4*d + 12*(2*
a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*a*b^4*d*cosh(d*x
 + c)^5 + a*b^4*d*cosh(d*x + c) + 4*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3)*
sinh(d*x + c)), 1/8*(a*b^2*cosh(d*x + c)^8 + 8*a*b^2*cosh(d*x + c)*sinh(d*x
 + c)^7 + a*b^2*sinh(d*x + c)^8 + 2*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2
```

```
)*d*x)*cosh(d*x + c)^6 + 2*(14*a*b^2*cosh(d*x + c)^2 + 2*a^2*b - a*b^2 - 2*
(4*a^2*b - 5*a*b^2)*d*x)*sinh(d*x + c)^6 + 4*(14*a*b^2*cosh(d*x + c)^3 + 3*
(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^
5 - 8*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*
cosh(d*x + c)^4 + 2*(35*a*b^2*cosh(d*x + c)^4 - 8*a^3 + 20*a^2*b - 16*a*b^2
 + 4*b^3 - 4*(8*a^3 - 14*a^2*b + 5*a*b^2)*d*x + 15*(2*a^2*b - a*b^2 - 2*(4*
a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*a*b^2*cosh(d*
x + c)^5 + 5*(2*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^3
 - 4*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*co
sh(d*x + c))*sinh(d*x + c)^3 - a*b^2 - 2*(6*a^2*b - 9*a*b^2 + 4*b^3 + 2*(4*
a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^2 + 2*(14*a*b^2*cosh(d*x + c)^6 + 15*(2
*a^2*b - a*b^2 - 2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c)^4 - 6*a^2*b + 9*a
*b^2 - 4*b^3 - 2*(4*a^2*b - 5*a*b^2)*d*x - 24*(2*a^3 - 5*a^2*b + 4*a*b^2 -
b^3 + (8*a^3 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 -
4*((4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a^2*b - 3*a*b^2 - b^3)*
cosh(d*x + c)*sinh(d*x + c)^5 + (4*a^2*b - 3*a*b^2 - b^3)*sinh(d*x + c)^6 +
 2*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^3 - 20*a^2*b +
2*a*b^2 + 2*b^3 + 15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x +
c)^4 + 4*(5*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(8*a^3 - 10*a^2*b
 + a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*
cosh(d*x + c)^2 + (15*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^4 + 4*a^2*b -
 3*a*b^2 - b^3 + 12*(8*a^3 - 10*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(
d*x + c)^2 + 2*(3*(4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x + c)^5 + 4*(8*a^3 - 10
*a^2*b + a*b^2 + b^3)*cosh(d*x + c)^3 + (4*a^2*b - 3*a*b^2 - b^3)*cosh(d*x
+ c))*sinh(d*x + c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*
cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)
/(a - b)) + 4*(2*a*b^2*cosh(d*x + c)^7 + 3*(2*a^2*b - a*b^2 - 2*(4*a^2*b -
5*a*b^2)*d*x)*cosh(d*x + c)^5 - 8*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3 + (8*a^3
 - 14*a^2*b + 5*a*b^2)*d*x)*cosh(d*x + c)^3 - (6*a^2*b - 9*a*b^2 + 4*b^3 +
2*(4*a^2*b - 5*a*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a*b^4*d*cosh(d*x
+ c)^6 + 6*a*b^4*d*cosh(d*x + c)*sinh(d*x + c)^5 + a*b^4*d*sinh(d*x + c)^6
+ a*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + (15*a
*b^4*d*cosh(d*x + c)^2 + 2*(2*a^2*b^3 - a*b^4)*d)*sinh(d*x + c)^4 + 4*(5*a*
b^4*d*cosh(d*x + c)^3 + 2*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c))*sinh(d*x + c
)^3 + (15*a*b^4*d*cosh(d*x + c)^4 + a*b^4*d + 12*(2*a^2*b^3 - a*b^4)*d*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*a*b^4*d*cosh(d*x + c)^5 + a*b^4*d*cosh(
d*x + c) + 4*(2*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17041, size = 421, normalized size = 2.66

$$-\frac{(dx+c)(4a-5b)}{2b^3d}+\frac{e^{(2dx+2c)}}{8b^2d}+\frac{\left(4a^3-7a^2b+2ab^2+b^3\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{2\sqrt{-a^2+ab}ab^3d}+\frac{8a^2be^{(6dx+6c)}-10ab^2e^{(6dx+6c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(d*x + c)*(4*a - 5*b)/(b^3*d) + 1/8*e^(2*d*x + 2*c)/(b^2*d) + 1/2*(4*a
^3 - 7*a^2*b + 2*a*b^2 + b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt
(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^3*d) + 1/24*(8*a^2*b*e^(6*d*x + 6*c) -
10*a*b^2*e^(6*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 64*a^2*b*e^(4*d*x + 4*c
) - 79*a*b^2*e^(4*d*x + 4*c) + 24*b^3*e^(4*d*x + 4*c) - 28*a^2*b*e^(2*d*x +
 2*c) + 44*a*b^2*e^(2*d*x + 2*c) - 24*b^3*e^(2*d*x + 2*c) - 3*a*b^2)/((b*e^
(6*d*x + 6*c) + 4*a*e^(4*d*x + 4*c) - 2*b*e^(4*d*x + 4*c) + b*e^(2*d*x + 2*
c))*a*b^3*d)
```

3.330 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=104

$$-\frac{(3a^2-2ab-b^2)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)}{b^2d}$$

```
[Out] -((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)
*b^(5/2)*d) + Sinh[c + d*x]/(b^2*d) + ((a - b)^2*Sinh[c + d*x])/(2*a*b^2*d*
(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.138491, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 390, 385, 205}

$$-\frac{(3a^2-2ab-b^2)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d}+\frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{\sinh(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```


```
[Out] -((3*a^2 - 2*a*b - b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)
*b^(5/2)*d) + Sinh[c + d*x]/(b^2*d) + ((a - b)^2*Sinh[c + d*x])/(2*a*b^2*d*
(a + b*Sinh[c + d*x]^2))
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a^2-b^2+2(a-b)bx^2}{b^2\left(a+bx^2\right)^2}\right) dx, x, \sinh(c+dx)\right)}{d} \\
&= \frac{\sinh(c+dx)}{b^2d} - \frac{\text{Subst}\left(\int \frac{a^2-b^2+2(a-b)bx^2}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{b^2d} \\
&= \frac{\sinh(c+dx)}{b^2d} + \frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)} - \frac{((a-b)(3a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh\right.}{2ab^2d} \\
&= -\frac{(a-b)(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}d} + \frac{\sinh(c+dx)}{b^2d} + \frac{(a-b)^2\sinh(c+dx)}{2ab^2d\left(a+b\sinh^2(c+dx)\right)}
\end{aligned}
$$

Mathematica [A] time = 0.28311, size = 106, normalized size = 1.02

$$
\frac{-\frac{\left(-3a^2+2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\,\text{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{2\sqrt{b}(a-b)^2\sinh(c+dx)}{a(2a+b\cosh(2(c+dx))-b)} + 2\sqrt{b}\sinh(c+dx)}{2b^{5/2}d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(((-3*a^2 + 2*a*b + b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2)
) + 2*Sqrt[b]*Sinh[c + d*x] + (2*(a - b)^2*Sqrt[b]*Sinh[c + d*x])/(a*(2*a -
 b + b*Cosh[2*(c + d*x)])))/(2*b^(5/2)*d)
```

Maple [B] time = 0.065, size = 1403, normalized size = 13.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d/b^2/(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/
2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a*tanh(1/2*d*x+1/2*c)^3+2/d/b
/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2
*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/b^2/(tanh(
1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*a
*tanh(1/2*d*x+1/2*c)-2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2
*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c
)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x
+1/2*c)+3/2/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d/
b^2*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((
```

```
2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/2/d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))+3/2/d/b^2*a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
))-3/2/d/b^2*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1
/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/2/d/b*a/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))-1/d/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1
/2/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2))+1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b+1/2/
d/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))*b-1/d/b^2/(tanh(1/2*d*x+1/2*c)-1)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{abe^{(6\,dx+6\,c)} - ab + \left(6\,a^2e^{(4\,c)} - 7\,abe^{(4\,c)} + 2\,b^2e^{(4\,c)}\right)e^{(4\,dx)} - \left(6\,a^2e^{(2\,c)} - 7\,abe^{(2\,c)} + 2\,b^2e^{(2\,c)}\right)e^{(2\,dx)}}{2\left(ab^3de^{(5\,dx+5\,c)} + ab^3de^{(dx+c)} + 2\left(2\,a^2b^2de^{(3\,c)} - ab^3de^{(3\,c)}\right)e^{(3\,dx)}\right)} - \frac{1}{32}\int \frac{32\left(\left(3\,a^2e^{(3\,c}\right.\right.}{ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*b*e^(6*d*x + 6*c) - a*b + (6*a^2*e^(4*c) - 7*a*b*e^(4*c) + 2*b^2*e^(
4*c))*e^(4*d*x) - (6*a^2*e^(2*c) - 7*a*b*e^(2*c) + 2*b^2*e^(2*c))*e^(2*d*x)
)/(a*b^3*d*e^(5*d*x + 5*c) + a*b^3*d*e^(d*x + c) + 2*(2*a^2*b^2*d*e^(3*c) -
 a*b^3*d*e^(3*c))*e^(3*d*x)) - 1/32*integrate(32*((3*a^2*e^(3*c) - 2*a*b*e^
(3*c) - b^2*e^(3*c))*e^(3*d*x) + (3*a^2*e^c - 2*a*b*e^c - b^2*e^c)*e^(d*x))
/(a*b^3*e^(4*d*x + 4*c) + a*b^3 + 2*(2*a^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(
2*d*x)), x)
```

Fricas [B] time = 1.82207, size = 6456, normalized size = 62.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*a^2*b^2*cosh(d*x + c)^6 + 12*a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^5
+ 2*a^2*b^2*sinh(d*x + c)^6 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x +
c)^4 + 2*(15*a^2*b^2*cosh(d*x + c)^2 + 6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*sinh(
d*x + c)^4 - 2*a^2*b^2 + 8*(5*a^2*b^2*cosh(d*x + c)^3 + (6*a^3*b - 7*a^2*b^
2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(6*a^3*b - 7*a^2*b^2 + 2*a*
b^3)*cosh(d*x + c)^2 + 2*(15*a^2*b^2*cosh(d*x + c)^4 - 6*a^3*b + 7*a^2*b^2
- 2*a*b^3 + 6*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
```

)^2 + ((3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (3*a^2*b - 2*a*b^2 - b^3)*sinh(d*x + c)^5 + 2*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^3 + 2*(6*a^3 - 7*a^2*b + b^3 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c) + (5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 3*a^2*b - 2*a*b^2 - b^3 + 6*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(3*a^2*b^2*cosh(d*x + c)^5 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 - (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*b^4*d*sinh(d*x + c)^5 + a^2*b^4*d*cosh(d*x + c) + 2*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(5*a^2*b^4*d*cosh(d*x + c)^2 + (2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^3 + 2*(5*a^2*b^4*d*cosh(d*x + c)^3 + 3*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^2*b^4*d*cosh(d*x + c)^4 + a^2*b^4*d + 6*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c)), 1/2*(a^2*b^2*cosh(d*x + c)^6 + 6*a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^5 + a^2*b^2*sinh(d*x + c)^6 + (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^4 + (15*a^2*b^2*cosh(d*x + c)^2 + 6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*sinh(d*x + c)^4 - a^2*b^2 + 4*(5*a^2*b^2*cosh(d*x + c)^3 + (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2 + (15*a^2*b^2*cosh(d*x + c)^4 - 6*a^3*b + 7*a^2*b^2 - 2*a*b^3 + 6*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - ((3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (3*a^2*b - 2*a*b^2 - b^3)*sinh(d*x + c)^5 + 2*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^3 + 2*(6*a^3 - 7*a^2*b + b^3 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c) + (5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 3*a^2*b - 2*a*b^2 - b^3 + 6*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - ((3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^5 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^4 + (3*a^2*b - 2*a*b^2 - b^3)*sinh(d*x + c)^5 + 2*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^3 + 2*(6*a^3 - 7*a^2*b + b^3 + 5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + (3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c) + (5*(3*a^2*b - 2*a*b^2 - b^3)*cosh(d*x + c)^4 + 3*a^2*b - 2*a*b^2 - b^3 + 6*(6*a^3 - 7*a^2*b + b^3)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) + 2*(3*a^2*b^2*cosh(d*x + c)^5 + 2*(6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c)^3 - (6*a^3*b - 7*a^2*b^2 + 2*a*b^3)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^4*d*cosh(d*x + c)^5 + 5*a^2*b^4*d*cosh(d*x + c)*sinh(d*x + c)^4 + a^2*b^4*d*sinh(d*x + c)^5 + a^2*b^4*d*cosh(d*x + c) + 2*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + 2*(5*a^2*b^4*d*cosh(d*x + c)^2 + (2*a^3*b^3 - a^2*b^4)*d)*sinh(d*x + c)^3 + 2*(5*a^2*b^4*d*cosh(d*x + c)^3 + 3*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^2 + (5*a^2*b^4*d*cosh(d*x + c)^4 + a^2*b^4*d + 6*(2*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2)*sinh(d*x + c))]

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.331 $\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=100

$$-\frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

```
[Out] x/b^2 - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(2*a^(3/2)*b^2*d) - ((a - b)*Tanh[c + d*x])/(2*a*b*d*(a - (a - b)*Tanh[c
+ d*x]^2))
```

Rubi [A] time = 0.1384, antiderivative size = 100, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 414, 522, 206, 208}

$$-\frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d}-\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a - b]*(2*a + b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(2*a^(3/2)*b^2*d) - ((a - b)*Tanh[c + d*x])/(2*a*b*d*(a - (a - b)*Tanh[c
+ d*x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a-b+(-a+b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)}\,dx, x, \tanh(c+dx)\right)}{2abd}$$

$$= -\frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(c+dx)\right)}{b^2d} - \frac{((a-b)(2a}{}$$

$$= \frac{x}{b^2} - \frac{\sqrt{a-b}(2a+b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^2d} - \frac{(a-b)\tanh(c+dx)}{2abd\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.590935, size = 108, normalized size = 1.08

$$\frac{-\frac{(2a^2-ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a-b}} + \frac{b(b-a)\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)} + 2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - ((2*a^2 - a*b - b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqr
t[a]])/(a^(3/2)*Sqrt[a - b]) + (b*(-a + b)*Sinh[2*(c + d*x)])/(a*(2*a - b +
 b*Cosh[2*(c + d*x)])))/(2*b^2*d)
```

Maple [B] time = 0.062, size = 1116, normalized size = 11.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2
*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
/a*tanh(1/2*d*x+1/2*c)^3-1/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*
c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1
```

```
/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2
*d*x+1/2*c)-1/d/b^2*a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b*a/(-b*(a-b))^(1/2
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/b^2*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/
b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/b/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))-1/2/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*ar
ctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/b/(
(2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))+1/2/d/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))*b-1/2/d/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arcta
n(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1
)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")

[Out] Exception raised: ValueError
```

Fricas [B] time = 1.75171, size = 3749, normalized size = 37.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")

[Out] [1/4*(4*a*b*d*x*cosh(d*x + c)^4 + 16*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3
+ 4*a*b*d*x*sinh(d*x + c)^4 + 4*a*b*d*x + 4*(2*(2*a^2 - a*b)*d*x + 2*a^2 -
3*a*b + b^2)*cosh(d*x + c)^2 + 4*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - a*
b)*d*x + 2*a^2 - 3*a*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c
)^4 + 4*(2*a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*
x + c)^4 + 2*(4*a^2 - b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x +
c)^2 + 4*a^2 - b^2)*sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d
*x + c)^3 + (4*a^2 - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt((a - b)/a)*log
((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
+ c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b
 - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2
*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x + c)^2 + 2*a*b*c
osh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 - a*b)*sqrt((a - b
)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x +
```

```
c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sin
h(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c
) + b)) + 4*a*b - 4*b^2 + 8*(2*a*b*d*x*cosh(d*x + c)^3 + (2*(2*a^2 - a*b)*d
*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(a*b^3*d*cosh(d*x +
 c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*d*sinh(d*x + c)^4 +
 a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*a*b^3*d*cosh(d*x
+ c)^2 + (2*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + 4*(a*b^3*d*cosh(d*x + c)^
3 + (2*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*d*x*cos
h(d*x + c)^4 + 8*a*b*d*x*cosh(d*x + c)*sinh(d*x + c)^3 + 2*a*b*d*x*sinh(d*x
 + c)^4 + 2*a*b*d*x + 2*(2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*
x + c)^2 + 2*(6*a*b*d*x*cosh(d*x + c)^2 + 2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a
*b + b^2)*sinh(d*x + c)^2 + ((2*a*b + b^2)*cosh(d*x + c)^4 + 4*(2*a*b + b^2
)*cosh(d*x + c)*sinh(d*x + c)^3 + (2*a*b + b^2)*sinh(d*x + c)^4 + 2*(4*a^2
- b^2)*cosh(d*x + c)^2 + 2*(3*(2*a*b + b^2)*cosh(d*x + c)^2 + 4*a^2 - b^2)*
sinh(d*x + c)^2 + 2*a*b + b^2 + 4*((2*a*b + b^2)*cosh(d*x + c)^3 + (4*a^2 -
 b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*
x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*s
qrt(-(a - b)/a)/(a - b)) + 2*a*b - 2*b^2 + 4*(2*a*b*d*x*cosh(d*x + c)^3 + (
2*(2*a^2 - a*b)*d*x + 2*a^2 - 3*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))/(a
*b^3*d*cosh(d*x + c)^4 + 4*a*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a*b^3*d*
sinh(d*x + c)^4 + a*b^3*d + 2*(2*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*
a*b^3*d*cosh(d*x + c)^2 + (2*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + 4*(a*b^3
*d*cosh(d*x + c)^3 + (2*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [B] time = 1.19301, size = 243, normalized size = 2.43

$$\frac{dx+c}{b^2d} - \frac{\left(2\,a^2 - ab - b^2\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{2\,\sqrt{-a^2+ab}ab^2d} + \frac{2\,a^2e^{(2\,dx+2\,c)} - 3\,abe^{(2\,dx+2\,c)} + b^2e^{(2\,dx+2\,c)} + ab - b^2}{\left(be^{(4\,dx+4\,c)} + 4\,ae^{(2\,dx+2\,c)} - 2\,be^{(2\,dx+2\,c)} + b\right)ab^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] (d*x + c)/(b^2*d) - 1/2*(2*a^2 - a*b - b^2)*arctan(1/2*(b*e^(2*d*x + 2*c) +
 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a*b^2*d) + (2*a^2*e^(2*d*x +
2*c) - 3*a*b*e^(2*d*x + 2*c) + b^2*e^(2*d*x + 2*c) + a*b - b^2)/((b*e^(4*d*
x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b^2*d)
```

3.332 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=77

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - (
(a - b)*Sinh[c + d*x])/(2*a*b*d*(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0781891, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 385, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d}-\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)*d) - (
(a - b)*Sinh[c + d*x])/(2*a*b*d*(a + b*Sinh[c + d*x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{2abd}$$

$$= \frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}d} - \frac{(a-b)\sinh(c+dx)}{2abd\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.327024, size = 75, normalized size = 0.97

$$\frac{\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}} - \frac{(a-b)\sinh(c+dx)}{2ab\left(a+b\sinh^2(c+dx)\right)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (((a + b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) - ((
a - b)*Sinh[c + d*x])/(2*a*b*(a + b*Sinh[c + d*x]^2)))/d
```

Maple [B] time = 0.057, size = 808, normalized size = 10.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3-1/d/b/(ta
nh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a
)*tanh(1/2*d*x+1/2*c)+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)-1/2/d/b*a/(-b*(a-b))^(
1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/b/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(
1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/
2/d/b*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/b/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))-1/2/d/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d/a/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))*b+1/2/d/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/
2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(ae^{(3c)} - be^{(3c)}\right)e^{(3dx)} - (ae^{c} - be^{c})e^{(dx)}}{ab^2de^{(4dx+4c)} + ab^2d + 2\left(2a^2bde^{(2c)} - ab^2de^{(2c)}\right)e^{(2dx)}} + \frac{1}{8}\int \frac{8\left(\left(ae^{(3c)} + be^{(3c)}\right)e^{(3dx)} + (ae^{c} + be^{c})e^{(dx)}\right)}{ab^2e^{(4dx+4c)} + ab^2 + 2\left(2a^2be^{(2c)} - ab^2e^{(2c)}\right)e^{(2dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -((a*e^(3*c) - b*e^(3*c))*e^(3*d*x) - (a*e^c - b*e^c)*e^(d*x))/(a*b^2*d*e^(
4*d*x + 4*c) + a*b^2*d + 2*(2*a^2*b*d*e^(2*c) - a*b^2*d*e^(2*c))*e^(2*d*x))
 + 1/8*integrate(8*((a*e^(3*c) + b*e^(3*c))*e^(3*d*x) + (a*e^c + b*e^c)*e^(
d*x))/(a*b^2*e^(4*d*x + 4*c) + a*b^2 + 2*(2*a^2*b*e^(2*c) - a*b^2*e^(2*c))*
e^(2*d*x)), x)
```

Fricas [B] time = 1.65626, size = 3937, normalized size = 51.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b - a*b^2)*cosh(d*x + c)^3 + 12*(a^2*b - a*b^2)*cosh(d*x + c)
*sinh(d*x + c)^2 + 4*(a^2*b - a*b^2)*sinh(d*x + c)^3 + ((a*b + b^2)*cosh(d*
x + c)^4 + 4*(a*b + b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d
*x + c)^4 + 2*(2*a^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d
*x + c)^2 + 2*a^2 + a*b - b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)
*cosh(d*x + c)^3 + (2*a^2 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-
a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*
x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*
sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x
+ c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)
^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)
/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
 + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b
)) - 4*(a^2*b - a*b^2)*cosh(d*x + c) - 4*(a^2*b - a*b^2 - 3*(a^2*b - a*b^2)
*cosh(d*x + c)^2)*sinh(d*x + c))/(a^2*b^3*d*cosh(d*x + c)^4 + 4*a^2*b^3*d*c
osh(d*x + c)*sinh(d*x + c)^3 + a^2*b^3*d*sinh(d*x + c)^4 + a^2*b^3*d + 2*(2
*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*cosh(d*x + c)^2 + (2
*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + 4*(a^2*b^3*d*cosh(d*x + c)^3 + (2*
a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(a^2*b - a*b^2)
*cosh(d*x + c)^3 + 6*(a^2*b - a*b^2)*cosh(d*x + c)*sinh(d*x + c)^2 + 2*(a^2
*b - a*b^2)*sinh(d*x + c)^3 - ((a*b + b^2)*cosh(d*x + c)^4 + 4*(a*b + b^2)*
cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a^2 + a*
b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 + 2*a^2 + a*b -
 b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(d*x + c)^3 + (2*a^2
 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)*
(cosh(d*x + c) + sinh(d*x + c))/a) - ((a*b + b^2)*cosh(d*x + c)^4 + 4*(a*b
+ b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (a*b + b^2)*sinh(d*x + c)^4 + 2*(2*a
^2 + a*b - b^2)*cosh(d*x + c)^2 + 2*(3*(a*b + b^2)*cosh(d*x + c)^2 + 2*a^2
+ a*b - b^2)*sinh(d*x + c)^2 + a*b + b^2 + 4*((a*b + b^2)*cosh(d*x + c)^3 +
 (2*a^2 + a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*(b*
cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (
```

```
4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqr
t(a*b)/(a*b)) - 2*(a^2*b - a*b^2)*cosh(d*x + c) - 2*(a^2*b - a*b^2 - 3*(a^2
*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^2*b^3*d*cosh(d*x + c)^4 + 4*
a^2*b^3*d*cosh(d*x + c)*sinh(d*x + c)^3 + a^2*b^3*d*sinh(d*x + c)^4 + a^2*b
^3*d + 2*(2*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^3*d*cosh(d*x
+ c)^2 + (2*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + 4*(a^2*b^3*d*cosh(d*x +
 c)^3 + (2*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.333 $\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=79

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Ta
nh[c + d*x]/(2*a*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0768361, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3191, 199, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d\sqrt{a-b}}+\frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a - b]*d) + Ta
nh[c + d*x]/(2*a*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\,dx, x, \tanh(c+dx)\right)}{2ad}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a-b}d} + \frac{\tanh(c+dx)}{2ad\left(a-(a-b)\tanh^2(c+dx)\right)}$$

Mathematica [A] time = 0.206394, size = 78, normalized size = 0.99

$$\frac{\frac{\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a}\sinh(2(c+dx))}{2a+b\cosh(2(c+dx))-b}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]/Sqrt[a - b] + (Sqrt[a]*Sinh[2
*(c + d*x)])/(2*a - b + b*Cosh[2*(c + d*x)]))/(2*a^(3/2)*d)
```

Maple [B] time = 0.053, size = 404, normalized size = 5.1

$$\frac{1}{da}\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^4 a-2\left(\tanh\left(1/2\,dx+c/2\right)\right)^2 a+4\left(\tanh\left(1/2\,dx+c/2\right)\right)^2 b+a\right)^{-1}+\frac{1}{da}\tanh$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)+1/2/d/a/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2))*b-1/2/d/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 1.68031, size = 3321, normalized size = 42.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*a^2*b - 4*a*b^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 8*
(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x + c)*sinh(d*x + c) + 4*(2*a^3 - 3*a^2*b
+ a*b^2)*sinh(d*x + c)^2 - (b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(
d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b
^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x +
 c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^
2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)
^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b
^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b
 - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
+ c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(
d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a
 - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 +
 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)))/((a^3
*b^2 - a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)*s
inh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4*b - 3*a^3
*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^3)*d*cosh(d*x + c
)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*sinh(d*x + c)^2 + (a^3*b^2 - a^2*b
^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b - 3*a^3*b^2 + a
^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*a^2*b - 2*a*b^2 + 2*(2*a^3
 - 3*a^2*b + a*b^2)*cosh(d*x + c)^2 + 4*(2*a^3 - 3*a^2*b + a*b^2)*cosh(d*x
+ c)*sinh(d*x + c) + 2*(2*a^3 - 3*a^2*b + a*b^2)*sinh(d*x + c)^2 + (b^2*cos
h(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 +
2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*s
inh(d*x + c)^2 + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c)
)*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh
(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^
2 - a*b)))/((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b^2 - a^2*b^3)*d
*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b^2 - a^2*b^3)*d*sinh(d*x + c)^4 + 2*
(2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b^2 - a^2*b^3
)*d*cosh(d*x + c)^2 + (2*a^4*b - 3*a^3*b^2 + a^2*b^3)*d)*sinh(d*x + c)^2 +
(a^3*b^2 - a^2*b^3)*d + 4*((a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b
 - 3*a^3*b^2 + a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29874, size = 173, normalized size = 2.19

$$\frac{\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{2\,\sqrt{-a^2+ab}ad}-\frac{2\,ae^{(2\,dx+2\,c)}-be^{(2\,dx+2\,c)}+b}{\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+b\right)abd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*a*d) - (2*a*e^(2*d*x + 2*c) - b*e^(2*d*x + 2*c) + b)/((b*e^(4*d*x + 4
*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)*a*b*d)
```

3.334 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=66

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Sinh[c + d*
x]/(2*a*d*(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0466733, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d}+\frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]*d) + Sinh[c + d*
x]/(2*a*d*(a + b*Sinh[c + d*x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sinh(c+dx)\right)}{2ad}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}d} + \frac{\sinh(c+dx)}{2ad\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.0456064, size = 64, normalized size = 0.97

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}} + \frac{\sinh(c+dx)}{2a\left(a+b\sinh^2(c+dx)\right)}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sinh[c + d*x
]/(2*a*(a + b*Sinh[c + d*x]^2)))/d
```

Maple [A] time = 0.013, size = 57, normalized size = 0.9

$$\frac{\sinh(dx+c)}{2\,da\left(a+b\left(\sinh(dx+c)\right)^2\right)} + \frac{1}{2\,da}\arctan\left(b\sinh(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/2*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)+1/2/d/a/(a*b)^(1/2)*arctan(sinh(d*x
+c)*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{e^{(3\,dx+3\,c)} - e^{(dx+c)}}{abde^{(4\,dx+4\,c)} + abd + 2\left(2\,a^2de^{(2\,c)} - abde^{(2\,c)}\right)e^{(2\,dx)}} + \frac{1}{2}\int \frac{2\left(e^{(3\,dx+3\,c)} + e^{(dx+c)}\right)}{abe^{(4\,dx+4\,c)} + ab + 2\left(2\,a^2e^{(2\,c)} - abe^{(2\,c)}\right)e^{(2\,dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (e^(3*d*x + 3*c) - e^(d*x + c))/(a*b*d*e^(4*d*x + 4*c) + a*b*d + 2*(2*a^2*d
*e^(2*c) - a*b*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(2*(e^(3*d*x + 3*c) + e
^(d*x + c))/(a*b*e^(4*d*x + 4*c) + a*b + 2*(2*a^2*e^(2*c) - a*b*e^(2*c))*e^
(2*d*x)), x)
```

Fricas [B] time = 1.6321, size = 3372, normalized size = 51.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*a*b*cosh(d*x + c)^3 + 12*a*b*cosh(d*x + c)*sinh(d*x + c)^2 + 4*a*b*
sinh(d*x + c)^3 - 4*a*b*cosh(d*x + c) - (b*cosh(d*x + c)^4 + 4*b*cosh(d*x +
 c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(
3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*
a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(-a*b)*log((b*cosh(d*x + c)^4
+ 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(
d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(
d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)
*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*co
sh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)
^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)
^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*(3*a*b*cosh(d*x + c)^
2 - a*b)*sinh(d*x + c))/(a^2*b^2*d*cosh(d*x + c)^4 + 4*a^2*b^2*d*cosh(d*x +
 c)*sinh(d*x + c)^3 + a^2*b^2*d*sinh(d*x + c)^4 + a^2*b^2*d + 2*(2*a^3*b -
a^2*b^2)*d*cosh(d*x + c)^2 + 2*(3*a^2*b^2*d*cosh(d*x + c)^2 + (2*a^3*b - a^
2*b^2)*d)*sinh(d*x + c)^2 + 4*(a^2*b^2*d*cosh(d*x + c)^3 + (2*a^3*b - a^2*b
^2)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*a*b*cosh(d*x + c)^3 + 6*a*b*cos
h(d*x + c)*sinh(d*x + c)^2 + 2*a*b*sinh(d*x + c)^3 - 2*a*b*cosh(d*x + c) +
(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4
+ 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x
+ c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)
*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + (b*cos
h(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2
*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)*sqrt(
a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*
sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)
*sinh(d*x + c))*sqrt(a*b)/(a*b)) + 2*(3*a*b*cosh(d*x + c)^2 - a*b)*sinh(d*x
 + c))/(a^2*b^2*d*cosh(d*x + c)^4 + 4*a^2*b^2*d*cosh(d*x + c)*sinh(d*x + c)
^3 + a^2*b^2*d*sinh(d*x + c)^4 + a^2*b^2*d + 2*(2*a^3*b - a^2*b^2)*d*cosh(d
*x + c)^2 + 2*(3*a^2*b^2*d*cosh(d*x + c)^2 + (2*a^3*b - a^2*b^2)*d)*sinh(d*
x + c)^2 + 4*(a^2*b^2*d*cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2)*d*cosh(d*x +
c))*sinh(d*x + c))]
```

Sympy [A] time = 31.6704, size = 428, normalized size = 6.48

$$\begin{cases} \frac{\tilde{\infty} x \cosh(c)}{\sinh^4(c)} \\ \frac{\sinh(c+dx)}{a^2 d} \\ -\frac{1}{3b^2 d \sinh^3(c+dx)} \\ \frac{x \cosh(c)}{\left(a+b\sinh^2(c)\right)^2} \\ \frac{2i\sqrt{a}b\sqrt{\frac{1}{b}}\sinh(c+dx)}{4ia^{\frac{5}{2}}bd\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}}\sinh^2(c+dx)} + \frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right)}{4ia^{\frac{5}{2}}bd\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}}\sinh^2(c+dx)} - \frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right)}{4ia^{\frac{5}{2}}bd\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}}\sinh^2(c+dx)} + \frac{b\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sinh(c+dx)\right) \mathrm{si}}{4ia^{\frac{5}{2}}bd\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2d\sqrt{\frac{1}{b}}\,\mathrm{sinh}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh
(c + d*x)/(a**2*d), Eq(b, 0)), (-1/(3*b**2*d*sinh(c + d*x)**3), Eq(a, 0)),
(x*cosh(c)/(a + b*sinh(c)**2)**2, Eq(d, 0)), (2*I*sqrt(a)*b*sqrt(1/b)*sinh(
c + d*x)/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sinh(c
 + d*x)**2) + a*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))/(4*I*a**(5/2)*b*d
*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2) - a*log(I*sqrt
(a)*sqrt(1/b) + sinh(c + d*x))/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b
**2*d*sqrt(1/b)*sinh(c + d*x)**2) + b*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d
*x))*sinh(c + d*x)**2/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqr
t(1/b)*sinh(c + d*x)**2) - b*log(I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sinh(
c + d*x)**2/(4*I*a**(5/2)*b*d*sqrt(1/b) + 4*I*a**(3/2)*b**2*d*sqrt(1/b)*sin
h(c + d*x)**2), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.335 $\int \frac{\text{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=106

$$-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\sinh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)^2}$$

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)^2*d) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Si
nh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^2*d) - (b*Sinh[c + d*x])/(2*a*(a
- b)*d*(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.107907, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3190, 414, 522, 203, 205}

$$-\frac{\sqrt{b}(3a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^2}-\frac{b\sinh(c+dx)}{2ad(a-b)\left(a+b\sinh^2(c+dx)\right)}+\frac{\tan^{-1}(\sinh(c+dx))}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)^2*d) - ((3*a - b)*Sqrt[b]*ArcTan[(Sqrt[b]*Si
nh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^2*d) - (b*Sinh[c + d*x])/(2*a*(a
- b)*d*(a + b*Sinh[c + d*x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{2a-b-bx^2}{(1+x^2)(a+bx^2)}\,dx, x, \sinh(c+dx)\right)}{2a(a-b)d}$$

$$= -\frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{(a-b)^2d} - \frac{((3a-b)b)\,\mathrm{S}}{}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)^2d} - \frac{(3a-b)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^2d} - \frac{b\sinh(c+dx)}{2a(a-b)d\left(a+b\sinh^2(c+\right.}$$

Mathematica [A] time = 0.377087, size = 174, normalized size = 1.64

$$\frac{\cosh(2(c+dx))\left(4a^{3/2}b\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right) - b^{3/2}(b-3a)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)\right) + (2a-b)\left(4a^{3/2}\tan^{-1}\left(\tanh\left(\frac{1}{2}\right.\right.\right.}{2a^{3/2}d(a-b)^2(2a+b\cosh(2(c+dx))-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((2*a - b)*(-(Sqrt[b]*(-3*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) +
 4*a^(3/2)*ArcTan[Tanh[(c + d*x)/2]]) + (-(b^(3/2)*(-3*a + b)*ArcTan[(Sqrt[
a]*Csch[c + d*x])/Sqrt[b]]) + 4*a^(3/2)*b*ArcTan[Tanh[(c + d*x)/2]])*Cosh[2
*(c + d*x)] - 2*Sqrt[a]*(a - b)*b*Sinh[c + d*x])/(2*a^(3/2)*(a - b)^2*d*(2*
a - b + b*Cosh[2*(c + d*x)]))
```

Maple [B] time = 0.082, size = 1080, normalized size = 10.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*b/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2
*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3-1/d*b^2/(a-b)^2/(tanh(1/2*d*x+1/2*
c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*
x+1/2*c)^3-1/d*b/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a
```

```
+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)+1/d*b^2/(a-b)^2/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*ta
nh(1/2*d*x+1/2*c)+3/2/d*b/(a-b)^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))+3/2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh
(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-2/d*b^2/(a-b)^2/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d*b/(a-b)^2*a/(-b*(a-b))^(1/2
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
-2/d*b^2/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d*b^2/(a
-b)^2/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d*b^3/(a-b)^2/a/(-b*(a-b))^(1/2)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d*b^2/(a-b)^2/a/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2))+1/2/d*b^3/(a-b)^2/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/
2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d/(
a-b)^2*arctan(tanh(1/2*d*x+1/2*c))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{be^{(3\,dx+3\,c)} - be^{(dx+c)}}{a^2bd - ab^2d + \left(a^2bde^{(4\,c)} - ab^2de^{(4\,c)}\right)e^{(4\,dx)} + 2\left(2\,a^3de^{(2\,c)} - 3\,a^2bde^{(2\,c)} + ab^2de^{(2\,c)}\right)e^{(2\,dx)}} + \frac{2\,\arctan\left(e^{(dx+c)}\right)}{a^2d - 2\,abd + b^2d} - 2\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -(b*e^(3*d*x + 3*c) - b*e^(d*x + c))/(a^2*b*d - a*b^2*d + (a^2*b*d*e^(4*c)
 - a*b^2*d*e^(4*c))*e^(4*d*x) + 2*(2*a^3*d*e^(2*c) - 3*a^2*b*d*e^(2*c) + a*b
^2*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(a^2*d - 2*a*b*d + b^2*d)
 - 2*integrate(1/2*((3*a*b*e^(3*c) - b^2*e^(3*c))*e^(3*d*x) + (3*a*b*e^c - b
^2*e^c)*e^(d*x))/(a^3*b - 2*a^2*b^2 + a*b^3 + (a^3*b*e^(4*c) - 2*a^2*b^2*e^
(4*c) + a*b^3*e^(4*c))*e^(4*d*x) + 2*(2*a^4*e^(2*c) - 5*a^3*b*e^(2*c) + 4*a
^2*b^2*e^(2*c) - a*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.00276, size = 5192, normalized size = 48.98

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a*b - b^2)*cosh(d*x + c)^3 + 12*(a*b - b^2)*cosh(d*x + c)*sinh(d*
x + c)^2 + 4*(a*b - b^2)*sinh(d*x + c)^3 + ((3*a*b - b^2)*cosh(d*x + c)^4 +
 4*(3*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*sinh(d*x + c
)^4 + 2*(6*a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b^2)*cosh(d*x
 + c)^2 + 6*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 + 4*((3*a*b -
b^2)*cosh(d*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*s
```

```
inh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a
 - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sin
h(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x + c)^2 + a*s
inh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*sinh(d*x + c))
*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b
*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2
*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*s
inh(d*x + c) + b)) - 8*(a*b*cosh(d*x + c)^4 + 4*a*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + a*b*sinh(d*x + c)^4 + 2*(2*a^2 - a*b)*cosh(d*x + c)^2 + 2*(3*a*b*c
osh(d*x + c)^2 + 2*a^2 - a*b)*sinh(d*x + c)^2 + a*b + 4*(a*b*cosh(d*x + c)^
3 + (2*a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh
(d*x + c)) - 4*(a*b - b^2)*cosh(d*x + c) + 4*(3*(a*b - b^2)*cosh(d*x + c)^2
 - a*b + b^2)*sinh(d*x + c))/((a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^4
 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)*sinh(d*x + c)^3 + (a^3*b -
 2*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*
b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^2
 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d)*sinh(d*x + c)^2 + (a^3*b - 2*a^
2*b^2 + a*b^3)*d + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)^3 + (2*a^
4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(
a*b - b^2)*cosh(d*x + c)^3 + 6*(a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^2 +
2*(a*b - b^2)*sinh(d*x + c)^3 + ((3*a*b - b^2)*cosh(d*x + c)^4 + 4*(3*a*b -
 b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*sinh(d*x + c)^4 + 2*(6*
a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b^2)*cosh(d*x + c)^2 + 6
*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 + 4*((3*a*b - b^2)*cosh(d
*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*a
rctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + ((3*a*b - b^2)*cosh(
d*x + c)^4 + 4*(3*a*b - b^2)*cosh(d*x + c)*sinh(d*x + c)^3 + (3*a*b - b^2)*
sinh(d*x + c)^4 + 2*(6*a^2 - 5*a*b + b^2)*cosh(d*x + c)^2 + 2*(3*(3*a*b - b
^2)*cosh(d*x + c)^2 + 6*a^2 - 5*a*b + b^2)*sinh(d*x + c)^2 + 3*a*b - b^2 +
4*((3*a*b - b^2)*cosh(d*x + c)^3 + (6*a^2 - 5*a*b + b^2)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sin
h(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x
+ c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) - 4*(a*b*cosh(d*x + c)^4 + 4*
a*b*cosh(d*x + c)*sinh(d*x + c)^3 + a*b*sinh(d*x + c)^4 + 2*(2*a^2 - a*b)*c
osh(d*x + c)^2 + 2*(3*a*b*cosh(d*x + c)^2 + 2*a^2 - a*b)*sinh(d*x + c)^2 +
a*b + 4*(a*b*cosh(d*x + c)^3 + (2*a^2 - a*b)*cosh(d*x + c))*sinh(d*x + c))*
arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a*b - b^2)*cosh(d*x + c) + 2*(3*
(a*b - b^2)*cosh(d*x + c)^2 - a*b + b^2)*sinh(d*x + c))/((a^3*b - 2*a^2*b^2
 + a*b^3)*d*cosh(d*x + c)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*d*cosh(d*x + c)
*sinh(d*x + c)^3 + (a^3*b - 2*a^2*b^2 + a*b^3)*d*sinh(d*x + c)^4 + 2*(2*a^4
 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c)^2 + 2*(3*(a^3*b - 2*a^2*b^2
 + a*b^3)*d*cosh(d*x + c)^2 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d)*sinh
(d*x + c)^2 + (a^3*b - 2*a^2*b^2 + a*b^3)*d + 4*((a^3*b - 2*a^2*b^2 + a*b^3
)*d*cosh(d*x + c)^3 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*d*cosh(d*x + c)
)*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

<u>Giac</u> [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.336 $\int \frac{\mathbf{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=114

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\tanh(c+dx)}{2ad(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{d(a-b)^2}$$

```
[Out] -((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
 b)^(5/2)*d) + Tanh[c + d*x]/((a - b)^2*d) + (b^2*Tanh[c + d*x])/(2*a*(a -
b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.179353, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 208}

$$-\frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{5/2}}+\frac{b^2\tanh(c+dx)}{2ad(a-b)^2\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a -
 b)^(5/2)*d) + Tanh[c + d*x]/((a - b)^2*d) + (b^2*Tanh[c + d*x])/(2*a*(a -
b)^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^2} - \frac{(2a-b)b-2(a-b)bx^2}{(a-b)^2\left(a+(-a+b)x^2\right)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^2 d} - \frac{\text{Subst}\left(\int \frac{(2a-b)b-2(a-b)bx^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{(a-b)^2 d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^2 d} + \frac{b^2\tanh(c+dx)}{2a(a-b)^2 d\left(a-(a-b)\tanh^2(c+dx)\right)} - \frac{((4a-b)b)\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\right.}{2a(a-b)^2 c}$$

$$= -\frac{(4a-b)b\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{5/2}d} + \frac{\tanh(c+dx)}{(a-b)^2 d} + \frac{b^2\tanh(c+dx)}{2a(a-b)^2 d\left(a-(a-b)\tanh^2(c+\right.}$$

Mathematica [A] time = 0.855237, size = 105, normalized size = 0.92

$$\frac{\frac{\frac{b^2\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)}+2\tanh(c+dx)}{(a-b)^2} - \frac{b(4a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{5/2}}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (-(((4*a - b)*b*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a -
 b)^(5/2))) + ((b^2*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)]))
+ 2*Tanh[c + d*x])/(a - b)^2)/(2*d)
```

Maple [B] time = 0.075, size = 798, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] 1/d*b^2/(a-b)^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3+1/d*b^2/(a-b)^2/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/
2*d*x+1/2*c)-2/d*b/(a-b)^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d*b^2/(a-b)^2/(-
b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d*b/(a-b)^2/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*
b)*a)^(1/2))+2/d*b^2/(a-b)^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+
1/2/d*b^2/(a-b)^2/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
```

```
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d*b^3/(a-b)^2/a/(-b*(
a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d*b^2/(a-b)^2/a/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2))-1/2/d*b^3/(a-b)^2/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2))+2/d/(a-b)^2*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d*x+1/2*c)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.08948, size = 7185, normalized size = 63.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^4 + 16*(4*a^3*b - 5*a^
2*b^2 + a*b^3)*cosh(d*x + c)*sinh(d*x + c)^3 + 4*(4*a^3*b - 5*a^2*b^2 + a*b
^3)*sinh(d*x + c)^4 + 8*a^3*b - 4*a^2*b^2 - 4*a*b^3 + 8*(4*a^4 - 5*a^3*b +
a^2*b^2)*cosh(d*x + c)^2 + 8*(4*a^4 - 5*a^3*b + a^2*b^2 + 3*(4*a^3*b - 5*a^
2*b^2 + a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((4*a*b^2 - b^3)*cosh(d*x
 + c)^6 + 6*(4*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - b^3)
*sinh(d*x + c)^6 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^2*b -
 8*a*b^2 + b^3 + 15*(4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5
*(4*a*b^2 - b^3)*cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*a*b^2 - b^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c
)^2 + (15*(4*a*b^2 - b^3)*cosh(d*x + c)^4 + 16*a^2*b - 8*a*b^2 + b^3 + 6*(1
6*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 -
 b^3)*cosh(d*x + c)^5 + 2*(16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^3 + (16*
a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b
^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c
)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b -
b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*
b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x
 + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh
(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*
a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2
+ 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*
((4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + (4*a^4 - 5*a^3*b + a^2*b^2
)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*
d*cosh(d*x + c)^6 + 6*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x
+ c)*sinh(d*x + c)^5 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*sinh(d*x
 + c)^6 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x
+ c)^4 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^2 +
(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^4 +
```

```
(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + 4
*(5*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^3 + (4*a^6 -
13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^3 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^4 + 6*(4
*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^2 + (4*
a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*sinh(d*x + c)^2 + (a^
5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d + 2*(3*(a^5*b - 3*a^4*b^2 + 3*a^3*
b^3 - a^2*b^4)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3
*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*
b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), -1/2*(2*(4*a^3*b - 5*a^2*b^
2 + a*b^3)*cosh(d*x + c)^4 + 8*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)*
sinh(d*x + c)^3 + 2*(4*a^3*b - 5*a^2*b^2 + a*b^3)*sinh(d*x + c)^4 + 4*a^3*b
 - 2*a^2*b^2 - 2*a*b^3 + 4*(4*a^4 - 5*a^3*b + a^2*b^2)*cosh(d*x + c)^2 + 4*
(4*a^4 - 5*a^3*b + a^2*b^2 + 3*(4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 - ((4*a*b^2 - b^3)*cosh(d*x + c)^6 + 6*(4*a*b^2 - b^3)*c
osh(d*x + c)*sinh(d*x + c)^5 + (4*a*b^2 - b^3)*sinh(d*x + c)^6 + (16*a^2*b
- 8*a*b^2 + b^3)*cosh(d*x + c)^4 + (16*a^2*b - 8*a*b^2 + b^3 + 15*(4*a*b^2
- b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 4*(5*(4*a*b^2 - b^3)*cosh(d*x + c
)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*a*b^2 -
 b^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^2 + (15*(4*a*b^2 - b^3)*cos
h(d*x + c)^4 + 16*a^2*b - 8*a*b^2 + b^3 + 6*(16*a^2*b - 8*a*b^2 + b^3)*cosh
(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*(4*a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(16
*a^2*b - 8*a*b^2 + b^3)*cosh(d*x + c)^3 + (16*a^2*b - 8*a*b^2 + b^3)*cosh(d
*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2
*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a
*b)/(a^2 - a*b)) + 8*((4*a^3*b - 5*a^2*b^2 + a*b^3)*cosh(d*x + c)^3 + (4*a^
4 - 5*a^3*b + a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b - 3*a^4*b^2 +
3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^6 + 6*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 -
 a^2*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^5 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3
- a^2*b^4)*d*sinh(d*x + c)^6 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 +
 a^2*b^4)*d*cosh(d*x + c)^4 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)
*d*cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*
d)*sinh(d*x + c)^4 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*
d*cosh(d*x + c)^2 + 4*(5*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d
*x + c)^3 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + (15*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*c
osh(d*x + c)^4 + 6*(4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*
cosh(d*x + c)^2 + (4*a^6 - 13*a^5*b + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d)*
sinh(d*x + c)^2 + (a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d + 2*(3*(a^5*b
 - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cosh(d*x + c)^5 + 2*(4*a^6 - 13*a^5*b
 + 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c)^3 + (4*a^6 - 13*a^5*b
+ 15*a^4*b^2 - 7*a^3*b^3 + a^2*b^4)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

Giac [A] time = 2.44126, size = 200, normalized size = 1.75

$$-\frac{4\,abe^{(4\,dx+4\,c)} - b^2e^{(4\,dx+4\,c)} + 8\,a^2e^{(2\,dx+2\,c)} - 2\,abe^{(2\,dx+2\,c)} + 2\,ab + b^2}{\left(a^3d - 2\,a^2bd + ab^2d\right)\left(be^{(6\,dx+6\,c)} + 4\,ae^{(4\,dx+4\,c)} - be^{(4\,dx+4\,c)} + 4\,ae^{(2\,dx+2\,c)} - be^{(2\,dx+2\,c)} + b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -(4*a*b*e^(4*d*x + 4*c) - b^2*e^(4*d*x + 4*c) + 8*a^2*e^(2*d*x + 2*c) - 2*a
*b*e^(2*d*x + 2*c) + 2*a*b + b^2)/((a^3*d - 2*a^2*b*d + a*b^2*d)*(b*e^(6*d*
x + 6*c) + 4*a*e^(4*d*x + 4*c) - b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) -
b*e^(2*d*x + 2*c) + b))
```

3.337 $\int \frac{\text{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=157

$$\frac{b^{3/2}(5a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\sinh(c+dx)}{2ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{(a-5b)\tan^{-1}(\sinh(c+dx))}{2d(a-b)^3}+\frac{\tanh(c+dx)\text{se}}{2d(a-b)\left(a+b\,\text{si}\right.}$$

```
[Out] ((a - 5*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^3*d) + ((5*a - b)*b^(3/2)*ArcT
an[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (b*(a + b)*S
inh[c + d*x])/(2*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)) + (Sech[c + d*x]*Ta
nh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.19279, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}(5a-b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^3}+\frac{b(a+b)\sinh(c+dx)}{2ad(a-b)^2\left(a+b\sinh^2(c+dx)\right)}+\frac{(a-5b)\tan^{-1}(\sinh(c+dx))}{2d(a-b)^3}+\frac{\tanh(c+dx)\text{se}}{2d(a-b)\left(a+b\,\text{si}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```


```
[Out] ((a - 5*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^3*d) + ((5*a - b)*b^(3/2)*ArcT
an[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a - b)^3*d) + (b*(a + b)*S
inh[c + d*x])/(2*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)) + (Sech[c + d*x]*Ta
nh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]^2))
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a+2b-3bx^2}{\left(1+x^2\right)\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{2(a-b)d}$$

$$= \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2(a^2-}{(}\right.}{}$$

$$= \frac{b(a+b)\sinh(c+dx)}{2a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)} + \frac{(a-5b)\,\text{Subst}\left(\right.}{}$$

$$= \frac{(a-5b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^3d} + \frac{(5a-b)b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^3d} + \frac{b(a+b)\sinh}{2a(a-b)^2d\left(a+b\,s\right.}$$

Mathematica [A] time = 0.874894, size = 230, normalized size = 1.46

$$\frac{(2a-b)\left(2a^{3/2}(a-5b)\tan^{-1}\left(\tanh\left(\frac{1}{2}(c+dx)\right)\right)+a^{3/2}(a-b)\tanh(c+dx)\operatorname{sech}(c+dx)+b^{3/2}(b-5a)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c}{\sqrt{b}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] (2*Sqrt[a]*(a - b)*b^2*Sinh[c + d*x] + (2*a - b)*(b^(3/2)*(-5*a + b)*ArcTan
[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a - 5*b)*ArcTan[Tanh[(c + d*
x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c + d*x]) + b*Cosh[2*(c + d*x)]
*(b^(3/2)*(-5*a + b)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 2*a^(3/2)*(a
 - 5*b)*ArcTan[Tanh[(c + d*x)/2]] + a^(3/2)*(a - b)*Sech[c + d*x]*Tanh[c +
```

```
d*x]))/(2*a^(3/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)]))
```

Maple [B] time = 0.089, size = 1265, normalized size = 8.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)^3+1/d*b^3/(a-b)^3/(tanh(1/2*d*x+1
/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2
*d*x+1/2*c)^3+1/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)*tanh(1/2*d*x+1/2*c)-1/d*b^3/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
/a*tanh(1/2*d*x+1/2*c)-5/2/d*b^2/(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2))-5/2/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*b^3/(a-b
)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/2/d*b^2/(a-b)^3*a/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+5/2/d*b^2/(a-b)^3/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2))+3/d*b^3/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
+1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d*b^4/(a-b)^3/a/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b
))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2))-1/2/d*b^4/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-
a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)
^(1/2))-1/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*a+1/d
/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3*b+1/d/(a-b)^3/(t
anh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*a-1/d/(a-b)^3/(tanh(1/2*d*x+1
/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)*b+1/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))*
a-5/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))*b
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] (a*e^c - 5*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^3*d - 3*a^2*b*d + 3*a*b^2*d
 - b^3*d) + ((a*b*e^(7*c) + b^2*e^(7*c))*e^(7*d*x) + (4*a^2*e^(5*c) - 3*a*b
*e^(5*c) + b^2*e^(5*c))*e^(5*d*x) - (4*a^2*e^(3*c) - 3*a*b*e^(3*c) + b^2*e^
(3*c))*e^(3*d*x) - (a*b*e^c + b^2*e^c)*e^(d*x))/(a^3*b*d - 2*a^2*b^2*d + a*
b^3*d + (a^3*b*d*e^(8*c) - 2*a^2*b^2*d*e^(8*c) + a*b^3*d*e^(8*c))*e^(8*d*x)
 + 4*(a^4*d*e^(6*c) - 2*a^3*b*d*e^(6*c) + a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*
```

```
(4*a^4*d*e^(4*c) - 9*a^3*b*d*e^(4*c) + 6*a^2*b^2*d*e^(4*c) - a*b^3*d*e^(4*c
))*e^(4*d*x) + 4*(a^4*d*e^(2*c) - 2*a^3*b*d*e^(2*c) + a^2*b^2*d*e^(2*c))*e^
(2*d*x)) + 8*integrate(1/8*((5*a*b^2*e^(3*c) - b^3*e^(3*c))*e^(3*d*x) + (5*
a*b^2*e^c - b^3*e^c)*e^(d*x))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4 + (a^4
*b*e^(4*c) - 3*a^3*b^2*e^(4*c) + 3*a^2*b^3*e^(4*c) - a*b^4*e^(4*c))*e^(4*d*
x) + 2*(2*a^5*e^(2*c) - 7*a^4*b*e^(2*c) + 9*a^3*b^2*e^(2*c) - 5*a^2*b^3*e^(
2*c) + a*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.00745, size = 15107, normalized size = 96.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(a^2*b - b^3)*cosh(d*x + c)^7 + 28*(a^2*b - b^3)*cosh(d*x + c)*sinh
(d*x + c)^6 + 4*(a^2*b - b^3)*sinh(d*x + c)^7 + 4*(4*a^3 - 7*a^2*b + 4*a*b^
2 - b^3)*cosh(d*x + c)^5 + 4*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3 + 21*(a^2*b -
 b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*b - b^3)*cosh(d*x + c)^
3 + (4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(4
*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 + 4*(35*(a^2*b - b^3)*cosh(
d*x + c)^4 - 4*a^3 + 7*a^2*b - 4*a*b^2 + b^3 + 10*(4*a^3 - 7*a^2*b + 4*a*b^
2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b - b^3)*cosh(d*x +
c)^5 + 10*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^3 - 7*
a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((5*a*b^2 - b^3)*co
sh(d*x + c)^8 + 8*(5*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2
- b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(5*a^2*b -
 a*b^2 + 7*(5*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2
 - b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^
5 + 2*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - b^3)*co
sh(d*x + c)^4 + 20*a^2*b - 9*a*b^2 + b^3 + 30*(5*a^2*b - a*b^2)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b
- a*b^2)*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c))*sinh(d
*x + c)^3 + 5*a*b^2 - b^3 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(5*a
*b^2 - b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - a*b^2)*cosh(d*x + c)^4 + 5*a^2*
b - a*b^2 + 3*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((5*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c)^5 +
 (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^3 + (5*a^2*b - a*b^2)*cosh(d*x +
c))*sinh(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*si
nh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*co
sh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)
*cosh(d*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*si
nh(d*x + c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2
- a)*sinh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*
sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*
cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a -
b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 4*((a^2*b - 5*a*b^2)*cosh(d*x + c)^
8 + 8*(a^2*b - 5*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - 5*a*b^2)*s
inh(d*x + c)^8 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^6 + 4*(a^3 - 5*a^2*b + 7*(
a^2*b - 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b - 5*a*b^2)*
cosh(d*x + c)^3 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(4*a
^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2*b - 5*a*b^2)*cosh(d*x
 + c)^4 + 4*a^3 - 21*a^2*b + 5*a*b^2 + 30*(a^3 - 5*a^2*b)*cosh(d*x + c)^2)*
sinh(d*x + c)^4 + 8*(7*(a^2*b - 5*a*b^2)*cosh(d*x + c)^5 + 10*(a^3 - 5*a^2*
b)*cosh(d*x + c)^3 + (4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x +
 c)^3 + a^2*b - 5*a*b^2 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^2 + 4*(7*(a^2*b -
```

5*a*b^2)*cosh(d*x + c)^6 + 15*(a^3 - 5*a^2*b)*cosh(d*x + c)^4 + a^3 - 5*a^2*b + 3*(4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b - 5*a*b^2)*cosh(d*x + c)^7 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c)^5 + (4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^3 + (a^3 - 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 4*(a^2*b - b^3)*cosh(d*x + c) + 4*(7*(a^2*b - b^3)*cosh(d*x + c)^6 + 5*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^4 - a^2*b + b^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*sinh(d*x + c)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d + 8*((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*cosh(d*x + c)^3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh(d*x + c)), 1/2*(2*(a^2*b - b^3)*cosh(d*x + c)^7 + 14*(a^2*b - b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b - b^3)*sinh(d*x + c)^7 + 2*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^5 + 2*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3 + 21*(a^2*b - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(a^2*b - b^3)*cosh(d*x + c)^3 + (4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 + 2*(35*(a^2*b - b^3)*cosh(d*x + c)^4 - 4*a^3 + 7*a^2*b - 4*a*b^2 + b^3 + 10*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(a^2*b - b^3)*cosh(d*x + c)^5 + 10*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c)^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((5*a*b^2 - b^3)*cosh(d*x + c)^8 + 8*(5*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(5*a^2*b - a*b^2 + 7*(5*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - b^3)*cosh(d*x + c)^4 + 20*a^2*b - 9*a*b^2 + b^3 + 30*(5*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - a*b^2)*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^2 - b^3 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^6 + 15*(5*a^2*b - a*b^2)*cosh(d*x + c)^4 + 5*a^2*b - a*b^2 + 3*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - b^3)*cosh(d*x + c)^7 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c)^5 + (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^3 + (5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*sqrt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + ((5*a*b^2 - b^3)*cosh(d*x + c)^8 + 8*(5*a*b^2 - b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (5*a*b^2 - b^3)*sinh(d*x + c)^8 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^6 + 4*(5*a^2*b - a*b^2 + 7*(5*a*b^2 - b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^3 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c)^4 + 2*(35*(5*a*b^2 - b^3)*cosh(d*x + c)^4 + 20*a^2*b

```
- 9*a*b^2 + b^3 + 30*(5*a^2*b - a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8
*(7*(5*a*b^2 - b^3)*cosh(d*x + c)^5 + 10*(5*a^2*b - a*b^2)*cosh(d*x + c)^3
+ (20*a^2*b - 9*a*b^2 + b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + 5*a*b^2 - b^3
 + 4*(5*a^2*b - a*b^2)*cosh(d*x + c)^2 + 4*(7*(5*a*b^2 - b^3)*cosh(d*x + c)
^6 + 15*(5*a^2*b - a*b^2)*cosh(d*x + c)^4 + 5*a^2*b - a*b^2 + 3*(20*a^2*b -
 9*a*b^2 + b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((5*a*b^2 - b^3)*cosh(
d*x + c)^7 + 3*(5*a^2*b - a*b^2)*cosh(d*x + c)^5 + (20*a^2*b - 9*a*b^2 + b^
3)*cosh(d*x + c)^3 + (5*a^2*b - a*b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b
/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*s
inh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*
sinh(d*x + c))*sqrt(b/a)/b) + 2*((a^2*b - 5*a*b^2)*cosh(d*x + c)^8 + 8*(a^2
*b - 5*a*b^2)*cosh(d*x + c)*sinh(d*x + c)^7 + (a^2*b - 5*a*b^2)*sinh(d*x +
c)^8 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^6 + 4*(a^3 - 5*a^2*b + 7*(a^2*b - 5*
a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a^2*b - 5*a*b^2)*cosh(d*x +
 c)^3 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(4*a^3 - 21*a^
2*b + 5*a*b^2)*cosh(d*x + c)^4 + 2*(35*(a^2*b - 5*a*b^2)*cosh(d*x + c)^4 +
4*a^3 - 21*a^2*b + 5*a*b^2 + 30*(a^3 - 5*a^2*b)*cosh(d*x + c)^2)*sinh(d*x +
 c)^4 + 8*(7*(a^2*b - 5*a*b^2)*cosh(d*x + c)^5 + 10*(a^3 - 5*a^2*b)*cosh(d*
x + c)^3 + (4*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + a^
2*b - 5*a*b^2 + 4*(a^3 - 5*a^2*b)*cosh(d*x + c)^2 + 4*(7*(a^2*b - 5*a*b^2)*
cosh(d*x + c)^6 + 15*(a^3 - 5*a^2*b)*cosh(d*x + c)^4 + a^3 - 5*a^2*b + 3*(4
*a^3 - 21*a^2*b + 5*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a^2*b - 5
*a*b^2)*cosh(d*x + c)^7 + 3*(a^3 - 5*a^2*b)*cosh(d*x + c)^5 + (4*a^3 - 21*a
^2*b + 5*a*b^2)*cosh(d*x + c)^3 + (a^3 - 5*a^2*b)*cosh(d*x + c))*sinh(d*x +
 c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 2*(a^2*b - b^3)*cosh(d*x + c)
+ 2*(7*(a^2*b - b^3)*cosh(d*x + c)^6 + 5*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*
cosh(d*x + c)^4 - a^2*b + b^3 - 3*(4*a^3 - 7*a^2*b + 4*a*b^2 - b^3)*cosh(d*
x + c)^2)*sinh(d*x + c))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*
x + c)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)*sinh(d
*x + c)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*sinh(d*x + c)^8 + 4*(
a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b - 3*a^
3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*d)*sinh(d*x + c)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3
 + a*b^4)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*
d*cosh(d*x + c)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)
)*sinh(d*x + c)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*
x + c)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + (4*
a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d)*sinh(d*x + c)^4 + 4*(a^
5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b - 3*a^3*
b^2 + 3*a^2*b^3 - a*b^4)*d*cosh(d*x + c)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2
- a^2*b^3)*d*cosh(d*x + c)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 +
 a*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*
b^3 - a*b^4)*d*cosh(d*x + c)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d
*cosh(d*x + c)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*
cosh(d*x + c)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d)*sinh(d*x + c)^2
+ (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d + 8*((a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*d*cosh(d*x + c)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d
*cosh(d*x + c)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*d*co
sh(d*x + c)^3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d*cosh(d*x + c))*sinh
(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**2,x)
```

[Out] Timed out

Giac [B] time = 1.61301, size = 1322, normalized size = 8.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a^5*b*d - 12*a^4*b^2*d + 22*a^3*b^3*d - 12*a^2*b^4*d + a*b^5*d - a*b*
abs(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d) - b^2*abs(-a^4*d + 3*a^3*b*
d - 3*a^2*b^2*d + a*b^3*d))*arctan(1/2*sqrt(2)*(e^(d*x + c) - e^(-d*x - c))
/sqrt((a^4*d - a^3*b*d - a^2*b^2*d + a*b^3*d + sqrt((a^4*d - a^3*b*d - a^2*
b^2*d + a*b^3*d)^2 - 4*(a^4*d - 2*a^3*b*d + a^2*b^2*d)*(a^3*b*d - 2*a^2*b^2
*d + a*b^3*d)))/(a^3*b*d - 2*a^2*b^2*d + a*b^3*d)))/(a^4*d*abs(-a^4*d + 3*a
^3*b*d - 3*a^2*b^2*d + a*b^3*d) - a^3*b*d*abs(-a^4*d + 3*a^3*b*d - 3*a^2*b^
2*d + a*b^3*d) - a^2*b^2*d*abs(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d)
+ a*b^3*d*abs(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d) + (a^4*d - 3*a^3*
b*d + 3*a^2*b^2*d - a*b^3*d)^2) - 1/2*((a^5 - 12*a^4*b + 22*a^3*b^2 - 12*a^
2*b^3 + a*b^4)*sqrt(a*b)*d*abs(b) + sqrt(a*b)*(a + b)*abs(-a^4*d + 3*a^3*b*
d - 3*a^2*b^2*d + a*b^3*d)*abs(b))*arctan(1/2*sqrt(2)*(e^(d*x + c) - e^(-d*
x - c))/sqrt((a^4*d - a^3*b*d - a^2*b^2*d + a*b^3*d - sqrt((a^4*d - a^3*b*d
 - a^2*b^2*d + a*b^3*d)^2 - 4*(a^4*d - 2*a^3*b*d + a^2*b^2*d)*(a^3*b*d - 2*
a^2*b^2*d + a*b^3*d)))/(a^3*b*d - 2*a^2*b^2*d + a*b^3*d)))/((a^4*d - 3*a^3*
b*d + 3*a^2*b^2*d - a*b^3*d)^2*b - (a^4*b - a^3*b^2 - a^2*b^3 + a*b^4)*d*ab
s(-a^4*d + 3*a^3*b*d - 3*a^2*b^2*d + a*b^3*d)) + (a*b*(e^(d*x + c) - e^(-d*
x - c))^3 + b^2*(e^(d*x + c) - e^(-d*x - c))^3 + 4*a^2*(e^(d*x + c) - e^(-d
*x - c)) + 4*b^2*(e^(d*x + c) - e^(-d*x - c)))/((b*(e^(d*x + c) - e^(-d*x -
 c))^4 + 4*a*(e^(d*x + c) - e^(-d*x - c))^2 + 4*b*(e^(d*x + c) - e^(-d*x -
c))^2 + 16*a)*(a^3*d - 2*a^2*b*d + a*b^2*d))
```

3.338 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=143

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{2ad(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\tanh^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\tanh(c+dx)}{d(a-b)^3}$$

```
[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a
- b)^(7/2)*d) + ((a - 3*b)*Tanh[c + d*x])/((a - b)^3*d) - Tanh[c + d*x]^3/(
3*(a - b)^2*d) - (b^3*Tanh[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Tanh[c +
 d*x]^2))
```

Rubi [A] time = 0.207041, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 208}

$$\frac{b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{2ad(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{\tanh^3(c+dx)}{3d(a-b)^2}+\frac{(a-3b)\tanh(c+dx)}{d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a
- b)^(7/2)*d) + ((a - 3*b)*Tanh[c + d*x])/((a - b)^3*d) - Tanh[c + d*x]^3/(
3*(a - b)^2*d) - (b^3*Tanh[c + d*x])/(2*a*(a - b)^3*d*(a - (a - b)*Tanh[c +
 d*x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{(a-b)^3}-\frac{x^2}{(a-b)^2}+\frac{(3a-b)b^2-3(a-b)b^2x^2}{(a-b)^3\left(a+(-a+b)x^2\right)^2}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a-3b)\tanh(c+dx)}{(a-b)^3d}-\frac{\tanh^3(c+dx)}{3(a-b)^2d}+\frac{\text{Subst}\left(\int \frac{(3a-b)b^2-3(a-b)b^2x^2}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{(a-b)^3d}$$

$$= \frac{(a-3b)\tanh(c+dx)}{(a-b)^3d}-\frac{\tanh^3(c+dx)}{3(a-b)^2d}-\frac{b^3\tanh(c+dx)}{2a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)}+ \frac{((6a}{}$$

$$= \frac{(6a-b)b^2\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a-b)^{7/2}d}+\frac{(a-3b)\tanh(c+dx)}{(a-b)^3d}-\frac{\tanh^3(c+dx)}{3(a-b)^2d}-\frac{}{2a(a-b)}$$

Mathematica [A] time = 1.89396, size = 130, normalized size = 0.91

$$\frac{\frac{3b^2(6a-b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a-b)^{7/2}}+\frac{2\tanh(c+dx)\left((a-b)\operatorname{sech}^2(c+dx)+2(a-4b)\right)-\frac{3b^3\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)}}{(a-b)^3}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^2,x]
```

```
[Out] ((3*(6*a - b)*b^2*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a^(3/2)*(a
 - b)^(7/2)) + ((-3*b^3*Sinh[2*(c + d*x)])/(a*(2*a - b + b*Cosh[2*(c + d*x)
])) + 2*(2*(a - 4*b) + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b)^3)/(
6*d)
```

Maple [B] time = 0.092, size = 998, normalized size = 7.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x)
```

```
[Out] -1/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(
1/2*d*x+1/2*c)^2*b+a)/a*tanh(1/2*d*x+1/2*c)^3-1/d*b^3/(a-b)^3/(tanh(1/2*d*x
+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)/a*tanh(1
/2*d*x+1/2*c)+3/d*b^2/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(
a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*b^3/(a-b)^3
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/d*b^2/(a-b)^3/((2*(-b*(a-
```

```
b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
-a+2*b)*a)^(1/2))-3/d*b^3/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2
*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1
/2))-1/2/d*b^3/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tan
h(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d*b^4/(a-b)^3/a/
(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/2/d*b^3/(a-b)^3/a/((2*(-b*
(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2))+1/2/d*b^4/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1
/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b
)*a)^(1/2))+2/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a
-6/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*b+4/3/d/(a-b
)^3/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*a-28/3/d/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^3*b+2/d/(a-b)^3/(tanh(1/2*d*x+1
/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)*a-6/d/(a-b)^3/(tanh(1/2*d*x+1/2*c)^2+1)^3*
tanh(1/2*d*x+1/2*c)*b
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.66689, size = 17920, normalized size = 125.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^8 + 96*(6*a^3*b^2 -
 7*a^2*b^3 + a*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + 12*(6*a^3*b^2 - 7*a^2*b
^3 + a*b^4)*sinh(d*x + c)^8 + 24*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*co
sh(d*x + c)^6 + 24*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4 + 14*(6*a^3*b^2 -
 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 48*(14*(6*a^3*b^2 -
7*a^2*b^3 + a*b^4)*cosh(d*x + c)^3 + 3*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b
^4)*cosh(d*x + c))*sinh(d*x + c)^5 - 16*a^4*b + 80*a^3*b^2 - 52*a^2*b^3 - 1
2*a*b^4 - 8*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^4
- 8*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3 - 105*(6*a^3*b^2 - 7*a^2*
b^3 + a*b^4)*cosh(d*x + c)^4 - 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*co
sh(d*x + c)^5 + 15*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^3
- (24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c))*sinh(d*x +
c)^3 - 8*(8*a^5 - 38*a^4*b + 25*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c
)^2 + 8*(42*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 - 8*a^5 + 38*a^
4*b - 25*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4 + 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3
+ a*b^4)*cosh(d*x + c)^4 - 6*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)
*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((6*a*b^3 - b^4)*cosh(d*x + c)^10 + 1
0*(6*a*b^3 - b^4)*cosh(d*x + c)*sinh(d*x + c)^9 + (6*a*b^3 - b^4)*sinh(d*x
+ c)^10 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 + 2*a*
```

```
b^3 - b^4 + 45*(6*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(6*
a*b^3 - b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c))*
sinh(d*x + c)^7 + 2*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^6 + 2*(105*
(6*a*b^3 - b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 14*(24*a^2*
b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - b^
4)*cosh(d*x + c)^5 + 14*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(3
6*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2*b^2
 - 12*a*b^3 + b^4)*cosh(d*x + c)^4 + 2*(105*(6*a*b^3 - b^4)*cosh(d*x + c)^6
 + 35*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 +
 b^4 + 15*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 +
6*a*b^3 - b^4 + 8*(15*(6*a*b^3 - b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 + 2*a
*b^3 - b^4)*cosh(d*x + c)^5 + 5*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)
^3 + (36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (24*a^2
*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^2 + (45*(6*a*b^3 - b^4)*cosh(d*x + c)^8
 + 28*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^6 + 30*(36*a^2*b^2 - 12*a*
b^3 + b^4)*cosh(d*x + c)^4 + 24*a^2*b^2 + 2*a*b^3 - b^4 + 12*(36*a^2*b^2 -
12*a*b^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - b^4)*cos
h(d*x + c)^9 + 4*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^7 + 6*(36*a^2*b
^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^5 + 4*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh
(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(a^2 - a*b)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x +
 c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt
(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sin
h(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a -
 b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(
d*x + c) + b)) + 16*(6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^7 + 9*
(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 - 2*(24*a^5 - 106*a
^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^3 - (8*a^5 - 38*a^4*b + 25*a^
3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^6*b - 4*a^5*
b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^10 + 10*(a^6*b - 4*a
^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^9 +
 (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*sinh(d*x + c)^10 +
 (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^8 +
 (45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^
2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)
^8 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*
d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*
b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
b^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b - 4*a^5*b^2 + 6*a^4*b
^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 14*(4*a^7 - 15*a^6*b + 20*a^5
*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^2 + (6*a^7 - 25*a^6*b + 40*a^5
*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(6*a^7 - 2
5*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4
 + 4*(63*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x +
 c)^5 + 14*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*
x + c)^3 + 3*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6*b - 4*a^5*b^2 + 6*a^4*
b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 35*(4*a^7 - 15*a^6*b + 20*a^
5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^4 + 15*(6*a^7 - 25*a^6*b + 40
*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (6*a^7 -
25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)
^4 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)
^2 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x
 + c)^7 + 7*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d
*x + c)^5 + 5*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^
2*b^5)*d*cosh(d*x + c)^3 + (6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10
```

*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + (45*(a^6*b - 4*a^5*b
^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8 + 28*(4*a^7 - 15*a^
6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^6 + 30*(6*a^7 - 25
*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4
+ 12*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*
cosh(d*x + c)^2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d)
*sinh(d*x + c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d
+ 2*(5*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c
)^9 + 4*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x +
c)^7 + 6*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^
5)*d*cosh(d*x + c)^5 + 4*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a
^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a
^4*b^3 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), 1/6*(6*(6*a^3*b^2 - 7*a^
2*b^3 + a*b^4)*cosh(d*x + c)^8 + 48*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*
x + c)*sinh(d*x + c)^7 + 6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*sinh(d*x + c)^8
+ 12*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^6 + 12*(6*a^4*b
- a^3*b^2 - 6*a^2*b^3 + a*b^4 + 14*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x
+ c)^2)*sinh(d*x + c)^6 + 24*(14*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x
+ c)^3 + 3*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c))*sinh(d*x
+ c)^5 - 8*a^4*b + 40*a^3*b^2 - 26*a^2*b^3 - 6*a*b^4 - 4*(24*a^5 - 106*a^4*
b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^4 - 4*(24*a^5 - 106*a^4*b + 95*a
^3*b^2 - 13*a^2*b^3 - 105*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 -
45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + 16*(21*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*cosh(d*x + c)^5 + 15*(6*a^4*b -
a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^3 - (24*a^5 - 106*a^4*b + 95*a^3
*b^2 - 13*a^2*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(8*a^5 - 38*a^4*b + 2
5*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2 + 4*(42*(6*a^3*b^2 - 7*a^2
*b^3 + a*b^4)*cosh(d*x + c)^6 - 8*a^5 + 38*a^4*b - 25*a^3*b^2 - 2*a^2*b^3 -
3*a*b^4 + 45*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^4 - 6*(
24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^2)*sinh(d*x + c
)^2 - 3*((6*a*b^3 - b^4)*cosh(d*x + c)^10 + 10*(6*a*b^3 - b^4)*cosh(d*x + c
)*sinh(d*x + c)^9 + (6*a*b^3 - b^4)*sinh(d*x + c)^10 + (24*a^2*b^2 + 2*a*b^
3 - b^4)*cosh(d*x + c)^8 + (24*a^2*b^2 + 2*a*b^3 - b^4 + 45*(6*a*b^3 - b^4)
*cosh(d*x + c)^2)*sinh(d*x + c)^8 + 8*(15*(6*a*b^3 - b^4)*cosh(d*x + c)^3 +
(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(36*a^2*b^
2 - 12*a*b^3 + b^4)*cosh(d*x + c)^6 + 2*(105*(6*a*b^3 - b^4)*cosh(d*x + c)^
4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 14*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x
+ c)^2)*sinh(d*x + c)^6 + 4*(63*(6*a*b^3 - b^4)*cosh(d*x + c)^5 + 14*(24*a^
2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^3 + 3*(36*a^2*b^2 - 12*a*b^3 + b^4)*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c
)^4 + 2*(105*(6*a*b^3 - b^4)*cosh(d*x + c)^6 + 35*(24*a^2*b^2 + 2*a*b^3 - b
^4)*cosh(d*x + c)^4 + 36*a^2*b^2 - 12*a*b^3 + b^4 + 15*(36*a^2*b^2 - 12*a*b
^3 + b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 6*a*b^3 - b^4 + 8*(15*(6*a*b^3
- b^4)*cosh(d*x + c)^7 + 7*(24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x + c)^5 +
5*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^3 + (36*a^2*b^2 - 12*a*b^3 +
b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + (24*a^2*b^2 + 2*a*b^3 - b^4)*cosh(d*x
+ c)^2 + (45*(6*a*b^3 - b^4)*cosh(d*x + c)^8 + 28*(24*a^2*b^2 + 2*a*b^3 -
b^4)*cosh(d*x + c)^6 + 30*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^4 + 2
4*a^2*b^2 + 2*a*b^3 - b^4 + 12*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 2*(5*(6*a*b^3 - b^4)*cosh(d*x + c)^9 + 4*(24*a^2*b^2 +
2*a*b^3 - b^4)*cosh(d*x + c)^7 + 6*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x
+ c)^5 + 4*(36*a^2*b^2 - 12*a*b^3 + b^4)*cosh(d*x + c)^3 + (24*a^2*b^2 + 2*
a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*
cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a
- b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 8*(6*(6*a^3*b^2 - 7*a^2*b^3 + a*b^4)*
cosh(d*x + c)^7 + 9*(6*a^4*b - a^3*b^2 - 6*a^2*b^3 + a*b^4)*cosh(d*x + c)^5
- 2*(24*a^5 - 106*a^4*b + 95*a^3*b^2 - 13*a^2*b^3)*cosh(d*x + c)^3 - (8*a^
5 - 38*a^4*b + 25*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4)*cosh(d*x + c))*sinh(d*x +
c))/((a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^

```
10 + 10*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x +
c)*sinh(d*x + c)^9 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*
d*sinh(d*x + c)^10 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)
*d*cosh(d*x + c)^8 + (45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b
^5)*d*cosh(d*x + c)^2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b
^5)*d)*sinh(d*x + c)^8 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10
*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^
3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b + 20*a^5*b^2
 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^6*b -
 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 14*(4*a^7
 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^2 + (6*a^7
 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d)*sinh(d*x +
 c)^6 + 2*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^
5)*d*cosh(d*x + c)^4 + 4*(63*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a
^2*b^5)*d*cosh(d*x + c)^5 + 14*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3
+ a^2*b^5)*d*cosh(d*x + c)^3 + 3*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^
3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^6*b
- 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 35*(4*a^
7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c)^4 + 15*(6
*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*
x + c)^2 + (6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b
^5)*d)*sinh(d*x + c)^4 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*
b^5)*d*cosh(d*x + c)^2 + 8*(15*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 +
 a^2*b^5)*d*cosh(d*x + c)^7 + 7*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3
 + a^2*b^5)*d*cosh(d*x + c)^5 + 5*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b
^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (6*a^7 - 25*a^6*b + 40*a^5*b
^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 +
(45*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^8
 + 28*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c
)^6 + 30*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5
)*d*cosh(d*x + c)^4 + 12*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 + 10*a
^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a
^4*b^3 + a^2*b^5)*d)*sinh(d*x + c)^2 + (a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a
^3*b^4 + a^2*b^5)*d + 2*(5*(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2
*b^5)*d*cosh(d*x + c)^9 + 4*(4*a^7 - 15*a^6*b + 20*a^5*b^2 - 10*a^4*b^3 + a
^2*b^5)*d*cosh(d*x + c)^7 + 6*(6*a^7 - 25*a^6*b + 40*a^5*b^2 - 30*a^4*b^3 +
 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + 4*(6*a^7 - 25*a^6*b + 40*a^5*b^2
 - 30*a^4*b^3 + 10*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (4*a^7 - 15*a^6*b
 + 20*a^5*b^2 - 10*a^4*b^3 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**2,x)

[Out] Timed out
```

Giac [B] time = 1.38008, size = 377, normalized size = 2.64

$$\frac{\left(6\,ab^2-b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{2\left(a^4d-3\,a^3bd+3\,a^2b^2d-ab^3d\right)\sqrt{-a^2+ab}}+\frac{2\,ab^2e^{(2\,dx+2\,c)}-b^3e^{(2\,dx+2\,c)}+b^3}{\left(a^4d-3\,a^3bd+3\,a^2b^2d-ab^3d\right)\left(be^{(4\,dx+4\,c)}+4\,ae^{(2\,dx+2\,c)}-2\,be^{(2\,dx+2\,c)}+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(6*a*b^2 - b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*
b))/((a^4*d - 3*a^3*b*d + 3*a^2*b^2*d - a*b^3*d)*sqrt(-a^2 + a*b)) + (2*a*b
^2*e^(2*d*x + 2*c) - b^3*e^(2*d*x + 2*c) + b^3)/((a^4*d - 3*a^3*b*d + 3*a^2
*b^2*d - a*b^3*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x +
 2*c) + b)) + 4/3*(3*b*e^(4*d*x + 4*c) - 3*a*e^(2*d*x + 2*c) + 9*b*e^(2*d*x
 + 2*c) - a + 4*b)/((a^3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d)*(e^(2*d*x + 2*c
) + 1)^3)
```

3.339 $\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=160

$$-\frac{\sqrt{a-b}\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}-\frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+d\right.}$$

```
[Out] x/b^3 - (Sqrt[a - b]*(8*a^2 + 4*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) - ((a - b)*Tanh[c + d*x])/(4*a*b*d*(a - (
a - b)*Tanh[c + d*x]^2)^2) - ((a - b)*(4*a + 3*b)*Tanh[c + d*x])/(8*a^2*b^2
*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.236404, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3191, 414, 527, 522, 206, 208}

$$-\frac{\sqrt{a-b}\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d}-\frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] x/b^3 - (Sqrt[a - b]*(8*a^2 + 4*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c +
d*x])/Sqrt[a]])/(8*a^(5/2)*b^3*d) - ((a - b)*Tanh[c + d*x])/(4*a*b*d*(a - (
a - b)*Tanh[c + d*x]^2)^2) - ((a - b)*(4*a + 3*b)*Tanh[c + d*x])/(8*a^2*b^2
*d*(a - (a - b)*Tanh[c + d*x]^2))
```


Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^6(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-a-3b-3(a-b)x^2}{(1-x^2)\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4abd}$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+dx)\right)^2} - \frac{(a-b)(4a+3b)\tanh(c+dx)}{8a^2b^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{\text{Subst}\left(\int\right.}{}$$

$$= \frac{x}{b^3} - \frac{\sqrt{a-b}\left(8a^2+4ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^3d} - \frac{(a-b)\tanh(c+dx)}{4abd\left(a-(a-b)\tanh^2(c+\right.}$$

Mathematica [A] time = 1.34049, size = 164, normalized size = 1.02

$$\frac{-\frac{\left(-4a^2b+8a^3-ab^2-3b^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a-b}} + \frac{3b\left(-2a^2+ab+b^2\right)\sinh(2(c+dx))}{a^2(2a+b\cosh(2(c+dx))-b)} + \frac{4b(a-b)^2\sinh(2(c+dx))}{a(2a+b\cosh(2(c+dx))-b)^2} + 8(c+dx)}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^6/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - ((8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*ArcTanh[(Sqrt[a - b]*Tanh
[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a - b]) + (4*(a - b)^2*b*Sinh[2*(c + d*x
)])/(a*(2*a - b + b*Cosh[2*(c + d*x)])^2) + (3*b*(-2*a^2 + a*b + b^2)*Sinh[
2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(c + d*x)])))/(8*b^3*d)
```

Maple [B] time = 0.076, size = 2048, normalized size = 12.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -1/d/b^3*ln(tanh(1/2*d*x+1/2*c)-1)+1/d/b^3*ln(tanh(1/2*d*x+1/2*c)+1)+1/d/b^
2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b/a^2/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/d/b^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2))-3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3
/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+1/2/d/b^2/((2*(-b*(a-b))^(1/2)+a-2*b)
*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
))-1/2/d/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-
2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c
)^7-23/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-23/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a
-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*
c)^3-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-
3/8/d/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d/a^2/((2*(-b*(a-b))^(1/2)+a-2*b
)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/
2))+7/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)
^7+7/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c
)+1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^5+3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*
c)^5+1/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^3-1/d/b^3*a/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2))+1/d/b^3*a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/d/b^2/(tanh(1/2*d*x+1/2*c
)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d
*x+1/2*c)^7-1/8/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8/d/a/
(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/8/d/b/a/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2))+1/8/d/b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2
*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/b/(-b*(a-b))^(1/2)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/2/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)
-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.55732, size = 12741, normalized size = 79.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(16*a^2*b^2*d*x*cosh(d*x + c)^8 + 128*a^2*b^2*d*x*cosh(d*x + c)*sinh(
d*x + c)^7 + 16*a^2*b^2*d*x*sinh(d*x + c)^8 + 4*(16*a^3*b - 20*a^2*b^2 + a*
b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 4*(112*a^2*b^2*
d*x*cosh(d*x + c)^2 + 16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b -
 a^2*b^2)*d*x)*sinh(d*x + c)^6 + 16*a^2*b^2*d*x + 8*(112*a^2*b^2*d*x*cosh(d
*x + c)^3 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^
2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(48*a^4 - 72*a^3*b + 18*a^2*b^2
+ 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c)^4 +
 4*(280*a^2*b^2*d*x*cosh(d*x + c)^4 + 48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a
*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x + 15*(16*a^3*b - 20*a^2*
b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 24*a^2*b^2 - 12*a*b^3 - 12*b^4 + 16*(56*a^2*b^2*d*x*cosh(d*x + c)
^5 + 5*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)
*cosh(d*x + c)^3 + (48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(
8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(32*a^
3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x
+ c)^2 + 4*(112*a^2*b^2*d*x*cosh(d*x + c)^6 + 15*(16*a^3*b - 20*a^2*b^2 + a
*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^4 + 32*a^3*b - 28*
a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x + 6*(48*a^4 - 72*a^
3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*
cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x
+ c)^8 + 8*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (8
*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4
)*cosh(d*x + c)^6 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 + 4*a*b^3
+ 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 + 4*a*b^3 + 3*b
^4)*cosh(d*x + c)^3 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*
x + c)^5 + 2*(64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x +
 c)^4 + 2*(35*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64*a^4 - 32*a
^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(8*a^2*b^
2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh
(d*x + c)^3 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^3 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*
(7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b + 2*a*b^3 -
 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b + 2*a*b^3 - 3*b^4 + 3*(64*a^4 - 32*a^3*b
 + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((8*
a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)
```

```
*cosh(d*x + c)^5 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh
(d*x + c)^3 + (16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sq
rt((a - b)/a)*log((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^
3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d
*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cos
h(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cosh(d*x
 + c)^2 + 2*a*b*cosh(d*x + c)*sinh(d*x + c) + a*b*sinh(d*x + c)^2 + 2*a^2 -
 a*b)*sqrt((a - b)/a))/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)
^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)
^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x +
 c))*sinh(d*x + c) + b)) + 8*(16*a^2*b^2*d*x*cosh(d*x + c)^7 + 3*(16*a^3*b
- 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^5
+ 2*(48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b
 + 3*a^2*b^2)*d*x)*cosh(d*x + c)^3 + (32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*
b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^5*d*
cosh(d*x + c)^8 + 8*a^2*b^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*b^5*d*sin
h(d*x + c)^8 + a^2*b^5*d + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7
*a^2*b^5*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(
8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*a^2*b^5*d*cosh(
d*x + c)^3 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
35*a^2*b^5*d*cosh(d*x + c)^4 + 30*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 +
 (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^3*b^4 - a^
2*b^5)*d*cosh(d*x + c)^2 + 8*(7*a^2*b^5*d*cosh(d*x + c)^5 + 10*(2*a^3*b^4 -
 a^2*b^5)*d*cosh(d*x + c)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*
x + c))*sinh(d*x + c)^3 + 4*(7*a^2*b^5*d*cosh(d*x + c)^6 + 15*(2*a^3*b^4 -
a^2*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d
*x + c)^2 + (2*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^2 + 8*(a^2*b^5*d*cosh(d*
x + c)^7 + 3*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + (8*a^4*b^3 - 8*a^3*b
^4 + 3*a^2*b^5)*d*cosh(d*x + c)^3 + (2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*
sinh(d*x + c)), 1/8*(8*a^2*b^2*d*x*cosh(d*x + c)^8 + 64*a^2*b^2*d*x*cosh(d*
x + c)*sinh(d*x + c)^7 + 8*a^2*b^2*d*x*sinh(d*x + c)^8 + 2*(16*a^3*b - 20*a
^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^6 + 2*(1
12*a^2*b^2*d*x*cosh(d*x + c)^2 + 16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16
*(2*a^3*b - a^2*b^2)*d*x)*sinh(d*x + c)^6 + 8*a^2*b^2*d*x + 4*(112*a^2*b^2*
d*x*cosh(d*x + c)^3 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*
b - a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(48*a^4 - 72*a^3*b + 1
8*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*
x + c)^4 + 2*(280*a^2*b^2*d*x*cosh(d*x + c)^4 + 48*a^4 - 72*a^3*b + 18*a^2*
b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x + 15*(16*a^3*b
 - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^2
)*sinh(d*x + c)^4 + 12*a^2*b^2 - 6*a*b^3 - 6*b^4 + 8*(56*a^2*b^2*d*x*cosh(d
*x + c)^5 + 5*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^
2)*d*x)*cosh(d*x + c)^3 + (48*a^4 - 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^
4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c))*sinh(d*x + c)^3 + 2
*(32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*co
sh(d*x + c)^2 + 2*(112*a^2*b^2*d*x*cosh(d*x + c)^6 + 15*(16*a^3*b - 20*a^2*
b^2 + a*b^3 + 3*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^4 + 32*a^3*
b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*x + 6*(48*a^4
- 72*a^3*b + 18*a^2*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2
)*d*x)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + ((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*co
sh(d*x + c)^8 + 8*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)
^7 + (8*a^2*b^2 + 4*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(16*a^3*b + 2*a*b^3
- 3*b^4)*cosh(d*x + c)^6 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4 + 7*(8*a^2*b^2 + 4
*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(8*a^2*b^2 + 4*a*b^
3 + 3*b^4)*cosh(d*x + c)^3 + 3*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*
sinh(d*x + c)^5 + 2*(64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cos
h(d*x + c)^4 + 2*(35*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 64*a^4
 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4 + 30*(16*a^3*b + 2*a*b^3 - 3*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*a^2*b^2 + 4*a*b^3 + 3*b^4 + 8*(7*(8
```

```
*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(16*a^3*b + 2*a*b^3 - 3*b^
4)*cosh(d*x + c)^3 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*co
sh(d*x + c))*sinh(d*x + c)^3 + 4*(16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c)
^2 + 4*(7*(8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(16*a^3*b + 2*
a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 16*a^3*b + 2*a*b^3 - 3*b^4 + 3*(64*a^4 - 3
2*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
 8*((8*a^2*b^2 + 4*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(16*a^3*b + 2*a*b^3 -
 3*b^4)*cosh(d*x + c)^5 + (64*a^4 - 32*a^3*b + 16*a^2*b^2 - 12*a*b^3 + 9*b^
4)*cosh(d*x + c)^3 + (16*a^3*b + 2*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x +
 c))*sqrt(-(a - b)/a)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*si
nh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-(a - b)/a)/(a - b)) + 4*(1
6*a^2*b^2*d*x*cosh(d*x + c)^7 + 3*(16*a^3*b - 20*a^2*b^2 + a*b^3 + 3*b^4 +
16*(2*a^3*b - a^2*b^2)*d*x)*cosh(d*x + c)^5 + 2*(48*a^4 - 72*a^3*b + 18*a^2
*b^2 + 15*a*b^3 - 9*b^4 + 8*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*d*x)*cosh(d*x + c
)^3 + (32*a^3*b - 28*a^2*b^2 - 13*a*b^3 + 9*b^4 + 16*(2*a^3*b - a^2*b^2)*d*
x)*cosh(d*x + c))*sinh(d*x + c))/(a^2*b^5*d*cosh(d*x + c)^8 + 8*a^2*b^5*d*c
osh(d*x + c)*sinh(d*x + c)^7 + a^2*b^5*d*sinh(d*x + c)^8 + a^2*b^5*d + 4*(2
*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*a^2*b^5*d*cosh(d*x + c)^2 + (2
*a^3*b^4 - a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b
^5)*d*cosh(d*x + c)^4 + 8*(7*a^2*b^5*d*cosh(d*x + c)^3 + 3*(2*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^2*b^5*d*cosh(d*x + c)^4 +
30*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2
*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + 8*(7
*a^2*b^5*d*cosh(d*x + c)^5 + 10*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + (
8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*
a^2*b^5*d*cosh(d*x + c)^6 + 15*(2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 + 3*
(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)^2 + (2*a^3*b^4 - a^2*b^
5)*d)*sinh(d*x + c)^2 + 8*(a^2*b^5*d*cosh(d*x + c)^7 + 3*(2*a^3*b^4 - a^2*b
^5)*d*cosh(d*x + c)^5 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*d*cosh(d*x + c)
^3 + (2*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**6/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.19356, size = 481, normalized size = 3.01

$$\frac{dx+c}{b^3 d} - \frac{\left(8\,a^3 - 4\,a^2 b - ab^2 - 3\,b^3\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{8\,\sqrt{-a^2+ab}a^2 b^3 d} + \frac{16\,a^3 be^{(6\,dx+6\,c)} - 20\,a^2 b^2 e^{(6\,dx+6\,c)} + ab^3 e^{(6\,dx+6\,c)} + 3\,b^4 e}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^6/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] (d*x + c)/(b^3*d) - 1/8*(8*a^3 - 4*a^2*b - a*b^2 - 3*b^3)*arctan(1/2*(b*e^(
2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 + a*b)*a^2*b^3*d) + 1/
4*(16*a^3*b*e^(6*d*x + 6*c) - 20*a^2*b^2*e^(6*d*x + 6*c) + a*b^3*e^(6*d*x +
```

```
6*c) + 3*b^4*e^(6*d*x + 6*c) + 48*a^4*e^(4*d*x + 4*c) - 72*a^3*b*e^(4*d*x
+ 4*c) + 18*a^2*b^2*e^(4*d*x + 4*c) + 15*a*b^3*e^(4*d*x + 4*c) - 9*b^4*e^(4
*d*x + 4*c) + 32*a^3*b*e^(2*d*x + 2*c) - 28*a^2*b^2*e^(2*d*x + 2*c) - 13*a*
b^3*e^(2*d*x + 2*c) + 9*b^4*e^(2*d*x + 2*c) + 6*a^2*b^2 - 3*a*b^3 - 3*b^4)/
((b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2*a^2*
b^3*d)
```

3.340 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=133

$$\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\sinh(c+dx)}{8d\left(a+b\sinh^2(c+dx)\right)}+\frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}-\frac{(a-b)\sinh(c+dx)\cosh^2(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2
)*b^(5/2)*d) - ((a - b)*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh
[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Sinh[c + d*x])/(8*d*(a + b*Sinh[c +
d*x]^2))
```

Rubi [A] time = 0.124537, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 413, 385, 205}

$$\frac{3\left(\frac{1}{a^2}-\frac{1}{b^2}\right)\sinh(c+dx)}{8d\left(a+b\sinh^2(c+dx)\right)}+\frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d}-\frac{(a-b)\sinh(c+dx)\cosh^2(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] ((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2
)*b^(5/2)*d) - ((a - b)*Cosh[c + d*x]^2*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh
[c + d*x]^2)^2) + (3*(a^(-2) - b^(-2))*Sinh[c + d*x])/(8*d*(a + b*Sinh[c +
d*x]^2))
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{a+3b+(3a+b)x^2}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{4abd}$$

$$= -\frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3\left(a^2-b^2\right)\sinh(c+dx)}{8a^2b^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\left(3a^2+2ab+3b^2\right)\ldots}{\ldots}$$

$$= \frac{\left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{5/2}d} - \frac{(a-b)\cosh^2(c+dx)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} - \frac{3\left(a^2-\ldots\right)}{8a^2b^2d\left(a\ldots\right)}$$

Mathematica [A] time = 0.352346, size = 149, normalized size = 1.12

$$\frac{-\frac{2\sqrt{a}\sqrt{b}\left(5a^2-2ab-3b^2\right)\sinh(c+dx)}{2a+b\cosh(2(c+dx))-b} - \left(3a^2+2ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right) + \frac{8a^{3/2}\sqrt{b}(a-b)^2\sinh(c+dx)}{(2a+b\cosh(2(c+dx))-b)^2}}{8a^{5/2}b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-((3*a^2 + 2*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]]) + (8*a^
(3/2)*(a - b)^2*Sqrt[b]*Sinh[c + d*x])/(2*a - b + b*Cosh[2*(c + d*x)])^2 -
(2*Sqrt[a]*Sqrt[b]*(5*a^2 - 2*a*b - 3*b^2)*Sinh[c + d*x])/(2*a - b + b*Cosh
[2*(c + d*x)]))/(8*a^(5/2)*b^(5/2)*d)
```

Maple [B] time = 0.062, size = 1884, normalized size = 14.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -3/8/d/b^2*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b/a^2/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/b^2*a/(-b*(a-b))^(1/2)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*
a)^(1/2))+3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
```

```
/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3-3/8/d/b^2/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2
*b)*a)^(1/2))+3/8/d/b^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/2/d/b/(tanh(1/2*d*x+
1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/
2*d*x+1/2*c)^7+7/2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-7/2/d/b/(tanh(1/2*d*x+1
/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2
*d*x+1/2*c)^3-1/2/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)
^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*
x+1/2*c)+3/8/d/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/a^2/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))-7/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3-5/4/d/(tanh(1/2*d*x+1/2*c
)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d
*x+1/2*c)^7+7/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh
(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-3/4/d/b^2/(tanh(1/2*d*x+1/
2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/
2*d*x+1/2*c)-9/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^5-3/d*b/(tanh(1/2*d*x+1
/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh
(1/2*d*x+1/2*c)^5+9/4/d/b^2/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^
2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^3+3/4/d/b^2/(tanh(
1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2
*a*tanh(1/2*d*x+1/2*c)^7-1/8/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2))-1/8/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/4/d/b/a/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)-a+2*b)*a)^(1/2))-1/4/d/b/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arcta
nh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/8/d/b/(-b*
(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/
2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/8/d/b/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$-\frac{\left(5\,a^2be^{(7\,c)} - 2\,ab^2e^{(7\,c)} - 3\,b^3e^{(7\,c)}\right)e^{(7\,dx)} + \left(12\,a^3e^{(5\,c)} - 7\,a^2be^{(5\,c)} - 14\,ab^2e^{(5\,c)} + 9\,b^3e^{(5\,c)}\right)e^{(5\,dx)} - \left(12\,a^3e^{(3\,c)} - 7\,a^2be^{\cdots}\right.}{4\left(a^2b^4de^{(8\,dx+8\,c)} + a^2b^4d + 4\left(2\,a^3b^3de^{(6\,c)} - a^2b^4de^{(6\,c)}\right)e^{(6\,dx)} + 2\left(8\,a^4b^2de^{(4\,c)} - 8\,a^3b^3de^{(4\,c)} + 3\,a^2b\cdots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((5*a^2*b*e^(7*c) - 2*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e^(7*d*x) + (12*a
^3*e^(5*c) - 7*a^2*b*e^(5*c) - 14*a*b^2*e^(5*c) + 9*b^3*e^(5*c))*e^(5*d*x)
- (12*a^3*e^(3*c) - 7*a^2*b*e^(3*c) - 14*a*b^2*e^(3*c) + 9*b^3*e^(3*c))*e^(
3*d*x) - (5*a^2*b*e^c - 2*a*b^2*e^c - 3*b^3*e^c)*e^(d*x))/(a^2*b^4*d*e^(8*d
*x + 8*c) + a^2*b^4*d + 4*(2*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*
x) + 2*(8*a^4*b^2*d*e^(4*c) - 8*a^3*b^3*d*e^(4*c) + 3*a^2*b^4*d*e^(4*c))*e^
(4*d*x) + 4*(2*a^3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 1/32*int
egrate(8*((3*a^2*e^(3*c) + 2*a*b*e^(3*c) + 3*b^2*e^(3*c))*e^(3*d*x) + (3*a^
2*e^c + 2*a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*e^(4*d*x + 4*c) + a^2*b^3
```

```
+ 2*(2*a^3*b^2*e^(2*c) - a^2*b^3*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.45113, size = 13445, normalized size = 101.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^7 + 28*(5*a^3*b^2
 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(5*a^3*b^2 - 2*a^
2*b^3 - 3*a*b^4)*sinh(d*x + c)^7 + 4*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9
*a*b^4)*cosh(d*x + c)^5 + 4*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 +
21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*
(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (12*a^4*b - 7*a^3*b^
2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(12*a^4*b - 7*
a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 4*(12*a^4*b - 7*a^3*b^2 -
 14*a^2*b^3 + 9*a*b^4 - 35*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^
4 - 10*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^3 + 4*(21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 10*(
12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 3*(12*a^4*b
- 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((3*a^
2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*
cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)
^8 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(6*a^3*b +
 a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3
*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(
24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(
3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^
2 - 18*a*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
c)^2)*sinh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a
*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*co
sh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x
 + c))*sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x +
 c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(6*a^3*b +
a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 6*a^3*b + a^2*b^2 + 4*a*b^3 -
3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^
2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(
6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 - 8*a^3*b +
17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b
^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c)^4
 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh
(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh
(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 +
 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1
)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*c
osh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c
)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c
)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - 4*(5*a^3*b^2 - 2*a^2*b
^3 - 3*a*b^4)*cosh(d*x + c) + 4*(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d
*x + c)^6 - 5*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4 + 5*(12*a^4*b - 7*a^3*b^2 - 14*
a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 - 3*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 +
 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^3*b^5*d*cosh(d*x + c)^8 + 8*a^
3*b^5*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*b^5*d*sinh(d*x + c)^8 + a^3*b^5
*d + 4*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*a^3*b^5*d*cosh(d*x +
```

c)^2 + (2*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*a^3*b^5*d*cosh(d*x + c)^3 + 3*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*b^5*d*cosh(d*x + c)^4 + 30*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^5*d*cosh(d*x + c)^5 + 10*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^5*d*cosh(d*x + c)^6 + 15*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + 8*(a^3*b^5*d*cosh(d*x + c)^7 + 3*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^7 + 14*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*sinh(d*x + c)^7 + 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^5 + 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 + 21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^3 + (12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 - 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 2*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4 - 35*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^4 - 10*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^5 + 10*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - 3*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 - ((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) - ((3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (3*a^2*b^2 + 2*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4 + 30*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 3*a^2*b^2 + 2*a*b^3 + 3*b^4 + 8*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(3*a^2*b^2 + 2*a*b^3 + 3*b^4)

```
*cosh(d*x + c)^6 + 15*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c)^4
 + 6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4 + 3*(24*a^4 - 8*a^3*b + 17*a^2*b^2 -
 18*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((3*a^2*b^2 + 2*a*b
^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(
d*x + c)^5 + (24*a^4 - 8*a^3*b + 17*a^2*b^2 - 18*a*b^3 + 9*b^4)*cosh(d*x +
c)^3 + (6*a^3*b + a^2*b^2 + 4*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2
 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a
 - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) - 2*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)
*cosh(d*x + c) + 2*(7*(5*a^3*b^2 - 2*a^2*b^3 - 3*a*b^4)*cosh(d*x + c)^6 - 5
*a^3*b^2 + 2*a^2*b^3 + 3*a*b^4 + 5*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a
*b^4)*cosh(d*x + c)^4 - 3*(12*a^4*b - 7*a^3*b^2 - 14*a^2*b^3 + 9*a*b^4)*cos
h(d*x + c)^2)*sinh(d*x + c))/(a^3*b^5*d*cosh(d*x + c)^8 + 8*a^3*b^5*d*cosh(
d*x + c)*sinh(d*x + c)^7 + a^3*b^5*d*sinh(d*x + c)^8 + a^3*b^5*d + 4*(2*a^4
*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*a^3*b^5*d*cosh(d*x + c)^2 + (2*a^4
*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*
d*cosh(d*x + c)^4 + 8*(7*a^3*b^5*d*cosh(d*x + c)^3 + 3*(2*a^4*b^4 - a^3*b^5
)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*b^5*d*cosh(d*x + c)^4 + 30*(
2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5
)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*a^3
*b^5*d*cosh(d*x + c)^5 + 10*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^
5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*
b^5*d*cosh(d*x + c)^6 + 15*(2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a
^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^4*b^4 - a^3*b^5)*d
)*sinh(d*x + c)^2 + 8*(a^3*b^5*d*cosh(d*x + c)^7 + 3*(2*a^4*b^4 - a^3*b^5)*
d*cosh(d*x + c)^5 + (8*a^5*b^3 - 8*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^3 +
 (2*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.341 $\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3} dx$

Optimal. Leaf size=114

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a-b}}+\frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

```
[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a - b]*d)
+ Tanh[c + d*x]/(4*a*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])
/(8*a^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.0954025, antiderivative size = 114, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3191, 199, 208}

$$\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d\sqrt{a-b}}+\frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[a - b]*d)
+ Tanh[c + d*x]/(4*a*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*Tanh[c + d*x])
/(8*a^2*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{\left(a+(-a+b)x^2\right)^2}\,dx, x, \tanh(c+dx)\right)}{4ad}$$

$$= \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c+dx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+(-a+b)x^2}\right.}{}$$

$$= \frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{a-b}d} + \frac{\tanh(c+dx)}{4ad\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3\tanh(c+dx)}{8a^2d\left(a-(a-b)\tanh^2(c\right.}$$

Mathematica [A] time = 0.654372, size = 102, normalized size = 0.89

$$\frac{\frac{3\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{\sqrt{a-b}} + \frac{\sqrt{a}\sinh(2(c+dx))((2a+3b)\cosh(2(c+dx))+8a-3b)}{(2a+b\cosh(2(c+dx))-b)^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/Sqrt[a - b] + (Sqrt[a]*(8
*a - 3*b + (2*a + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(2*a - b + b*C
osh[2*(c + d*x)])^2)/(8*a^(5/2)*d)
```

Maple [B] time = 0.064, size = 664, normalized size = 5.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+3
/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5+3/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+
3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2
*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)+3
/8/d/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b
*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^
(1/2)+a-2*b)*a)^(1/2))-3/8/d/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arcta
n(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b/a^2/(
-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+
```

```
1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.28789, size = 10175, normalized size = 89.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*
(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 +
 4*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 + 8*a^3*b^2
+ 4*a^2*b^3 - 12*a*b^4 + 4*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*
a*b^4)*cosh(d*x + c)^4 + 4*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*
a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^4 + 16*(5*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x +
 c)^3 + (16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c
))*sinh(d*x + c)^3 + 4*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*cosh(d*
x + c)^2 + 4*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4 + 15*(8*a^4*b - 8*
a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 6*(16*a^5 - 8*a^4*b - 26*a
^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*(b^4*co
sh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c)^7 + b^4*sinh(d*x + c)^8 +
 4*(2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(7*b^4*cosh(d*x + c)^2 + 2*a*b^3 - b
^4)*sinh(d*x + c)^6 + 8*(7*b^4*cosh(d*x + c)^3 + 3*(2*a*b^3 - b^4)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^4 +
2*(35*b^4*cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^3 + 3*b^4 + 30*(2*a*b^3 - b^4
)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + b^4 + 8*(7*b^4*cosh(d*x + c)^5 + 10*(2
*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)
)*sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*cosh(d*x + c)^2 + 4*(7*b^4*cosh(d*x +
 c)^6 + 15*(2*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a*b^3 - b^4 + 3*(8*a^2*b^2 -
 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(b^4*cosh(d*x + c)^7
 + 3*(2*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d
*x + c)^3 + (2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*l
og((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*
x + c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 +
(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cos
h(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b
*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 +
2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x +
c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b))
+ 8*(3*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 2*(16*
a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^3 + (8*a^4
*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^4*
```

b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)*si
nh(d*x + c)^7 + (a^4*b^4 - a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^5*b^3 - 3*a^
4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x +
c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^6*b^2
- 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4 -
a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 30*(
2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 16*a^5*b^
3 + 11*a^4*b^4 - 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 3*a^4*b^4 +
 a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 +
10*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 16*a^
5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^
4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d
*cosh(d*x + c)^4 + 3*(8*a^6*b^2 - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*co
sh(d*x + c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^2 + (a^4
*b^4 - a^3*b^5)*d + 8*((a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^5*b^3
 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^6*b^2 - 16*a^5*b^3 + 11*a^
4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3
*a*b^4)*cosh(d*x + c)^6 + 12*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*co
sh(d*x + c)*sinh(d*x + c)^5 + 2*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)
*sinh(d*x + c)^6 + 4*a^3*b^2 + 2*a^2*b^3 - 6*a*b^4 + 2*(16*a^5 - 8*a^4*b -
26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^4 + 2*(16*a^5 - 8*a^4*b -
26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3
*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(8*a^4*b - 8*a^3*b^2 - 3*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 + (16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*
b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 2*(8*a^4*b + 8*a^3*b^2 - 25
*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2 + 2*(8*a^4*b + 8*a^3*b^2 - 25*a^2*b^3 +
 9*a*b^4 + 15*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 +
 6*(16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2)*
sinh(d*x + c)^2 + 3*(b^4*cosh(d*x + c)^8 + 8*b^4*cosh(d*x + c)*sinh(d*x + c
)^7 + b^4*sinh(d*x + c)^8 + 4*(2*a*b^3 - b^4)*cosh(d*x + c)^6 + 4*(7*b^4*co
sh(d*x + c)^2 + 2*a*b^3 - b^4)*sinh(d*x + c)^6 + 8*(7*b^4*cosh(d*x + c)^3 +
 3*(2*a*b^3 - b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2*b^2 - 8*a*b^3
+ 3*b^4)*cosh(d*x + c)^4 + 2*(35*b^4*cosh(d*x + c)^4 + 8*a^2*b^2 - 8*a*b^3
+ 3*b^4 + 30*(2*a*b^3 - b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + b^4 + 8*(7*
b^4*cosh(d*x + c)^5 + 10*(2*a*b^3 - b^4)*cosh(d*x + c)^3 + (8*a^2*b^2 - 8*a
*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b^3 - b^4)*cosh(d*x +
 c)^2 + 4*(7*b^4*cosh(d*x + c)^6 + 15*(2*a*b^3 - b^4)*cosh(d*x + c)^4 + 2*a
b^3 - b^4 + 3(8*a^2*b^2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)
^2 + 8*(b^4*cosh(d*x + c)^7 + 3*(2*a*b^3 - b^4)*cosh(d*x + c)^5 + (8*a^2*b^
2 - 8*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (2*a*b^3 - b^4)*cosh(d*x + c))*sinh(
d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x +
c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b
)) + 4*(3*(8*a^4*b - 8*a^3*b^2 - 3*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 + 2*(
16*a^5 - 8*a^4*b - 26*a^3*b^2 + 27*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^3 + (8*
a^4*b + 8*a^3*b^2 - 25*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a
^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)
*sinh(d*x + c)^7 + (a^4*b^4 - a^3*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^5*b^3 - 3
*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x
 + c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^6*b
^2 - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^4*b^4
 - a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(
d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3
0*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (8*a^6*b^2 - 16*a^5
*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^5*b^3 - 3*a^4*b^
4 + a^3*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5
 + 10*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^6*b^2 - 16
*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*

```
(a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5
)*d*cosh(d*x + c)^4 + 3*(8*a^6*b^2 - 16*a^5*b^3 + 11*a^4*b^4 - 3*a^3*b^5)*d
*cosh(d*x + c)^2 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)*d)*sinh(d*x + c)^2 + (
a^4*b^4 - a^3*b^5)*d + 8*((a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^5*
b^3 - 3*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + (8*a^6*b^2 - 16*a^5*b^3 + 11
*a^4*b^4 - 3*a^3*b^5)*d*cosh(d*x + c)^3 + (2*a^5*b^3 - 3*a^4*b^4 + a^3*b^5)
*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.36082, size = 331, normalized size = 2.9

$$\frac{3\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{8\,\sqrt{-a^2+ab}a^2d} - \frac{8\,a^2be^{(6\,dx+6\,c)} - 3\,b^3e^{(6\,dx+6\,c)} + 16\,a^3e^{(4\,dx+4\,c)} + 8\,a^2be^{(4\,dx+4\,c)} - 18\,ab^2e^{(4\,dx+4\,c)} + 9\,b^3e^{(4\,d\cdots}}{4\,\left(be^{(4\,dx+4\,c)} + 4\,ae^{(2\,dx+2\,c)} - 2\,be^{(2\,dx\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 3/8*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/(sqrt(-a^2 +
 a*b)*a^2*d) - 1/4*(8*a^2*b*e^(6*d*x + 6*c) - 3*b^3*e^(6*d*x + 6*c) + 16*a^
3*e^(4*d*x + 4*c) + 8*a^2*b*e^(4*d*x + 4*c) - 18*a*b^2*e^(4*d*x + 4*c) + 9*
b^3*e^(4*d*x + 4*c) + 8*a^2*b*e^(2*d*x + 2*c) + 16*a*b^2*e^(2*d*x + 2*c) -
9*b^3*e^(2*d*x + 2*c) + 2*a*b^2 + 3*b^3)/((b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x
 + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2*a^2*b^2*d)
```

3.342 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=117

$$\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}+\frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)}-\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*d) -
 ((a - b)*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh[c + d*x]^2)^2) + ((a + 3*b)*S
inh[c + d*x])/(8*a^2*b*d*(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0925692, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 385, 199, 205}

$$\frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d}+\frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)}-\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a + 3*b)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*b^(3/2)*d) -
 ((a - b)*Sinh[c + d*x])/(4*a*b*d*(a + b*Sinh[c + d*x]^2)^2) + ((a + 3*b)*S
inh[c + d*x])/(8*a^2*b*d*(a + b*Sinh[c + d*x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\mathrm{Subst}\left(\int \frac{1+x^2}{\left(a+bx^2\right)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(a+3b)\,\mathrm{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx, x, \sinh(c+dx)\right)}{4abd}$$

$$= -\frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(a+3b)\sinh(c+dx)}{8a^2bd\left(a+b\sinh^2(c+dx)\right)} + \frac{(a+3b)\,\mathrm{Subst}\left(\int \frac{1}{a+bx^2}\right.}{8a^2b}$$

$$= \frac{(a+3b)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}b^{3/2}d} - \frac{(a-b)\sinh(c+dx)}{4abd\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(a+3b)\sinh(c+dx}{8a^2bd\left(a+b\sinh^2(c+\right.}$$

Mathematica [A] time = 0.665558, size = 114, normalized size = 0.97

$$\frac{(a+3b)\left(\frac{\sinh(c+dx)\left(5a+3b\sinh^2(c+dx)\right)}{8a^2\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}}\right) - \frac{\sinh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^2}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-(Sinh[c + d*x]/(a + b*Sinh[c + d*x]^2)^2) + (a + 3*b)*((3*ArcTan[(Sqrt[b]
*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]) + (Sinh[c + d*x]*(5*a + 3*b*S
inh[c + d*x]^2))/(8*a^2*(a + b*Sinh[c + d*x]^2)^2)))/(3*b*d)
```

Maple [B] time = 0.068, size = 1348, normalized size = 11.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7-3
/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+11/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-3
/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5+3/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-
11/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/
```

```
2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3+3/d*b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^
3-1/4/d/b/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(
1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-1/8
/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8/d/b/a/((2*(-b*(a-b)
)^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+
a-2*b)*a)^(1/2))-1/4/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1
/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-1/8
/d/b/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+1/8/d/b/a/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+
2*b)*a)^(1/2))-1/4/d/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2
)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d/
a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*
(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))+3/8/d/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b/a^2/(-b*(a
-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{(abe^{(7c)} + 3b^2e^{(7c)})e^{(7dx)} - (4a^2e^{(5c)} - 17abe^{(5c)} + 9b^2e^{(5c)})e^{(5dx)} + (4a^2e^{(3c)} - 17abe^{(3c)} + 9b^2e^{(3c)})e^{(3dx)} - (}{4(a^2b^3de^{(8dx+8c)} + a^2b^3d + 4(2a^3b^2de^{(6c)} - a^2b^3de^{(6c)})e^{(6dx)} + 2(8a^4bde^{(4c)} - 8a^3b^2de^{(4c)} + 3a^2b^3de^{(4c)})e^{(4dx)} + 4(2a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((a*b*e^(7*c) + 3*b^2*e^(7*c))*e^(7*d*x) - (4*a^2*e^(5*c) - 17*a*b*e^(5
*c) + 9*b^2*e^(5*c))*e^(5*d*x) + (4*a^2*e^(3*c) - 17*a*b*e^(3*c) + 9*b^2*e^
(3*c))*e^(3*d*x) - (a*b*e^c + 3*b^2*e^c)*e^(d*x))/(a^2*b^3*d*e^(8*d*x + 8*c
) + a^2*b^3*d + 4*(2*a^3*b^2*d*e^(6*c) - a^2*b^3*d*e^(6*c))*e^(6*d*x) + 2*(
8*a^4*b*d*e^(4*c) - 8*a^3*b^2*d*e^(4*c) + 3*a^2*b^3*d*e^(4*c))*e^(4*d*x) +
4*(2*a^3*b^2*d*e^(2*c) - a^2*b^3*d*e^(2*c))*e^(2*d*x)) + 1/8*integrate(2*((
a*e^(3*c) + 3*b*e^(3*c))*e^(3*d*x) + (a*e^c + 3*b*e^c)*e^(d*x))/(a^2*b^2*e^
(4*d*x + 4*c) + a^2*b^2 + 2*(2*a^3*b*e^(2*c) - a^2*b^2*e^(2*c))*e^(2*d*x)),
 x)
```

Fricas [B] time = 2.33105, size = 11478, normalized size = 98.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 28*(a^2*b^2 + 3*a*b^3)*cosh(
d*x + c)*sinh(d*x + c)^6 + 4*(a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^7 - 4*(4*a^3
*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^5 - 4*(4*a^3*b - 17*a^2*b^2 + 9*a*
b^3 - 21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(a^2*
```

b^2 + 3*a*b^3)*cosh(d*x + c)^3 - (4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3 + 4*(35*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^3*b - 17*a^2*b^2 + 9*a*b^3 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 - ((a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3 + 7*(a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^2 + 3*b^3 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 2*a^2*b + 5*a*b^2 - 3*b^3 + 3*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 + 3*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) - 4*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c) + 4*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^6 - 5*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^4 - a^2*b^2 - 3*a*b^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^3*b^4*d*cosh(d*x + c)^8 + 8*a^3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*b^4*d*sinh(d*x + c)^8 + a^3*b^4*d + 4*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^6 + 4*(7*a^3*b^4*d*cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*a^3*b^4*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*a^3*b^4*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^4*d*cosh(d*x + c)^5 + 10*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^4*d*cosh(d*x + c)^6 + 15*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4 + 3*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + 8*(a^3*b^4*d*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)), 1/8*(2*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^7 + 14*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)*sinh(d*x + c)^6 + 2*(a^2*b^2 + 3*a*b^3)*sinh(d*x + c)^7 - 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^5 - 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3 - 21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 10*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^3 - (4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^4 + 2*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3 + 2*(35*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^4 + 4*a^3*b - 17*a^2*b^2 + 9*a*b^3 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(21*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c)^5 - 10*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c))*sinh(d*x + c)^2 + ((a*b^2 + 3*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7 + (a*b^2 + 3*b^3)*sinh(d*x + c)^8 +

```
 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b
^3 + 7*(a*b^2 + 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b
^3)*cosh(d*x + c)^3 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x
 + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(
a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(
2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 +
 3*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^3 +
(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c))*sinh(d*x + c)^3 + a*b^
2 + 3*b^3 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3
*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 2*
a^2*b + 5*a*b^2 - 3*b^3 + 3*(8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x
+ c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 + 3*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b +
 5*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*c
osh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c))*
sqrt(a*b)*arctan(1/2*sqrt(a*b)*(cosh(d*x + c) + sinh(d*x + c))/a) + ((a*b^2
 + 3*b^3)*cosh(d*x + c)^8 + 8*(a*b^2 + 3*b^3)*cosh(d*x + c)*sinh(d*x + c)^7
 + (a*b^2 + 3*b^3)*sinh(d*x + c)^8 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x
 + c)^6 + 4*(2*a^2*b + 5*a*b^2 - 3*b^3 + 7*(a*b^2 + 3*b^3)*cosh(d*x + c)^2)
*sinh(d*x + c)^6 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^3 + 3*(2*a^2*b + 5*a*
b^2 - 3*b^3)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^3 + 16*a^2*b - 21*a*b^
2 + 9*b^3)*cosh(d*x + c)^4 + 2*(35*(a*b^2 + 3*b^3)*cosh(d*x + c)^4 + 8*a^3
+ 16*a^2*b - 21*a*b^2 + 9*b^3 + 30*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c
)^2)*sinh(d*x + c)^4 + 8*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^5 + 10*(2*a^2*b +
 5*a*b^2 - 3*b^3)*cosh(d*x + c)^3 + (8*a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*c
osh(d*x + c))*sinh(d*x + c)^3 + a*b^2 + 3*b^3 + 4*(2*a^2*b + 5*a*b^2 - 3*b^
3)*cosh(d*x + c)^2 + 4*(7*(a*b^2 + 3*b^3)*cosh(d*x + c)^6 + 15*(2*a^2*b + 5
*a*b^2 - 3*b^3)*cosh(d*x + c)^4 + 2*a^2*b + 5*a*b^2 - 3*b^3 + 3*(8*a^3 + 16
*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((a*b^2 + 3
*b^3)*cosh(d*x + c)^7 + 3*(2*a^2*b + 5*a*b^2 - 3*b^3)*cosh(d*x + c)^5 + (8*
a^3 + 16*a^2*b - 21*a*b^2 + 9*b^3)*cosh(d*x + c)^3 + (2*a^2*b + 5*a*b^2 - 3
*b^3)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*(b*cosh(d*x + c)^3
 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d
*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(a*b)/(a*b)) -
 2*(a^2*b^2 + 3*a*b^3)*cosh(d*x + c) + 2*(7*(a^2*b^2 + 3*a*b^3)*cosh(d*x +
c)^6 - 5*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^4 - a^2*b^2 - 3*a*b
^3 + 3*(4*a^3*b - 17*a^2*b^2 + 9*a*b^3)*cosh(d*x + c)^2)*sinh(d*x + c))/(a^
3*b^4*d*cosh(d*x + c)^8 + 8*a^3*b^4*d*cosh(d*x + c)*sinh(d*x + c)^7 + a^3*b
^4*d*sinh(d*x + c)^8 + a^3*b^4*d + 4*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^
6 + 4*(7*a^3*b^4*d*cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)
^6 + 2*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^4 + 8*(7*a^3*b^4
*d*cosh(d*x + c)^3 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c))*sinh(d*x + c)
^5 + 2*(35*a^3*b^4*d*cosh(d*x + c)^4 + 30*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x
+ c)^2 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d)*sinh(d*x + c)^4 + 4*(2*a^4*
b^3 - a^3*b^4)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^4*d*cosh(d*x + c)^5 + 10*(2*a
^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^3 + (8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*d
*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^4*d*cosh(d*x + c)^6 + 15*(2*a^
4*b^3 - a^3*b^4)*d*cosh(d*x + c)^4 + 3*(8*a^5*b^2 - 8*a^4*b^3 + 3*a^3*b^4)*
d*cosh(d*x + c)^2 + (2*a^4*b^3 - a^3*b^4)*d)*sinh(d*x + c)^2 + 8*(a^3*b^4*d
*cosh(d*x + c)^7 + 3*(2*a^4*b^3 - a^3*b^4)*d*cosh(d*x + c)^5 + (8*a^5*b^2 -
 8*a^4*b^3 + 3*a^3*b^4)*d*cosh(d*x + c)^3 + (2*a^4*b^3 - a^3*b^4)*d*cosh(d*
x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.343 $\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=143

$$\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{3/2}}+\frac{(4a-3b)\tanh(c+dx)}{8a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b\tanh(c+dx)}{4ad(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

```
[Out] ((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a -
b)^(3/2)*d) - (b*Tanh[c + d*x])/(4*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2
)^2) + ((4*a - 3*b)*Tanh[c + d*x])/(8*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d
*x]^2))
```

Rubi [A] time = 0.125523, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 385, 199, 208}

$$\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{3/2}}+\frac{(4a-3b)\tanh(c+dx)}{8a^2d(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{b\tanh(c+dx)}{4ad(a-b)\left(a-(a-b)\tanh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] ((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a -
b)^(3/2)*d) - (b*Tanh[c + d*x])/(4*a*(a - b)*d*(a - (a - b)*Tanh[c + d*x]^2
)^2) + ((4*a - 3*b)*Tanh[c + d*x])/(8*a^2*(a - b)*d*(a - (a - b)*Tanh[c + d
*x]^2))
```


Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\cosh^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx=\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-(a-b)x^2\right)^3}\,dx,x,\tanh(c+dx)\right)}{d}$$

$$=-\frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2}+\frac{(4a-3b)\,\text{Subst}\left(\int \frac{1}{\left(a+(-a+b)x^2\right)^2}\,dx,x,\tanh\right.}{4a(a-b)d}$$

$$=-\frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2}+\frac{(4a-3b)\tanh(c+dx)}{8a^2(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)}+$$

$$=\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{3/2}d}-\frac{b\tanh(c+dx)}{4a(a-b)d\left(a-(a-b)\tanh^2(c+dx)\right)^2}+\frac{}{8a^2(a-}$$

Mathematica [A] time = 0.888273, size = 124, normalized size = 0.87

$$\frac{\frac{\sqrt{a}\sinh(2(c+dx))\left(8a^2+b(2a-3b)\cosh(2(c+dx))-12ab+3b^2\right)}{(a-b)(2a+b\cosh(2(c+dx))-b)^2}+\frac{(4a-3b)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{(a-b)^{3/2}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (((4*a - 3*b)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(a - b)^(3/2) +
 (Sqrt[a]*(8*a^2 - 12*a*b + 3*b^2 + (2*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[2
*(c + d*x)])/((a - b)*(2*a - b + b*Cosh[2*(c + d*x)])^2))/(8*a^(5/2)*d)
```

Maple [B] time = 0.06, size = 1322, normalized size = 9.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c
)^2*b+a)^2/(a-b)*tanh(1/2*d*x+1/2*c)^7-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/
2*c)^7*b-1/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*
d*x+1/2*c)^2*b+a)^2/(a-b)*tanh(1/2*d*x+1/2*c)^5+13/4/d/(tanh(1/2*d*x+1/2*c)
^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(
1/2*d*x+1/2*c)^5*b-3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2/(a-b)*tanh(1/2*d*x+1/2*c)^5*b^2-1/d/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/(a-b)*tanh(1/2*d*x+1/2*c)^3+13/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*
```

```
x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/2*c)^3*b
-3/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*
c)^2*b+a)^2/a^2/(a-b)*tanh(1/2*d*x+1/2*c)^3*b^2+1/d/(tanh(1/2*d*x+1/2*c)^4*
a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/(a-b)*tanh(1/2*d
*x+1/2*c)-5/4/d/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2/a/(a-b)*tanh(1/2*d*x+1/2*c)*b+1/2/d/a/(a-b)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)+a-2*b)*a)^(1/2))-1/2/d/a/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a
-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)
^(1/2))*b-1/2/d/a/(a-b)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-1/2/d/a/(a-b)/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b-3/8/d/a^2/(a-b)*b/((2*(-b*(a-b))^(1
/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*
b)*a)^(1/2))+3/8/d/a^2/(a-b)/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a
)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
*b^2+3/8/d/a^2/(a-b)*b/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d/a^2/(a-b)/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))*b^2
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.35308, size = 11760, normalized size = 82.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(4*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6 + 24*(4*a^3*b^2
 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 4*(4*a^3*b^2 - 7*a^2
*b^3 + 3*a*b^4)*sinh(d*x + c)^6 - 8*a^3*b^2 + 20*a^2*b^3 - 12*a*b^4 - 4*(16
*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^4 - 4*(1
6*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(5*(4*a^3*b^2 - 7*a^
2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 - (16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2
*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 4*(16*a^4*b - 44*a^3*b^2 +
 37*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 - 4*(16*a^4*b - 44*a^3*b^2 + 37*a^2*
b^3 - 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^4 + 6*(1
6*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + ((4*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(4*a*b^3 - 3*b^4)*cosh
(d*x + c)*sinh(d*x + c)^7 + (4*a*b^3 - 3*b^4)*sinh(d*x + c)^8 + 4*(8*a^2*b^
2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 7
*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4*a*b^3 - 3*b^4
)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*
```

x + c)^5 + 2*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^4 + 2
(35(4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 32*a^3*b - 56*a^2*b^2 + 36*a*b^3 -
9*b^4 + 30*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4
+ 4*a*b^3 - 3*b^4 + 8*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + 10*(8*a^2*b^2
- 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 -
9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*co
sh(d*x + c)^2 + 4*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 15*(8*a^2*b^2 - 10
*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 10*a*b^3 + 3*b^4 + 3*(32*a^3*
b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((4
*a*b^3 - 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x
+ c)^5 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + (8*a
^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*lo
g((b^2*cosh(d*x + c)^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x
+ c)^4 + 2*(2*a*b - b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*
b - b^2)*sinh(d*x + c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (
2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh
(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*
cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2
*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c
)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) +
8*(3*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 - 2*(16*a^5 - 56*a^
4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - (16*a^4*b - 44*a
^3*b^2 + 37*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 2*
a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*
cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*sinh(d*x
+ c)^8 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6
+ 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 5*a
^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 24*a^6*b^2
+ 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 -
2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b
^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3 - 2*a^4*b^4
+ a^3*b^5)*d*cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3
*b^5)*d*cosh(d*x + c)^2 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 +
3*a^3*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3
*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x +
c)^5 + 10*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3
+ (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x +
c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)
^6 + 15*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3
*(8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x +
c)^2 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (
a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d + 8*((a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cos
h(d*x + c)^7 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x +
c)^5 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh
(d*x + c)^3 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)
)*sinh(d*x + c)), 1/8*(2*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^6
+ 12*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)*sinh(d*x + c)^5 + 2*(4
*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*sinh(d*x + c)^6 - 4*a^3*b^2 + 10*a^2*b^3 -
6*a*b^4 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*
x + c)^4 - 2*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4 - 15*(4
*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 8*(5*(4*
a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^3 - (16*a^5 - 56*a^4*b + 70*a^
3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 - 2*(16*a^4*b
- 44*a^3*b^2 + 37*a^2*b^3 - 9*a*b^4)*cosh(d*x + c)^2 - 2*(16*a^4*b - 44*a^3
*b^2 + 37*a^2*b^3 - 9*a*b^4 - 15*(4*a^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x
+ c)^4 + 6*(16*a^5 - 56*a^4*b + 70*a^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*
x + c)^2)*sinh(d*x + c)^2 - ((4*a*b^3 - 3*b^4)*cosh(d*x + c)^8 + 8*(4*a*b^3
- 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (4*a*b^3 - 3*b^4)*sinh(d*x + c)^8
+ 4*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 4*(8*a^2*b^2 - 10*a*b

```
^3 + 3*b^4 + 7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(4
*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + 3*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(
d*x + c)^4 + 2*(35*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 32*a^3*b - 56*a^2*b^
2 + 36*a*b^3 - 9*b^4 + 30*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*s
inh(d*x + c)^4 + 4*a*b^3 - 3*b^4 + 8*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^5 +
 10*(8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (32*a^3*b - 56*a^2*b^2
 + 36*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(8*a^2*b^2 - 10*a*b
^3 + 3*b^4)*cosh(d*x + c)^2 + 4*(7*(4*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 15*(
8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 8*a^2*b^2 - 10*a*b^3 + 3*b^
4 + 3*(32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x
+ c)^2 + 8*((4*a*b^3 - 3*b^4)*cosh(d*x + c)^7 + 3*(8*a^2*b^2 - 10*a*b^3 + 3
*b^4)*cosh(d*x + c)^5 + (32*a^3*b - 56*a^2*b^2 + 36*a*b^3 - 9*b^4)*cosh(d*x
 + c)^3 + (8*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt
(-a^2 + a*b)*arctan(-1/2*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(-a^2 + a*b)/(a^2 - a*b)) + 4*(3*(4*a
^3*b^2 - 7*a^2*b^3 + 3*a*b^4)*cosh(d*x + c)^5 - 2*(16*a^5 - 56*a^4*b + 70*a
^3*b^2 - 39*a^2*b^3 + 9*a*b^4)*cosh(d*x + c)^3 - (16*a^4*b - 44*a^3*b^2 + 3
7*a^2*b^3 - 9*a*b^4)*cosh(d*x + c))*sinh(d*x + c))/((a^5*b^3 - 2*a^4*b^4 +
a^3*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x +
 c)*sinh(d*x + c)^7 + (a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*sinh(d*x + c)^8 + 4
*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^
5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b^2 - 5*a^5*b^3 + 4
*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7*b - 24*a^6*b^2 + 27*a^5*b
^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^3 - 2*a^4*b^4
+ a^3*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b
^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5
)*d*cosh(d*x + c)^4 + 30*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*co
sh(d*x + c)^2 + (8*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5
)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*co
sh(d*x + c)^2 + 8*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^5 + 10
*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^3 + (8*a^7*b
 - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c))*sinh(
d*x + c)^3 + 4*(7*(a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)^6 + 15*(2
*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^7*b
- 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^2 + (2*
a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d)*sinh(d*x + c)^2 + (a^5*b^3 -
2*a^4*b^4 + a^3*b^5)*d + 8*((a^5*b^3 - 2*a^4*b^4 + a^3*b^5)*d*cosh(d*x + c)
^7 + 3*(2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c)^5 + (8
*a^7*b - 24*a^6*b^2 + 27*a^5*b^3 - 14*a^4*b^4 + 3*a^3*b^5)*d*cosh(d*x + c)^
3 + (2*a^6*b^2 - 5*a^5*b^3 + 4*a^4*b^4 - a^3*b^5)*d*cosh(d*x + c))*sinh(d*x
 + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.39818, size = 365, normalized size = 2.55

$$\frac{(4\,a-3\,b)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{8\,\left(a^3d-a^2bd\right)\sqrt{-a^2+ab}}+\frac{4\,ab^2e^{(6\,dx+6\,c)}-3\,b^3e^{(6\,dx+6\,c)}-16\,a^3e^{(4\,dx+4\,c)}+40\,a^2be^{(4\,dx+4\,c)}-30\,ab^2e^{(4\,dx+4\,c)}}{4\,\left(a^3bd-a^2b^2d\right)\left(be^{(4\,dx+4\,c)}+4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(4*a - 3*b)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqrt(-a^2 + a*b))/
((a^3*d - a^2*b*d)*sqrt(-a^2 + a*b)) + 1/4*(4*a*b^2*e^(6*d*x + 6*c) - 3*b^3
*e^(6*d*x + 6*c) - 16*a^3*e^(4*d*x + 4*c) + 40*a^2*b*e^(4*d*x + 4*c) - 30*a
*b^2*e^(4*d*x + 4*c) + 9*b^3*e^(4*d*x + 4*c) - 16*a^2*b*e^(2*d*x + 2*c) + 2
8*a*b^2*e^(2*d*x + 2*c) - 9*b^3*e^(2*d*x + 2*c) - 2*a*b^2 + 3*b^3)/((a^3*b*
d - a^2*b^2*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*
c) + b)^2)
```

3.344 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=96

$$\frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}+\frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2}$$

```
[Out] (3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Sinh[c
+ d*x]/(4*a*d*(a + b*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/(8*a^2*d*(a +
b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.0571966, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 199, 205}

$$\frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d}+\frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*Sqrt[b]*d) + Sinh[c
+ d*x]/(4*a*d*(a + b*Sinh[c + d*x]^2)^2) + (3*Sinh[c + d*x])/(8*a^2*d*(a +
b*Sinh[c + d*x]^2))
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{4ad}$$

$$= \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin\right.}{8a^2d}$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}\sqrt{b}d} + \frac{\sinh(c+dx)}{4ad\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\sinh(c+dx)}{8a^2d\left(a+b\sinh^2(c+dx)\right)}$$

Mathematica [A] time = 0.170788, size = 79, normalized size = 0.82

$$\frac{\frac{\sqrt{a}\sinh(c+dx)\left(5a+3b\sinh^2(c+dx)\right)}{\left(a+b\sinh^2(c+dx)\right)^2} + \frac{3\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{\sqrt{b}}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/Sqrt[b] + (Sqrt[a]*Sinh[c + d*
x]*(5*a + 3*b*Sinh[c + d*x]^2))/(a + b*Sinh[c + d*x]^2)^2)/(8*a^(5/2)*d)
```

Maple [A] time = 0.013, size = 85, normalized size = 0.9

$$\frac{\sinh(dx+c)}{4\,da\left(a+b\left(\sinh(dx+c)\right)^2\right)^2} + \frac{3\,\sinh(dx+c)}{8\,da^2\left(a+b\left(\sinh(dx+c)\right)^2\right)} + \frac{3}{8\,da^2}\arctan\left(b\sinh(dx+c)\frac{1}{\sqrt{ab}}\right)\frac{1}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] 1/4*sinh(d*x+c)/a/d/(a+b*sinh(d*x+c)^2)^2+3/8*sinh(d*x+c)/a^2/d/(a+b*sinh(d
*x+c)^2)+3/8/d/a^2/(a*b)^(1/2)*arctan(sinh(d*x+c)*b/(a*b)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(20\,ae^{(5\,c)}-9\,be^{(5\,c)}\right)e^{(5\,dx)}-\left(20\,ae^{(3\,c)}-9\,be^{(3\,c)}\right)e^{(3\,dx)}+3\,be^{(7\,dx+7\,c)}-3\,be^{(dx+c)}}{4\left(a^2b^2de^{(8\,dx+8\,c)}+a^2b^2d+4\left(2\,a^3bde^{(6\,c)}-a^2b^2de^{(6\,c)}\right)e^{(6\,dx)}+2\left(8\,a^4de^{(4\,c)}-8\,a^3bde^{(4\,c)}+3\,a^2b^2de^{(4\,c)}\right)e^{(4\,dx)}+4\left(2\,a^3\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/4*((20*a*e^(5*c) - 9*b*e^(5*c))*e^(5*d*x) - (20*a*e^(3*c) - 9*b*e^(3*c))*
e^(3*d*x) + 3*b*e^(7*d*x + 7*c) - 3*b*e^(d*x + c))/(a^2*b^2*d*e^(8*d*x + 8*
c) + a^2*b^2*d + 4*(2*a^3*b*d*e^(6*c) - a^2*b^2*d*e^(6*c))*e^(6*d*x) + 2*(8
*a^4*d*e^(4*c) - 8*a^3*b*d*e^(4*c) + 3*a^2*b^2*d*e^(4*c))*e^(4*d*x) + 4*(2*
a^3*b*d*e^(2*c) - a^2*b^2*d*e^(2*c))*e^(2*d*x)) + 1/2*integrate(3/2*(e^(3*d
*x + 3*c) + e^(d*x + c))/(a^2*b*e^(4*d*x + 4*c) + a^2*b + 2*(2*a^3*e^(2*c)
- a^2*b*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 2.18802, size = 9451, normalized size = 98.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/16*(12*a*b^2*cosh(d*x + c)^7 + 84*a*b^2*cosh(d*x + c)*sinh(d*x + c)^6 +
12*a*b^2*sinh(d*x + c)^7 + 4*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^5 + 4*(63*a
*b^2*cosh(d*x + c)^2 + 20*a^2*b - 9*a*b^2)*sinh(d*x + c)^5 + 20*(21*a*b^2*c
osh(d*x + c)^3 + (20*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^4 - 12*a
*b^2*cosh(d*x + c) - 4*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 + 4*(105*a*b^2*
cosh(d*x + c)^4 - 20*a^2*b + 9*a*b^2 + 10*(20*a^2*b - 9*a*b^2)*cosh(d*x + c
)^2)*sinh(d*x + c)^3 + 4*(63*a*b^2*cosh(d*x + c)^5 + 10*(20*a^2*b - 9*a*b^2
)*cosh(d*x + c)^3 - 3*(20*a^2*b - 9*a*b^2)*cosh(d*x + c))*sinh(d*x + c)^2 -
 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*
x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a
*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cosh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh
(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*
(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b
+ 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh
(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2
*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*c
osh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*s
inh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 + 3*(2*a*b - b^2)*cosh(d*x +
c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*x + c
))*sinh(d*x + c))*sqrt(-a*b)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sin
h(d*x + c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cos
h(d*x + c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*
cosh(d*x + c))*sinh(d*x + c) - 4*(cosh(d*x + c)^3 + 3*cosh(d*x + c)*sinh(d*
x + c)^2 + sinh(d*x + c)^3 + (3*cosh(d*x + c)^2 - 1)*sinh(d*x + c) - cosh(d
*x + c))*sqrt(-a*b) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x +
c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x +
c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh(d*x
 + c))*sinh(d*x + c) + b)) + 4*(21*a*b^2*cosh(d*x + c)^6 + 5*(20*a^2*b - 9*
a*b^2)*cosh(d*x + c)^4 - 3*a*b^2 - 3*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*
sinh(d*x + c))/(a^3*b^3*d*cosh(d*x + c)^8 + 8*a^3*b^3*d*cosh(d*x + c)*sinh(
d*x + c)^7 + a^3*b^3*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x
 + c)^6 + a^3*b^3*d + 4*(7*a^3*b^3*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3
)*d)*sinh(d*x + c)^6 + 2*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^
4 + 8*(7*a^3*b^3*d*cosh(d*x + c)^3 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c
))*sinh(d*x + c)^5 + 2*(35*a^3*b^3*d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - a^3*
b^3)*d*cosh(d*x + c)^2 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d)*sinh(d*x + c)
^4 + 4*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^3*d*cosh(d*x +
c)^5 + 10*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^3 + (8*a^5*b - 8*a^4*b^2 +
3*a^3*b^3)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^3*d*cosh(d*x + c)^
6 + 15*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 8*a^4*b^2 + 3
*a^3*b^3)*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^2 + 8*
```

```
(a^3*b^3*d*cosh(d*x + c)^7 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^5 + (8
*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b^2 - a^3*b^3)*d
*cosh(d*x + c))*sinh(d*x + c)), 1/8*(6*a*b^2*cosh(d*x + c)^7 + 42*a*b^2*cos
h(d*x + c)*sinh(d*x + c)^6 + 6*a*b^2*sinh(d*x + c)^7 + 2*(20*a^2*b - 9*a*b^
2)*cosh(d*x + c)^5 + 2*(63*a*b^2*cosh(d*x + c)^2 + 20*a^2*b - 9*a*b^2)*sinh
(d*x + c)^5 + 10*(21*a*b^2*cosh(d*x + c)^3 + (20*a^2*b - 9*a*b^2)*cosh(d*x
+ c))*sinh(d*x + c)^4 - 6*a*b^2*cosh(d*x + c) - 2*(20*a^2*b - 9*a*b^2)*cosh
(d*x + c)^3 + 2*(105*a*b^2*cosh(d*x + c)^4 - 20*a^2*b + 9*a*b^2 + 10*(20*a^
2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 2*(63*a*b^2*cosh(d*x + c)
^5 + 10*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^3 - 3*(20*a^2*b - 9*a*b^2)*cosh(
d*x + c))*sinh(d*x + c)^2 + 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d*x + c)*si
nh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x + c)^6 + 4*(
7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*cosh(d*x +
c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2 - 8*a*b +
3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b - b^2)*cosh(
d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*cosh(d*x + c
)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x +
 c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2*cosh(d*x
+ c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(
d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x + c)^7 +
3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*x + c)^3 +
 (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2*sqrt(a*b)
*(cosh(d*x + c) + sinh(d*x + c))/a) + 3*(b^2*cosh(d*x + c)^8 + 8*b^2*cosh(d
*x + c)*sinh(d*x + c)^7 + b^2*sinh(d*x + c)^8 + 4*(2*a*b - b^2)*cosh(d*x +
c)^6 + 4*(7*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^6 + 8*(7*b^2*c
osh(d*x + c)^3 + 3*(2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^2
- 8*a*b + 3*b^2)*cosh(d*x + c)^4 + 2*(35*b^2*cosh(d*x + c)^4 + 30*(2*a*b -
b^2)*cosh(d*x + c)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(d*x + c)^4 + 8*(7*b^2*co
sh(d*x + c)^5 + 10*(2*a*b - b^2)*cosh(d*x + c)^3 + (8*a^2 - 8*a*b + 3*b^2)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a*b - b^2)*cosh(d*x + c)^2 + 4*(7*b^2
*cosh(d*x + c)^6 + 15*(2*a*b - b^2)*cosh(d*x + c)^4 + 3*(8*a^2 - 8*a*b + 3*
b^2)*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x + c)^2 + b^2 + 8*(b^2*cosh(d*x
 + c)^7 + 3*(2*a*b - b^2)*cosh(d*x + c)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(d*
x + c)^3 + (2*a*b - b^2)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a*b)*arctan(1/2
*(b*cosh(d*x + c)^3 + 3*b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3
 + (4*a - b)*cosh(d*x + c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))
*sqrt(a*b)/(a*b)) + 2*(21*a*b^2*cosh(d*x + c)^6 + 5*(20*a^2*b - 9*a*b^2)*co
sh(d*x + c)^4 - 3*a*b^2 - 3*(20*a^2*b - 9*a*b^2)*cosh(d*x + c)^2)*sinh(d*x
+ c))/(a^3*b^3*d*cosh(d*x + c)^8 + 8*a^3*b^3*d*cosh(d*x + c)*sinh(d*x + c)^
7 + a^3*b^3*d*sinh(d*x + c)^8 + 4*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^6 +
 a^3*b^3*d + 4*(7*a^3*b^3*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3)*d)*sinh
(d*x + c)^6 + 2*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^4 + 8*(7*
a^3*b^3*d*cosh(d*x + c)^3 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c))*sinh(d
*x + c)^5 + 2*(35*a^3*b^3*d*cosh(d*x + c)^4 + 30*(2*a^4*b^2 - a^3*b^3)*d*co
sh(d*x + c)^2 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)*d)*sinh(d*x + c)^4 + 4*(2
*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^2 + 8*(7*a^3*b^3*d*cosh(d*x + c)^5 + 10
*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^3 + (8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3
)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*a^3*b^3*d*cosh(d*x + c)^6 + 15*(2
*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^4 + 3*(8*a^5*b - 8*a^4*b^2 + 3*a^3*b^3)
*d*cosh(d*x + c)^2 + (2*a^4*b^2 - a^3*b^3)*d)*sinh(d*x + c)^2 + 8*(a^3*b^3*
d*cosh(d*x + c)^7 + 3*(2*a^4*b^2 - a^3*b^3)*d*cosh(d*x + c)^5 + (8*a^5*b -
8*a^4*b^2 + 3*a^3*b^3)*d*cosh(d*x + c)^3 + (2*a^4*b^2 - a^3*b^3)*d*cosh(d*x
 + c))*sinh(d*x + c))]
```

Sympy [A] time = 175.048, size = 915, normalized size = 9.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Piecewise((zoo*x*cosh(c)/sinh(c)**6, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (sinh
(c + d*x)/(a**3*d), Eq(b, 0)), (-1/(5*b**3*d*sinh(c + d*x)**5), Eq(a, 0)),
(x*cosh(c)/(a + b*sinh(c)**2)**3, Eq(d, 0)), (10*I*a**(3/2)*b*sqrt(1/b)*sin
h(c + d*x)/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*si
nh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4) + 6*I*sqr
t(a)*b**2*sqrt(1/b)*sinh(c + d*x)**3/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a*
*(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*s
inh(c + d*x)**4) + 3*a**2*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))/(16*I*a
**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 + 1
6*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4) - 3*a**2*log(I*sqrt(a)*sqrt
(1/b) + sinh(c + d*x))/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*
sqrt(1/b)*sinh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**
4) + 6*a*b*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sinh(c + d*x)**2/(16*I
*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 +
 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4) - 6*a*b*log(I*sqrt(a)*sqr
t(1/b) + sinh(c + d*x))*sinh(c + d*x)**2/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*
I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/
b)*sinh(c + d*x)**4) + 3*b**2*log(-I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sin
h(c + d*x)**4/(16*I*a**(9/2)*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)
*sinh(c + d*x)**2 + 16*I*a**(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4) - 3*b*
*2*log(I*sqrt(a)*sqrt(1/b) + sinh(c + d*x))*sinh(c + d*x)**4/(16*I*a**(9/2)
*b*d*sqrt(1/b) + 32*I*a**(7/2)*b**2*d*sqrt(1/b)*sinh(c + d*x)**2 + 16*I*a**
(5/2)*b**3*d*sqrt(1/b)*sinh(c + d*x)**4), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.345 $\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=159

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b(7a-3b)\sinh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}-\frac{b\sinh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)^3*d) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*Ar
cTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Sinh[c
+ d*x])/(4*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - ((7*a - 3*b)*b*Sinh[c +
 d*x])/(8*a^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))
```

Rubi [A] time = 0.180384, antiderivative size = 159, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3190, 414, 527, 522, 203, 205}

$$-\frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^3}-\frac{b(7a-3b)\sinh(c+dx)}{8a^2d(a-b)^2\left(a+b\sinh^2(c+dx)\right)}-\frac{b\sinh(c+dx)}{4ad(a-b)\left(a+b\sinh^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] ArcTan[Sinh[c + d*x]]/((a - b)^3*d) - (Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*Ar
cTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^3*d) - (b*Sinh[c
+ d*x])/(4*a*(a - b)*d*(a + b*Sinh[c + d*x]^2)^2) - ((7*a - 3*b)*b*Sinh[c +
 d*x])/(8*a^2*(a - b)^2*d*(a + b*Sinh[c + d*x]^2))
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
```

```
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{\operatorname{Subst}\left(\int \frac{4a-3b-3bx^2}{(1+x^2)(a+bx^2)^2}\,dx, x, \sinh(c+dx)\right)}{4a(a-b)d}$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{8a^2-}{}\right.}{}$$

$$= -\frac{b\sinh(c+dx)}{4a(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{(7a-3b)b\sinh(c+dx)}{8a^2(a-b)^2d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{Subst}\left(\int \frac{1}{1+x^2}\right.}{(a}$$

$$= \frac{\tan^{-1}(\sinh(c+dx))}{(a-b)^3d} - \frac{\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^3d} - \frac{b\sinh(c}{4a(a-b)d\left(a+b\sin\right.}$$

Mathematica [B] time = 0.730706, size = 321, normalized size = 2.02

$$\frac{-2\sqrt{a}b\left(-35a^2b+18a^3+20ab^2-3b^3\right)\sinh(c+dx)+(b-2a)^2\left(\sqrt{b}\left(15a^2-10ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)+16a^{5/2}\tan\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-2*a + b)^2*(Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d
*x])/Sqrt[b]] + 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]]) + (b^(5/2)*(15*a^2 -
10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] + 16*a^(5/2)*b^2*Ar
cTan[Tanh[(c + d*x)/2]])*Cosh[2*(c + d*x)]^2 - 2*Sqrt[a]*b*(18*a^3 - 35*a^2
*b + 20*a*b^2 - 3*b^3)*Sinh[c + d*x] - 2*b*Cosh[2*(c + d*x)]*(-((2*a - b)*(
```

```
Sqrt[b]*(15*a^2 - 10*a*b + 3*b^2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]] +
 16*a^(5/2)*ArcTan[Tanh[(c + d*x)/2]])) + Sqrt[a]*b*(7*a^2 - 10*a*b + 3*b^2
)*Sinh[c + d*x]))/(8*a^(5/2)*(a - b)^3*d*(2*a - b + b*Cosh[2*(c + d*x)])^2)
```

Maple [B] time = 0.092, size = 2118, normalized size = 13.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1
/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-55/4/d*b^3/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2/a*tanh(1/2*d*x+1/2*c)^5+5/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7
-25/8/d*b^2/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*a
rctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-5/4/d*b^2
/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*
c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))+5/4/d*b^2/(a-b)^3/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*
arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+9/4/d*b/
(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1
/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^7-27/4/d*b/(a-b)^3/(tanh(1/2*d*x+1/2*c
)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d
*x+1/2*c)^5+27/4/d*b/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)
^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^3-9/4/d*b/(a-b)^3
/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2
*b+a)^2*a*tanh(1/2*d*x+1/2*c)+3/d*b^4/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c
)^5+55/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3-3/d*b^4/(a-b)^3/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/a^2*tanh(1/2*d*x+1/2*c)^3-5/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*ta
nh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-
25/8/d*b^2/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*ar
ctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+15/8/d*b/
(a-b)^3*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*t
anh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+15/8/d*b/(a-b)^3*a
/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*
x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+13/8/d*b^3/(a-b)^3/a/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d*b^4/(a-b)^3/a^2/(-b*(a-b))^(1
/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b^4/(a-b)^3/a^2/(-b*(a-b))^(1/2)/((2
*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2))+13/8/d*b^3/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a
-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2))+2/d/(a-b)^3*arctan(tanh(1/2*d*x+1/2*c))-7/2/d*b^2/(a-b)^
3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^
2*b+a)^2*tanh(1/2*d*x+1/2*c)^7+35/2/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-
2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c
)^5-35/2/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+7/2/d*b^2/(a-b)^3/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
```

```
2*tanh(1/2*d*x+1/2*c)+15/8/d*b/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-15/8/d
*b/(a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c
)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/4*((7*a*b^2*e^(7*c) - 3*b^3*e^(7*c))*e^(7*d*x) + (36*a^2*b*e^(5*c) - 41*
a*b^2*e^(5*c) + 9*b^3*e^(5*c))*e^(5*d*x) - (36*a^2*b*e^(3*c) - 41*a*b^2*e^(
3*c) + 9*b^3*e^(3*c))*e^(3*d*x) - (7*a*b^2*e^c - 3*b^3*e^c)*e^(d*x))/(a^4*b
^2*d - 2*a^3*b^3*d + a^2*b^4*d + (a^4*b^2*d*e^(8*c) - 2*a^3*b^3*d*e^(8*c) +
 a^2*b^4*d*e^(8*c))*e^(8*d*x) + 4*(2*a^5*b*d*e^(6*c) - 5*a^4*b^2*d*e^(6*c)
+ 4*a^3*b^3*d*e^(6*c) - a^2*b^4*d*e^(6*c))*e^(6*d*x) + 2*(8*a^6*d*e^(4*c) -
 24*a^5*b*d*e^(4*c) + 27*a^4*b^2*d*e^(4*c) - 14*a^3*b^3*d*e^(4*c) + 3*a^2*b
^4*d*e^(4*c))*e^(4*d*x) + 4*(2*a^5*b*d*e^(2*c) - 5*a^4*b^2*d*e^(2*c) + 4*a^
3*b^3*d*e^(2*c) - a^2*b^4*d*e^(2*c))*e^(2*d*x)) + 2*arctan(e^(d*x + c))/(a^
3*d - 3*a^2*b*d + 3*a*b^2*d - b^3*d) - 2*integrate(1/8*((15*a^2*b*e^(3*c) -
 10*a*b^2*e^(3*c) + 3*b^3*e^(3*c))*e^(3*d*x) + (15*a^2*b*e^c - 10*a*b^2*e^c
 + 3*b^3*e^c)*e^(d*x))/(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4 + (a^5*b*e^
(4*c) - 3*a^4*b^2*e^(4*c) + 3*a^3*b^3*e^(4*c) - a^2*b^4*e^(4*c))*e^(4*d*x)
+ 2*(2*a^6*e^(2*c) - 7*a^5*b*e^(2*c) + 9*a^4*b^2*e^(2*c) - 5*a^3*b^3*e^(2*c
) + a^2*b^4*e^(2*c))*e^(2*d*x)), x)
```

Fricas [B] time = 3.15793, size = 18737, normalized size = 117.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(4*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 28*(7*a^2*b^2 -
10*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^6 + 4*(7*a^2*b^2 - 10*a*b^3 +
 3*b^4)*sinh(d*x + c)^7 + 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh
(d*x + c)^5 + 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4 + 21*(7*a^2*b^2 -
 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^5 + 20*(7*(7*a^2*b^2 - 10
*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4
)*cosh(d*x + c))*sinh(d*x + c)^4 - 4*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*
b^4)*cosh(d*x + c)^3 + 4*(35*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4
 - 36*a^3*b + 77*a^2*b^2 - 50*a*b^3 + 9*b^4 + 10*(36*a^3*b - 77*a^2*b^2 + 5
0*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^3 + 4*(21*(7*a^2*b^2 - 10*a
*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^
4)*cosh(d*x + c)^3 - 3*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x
+ c))*sinh(d*x + c)^2 + ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^8 +
8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b
^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^
3 - 3*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 +
7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(
15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(30*a^3*b - 35*a^2*b^2 +
```

```
16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(120*a^4 - 200*a^3*b
+ 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 10*
a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b
^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^4 + 15*a^2*b^2 - 10*a*b^3 + 3*b^4 + 8*(7*(15*a^2*b^2 - 10*a*
b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4
)*cosh(d*x + c)^3 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*
cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^
4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 +
 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^4 + 30*a^3*b -
 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*
a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15*a^2*b^2 - 10*a*b^3
 + 3*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*co
sh(d*x + c)^5 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh
(d*x + c)^3 + (30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sin
h(d*x + c))*sqrt(-b/a)*log((b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*x
+ c)^3 + b*sinh(d*x + c)^4 - 2*(2*a + b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*x
+ c)^2 - 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 - (2*a + b)*cosh(d
*x + c))*sinh(d*x + c) + 4*(a*cosh(d*x + c)^3 + 3*a*cosh(d*x + c)*sinh(d*x
+ c)^2 + a*sinh(d*x + c)^3 - a*cosh(d*x + c) + (3*a*cosh(d*x + c)^2 - a)*si
nh(d*x + c))*sqrt(-b/a) + b)/(b*cosh(d*x + c)^4 + 4*b*cosh(d*x + c)*sinh(d*
x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x + c)^2 + 2*(3*b*cosh(d*
x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x + c)^3 + (2*a - b)*cosh
(d*x + c))*sinh(d*x + c) + b)) - 32*(a^2*b^2*cosh(d*x + c)^8 + 8*a^2*b^2*co
sh(d*x + c)*sinh(d*x + c)^7 + a^2*b^2*sinh(d*x + c)^8 + 4*(2*a^3*b - a^2*b^
2)*cosh(d*x + c)^6 + 4*(7*a^2*b^2*cosh(d*x + c)^2 + 2*a^3*b - a^2*b^2)*sinh
(d*x + c)^6 + 8*(7*a^2*b^2*cosh(d*x + c)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(d*x
 + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^4 +
2*(35*a^2*b^2*cosh(d*x + c)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b -
 a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(d*
x + c)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^3 + (8*a^4 - 8*a^3*b + 3*a^
2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(d*x + c)
^2 + 4*(7*a^2*b^2*cosh(d*x + c)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^4
+ 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^2)*sinh
(d*x + c)^2 + 8*(a^2*b^2*cosh(d*x + c)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(d*x +
 c)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c)^3 + (2*a^3*b - a^2*b^2)
*cosh(d*x + c))*sinh(d*x + c))*arctan(cosh(d*x + c) + sinh(d*x + c)) - 4*(7
*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c) + 4*(7*(7*a^2*b^2 - 10*a*b^3 + 3
*b^4)*cosh(d*x + c)^6 + 5*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d
*x + c)^4 - 7*a^2*b^2 + 10*a*b^3 - 3*b^4 - 3*(36*a^3*b - 77*a^2*b^2 + 50*a*
b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*
b^4 - a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 -
a^2*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2
*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^
2*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 +
 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*
a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 -
 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(35*(a^5*b^2 - 3
*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^4 + 30*(2*a^6*b - 7*a^5*b^2
 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 + (8*a^7 - 32*a^6*b +
 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(
2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^2 +
8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^5 + 10*(2*
a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^3 + (8
*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(
d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5
)*d*cosh(d*x + c)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2
```

```
*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 1
7*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3
 - 5*a^3*b^4 + a^2*b^5)*d)*sinh(d*x + c)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b
^4 - a^2*b^5)*d + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x
 + c)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(
d*x + c)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a
^2*b^5)*d*cosh(d*x + c)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 +
a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)), -1/8*(2*(7*a^2*b^2 - 10*a*b^3 + 3
*b^4)*cosh(d*x + c)^7 + 14*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)*sin
h(d*x + c)^6 + 2*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c)^7 + 2*(36*a^3
*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^5 + 2*(36*a^3*b - 77*a^2*
b^2 + 50*a*b^3 - 9*b^4 + 21*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)
*sinh(d*x + c)^5 + 10*(7*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + (
36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^4 -
2*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^3 + 2*(35*(7*a^2
*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 - 36*a^3*b + 77*a^2*b^2 - 50*a*b^3
 + 9*b^4 + 10*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*s
inh(d*x + c)^3 + 2*(21*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*
(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^3 - 3*(36*a^3*b -
77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + ((15*a^2*b^
2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*c
osh(d*x + c)*sinh(d*x + c)^7 + (15*a^2*b^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c
)^8 + 4*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(30*
a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*c
osh(d*x + c)^2)*sinh(d*x + c)^6 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh
(d*x + c)^3 + 3*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*s
inh(d*x + c)^5 + 2*(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*c
osh(d*x + c)^4 + 2*(35*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 12
0*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*
b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 10*
a*b^3 + 3*b^4 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(
30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (120*a^4 - 200*
a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*
(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b
^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b
^3 - 3*b^4)*cosh(d*x + c)^4 + 30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*
(120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sin
h(d*x + c)^2 + 8*((15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(30*a
^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (120*a^4 - 200*a^3*
b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 35*a^2*b^
2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*sq
rt(b/a)*(cosh(d*x + c) + sinh(d*x + c))) + ((15*a^2*b^2 - 10*a*b^3 + 3*b^4)
*cosh(d*x + c)^8 + 8*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)*sinh(d*x
 + c)^7 + (15*a^2*b^2 - 10*a*b^3 + 3*b^4)*sinh(d*x + c)^8 + 4*(30*a^3*b - 3
5*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^6 + 4*(30*a^3*b - 35*a^2*b^2 +
16*a*b^3 - 3*b^4 + 7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^2)*sinh(
d*x + c)^6 + 8*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^3 + 3*(30*a
^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(1
20*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^4 + 2*(3
5*(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^4 + 120*a^4 - 200*a^3*b + 1
49*a^2*b^2 - 54*a*b^3 + 9*b^4 + 30*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^
4)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 15*a^2*b^2 - 10*a*b^3 + 3*b^4 + 8*(7*
(15*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^5 + 10*(30*a^3*b - 35*a^2*b^2
 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^3 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 -
 54*a*b^3 + 9*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(30*a^3*b - 35*a^2*b^
2 + 16*a*b^3 - 3*b^4)*cosh(d*x + c)^2 + 4*(7*(15*a^2*b^2 - 10*a*b^3 + 3*b^4
)*cosh(d*x + c)^6 + 15*(30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)*cosh(d*x
+ c)^4 + 30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4 + 3*(120*a^4 - 200*a^3*b
+ 149*a^2*b^2 - 54*a*b^3 + 9*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*((15
```

```
*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^7 + 3*(30*a^3*b - 35*a^2*b^2 + 1
6*a*b^3 - 3*b^4)*cosh(d*x + c)^5 + (120*a^4 - 200*a^3*b + 149*a^2*b^2 - 54*
a*b^3 + 9*b^4)*cosh(d*x + c)^3 + (30*a^3*b - 35*a^2*b^2 + 16*a*b^3 - 3*b^4)
*cosh(d*x + c))*sinh(d*x + c))*sqrt(b/a)*arctan(1/2*(b*cosh(d*x + c)^3 + 3*
b*cosh(d*x + c)*sinh(d*x + c)^2 + b*sinh(d*x + c)^3 + (4*a - b)*cosh(d*x +
c) + (3*b*cosh(d*x + c)^2 + 4*a - b)*sinh(d*x + c))*sqrt(b/a)/b) - 16*(a^2*
b^2*cosh(d*x + c)^8 + 8*a^2*b^2*cosh(d*x + c)*sinh(d*x + c)^7 + a^2*b^2*sin
h(d*x + c)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^6 + 4*(7*a^2*b^2*cosh(d*
x + c)^2 + 2*a^3*b - a^2*b^2)*sinh(d*x + c)^6 + 8*(7*a^2*b^2*cosh(d*x + c)^
3 + 3*(2*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(8*a^4 - 8*a^3
*b + 3*a^2*b^2)*cosh(d*x + c)^4 + 2*(35*a^2*b^2*cosh(d*x + c)^4 + 8*a^4 - 8
*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^
4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(d*x + c)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(d*
x + c)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(d*x + c))*sinh(d*x + c)^3 + 4
*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^2 + 4*(7*a^2*b^2*cosh(d*x + c)^6 + 15*(2
*a^3*b - a^2*b^2)*cosh(d*x + c)^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b
+ 3*a^2*b^2)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 8*(a^2*b^2*cosh(d*x + c)^7
+ 3*(2*a^3*b - a^2*b^2)*cosh(d*x + c)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cos
h(d*x + c)^3 + (2*a^3*b - a^2*b^2)*cosh(d*x + c))*sinh(d*x + c))*arctan(cos
h(d*x + c) + sinh(d*x + c)) - 2*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c
) + 2*(7*(7*a^2*b^2 - 10*a*b^3 + 3*b^4)*cosh(d*x + c)^6 + 5*(36*a^3*b - 77*
a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^4 - 7*a^2*b^2 + 10*a*b^3 - 3*b^4
- 3*(36*a^3*b - 77*a^2*b^2 + 50*a*b^3 - 9*b^4)*cosh(d*x + c)^2)*sinh(d*x +
c))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^8 + 8*(a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)*sinh(d*x + c)^7 + (
a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*sinh(d*x + c)^8 + 4*(2*a^6*b -
 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^6 + 4*(7*(a^5
*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^2 + (2*a^6*b - 7*a^
5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d)*sinh(d*x + c)^6 + 2*(8*a^7 - 32
*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^
4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^3 + 3*
(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c))*si
nh(d*x + c)^5 + 2*(35*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*
x + c)^4 + 30*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cos
h(d*x + c)^2 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3
*a^2*b^5)*d)*sinh(d*x + c)^4 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b
^4 + a^2*b^5)*d*cosh(d*x + c)^2 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*d*cosh(d*x + c)^5 + 10*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4
 + a^2*b^5)*d*cosh(d*x + c)^3 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3
 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(7*(a^5*b^2
 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^6 + 15*(2*a^6*b - 7*a^5
*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^4 + 3*(8*a^7 - 32*a
^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^2
+ (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d)*sinh(d*x + c)^
2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*d + 8*((a^5*b^2 - 3*a^4*b^3
 + 3*a^3*b^4 - a^2*b^5)*d*cosh(d*x + c)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*
b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b
^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*d*cosh(d*x + c)^3 + (2*a^6*b - 7*
a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.346 $\int \frac{\text{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=172

$$\frac{3b^2(4a-b)\tanh(c+dx)}{8a^2d(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{4ad(a-b)^3\left(a-(a-b)\tan\right.}$$

```
[Out] (-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(
8*a^(5/2)*(a - b)^(7/2)*d) + Tanh[c + d*x]/((a - b)^3*d) - (b^3*Tanh[c + d*
x])/(4*a*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*(4*a - b)*b^2*Ta
nh[c + d*x])/(8*a^2*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.264594, antiderivative size = 172, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 390, 1157, 385, 208}

$$\frac{3b^2(4a-b)\tanh(c+dx)}{8a^2d(a-b)^3\left(a-(a-b)\tanh^2(c+dx)\right)}-\frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{7/2}}-\frac{b^3\tanh(c+dx)}{4ad(a-b)^3\left(a-(a-b)\tan\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/(
8*a^(5/2)*(a - b)^(7/2)*d) + Tanh[c + d*x]/((a - b)^3*d) - (b^3*Tanh[c + d*
x])/(4*a*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2)^2) + (3*(4*a - b)*b^2*Ta
nh[c + d*x])/(8*a^2*(a - b)^3*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
 b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a-b)^3} - \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{(a-b)^3\left(a+(-a+b)x^2\right)^3}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{\text{Subst}\left(\int \frac{b\left(3a^2-3ab+b^2\right)-3(a-b)(2a-b)bx^2+3(a-b)^2bx^4}{\left(a+(-a+b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{(a-b)^3d}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3\tanh(c+dx)}{4a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-3(2a-b)^2b+12a(a-b)bx^2}{\left(a+(-a+b)x^2\right)^2} \right.}{4a(a-b)^3}$$

$$= \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3\tanh(c+dx)}{4a(a-b)^3d\left(a-(a-b)\tanh^2(c+dx)\right)^2} + \frac{3(4a-b)b^2\tanh(c+}{8a^2(a-b)^3d\left(a-(a-b)\tanh\right.}$$

$$= -\frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{7/2}d} + \frac{\tanh(c+dx)}{(a-b)^3d} - \frac{b^3\tanh(c+}{4a(a-b)^3d\left(a-(a-b)\right.}$$

Mathematica [A] time = 1.07899, size = 165, normalized size = 0.96

$$\frac{-\frac{3b\left(8a^2-4ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{7/2}} + \frac{b^2(10a-3b)\sinh(2(c+dx))}{a^2(a-b)^3(2a+b\cosh(2(c+dx))-b)} + \frac{4b^2\sinh(2(c+dx))}{a(a-b)^2(2a+b\cosh(2(c+dx))-b)^2} + \frac{8\tanh(c+dx)}{(a-b)^3}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^2/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-3*b*(8*a^2 - 4*a*b + b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]])/
(a^(5/2)*(a - b)^(7/2)) + (4*b^2*Sinh[2*(c + d*x)])/(a*(a - b)^2*(2*a - b +
 b*Cosh[2*(c + d*x)])^2) + ((10*a - 3*b)*b^2*Sinh[2*(c + d*x)])/(a^2*(a - b
)^3*(2*a - b + b*Cosh[2*(c + d*x)])) + (8*Tanh[c + d*x])/(a - b)^3)/(8*d)
```

Maple [B] time = 0.084, size = 1694, normalized size = 9.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -5/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^7+3/d*b^2/(a-b)^3/(tanh(1/2
*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*ta
nh(1/2*d*x+1/2*c)^7-3/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x
+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+45/4/d*b^3
/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-3/d*b^4/(a-b)^3/(tanh(1/2*d*x+1/2*c
)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2
*d*x+1/2*c)^5-3/d*b^2/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c
)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3+45/4/d*b^3/(a-b)
^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)
^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^3-3/d*b^4/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-
2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1
/2*c)^3-5/4/d*b^3/(a-b)^3/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*
a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)+3/d*b^2/(a-b)^3/(tan
h(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)
^2*tanh(1/2*d*x+1/2*c)-3/d*b/(a-b)^3/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*a
rctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/2/d*b^
2/(a-b)^3/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-
b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2
)+a-2*b)*a)^(1/2))+3/d*b^2/(a-b)^3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-
2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2))-3/2/d*b^3/(a-b)^3/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3
/8/d*b^4/(a-b)^3/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*
arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/d*b/(
a-b)^3/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((
2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/2/d*b^2/(a-b)^3/a/((2*(-b*(a-b))^(1/2
)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2))+3/8/d*b^3/(a-b)^3/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/d*b^2/(a-b)^
3/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d
*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/2/d*b^3/(a-b)^3/a/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b^4/(a-b)^3/a^2/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-
b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+2/d/(a-b)^3*tanh(1/2*d*x+1/2*c)/(tanh(1/2*d
*x+1/2*c)^2+1)
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [B] time = 2.84346, size = 21442, normalized size = 124.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/16*(12*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^8 + 9
6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)*sinh(d*x + c)^
7 + 12*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*sinh(d*x + c)^8 + 24*(2
4*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^6 + 24
*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5 + 14*(8*a^4*b^2 -
12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 32*a^4*b
^2 + 8*a^3*b^3 - 52*a^2*b^4 + 12*a*b^5 + 48*(14*(8*a^4*b^2 - 12*a^3*b^3 + 5
*a^2*b^4 - a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 -
 8*a^2*b^4 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 8*(64*a^6 - 88*a^5*b +
 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^4 + 8*(64*a^6 - 88*a^5*b +
 28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4 + 105*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4
 - a*b^5)*cosh(d*x + c)^4 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*
b^4 + a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 32*(21*(8*a^4*b^2 - 12*a^3*
b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 44*a^4*b^2 + 27*a
^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 88*a^5*b + 28*a^4*b
^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 8*(32*a^5*b - 16
*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c)^2 + 8*(42*(8*a^
4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^6 + 32*a^5*b - 16*a^4
*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a
^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^4 + 6*(64*a^6 - 88*a^5*b + 28*a^4
*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 3*((8*a^2*b^
3 - 4*a*b^4 + b^5)*cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d
*x + c)*sinh(d*x + c)^9 + (8*a^2*b^3 - 4*a*b^4 + b^5)*sinh(d*x + c)^10 + (6
4*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 -
56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 45*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c
)^2)*sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^3 +
(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^7
 + 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^6 +
 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5 + 105*(8*a^2*b^3 - 4
*a*b^4 + b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*
b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(8*a^2*b^3 - 4*a*b^4 + b^5)*c
osh(d*x + c)^5 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x +
 c)^3 + 3*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c
))*sinh(d*x + c)^5 + 8*a^2*b^3 - 4*a*b^4 + b^5 + 2*(64*a^4*b - 64*a^3*b^2 +
 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 4*a*b^4
+ b^5)*cosh(d*x + c)^6 + 64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5
 + 35*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^4 + 15*(64
*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x
 + c)^4 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2
 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 64*a^3*b^
2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^4*b - 64*a^3*b^2 +
32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + (64*a^3*b^2 -
56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 4*a*b^4 +
 b^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cos
h(d*x + c)^6 + 64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 30*(64*a^4*b -
64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 12*(64*a^4*b - 6
4*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
2*(5*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 56*a^2*b
^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^7 + 6*(64*a^4*b - 64*a^3*b^2 + 32*a^2*
b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^5 + 4*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^
3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 -
```

```
3*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(a^2 - a*b)*log((b^2*cosh(d*x + c)
^4 + 4*b^2*cosh(d*x + c)*sinh(d*x + c)^3 + b^2*sinh(d*x + c)^4 + 2*(2*a*b -
 b^2)*cosh(d*x + c)^2 + 2*(3*b^2*cosh(d*x + c)^2 + 2*a*b - b^2)*sinh(d*x +
c)^2 + 8*a^2 - 8*a*b + b^2 + 4*(b^2*cosh(d*x + c)^3 + (2*a*b - b^2)*cosh(d*
x + c))*sinh(d*x + c) - 4*(b*cosh(d*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x +
 c) + b*sinh(d*x + c)^2 + 2*a - b)*sqrt(a^2 - a*b))/(b*cosh(d*x + c)^4 + 4*
b*cosh(d*x + c)*sinh(d*x + c)^3 + b*sinh(d*x + c)^4 + 2*(2*a - b)*cosh(d*x
+ c)^2 + 2*(3*b*cosh(d*x + c)^2 + 2*a - b)*sinh(d*x + c)^2 + 4*(b*cosh(d*x
+ c)^3 + (2*a - b)*cosh(d*x + c))*sinh(d*x + c) + b)) + 16*(6*(8*a^4*b^2 -
12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^7 + 9*(24*a^5*b - 44*a^4*b^2
+ 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^5 + 2*(64*a^6 - 88*a^5*b +
28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^3 + (32*a^5*b - 16*a^4*b^2
- 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b^
2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^10 + 10*(a
^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)*sinh(
d*x + c)^9 + (a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*sinh
(d*x + c)^10 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5
 - 3*a^3*b^6)*d*cosh(d*x + c)^8 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*
a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 -
 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d)*sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^
8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*
x + c)^6 + 8*(15*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*
cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*
b^5 - 3*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7*b^2 - 4*a^6
*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 14*(8*a^8*b - 3
5*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x +
c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5
 + a^3*b^6)*d)*sinh(d*x + c)^6 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*
b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 4*(63*(a^7*b^2
- 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 14*(8*a^
8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh
(d*x + c)^3 + 3*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 -
8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7*b^2 - 4
*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^6 + 35*(8*a^8*b
 - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*
x + c)^4 + 15*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*
a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*
a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d)*sinh(d*x + c)^4 + (8*a^8*b -
 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x
+ c)^2 + 8*(15*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*co
sh(d*x + c)^7 + 7*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*
b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*
a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^3 + (8*a^9 - 36
*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh
(d*x + c))*sinh(d*x + c)^3 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b
^5 + a^3*b^6)*d*cosh(d*x + c)^8 + 28*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 5
0*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^6 + 30*(8*a^9 - 36*a^8*
b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x
+ c)^4 + 12*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^
4*b^5 + a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 5
0*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d)*sinh(d*x + c)^2 + (a^7*b^2 - 4*a^6*b
^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d + 2*(5*(a^7*b^2 - 4*a^6*b^3 + 6*a^5
*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^9 + 4*(8*a^8*b - 35*a^7*b^2 + 6
0*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^7 + 6*(8*a
^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)
*d*cosh(d*x + c)^5 + 4*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5
*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*
a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x +
c)), -1/8*(6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^8 +
```

48*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)*sinh(d*x + c
)^7 + 6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*sinh(d*x + c)^8 + 12*(
24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^6 + 1
2*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4 + a*b^5 + 14*(8*a^4*b^2 -
12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 16*a^4*
b^2 + 4*a^3*b^3 - 26*a^2*b^4 + 6*a*b^5 + 24*(14*(8*a^4*b^2 - 12*a^3*b^3 + 5
*a^2*b^4 - a*b^5)*cosh(d*x + c)^3 + 3*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 -
8*a^2*b^4 + a*b^5)*cosh(d*x + c))*sinh(d*x + c)^5 + 4*(64*a^6 - 88*a^5*b +
28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^4 + 4*(64*a^6 - 88*a^5*b +
28*a^4*b^2 - 3*a^3*b^3 - a^2*b^4 + 105*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4
- a*b^5)*cosh(d*x + c)^4 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*
b^4 + a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^4 + 16*(21*(8*a^4*b^2 - 12*a^3*
b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^5 + 15*(24*a^5*b - 44*a^4*b^2 + 27*a
^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^3 + (64*a^6 - 88*a^5*b + 28*a^4*b
^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^3 + 4*(32*a^5*b - 16
*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5)*cosh(d*x + c)^2 + 4*(42*(8*a^
4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 - a*b^5)*cosh(d*x + c)^6 + 32*a^5*b - 16*a^4
*b^2 - 37*a^3*b^3 + 24*a^2*b^4 - 3*a*b^5 + 45*(24*a^5*b - 44*a^4*b^2 + 27*a
^3*b^3 - 8*a^2*b^4 + a*b^5)*cosh(d*x + c)^4 + 6*(64*a^6 - 88*a^5*b + 28*a^4
*b^2 - 3*a^3*b^3 - a^2*b^4)*cosh(d*x + c)^2)*sinh(d*x + c)^2 - 3*((8*a^2*b^
3 - 4*a*b^4 + b^5)*cosh(d*x + c)^10 + 10*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d
*x + c)*sinh(d*x + c)^9 + (8*a^2*b^3 - 4*a*b^4 + b^5)*sinh(d*x + c)^10 + (6
4*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^8 + (64*a^3*b^2 -
56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 45*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c
)^2)*sinh(d*x + c)^8 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^3 +
(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c))*sinh(d*x + c)^7
+ 2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^6 +
2*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5 + 105*(8*a^2*b^3 - 4
*a*b^4 + b^5)*cosh(d*x + c)^4 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*
b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^6 + 4*(63*(8*a^2*b^3 - 4*a*b^4 + b^5)*c
osh(d*x + c)^5 + 14*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x +
c)^3 + 3*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c
))*sinh(d*x + c)^5 + 8*a^2*b^3 - 4*a*b^4 + b^5 + 2*(64*a^4*b - 64*a^3*b^2 +
32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 2*(105*(8*a^2*b^3 - 4*a*b^4
+ b^5)*cosh(d*x + c)^6 + 64*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5
+ 35*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^4 + 15*(64
*a^4*b - 64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x
+ c)^4 + 8*(15*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^7 + 7*(64*a^3*b^2
- 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^5 + 5*(64*a^4*b - 64*a^3*b^
2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^4*b - 64*a^3*b^2 +
32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c))*sinh(d*x + c)^3 + (64*a^3*b^2 -
56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^2 + (45*(8*a^2*b^3 - 4*a*b^4 +
b^5)*cosh(d*x + c)^8 + 28*(64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5)*cos
h(d*x + c)^6 + 64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 - 3*b^5 + 30*(64*a^4*b -
64*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^4 + 12*(64*a^4*b - 6
4*a^3*b^2 + 32*a^2*b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^2)*sinh(d*x + c)^2 +
2*(5*(8*a^2*b^3 - 4*a*b^4 + b^5)*cosh(d*x + c)^9 + 4*(64*a^3*b^2 - 56*a^2*b
^3 + 20*a*b^4 - 3*b^5)*cosh(d*x + c)^7 + 6*(64*a^4*b - 64*a^3*b^2 + 32*a^2*
b^3 - 8*a*b^4 + b^5)*cosh(d*x + c)^5 + 4*(64*a^4*b - 64*a^3*b^2 + 32*a^2*b^
3 - 8*a*b^4 + b^5)*cosh(d*x + c)^3 + (64*a^3*b^2 - 56*a^2*b^3 + 20*a*b^4 -
3*b^5)*cosh(d*x + c))*sinh(d*x + c))*sqrt(-a^2 + a*b)*arctan(-1/2*(b*cosh(d
*x + c)^2 + 2*b*cosh(d*x + c)*sinh(d*x + c) + b*sinh(d*x + c)^2 + 2*a - b)*
sqrt(-a^2 + a*b)/(a^2 - a*b)) + 8*(6*(8*a^4*b^2 - 12*a^3*b^3 + 5*a^2*b^4 -
a*b^5)*cosh(d*x + c)^7 + 9*(24*a^5*b - 44*a^4*b^2 + 27*a^3*b^3 - 8*a^2*b^4
+ a*b^5)*cosh(d*x + c)^5 + 2*(64*a^6 - 88*a^5*b + 28*a^4*b^2 - 3*a^3*b^3 -
a^2*b^4)*cosh(d*x + c)^3 + (32*a^5*b - 16*a^4*b^2 - 37*a^3*b^3 + 24*a^2*b^4
- 3*a*b^5)*cosh(d*x + c))*sinh(d*x + c))/((a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4
- 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^10 + 10*(a^7*b^2 - 4*a^6*b^3 + 6*a^
5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)*sinh(d*x + c)^9 + (a^7*b^2 - 4

```
*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*sinh(d*x + c)^10 + (8*a^8*b -
 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x
+ c)^8 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh
(d*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5
- 3*a^3*b^6)*d)*sinh(d*x + c)^8 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6
*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^6 + 8*(15*(a^7*b^2
 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^3 + (8*a^8*
b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d
*x + c))*sinh(d*x + c)^7 + 2*(105*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*
b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 14*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 -
50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^2 + (8*a^9 - 36*a^8*b
+ 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d)*sinh(d*x +
 c)^6 + 2*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*
b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 4*(63*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 -
 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 14*(8*a^8*b - 35*a^7*b^2 + 60*a^6
*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^3 + 3*(8*a^9 -
36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*co
sh(d*x + c))*sinh(d*x + c)^5 + 2*(105*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*
a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^6 + 35*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^
3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^4 + 15*(8*a^9 - 36
*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh
(d*x + c)^2 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*
a^4*b^5 + a^3*b^6)*d)*sinh(d*x + c)^4 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3
- 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^2 + 8*(15*(a^7*b^2 -
 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^7 + 7*(8*a^8*
b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d
*x + c)^5 + 5*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*
a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^3 + (8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*
a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c)^
3 + (45*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^6)*d*cosh(d*x
+ c)^8 + 28*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 -
3*a^3*b^6)*d*cosh(d*x + c)^6 + 30*(8*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b
^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^4 + 12*(8*a^9 - 36*a
^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d
*x + c)^2 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20*a^4*b^5 -
3*a^3*b^6)*d)*sinh(d*x + c)^2 + (a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^
5 + a^3*b^6)*d + 2*(5*(a^7*b^2 - 4*a^6*b^3 + 6*a^5*b^4 - 4*a^4*b^5 + a^3*b^
6)*d*cosh(d*x + c)^9 + 4*(8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 +
20*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c)^7 + 6*(8*a^9 - 36*a^8*b + 65*a^7*b^
2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^6)*d*cosh(d*x + c)^5 + 4*(8
*a^9 - 36*a^8*b + 65*a^7*b^2 - 60*a^6*b^3 + 30*a^5*b^4 - 8*a^4*b^5 + a^3*b^
6)*d*cosh(d*x + c)^3 + (8*a^8*b - 35*a^7*b^2 + 60*a^6*b^3 - 50*a^5*b^4 + 20
*a^4*b^5 - 3*a^3*b^6)*d*cosh(d*x + c))*sinh(d*x + c))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**2/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.398, size = 509, normalized size = 2.96

$$-\frac{3\left(8a^2b-4ab^2+b^3\right)\arctan\left(\frac{be^{(2dx+2c)}+2a-b}{2\sqrt{-a^2+ab}}\right)}{8\left(a^5d-3a^4bd+3a^3b^2d-a^2b^3d\right)\sqrt{-a^2+ab}}-\frac{16a^2b^2e^{(6dx+6c)}-12ab^3e^{(6dx+6c)}+3b^4e^{(6dx+6c)}+80a^3be^{(4dx+4c)}-10}{4\left(a^5d-3a^4bd+3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^2/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -3/8*(8*a^2*b - 4*a*b^2 + b^3)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b)/sqr
t(-a^2 + a*b))/((a^5*d - 3*a^4*b*d + 3*a^3*b^2*d - a^2*b^3*d)*sqrt(-a^2 + a
*b)) - 1/4*(16*a^2*b^2*e^(6*d*x + 6*c) - 12*a*b^3*e^(6*d*x + 6*c) + 3*b^4*e
^(6*d*x + 6*c) + 80*a^3*b*e^(4*d*x + 4*c) - 104*a^2*b^2*e^(4*d*x + 4*c) + 5
4*a*b^3*e^(4*d*x + 4*c) - 9*b^4*e^(4*d*x + 4*c) + 64*a^2*b^2*e^(2*d*x + 2*c
) - 52*a*b^3*e^(2*d*x + 2*c) + 9*b^4*e^(2*d*x + 2*c) + 10*a*b^3 - 3*b^4)/((
a^5*d - 3*a^4*b*d + 3*a^3*b^2*d - a^2*b^3*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*
d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2) - 2/((a^3*d - 3*a^2*b*d + 3*a*b^2*
d - b^3*d)*(e^(2*d*x + 2*c) + 1))
```

3.347 $\int \frac{\mathbf{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=217

$$\frac{b^{3/2}\left(35a^2-14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^4}+\frac{b(4a-b)(a+3b)\sinh(c+dx)}{8a^2d(a-b)^3\left(a+b\sinh^2(c+dx)\right)}+\frac{b(2a+b)\sinh(c+dx)}{4ad(a-b)^2\left(a+b\sinh^2(c+dx)\right.}$$

```
[Out] ((a - 7*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^4*d) + (b^(3/2)*(35*a^2 - 14*a
*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^4*d
) + (b*(2*a + b)*Sinh[c + d*x])/(4*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)^2)
 + ((4*a - b)*b*(a + 3*b)*Sinh[c + d*x])/(8*a^2*(a - b)^3*d*(a + b*Sinh[c +
 d*x]^2)) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]
^2)^2)
```

Rubi [A] time = 0.303491, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 522, 203, 205}

$$\frac{b^{3/2}\left(35a^2-14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^4}+\frac{b(4a-b)(a+3b)\sinh(c+dx)}{8a^2d(a-b)^3\left(a+b\sinh^2(c+dx)\right)}+\frac{b(2a+b)\sinh(c+dx)}{4ad(a-b)^2\left(a+b\sinh^2(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((a - 7*b)*ArcTan[Sinh[c + d*x]])/(2*(a - b)^4*d) + (b^(3/2)*(35*a^2 - 14*a
*b + 3*b^2)*ArcTan[(Sqrt[b]*Sinh[c + d*x])/Sqrt[a]])/(8*a^(5/2)*(a - b)^4*d
) + (b*(2*a + b)*Sinh[c + d*x])/(4*a*(a - b)^2*d*(a + b*Sinh[c + d*x]^2)^2)
 + ((4*a - b)*b*(a + 3*b)*Sinh[c + d*x])/(8*a^2*(a - b)^3*d*(a + b*Sinh[c +
 d*x]^2)) + (Sech[c + d*x]*Tanh[c + d*x])/(2*(a - b)*d*(a + b*Sinh[c + d*x]
^2)^2)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-5bx^2}{(1+x^2)(a+bx^2)^3}\,dx, x, \sinh(c+dx)\right)}{2(a-b)d}$$

$$= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{\operatorname{sech}(c+dx)\tanh(c+dx)}{2(a-b)d\left(a+b\sinh^2(c+dx)\right)^2} - \frac{\operatorname{Subst}\left(\int \frac{-2(2a^2-\ldots}{(1\ldots}\right.}{\ldots}$$

$$= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx\ldots}{2(a-b)d\left(a+\ldots\right.}$$

$$= \frac{b(2a+b)\sinh(c+dx)}{4a(a-b)^2d\left(a+b\sinh^2(c+dx)\right)^2} + \frac{(4a-b)b(a+3b)\sinh(c+dx)}{8a^2(a-b)^3d\left(a+b\sinh^2(c+dx)\right)} + \frac{\operatorname{sech}(c+dx\ldots}{2(a-b)d\left(a+\ldots\right.}$$

$$= \frac{(a-7b)\tan^{-1}(\sinh(c+dx))}{2(a-b)^4d} + \frac{b^{3/2}\left(35a^2-14ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sinh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^4d} + \frac{b(2\ldots}{4a(a-b)\ldots}$$

Mathematica [A] time = 1.79072, size = 222, normalized size = 1.02

$$\frac{\frac{2b^2(a-b)\sinh(c+dx)\left(26a^2+b(11a-3b)\cosh(2(c+dx))-21ab+3b^2\right)}{a^2(2a+b\cosh(2(c+dx))-b)^2} - \frac{3b^{7/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{5/2}} + \frac{14b^{5/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{a^{3/2}} - \frac{35b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\operatorname{csch}(c+dx)}{\sqrt{b}}\right)}{\sqrt{a}} \ldots}{8d(a-b)^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^3/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((-35*b^(3/2)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/Sqrt[a] + (14*b^(5/2
)*ArcTan[(Sqrt[a]*Csch[c + d*x])/Sqrt[b]])/a^(3/2) - (3*b^(7/2)*ArcTan[(Sqr
t[a]*Csch[c + d*x])/Sqrt[b]])/a^(5/2) + 8*a*ArcTan[Tanh[(c + d*x)/2]] - 56*
b*ArcTan[Tanh[(c + d*x)/2]] + (2*(a - b)*b^2*(26*a^2 - 21*a*b + 3*b^2 + (11
*a - 3*b)*b*Cosh[2*(c + d*x)])*Sinh[c + d*x])/(a^2*(2*a - b + b*Cosh[2*(c +
 d*x)])^2) + 4*(a - b)*Sech[c + d*x]*Tanh[c + d*x])/(8*(a - b)^4*d)
```

Maple [B] time = 0.104, size = 2307, normalized size = 10.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -13/4/d*b^2/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^7+39/4/d*b^2/(a-b)^4/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2*a*tanh(1/2*d*x+1/2*c)^5-39/4/d*b^2/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tan
h(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)^3
+13/4/d*b^2/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*ta
nh(1/2*d*x+1/2*c)^2*b+a)^2*a*tanh(1/2*d*x+1/2*c)+49/8/d*b^3/(a-b)^4/(-b*(a-
b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+49/8/d*b^3/(a-b)^4/(-b*(a-b))^(1/2)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*
(a-b))^(1/2)+a-2*b)*a)^(1/2))-5/4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*
tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c
)^7+71/4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5-3/d*b^5/(a-b)^4/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/a^2*tanh(1/2*d*x+1/2*c)^5-71/4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*t
anh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)
^3+3/d*b^5/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tan
h(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+5/4/d*b^4/(a-b)^4/(tanh
(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^
2/a*tanh(1/2*d*x+1/2*c)+7/4/d*b^3/(a-b)^4/a/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^
(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3
/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-7/4/d*b^3/(a-b)^4/a/((2*
(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b)
)^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^(1/2)-a+2*b)*
a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))
-35/8/d*b^2/(a-b)^4*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)
*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-35/8/d
*b^2/(a-b)^4*a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan
(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-17/8/d*b^4/(a-
b)^4/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(
1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+3/8/d*b^5/(a-b)^4/a^2/
(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x
+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-17/8/d*b^4/(a-b)^4/a/(-b*(a-b
))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)
/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+3/8/d*b^5/(a-b)^4/a^2/(-b*(a-b))^(1/
2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(
-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+1/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^2*a*
tanh(1/2*d*x+1/2*c)-9/2/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d
```

```
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+49/2/d*b^3
/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+
1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^3-1/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)
^2*a*tanh(1/2*d*x+1/2*c)^3-1/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2
*d*x+1/2*c)*b+1/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^2*tanh(1/2*d*x+1/2*c)^3
*b-49/2/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*
tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5+35/8/d*b^2/(a-b)^4/((2*(
-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))
^(1/2)-a+2*b)*a)^(1/2))+9/2/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1
/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7-35/8
/d*b^2/(a-b)^4/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+
1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-7/d/(a-b)^4*arctan(tanh(1/2*d*
x+1/2*c))*b+1/d/(a-b)^4*arctan(tanh(1/2*d*x+1/2*c))*a
```

Maxima [F] time = 0., size = 0, normalized size = 0.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] (a*e^c - 7*b*e^c)*arctan(e^(d*x + c))*e^(-c)/(a^4*d - 4*a^3*b*d + 6*a^2*b^2
*d - 4*a*b^3*d + b^4*d) + 1/4*((4*a^2*b^2*e^(11*c) + 11*a*b^3*e^(11*c) - 3*
b^4*e^(11*c))*e^(11*d*x) + (32*a^3*b*e^(9*c) + 32*a^2*b^2*e^(9*c) - 31*a*b^
3*e^(9*c) + 3*b^4*e^(9*c))*e^(9*d*x) + 2*(32*a^4*e^(7*c) - 48*a^3*b*e^(7*c)
 + 46*a^2*b^2*e^(7*c) - 21*a*b^3*e^(7*c) + 3*b^4*e^(7*c))*e^(7*d*x) - 2*(32
*a^4*e^(5*c) - 48*a^3*b*e^(5*c) + 46*a^2*b^2*e^(5*c) - 21*a*b^3*e^(5*c) + 3
*b^4*e^(5*c))*e^(5*d*x) - (32*a^3*b*e^(3*c) + 32*a^2*b^2*e^(3*c) - 31*a*b^3
*e^(3*c) + 3*b^4*e^(3*c))*e^(3*d*x) - (4*a^2*b^2*e^c + 11*a*b^3*e^c - 3*b^4
*e^c)*e^(d*x))/(a^5*b^2*d - 3*a^4*b^3*d + 3*a^3*b^4*d - a^2*b^5*d + (a^5*b^
2*d*e^(12*c) - 3*a^4*b^3*d*e^(12*c) + 3*a^3*b^4*d*e^(12*c) - a^2*b^5*d*e^(1
2*c))*e^(12*d*x) + 2*(4*a^6*b*d*e^(10*c) - 13*a^5*b^2*d*e^(10*c) + 15*a^4*b
^3*d*e^(10*c) - 7*a^3*b^4*d*e^(10*c) + a^2*b^5*d*e^(10*c))*e^(10*d*x) + (16
*a^7*d*e^(8*c) - 48*a^6*b*d*e^(8*c) + 47*a^5*b^2*d*e^(8*c) - 13*a^4*b^3*d*e
^(8*c) - 3*a^3*b^4*d*e^(8*c) + a^2*b^5*d*e^(8*c))*e^(8*d*x) + 4*(8*a^7*d*e^
(6*c) - 28*a^6*b*d*e^(6*c) + 37*a^5*b^2*d*e^(6*c) - 23*a^4*b^3*d*e^(6*c) +
7*a^3*b^4*d*e^(6*c) - a^2*b^5*d*e^(6*c))*e^(6*d*x) + (16*a^7*d*e^(4*c) - 48
*a^6*b*d*e^(4*c) + 47*a^5*b^2*d*e^(4*c) - 13*a^4*b^3*d*e^(4*c) - 3*a^3*b^4*
d*e^(4*c) + a^2*b^5*d*e^(4*c))*e^(4*d*x) + 2*(4*a^6*b*d*e^(2*c) - 13*a^5*b^
2*d*e^(2*c) + 15*a^4*b^3*d*e^(2*c) - 7*a^3*b^4*d*e^(2*c) + a^2*b^5*d*e^(2*c
))*e^(2*d*x)) + 8*integrate(1/32*((35*a^2*b^2*e^(3*c) - 14*a*b^3*e^(3*c) +
3*b^4*e^(3*c))*e^(3*d*x) + (35*a^2*b^2*e^c - 14*a*b^3*e^c + 3*b^4*e^c)*e^(d
*x))/(a^6*b - 4*a^5*b^2 + 6*a^4*b^3 - 4*a^3*b^4 + a^2*b^5 + (a^6*b*e^(4*c)
- 4*a^5*b^2*e^(4*c) + 6*a^4*b^3*e^(4*c) - 4*a^3*b^4*e^(4*c) + a^2*b^5*e^(4*
c))*e^(4*d*x) + 2*(2*a^7*e^(2*c) - 9*a^6*b*e^(2*c) + 16*a^5*b^2*e^(2*c) - 1
4*a^4*b^3*e^(2*c) + 6*a^3*b^4*e^(2*c) - a^2*b^5*e^(2*c))*e^(2*d*x)), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**3/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^3/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.348 $\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=203

$$-\frac{b^3(16a-3b)\tanh(c+dx)}{8a^2d(a-b)^4\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{b^2\left(48a^2-16ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{9/2}}+\frac{b^4\tanh(c+d}{4ad(a-b)^4\left(a-(a-b)\tanh\right.}$$

```
[Out] (b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(8*a^(5/2)*(a - b)^(9/2)*d) + ((a - 4*b)*Tanh[c + d*x])/((a - b)^4*d) - T
anh[c + d*x]^3/(3*(a - b)^3*d) + (b^4*Tanh[c + d*x])/(4*a*(a - b)^4*d*(a -
(a - b)*Tanh[c + d*x]^2)^2) - ((16*a - 3*b)*b^3*Tanh[c + d*x])/(8*a^2*(a -
b)^4*d*(a - (a - b)*Tanh[c + d*x]^2))
```

Rubi [A] time = 0.340881, antiderivative size = 203, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 390, 1157, 385, 208}

$$-\frac{b^3(16a-3b)\tanh(c+dx)}{8a^2d(a-b)^4\left(a-(a-b)\tanh^2(c+dx)\right)}+\frac{b^2\left(48a^2-16ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a-b)^{9/2}}+\frac{b^4\tanh(c+d}{4ad(a-b)^4\left(a-(a-b)\tanh\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```


```
[Out] (b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[a]]
)/(8*a^(5/2)*(a - b)^(9/2)*d) + ((a - 4*b)*Tanh[c + d*x])/((a - b)^4*d) - T
anh[c + d*x]^3/(3*(a - b)^3*d) + (b^4*Tanh[c + d*x])/(4*a*(a - b)^4*d*(a -
(a - b)*Tanh[c + d*x]^2)^2) - ((16*a - 3*b)*b^3*Tanh[c + d*x])/(8*a^2*(a -
b)^4*d*(a - (a - b)*Tanh[c + d*x]^2))
```


Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 1157

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.),
x_Symbol] :> With[{Qx = PolynomialQuotient[(a + b*x^2 + c*x^4)^p, d + e*x^2
, x], R = Coeff[PolynomialRemainder[(a + b*x^2 + c*x^4)^p, d + e*x^2, x], x
, 0]}, -Simp[(R*x*(d + e*x^2)^(q + 1))/(2*d*(q + 1)), x] + Dist[1/(2*d*(q +
 1)), Int[(d + e*x^2)^(q + 1)*ExpandToSum[2*d*(q + 1)*Qx + R*(2*q + 3), x],
 x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 -
```

```
b*d*e + a*e^2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(c+dx)}{\left(a+b\sinh^2(c+dx)\right)^3}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-(a-b)x^2\right)^3}\,dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{\operatorname{Subst}\left(\int \left(\frac{a-4b}{(a-b)^4}-\frac{x^2}{(a-b)^3}+\frac{b^2\left(6a^2-4ab+b^2\right)-4(a-b)(3a-b)b^2x^2+6(a-b)^2b^2x^4}{(a-b)^4\left(a+(-a+b)x^2\right)^3}\right)dx, x, \tanh(c+dx)\right)}{d}$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d}-\frac{\tanh^3(c+dx)}{3(a-b)^3d}+\frac{\operatorname{Subst}\left(\int \frac{b^2\left(6a^2-4ab+b^2\right)-4(a-b)(3a-b)b^2x^2+6(a-b)^2}{\left(a+(-a+b)x^2\right)^3}\right.}{(a-b)^4d}$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d}-\frac{\tanh^3(c+dx)}{3(a-b)^3d}+\frac{b^4\tanh(c+dx)}{4a(a-b)^4d\left(a-(a-b)\tanh^2(c+dx)\right)^2}-$$

$$= \frac{(a-4b)\tanh(c+dx)}{(a-b)^4d}-\frac{\tanh^3(c+dx)}{3(a-b)^3d}+\frac{b^4\tanh(c+dx)}{4a(a-b)^4d\left(a-(a-b)\tanh^2(c+dx)\right)^2}-$$

$$= \frac{b^2\left(48a^2-16ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a-b)^{9/2}d}+\frac{(a-4b)\tanh(c+dx)}{(a-b)^4d}-\frac{\tanh^3(c+}{3(a-b)^3}$$

Mathematica [A] time = 2.19955, size = 169, normalized size = 0.83

$$\frac{\frac{3b^2\left(48a^2-16ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tanh(c+dx)}{\sqrt{a}}\right)}{a^{5/2}(a-b)^{9/2}}+\frac{\frac{3b^3\sinh(2(c+dx))\left(-32a^2+b(3b-14a)\cosh(2(c+dx))+24ab-3b^2\right)}{a^2(2a+b\cosh(2(c+dx))-b)^2}+8\tanh(c+dx)\left((a-b)\operatorname{sech}^2(c+dx)+2a-11b\right)}{(a-b)^4}}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]^4/(a + b*Sinh[c + d*x]^2)^3,x]
```

```
[Out] ((3*b^2*(48*a^2 - 16*a*b + 3*b^2)*ArcTanh[(Sqrt[a - b]*Tanh[c + d*x])/Sqrt[
a]])/(a^(5/2)*(a - b)^(9/2)) + ((3*b^3*(-32*a^2 + 24*a*b - 3*b^2 + b*(-14*a
 + 3*b)*Cosh[2*(c + d*x)])*Sinh[2*(c + d*x)])/(a^2*(2*a - b + b*Cosh[2*(c +
 d*x)])^2) + 8*(2*a - 11*b + (a - b)*Sech[c + d*x]^2)*Tanh[c + d*x])/(a - b
)^4)/(24*d)
```

Maple [B] time = 0.108, size = 1892, normalized size = 9.3

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x)
```

```
[Out] -6/d*b^3/(a-b)^4/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arct
an(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-6/d*b^3/(a-b
)^4/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/
2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+5/4/d*b^4/(a-b)^4/(tanh(
1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2
/a*tanh(1/2*d*x+1/2*c)^7-61/4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh
(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)^5+
3/d*b^5/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^5-61/4/d*b^4/(a-b)^4/(tanh(1
/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/
a*tanh(1/2*d*x+1/2*c)^3+3/d*b^5/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2
*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2/a^2*tanh(1/2*d*x+1/2*c)^3+5/
4/d*b^4/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1
/2*d*x+1/2*c)^2*b+a)^2/a*tanh(1/2*d*x+1/2*c)-2/d*b^3/(a-b)^4/a/((2*(-b*(a-b
))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
+a-2*b)*a)^(1/2))+3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2
)*arctanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d*b
^3/(a-b)^4/a/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2
*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))-3/8/d*b^4/(a-b)^4/a^2/((2*(-b*(a-
b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)
-a+2*b)*a)^(1/2))+2/d*b^4/(a-b)^4/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a
+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^
(1/2))-3/8/d*b^5/(a-b)^4/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)-a+2*b)*a
)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)-a+2*b)*a)^(1/2))+
2/d*b^4/(a-b)^4/a/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arc
tanh(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))-3/8/d*b^5/
(a-b)^4/a^2/(-b*(a-b))^(1/2)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a
*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))+2/d/(a-b)^4/(tan
h(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/2*c)^5*a-8/d/(a-b)^4/(tanh(1/2*d*x+1
/2*c)^2+1)^3*b*tanh(1/2*d*x+1/2*c)^5+4/3/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1
)^3*tanh(1/2*d*x+1/2*c)^3*a-40/3/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^3*b*ta
nh(1/2*d*x+1/2*c)^3+2/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^3*tanh(1/2*d*x+1/
2*c)*a-8/d/(a-b)^4/(tanh(1/2*d*x+1/2*c)^2+1)^3*b*tanh(1/2*d*x+1/2*c)-4/d*b^
3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x
+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)+4/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4
*a-2*tanh(1/2*d*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/
2*c)^3+4/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d*x+1/2*c)^2*a+4
*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^5-6/d*b^2/(a-b)^4/((2*(-b
*(a-b))^(1/2)-a+2*b)*a)^(1/2)*arctan(a*tanh(1/2*d*x+1/2*c)/((2*(-b*(a-b))^(
1/2)-a+2*b)*a)^(1/2))-4/d*b^3/(a-b)^4/(tanh(1/2*d*x+1/2*c)^4*a-2*tanh(1/2*d
*x+1/2*c)^2*a+4*tanh(1/2*d*x+1/2*c)^2*b+a)^2*tanh(1/2*d*x+1/2*c)^7+6/d*b^2/
(a-b)^4/((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2)*arctanh(a*tanh(1/2*d*x+1/2*c)/
((2*(-b*(a-b))^(1/2)+a-2*b)*a)^(1/2))
```

Maxima [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)**4/(a+b*sinh(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

Giac [B] time = 1.49803, size = 608, normalized size = 3.

$$\frac{\left(48\,a^2b^2-16\,ab^3+3\,b^4\right)\arctan\left(\frac{be^{(2\,dx+2\,c)}+2\,a-b}{2\,\sqrt{-a^2+ab}}\right)}{8\left(a^6d-4\,a^5bd+6\,a^4b^2d-4\,a^3b^3d+a^2b^4d\right)\sqrt{-a^2+ab}}+\frac{24\,a^2b^3e^{(6\,dx+6\,c)}-16\,ab^4e^{(6\,dx+6\,c)}+3\,b^5e^{(6\,dx+6\,c)}+112\,a^3b^2}{4\left(a^6d-4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)^4/(a+b*sinh(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(48*a^2*b^2 - 16*a*b^3 + 3*b^4)*arctan(1/2*(b*e^(2*d*x + 2*c) + 2*a - b
)/sqrt(-a^2 + a*b))/((a^6*d - 4*a^5*b*d + 6*a^4*b^2*d - 4*a^3*b^3*d + a^2*b
^4*d)*sqrt(-a^2 + a*b)) + 1/4*(24*a^2*b^3*e^(6*d*x + 6*c) - 16*a*b^4*e^(6*d
*x + 6*c) + 3*b^5*e^(6*d*x + 6*c) + 112*a^3*b^2*e^(4*d*x + 4*c) - 136*a^2*b
^3*e^(4*d*x + 4*c) + 66*a*b^4*e^(4*d*x + 4*c) - 9*b^5*e^(4*d*x + 4*c) + 88*
a^2*b^3*e^(2*d*x + 2*c) - 64*a*b^4*e^(2*d*x + 2*c) + 9*b^5*e^(2*d*x + 2*c)
+ 14*a*b^4 - 3*b^5)/((a^6*d - 4*a^5*b*d + 6*a^4*b^2*d - 4*a^3*b^3*d + a^2*b
^4*d)*(b*e^(4*d*x + 4*c) + 4*a*e^(2*d*x + 2*c) - 2*b*e^(2*d*x + 2*c) + b)^2
) + 2/3*(9*b*e^(4*d*x + 4*c) - 6*a*e^(2*d*x + 2*c) + 24*b*e^(2*d*x + 2*c) -
 2*a + 11*b)/((a^4*d - 4*a^3*b*d + 6*a^2*b^2*d - 4*a*b^3*d + b^4*d)*(e^(2*d
*x + 2*c) + 1)^3)
```

3.349 $\int \frac{\cosh^2(x)}{1-\sinh^2(x)} \, dx$

Optimal. Leaf size=19

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rubi [A] time = 0.0450643, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3191, 391, 206}

$$\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-x$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^2/(1 - Sinh[x]^2),x]
```

```
[Out] -x + Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\cosh^2(x)}{1-\sinh^2(x)} \, dx &= \text{Subst}\left(\int \frac{1}{\left(1-2x^2\right)\left(1-x^2\right)} \, dx, x, \tanh(x)\right)\\
&= 2\,\text{Subst}\left(\int \frac{1}{1-2x^2} \, dx, x, \tanh(x)\right)-\text{Subst}\left(\int \frac{1}{1-x^2} \, dx, x, \tanh(x)\right)\\
&= -x+\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.0756311, size = 24, normalized size = 1.26

$$-2\left(\frac{x}{2}-\frac{\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^2/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(x/2 - ArcTanh[Sqrt[2]*Tanh[x]]/Sqrt[2])
```

Maple [B] time = 0.023, size = 54, normalized size = 2.8

$$\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)+2)\right)-\ln\left(\tanh\left(\frac{x}{2}\right)+1\right)+\sqrt{2}\text{Artanh}\left(\frac{\sqrt{2}}{4}(2\tanh(x/2)-2)\right)+\ln\left(\tanh\left(\frac{x}{2}\right)-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^2/(1-sinh(x)^2),x)
```

```
[Out] 2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))-ln(tanh(1/2*x)+1)+2^(1/2)*ar
ctanh(1/4*(2*tanh(1/2*x)-2)*2^(1/2))+ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.70328, size = 86, normalized size = 4.53

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right)-\frac{1}{2}\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - 1/2*sqrt(
2)*log(-(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - x
```

Fricas [B] time = 1.56854, size = 220, normalized size = 11.58

$$\frac{1}{2}\sqrt{2}\log\left(-\frac{3\left(2\sqrt{2}-3\right)\cosh(x)^2-4\left(3\sqrt{2}-4\right)\cosh(x)\sinh(x)+3\left(2\sqrt{2}-3\right)\sinh(x)^2-2\sqrt{2}+3}{\cosh(x)^2+\sinh(x)^2-3}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*sqrt(2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) - 4)*cosh(x)*s
inh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^2 + sinh(x)^
2 - 3)) - x
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**2/(1-sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.17485, size = 55, normalized size = 2.89

$$-\frac{1}{2}\sqrt{2}\log\left(\frac{\left|-4\sqrt{2}+2e^{(2x)}-6\right|}{\left|4\sqrt{2}+2e^{(2x)}-6\right|}\right)-x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^2/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)/abs(4*sqrt(2) + 2*e^(2*x)
- 6)) - x
```

3.350 $\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=10

$$2\tanh^{-1}(\sinh(x))-\sinh(x)$$

```
[Out] 2*ArcTanh[Sinh[x]] - Sinh[x]
```

Rubi [A] time = 0.0389517, antiderivative size = 10, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 388, 206}

$$2\tanh^{-1}(\sinh(x))-\sinh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^3/(1 - Sinh[x]^2),x]
```

```
[Out] 2*ArcTanh[Sinh[x]] - Sinh[x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh^3(x)}{1-\sinh^2(x)}\,dx&=\text{Subst}\left(\int \frac{1+x^2}{1-x^2}\,dx,x,\sinh(x)\right)\\&=-\sinh(x)+2\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sinh(x)\right)\\&=2\tanh^{-1}(\sinh(x))-\sinh(x)\end{aligned}$$

Mathematica [A] time = 0.0095023, size = 14, normalized size = 1.4

$$-2\left(\frac{\sinh(x)}{2}-\tanh^{-1}(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^3/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(-ArcTanh[Sinh[x]] + Sinh[x]/2)
```

Maple [B] time = 0.028, size = 50, normalized size = 5.

$$-\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}+2\tanh(x/2)-1\right)+\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1}+\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^{2}-2\tanh(x/2)-1\right)+\left(\tanh\left(\frac{x}{2}\right)-1\right)^{-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^3/(1-sinh(x)^2),x)
```

```
[Out] -ln(tanh(1/2*x)^2+2*tanh(1/2*x)-1)+1/(tanh(1/2*x)+1)+ln(tanh(1/2*x)^2-2*tan
h(1/2*x)-1)+1/(tanh(1/2*x)-1)
```

Maxima [B] time = 1.10616, size = 53, normalized size = 5.3

$$\frac{1}{2}e^{(-x)}-\frac{1}{2}e^{x}-\log\left(2e^{(-x)}+e^{(-2x)}-1\right)+\log\left(-2e^{(-x)}+e^{(-2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*e^(-x) - 1/2*e^x - log(2*e^(-x) + e^(-2*x) - 1) + log(-2*e^(-x) + e^(-2
*x) - 1)
```

Fricas [B] time = 1.49507, size = 275, normalized size = 27.5

$$-\frac{\cosh(x)^{2}-2(\cosh(x)+\sinh(x))\log\left(\frac{2(\sinh(x)+1)}{\cosh(x)-\sinh(x)}\right)+2(\cosh(x)+\sinh(x))\log\left(\frac{2(\sinh(x)-1)}{\cosh(x)-\sinh(x)}\right)+2\cosh(x)\sinh(x}{2(\cosh(x)+\sinh(x))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(cosh(x)^2 - 2*(cosh(x) + sinh(x))*log(2*(sinh(x) + 1)/(cosh(x) - sinh
(x))) + 2*(cosh(x) + sinh(x))*log(2*(sinh(x) - 1)/(cosh(x) - sinh(x))) + 2*
cosh(x)*sinh(x) + sinh(x)^2 - 1)/(cosh(x) + sinh(x))
```

Sympy [B] time = 2.30906, size = 129, normalized size = 12.9

$$\frac{\log\left(\tanh^{2}\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)\tanh^{2}\left(\frac{x}{2}\right)}{\tanh^{2}\left(\frac{x}{2}\right)-1}-\frac{\log\left(\tanh^{2}\left(\frac{x}{2}\right)-2\tanh\left(\frac{x}{2}\right)-1\right)}{\tanh^{2}\left(\frac{x}{2}\right)-1}-\frac{\log\left(\tanh^{2}\left(\frac{x}{2}\right)+2\tanh\left(\frac{x}{2}\right)-1\right)\tanl}{\tanh^{2}\left(\frac{x}{2}\right)-1}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**3/(1-sinh(x)**2),x)
```

```
[Out] log(tanh(x/2)**2 - 2*tanh(x/2) - 1)*tanh(x/2)**2/(tanh(x/2)**2 - 1) - log(t
anh(x/2)**2 - 2*tanh(x/2) - 1)/(tanh(x/2)**2 - 1) - log(tanh(x/2)**2 + 2*ta
nh(x/2) - 1)*tanh(x/2)**2/(tanh(x/2)**2 - 1) + log(tanh(x/2)**2 + 2*tanh(x/
2) - 1)/(tanh(x/2)**2 - 1) + 2*tanh(x/2)/(tanh(x/2)**2 - 1)
```

Giac [B] time = 1.1708, size = 50, normalized size = 5.

$$\frac{1}{2}\,e^{(-x)} - \frac{1}{2}\,e^{x} + \log\left(\left|-e^{(-x)} + e^{x} + 2\right|\right) - \log\left(\left|-e^{(-x)} + e^{x} - 2\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^3/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*e^(-x) - 1/2*e^x + log(abs(-e^(-x) + e^x + 2)) - log(abs(-e^(-x) + e^x
 - 2))
```

3.351 $\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{5x}{2}+2\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-\frac{1}{2}\sinh(x)\cosh(x)$$

```
[Out] (-5*x)/2 + 2*Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] - (Cosh[x]*Sinh[x])/2
```

Rubi [A] time = 0.0589306, antiderivative size = 30, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 414, 522, 206}

$$-\frac{5x}{2}+2\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)-\frac{1}{2}\sinh(x)\cosh(x)$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[x]^4/(1 - Sinh[x]^2),x]
```

```
[Out] (-5*x)/2 + 2*Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] - (Cosh[x]*Sinh[x])/2
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^4(x)}{1-\sinh^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-2x^2)(1-x^2)^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}\cosh(x)\sinh(x) - \frac{1}{2}\text{Subst}\left(\int \frac{-3-2x^2}{(1-2x^2)(1-x^2)}\,dx, x, \tanh(x)\right)$$
$$= -\frac{1}{2}\cosh(x)\sinh(x) - \frac{5}{2}\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \tanh(x)\right) + 4\,\text{Subst}\left(\int \frac{1}{1-2x^2}\,dx, x, \tanh(x)\right)$$
$$= -\frac{5x}{2} + 2\sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right) - \frac{1}{2}\cosh(x)\sinh(x)$$

Mathematica [A] time = 0.0686895, size = 32, normalized size = 1.07

$$-2\left(\frac{5x}{4} + \frac{1}{8}\sinh(2x) - \sqrt{2}\tanh^{-1}\left(\sqrt{2}\tanh(x)\right)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[x]^4/(1 - Sinh[x]^2),x]
```

```
[Out] -2*((5*x)/4 - Sqrt[2]*ArcTanh[Sqrt[2]*Tanh[x]] + Sinh[2*x]/8)
```

Maple [B] time = 0.032, size = 98, normalized size = 3.3

$$2\sqrt{2}\text{Artanh}\left(1/4\ (2\ \tanh(x/2)+2)\sqrt{2}\right) + \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-2} - \frac{1}{2}\left(\tanh\left(\frac{x}{2}\right)+1\right)^{-1} - \frac{5}{2}\ln\left(\tanh\left(\frac{x}{2}\right)+1\right) + 2\sqrt{2}A$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(x)^4/(1-sinh(x)^2),x)
```

```
[Out] 2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)+2)*2^(1/2))+1/2/(tanh(1/2*x)+1)^2-1/2/
(tanh(1/2*x)+1)-5/2*ln(tanh(1/2*x)+1)+2*2^(1/2)*arctanh(1/4*(2*tanh(1/2*x)-
2)*2^(1/2))-1/2/(tanh(1/2*x)-1)^2-1/2/(tanh(1/2*x)-1)+5/2*ln(tanh(1/2*x)-1)
```

Maxima [B] time = 1.53231, size = 101, normalized size = 3.37

$$\sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}+1}{\sqrt{2}+e^{(-x)}-1}\right) - \sqrt{2}\log\left(-\frac{\sqrt{2}-e^{(-x)}-1}{\sqrt{2}+e^{(-x)}+1}\right) - \frac{5}{2}x - \frac{1}{8}e^{(2x)} + \frac{1}{8}e^{(-2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] sqrt(2)*log(-(sqrt(2) - e^(-x) + 1)/(sqrt(2) + e^(-x) - 1)) - sqrt(2)*log(-
(sqrt(2) - e^(-x) - 1)/(sqrt(2) + e^(-x) + 1)) - 5/2*x - 1/8*e^(2*x) + 1/8*
e^(-2*x)
```

Fricas [B] time = 1.51314, size = 552, normalized size = 18.4

$$-\frac{\cosh(x)^4 + 4\cosh(x)\sinh(x)^3 + \sinh(x)^4 + 20x\cosh(x)^2 + 2\left(3\cosh(x)^2 + 10x\right)\sinh(x)^2 - 8\left(\sqrt{2}\cosh(x)^2 + 2\sqrt{\cdots}\right.}{8\left(\cosh(x\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(cosh(x)^4 + 4*cosh(x)*sinh(x)^3 + sinh(x)^4 + 20*x*cosh(x)^2 + 2*(3*c
osh(x)^2 + 10*x)*sinh(x)^2 - 8*(sqrt(2)*cosh(x)^2 + 2*sqrt(2)*cosh(x)*sinh(
x) + sqrt(2)*sinh(x)^2)*log(-(3*(2*sqrt(2) - 3)*cosh(x)^2 - 4*(3*sqrt(2) -
4)*cosh(x)*sinh(x) + 3*(2*sqrt(2) - 3)*sinh(x)^2 - 2*sqrt(2) + 3)/(cosh(x)^
2 + sinh(x)^2 - 3)) + 4*(cosh(x)^3 + 10*x*cosh(x))*sinh(x) - 1)/(cosh(x)^2
+ 2*cosh(x)*sinh(x) + sinh(x)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)**4/(1-sinh(x)**2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.15401, size = 82, normalized size = 2.73

$$\frac{1}{8}\left(10e^{(2x)} + 1\right)e^{(-2x)} - \sqrt{2}\log\left(\frac{\left|-4\sqrt{2} + 2e^{(2x)} - 6\right|}{\left|4\sqrt{2} + 2e^{(2x)} - 6\right|}\right) - \frac{5}{2}x - \frac{1}{8}e^{(2x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(x)^4/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] 1/8*(10*e^(2*x) + 1)*e^(-2*x) - sqrt(2)*log(abs(-4*sqrt(2) + 2*e^(2*x) - 6)
/abs(4*sqrt(2) + 2*e^(2*x) - 6)) - 5/2*x - 1/8*e^(2*x)
```

3.352 $\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf}-\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf}$$

```
[Out] -(a*(a - 4*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(8*b^(3/2)*f) - ((a - 4*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*b
*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*b*f)
```

Rubi [A] time = 0.113507, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 388, 195, 217, 206}

$$-\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf}-\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] -(a*(a - 4*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(8*b^(3/2)*f) - ((a - 4*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*b
*f) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*b*f)
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)\sqrt{a+bx^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4bf} - \frac{(a-4b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sin\right.}{4bf}$$

$$= -\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf} + \frac{\sinh(e+fx)\left(a+b\sinh^2(e+\right.}{4bf}$$

$$= -\frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8bf} + \frac{\sinh(e+fx)\left(a+b\sinh^2(e+\right.}{4bf}$$

$$= -\frac{a(a-4b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8b^{3/2}f} - \frac{(a-4b)\sinh(e+fx)\sqrt{a+b\sinh^2(e-}}{8bf}$$

Mathematica [A] time = 0.663081, size = 124, normalized size = 1.06

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(\sqrt{b}\sinh(e+fx)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}(a+b\cosh(2(e+fx))+3b)-\sqrt{a}(a-4b)\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right.}{8b^{3/2}f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(-(Sqrt[a]*(a - 4*b)*ArcSinh[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]]) + Sqrt[b]*(a + 3*b + b*Cosh[2*(e + f*x)])*Sinh[e + f*x]*Sq
rt[1 + (b*Sinh[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
])
```

Maple [C] time = 0.09, size = 52, normalized size = 0.4

$$\frac{1}{f}\text{`int/indef0`}\left(\left(b\left(\cosh\left(fx+e\right)\right)^4+(a-b)\left(\cosh\left(fx+e\right)\right)^2\right)\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}}, \sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2
),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \cosh\left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^3, x)
```

Fricas [B] time = 2.45862, size = 8343, normalized size = 71.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(2*((a^2 - 4*a*b)*cosh(f*x + e)^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*
sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 -
4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(
b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3
)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 +
 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5
*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^
6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*
b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*
cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 1
4*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*
a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cos
h(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2
*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2
- 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x +
 e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b
+ b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2
- 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*
sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)
*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*
sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 -
 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e
))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b -
 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(
f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^
3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(
f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x
 + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e
)^5 + sinh(f*x + e)^6)) + 2*((a^2 - 4*a*b)*cosh(f*x + e)^4 + 4*(a^2 - 4*a*b
)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f*x + e)^2*sinh(f*x
+ e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 4*a*b)*sinh
(f*x + e)^4)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
```

```
e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a
)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b^
2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)
^6 + (2*a*b + 5*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 2*a*b + 5*
b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (2*a*b + 5*b^2)*cosh(f*x
+ e))*sinh(f*x + e)^3 - (2*a*b + 5*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x
+ e)^4 + 6*(2*a*b + 5*b^2)*cosh(f*x + e)^2 - 2*a*b - 5*b^2)*sinh(f*x + e)^2
 - b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(2*a*b + 5*b^2)*cosh(f*x + e)^3 - (2*
a*b + 5*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh
(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2*f*cosh(f*x + e)^3*sinh(f*x
 + e) + 6*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b^2*f*cosh(f*x + e)*sin
h(f*x + e)^3 + b^2*f*sinh(f*x + e)^4), 1/64*(4*((a^2 - 4*a*b)*cosh(f*x + e)
^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4*a*b)*cosh(f
*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x +
 e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b
)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*co
sh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*s
inh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x
+ e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x +
 e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 4*((a^2 - 4*a*b)*c
osh(f*x + e)^4 + 4*(a^2 - 4*a*b)*cosh(f*x + e)^3*sinh(f*x + e) + 6*(a^2 - 4
*a*b)*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*(a^2 - 4*a*b)*cosh(f*x + e)*sinh(
f*x + e)^3 + (a^2 - 4*a*b)*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*co
sh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)
^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)
^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b^2*cosh(f*x +
 e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (2*a*b
+ 5*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 2*a*b + 5*b^2)*sinh(f*
x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (2*a*b + 5*b^2)*cosh(f*x + e))*sinh(f
*x + e)^3 - (2*a*b + 5*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(
2*a*b + 5*b^2)*cosh(f*x + e)^2 - 2*a*b - 5*b^2)*sinh(f*x + e)^2 - b^2 + 2*(
3*b^2*cosh(f*x + e)^5 + 2*(2*a*b + 5*b^2)*cosh(f*x + e)^3 - (2*a*b + 5*b^2)
*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(b^2*f*cosh(f*x + e)^4 + 4*b^2*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b^
2*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^3
 + b^2*f*sinh(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\cosh\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^3, x)
```

3.353 $\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}f}$$

```
[Out] (a*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]
*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)
```

Rubi [A] time = 0.0555239, antiderivative size = 72, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 195, 217, 206}

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt[b]
*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x, \sinh(e+fx)\right)}{f}$$
$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx\right.}{2f}$$
$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx}}\right.}{2f}$$
$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Mathematica [A] time = 0.245125, size = 96, normalized size = 1.33

$$\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}+\sqrt{b}\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)}{2\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*S
inh[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt[a
 + b*Sinh[e + f*x]^2])
```

Maple [A] time = 0.013, size = 62, normalized size = 0.9

$$\frac{\sinh(fx+e)}{2f}\sqrt{a+b\left(\sinh(fx+e)\right)^2}+\frac{a}{2f}\ln\left(\sinh(fx+e)\sqrt{b}+\sqrt{a+b\left(\sinh(fx+e)\right)^2}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/f+1/2/f*a/b^(1/2)*ln(sinh(f*x+e)*
b^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(fx+e)^2+a}\cosh(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e), x)
```

Fricas [B] time = 2.09124, size = 6229, normalized size = 86.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*x + e
)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a
*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*
x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4
*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh
(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^
2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2
 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9
*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x +
 e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10
*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b
+ 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*
cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^
2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 +
3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)
^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*
a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b
 + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
- 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*si
nh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*
b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*
b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x +
e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^
3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*si
nh(f*x + e)^5 + sinh(f*x + e)^6)) + (a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*
sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cos
h(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3
*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh
(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b
*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*
f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e)*sinh(f*x + e) + b*f*sinh(f*x + e)^2
), -1/8*(2*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)*sinh(f*x + e) + a*sinh(f*
x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh
(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b
```

```
 - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*a*
b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2
)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2
)*cosh(f*x + e))*sinh(f*x + e))) + 2*(a*cosh(f*x + e)^2 + 2*a*cosh(f*x + e)
*sinh(f*x + e) + a*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
 e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(
3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*
a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*
b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^2 + 2*b*f*cosh(f*x + e)*si
nh(f*x + e) + b*f*sinh(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^{2}+a} \cosh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e), x)
```

3.354 $\int \operatorname{sech}(e + fx)\sqrt{a + b\sinh^2(e + fx)}\,dx$

Optimal. Leaf size=85

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/f
```

Rubi [A] time = 0.0920647, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 402, 217, 206, 377, 203}

$$\frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/f + (Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/f
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1+x^2}\,dx,x,\sinh(e+fx)\right)}{f}\\ &= \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx,x,\sinh(e+fx)\right)}{f}+\frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,\right.}{f}\\ &= \frac{(a-b)\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx,x,\frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{b\operatorname{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x\right.}{f}\\ &= \frac{\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}\end{aligned}$$

Mathematica [A] time = 0.710105, size = 130, normalized size = 1.53

$$\frac{\frac{\sqrt{a}\sqrt{b}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}{\sqrt{2a+b\cosh(2(e+fx))-b}}+\sqrt{a-b}\tan^{-1}\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a - b]*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2
*(e + f*x)]]] + (Sqrt[a]*Sqrt[b]*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]]*S
qrt[(2*a - b + b*Cosh[2*(e + f*x)])/a])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
)/f
```

Maple [C] time = 0.091, size = 51, normalized size = 0.6

$$\frac{1}{f}\,\text{`int/indef0`}\left(-\frac{-b\left(\sinh\left(fx+e\right)\right)^{2}-a}{\left(\cosh\left(fx+e\right)\right)^{2}}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(-(-b*sinh(f*x+e)^2-a)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),
sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \operatorname{sech}\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e), x)
```

Fricas [B] time = 2.644, size = 13256, normalized size = 155.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*
a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f
*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 +
9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 1
0*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
 + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b
 + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b
^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)
*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a
^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 +
 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e
)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*
b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*s
inh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2
*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a
*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x +
 e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)
^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*s
inh(f*x + e)^5 + sinh(f*x + e)^6)) + 2*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x
 + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x +
e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a
 + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh
```

```
(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x +
 e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x +
 e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cos
h(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + sq
rt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f
*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)
^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*c
osh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*c
osh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/f, 1/4*(4*sqrt(a - b)*arcta
n(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(
cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh
(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*
a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2
+ 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqr
t(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b
^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8
 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b +
 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e
)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*
a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3
)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b -
 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*s
inh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*c
osh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 +
 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^
2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^
3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x
 + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*
b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^
2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(
5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)
)*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^
2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2
)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2
 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b
 - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos
h(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*
b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sin
h(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f
*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x +
 e)^5 + sinh(f*x + e)^6)) + sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
 e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh
(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x +
e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
))/f, -1/2*(sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*co
sh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a
```

```
*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*
a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b
^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b
^2)*cosh(f*x + e))*sinh(f*x + e))) + sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)
^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x
+ e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2
*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(-a + b)*log(((a - 2*b)*co
sh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(
f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2
 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh
(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 +
 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)
))/f, 1/2*(2*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)
^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x +
 e))*sinh(f*x + e) + b)) - sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2
 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqr
t(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*
x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f
*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^
2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3
 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) - sqrt(-b)*arctan(sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sq
rt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4
+ 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(
f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)))/f]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2 \left(e + fx\right)} \operatorname{sech}\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx + e\right)^2 + a} \operatorname{sech}\left(fx + e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e), x)
```

3.355 $\int \mathbf{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=86

$$\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\tanh(e+fx)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

```
[Out] (a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt
[a - b]*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*f
)
```

Rubi [A] time = 0.0964859, antiderivative size = 86, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 378, 377, 203}

$$\frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f\sqrt{a-b}}+\frac{\tanh(e+fx)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(2*Sqrt
[a - b]*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(2*f
)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+b}}\right.}{2}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\right.}{2}$$

$$= \frac{a\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2\sqrt{a-b}f} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+}{2f}$$

Mathematica [B] time = 1.51378, size = 175, normalized size = 2.03

$$\frac{\sinh(e+fx)\left(\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\tanh^{-1}\left(\frac{\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}}{\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}\right)+\operatorname{sech}^2(e+fx)\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}(2a+b\cosh(2(e\right.}{4f\sqrt{-\frac{(a-b)\sinh^2(e+fx)}{a}}\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sinh[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[-(((a - b)*Sinh[e + f*x]^2)/a)]/Sqrt
[1 + (b*Sinh[e + f*x]^2)/a]]*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a] + (2*a
 - b + b*Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[-(((a - b)*Sinh[e + f*x]^2
)/a)]))/(4*f*Sqrt[-(((a - b)*Sinh[e + f*x]^2)/a)]*Sqrt[a + b*Sinh[e + f*x]^
2])
```

Maple [C] time = 0.085, size = 35, normalized size = 0.4

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{1}{(\cosh(fx+e))^4}\sqrt{a+b(\sinh(fx+e))^2}, \sinh(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(fx+e)^2+a}\,\operatorname{sech}(fx+e)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^3, x)
```

Fricas [B] time = 1.97148, size = 3534, normalized size = 41.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*((a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh(f*x
+ e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2
+ 4*(a*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt(-a + b)*l
og(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)
*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2
+ 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 -
(3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*co
sh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*s
inh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*si
nh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x
 + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e
)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^4 + 4*(a - b)*f*cosh
(f*x + e)*sinh(f*x + e)^3 + (a - b)*f*sinh(f*x + e)^4 + 2*(a - b)*f*cosh(f*
x + e)^2 + 2*(3*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^2 + (a
 - b)*f + 4*((a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x
+ e)), 1/2*((a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh
(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 + a)*sinh(f*x +
e)^2 + 4*(a*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt(a -
b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4
 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x
 + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)) + sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x +
e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)))/((a - b)*f*cosh(f*x + e)^4 + 4*(a - b)*f*cosh(f*x + e)*sinh(f*
x + e)^3 + (a - b)*f*sinh(f*x + e)^4 + 2*(a - b)*f*cosh(f*x + e)^2 + 2*(3*(
a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^2 + (a - b)*f + 4*((a -
 b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2\left(e + fx\right)} \operatorname{sech}^3\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**3, x)
```

Giac [B] time = 1.29754, size = 938, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(sqrt(a - b)*
f) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a - 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b - 5*(
sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*
b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(3/2)
- 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e)
 - 2*b*e^(2*f*x + 2*e) + b))*a^2 - (sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f
*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b + 2*(sqrt(b
)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2
*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 5*a*b^(3/2) - 2*b^(5/2))/(((sqrt(b)
*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*
f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^2*f)
```

3.356 $\int \mathbf{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=151

$$\frac{a(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\tanh(e+fx)\mathbf{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f(a-b)}+\frac{(3a-4b)\tanh(e+fx)\mathbf{sech}(}{8f(a}$$

```
[Out] (a*(3*a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/(8*(a - b)^(3/2)*f) + ((3*a - 4*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2]*Tanh[e + f*x])/(8*(a - b)*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]
^2)^(3/2)*Tanh[e + f*x])/(4*(a - b)*f)
```

Rubi [A] time = 0.143462, antiderivative size = 151, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 382, 378, 377, 203}

$$\frac{a(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f(a-b)^{3/2}}+\frac{\tanh(e+fx)\mathbf{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f(a-b)}+\frac{(3a-4b)\tanh(e+fx)\mathbf{sech}(}{8f(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (a*(3*a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/(8*(a - b)^(3/2)*f) + ((3*a - 4*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2]*Tanh[e + f*x])/(8*(a - b)*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]
^2)^(3/2)*Tanh[e + f*x])/(4*(a - b)*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1+x^2)^3}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{4(a-b)f} + \frac{(3a-4b)\operatorname{Subst}\left(\right.}{}$$

$$= \frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{8(a-b)f} + \frac{\operatorname{sech}^3(e+f}{}$$

$$= \frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{8(a-b)f} + \frac{\operatorname{sech}^3(e+f}{}$$

$$= \frac{a(3a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8(a-b)^{3/2}f} + \frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2}}{8(a-b)f}$$

Mathematica [C] time = 12.4229, size = 684, normalized size = 4.53

$$-\frac{\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)\left(10b\sinh^2(e+fx)\sqrt{\frac{(a-b)\tanh^2(e+fx)\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a^2}}+15a\sqrt{\frac{(a}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(-15*a*ArcSin[S
qrt[((a - b)*Tanh[e + f*x]^2)/a]] - 10*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]
^2)/a]]*Sinh[e + f*x]^2 - 30*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) - 20*b*Sinh[e + f*x]^2*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2)
 - 32*a*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sec
h[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2
) - 32*b*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e +
 f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e
+ f*x]^2)/a)^(5/2) + 32*a*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f*
x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e
 + f*x]^2)/a)^(7/2) + 32*b*Hypergeometric2F1[2, 4, 7/2, ((a - b)*Tanh[e + f
*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 15*a*Sqrt[((a - b)*Sech[e + f*x]^2*(
```

```
a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] + 10*b*Sinh[e + f*x]^2*Sqrt[((
a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2]))/(40*
f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)
)/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))
```

Maple [C] time = 0.104, size = 35, normalized size = 0.2

$$\frac{1}{f}`\ \mathrm{int/indef0}`\left(\frac{1}{\left(\cosh\left(fx+e\right)\right)^{6}}\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^6*(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\mathrm{sech}\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^5, x)
```

Fricas [B] time = 3.42903, size = 9495, normalized size = 62.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((3*a^2 - 4*a*b)*cosh(f*x + e)^8 + 8*(3*a^2 - 4*a*b)*cosh(f*x + e)*
sinh(f*x + e)^7 + (3*a^2 - 4*a*b)*sinh(f*x + e)^8 + 4*(3*a^2 - 4*a*b)*cosh(
f*x + e)^6 + 4*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x
 + e)^6 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x
 + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 2*(35*(3*a^2 -
 4*a*b)*cosh(f*x + e)^4 + 30*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 9*a^2 - 12*a
*b)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^5 + 10*(3*a^2 - 4*
a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 4
*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 4*(7*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 1
5*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 9*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a
^2 - 4*a*b)*sinh(f*x + e)^2 + 3*a^2 - 4*a*b + 8*((3*a^2 - 4*a*b)*cosh(f*x +
 e)^7 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)^5 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)
^3 + (3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2
*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)
*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x
+ e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
```

```
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x
+ e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b
)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e
)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x +
e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e
) + 1)) - 2*sqrt(2)*((3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6 + 6*(3*a^2 - 5
*a*b + 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b + 2*b^2)*sinh(
f*x + e)^6 + (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^4 + (15*(3*a^2 - 5*a*
b + 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 21*a*b + 10*b^2)*sinh(f*x + e)^4 + 4*
(5*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 21*a*b + 10*b^2)*cos
h(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^2 +
(15*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 21*a*b + 10*b^2)*
cosh(f*x + e)^2 - 11*a^2 + 21*a*b - 10*b^2)*sinh(f*x + e)^2 - 3*a^2 + 5*a*b
 - 2*b^2 + 2*(3*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 21*a*
b + 10*b^2)*cosh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b
 + b^2)*f*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x
+ e)^7 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^8 + 4*(a^2 - 2*a*b + b^2)*f*co
sh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b +
 b^2)*f)*sinh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 8*(7*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e
))*sinh(f*x + e)^5 + 2*(35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 30*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^2 + 3*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4
 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^5 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 9*(a^2 - 2*a*b + b^2)*
f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b +
 b^2)*f + 8*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 3*(a^2 - 2*a*b + b^2)*
f*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((3*a^2 - 4*a*b)*cosh(f*x + e)
^8 + 8*(3*a^2 - 4*a*b)*cosh(f*x + e)*sinh(f*x + e)^7 + (3*a^2 - 4*a*b)*sinh
(f*x + e)^8 + 4*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 4*(7*(3*a^2 - 4*a*b)*cosh
(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x + e)^6 + 8*(7*(3*a^2 - 4*a*b)*cosh(f*
x + e)^3 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(3*a^2 - 4*
a*b)*cosh(f*x + e)^4 + 2*(35*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 30*(3*a^2 -
4*a*b)*cosh(f*x + e)^2 + 9*a^2 - 12*a*b)*sinh(f*x + e)^4 + 8*(7*(3*a^2 - 4*
a*b)*cosh(f*x + e)^5 + 10*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + 3*(3*a^2 - 4*a*
b)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 4*(
7*(3*a^2 - 4*a*b)*cosh(f*x + e)^6 + 15*(3*a^2 - 4*a*b)*cosh(f*x + e)^4 + 9*
(3*a^2 - 4*a*b)*cosh(f*x + e)^2 + 3*a^2 - 4*a*b)*sinh(f*x + e)^2 + 3*a^2 -
4*a*b + 8*((3*a^2 - 4*a*b)*cosh(f*x + e)^7 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e
)^5 + 3*(3*a^2 - 4*a*b)*cosh(f*x + e)^3 + (3*a^2 - 4*a*b)*cosh(f*x + e))*si
nh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)
^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x +
 e))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^6
 + 6*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - 5*a*b
 + 2*b^2)*sinh(f*x + e)^6 + (11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^4 + (1
5*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 21*a*b + 10*b^2)*sinh(
f*x + e)^4 + 4*(5*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 21*a*
b + 10*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cos
h(f*x + e)^2 + (15*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 21
*a*b + 10*b^2)*cosh(f*x + e)^2 - 11*a^2 + 21*a*b - 10*b^2)*sinh(f*x + e)^2
```

```
- 3*a^2 + 5*a*b - 2*b^2 + 2*(3*(3*a^2 - 5*a*b + 2*b^2)*cosh(f*x + e)^5 + 2*
(11*a^2 - 21*a*b + 10*b^2)*cosh(f*x + e)^3 - (11*a^2 - 21*a*b + 10*b^2)*cos
h(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
)/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 8*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)*sinh(f*x + e)^7 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^8 + 4*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^6 + 4*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 +
 (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^4 + 8*(7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)
*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^2 - 2*a*b + b^2)*f*cosh(f*x +
e)^4 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 3*(a^2 - 2*a*b + b^2)*f)*
sinh(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 8*(7*(a^2 - 2*a
*b + b^2)*f*cosh(f*x + e)^5 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 9*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 +
 (a^2 - 2*a*b + b^2)*f + 8*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 3*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^5 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3
+ (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.41716, size = 3009, normalized size = 19.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*a^2 - 4*a*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
+ 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a -
 b))/((a*f - b*f)*sqrt(a - b)) - 1/2*(3*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e
^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^7*a^2 - 4*
(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2
*b*e^(2*f*x + 2*e) + b))^7*a*b + 21*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*
f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*a^2*sqrt(b)
- 60*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b))^6*a*b^(3/2) + 32*(sqrt(b)*e^(2*f*x + 2*e) - s
qrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^6*b
^(5/2) + 44*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*
x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a^3 - 253*(sqrt(b)*e^(2*f*x + 2*e) -
 sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5
*a^2*b + 252*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f
*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^5*a*b^2 - 64*(sqrt(b)*e^(2*f*x + 2*e)
 - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))
```

^5*b^3 - 292*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^3*sqrt(b) + 317*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a^2*b^(3/2) - 28*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*a*b^(5/2) - 32*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^4*b^(7/2) - 176*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^4 - 168*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^3*b + 737*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a^2*b^2 - 556*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a*b^3 + 128*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*b^4 - 528*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^4*sqrt(b) + 1176*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^3*b^(3/2) - 937*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a^2*b^(5/2) + 300*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*b^(7/2) - 32*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*b^(9/2) - 192*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^5 + 304*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^4*b + 124*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^3*b^2 - 487*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2*b^3 + 308*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a*b^4 - 64*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^5 - 192*a^5*sqrt(b) + 656*a^4*b^(3/2) - 884*a^3*b^(5/2) + 599*a^2*b^(7/2) - 212*a*b^(9/2) + 32*b^(11/2))/(((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*sqrt(b) + 4*a - 3*b)^4*(a*f - b*f))

3.357 $\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=301

$$-\frac{(a-9b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{15bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\left(2a^2-7ab-3b^2\right)\tanh(e+fx)\sqrt{a}}{15b^2f}$$

```
[Out] (-2*(a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*
b*f) + (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*b*f)
+ ((2*a^2 - 7*a*b - 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e
 + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) - ((a - 9*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/
a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2
*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a^2 - 7*a*b - 3*b^2)*Sqrt[a + b*Sinh[e
+ f*x]^2]*Tanh[e + f*x])/(15*b^2*f)
```

Rubi [A] time = 0.297575, antiderivative size = 301, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 416, 528, 531, 418, 492, 411}

$$-\frac{\left(2a^2-7ab-3b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15b^2f}+\frac{\left(2a^2-7ab-3b^2\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(s\right.}{15b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] (-2*(a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*
b*f) + (Cosh[e + f*x]*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(5*b*f)
+ ((2*a^2 - 7*a*b - 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e
 + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) - ((a - 9*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/
a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2
*(a + b*Sinh[e + f*x]^2))/a]) - ((2*a^2 - 7*a*b - 3*b^2)*Sqrt[a + b*Sinh[e
+ f*x]^2]*Tanh[e + f*x])/(15*b^2*f)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \cosh^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int\left(1+x^2\right)^{3/2}\sqrt{a+bx^2}\,dx,x,\sinh(e$$

$$f$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{5bf} + \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}\right.}{}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+fx)\,\text{s}}{}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+fx)\,\text{s}}{}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+fx)\,\text{s}}{}$$

$$= -\frac{2(a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf} + \frac{\cosh(e+fx)\,\text{s}}{}$$

Mathematica [C] time = 1.37619, size = 211, normalized size = 0.7

$$\frac{-32ia\left(a^2-4ab+3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(8a^2+4b(4a+3b)\cosh(2(e+\right.}{240b^2f\sqrt{2a+b\cosh(2(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((16*I)*a*(2*a^2 - 7*a*b - 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (32*I)*a*(a^2 - 4*a*b + 3*b^2)*Sqrt[(2*a - b +
 b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 + 3
2*a*b - 15*b^2 + 4*b*(4*a + 3*b)*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x)
])*Sinh[2*(e + f*x)])/(240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.106, size = 521, normalized size = 1.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+4*(-1/a*b)^(1/2)*a*b*c
osh(f*x+e)^4*sinh(f*x+e)+((-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-3*(-1/a*b
)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2
*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(co
```

```
sh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*
(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+7*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+3*(
b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f
*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^2 + a} \cosh \left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b \sinh \left(fx+e\right)^2 + a} \cosh \left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^2 + a} \cosh \left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4, x)
```

3.358 $\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=223

$$\frac{2\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}$$

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((a + b)*
EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e +
 f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*
Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b*f)
```

Rubi [A] time = 0.203341, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3192, 417, 531, 418, 492, 411}

$$\frac{(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf}+\frac{\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{2\operatorname{sech}(e+fx)\sqrt{a+b\sin}}{3f\sqrt{\frac{s}{-}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((a + b)*
EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e +
 f*x]^2])/(3*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*
Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \cosh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \sqrt{1+x^2}\sqrt{a+bx^2}\,dx, x, \sinh(e\cdots\right.}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\operatorname{sec}\cdots\right.}{\cdots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2a\sqrt{\cosh^2(e+fx)}\operatorname{se}\cdots\right.}{\cdots}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{2F\left(\tan^{-1}(\sinh(e+f\cdots\right.}{3f\sqrt{\cdots}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(a+b)E\left(\tan^{-1}(\sinh(\cdots\right.}{3\cdots}$$

Mathematica [C] time = 0.697644, size = 168, normalized size = 0.75

$$\frac{2i\sqrt{2}a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-2i\sqrt{2}a(a+\cdots}{6bf\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE
[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e +
f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])*S
inh[2*(e + f*x)])/(6*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.085, size = 339, normalized size = 1.5

$$\frac{1}{3f\cosh\left(fx+e\right)}\left(\sqrt{-\frac{b}{a}}b\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^{4}+\left(\sqrt{-\frac{b}{a}}a-\sqrt{-\frac{b}{a}}b\right)\left(\cosh\left(fx+e\right)\right)^{2}\sinh\left(fx+e\right)+a\sqrt{\frac{b\left(\cosh\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^
(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)
^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+(b/a*cosh(f*x+e)
^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2
),(a/b)^(1/2))*b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\cosh\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\cosh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^2 + a} \cosh \left(fx+e\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2, x)
```

3.359 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rubi [A] time = 0.036322, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\\ &= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0788632, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0., size = 140, normalized size = 2.3

$$\frac{1}{\cosh\left(fx+e\right)f}\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\left(a\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\mathrm{EllipticF}\left(\mathrm{si}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.360 $\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=70

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

```
[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rubi [A] time = 0.0844919, antiderivative size = 70, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3192, 411}

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\begin{aligned}\int \operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1+x^2\right)^{3/2}}\,dx,x,\sinh(e+fx)\right)}{f}\\ &= \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}\end{aligned}$$

Mathematica [C] time = 0.484483, size = 148, normalized size = 2.11

$$\frac{-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}\tanh(e+fx)(2a+b\cosh(2(e+fx))-b)+2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
] - (2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x),
b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*f*Sqrt[2*a
 - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.101, size = 177, normalized size = 2.5

$$\frac{1}{f\cosh(fx+e)}\left(\sqrt{-\frac{b}{a}}b\left(\sinh(fx+e)\right)^{3}+b\sqrt{\frac{a+b\left(\sinh(fx+e)\right)^{2}}{a}}\sqrt{\left(\cosh(fx+e)\right)^{2}}\text{EllipticF}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((-1/a*b)^(1/2)*b*sinh(f*x+e)^3+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*((a+b*sinh(f
*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2
),(a/b)^(1/2))+(-1/a*b)^(1/2)*a*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+
b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh(fx+e)^{2}+a}\,\text{sech}(fx+e)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sinh(fx+e)^{2}+a}\,\text{sech}(fx+e)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\,\text{sech}^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*sech(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx + e\right)^2 + a}\,\text{sech}\left(fx + e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2, x)
```

3.361 $\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=206

$$-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(}}{3f}$$

```
[Out] ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(3*f)
```

Rubi [A] time = 0.166938, antiderivative size = 206, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3192, 412, 525, 418, 411}

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])
/(3*f)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
```

```
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1+x^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sec}\right.}{}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f} + \frac{\left((2a-b)\sqrt{\cosh^2(e+\right.}{}$$

$$= \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 2.83669, size = 204, normalized size = 0.99

$$\frac{-16ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}\tanh(e+fx)\operatorname{sech}^2(e+fx)\left(\left(8a^2-4b^2\right)\cosh(2(e+fx)\right.}{24f(a-b)\sqrt{2a+b\cosh(2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((8*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] - (16*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] + Sqrt[2]*((8*a^2 - 4*b^2)*Cosh[2*(e + f*x)] + (2*a
 - b)*(8*a - 5*b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x])/(24
*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.132, size = 318, normalized size = 1.5

$$\frac{1}{3\left(\cosh\left(fx+e\right)\right)^{3}(a-b)f}\left(\left(2\sqrt{-\frac{b}{a}}ab-\sqrt{-\frac{b}{a}}b^{2}\right)\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^{4}+\left(2\sqrt{-\frac{b}{a}}a^{2}-2\sqrt{-\frac{b}{a}}ab\right)\left(\cosh\left(fx\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(2
*(-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)+((-1/a*
b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)+(cosh(f*x
+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))-2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+b*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2)/cosh(f*x+e)^3/(a-b)/(-1/
a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\mathrm{sech}\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\mathrm{sech}\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^2+a}\,\mathrm{sech}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^4, x)
```

3.362 $\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=157

$$-\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf}-\frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx\right.}{24bf}$$

```
[Out] -(a^2*(a - 6*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/(16*b^(3/2)*f) - (a*(a - 6*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(16*b*f) - ((a - 6*b)*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*b*f
) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(5/2))/(6*b*f)
```

Rubi [A] time = 0.128896, antiderivative size = 157, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 388, 195, 217, 206}

$$-\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf}-\frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx\right.}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(a^2*(a - 6*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]
])/(16*b^(3/2)*f) - (a*(a - 6*b)*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(16*b*f) - ((a - 6*b)*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*b*f
) + (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^{3/2}dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6bf} - \frac{(a-6b)\,\text{Subst}\left(\int\left(a+bx^2\right)^{3/2}\right.}{6bf}$$

$$= -\frac{(a-6b)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24bf} + \frac{\sinh(e+fx)\left(a+b\right.}{6bf}$$

$$= -\frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf} - \frac{(a-6b)\sinh(e+fx)}{2}$$

$$= -\frac{a(a-6b)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{16bf} - \frac{(a-6b)\sinh(e+fx)}{2}$$

$$= -\frac{a^2(a-6b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16b^{3/2}f} - \frac{a(a-6b)\sinh(e+fx)\sqrt{a+b}}{16bf}$$

Mathematica [A] time = 1.19984, size = 149, normalized size = 0.95

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(\sqrt{b}\sinh(e+fx)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\left(2b(7a+6b)\sinh^2(e+fx)+3a(a+10b)+8b^2\sinh^4(e+fx\right.\right.}{48b^{3/2}f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(-3*a^(3/2)*(a - 6*b)*ArcSinh[(Sqrt[b]*Sinh[e
+ f*x])/Sqrt[a]] + Sqrt[b]*Sinh[e + f*x]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a]*(3
*a*(a + 10*b) + 2*b*(7*a + 6*b)*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4)))/
(48*b^(3/2)*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Maple [C] time = 0.085, size = 77, normalized size = 0.5

$$\frac{1}{f}\text{` int/indef0`}\left(\left(b^2\left(\sinh\left(fx+e\right)\right)^6+\left(2ab+b^2\right)\left(\sinh\left(fx+e\right)\right)^4+\left(a^2+2ab\right)\left(\sinh\left(fx+e\right)\right)^2+a^2\right)\frac{}{\sqrt{a+b\left(\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)

[Out] `int/indef0`((b^2*sinh(f*x+e)^6+(2*a*b+b^2)*sinh(f*x+e)^4+(a^2+2*a*b)*sinh(f*x+e)^2+a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f

---

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \cosh \left(fx+e\right)^3\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^3, x)

Fricas [B] time = 3.63228, size = 11570, normalized size = 73.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")

[Out] [-1/384*(6*((a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2

```
 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))
/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*si
nh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sin
h(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*((a^
3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)^5*sinh(f*x +
 e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 - 6*a^2*
b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^2*sin
h(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 - 6*a
^2*b)*sinh(f*x + e)^6)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
+ e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh
(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) +
b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - s
qrt(2)*(b^3*cosh(f*x + e)^10 + 10*b^3*cosh(f*x + e)*sinh(f*x + e)^9 + b^3*s
inh(f*x + e)^10 + (7*a*b^2 + b^3)*cosh(f*x + e)^8 + (45*b^3*cosh(f*x + e)^2
 + 7*a*b^2 + b^3)*sinh(f*x + e)^8 + 8*(15*b^3*cosh(f*x + e)^3 + (7*a*b^2 +
b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x
 + e)^6 + (210*b^3*cosh(f*x + e)^4 + 6*a^2*b + 39*a*b^2 - 8*b^3 + 28*(7*a*b
^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 2
8*(7*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x
 + e))*sinh(f*x + e)^5 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + (21
0*b^3*cosh(f*x + e)^6 + 70*(7*a*b^2 + b^3)*cosh(f*x + e)^4 - 6*a^2*b - 39*a
*b^2 + 8*b^3 + 15*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*x +
e)^4 + 4*(30*b^3*cosh(f*x + e)^7 + 14*(7*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(
6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (6*a^2*b + 39*a*b^2 - 8*b^3)*
cosh(f*x + e))*sinh(f*x + e)^3 - b^3 - (7*a*b^2 + b^3)*cosh(f*x + e)^2 + (4
5*b^3*cosh(f*x + e)^8 + 28*(7*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b +
39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 - 7*a*b^2 - b^3 - 6*(6*a^2*b + 39*a*b^2 -
 8*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*
a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)
^5 - 2*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (7*a*b^2 + b^3)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
(b^2*f*cosh(f*x + e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*c
osh(f*x + e)^4*sinh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 +
 15*b^2*f*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x
+ e)^5 + b^2*f*sinh(f*x + e)^6), 1/384*(12*((a^3 - 6*a^2*b)*cosh(f*x + e)^6
 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cos
h(f*x + e)^4*sinh(f*x + e)^2 + 20*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x
+ e)^3 + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^
2*b)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(
-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(
f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*
x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh
(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)
^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e)
)*sinh(f*x + e))) + 12*((a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 6*(a^3 - 6*a^2*b)
*cosh(f*x + e)^5*sinh(f*x + e) + 15*(a^3 - 6*a^2*b)*cosh(f*x + e)^4*sinh(f*
x + e)^2 + 20*(a^3 - 6*a^2*b)*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*(a^3 - 6
*a^2*b)*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*(a^3 - 6*a^2*b)*cosh(f*x + e)*s
inh(f*x + e)^5 + (a^3 - 6*a^2*b)*sinh(f*x + e)^6)*sqrt(-b)*arctan(sqrt(2)*(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt
(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 +
4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*
```

```
x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*
x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*(b^3*cosh
(f*x + e)^10 + 10*b^3*cosh(f*x + e)*sinh(f*x + e)^9 + b^3*sinh(f*x + e)^10
+ (7*a*b^2 + b^3)*cosh(f*x + e)^8 + (45*b^3*cosh(f*x + e)^2 + 7*a*b^2 + b^3
)*sinh(f*x + e)^8 + 8*(15*b^3*cosh(f*x + e)^3 + (7*a*b^2 + b^3)*cosh(f*x +
e))*sinh(f*x + e)^7 + (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^6 + (210*b
^3*cosh(f*x + e)^4 + 6*a^2*b + 39*a*b^2 - 8*b^3 + 28*(7*a*b^2 + b^3)*cosh(f
*x + e)^2)*sinh(f*x + e)^6 + 2*(126*b^3*cosh(f*x + e)^5 + 28*(7*a*b^2 + b^3
)*cosh(f*x + e)^3 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e))*sinh(f*x
+ e)^5 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^4 + (210*b^3*cosh(f*x +
 e)^6 + 70*(7*a*b^2 + b^3)*cosh(f*x + e)^4 - 6*a^2*b - 39*a*b^2 + 8*b^3 + 1
5*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(30*b^3
*cosh(f*x + e)^7 + 14*(7*a*b^2 + b^3)*cosh(f*x + e)^5 + 5*(6*a^2*b + 39*a*b
^2 - 8*b^3)*cosh(f*x + e)^3 - (6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e))*s
inh(f*x + e)^3 - b^3 - (7*a*b^2 + b^3)*cosh(f*x + e)^2 + (45*b^3*cosh(f*x +
 e)^8 + 28*(7*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(6*a^2*b + 39*a*b^2 - 8*b^3
)*cosh(f*x + e)^4 - 7*a*b^2 - b^3 - 6*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x
 + e)^2)*sinh(f*x + e)^2 + 2*(5*b^3*cosh(f*x + e)^9 + 4*(7*a*b^2 + b^3)*cos
h(f*x + e)^7 + 3*(6*a^2*b + 39*a*b^2 - 8*b^3)*cosh(f*x + e)^5 - 2*(6*a^2*b
+ 39*a*b^2 - 8*b^3)*cosh(f*x + e)^3 - (7*a*b^2 + b^3)*cosh(f*x + e))*sinh(f
*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x
+ e)^6 + 6*b^2*f*cosh(f*x + e)^5*sinh(f*x + e) + 15*b^2*f*cosh(f*x + e)^4*s
inh(f*x + e)^2 + 20*b^2*f*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*b^2*f*cosh(f
*x + e)^2*sinh(f*x + e)^4 + 6*b^2*f*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*f*s
inh(f*x + e)^6)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^3, x)
```

3.363 $\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}$$

```
[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*Sqr
t[b]*f) + (3*a*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*f) + (Sinh[e +
 f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*f)
```

Rubi [A] time = 0.0687584, antiderivative size = 104, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f}+\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*Sqr
t[b]*f) + (3*a*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(8*f) + (Sinh[e +
 f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx,x,\sinh(e+fx)\right)}{f}$$
$$= \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f} + \frac{(3a)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,\text{sinh}\right.}{4f}$$
$$= \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right.}{4f}$$
$$= \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f} + \frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right.}{4f}$$
$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3a\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8f} + \text{si}$$

Mathematica [A] time = 0.485459, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}+5a\sinh(e+fx)+2b\sinh^3(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sinh[e + f*x]^2]*(5*a*Sinh[e + f*x] + 2*b*Sinh[e + f*x]^3 + (3*
a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Sinh
[e + f*x]^2)/a])))/(8*f)
```

Maple [A] time = 0.011, size = 90, normalized size = 0.9

$$\frac{\sinh\left(fx+e\right)}{4f}\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^{\frac{3}{2}}+\frac{3a\sinh\left(fx+e\right)}{8f}\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}+\frac{3a^2}{8f}\ln\left(\sinh\left(fx+e\right)\sqrt{b}+\sqrt{a+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2)/f+3/8*a*sinh(f*x+e)*(a+b*sinh(f*x
+e)^2)^(1/2)/f+3/8/f*a^2/b^(1/2)*ln(sinh(f*x+e)*b^(1/2)+(a+b*sinh(f*x+e)^2)
^(1/2))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e), x)
```

Fricas [B] time = 2.75815, size = 8070, normalized size = 77.6

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(6*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2
*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^
2*sinh(f*x + e)^4)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 +
8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2
+ b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^
6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*
x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 +
 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^
2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b
 - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*
b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(
a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a
^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 -
 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*
x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)
*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*
b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 +
 (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*co
sh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cos
h(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e
)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b
+ 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x
+ e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 +
 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*
cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*co
sh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 6*(a^2*cosh(f*x + e)^4 +
4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x + e)^2
 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(b)*log((
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2
)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*s
qrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e
)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*
cosh(f*x + e)*sinh(f*x + e)^5 + b^2*sinh(f*x + e)^6 + (10*a*b - 3*b^2)*cosh
(f*x + e)^4 + (15*b^2*cosh(f*x + e)^2 + 10*a*b - 3*b^2)*sinh(f*x + e)^4 + 4
```

```
*(5*b^2*cosh(f*x + e)^3 + (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 -
 (10*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*b^2*cosh(f*x + e)^4 + 6*(10*a*b - 3
*b^2)*cosh(f*x + e)^2 - 10*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*b^2*co
sh(f*x + e)^5 + 2*(10*a*b - 3*b^2)*cosh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(
f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/
(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*x + e)^3*sinh(f*x + e) + 6*b*f*cosh(f*x
 + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + b*f*sinh(f*
x + e)^4), -1/64*(12*(a^2*cosh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x
+ e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x
 + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x +
e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)
*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cos
h(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*si
nh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x +
 e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x +
e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 12*(a^2*cosh(f*x +
e)^4 + 4*a^2*cosh(f*x + e)^3*sinh(f*x + e) + 6*a^2*cosh(f*x + e)^2*sinh(f*x
 + e)^2 + 4*a^2*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*sinh(f*x + e)^4)*sqrt(-
b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b
*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x +
e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))
- sqrt(2)*(b^2*cosh(f*x + e)^6 + 6*b^2*cosh(f*x + e)*sinh(f*x + e)^5 + b^2*
sinh(f*x + e)^6 + (10*a*b - 3*b^2)*cosh(f*x + e)^4 + (15*b^2*cosh(f*x + e)^
2 + 10*a*b - 3*b^2)*sinh(f*x + e)^4 + 4*(5*b^2*cosh(f*x + e)^3 + (10*a*b -
3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(f*x + e)^2 +
(15*b^2*cosh(f*x + e)^4 + 6*(10*a*b - 3*b^2)*cosh(f*x + e)^2 - 10*a*b + 3*b
^2)*sinh(f*x + e)^2 - b^2 + 2*(3*b^2*cosh(f*x + e)^5 + 2*(10*a*b - 3*b^2)*c
osh(f*x + e)^3 - (10*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
 e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b*f*cosh(f*x + e)^4 + 4*b*f*cosh(f*
x + e)^3*sinh(f*x + e) + 6*b*f*cosh(f*x + e)^2*sinh(f*x + e)^2 + 4*b*f*cosh
(f*x + e)*sinh(f*x + e)^3 + b*f*sinh(f*x + e)^4)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e), x)
```

3.364 $\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=125

$$\frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}$$

```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Si
nh[e + f*x]^2]])/(2*f) + (b*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f
)
```

Rubi [A] time = 0.138583, antiderivative size = 125, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3190, 416, 523, 217, 206, 377, 203}

$$\frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a - b)^(3/2)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^
2]])/f + ((3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Si
nh[e + f*x]^2]])/(2*f) + (b*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f
)
```


Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{1+x^2}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$=\frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{\operatorname{Subst}\left(\int\frac{a(2a-b)+(3a-2b)bx^2}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx,x,$$

$$=\frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(a-b)^2\operatorname{Subst}\left(\int\frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,d$$

$$=\frac{b\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f}+\frac{(a-b)^2\operatorname{Subst}\left(\int\frac{1}{1-(-a+b)x^2}\,dx,$$

$$=\frac{(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{(3a-2b)\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+f}$$

Mathematica [A] time = 0.45291, size = 142, normalized size = 1.14

$$\frac{b\sinh(e+fx)\sqrt{4a+2b\cosh(2(e+fx))-2b}+4(a-b)^{3/2}\tan^{-1}\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)+2\sqrt{b}(3a-2b)\tanh^{-1}\left(\frac{\sqrt{2}}{\sqrt{2a+b}}\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*(a - b)^(3/2)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Co
sh[2*(e + f*x)]]] + 2*(3*a - 2*b)*Sqrt[b]*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e +
 f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + b*Sqrt[4*a - 2*b + 2*b*Cosh[2
*(e + f*x)]]*Sinh[e + f*x])/(4*f)
```

Maple [C] time = 0.085, size = 63, normalized size = 0.5

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{b^2\left(\sinh\left(fx+e\right)\right)^4+2\,ab\left(\sinh\left(fx+e\right)\right)^2+a^2}{\left(\cosh\left(fx+e\right)\right)^2}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^2/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e), x)
```

Fricas [B] time = 4.02219, size = 16424, normalized size = 131.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x
+ e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*c
osh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 +
(a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b
^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*
cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f
*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a
*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*
b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*
b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3
+ 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2
 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2
- sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh
(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 -
2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a
^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^
3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*
```

cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 4*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + ((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x + e)^2), 1/8*(8*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - ((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 - sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a

```
*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a
^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x
 + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*
cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3
*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 -
 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*
cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x
 + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e
)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)
^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) - ((3*a - 2*b)*cos
h(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh
(f*x + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a
)*sinh(f*x + e)^2 - sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + sqrt(2)*(b*
cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*
cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x + e)^2), -1/8*(2*((3*a - 2*b)*cosh
(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh(
f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*co
sh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a
*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*
a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b
^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b
^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*
a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(-b
)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*
cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2
*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e
)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) +
 4*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a -
b)*sinh(f*x + e)^2)*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*
b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b
)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x +
e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
 + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e)
+ a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(c
osh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x +
 e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x +
 e)*sinh(f*x + e) + f*sinh(f*x + e)^2), 1/8*(8*((a - b)*cosh(f*x + e)^2 + 2
*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2)*sqrt(a - b)
*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
```

```
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x +
 e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))
 - 2*((3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x +
e) + (3*a - 2*b)*sinh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x
 + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 +
 b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*
cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)
*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*
x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x
 + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) - 2*((3*a - 2*b)*c
osh(f*x + e)^2 + 2*(3*a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (3*a - 2*b)*si
nh(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
 b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))
*sinh(f*x + e) + b)) + sqrt(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(
f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2)))/(f*cosh(f*x + e)^2 + 2*f*cosh(f*x + e)*sinh(f*x + e) + f*sinh(f*x
+ e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e), x)
```

3.365 $\int \mathrm{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=133

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{a-b}(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+\frac{(a-b)\tanh(e+fx)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2}}{2f}$$

```
[Out] (Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e
 + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Si
nh[e + f*x]^2]])/f + ((a - b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(2*f)
```

Rubi [A] time = 0.146103, antiderivative size = 133, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3190, 413, 523, 217, 206, 377, 203}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}+\frac{\sqrt{a-b}(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+\frac{(a-b)\tanh(e+fx)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e
 + f*x]^2]])/(2*f) + (b^(3/2)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Si
nh[e + f*x]^2]])/f + ((a - b)*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(2*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx=\frac{\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^2}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$=\frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f}+\frac{\operatorname{Subst}\left(\int\frac{a}{(1\ldots}\right.}{\ldots}$$

$$=\frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f}+\frac{b^2\operatorname{Subst}\left(\int\ldots\right.}{\ldots}$$

$$=\frac{(a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2f}+\frac{b^2\operatorname{Subst}\left(\int\ldots\right.}{\ldots}$$

$$=\frac{\sqrt{a-b}(a+2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f}+\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}\ldots$$

Mathematica [A] time = 0.727985, size = 150, normalized size = 1.13

$$\frac{4b^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)+2\sqrt{a-b}(a+2b)\tan^{-1}\left(\frac{\sqrt{2a-2b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)+(a-b)\tanh(e+fx)\operatorname{sech}(e+fx)\ldots}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*Sqrt[a - b]*(a + 2*b)*ArcTan[(Sqrt[2*a - 2*b]*Sinh[e + f*x])/Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]]] + 4*b^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x
])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]] + (a - b)*Sqrt[4*a - 2*b + 2*b*Cosh
[2*(e + f*x)]]*Sech[e + f*x]*Tanh[e + f*x])/(4*f)
```

Maple [C] time = 0.091, size = 63, normalized size = 0.5

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{b^2\left(\sinh\left(fx+e\right)\right)^4+2\,ab\left(\sinh\left(fx+e\right)\right)^2+a^2}{\left(\cosh\left(fx+e\right)\right)^4}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^4/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^3, x)
```

Fricas [B] time = 4.64181, size = 19290, normalized size = 145.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
 e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 +
 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(b)*log(-((
a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x
 + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2
*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*
(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x +
 e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 +
6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)
^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*
a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)
)*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*
b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*c
osh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sin
h(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b +
b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a
```

```
*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x +
 e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x +
 e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b
^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 +
 b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5
 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*
x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) +
 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 +
15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh
(f*x + e)^6)) + ((a + 2*b)*cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*sinh
(f*x + e)^3 + (a + 2*b)*sinh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(
3*(a + 2*b)*cosh(f*x + e)^2 + a + 2*b)*sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(
f*x + e)^3 + (a + 2*b)*cosh(f*x + e))*sinh(f*x + e) + a + 2*b)*sqrt(-a + b)
*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3
 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*
b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt(
(b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh
(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3
- (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*
cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)
*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*
sinh(f*x + e) + 1)) + (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^
3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*s
inh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)
*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sin
h(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x +
 e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(
b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 +
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 2*sqrt(2)*((a - b)*cosh
(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)
^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x +
 e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 + 2*f*cosh(f*
x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)
^3 + f*cosh(f*x + e))*sinh(f*x + e) + f), 1/4*(2*((a + 2*b)*cosh(f*x + e)^4
 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*sinh(f*x + e)^4 +
2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e)^2 + a + 2*b)*sin
h(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*cosh(f*x + e))*sinh
(f*x + e) + a + 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(
f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(
f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*co
sh(f*x + e))*sinh(f*x + e) + b)) + (b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
+ e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(
f*x + e) + b)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^
2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3
)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2
*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e
)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a
^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b -
```

14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*c
osh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14
*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*
cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b
- 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*
b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e
)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh
(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b
^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*
(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^
2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*
x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x
+ e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a
*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^
3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*co
sh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(
f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*
x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + (b*cosh(f*x + e)^4 + 4*b*cosh(
f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b
*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*cosh(f*x +
e))*sinh(f*x + e) + b)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*
sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
+ e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) +
b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) +
2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e)
+ (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e
)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
+ e)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f
*x + e)^4 + 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)
^2 + 4*(f*cosh(f*x + e)^3 + f*cosh(f*x + e))*sinh(f*x + e) + f), -1/4*(2*(b
*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*co
sh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)
*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)
*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x +
e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(
a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a
*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e)))
+ 2*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e
)^4 + 2*b*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4
*(b*cosh(f*x + e)^3 + b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(s
qrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 +
1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x +
e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b
*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - ((a + 2*b
)*cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*s

```
inh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e)
^2 + a + 2*b)*sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*co
sh(f*x + e))*sinh(f*x + e) + a + 2*b)*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x
+ e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e
)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a
+ 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(
f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x +
e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x +
e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh
(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*s
qrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (
a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)))/(f*cosh(f*x + e)^4 + 4*f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x
+ e)^4 + 2*f*cosh(f*x + e)^2 + 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^2
+ 4*(f*cosh(f*x + e)^3 + f*cosh(f*x + e))*sinh(f*x + e) + f), 1/2*(((a + 2*
b)*cosh(f*x + e)^4 + 4*(a + 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a + 2*b)*
sinh(f*x + e)^4 + 2*(a + 2*b)*cosh(f*x + e)^2 + 2*(3*(a + 2*b)*cosh(f*x + e
)^2 + a + 2*b)*sinh(f*x + e)^2 + 4*((a + 2*b)*cosh(f*x + e)^3 + (a + 2*b)*c
osh(f*x + e))*sinh(f*x + e) + a + 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x
 + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - (b*cosh(f*x + e)^4 + 4
*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e)^2
+ 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + b*co
sh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x +
 e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b
)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*co
sh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*s
inh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x
+ e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x +
 e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) - (b*cosh(f*x + e)^4
 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*b*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 +
b*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^
2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
 e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(
3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*
a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((a - b)*cosh(f*x + e)^
2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^4 + 4*
f*cosh(f*x + e)*sinh(f*x + e)^3 + f*sinh(f*x + e)^4 + 2*f*cosh(f*x + e)^2 +
 2*(3*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^2 + 4*(f*cosh(f*x + e)^3 + f*cos
h(f*x + e))*sinh(f*x + e) + f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^3, x)
```

3.366 $\int \mathbf{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=126

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f\sqrt{a-b}}+\frac{\tanh(e+fx)\text{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tanh(e+fx)\text{sech}(e+fx)\sqrt{a\ldots}}{8f}$$

```
[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*
Sqrt[a - b]*f) + (3*a*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*
x])/(8*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(
4*f)
```

Rubi [A] time = 0.120855, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 378, 377, 203}

$$\frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8f\sqrt{a-b}}+\frac{\tanh(e+fx)\text{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tanh(e+fx)\text{sech}(e+fx)\sqrt{a\ldots}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(8*
Sqrt[a - b]*f) + (3*a*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*
x])/(8*f) + (Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(
4*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \operatorname{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\operatorname{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^3}dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{4f} + \frac{(3a)\operatorname{Subst}\left(\int \frac{\sqrt{a+}}{(1+}\right.}{}$$

$$= \frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{8f} + \frac{\operatorname{sech}^3(e+fx)\left(a+\right.}{}$$

$$= \frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{8f} + \frac{\operatorname{sech}^3(e+fx)\left(a+\right.}{}$$

$$= \frac{3a^2\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{8\sqrt{a-b}f} + \frac{3a\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh}{8f}$$

Mathematica [C] time = 0.108615, size = 66, normalized size = 0.52

$$\frac{a^2\sinh(e+fx)\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};-\frac{(a-b)\sinh^2(e+fx)}{b\sinh^2(e+fx)+a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Hypergeometric2F1[1/2, 3, 3/2, -(((a - b)*Sinh[e + f*x]^2)/(a + b*Sinh
[e + f*x]^2))]*Sinh[e + f*x])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] time = 0.107, size = 63, normalized size = 0.5

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{b^2\left(\sinh\left(fx+e\right)\right)^4+2ab\left(\sinh\left(fx+e\right)\right)^2+a^2}{\left(\cosh\left(fx+e\right)\right)^6}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^6/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^5, x)
```

Fricas [B] time = 3.80103, size = 7839, normalized size = 62.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(3*(a^2*cosh(f*x + e)^8 + 8*a^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*
sinh(f*x + e)^8 + 4*a^2*cosh(f*x + e)^6 + 4*(7*a^2*cosh(f*x + e)^2 + a^2)*s
inh(f*x + e)^6 + 6*a^2*cosh(f*x + e)^4 + 8*(7*a^2*cosh(f*x + e)^3 + 3*a^2*c
osh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a^2*cosh(f*x + e)^4 + 30*a^2*cosh(f*x
 + e)^2 + 3*a^2)*sinh(f*x + e)^4 + 4*a^2*cosh(f*x + e)^2 + 8*(7*a^2*cosh(f*
x + e)^5 + 10*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^3 +
4*(7*a^2*cosh(f*x + e)^6 + 15*a^2*cosh(f*x + e)^4 + 9*a^2*cosh(f*x + e)^2 +
 a^2)*sinh(f*x + e)^2 + a^2 + 8*(a^2*cosh(f*x + e)^7 + 3*a^2*cosh(f*x + e)^
5 + 3*a^2*cosh(f*x + e)^3 + a^2*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*
log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3
+ (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b
)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2
 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((
b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(
f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 -
 (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*c
osh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*
sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*s
inh(f*x + e) + 1)) - 2*sqrt(2)*((3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^6 + 6*(
3*a^2 - a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b - 2*b^2)*
sinh(f*x + e)^6 + (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^4 + (15*(3*a^2 -
a*b - 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 17*a*b + 6*b^2)*sinh(f*x + e)^4 + 4
*(5*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 17*a*b + 6*b^2)*cosh(
f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^2 + (15
*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 17*a*b + 6*b^2)*cosh(f
*x + e)^2 - 11*a^2 + 17*a*b - 6*b^2)*sinh(f*x + e)^2 - 3*a^2 + a*b + 2*b^2
+ 2*(3*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 17*a*b + 6*b^2)*
cosh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e))*sinh(f*x + e))*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^8
 + 8*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a - b)*f*sinh(f*x + e)^8 +
4*(a - b)*f*cosh(f*x + e)^6 + 4*(7*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*s
inh(f*x + e)^6 + 6*(a - b)*f*cosh(f*x + e)^4 + 8*(7*(a - b)*f*cosh(f*x + e)
^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a - b)*f*cosh(f*x
+ e)^4 + 30*(a - b)*f*cosh(f*x + e)^2 + 3*(a - b)*f)*sinh(f*x + e)^4 + 4*(a
 - b)*f*cosh(f*x + e)^2 + 8*(7*(a - b)*f*cosh(f*x + e)^5 + 10*(a - b)*f*cos
h(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a - b)*f*
cosh(f*x + e)^6 + 15*(a - b)*f*cosh(f*x + e)^4 + 9*(a - b)*f*cosh(f*x + e)^
```

```
2 + (a - b)*f)*sinh(f*x + e)^2 + (a - b)*f + 8*((a - b)*f*cosh(f*x + e)^7 +
 3*(a - b)*f*cosh(f*x + e)^5 + 3*(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh
(f*x + e))*sinh(f*x + e)), 1/8*(3*(a^2*cosh(f*x + e)^8 + 8*a^2*cosh(f*x + e
)*sinh(f*x + e)^7 + a^2*sinh(f*x + e)^8 + 4*a^2*cosh(f*x + e)^6 + 4*(7*a^2*
cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^6 + 6*a^2*cosh(f*x + e)^4 + 8*(7*a^2*c
osh(f*x + e)^3 + 3*a^2*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a^2*cosh(f*x
+ e)^4 + 30*a^2*cosh(f*x + e)^2 + 3*a^2)*sinh(f*x + e)^4 + 4*a^2*cosh(f*x +
 e)^2 + 8*(7*a^2*cosh(f*x + e)^5 + 10*a^2*cosh(f*x + e)^3 + 3*a^2*cosh(f*x
+ e))*sinh(f*x + e)^3 + 4*(7*a^2*cosh(f*x + e)^6 + 15*a^2*cosh(f*x + e)^4 +
 9*a^2*cosh(f*x + e)^2 + a^2)*sinh(f*x + e)^2 + a^2 + 8*(a^2*cosh(f*x + e)^
7 + 3*a^2*cosh(f*x + e)^5 + 3*a^2*cosh(f*x + e)^3 + a^2*cosh(f*x + e))*sinh
(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*si
nh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2
 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e
))*sinh(f*x + e) + b)) + sqrt(2)*((3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^6 + 6
*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (3*a^2 - a*b - 2*b^2
)*sinh(f*x + e)^6 + (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^4 + (15*(3*a^2
- a*b - 2*b^2)*cosh(f*x + e)^2 + 11*a^2 - 17*a*b + 6*b^2)*sinh(f*x + e)^4 +
 4*(5*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^3 + (11*a^2 - 17*a*b + 6*b^2)*cos
h(f*x + e))*sinh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e)^2 + (
15*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^4 + 6*(11*a^2 - 17*a*b + 6*b^2)*cosh
(f*x + e)^2 - 11*a^2 + 17*a*b - 6*b^2)*sinh(f*x + e)^2 - 3*a^2 + a*b + 2*b^
2 + 2*(3*(3*a^2 - a*b - 2*b^2)*cosh(f*x + e)^5 + 2*(11*a^2 - 17*a*b + 6*b^2
)*cosh(f*x + e)^3 - (11*a^2 - 17*a*b + 6*b^2)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)
^8 + 8*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a - b)*f*sinh(f*x + e)^8
+ 4*(a - b)*f*cosh(f*x + e)^6 + 4*(7*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)
*sinh(f*x + e)^6 + 6*(a - b)*f*cosh(f*x + e)^4 + 8*(7*(a - b)*f*cosh(f*x +
e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a - b)*f*cosh(f*
x + e)^4 + 30*(a - b)*f*cosh(f*x + e)^2 + 3*(a - b)*f)*sinh(f*x + e)^4 + 4*
(a - b)*f*cosh(f*x + e)^2 + 8*(7*(a - b)*f*cosh(f*x + e)^5 + 10*(a - b)*f*c
osh(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a - b)*
f*cosh(f*x + e)^6 + 15*(a - b)*f*cosh(f*x + e)^4 + 9*(a - b)*f*cosh(f*x + e
)^2 + (a - b)*f)*sinh(f*x + e)^2 + (a - b)*f + 8*((a - b)*f*cosh(f*x + e)^7
 + 3*(a - b)*f*cosh(f*x + e)^5 + 3*(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*co
sh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^5, x)
```

3.367 $\int \textbf{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=205

$$\frac{a^2(5a-6b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16f(a-b)^{3/2}}+\frac{\tanh(e+fx)\text{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6f(a-b)}+\frac{(5a-6b)\tanh(e+fx)\text{sech}}{24}$$

```
[Out] (a^2*(5*a - 6*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x
]^2]])/(16*(a - b)^(3/2)*f) + (a*(5*a - 6*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2]*Tanh[e + f*x])/(16*(a - b)*f) + ((5*a - 6*b)*Sech[e + f*x]^3*(a
 + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(24*(a - b)*f) + (Sech[e + f*x]^
5*(a + b*Sinh[e + f*x]^2)^(5/2)*Tanh[e + f*x])/(6*(a - b)*f)
```

Rubi [A] time = 0.172294, antiderivative size = 205, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 382, 378, 377, 203}

$$\frac{a^2(5a-6b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16f(a-b)^{3/2}}+\frac{\tanh(e+fx)\text{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{6f(a-b)}+\frac{(5a-6b)\tanh(e+fx)\text{sech}}{24}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*(5*a - 6*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x
]^2]])/(16*(a - b)^(3/2)*f) + (a*(5*a - 6*b)*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2]*Tanh[e + f*x])/(16*(a - b)*f) + ((5*a - 6*b)*Sech[e + f*x]^3*(a
 + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/(24*(a - b)*f) + (Sech[e + f*x]^
5*(a + b*Sinh[e + f*x]^2)^(5/2)*Tanh[e + f*x])/(6*(a - b)*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \text{sech}^7(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^4}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\text{sech}^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}\tanh(e+fx)}{6(a-b)f} + \frac{(5a-6b)\,\text{Subs}}{}$$

$$= \frac{(5a-6b)\text{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{24(a-b)f} + \text{sech}^5$$

$$= \frac{a(5a-6b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{16(a-b)f} + \frac{(5a-6b}{}$$

$$= \frac{a(5a-6b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{16(a-b)f} + \frac{(5a-6b}{}$$

$$= \frac{a^2(5a-6b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{16(a-b)^{3/2}f} + \frac{a(5a-6b)\text{sech}(e+fx)\sqrt{a+b\,\text{s}}}{16(a-b}$$

Mathematica [C] time = 15.325, size = 959, normalized size = 4.68

$$\frac{a^2\text{sech}^3(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^2\tanh(e+fx)\left(256b\,{}_2F_1\left(2,5;\frac{7}{2};\frac{(a-b)\tanh^2(e+fx)}{a}\right)\sinh^2(e+fx)\sqrt{\frac{\text{sech}^2(e+fx)\left(b\sinh^2(e\right.}{a}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^7*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)^2*Tanh[e + f*x]*(45*a*ArcS
in[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e +
f*x]^2)/a]]*Sinh[e + f*x]^2 + 210*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + 140*b*Sinh[e + f*x]^2*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)
^(3/2) - 120*a*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*
Tanh[e + f*x]^2)/a)^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tan
h[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b
```

```
)*Tanh[e + f*x]^2)/a)^(5/2) - 80*b*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) + 256*b*Hyper
geometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(
5/2) - 512*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)
^(7/2) - 512*b*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Si
nh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*T
anh[e + f*x]^2)/a)^(7/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Tanh
[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)
*Tanh[e + f*x]^2)/a)^(9/2) + 256*b*Hypergeometric2F1[2, 5, 7/2, ((a - b)*Ta
nh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(9/2) - 45*a*Sqrt[((a - b)*Sech[e +
f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2] - 30*b*Sinh[e + f*x]^2
*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2
]))/(240*f*(a + b*Sinh[e + f*x]^2)^(3/2)*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[
e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))
```

Maple [C] time = 0.154, size = 63, normalized size = 0.3

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{b^2\left(\sinh\left(fx+e\right)\right)^4+2\,ab\left(\sinh\left(fx+e\right)\right)^2+a^2}{\left(\cosh\left(fx+e\right)\right)^8}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f*x+e)^8/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^7, x)
```

Fricas [B] time = 10.1145, size = 19302, normalized size = 94.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/96*(3*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^12 + 12*(5*a^3 - 6*a^2*b)*cosh(f
*x + e)*sinh(f*x + e)^11 + (5*a^3 - 6*a^2*b)*sinh(f*x + e)^12 + 6*(5*a^3 -
```

6*a^2*b)*cosh(f*x + e)^10 + 6*(5*a^3 - 6*a^2*b + 11*(5*a^3 - 6*a^2*b)*cosh(
f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3
*(5*a^3 - 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 6*a^2*b)*co
sh(f*x + e)^8 + 15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b
+ 18*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 6
*a^2*b)*cosh(f*x + e)^5 + 30*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 -
6*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^7 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x +
e)^6 + 4*(231*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 315*(5*a^3 - 6*a^2*b)*cos
h(f*x + e)^4 + 25*a^3 - 30*a^2*b + 105*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*s
inh(f*x + e)^6 + 24*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 63*(5*a^3 - 6*a
^2*b)*cosh(f*x + e)^5 + 35*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 - 6
*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 6*a^2*b)*cosh(f*x + e)
^4 + 15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^8 + 84*(5*a^3 - 6*a^2*b)*cosh(f
x + e)^6 + 70(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 20*(5*
a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 20*(11*(5*a^3 - 6*a^2*b)*
cosh(f*x + e)^9 + 36*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 42*(5*a^3 - 6*a^2*
b)*cosh(f*x + e)^5 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3*(5*a^3 - 6*a^
2*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 5*a^3 - 6*a^2*b + 6*(5*a^3 - 6*a^2*b)
*cosh(f*x + e)^2 + 6*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^10 + 45*(5*a^3 - 6
*a^2*b)*cosh(f*x + e)^8 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 50*(5*a^3
- 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 15*(5*a^3 - 6*a^2*b)*cosh(f*
x + e)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^11 + 5*(5*a
^3 - 6*a^2*b)*cosh(f*x + e)^9 + 10*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 10*(
5*a^3 - 6*a^2*b)*cosh(f*x + e)^5 + 5*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + (5
*a^3 - 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2*b)*c
osh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh
(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^
2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cos
h(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sin
h(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2
+ 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1
)) - 2*sqrt(2)*((15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^10 + 10
*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*
a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*sinh(f*x + e)^10 + (85*a^3 - 133*a^2*b +
20*a*b^2 + 28*b^3)*cosh(f*x + e)^8 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^
3 + 45*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)
^8 + 8*(15*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^3 + (85*a^3
- 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(99*a^3
- 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 23*a^
2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^4 + 99*a^3 - 247*a^2*b + 200*a*b^2 - 5
2*b^3 + 14*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^2)*sinh(f
x + e)^6 + 4(63*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^5 + 1
4*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^3 + 3*(99*a^3 - 24
7*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(99*a^3 -
247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 23*a^2*b
+ 4*a*b^2 + 4*b^3)*cosh(f*x + e)^6 + 35*(85*a^3 - 133*a^2*b + 20*a*b^2 + 2
8*b^3)*cosh(f*x + e)^4 - 99*a^3 + 247*a^2*b - 200*a*b^2 + 52*b^3 + 15*(99*a
^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(
15*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^7 + 7*(85*a^3 - 133*
a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^5 + 5*(99*a^3 - 247*a^2*b + 200*a*
b^2 - 52*b^3)*cosh(f*x + e)^3 - (99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*c
osh(f*x + e))*sinh(f*x + e)^3 - 15*a^3 + 23*a^2*b - 4*a*b^2 - 4*b^3 - (85*a
^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^2 + (45*(15*a^3 - 23*a^2*
b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^8 + 28*(85*a^3 - 133*a^2*b + 20*a*b^2 +
28*b^3)*cosh(f*x + e)^6 + 30*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh
(f*x + e)^4 - 85*a^3 + 133*a^2*b - 20*a*b^2 - 28*b^3 - 12*(99*a^3 - 247*a^2

*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 -
23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^9 + 4*(85*a^3 - 133*a^2*b + 20*a*
b^2 + 28*b^3)*cosh(f*x + e)^7 + 6*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)
*cosh(f*x + e)^5 - 4*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e
)^3 - (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e))*sinh(f*x + e)
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*
cosh(f*x + e)^12 + 12*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^11
+ (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^12 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^10 + 6*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2
)*f)*sinh(f*x + e)^10 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 20*(11*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e
))*sinh(f*x + e)^9 + 15*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 18*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^8
+ 20*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 24*(33*(a^2 - 2*a*b + b^2)*f*c
osh(f*x + e)^5 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 5*(a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231*(a^2 - 2*a*b + b^2)*f*cosh
(f*x + e)^6 + 315*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 105*(a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e)^2 + 5*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^6 + 15*(a
^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 24*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^7 + 63*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 35*(a^2 - 2*a*b + b^2)
*f*cosh(f*x + e)^3 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5
+ 15*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 84*(a^2 - 2*a*b + b^2)*f*
cosh(f*x + e)^6 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 20*(a^2 - 2*a*
b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4 + 6*(a^
2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 20*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^9 + 36*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 42*(a^2 - 2*a*b + b^2)*
f*cosh(f*x + e)^5 + 20*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a
*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 6*(11*(a^2 - 2*a*b + b^2)*f*co
sh(f*x + e)^10 + 45*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 70*(a^2 - 2*a*b
+ b^2)*f*cosh(f*x + e)^6 + 50*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(
a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)
^2 + (a^2 - 2*a*b + b^2)*f + 12*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^11 + 5
*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^7 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f
*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/4
8*(3*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^12 + 12*(5*a^3 - 6*a^2*b)*cosh(f*x +
e)*sinh(f*x + e)^11 + (5*a^3 - 6*a^2*b)*sinh(f*x + e)^12 + 6*(5*a^3 - 6*a^2
*b)*cosh(f*x + e)^10 + 6*(5*a^3 - 6*a^2*b + 11*(5*a^3 - 6*a^2*b)*cosh(f*x +
e)^2)*sinh(f*x + e)^10 + 20*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3*(5*a
^3 - 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e)^9 + 15*(5*a^3 - 6*a^2*b)*cosh(f*
x + e)^8 + 15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 18*
(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 24*(33*(5*a^3 - 6*a^2*
b)*cosh(f*x + e)^5 + 30*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 - 6*a^
2*b)*cosh(f*x + e))*sinh(f*x + e)^7 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6
+ 4*(231*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 315*(5*a^3 - 6*a^2*b)*cosh(f*x
+ e)^4 + 25*a^3 - 30*a^2*b + 105*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^2)*sinh(f
x + e)^6 + 24(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 63*(5*a^3 - 6*a^2*b)
*cosh(f*x + e)^5 + 35*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 5*(5*a^3 - 6*a^2*
b)*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 +
15*(33*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^8 + 84*(5*a^3 - 6*a^2*b)*cosh(f*x +
e)^6 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 20*(5*a^3 -
6*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 20*(11*(5*a^3 - 6*a^2*b)*cosh(
f*x + e)^9 + 36*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 42*(5*a^3 - 6*a^2*b)*co
sh(f*x + e)^5 + 20*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + 3*(5*a^3 - 6*a^2*b)*
cosh(f*x + e))*sinh(f*x + e)^3 + 5*a^3 - 6*a^2*b + 6*(5*a^3 - 6*a^2*b)*cosh
(f*x + e)^2 + 6*(11*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^10 + 45*(5*a^3 - 6*a^2*
b)*cosh(f*x + e)^8 + 70*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^6 + 50*(5*a^3 - 6*a
^2*b)*cosh(f*x + e)^4 + 5*a^3 - 6*a^2*b + 15*(5*a^3 - 6*a^2*b)*cosh(f*x + e

)^2)*sinh(f*x + e)^2 + 12*((5*a^3 - 6*a^2*b)*cosh(f*x + e)^11 + 5*(5*a^3 -
6*a^2*b)*cosh(f*x + e)^9 + 10*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^7 + 10*(5*a^3
 - 6*a^2*b)*cosh(f*x + e)^5 + 5*(5*a^3 - 6*a^2*b)*cosh(f*x + e)^3 + (5*a^3
- 6*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f
*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b
*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x +
 e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
 e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((15*a^3 - 2
3*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^10 + 10*(15*a^3 - 23*a^2*b + 4*a*b
^2 + 4*b^3)*cosh(f*x + e)*sinh(f*x + e)^9 + (15*a^3 - 23*a^2*b + 4*a*b^2 +
4*b^3)*sinh(f*x + e)^10 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x
 + e)^8 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3 + 45*(15*a^3 - 23*a^2*b +
 4*a*b^2 + 4*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(15*(15*a^3 - 23*a^2
*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^3 + (85*a^3 - 133*a^2*b + 20*a*b^2 + 28
*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 2*(99*a^3 - 247*a^2*b + 200*a*b^2 -
52*b^3)*cosh(f*x + e)^6 + 2*(105*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh
(f*x + e)^4 + 99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3 + 14*(85*a^3 - 133*a^
2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(63*(15*a^3 -
 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*x + e)^5 + 14*(85*a^3 - 133*a^2*b + 20*
a*b^2 + 28*b^3)*cosh(f*x + e)^3 + 3*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^
3)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*
b^3)*cosh(f*x + e)^4 + 2*(105*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f*
x + e)^6 + 35*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^4 - 99
*a^3 + 247*a^2*b - 200*a*b^2 + 52*b^3 + 15*(99*a^3 - 247*a^2*b + 200*a*b^2
- 52*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(15*(15*a^3 - 23*a^2*b + 4*a
*b^2 + 4*b^3)*cosh(f*x + e)^7 + 7*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*
cosh(f*x + e)^5 + 5*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)
^3 - (99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e))*sinh(f*x + e)
^3 - 15*a^3 + 23*a^2*b - 4*a*b^2 - 4*b^3 - (85*a^3 - 133*a^2*b + 20*a*b^2 +
 28*b^3)*cosh(f*x + e)^2 + (45*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)*cosh(f
x + e)^8 + 28(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)^6 + 3
0*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^4 - 85*a^3 + 133*
a^2*b - 20*a*b^2 - 28*b^3 - 12*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*co
sh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(5*(15*a^3 - 23*a^2*b + 4*a*b^2 + 4*b^3)
*cosh(f*x + e)^9 + 4*(85*a^3 - 133*a^2*b + 20*a*b^2 + 28*b^3)*cosh(f*x + e)
^7 + 6*(99*a^3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^5 - 4*(99*a^
3 - 247*a^2*b + 200*a*b^2 - 52*b^3)*cosh(f*x + e)^3 - (85*a^3 - 133*a^2*b +
 20*a*b^2 + 28*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^12 + 12*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^2 - 2*a*b + b^2)*f*sin
h(f*x + e)^12 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^10 + 6*(11*(a^2 - 2*a
*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^10 + 15*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 20*(11*(a^2 - 2*a*b + b^2)*f*cosh(f
x + e)^3 + 3(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^9 + 15*(33
*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 18*(a^2 - 2*a*b + b^2)*f*cosh(f*x
+ e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^8 + 20*(a^2 - 2*a*b + b^2)*f*
cosh(f*x + e)^6 + 24*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 30*(a^2 -
2*a*b + b^2)*f*cosh(f*x + e)^3 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sin
h(f*x + e)^7 + 4*(231*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 315*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^4 + 105*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 +
5*(a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^6 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*
x + e)^4 + 24*(33*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 63*(a^2 - 2*a*b +
 b^2)*f*cosh(f*x + e)^5 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 5*(a^2
 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 15*(33*(a^2 - 2*a*b + b^
2)*f*cosh(f*x + e)^8 + 84*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 70*(a^2 -
 2*a*b + b^2)*f*cosh(f*x + e)^4 + 20*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2

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+ (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^4 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^2 + 20*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 36*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^7 + 42*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 20*(a^2
 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*
sinh(f*x + e)^3 + 6*(11*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^10 + 45*(a^2 -
2*a*b + b^2)*f*cosh(f*x + e)^8 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 +
 50*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f
*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f
+ 12*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^11 + 5*(a^2 - 2*a*b + b^2)*f*cosh
(f*x + e)^9 + 10*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 10*(a^2 - 2*a*b +
b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2
*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**7*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^7*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^7, x)
```

3.368 $\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=357

$$-\frac{\left(a^2-18ab+b^2\right)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{35bf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a+b)\left(a^2-6ab+b^2\right.}{}$$

```
[Out] ((a^2 + 9*a*b - 2*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]
^2])/(35*b*f) + (2*(4*a - b)*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]^5*Sinh[e + f*x]*Sqrt[a + b*Sinh[e +
f*x]^2])/(7*f) + (2*(a + b)*(a^2 - 6*a*b + b^2)*EllipticE[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 18*a*b + b^2)*Ellipti
cF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*
(a^2 - 6*a*b + b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(35*b^2*f)
```

Rubi [A] time = 0.392957, antiderivative size = 357, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 416, 528, 531, 418, 492, 411}

$$-\frac{2(a+b)\left(a^2-6ab+b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35b^2f}+\frac{\left(a^2+9ab-2b^2\right)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sin}}{35bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] ((a^2 + 9*a*b - 2*b^2)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]
^2])/(35*b*f) + (2*(4*a - b)*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(35*f) + (b*Cosh[e + f*x]^5*Sinh[e + f*x]*Sqrt[a + b*Sinh[e +
f*x]^2])/(7*f) + (2*(a + b)*(a^2 - 6*a*b + b^2)*EllipticE[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(35*b^2*f*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a^2 - 18*a*b + b^2)*Ellipti
cF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2
])/(35*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(a + b)*
(a^2 - 6*a*b + b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(35*b^2*f)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}\,dx,x\right.}{f}$$

$$= \frac{b\cosh^5(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{7f} + \frac{\left(\sqrt{\cosh^2(e+fx}\right.}{}$$

$$= \frac{2(4a-b)\cosh^3(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35f} + \frac{b\cosh^5(}{}$$

$$= \frac{\left(a^2+9ab-2b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(}{}$$

$$= \frac{\left(a^2+9ab-2b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(}{}$$

$$= \frac{\left(a^2+9ab-2b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(}{}$$

$$= \frac{\left(a^2+9ab-2b^2\right)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{35bf} + \frac{2(}{}$$

Mathematica [C] time = 2.53138, size = 256, normalized size = 0.72

$$\frac{-64ia\left(-11a^2b+2a^3+8ab^2+b^3\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(b\left(144a^2+192a\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((128*I)*a*(a^3 - 5*a^2*b - 5*a*b^2 + b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*
x)])/a]*EllipticE[I*(e + f*x), b/a] - (64*I)*a*(2*a^3 - 11*a^2*b + 8*a*b^2
+ b^3)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a]
+ Sqrt[2]*b*(32*a^3 + 400*a^2*b - 212*a*b^2 + 30*b^3 + b*(144*a^2 + 192*a*b
 - 37*b^2)*Cosh[2*(e + f*x)] + 2*b^2*(26*a + b)*Cosh[4*(e + f*x)] + 5*b^3*C
osh[6*(e + f*x)])*Sinh[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

Maple [A] time = 0.167, size = 730, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/35*(5*(-1/a*b)^(1/2)*b^3*sinh(f*x+e)*cosh(f*x+e)^8+(13*(-1/a*b)^(1/2)*a*b
^2-7*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^6*sinh(f*x+e)+(9*(-1/a*b)^(1/2)*a^2*b-
```

```
(-1/a*b)^(1/2)*a*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+((-1/a*b)^(1/2)*a^3+8*(-1/a
*b)^(1/2)*a^2*b-11*(-1/a*b)^(1/2)*a*b^2+2*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2
*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3+8*(b/a*cosh(f*x+e)^2+(a-b)/
a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a^2*b-11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+2*(b/a*cosh(f*x+e)^2+(a
-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*b^3-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ell
ipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3+10*(b/a*cosh(f*x+e)^2+(a
-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a^2*b+10*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-2*(b/a*cosh(f*x+e)^
2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b^3)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \cosh \left(fx+e\right)^{4} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \cosh \left(fx+e\right)^{4} \sinh \left(fx+e\right)^{2}+a \cosh \left(fx+e\right)^{4}\right) \sqrt{b \sinh \left(fx+e\right)^{2}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(f*x + e)^4*sinh(f*x + e)^2 + a*cosh(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\cosh \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^4, x)
```

3.369 $\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=299

$$\frac{(9a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{15f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\left(3a^2+7ab-2b^2\right)\tanh(e+fx)\sqrt{a-}}{15bf}$$

```
[Out] (2*(3*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f
) + (b*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) - (
(3*a^2 + 7*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e +
f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((9*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + ((3*a^2 + 7*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/(15*b*f)
```

Rubi [A] time = 0.290854, antiderivative size = 299, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 416, 528, 531, 418, 492, 411}

$$\frac{\left(3a^2+7ab-2b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15bf}-\frac{\left(3a^2+7ab-2b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\text{si}\right.}{15bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(3*a - b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f
) + (b*Cosh[e + f*x]^3*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(5*f) - (
(3*a^2 + 7*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e +
f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((9*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Se
ch[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(15*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + ((3*a^2 + 7*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/(15*b*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
```

```
, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2} dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\right)\text{Subst}\left(\int \sqrt{1+x^2}\left(a+bx^2\right)^{3/2} dx, x, \sinh\right.}{f}$$

$$= \frac{b\cosh^3(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{5f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\text{se}\right.}{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{b\cosh^3(e+}{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{b\cosh^3(e+}{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{b\cosh^3(e+}{}$$

$$= \frac{2(3a-b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{15f} + \frac{b\cosh^3(e+}{}$$

Mathematica [C] time = 1.32657, size = 213, normalized size = 0.71

$$\frac{16ia\left(3a^2-2ab-b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(48a^2+4b(9a-2b)\cosh(2(e+\right.}{240bf\sqrt{2a+b\cosh(2(e+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-16*I)*a*(3*a^2 + 7*a*b - 2*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*
EllipticE[I*(e + f*x), b/a] + (16*I)*a*(3*a^2 - 2*a*b - b^2)*Sqrt[(2*a - b
+ b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(48*a^2 -
 28*a*b + 5*b^2 + 4*(9*a - 2*b)*b*Cosh[2*(e + f*x)] + 3*b^2*Cosh[4*(e + f*x
)])*Sinh[2*(e + f*x)])/(240*b*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.12, size = 535, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/15*(3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+(9*(-1/a*b)^(1/2)*a*b-
5*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(6*(-1/a*b)^(1/2)*a^2-8*(-1
/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+6*a^2*(b/a*
cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))-8*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(
```

```
f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*b^2+3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+7*(b/a*cosh(f
*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)
^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/(-1/a*b)^(1/2
)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\cosh\left(fx+e\right)^2\sinh\left(fx+e\right)^2+a\cosh\left(fx+e\right)^2\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cosh(f*x + e)^2*sinh(f*x + e)^2 + a*cosh(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cosh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*cosh(f*x + e)^2, x)
```

3.370 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=174

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)\sqrt{a+b}}{3f}$$

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.189529, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(}}{3f\sqrt{\frac{b\sinh^2(}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a-b)+2(2a-b)b\sinh}{\sqrt{a+b\sinh^2(e+f}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}(a(a-b))\int \frac{1}{\sqrt{a+b\sinh^2(e+}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{\left(2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\right.}{3\sqrt{1+\frac{b\sinh^2}{a}}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b}}{3f\sqrt{1+\frac{b\sinh^2(e+fx}{a}}}$$

Mathematica [A] time = 0.633349, size = 169, normalized size = 0.97

$$\frac{2i\sqrt{2}a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-4i\sqrt{2}a(2a}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0., size = 416, normalized size = 2.4

$$\frac{1}{3\cosh\left(fx+e\right)f}\left(\sqrt{-\frac{b}{a}}b^2\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^4+\left(\sqrt{-\frac{b}{a}}ab-\sqrt{-\frac{b}{a}}b^2\right)\left(\cosh\left(fx+e\right)\right)^2\sinh\left(fx+e\right)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a*b-(-1/a
*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^
2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx + e\right)^{2} + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.371 $\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=210

$$\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

```
[Out] ((a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
+ (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sin
h[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a
- 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f + ((a - b)*Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rubi [A] time = 0.201249, antiderivative size = 210, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3192, 413, 531, 418, 492, 411}

$$-\frac{(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{(a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2}}{f\sqrt{\operatorname{sech}^2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
+ (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sin
h[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((a
- 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/f + ((a - b)*Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{3/2}}dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e\right.}{}$$

$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f} + \frac{\left(ab\sqrt{\cosh^2(e+fx)}\operatorname{sech}\right.}{}$$

$$= \frac{bF\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{(}{}$$

$$= \frac{(a-2b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 0.939001, size = 160, normalized size = 0.76

$$\frac{(a-b)\left(\sqrt{2}\tanh(e+fx)(2a+b\cosh(2(e+fx))-b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)\right)+2ia(a-2b)}{2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] + (a - b)*((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
ipticF[I*(e + f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e +
 f*x]))/(2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.115, size = 334, normalized size = 1.6

$$\frac{1}{f\cosh\left(fx+e\right)}\left(\sqrt{-\frac{b}{a}}ab\left(\sinh\left(fx+e\right)\right)^{3}-\sqrt{-\frac{b}{a}}b^{2}\left(\sinh\left(fx+e\right)\right)^{3}+2\,a\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\mathrm{E}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+2*a*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-((a+b*sinh(f*x+e)
^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a*b+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+(-1/a*b)^(1/2)*a^2*sinh(f*x+
e)-(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\operatorname{sech}\left(fx+e\right)^{2}\sinh\left(fx+e\right)^{2}+a\operatorname{sech}\left(fx+e\right)^{2}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sech(f*x + e)^2*sinh(f*x + e)^2 + a*sech(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^2, x)
```

3.372 $\int \mathbf{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=193

$$-\frac{b\,\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\mathrm{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a-b)\tanh(e+fx)\mathrm{sech}^2(e+fx)\sqrt{a+b}}{3f}$$

```
[Out] (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*S
inh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
((a - b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rubi [A] time = 0.187938, antiderivative size = 193, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3192, 413, 525, 418, 411}

$$\frac{(a-b)\tanh(e+fx)\mathrm{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{b\,\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))|1-\right.}{3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] (2*(a + b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
) - (b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*S
inh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
((a - b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
```

```
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \mathrm{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{5/2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{(a-b)\mathrm{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e\right.}}{}$$

$$= \frac{(a-b)\mathrm{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3f} - \frac{\left(ab\sqrt{\cosh^2\right.}}{}$$

$$= \frac{2(a+b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+}}{3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 1.96538, size = 197, normalized size = 1.02

$$\frac{-2ia(2a+b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\frac{\tanh(e+fx)\mathrm{sech}^2(e+fx)\left(\left(4a^2+6ab-2b^2\right)\cosh(2(e+fx))+8a^2+b(a+b)\cosh(4(}{\sqrt{2}}}{6f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((4*I)*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f
*x), b/a] - (2*I)*a*(2*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellip
ticF[I*(e + f*x), b/a] + ((8*a^2 - 3*a*b + b^2 + (4*a^2 + 6*a*b - 2*b^2)*Co
sh[2*(e + f*x)] + b*(a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x
])/Sqrt[2])/(6*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.116, size = 324, normalized size = 1.7

$$\frac{1}{3\left(\cosh\left(fx+e\right)\right)^3 f}\left(\left(2\sqrt{-\frac{b}{a}}ab+2\sqrt{-\frac{b}{a}}b^2\right)\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^4+\left(2\sqrt{-\frac{b}{a}}a^2+\sqrt{-\frac{b}{a}}ab-3\sqrt{-\frac{b}{a}}b^2\right)\left(c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+
(2*(-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b-3*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^
2*sinh(f*x+e)+((-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*
sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a-2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2)
/cosh(f*x+e)^3/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\operatorname{sech}\left(fx+e\right)^{4}\sinh\left(fx+e\right)^{2}+a\operatorname{sech}\left(fx+e\right)^{4}\right)\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sech(f*x + e)^4*sinh(f*x + e)^2 + a*sech(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\operatorname{sech}\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*sech(f*x + e)^4, x)
```

3.373 $\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f}$$

```
[Out] -((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(
2*b^(3/2)*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b*f)
```

Rubi [A] time = 0.0897847, antiderivative size = 79, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 388, 217, 206}

$$\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}-\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/(
2*b^(3/2)*f) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf} - \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{2bf}$$

$$= \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf} - \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2bf}$$

$$= -\frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}f} + \frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2bf}$$

Mathematica [A] time = 0.102935, size = 77, normalized size = 0.97

$$\frac{\frac{\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2b} - \frac{(a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2b^{3/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-((a - 2*b)*ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])/
(2*b^(3/2)) + (Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(2*b))/f
```

Maple [C] time = 0.073, size = 35, normalized size = 0.4

$$\frac{1}{f}\text{`int/indef0`}\left(\left(\cosh\left(fx+e\right)\right)^2\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}}, \sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.48203, size = 6375, normalized size = 80.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((a - 2*b)*cosh(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)
 + (a - 2*b)*sinh(f*x + e)^2)*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*
x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b
 - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cos
h(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e
)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 +
 b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*
a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 +
b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3
 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3
*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b
^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt
(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x +
 e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b
+ b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2
*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*
(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f
*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2
)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*
b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b -
 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3
 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3
)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f
*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x +
 e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + ((a - 2*b)*cos
h(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh(f*x
 + e)^2)*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*si
nh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x +
 e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - sqrt(2)*(b*cosh
(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^2 - b)*sqrt
((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x + e)^2 + 2*b^
2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2), 1/8*(2*((a - 2*b)
*cosh(f*x + e)^2 + 2*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh
(f*x + e)^2)*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*c
osh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(-b)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
```

```
x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x + e)^4 + 4*(
a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x + e)^4 - (3
*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2 - 3*a*b + 2*
b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 - (3*a*b - 2*
b^2)*cosh(f*x + e))*sinh(f*x + e))) + 2*((a - 2*b)*cosh(f*x + e)^2 + 2*(a -
 2*b)*cosh(f*x + e)*sinh(f*x + e) + (a - 2*b)*sinh(f*x + e)^2)*sqrt(-b)*arc
tan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(
f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a
 - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 +
 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt
(2)*(b*cosh(f*x + e)^2 + 2*b*cosh(f*x + e)*sinh(f*x + e) + b*sinh(f*x + e)^
2 - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(b^2*f*cosh(f*x +
e)^2 + 2*b^2*f*cosh(f*x + e)*sinh(f*x + e) + b^2*f*sinh(f*x + e)^2)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(cosh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.374 $\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

Rubi [A] time = 0.0465059, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

Mathematica [A] time = 0.0158504, size = 38, normalized size = 1.

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[b]*f)
```

Maple [A] time = 0.013, size = 34, normalized size = 0.9

$$\frac{1}{f}\ln\left(\sinh\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}\right)\frac{1}{\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*ln(sinh(f*x+e)*b^(1/2)+(a+b*sinh(f*x+e)^2)^(1/2))/b^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.10497, size = 5052, normalized size = 132.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*
a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f
*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 -
4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a
^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^
2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 +
9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 1
0*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 +
 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x
 + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b
 + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b
^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)
*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a
^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 +
 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e
)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh
(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*
b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*s
inh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*
(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2
*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a
*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x +
 e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)
^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*s
inh(f*x + e)^5 + sinh(f*x + e)^6)) + sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*co
sh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*si
nh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2)))/(b*f), -1/2*(sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x + e)^2 +
 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 + b)*sqrt(
-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*cosh(f*x
+ e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)*sinh(f*x
 + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*x + e)^2
- 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x + e)^3 -
 (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + sqrt(-b)*arctan(sqrt(2)*(
cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt
(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 +
4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*
x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*
x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)))/(b*f)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.375 $\int \frac{\mathrm{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=46

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b
]*f)
```

Rubi [A] time = 0.0688678, antiderivative size = 46, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b
]*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{\sqrt{a-b}f}$$

Mathematica [A] time = 0.0307613, size = 46, normalized size = 1.

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(Sqrt[a - b
]*f)
```

Maple [C] time = 0.079, size = 35, normalized size = 0.8

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{1}{\left(\cosh\left(fx+e\right)\right)^2}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 1.87393, size = 1615, normalized size = 35.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x +
e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e
)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(
2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*
sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b
)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(co
sh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*co
sh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3
 + cosh(f*x + e))*sinh(f*x + e) + 1))/((a - b)*f), arctan(sqrt(2)*(cosh(f*x
 + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*c
osh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e
)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e
)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b))/(sqrt(a - b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(e + fx\right)}{\sqrt{a + b\sinh^2\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [B] time = 1.43875, size = 113, normalized size = 2.46

$$\frac{2\arctan\left(-\frac{\sqrt{b}e^{\left(2fx+2e\right)}-\sqrt{be^{\left(4fx+4e\right)}+4ae^{\left(2fx+2e\right)}-2be^{\left(2fx+2e\right)}+b}+\sqrt{b}}{2\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*
f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/(sqrt(a - b)*
f)
```

3.376 $\int \frac{\text{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=97

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{3/2}}+\frac{\tanh(e+fx)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}$$

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(2*(a - b)^(3/2)*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e +
f*x])/(2*(a - b)*f)
```

Rubi [A] time = 0.107228, antiderivative size = 97, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 382, 377, 203}

$$\frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{3/2}}+\frac{\tanh(e+fx)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((a - 2*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(2*(a - b)^(3/2)*f) + (Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e +
f*x])/(2*(a - b)*f)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2(a-b)f} + \frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)\sqrt{a+bx^2}}\,dx, x, \mathrm{si}\ldots\right.}{2(a-b)f}$$

$$= \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2(a-b)f} + \frac{(a-2b)\operatorname{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\mathrm{si}\ldots}{\sqrt{a+\ldots}}\right.}{2(a-b)f}$$

$$= \frac{(a-2b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{3/2}f} + \frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{2(a-b)f}$$

Mathematica [C] time = 9.47891, size = 443, normalized size = 4.57

$$\tanh(e+fx)\operatorname{sech}^3(e+fx)\left(\frac{b\sinh^2(e+fx)}{a}+1\right)\left(-30b\sinh^2(e+fx)\sqrt{\frac{\tanh^2(e+fx)\operatorname{sech}^2(e+fx)\left(a^2+ab\left(\sinh^2(e+fx)-1\right)-b^2\sinh^2(e+fx)\right)}{a^2}}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sech[e + f*x]^3*(1 + (b*Sinh[e + f*x]^2)/a)*Tanh[e + f*x]*(45*a*ArcSin[Sqr
t[((a - b)*Tanh[e + f*x]^2)/a]] + 30*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2
)/a]]*Sinh[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[e +
 f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tan
h[e + f*x]^2)/a)^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, ((a - b)*Tanh[e
+ f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(5/2) - 45*a*Sqrt[(Sech[e + f*x]^2*(a^2 -
 b^2*Sinh[e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2] -
30*b*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b
*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2]))/(30*a*f*Sqrt[a + b*Sinh[e
+ f*x]^2]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[
e + f*x]^2)/a)^(3/2))
```

Maple [C] time = 0.096, size = 35, normalized size = 0.4

$$\frac{1}{f}\,\text{`int/indef0`}\left(\frac{1}{\left(\cosh\left(fx+e\right)\right)^4}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}}, \sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{3}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.36256, size = 4010, normalized size = 41.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (a - 2*b)*sinh(f*x + e)^4 + 2*(a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*
b)*cosh(f*x + e)^2 + a - 2*b)*sinh(f*x + e)^2 + 4*((a - 2*b)*cosh(f*x + e)^
3 + (a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)*sqrt(-a + b)*log(((a
- 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2
*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f
*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a -
2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x
+ e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x
 + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
+ e) + 1)) - 2*sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*s
inh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*sinh(f*x
 + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*a*b + b^2
)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^2 - 2*a*b
 + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*
f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2
*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 + 2*(a - 2*b)
*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 + a - 2*b)*sinh(f*x + e)^
2 + 4*((a - 2*b)*cosh(f*x + e)^3 + (a - 2*b)*cosh(f*x + e))*sinh(f*x + e) +
 a - 2*b)*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3
+ b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2
+ 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e)
)*sinh(f*x + e) + b)) + sqrt(2)*((a - b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f
*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 - a + b)*sqrt((b*cosh(f*x +
```

```
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f
*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^
2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*
b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [B] time = 2.01015, size = 976, normalized size = 10.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] f*((a - 2*b)*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e)
+ 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/((
a*f^2 - b*f^2)*sqrt(a - b)) - 2*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^3*a - 2*(sqrt(b)*
e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f
*x + 2*e) + b))^3*b - 5*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) +
 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2*a*sqrt(b) + 2*(sqrt(b)*e
^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*
x + 2*e) + b))^2*b^(3/2) - 4*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4
*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*a^2 - (sqrt(b)*e^(2*f
*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2
*e) + b))*a*b + 2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e
^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))*b^2 - 4*a^2*sqrt(b) + 5*a*b^(3/2
) - 2*b^(5/2))/((a*f^2 - b*f^2)*((sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x
 + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b))^2 + 2*(sqrt(b)*e^
(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x
 + 2*e) + b))*sqrt(b) + 4*a - 3*b)^2))
```

3.377 $\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=241

$$-\frac{(a-3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3abf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\,\text{s}}}{3b^2f}$$

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a -
 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Si
nh[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
 - ((a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]
^2))/a]) - (2*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f
)
```

Rubi [A] time = 0.218549, antiderivative size = 241, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3192, 416, 531, 418, 492, 411}

$$-\frac{2(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3b^2f}+\frac{2(a-2b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))|1\right.}{3b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] (Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*b*f) + (2*(a -
 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Si
nh[e + f*x]^2])/(3*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
 - ((a - 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]
^2))/a]) - (2*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*b^2*f
)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}}{3b}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{\left(2(a-2b)\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right.}{}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} - \frac{(a-3b)F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3abf\sqrt{\frac{\operatorname{sech}^2(e+fx)(}{}}}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3bf} + \frac{2(a-2b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)(}{}}}$$

Mathematica [C] time = 0.821807, size = 179, normalized size = 0.74

$$\frac{-2i\sqrt{2}\left(2a^2-5ab+3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)+4}{6b^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*Sqrt[2]*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elliptic
E[I*(e + f*x), b/a] - (2*I)*Sqrt[2]*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b +
 b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[
2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*b^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e +
f*x)]])
```

Maple [A] time = 0.109, size = 344, normalized size = 1.4

$$\frac{1}{3 b\cosh\left(fx+e\right) f}\left(\sqrt{-\frac{b}{a}}b\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^{4}+\left(\sqrt{-\frac{b}{a}}a-\sqrt{-\frac{b}{a}}b\right)\left(\cosh\left(fx+e\right)\right)^{2}\sinh\left(fx+e\right)-2\sqrt{\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^
(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+4*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*b+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b/a*cosh(f*
x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^
(1/2),(a/b)^(1/2))*b)/b/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2
)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cosh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.378 $\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf}-\frac{\text{sec}}{}$$

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])) + (El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a +
 b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(b*f)
```

Rubi [A] time = 0.14809, antiderivative size = 177, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3192, 422, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\operatorname{sech}(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])) + (El
lipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f
*x]^2])/(a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a +
 b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(b*f)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{a+bx^2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a+bx^2}}\,dx,x,\sinh(e+fx)\right)}{f} + \frac{\left(\sqrt{\cosh^2(e+\right.}{}$$

$$= \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{\sqrt{a+b\sinh^2(e+fx)}}{bf}$$

$$= -\frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{F\left(\tan^{-1}(\sinh(e+f\right.}{af}$$

Mathematica [C] time = 0.24074, size = 95, normalized size = 0.54

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\left((b-a)\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+aE\left(i(e+fx)\middle|\frac{b}{a}\right)\right)}{bf\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*(a*EllipticE[I*(e + f*x), b/a
] + (-a + b)*EllipticF[I*(e + f*x), b/a]))/(b*f*Sqrt[2*a - b + b*Cosh[2*(e
+ f*x)]])
```

Maple [A] time = 0.082, size = 86, normalized size = 0.5

$$\frac{1}{f\cosh(fx+e)}\sqrt{\frac{a+b\left(\sinh(fx+e)\right)^2}{a}}\sqrt{\left(\cosh(fx+e)\right)^2}\text{EllipticE}\left(\sinh(fx+e)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}\frac{1}{\sqrt{a+b\left(\sinh(f\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(
1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cosh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^2}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.379 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0383432, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}\\ &= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0690047, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0., size = 86, normalized size = 1.4

$$\frac{1}{\cosh\left(fx+e\right)f}\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^2}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^2}\text{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}\frac{1}{\sqrt{a+b\left(\sinh\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\sinh\left(fx+e\right)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sinh^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh\left(fx + e\right)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.380 $\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=160

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan\right.}{af(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(\right.}{a}}}$$

```
[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
(b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(a*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
)
```

Rubi [A] time = 0.180525, antiderivative size = 160, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 414, 21, 422, 418, 492, 411}

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx))}{a}}}-\frac{b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh\right.}{af(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)(a+b\sinh^2(e+fx)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
+ f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) -
(b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/(a*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
```

```
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{b+bx^2}{\sqrt{1+x^2}\sqrt{a+b\ldots}}\right.}{(-a+b)f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{\sqrt{a+bx^2}}\,d\ldots\right.}{(-a+b)f}$$

$$= \frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} + \frac{\left(b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a\ldots}}\right.}{(-a+b)f}$$

$$= -\frac{bF\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{se}\ldots\right.}{\ldots}$$

$$= \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{bF\left(\tan^{-1}(\sinh(e+f\ldots\right.}{a(a-b\ldots}$$

Mathematica [C] time = 0.622135, size = 159, normalized size = 0.99

$$\frac{-2i(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}\tanh(e+fx)(2a+b\cosh(2(e+fx))-b)+2ia\sqrt{\frac{2a+b\cosh}{}}}{2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a
] - (2*I)*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e +
f*x), b/a] + Sqrt[2]*(2*a - b + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*(a -
 b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.138, size = 133, normalized size = 0.8

$$-\frac{1}{(a-b)\cosh\left(fx+e\right)f}\left(-\sqrt{-\frac{b}{a}}b\left(\sinh\left(fx+e\right)\right)^{3}+b\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\text{EllipticE}\left(\sinh\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(-(-1/a*b)^(1/2)*b*sinh(f*x+e)^3+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(-1/a*b)^(1/
2)*a*sinh(f*x+e))/(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2
)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\operatorname{sech}\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sech(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.381 $\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=219

$$-\frac{b(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a}}{3f(a-b)}$$

```
[Out] (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
 + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech
[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^2*f*Sqrt[(Sech[e + f*x]
^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]
^2]*Tanh[e + f*x])/(3*(a - b)*f)
```

Rubi [A] time = 0.194929, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3192, 414, 525, 418, 411}

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}-\frac{b(a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx)\right.}{3af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] (2*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
 + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) - ((a - 3*b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech
[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*(a - b)^2*f*Sqrt[(Sech[e + f*x]
^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]
^2]*Tanh[e + f*x])/(3*(a - b)*f)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\operatorname{sech}^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{5/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subs}}{3(a}$$

$$= \frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} + \frac{\left(2(a-2b)\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+f\right.}{3(a}$$

$$= \frac{2(a-2b)E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2 f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} - \frac{(a-3b)bF\left(\right.}{}$$

Mathematica [C] time = 2.1519, size = 219, normalized size = 1.

$$\frac{-2i\left(2a^2-5ab+3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\left(\left(4a^2-6ab-2b^2\right)\cosh(2(e+fx))+8a^2+b(a-2b\right.}{\sqrt{2}}}{6f(a-b)^2\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((4*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
 f*x), b/a] - (2*I)*(2*a^2 - 5*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f
*x)])/a]*EllipticF[I*(e + f*x), b/a] + ((8*a^2 - 15*a*b + 4*b^2 + (4*a^2 -
6*a*b - 2*b^2)*Cosh[2*(e + f*x)] + (a - 2*b)*b*Cosh[4*(e + f*x)])*Sech[e +
f*x]^2*Tanh[e + f*x])/Sqrt[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*(e +
f*x)]])
```

Maple [A] time = 0.22, size = 343, normalized size = 1.6

$$\frac{1}{3\left(\cosh\left(fx+e\right)\right)^{3}\left(a^{2}-2ab+b^{2}\right)f}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\left(2\left(\cosh\left(fx+e\right)\right)^{4}\sqrt{-\frac{b}{a}}b\left(a-2b\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/cosh(f*x+e)^3/(-1/a*b)^(1/2)/
(b*cosh(f*x+e)^4+(a-b)*cosh(f*x+e)^2)^(1/2)/(a^2-2*a*b+b^2)*(2*cosh(f*x+e)^
4*(-1/a*b)^(1/2)*b*(a-2*b)*sinh(f*x+e)+cosh(f*x+e)^2*(-1/a*b)^(1/2)*(2*a^2-
5*a*b+3*b^2)*sinh(f*x+e)+(-1/a*b)^(1/2)*(a^2-2*a*b+b^2)*sinh(f*x+e)+(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(a*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))-2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a+4*b*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^
(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\frac{\operatorname{sech}\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{4}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sech(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sech(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.382 $\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f}-\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(3/2)*f) -
((a - b)*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0985304, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3190, 385, 217, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f}-\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(3/2)*f) -
((a - b)*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$
$$= -\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{bf}$$
$$= -\frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{bf}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{3/2}f} - \frac{(a-b)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.169992, size = 89, normalized size = 1.16

$$\frac{a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}+\sqrt{b}(b-a)\sinh(e+fx)}{ab^{3/2}f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(-a + b)*Sinh[e + f*x] + a^(3/2)*ArcSinh[(Sqrt[b]*Sinh[e + f*x])/S
qrt[a]]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sinh[e + f
*x]^2])
```

Maple [C] time = 0.065, size = 35, normalized size = 0.5

$$\frac{1}{f}\text{`int/indef0`}\left(\left(\cosh\left(fx+e\right)\right)^{2}\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)^{-\frac{3}{2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.72556, size = 7876, normalized size = 102.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh
(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 +
2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)
*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f
*x + e)^8 + 8*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*
b - 2*a*b^2 + b^3)*sinh(f*x + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*co
sh(f*x + e)^6 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 +
b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(
f*x + e)^3 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x +
e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2
+ b^3)*cosh(f*x + e)^4 + 9*a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5
*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 +
 b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^
3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(
3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x +
 e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*
b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqr
t(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x
+ e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b
 + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 +
2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3
*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(
f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^
2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a
*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b
- 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^
3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^
3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(
f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x
+ e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x +
 e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + (a*b*cosh(f*x
 + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*
a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f
*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sin
h(f*x + e))*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e
)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)
*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*
x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 4*sqrt(2)*((
```

```
a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a
*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
 + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/(a*b^3*f*cosh(f*x + e)^4 + 4*a*b^3*f*cosh(f*x + e)*sinh(f*x +
 e)^3 + a*b^3*f*sinh(f*x + e)^4 + a*b^3*f + 2*(2*a^2*b^2 - a*b^3)*f*cosh(f*
x + e)^2 + 2*(3*a*b^3*f*cosh(f*x + e)^2 + (2*a^2*b^2 - a*b^3)*f)*sinh(f*x +
 e)^2 + 4*(a*b^3*f*cosh(f*x + e)^3 + (2*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*s
inh(f*x + e)), -1/2*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x +
e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cos
h(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3
+ (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*((a -
 b)*cosh(f*x + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(
f*x + e)^2 + b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
- b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(
(a*b - b^2)*cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 +
 (a*b - b^2)*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b -
b^2)*cosh(f*x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b
^2)*cosh(f*x + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + (a*b
*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^
4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*
b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x
+ e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2
+ b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x
 + e))*sinh(f*x + e) + b)) + 2*sqrt(2)*((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*
b - b^2)*cosh(f*x + e)*sinh(f*x + e) + (a*b - b^2)*sinh(f*x + e)^2 - a*b +
b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*b^3*f*cosh(f*x +
e)^4 + 4*a*b^3*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*b^3*f*sinh(f*x + e)^4 +
a*b^3*f + 2*(2*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 2*(3*a*b^3*f*cosh(f*x +
 e)^2 + (2*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + 4*(a*b^3*f*cosh(f*x + e)^3
 + (2*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.383 $\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0442715, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3190, 191}

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}\\ &= \frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.0279143, size = 29, normalized size = 1.

$$\frac{\sinh(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Sinh[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [A] time = 0.012, size = 28, normalized size = 1.

$$\frac{\sinh\left(fx+e\right)}{af}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] sinh(f*x+e)/a/f/(a+b*sinh(f*x+e)^2)^(1/2)
```

Maxima [B] time = 1.65636, size = 319, normalized size = 11.

$$\frac{b^{2}e^{\left(-6\,fx-6\,e\right)}+2\,ab-b^{2}+\left(8\,a^{2}-8\,ab+3\,b^{2}\right)e^{\left(-2\,fx-2\,e\right)}+3\left(2\,ab-b^{2}\right)e^{\left(-4\,fx-4\,e\right)}}{2\left(a^{2}-ab\right)\left(2\,(2\,a-b)e^{\left(-2\,fx-2\,e\right)}+be^{\left(-4\,fx-4\,e\right)}+b\right)^{\frac{3}{2}}f}-\frac{b^{2}+3\left(2\,ab-b^{2}\right)e^{\left(-2\,fx-2\,e\right)}+\left(8\,a\right.}{2\left(a^{2}-ab\right)\left(2\,(2\,a-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(b^2*e^(-6*f*x - 6*e) + 2*a*b - b^2 + (8*a^2 - 8*a*b + 3*b^2)*e^(-2*f*x
 - 2*e) + 3*(2*a*b - b^2)*e^(-4*f*x - 4*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(3/2)*f) - 1/2*(b^2 + 3*(2*a*b - b^2)
*e^(-2*f*x - 2*e) + (8*a^2 - 8*a*b + 3*b^2)*e^(-4*f*x - 4*e) + (2*a*b - b^2
)*e^(-6*f*x - 6*e))/((a^2 - a*b)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*
x - 4*e) + b)^(3/2)*f)
```

Fricas [B] time = 1.82896, size = 641, normalized size = 22.1

$$\frac{\sqrt{2}\left(\cosh\left(fx+e\right)^{2}+2\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)+\mathrm{si}\right.}{abf\cosh\left(fx+e\right)^{4}+4\,abf\cosh\left(fx+e\right)\sinh\left(fx+e\right)^{3}+abf\sinh\left(fx+e\right)^{4}+2\left(2\,a^{2}-ab\right)f\cosh\left(fx+e\right)^{2}+abf-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
- 1)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*b*f*cosh(f*x + e)^
4 + 4*a*b*f*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*f*sinh(f*x + e)^4 + 2*(2*a^
2 - a*b)*f*cosh(f*x + e)^2 + a*b*f + 2*(3*a*b*f*cosh(f*x + e)^2 + (2*a^2 -
a*b)*f)*sinh(f*x + e)^2 + 4*(a*b*f*cosh(f*x + e)^3 + (2*a^2 - a*b)*f*cosh(f
```

```
*x + e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.2445, size = 211, normalized size = 7.28

$$\frac{\frac{(a^3f-2\,a^2bf+ab^2f)e^{(2\,fx+2\,e)}}{a^4b^3-2\,a^3b^4+a^2b^5}-\frac{a^3f-2\,a^2bf+ab^2f}{a^4b^3-2\,a^3b^4+a^2b^5}}{256\,\sqrt{be^{(4\,fx+4\,e)}+4\,ae^{(2\,fx+2\,e)}-2\,be^{(2\,fx+2\,e)}+b}}+\frac{f}{256\,ab^{\frac{7}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/256*((a^3*f - 2*a^2*b*f + a*b^2*f)*e^(2*f*x + 2*e)/(a^4*b^3 - 2*a^3*b^4 +
 a^2*b^5) - (a^3*f - 2*a^2*b*f + a*b^2*f)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5))/
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + 1
/256*f/(a*b^(7/2))
```

3.384 $\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=85

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\sinh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(3
/2)*f) - (b*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0965584, antiderivative size = 85, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 382, 377, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{3/2}}-\frac{b\sinh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(3
/2)*f) - (b*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{(a-b)f}$$

$$= -\frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(-a+b)x^2}\,dx, x, \frac{\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)f}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{3/2}f} - \frac{b\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] time = 7.55193, size = 315, normalized size = 3.71

$$\tanh(e+fx)\operatorname{sech}^7(e+fx)\sqrt{a+b\sinh^2(e+fx)}\left(4(a-b)^2\sinh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)\,{}_2F_1\left(2,2;\frac{7}{2};\frac{(a-b)\tanh^2(e+f\right.\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sech[e + f*x]^7*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]*(4*(a - b)^2*Hyp
ergeometric2F1[2, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^4*(a +
 b*Sinh[e + f*x]^2)*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[e + f*x]^2 + a*b*
(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2] + 15*a*Cosh[e + f*x]^2*(3*a +
 2*b*Sinh[e + f*x]^2)*(-(ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*(a + b*S
inh[e + f*x]^2)) + a*Cosh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a^2 - b^2*Sinh[
e + f*x]^2 + a*b*(-1 + Sinh[e + f*x]^2))*Tanh[e + f*x]^2)/a^2])))/(15*a^5*f
*(((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2)^(3
/2))
```

Maple [C] time = 0.115, size = 101, normalized size = 1.2

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{-b\left(\sinh\left(fx+e\right)\right)^2-a}{-b^2\left(\sinh\left(fx+e\right)\right)^6+\left(-2\,ab-b^2\right)\left(\sinh\left(fx+e\right)\right)^4+\left(-a^2-2\,ab\right)\left(\sinh\left(fx+e\right)\right)^2-a^2\,\sqrt{a+b(\mathrm{s}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((-b*sinh(f*x+e)^2-a)/(-b^2*sinh(f*x+e)^6+(-2*a*b-b^2)*sinh(f*x
+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e)
)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.48192, size = 4262, normalized size = 50.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((a*b*cosh(f*x + e)^4 + 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh
(f*x + e)^4 + 2*(2*a^2 - a*b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 +
2*a^2 - a*b)*sinh(f*x + e)^2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)
*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4
+ 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2
*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*
sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sin
h(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 +
sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x +
e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*
((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e) +
(a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f
*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sin
h(f*x + e)^2)))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^4 + 4*(a^3*b -
 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^3*b - 2*a^2*b^2 +
a*b^3)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*cosh(f
*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^2 + (2*a^4 - 5
*a^3*b + 4*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + (a^3*b - 2*a^2*b^2 + a*b^3
)*f + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)^3 + (2*a^4 - 5*a^3*b +
 4*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)), ((a*b*cosh(f*x + e)^4
+ 4*a*b*cosh(f*x + e)*sinh(f*x + e)^3 + a*b*sinh(f*x + e)^4 + 2*(2*a^2 - a*
b)*cosh(f*x + e)^2 + 2*(3*a*b*cosh(f*x + e)^2 + 2*a^2 - a*b)*sinh(f*x + e)^
2 + a*b + 4*(a*b*cosh(f*x + e)^3 + (2*a^2 - a*b)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x
+ e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*
x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*si
```

```
nh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a
- b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh
(f*x + e) + b)) - sqrt(2)*((a*b - b^2)*cosh(f*x + e)^2 + 2*(a*b - b^2)*cosh
(f*x + e)*sinh(f*x + e) + (a*b - b^2)*sinh(f*x + e)^2 - a*b + b^2)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b - 2*a^2*b^2 + a*b^3)*f*co
sh(f*x + e)^4 + 4*(a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x + e)*sinh(f*x + e)
^3 + (a^3*b - 2*a^2*b^2 + a*b^3)*f*sinh(f*x + e)^4 + 2*(2*a^4 - 5*a^3*b + 4
*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^3*b - 2*a^2*b^2 + a*b^3)*f*co
sh(f*x + e)^2 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 +
(a^3*b - 2*a^2*b^2 + a*b^3)*f + 4*((a^3*b - 2*a^2*b^2 + a*b^3)*f*cosh(f*x +
 e)^3 + (2*a^4 - 5*a^3*b + 4*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e
))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sech(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [B] time = 1.47401, size = 385, normalized size = 4.53

$$-\frac{\frac{\left(a^{4}bf-3\,a^{3}b^{2}f+3\,a^{2}b^{3}f-ab^{4}f\right)e^{\left(2\,fx+2\,e\right)}}{a^{4}b^{3}-2\,a^{3}b^{4}+a^{2}b^{5}}-\frac{a^{4}bf-3\,a^{3}b^{2}f+3\,a^{2}b^{3}f-ab^{4}f}{a^{4}b^{3}-2\,a^{3}b^{4}+a^{2}b^{5}}}{8\,\sqrt{be^{\left(4\,fx+4\,e\right)}+4\,ae^{\left(2\,fx+2\,e\right)}-2\,be^{\left(2\,fx+2\,e\right)}+b}}+\frac{2\,\arctan\left(-\frac{\sqrt{b}e^{\left(2\,fx+2\,e\right)}-\sqrt{be^{\left(4\,fx+4\,e\right)}+4\,ae^{\left(2\,fx+2\,e\right)}-2\,be^{\left(2\,fx+2\,e\right)}+b}+\sqrt{b}}{2\,\sqrt{a-b}}\right)}{\left(af-bf\right)\sqrt{a-b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*((a^4*b*f - 3*a^3*b^2*f + 3*a^2*b^3*f - a*b^4*f)*e^(2*f*x + 2*e)/(a^4*
b^3 - 2*a^3*b^4 + a^2*b^5) - (a^4*b*f - 3*a^3*b^2*f + 3*a^2*b^3*f - a*b^4*f
)/(a^4*b^3 - 2*a^3*b^4 + a^2*b^5))/sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x +
2*e) - 2*b*e^(2*f*x + 2*e) + b) + 2*arctan(-1/2*(sqrt(b)*e^(2*f*x + 2*e) -
sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b) + s
qrt(b))/sqrt(a - b))/((a*f - b*f)*sqrt(a - b)) - 1/8*(a*sqrt(b)*f - b^(3/2)
*f)/(a*b^3)
```

3.385 $\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=142

$$\frac{b(a+2b)\sinh(e+fx)}{2af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{5/2}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] ((a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(2*(a - b)^(5/2)*f) + (b*(a + 2*b)*Sinh[e + f*x])/(2*a*(a - b)^2*f*Sqrt[a
+ b*Sinh[e + f*x]^2]) + (Sech[e + f*x]*Tanh[e + f*x])/(2*(a - b)*f*Sqrt[a +
 b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.164915, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3190, 414, 527, 12, 377, 203}

$$\frac{b(a+2b)\sinh(e+fx)}{2af(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2f(a-b)^{5/2}}+\frac{\tanh(e+fx)\operatorname{sech}(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 4*b)*ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]])
/(2*(a - b)^(5/2)*f) + (b*(a + 2*b)*Sinh[e + f*x])/(2*a*(a - b)^2*f*Sqrt[a
+ b*Sinh[e + f*x]^2]) + (Sech[e + f*x]*Tanh[e + f*x])/(2*(a - b)*f*Sqrt[a +
 b*Sinh[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{(1+x^2)^2(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\operatorname{Subst}\left(\int \frac{-a+2b-2bx^2}{(1+x^2)(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{2(a-b)f}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{Subst}\left(\int \frac{a(a-4}{(1+x^2)\sqrt{}}\right.}{2a}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a-4b)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\tanh(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a-4b)\operatorname{Subst}\left(\int\right.}{}$$

$$= \frac{(a-4b)\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{2(a-b)^{5/2}f} + \frac{b(a+2b)\sinh(e+fx)}{2a(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}(e+fx)\,\mathrm{t}}{2(a-b)f\sqrt{a+b}}$$

Mathematica [C] time = 5.36355, size = 231, normalized size = 1.63

$$\frac{\tanh(e+fx)\operatorname{sech}^5(e+fx)\left(16(a-b)\sinh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^2 \operatorname{HypergeometricPFQ}\left(\{2,2,3\},\left\{1,\frac{9}{2}\right\},\frac{(a-b)}{}\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sech[e + f*x]^5*(16*(a - b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2}, ((a - b
)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^2 + 16*(a - b
)*Hypergeometric2F1[2, 3, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2
*(4*a^2 + 7*a*b*Sinh[e + f*x]^2 + 3*b^2*Sinh[e + f*x]^4) + 7*a*Cosh[e + f*x
]^2*Hypergeometric2F1[1, 2, 7/2, ((a - b)*Tanh[e + f*x]^2)/a]*(15*a^2 + 20*
a*b*Sinh[e + f*x]^2 + 8*b^2*Sinh[e + f*x]^4))*Tanh[e + f*x])/(105*a^4*f*Sqr
t[a + b*Sinh[e + f*x]^2])
```

Maple [C] time = 0.149, size = 95, normalized size = 0.7

$$\frac{1}{f}\text{` int/indef0`}\left(-\frac{\left(\cosh\left(fx+e\right)\right)^{2}}{-b^{2}\left(\cosh\left(fx+e\right)\right)^{10}+\left(-2\,ab+2\,b^{2}\right)\left(\cosh\left(fx+e\right)\right)^{8}+\left(-a^{2}+2\,ab-b^{2}\right)\left(\cosh\left(fx+e\right)\right)^{6}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*cosh(f*x+e)^10+
(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.65664, size = 11344, normalized size = 79.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((a^2*b - 4*a*b^2)*cosh(f*x + e)^8 + 8*(a^2*b - 4*a*b^2)*cosh(f*x + e
)*sinh(f*x + e)^7 + (a^2*b - 4*a*b^2)*sinh(f*x + e)^8 + 4*(a^3 - 4*a^2*b)*c
osh(f*x + e)^6 + 4*(a^3 - 4*a^2*b + 7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^2)*si
nh(f*x + e)^6 + 8*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b)*
cosh(f*x + e))*sinh(f*x + e)^5 + 2*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x +
e)^4 + 2*(35*(a^2*b - 4*a*b^2)*cosh(f*x + e)^4 + 4*a^3 - 17*a^2*b + 4*a*b^2
 + 30*(a^3 - 4*a^2*b)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b - 4*a*
b^2)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b)*cosh(f*x + e)^3 + (4*a^3 - 17*a^2
```

*b + 4*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + a^2*b - 4*a*b^2 + 4*(a^3 - 4
*a^2*b)*cosh(f*x + e)^2 + 4*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^6 + 15*(a^3
- 4*a^2*b)*cosh(f*x + e)^4 + a^3 - 4*a^2*b + 3*(4*a^3 - 17*a^2*b + 4*a*b^2)
*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*((a^2*b - 4*a*b^2)*cosh(f*x + e)^7 +
3*(a^3 - 4*a^2*b)*cosh(f*x + e)^5 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x +
e)^3 + (a^3 - 4*a^2*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*log(((a
- 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - 2
*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^2 + 2*(3*(a - 2*b)*cosh(f
*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 2*sqrt(2)*(cosh(f*x + e)^2 + 2*cos
h(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(-a + b)*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*cosh(f*x + e)^3 - (3*a -
2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh(f*x + e)^4 + 4*cosh(f*x
+ e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x
+ e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
+ e) + 1)) + 2*sqrt(2)*((a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 6*(a^2*b
+ a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2*b + a*b^2 - 2*b^3)*si
nh(f*x + e)^6 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + (4*a^
3 - 7*a^2*b + 5*a*b^2 - 2*b^3 + 15*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^4 + 4*(5*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (4*a^3 -
7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 - a^2*b - a*b^2 +
2*b^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + (15*(a^2*b +
a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^3 + 7*a^2*b - 5*a*b^2 + 2*b^3 + 6*(4*
a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 2*(3*(a
^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e)^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^8 + 8*(a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*f*sinh(f*x + e)^8 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f
*cosh(f*x + e)^6 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x
+ e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^6 + 2*(4*a^
5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^
4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^3 + 3*(a^5 - 3*a^4*b +
3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 30*(a^5 - 3*a^4*b + 3*a^3*b
^2 - a^2*b^3)*f*cosh(f*x + e)^2 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^
3 + a*b^4)*f)*sinh(f*x + e)^4 + 4*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*c
osh(f*x + e)^2 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x +
e)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^3 + (4*a^5
- 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e))*sinh(f*x + e)
^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^6 + 15*(a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^4 + 3*(4*a^5 - 13*a^4*b
+ 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a
^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^2 + (a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b
^4)*f + 8*((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^7 + 3*(a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^5 + (4*a^5 - 13*a^4*b +
15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^3 + (a^5 - 3*a^4*b + 3*a^3
*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e)), 1/2*(((a^2*b - 4*a*b^2)*co
sh(f*x + e)^8 + 8*(a^2*b - 4*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b
- 4*a*b^2)*sinh(f*x + e)^8 + 4*(a^3 - 4*a^2*b)*cosh(f*x + e)^6 + 4*(a^3 - 4
*a^2*b + 7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(7*(a^2*b
- 4*a*b^2)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2*b)*cosh(f*x + e))*sinh(f*x + e
)^5 + 2*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)^4 + 2*(35*(a^2*b - 4*a*b
^2)*cosh(f*x + e)^4 + 4*a^3 - 17*a^2*b + 4*a*b^2 + 30*(a^3 - 4*a^2*b)*cosh(
f*x + e)^2)*sinh(f*x + e)^4 + 8*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^5 + 10*(
a^3 - 4*a^2*b)*cosh(f*x + e)^3 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)
)*sinh(f*x + e)^3 + a^2*b - 4*a*b^2 + 4*(a^3 - 4*a^2*b)*cosh(f*x + e)^2 + 4

```
*(7*(a^2*b - 4*a*b^2)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b)*cosh(f*x + e)^4
+ a^3 - 4*a^2*b + 3*(4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)^2)*sinh(f*x
+ e)^2 + 8*((a^2*b - 4*a*b^2)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b)*cosh(f*x
+ e)^5 + (4*a^3 - 17*a^2*b + 4*a*b^2)*cosh(f*x + e)^3 + (a^3 - 4*a^2*b)*cos
h(f*x + e))*sinh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x + e)^2 + 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)
*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b
*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a -
 b)*cosh(f*x + e))*sinh(f*x + e) + b)) + sqrt(2)*((a^2*b + a*b^2 - 2*b^3)*c
osh(f*x + e)^6 + 6*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)*sinh(f*x + e)^5 +
(a^2*b + a*b^2 - 2*b^3)*sinh(f*x + e)^6 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^
3)*cosh(f*x + e)^4 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3 + 15*(a^2*b + a*b^2
 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(5*(a^2*b + a*b^2 - 2*b^3)*c
osh(f*x + e)^3 + (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*
x + e)^3 - a^2*b - a*b^2 + 2*b^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh
(f*x + e)^2 + (15*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^4 - 4*a^3 + 7*a^2*b
 - 5*a*b^2 + 2*b^3 + 6*(4*a^3 - 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)
*sinh(f*x + e)^2 + 2*(3*(a^2*b + a*b^2 - 2*b^3)*cosh(f*x + e)^5 + 2*(4*a^3
- 7*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 - (4*a^3 - 7*a^2*b + 5*a*b^2 -
 2*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^8 + 8
*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 +
(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*sinh(f*x + e)^8 + 4*(a^5 - 3*a^4*
b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*f*cosh(f*x + e)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)
*sinh(f*x + e)^6 + 2*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*
cosh(f*x + e)^4 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x +
 e)^3 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x +
 e)^5 + 2*(35*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f*cosh(f*x + e)^4 + 3
0*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^2 + (4*a^5 - 13*a^4
*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f)*sinh(f*x + e)^4 + 4*(a^5 - 3*a^4*b
+ 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*
b^3 - a*b^4)*f*cosh(f*x + e)^5 + 10*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f
*cosh(f*x + e)^3 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*co
sh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*f*cosh(f*x + e)^6 + 15*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x +
e)^4 + 3*(4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e
)^2 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f)*sinh(f*x + e)^2 + (a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*f + 8*((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4
)*f*cosh(f*x + e)^7 + 3*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x +
e)^5 + (4*a^5 - 13*a^4*b + 15*a^3*b^2 - 7*a^2*b^3 + a*b^4)*f*cosh(f*x + e)^
3 + (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sech(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.386 $\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=325

$$-\frac{2(2a-3b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3ab^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\left(8a^2-13ab+3b^2\right)\tanh(e+\ldots}{3ab^3\ldots}$$

```
[Out] -(((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*b^2*f) + ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^3*f*Sqrt[(Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(2*a - 3*b)*EllipticF[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^2*f
*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 - 13*a*b + 3*
b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*b^3*f)
```

Rubi [A] time = 0.300737, antiderivative size = 325, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 413, 528, 531, 418, 492, 411}

$$-\frac{\left(8a^2-13ab+3b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3ab^3f}+\frac{\left(8a^2-13ab+3b^2\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan\ldots\right.}{3ab^3f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) + ((4*a - 3*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])
/(3*a*b^2*f) + ((8*a^2 - 13*a*b + 3*b^2)*EllipticE[ArcTan[Sinh[e + f*x]], 1
 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^3*f*Sqrt[(Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (2*(2*a - 3*b)*EllipticF[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*b^2*f
*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - ((8*a^2 - 13*a*b + 3*
b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*b^3*f)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\left(1+x^2\right)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1+}}{}\right.}{abf}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\,\text{si}}}{3ab^2f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\,\text{si}}}{3ab^2f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\,\text{si}}}{3ab^2f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(4a-3b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\,\text{si}}}{3ab^2f}$$

Mathematica [C] time = 1.06951, size = 196, normalized size = 0.6

$$\frac{-4ia\left(8a^2-17ab+9b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(8a^2+ab\cosh(2(e+fx))\right.}{12ab^3f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((4*I)*a*(8*a^2 - 13*a*b + 3*b^2)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticE[I*(e + f*x), b/a] - (4*I)*a*(8*a^2 - 17*a*b + 9*b^2)*Sqrt[(2*a - b
 + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(8*a^2 -
 13*a*b + 6*b^2 + a*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(12*a*b^3*f*Sqr
t[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.118, size = 498, normalized size = 1.5

$$\frac{1}{3\,ab^2\cosh\left(fx+e\right)f}\left(\sqrt{-\frac{b}{a}}ab\left(\cosh\left(fx+e\right)\right)^4\sinh\left(fx+e\right)+\left(4\sqrt{-\frac{b}{a}}a^2-7\sqrt{-\frac{b}{a}}ab+3\sqrt{-\frac{b}{a}}b^2\right)\left(\cosh\left(fx+e\right)\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*cosh(f*x+e)^4*sinh(f*x+e)+(4*(-1/a*b)^(1/2)*a^2-7*(
-1/a*b)^(1/2)*a*b+3*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+4*a^2*(b/
```

```
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))-7*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x
+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+3*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b^2-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+13*(b/a*cos
h(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b-3*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)
^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/b^2/(-1/a*
b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{6}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\cosh\left(fx+e\right)^{6}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{6}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.387 $\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=244

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{abf\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{ab^2f}$$

```
[Out] -(((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(a*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
 + b*Sinh[e + f*x]^2])/(a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)
)/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*b^2*f)
```

Rubi [A] time = 0.218062, antiderivative size = 244, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3192, 413, 531, 418, 492, 411}

$$\frac{(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{ab^2f}-\frac{(2a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{ab^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] -(((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - ((2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt
[a + b*Sinh[e + f*x]^2])/(a*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x
]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
 + b*Sinh[e + f*x]^2])/(a*b*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)
)/a]) + ((2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a*b^2*f)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{a+(\cdots}{\sqrt{1+x\cdots}}\right.}{abf}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\cdots}{\sqrt{1+x\cdots}}\right.}{bf}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e+fx)\sqrt{a\cdots}}{abf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx\cdots\right.}{a}}}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(2a-b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e\cdots}{ab^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(\cdots\right.}{a}}}$$

Mathematica [C] time = 0.602297, size = 155, normalized size = 0.64

$$\frac{(a-b)\left(-\sqrt{2}b\sinh(2(e+fx))+4ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)\right)-2ia(2a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i\cdots\right.}{2ab^2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (a - b)*((4*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ell
ipticF[I*(e + f*x), b/a] - Sqrt[2]*b*Sinh[2*(e + f*x)]))/(2*a*b^2*f*Sqrt[2*
a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.116, size = 322, normalized size = 1.3

$$-\frac{1}{ab\cosh\left(fx+e\right)f}\left(\left(\sqrt{-\frac{b}{a}}a-\sqrt{-\frac{b}{a}}b\right)\left(\cosh\left(fx+e\right)\right)^{2}\sinh\left(fx+e\right)+a\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^{2}}{a}+\frac{a-b}{a}}\sqrt{\left(\cosh\left(fx+e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh(f*x+e)^2*sinh(f*x+e)+a*(b/a*cosh
(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/a*cosh(f*x+e)^2
+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),
(a/b)^(1/2))*a+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/b/(-1/a*b)^(1/2)/a/cosh(f*
x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\cosh\left(fx+e\right)^{4}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.388 $\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=91

$$\frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

```
[Out] (Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])
/(Sqrt[a]*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.1012, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.08, Rules used = {3192, 411}

$$\frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])
/(Sqrt[a]*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] time = 0.306912, size = 143, normalized size = 1.57

$$\frac{-i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)+b\sinh(2(e+fx))}{abf\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x),
 b/a] - I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e
+ f*x), b/a] + b*Sinh[2*(e + f*x)])/(a*b*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e +
 f*x)]])
```

Maple [A] time = 0.097, size = 181, normalized size = 2.

$$\frac{1}{a\cosh(fx+e)f}\left(\sqrt{-\frac{b}{a}}\sinh(fx+e)\left(\cosh(fx+e)\right)^2+\sqrt{\frac{b\left(\cosh(fx+e)\right)^2}{a}+\frac{a-b}{a}}\sqrt{\left(\cosh(fx+e)\right)^2}\mathrm{EllipticF}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] ((-1/a*b)^(1/2)*sinh(f*x+e)*cosh(f*x+e)^2+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(b
/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/a/cosh(f*x+e)/(a+b*sinh(f*x+e)
^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx+e)^2}{\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\cosh\left(fx+e\right)^{2}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.389 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rubi [A] time = 0.0611369, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3184, 21, 3178, 3177}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx=-\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$=-\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}$$

$$=-\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$=-\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}}-\frac{iE\left(ie+ifx\,\middle|\,\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.143378, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}b\sinh(2(e+fx))-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\,\middle|\,\frac{b}{a}\right)}{2af(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

Maple [A] time = 0.001, size = 252, normalized size = 2.2

$$\frac{1}{a\left(a-b\right)\cosh\left(fx+e\right)f}\left(-\sqrt{-\frac{b}{a}}b\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^2+a\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^2}{a}+\frac{a-b}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^2}\mathrm{El}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x
+ e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

3.390 $\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=217

$$-\frac{2b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{af(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{b}(a+\cdots}{\sqrt{a}f(\cdots}$$

```
[Out] (Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqr
t[a]], 1 - a/b])/(Sqrt[a]*(a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[
e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*b*EllipticF[ArcTan[Sinh[e +
f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*S
qrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*
f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.202535, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3192, 414, 525, 418, 411}

$$\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sqrt{b}(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{2b\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e\cdots}}{af(a-b)^2\sqrt{\frac{\operatorname{sech}\cdots}{\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqr
t[a]], 1 - a/b])/(Sqrt[a]*(a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[
e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*b*EllipticF[ArcTan[Sinh[e +
f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*S
qrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*
f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{b-bx^2}{\sqrt{1+x^2}\left(a+b\right.}\right.}{(-a+b)f}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2b\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{}}\right.}{(a-b)(-a+b)f}$$

$$= \frac{\sqrt{b}(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{\sqrt{a}(a-b)^2f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} - \frac{2bF\left(\tan^{-1}(\sinh(e+fx))|1\right.}{a(a-b)^2f\sqrt{}}$$

Mathematica [C] time = 1.17733, size = 178, normalized size = 0.82

$$\frac{-i\sqrt{2}a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\tanh(e+fx)\left(2a^2+b(a+b)\cosh(2(e+fx))-ab+b^2\right)+}{af(a-b)^2\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (I*Sqrt[2]*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] - I*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticF[I*(e + f*x), b/a] + (2*a^2 - a*b + b^2 + b*(a + b)*Cosh[2*(e + f
*x)])*Tanh[e + f*x])/(a*(a - b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]]
)
```

Maple [A] time = 0.148, size = 345, normalized size = 1.6

$$-\frac{1}{a\left(a-b\right)^{2}\cosh\left(fx+e\right)f}\left(-\sqrt{-\frac{b}{a}}ab\left(\sinh\left(fx+e\right)\right)^{3}-\sqrt{-\frac{b}{a}}b^{2}\left(\sinh\left(fx+e\right)\right)^{3}+a\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(-(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+a*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*b-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2
)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*a*b+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-(-1/a*b)^(1/2)*a^2*sinh(f*x+e)-(
-1/a*b)^(1/2)*b^2*sinh(f*x+e))/(a-b)^2/a/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\operatorname{sech}\left(fx+e\right)^{2}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}^{2}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sech(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.391 $\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f}-\frac{(a-b)\sinh(e+fx)\cosh^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(5/2)*f) -
((a - b)*Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3
/2)) - ((a - b)*(3*a + 2*b)*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sinh[e +
 f*x]^2])
```

Rubi [A] time = 0.141492, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3190, 413, 385, 217, 206}

$$-\frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f}-\frac{(a-b)\sinh(e+fx)\cosh^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/(b^(5/2)*f) -
((a - b)*Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3
/2)) - ((a - b)*(3*a + 2*b)*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sinh[e +
 f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
```

```
p, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\cosh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{a+2b+3ax^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{3abf}$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\ }{\sqrt{a\cdots}}\right.}{\ }$$

$$= -\frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(a-b)(3a+2b)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-b\cdots}\right.}{\ }$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{b^{5/2}f} - \frac{(a-b)\cosh^2(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(a-b)(3a+2b)\sin\cdots}{3a^2b^2f\sqrt{a+b\sinh\cdots}}$$

Mathematica [A] time = 0.863662, size = 126, normalized size = 0.94

$$\frac{\frac{2\sqrt{2}(b-a)\sinh(e+fx)\left(3a^2+b(2a+b)\cosh(2(e+fx))+ab-b^2\right)}{3a^2b^2(2a+b\cosh(2(e+fx))-b)^{3/2}} + \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{b}\sinh(e+fx)}{\sqrt{2a+b\cosh(2(e+fx))-b}}\right)}{b^{5/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (ArcTanh[(Sqrt[2]*Sqrt[b]*Sinh[e + f*x])/Sqrt[2*a - b + b*Cosh[2*(e + f*x)]
]]/b^(5/2) + (2*Sqrt[2]*(-a + b)*(3*a^2 + a*b - b^2 + b*(2*a + b)*Cosh[2*(e
 + f*x)])*Sinh[e + f*x])/(3*a^2*b^2*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2)))
/f
```

Maple [C] time = 0.082, size = 65, normalized size = 0.5

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\cosh\left(fx+e\right)\right)^{4}}{b^{2}\left(\sinh\left(fx+e\right)\right)^{4}+2\,ab\left(\sinh\left(fx+e\right)\right)^{2}+a^{2}}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{5}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 5.85418, size = 15815, normalized size = 118.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7
 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a
^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*
cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*
(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)
^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^
2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x
+ e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*
a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*co
sh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b +
3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(b)*log(-((a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^8 + 8*(a^2*b - 2*a*
b^2 + b^3)*cosh(f*x + e)*sinh(f*x + e)^7 + (a^2*b - 2*a*b^2 + b^3)*sinh(f*x
 + e)^8 + 2*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 2*(a^3 - 4*
a^2*b + 5*a*b^2 - 2*b^3 + 14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(
f*x + e)^6 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^3 + 3*(a^3 - 4*a^2
*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (9*a^2*b - 14*a*b^2
+ 6*b^3)*cosh(f*x + e)^4 + (70*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^4 + 9*
```

a^2*b - 14*a*b^2 + 6*b^3 + 30*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^5 + 10*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + b^3 + 2*(3*a*b^2 - 2*b^3)*cosh(f*x + e)^2 + 2*(14*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^6 + 15*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^4 + 3*a*b^2 - 2*b^3 + 3*(9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + sqrt(2)*((a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^5 + (a^2 - 2*a*b + b^2)*sinh(f*x + e)^6 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 3*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - a^2 + 2*a*b - b^2)*sinh(f*x + e)^4 + 4*(5*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - 3*(a^2 - 2*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e)^2 + (15*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 - 18*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 - 4*a*b + 3*b^2)*sinh(f*x + e)^2 - b^2 + 2*(3*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 - 6*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 - (4*a*b - 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(2*(a^2*b - 2*a*b^2 + b^3)*cosh(f*x + e)^7 + 3*(a^3 - 4*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5 + (9*a^2*b - 14*a*b^2 + 6*b^3)*cosh(f*x + e)^3 + (3*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e))/(cosh(f*x + e)^6 + 6*cosh(f*x + e)^5*sinh(f*x + e) + 15*cosh(f*x + e)^4*sinh(f*x + e)^2 + 20*cosh(f*x + e)^3*sinh(f*x + e)^3 + 15*cosh(f*x + e)^2*sinh(f*x + e)^4 + 6*cosh(f*x + e)*sinh(f*x + e)^5 + sinh(f*x + e)^6)) + 3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*a*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + a)*sinh(f*x + e)^2 + sqrt(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1)*sqrt(b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*(b*cosh(f*x + e)^3 + a*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) - 8*sqrt(2)*((2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^6 + 6*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b^2 - a*b^3 - b^4)*sinh(f*x + e)^6 + 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^4 + 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4 + 5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^4 - 2*a^2*b^2 + a*b^3 + b^4 + 4*(5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^3 + 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^2 + 3*(5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^4 - 2*a^3*b + 2*a^2*b^2 + a*b^3 - b^4 + 6*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^5 + 2*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^3 - (2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b^5*f*cosh(f*x + e)^8 + 8*a^2*b^5*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^5*f*sinh(f*x + e)^8 + a^2*b^5*f + 4*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^5*f*cosh(f*x + e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*

a^2*b^5*f*cosh(f*x + e)^3 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e))*sinh(f
x + e)^5 + 2(35*a^2*b^5*f*cosh(f*x + e)^4 + 30*(2*a^3*b^4 - a^2*b^5)*f*co
sh(f*x + e)^2 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f)*sinh(f*x + e)^4 + 4*
(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^2 + 8*(7*a^2*b^5*f*cosh(f*x + e)^5 +
10*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2
*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^5*f*cosh(f*x + e)^6 + 1
5*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^
2*b^5)*f*cosh(f*x + e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^2 + 8*(a^
2*b^5*f*cosh(f*x + e)^7 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 + (8*a^
4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x + e)^3 + (2*a^3*b^4 - a^2*b^5)*f*
cosh(f*x + e))*sinh(f*x + e)), -1/6*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2
*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2
*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*s
inh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(
f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4
 + 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*
b - a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh
(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3
*a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x +
 e)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)
^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*s
inh(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*
x + e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b
^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqrt(2)*((a - b)*cosh(f*x
 + e)^2 + 2*(a - b)*cosh(f*x + e)*sinh(f*x + e) + (a - b)*sinh(f*x + e)^2 +
 b)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a*b - b^2)*
cosh(f*x + e)^4 + 4*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^3 + (a*b - b^2)
*sinh(f*x + e)^4 - (3*a*b - 2*b^2)*cosh(f*x + e)^2 + (6*(a*b - b^2)*cosh(f*
x + e)^2 - 3*a*b + 2*b^2)*sinh(f*x + e)^2 - b^2 + 2*(2*(a*b - b^2)*cosh(f*x
 + e)^3 - (3*a*b - 2*b^2)*cosh(f*x + e))*sinh(f*x + e))) + 3*(a^2*b^2*cosh(
f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e
)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2
+ 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*
a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^
2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b +
3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b
^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3
+ (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b
 - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b -
a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b
^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^
3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e
)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-b)*arctan(sqr
t(2)*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 + 1
)*sqrt(-b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e
)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*c
osh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*c
osh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) + 4*sqrt(2)*(
(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^6 + 6*(2*a^2*b^2 - a*b^3 - b^4)*cos
h(f*x + e)*sinh(f*x + e)^5 + (2*a^2*b^2 - a*b^3 - b^4)*sinh(f*x + e)^6 + 3*
(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^4 + 3*(2*a^3*b - 2*a^2*b^
2 - a*b^3 + b^4 + 5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e)^2)*sinh(f*x + e
)^4 - 2*a^2*b^2 + a*b^3 + b^4 + 4*(5*(2*a^2*b^2 - a*b^3 - b^4)*cosh(f*x + e
)^3 + 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e))*sinh(f*x + e)^3
 - 3*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^2 + 3*(5*(2*a^2*b^2 -
 a*b^3 - b^4)*cosh(f*x + e)^4 - 2*a^3*b + 2*a^2*b^2 + a*b^3 - b^4 + 6*(2*a^
3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((2*a^2

```
*b^2 - a*b^3 - b^4)*cosh(f*x + e)^5 + 2*(2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)
*cosh(f*x + e)^3 - (2*a^3*b - 2*a^2*b^2 - a*b^3 + b^4)*cosh(f*x + e))*sinh(
f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b^5*f*cosh
(f*x + e)^8 + 8*a^2*b^5*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^5*f*sinh(f*
x + e)^8 + a^2*b^5*f + 4*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^6 + 4*(7*a^2
*b^5*f*cosh(f*x + e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^
4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*a^2*b^5*f*cosh(f*x
+ e)^3 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*a
^2*b^5*f*cosh(f*x + e)^4 + 30*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^2 + (8*
a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b^4 - a^2*b^
5)*f*cosh(f*x + e)^2 + 8*(7*a^2*b^5*f*cosh(f*x + e)^5 + 10*(2*a^3*b^4 - a^2
*b^5)*f*cosh(f*x + e)^3 + (8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x +
e))*sinh(f*x + e)^3 + 4*(7*a^2*b^5*f*cosh(f*x + e)^6 + 15*(2*a^3*b^4 - a^2*
b^5)*f*cosh(f*x + e)^4 + 3*(8*a^4*b^3 - 8*a^3*b^4 + 3*a^2*b^5)*f*cosh(f*x +
 e)^2 + (2*a^3*b^4 - a^2*b^5)*f)*sinh(f*x + e)^2 + 8*(a^2*b^5*f*cosh(f*x +
e)^7 + 3*(2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 + (8*a^4*b^3 - 8*a^3*b^4 +
 3*a^2*b^5)*f*cosh(f*x + e)^3 + (2*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e))*sinh
(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{5}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.392 $\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh^2(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] (Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*
Sinh[e + f*x])/(3*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0929841, antiderivative size = 73, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3190, 378, 191}

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)\cosh^2(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cosh[e + f*x]^2*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*
Sinh[e + f*x])/(3*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\cosh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$
$$= \frac{\cosh^2(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{3af}$$
$$= \frac{\cosh^2(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.105302, size = 50, normalized size = 0.68

$$\frac{(a+2b)\sinh^3(e+fx)+3a\sinh(e+fx)}{3a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*a*Sinh[e + f*x] + (a + 2*b)*Sinh[e + f*x]^3)/(3*a^2*f*(a + b*Sinh[e + f*
x]^2)^(3/2))
```

Maple [C] time = 0.082, size = 65, normalized size = 0.9

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\cosh\left(fx+e\right)\right)^2}{b^2\left(\sinh\left(fx+e\right)\right)^4+2\,ab\left(\sinh\left(fx+e\right)\right)^2+a^2}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [B] time = 1.75748, size = 1251, normalized size = 17.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/12*(b^4*e^(-10*f*x - 10*e) - 4*a^3*b + 6*a^2*b^2 - b^4 - (16*a^4 - 32*a^
3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-2*f*x - 2*e) + 10*(2*a^3*b - 3*a^2*
b^2 + 3*a*b^3 - b^4)*e^(-4*f*x - 4*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-
6*f*x - 6*e) + 5*(2*a*b^3 - b^4)*e^(-8*f*x - 8*e))/((a^4 - 2*a^3*b + a^2*b^
2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) + 1/4*(
```

```
2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-2*f
*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-4*f*x
 - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) + 5*(4*
a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x - 10
*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x
 - 4*e) + b)^(5/2)*f) - 1/4*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 + b^4)*
e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x - 4*e
) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x - 6*e)
+ 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^2 - 2
*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e
^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f) + 1/12*(b^4 + 5*(2*a*b^3
 - b^4)*e^(-2*f*x - 2*e) + 10*(3*a^2*b^2 - 3*a*b^3 + b^4)*e^(-4*f*x - 4*e)
+ 10*(2*a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*e^(-6*f*x - 6*e) - (16*a^4 - 32*
a^3*b + 6*a^2*b^2 + 10*a*b^3 - 5*b^4)*e^(-8*f*x - 8*e) - (4*a^3*b - 6*a^2*b
^2 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2
*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Fricas [B] time = 3.10231, size = 2267, normalized size = 31.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(2)*((a + 2*b)*cosh(f*x + e)^6 + 6*(a + 2*b)*cosh(f*x + e)*sinh(f*x
 + e)^5 + (a + 2*b)*sinh(f*x + e)^6 + 3*(3*a - 2*b)*cosh(f*x + e)^4 + 3*(5*
(a + 2*b)*cosh(f*x + e)^2 + 3*a - 2*b)*sinh(f*x + e)^4 + 4*(5*(a + 2*b)*cos
h(f*x + e)^3 + 3*(3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e)^3 - 3*(3*a - 2*b)
*cosh(f*x + e)^2 + 3*(5*(a + 2*b)*cosh(f*x + e)^4 + 6*(3*a - 2*b)*cosh(f*x
+ e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 + 6*((a + 2*b)*cosh(f*x + e)^5 + 2*(3*a
 - 2*b)*cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) - a - 2*
b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cosh(f*x + e
)^8 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(f*x + e)^8
 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh(f*x + e)^2
 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)
*f*cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)
*f*cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2*f*cosh(f*x +
e)^4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8*a^3*b + 3*a^2*
b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^
2*b^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (8*a^4
 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^2*f*c
osh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^4 - 8*a^
3*b + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^2
 + 8*(a^2*b^2*f*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^5 +
 (8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*f*co
sh(f*x + e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.63833, size = 801, normalized size = 10.97

$$\frac{\Bigg(\Bigg(\Bigg(\frac{\left(a^7b^2f^3-2\,a^6b^3f^3-2\,a^5b^4f^3+8\,a^4b^5f^3-7\,a^3b^6f^3+2\,a^2b^7f^3\right)e^{\left(2\,fx+2\,e\right)}}{a^8b^2f^4-4\,a^7b^3f^4+6\,a^6b^4f^4-4\,a^5b^5f^4+a^4b^6f^4}+\frac{3\left(3\,a^7b^2f^3-14\,a^6b^3f^3+26\,a^5b^4f^3-24\,a^4b^5f^3+11\,a^3b^6f^3-2\,a^2b^7f^3\right)}{a^8b^2f^4-4\,a^7b^3f^4+6\,a^6b^4f^4-4\,a^5b^5f^4+a^4b^6f^4}\Bigg)e^{\left(2\,fx+2\,e\right.}}{3\Big(be^{\left(4\,fx+4\,e\right)}+4\,ae^{\left(2\,fx+2\,e\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*((((a^7*b^2*f^3 - 2*a^6*b^3*f^3 - 2*a^5*b^4*f^3 + 8*a^4*b^5*f^3 - 7*a^3
*b^6*f^3 + 2*a^2*b^7*f^3)*e^(2*f*x + 2*e)/(a^8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*
a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4) + 3*(3*a^7*b^2*f^3 - 14*a^6*b^3*
f^3 + 26*a^5*b^4*f^3 - 24*a^4*b^5*f^3 + 11*a^3*b^6*f^3 - 2*a^2*b^7*f^3)/(a^
8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4))*e
^(2*f*x + 2*e) - 3*(3*a^7*b^2*f^3 - 14*a^6*b^3*f^3 + 26*a^5*b^4*f^3 - 24*a^
4*b^5*f^3 + 11*a^3*b^6*f^3 - 2*a^2*b^7*f^3)/(a^8*b^2*f^4 - 4*a^7*b^3*f^4 +
6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*b^6*f^4))*e^(2*f*x + 2*e) - (a^7*b^2*f^
3 - 2*a^6*b^3*f^3 - 2*a^5*b^4*f^3 + 8*a^4*b^5*f^3 - 7*a^3*b^6*f^3 + 2*a^2*b
^7*f^3)/(a^8*b^2*f^4 - 4*a^7*b^3*f^4 + 6*a^6*b^4*f^4 - 4*a^5*b^5*f^4 + a^4*
b^6*f^4))/(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) +
b)^(3/2) + 1/3*(a + 2*b)/(a^2*b^(3/2)*f)
```

3.393 $\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] Sinh[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*
a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0579766, antiderivative size = 65, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 192, 191}

$$\frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Sinh[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*Sinh[e + f*x])/(3*
a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rubi steps

$$\int \frac{\cosh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{3af}$$

$$= \frac{\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2\sinh(e+fx)}{3a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.0449284, size = 47, normalized size = 0.72

$$\frac{\sinh(e+fx)\left(3a+2b\sinh^2(e+fx)\right)}{3a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sinh[e + f*x]*(3*a + 2*b*Sinh[e + f*x]^2))/(3*a^2*f*(a + b*Sinh[e + f*x]^2
)^(3/2))
```

Maple [A] time = 0.011, size = 56, normalized size = 0.9

$$\frac{1}{f}\left(\frac{\sinh(fx+e)}{3a}\left(a+b\left(\sinh(fx+e)\right)^2\right)^{-\frac{3}{2}} + \frac{2\sinh(fx+e)}{3a^2}\frac{1}{\sqrt{a+b\left(\sinh(fx+e)\right)^2}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/f*(1/3*sinh(f*x+e)/a/(a+b*sinh(f*x+e)^2)^(3/2)+2/3/a^2*sinh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2))
```

Maxima [B] time = 1.6551, size = 655, normalized size = 10.08

$$\frac{2a^2b^2 - 2ab^3 + b^4 + 5\left(4a^3b - 6a^2b^2 + 4ab^3 - b^4\right)e^{(-2fx-2e)} + 2\left(24a^4 - 48a^3b + 49a^2b^2 - 25ab^3 + 5b^4\right)e^{(-4fx-4e)}}{3\left(a^4 - 2a^3b + a^2b^2\right)\left(2(2a-b)e^{(-2fx-}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*a^2*b^2 - 2*a*b^3 + b^4 + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^
(-2*f*x - 2*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-
4*f*x - 4*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-6*f*x - 6*e) +
```

```
5*(4*a^2*b^2 - 4*a*b^3 + b^4)*e^(-8*f*x - 8*e) + (2*a*b^3 - b^4)*e^(-10*f*x
 - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a - b)*e^(-2*f*x - 2*e) + b*e^(-
4*f*x - 4*e) + b)^(5/2)*f) - 1/3*(2*a*b^3 - b^4 + 5*(4*a^2*b^2 - 4*a*b^3 +
b^4)*e^(-2*f*x - 2*e) + 10*(6*a^3*b - 9*a^2*b^2 + 5*a*b^3 - b^4)*e^(-4*f*x
- 4*e) + 2*(24*a^4 - 48*a^3*b + 49*a^2*b^2 - 25*a*b^3 + 5*b^4)*e^(-6*f*x -
6*e) + 5*(4*a^3*b - 6*a^2*b^2 + 4*a*b^3 - b^4)*e^(-8*f*x - 8*e) + (2*a^2*b^
2 - 2*a*b^3 + b^4)*e^(-10*f*x - 10*e))/((a^4 - 2*a^3*b + a^2*b^2)*(2*(2*a -
 b)*e^(-2*f*x - 2*e) + b*e^(-4*f*x - 4*e) + b)^(5/2)*f)
```

Fricas [B] time = 3.09934, size = 2161, normalized size = 33.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 2/3*sqrt(2)*(b*cosh(f*x + e)^6 + 6*b*cosh(f*x + e)*sinh(f*x + e)^5 + b*sinh
(f*x + e)^6 + 3*(2*a - b)*cosh(f*x + e)^4 + 3*(5*b*cosh(f*x + e)^2 + 2*a -
b)*sinh(f*x + e)^4 + 4*(5*b*cosh(f*x + e)^3 + 3*(2*a - b)*cosh(f*x + e))*si
nh(f*x + e)^3 - 3*(2*a - b)*cosh(f*x + e)^2 + 3*(5*b*cosh(f*x + e)^4 + 6*(2
*a - b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 6*(b*cosh(f*x + e)^5 +
 2*(2*a - b)*cosh(f*x + e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) - b)*
sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2
*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a^2*b^2*f*cosh(f*x + e)^8
 + 8*a^2*b^2*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*f*sinh(f*x + e)^8 +
4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^6 + 4*(7*a^2*b^2*f*cosh(f*x + e)^2 +
(2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^6 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*f*
cosh(f*x + e)^4 + 8*(7*a^2*b^2*f*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*f*
cosh(f*x + e))*sinh(f*x + e)^5 + a^2*b^2*f + 2*(35*a^2*b^2*f*cosh(f*x + e)^
4 + 30*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + (8*a^4 - 8*a^3*b + 3*a^2*b^2
)*f)*sinh(f*x + e)^4 + 4*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^2*b
^2*f*cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^3 + (8*a^4 -
8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^2*b^2*f*cosh
(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^4 - 8*a^3*b
 + 3*a^2*b^2)*f*cosh(f*x + e)^2 + (2*a^3*b - a^2*b^2)*f)*sinh(f*x + e)^2 +
8*(a^2*b^2*f*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*f*cosh(f*x + e)^5 + (8
*a^4 - 8*a^3*b + 3*a^2*b^2)*f*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*f*cosh(
f*x + e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 1.69479, size = 702, normalized size = 10.8

$$\frac{2\left(\left(\left(\frac{\left(a^6b^3f-4\,a^5b^4f+6\,a^4b^5f-4\,a^3b^6f+a^2b^7f\right)e^{\left(2\,fx+2\,e\right)}}{a^8b^2f^2-4\,a^7b^3f^2+6\,a^6b^4f^2-4\,a^5b^5f^2+a^4b^6f^2}+\frac{3\left(2\,a^7b^2f-9\,a^6b^3f+16\,a^5b^4f-14\,a^4b^5f+6\,a^3b^6f-a^2b^7f\right)}{a^8b^2f^2-4\,a^7b^3f^2+6\,a^6b^4f^2-4\,a^5b^5f^2+a^4b^6f^2}\right)e^{\left(2\,fx+2\,e\right)}-\frac{3\left(2\,a^7b^2f-9\,a^6b^3f\cdots\right.}{a^8b^2f^2-4\,a^7b\cdots}\right.\right.}{3\left(be^{\left(4\,fx+4\,e\right)}+4\,ae^{\left(2\,fx+2\,e\right)}-2\,be^{\left(2\,fx+2\,e\right)}+\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 2/3*((((a^6*b^3*f - 4*a^5*b^4*f + 6*a^4*b^5*f - 4*a^3*b^6*f + a^2*b^7*f)*e^
(2*f*x + 2*e)/(a^8*b^2*f^2 - 4*a^7*b^3*f^2 + 6*a^6*b^4*f^2 - 4*a^5*b^5*f^2
+ a^4*b^6*f^2) + 3*(2*a^7*b^2*f - 9*a^6*b^3*f + 16*a^5*b^4*f - 14*a^4*b^5*f
 + 6*a^3*b^6*f - a^2*b^7*f)/(a^8*b^2*f^2 - 4*a^7*b^3*f^2 + 6*a^6*b^4*f^2 -
4*a^5*b^5*f^2 + a^4*b^6*f^2))*e^(2*f*x + 2*e) - 3*(2*a^7*b^2*f - 9*a^6*b^3*
f + 16*a^5*b^4*f - 14*a^4*b^5*f + 6*a^3*b^6*f - a^2*b^7*f)/(a^8*b^2*f^2 - 4
*a^7*b^3*f^2 + 6*a^6*b^4*f^2 - 4*a^5*b^5*f^2 + a^4*b^6*f^2))*e^(2*f*x + 2*e
) - (a^6*b^3*f - 4*a^5*b^4*f + 6*a^4*b^5*f - 4*a^3*b^6*f + a^2*b^7*f)/(a^8*
b^2*f^2 - 4*a^7*b^3*f^2 + 6*a^6*b^4*f^2 - 4*a^5*b^5*f^2 + a^4*b^6*f^2))/(b*
e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2) + 2/
3/(a^2*sqrt(b)*f)
```

3.394 $\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(5a-2b)\sinh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{b\sinh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(5
/2)*f) - (b*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) -
((5*a - 2*b)*b*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2
])
```

Rubi [A] time = 0.148909, antiderivative size = 134, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 12, 377, 203}

$$-\frac{b(5a-2b)\sinh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{b\sinh(e+fx)}{3af(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTan[(Sqrt[a - b]*Sinh[e + f*x])/Sqrt[a + b*Sinh[e + f*x]^2]]/((a - b)^(5
/2)*f) - (b*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) -
((5*a - 2*b)*b*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2
])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
```

```
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \frac{\operatorname{sech}(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{3a-2b-2bx^2}{(1+x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{3a(a-b)f}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\Big(}{}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\Big(}{}$$

$$= -\frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\Big(}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{a-b}\sinh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\right)}{(a-b)^{5/2}f} - \frac{b\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(5a-2b)b\sinh}{3a^2(a-b)^2f\sqrt{a+b}}$$

Mathematica [C] time = 9.30254, size = 1331, normalized size = 9.93

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sech[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sech[e + f*x]*Tanh[e + f*x]*(1575*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]
] + (2100*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2)/a +
(840*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^4)/a^2 - (
3150*(a - b)*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^2)/a -
 (4200*(a - b)*b*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*
Tanh[e + f*x]^2)/a^2 - (1680*(a - b)*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]
^2)/a]]*Sinh[e + f*x]^4*Tanh[e + f*x]^2)/a^3 + (1575*(a - b)^2*ArcSin[Sqrt[
((a - b)*Tanh[e + f*x]^2)/a]]*Tanh[e + f*x]^4)/a^2 + (2100*(a - b)^2*b*ArcS
in[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x]^2*Tanh[e + f*x]^4)/a^3
+ (840*(a - b)^2*b^2*ArcSin[Sqrt[((a - b)*Tanh[e + f*x]^2)/a]]*Sinh[e + f*x
]^4*Tanh[e + f*x]^4)/a^4 + 2100*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2) + (2800*b*Sinh[e + f*x]^2*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(
3/2))/a + (1120*b^2*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f
*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(3/2))/a^2 + 96*Hypergeometric2F1[
2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
 + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + 24*HypergeometricPFQ[{
2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sqrt[(Sech[e + f*x]^2*(a +
 b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2) + (168*b*Hyperg
eometric2F1[2, 2, 9/2, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^2*Sqrt[(S
ech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7
/2))/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x
]^2)/a]*Sinh[e + f*x]^2*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(
((a - b)*Tanh[e + f*x]^2)/a)^(7/2))/a + (72*b^2*Hypergeometric2F1[2, 2, 9/2
, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e + f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e + f*x]^2)/a)^(7/2))/a^2 + (24*b^2*Hy
pergeometricPFQ[{2, 2, 2}, {1, 9/2}, ((a - b)*Tanh[e + f*x]^2)/a]*Sinh[e +
f*x]^4*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(((a - b)*Tanh[e +
 f*x]^2)/a)^(7/2))/a^2 - 1575*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2)*Tanh[e + f*x]^2)/a^2] - (2100*b*Sinh[e + f*x]^2*Sqrt[((a - b)*Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e + f*x]^2)/a^2])/a - (840*b^2*Sin
h[e + f*x]^4*Sqrt[((a - b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)*Tanh[e +
 f*x]^2)/a^2])/a^2))/(315*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]*Sqrt[(Sech[e +
f*x]^2*(a + b*Sinh[e + f*x]^2))/a]*(1 + (b*Sinh[e + f*x]^2)/a)*(((a - b)*Ta
nh[e + f*x]^2)/a)^(5/2))
```

Maple [C] time = 0.133, size = 169, normalized size = 1.3

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{-b^2\left(\sinh\left(fx+e\right)\right)^4-2\,ab\left(\sinh\left(fx+e\right.\right.}{-b^4\left(\sinh\left(fx+e\right)\right)^{10}+\left(-4\,ab^3-b^4\right)\left(\sinh\left(fx+e\right)\right)^8+\left(-6\,a^2b^2-4\,ab^3\right)\left(\sinh\left(fx+e\right)\right)^6+\left(-4\,a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`((-b^2*sinh(f*x+e)^4-2*a*b*sinh(f*x+e)^2-a^2)/(-b^4*sinh(f*x+e)
^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+e)^6+(-4*a^3
*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(a+b*sinh(f*x
+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 5.13512, size = 12320, normalized size = 91.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2*b^2*cosh(f*x + e)*sinh(f*x + e)^7
 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a
^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*
cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*
(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)
^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*cosh(f*x + e)^5 + 10*(2*a^3*b - a^
2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x
+ e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*
a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 8*(a^2*b^2*co
sh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^5 + (8*a^4 - 8*a^3*b +
3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a^2*b^2)*cosh(f*x + e))*sinh(f*x +
e))*sqrt(-a + b)*log(((a - 2*b)*cosh(f*x + e)^4 + 4*(a - 2*b)*cosh(f*x + e)
*sinh(f*x + e)^3 + (a - 2*b)*sinh(f*x + e)^4 - 2*(3*a - 2*b)*cosh(f*x + e)^
2 + 2*(3*(a - 2*b)*cosh(f*x + e)^2 - 3*a + 2*b)*sinh(f*x + e)^2 - 2*sqrt(2)
*(cosh(f*x + e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sq
rt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)) + 4*((a - 2*b)*
cosh(f*x + e)^3 - (3*a - 2*b)*cosh(f*x + e))*sinh(f*x + e) + a - 2*b)/(cosh
(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh
(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 +
 cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*((5*a^2*b^2 - 7*a*b^3 + 2*b
^4)*cosh(f*x + e)^6 + 6*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)*sinh(f*
x + e)^5 + (5*a^2*b^2 - 7*a*b^3 + 2*b^4)*sinh(f*x + e)^6 + 3*(8*a^3*b - 17*
a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^4 + 3*(8*a^3*b - 17*a^2*b^2 + 11*
a*b^3 - 2*b^4 + 5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^2)*sinh(f*x +
 e)^4 - 5*a^2*b^2 + 7*a*b^3 - 2*b^4 + 4*(5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*co
sh(f*x + e)^3 + 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e))*
sinh(f*x + e)^3 - 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)
^2 + 3*(5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^4 - 8*a^3*b + 17*a^2*
b^2 - 11*a*b^3 + 2*b^4 + 6*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f
*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)
^5 + 2*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^3 - (8*a^3*b
 - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cos
h(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
```

```
e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 -
a^2*b^5)*f*cosh(f*x + e)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*
f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^
5)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2
*b^5)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*
f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)
*f)*sinh(f*x + e)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^
3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^
4 - a^2*b^5)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3
*b^4 + a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^5*b^2 - 3*a^4*b
^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 7*a^5*b^2 + 9*a
^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^2 + (8*a^7 - 32*a^6*b + 51*a^
5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f)*sinh(f*x + e)^4 + 4*(2*a^6*
b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^2 + 8*(7*(
a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 + 10*(2*a^6*b
- 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^3 + (8*a^7 -
 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x +
e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*co
sh(f*x + e)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*
f*cosh(f*x + e)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*
b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a
^3*b^4 + a^2*b^5)*f)*sinh(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^
7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x +
e)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5
)*f*cosh(f*x + e)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^
5)*f*cosh(f*x + e))*sinh(f*x + e)), 1/3*(3*(a^2*b^2*cosh(f*x + e)^8 + 8*a^2
*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + a^2*b^2*sinh(f*x + e)^8 + 4*(2*a^3*b -
 a^2*b^2)*cosh(f*x + e)^6 + 4*(7*a^2*b^2*cosh(f*x + e)^2 + 2*a^3*b - a^2*b^
2)*sinh(f*x + e)^6 + 8*(7*a^2*b^2*cosh(f*x + e)^3 + 3*(2*a^3*b - a^2*b^2)*c
osh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x +
e)^4 + 2*(35*a^2*b^2*cosh(f*x + e)^4 + 8*a^4 - 8*a^3*b + 3*a^2*b^2 + 30*(2*
a^3*b - a^2*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + a^2*b^2 + 8*(7*a^2*b^2*
cosh(f*x + e)^5 + 10*(2*a^3*b - a^2*b^2)*cosh(f*x + e)^3 + (8*a^4 - 8*a^3*b
 + 3*a^2*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^3*b - a^2*b^2)*cosh(f
*x + e)^2 + 4*(7*a^2*b^2*cosh(f*x + e)^6 + 15*(2*a^3*b - a^2*b^2)*cosh(f*x
+ e)^4 + 2*a^3*b - a^2*b^2 + 3*(8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^
2)*sinh(f*x + e)^2 + 8*(a^2*b^2*cosh(f*x + e)^7 + 3*(2*a^3*b - a^2*b^2)*cos
h(f*x + e)^5 + (8*a^4 - 8*a^3*b + 3*a^2*b^2)*cosh(f*x + e)^3 + (2*a^3*b - a
^2*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*arctan(sqrt(2)*(cosh(f*x
+ e)^2 + 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2 - 1)*sqrt(a - b)*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(b*cosh(f*x + e)^4 + 4*b*co
sh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)
^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)
^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)) - sqrt(2)*((5*a^2*b^2 - 7
*a*b^3 + 2*b^4)*cosh(f*x + e)^6 + 6*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x
+ e)*sinh(f*x + e)^5 + (5*a^2*b^2 - 7*a*b^3 + 2*b^4)*sinh(f*x + e)^6 + 3*(8
*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^4 + 3*(8*a^3*b - 17*a
^2*b^2 + 11*a*b^3 - 2*b^4 + 5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^2
)*sinh(f*x + e)^4 - 5*a^2*b^2 + 7*a*b^3 - 2*b^4 + 4*(5*(5*a^2*b^2 - 7*a*b^3
 + 2*b^4)*cosh(f*x + e)^3 + 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cos
h(f*x + e))*sinh(f*x + e)^3 - 3*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*c
osh(f*x + e)^2 + 3*(5*(5*a^2*b^2 - 7*a*b^3 + 2*b^4)*cosh(f*x + e)^4 - 8*a^3
*b + 17*a^2*b^2 - 11*a*b^3 + 2*b^4 + 6*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2
*b^4)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 6*((5*a^2*b^2 - 7*a*b^3 + 2*b^4)*c
osh(f*x + e)^5 + 2*(8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e)^
3 - (8*a^3*b - 17*a^2*b^2 + 11*a*b^3 - 2*b^4)*cosh(f*x + e))*sinh(f*x + e))
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
```

```
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^5*b^2 - 3*a^4*b^3 +
3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^8 + 8*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
 - a^2*b^5)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^5*b^2 - 3*a^4*b^3 + 3*a^3*
b^4 - a^2*b^5)*f*sinh(f*x + e)^8 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a
^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^6 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4
 - a^2*b^5)*f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^
4 + a^2*b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4
*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^4 + 8*(7*(a^5*b^2 - 3*a^4*b^
3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^3 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4
*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^5*b
^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^4 + 30*(2*a^6*b - 7*a
^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^2 + (8*a^7 - 32*a
^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f)*sinh(f*x + e)^4
 + 4*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e
)^2 + 8*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*cosh(f*x + e)^5 +
10*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f*cosh(f*x + e)^
3 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4 - 3*a^2*b^5)*f
*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a
^2*b^5)*f*cosh(f*x + e)^6 + 15*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4
 + a^2*b^5)*f*cosh(f*x + e)^4 + 3*(8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b
^3 + 17*a^3*b^4 - 3*a^2*b^5)*f*cosh(f*x + e)^2 + (2*a^6*b - 7*a^5*b^2 + 9*a
^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f)*sinh(f*x + e)^2 + (a^5*b^2 - 3*a^4*b^3 + 3
*a^3*b^4 - a^2*b^5)*f + 8*((a^5*b^2 - 3*a^4*b^3 + 3*a^3*b^4 - a^2*b^5)*f*co
sh(f*x + e)^7 + 3*(2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*b^4 + a^2*b^5)*f
*cosh(f*x + e)^5 + (8*a^7 - 32*a^6*b + 51*a^5*b^2 - 41*a^4*b^3 + 17*a^3*b^4
 - 3*a^2*b^5)*f*cosh(f*x + e)^3 + (2*a^6*b - 7*a^5*b^2 + 9*a^4*b^3 - 5*a^3*
b^4 + a^2*b^5)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [B] time = 3.25782, size = 1831, normalized size = 13.66

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(5*a*b - 2*b^2)/(a^4*sqrt(b)*f - 2*a^3*b^(3/2)*f + a^2*b^(5/2)*f) - 1/
3*((((5*a^13*b^4*f^3 - 52*a^12*b^5*f^3 + 245*a^11*b^6*f^3 - 690*a^10*b^7*f^
3 + 1290*a^9*b^8*f^3 - 1680*a^8*b^9*f^3 + 1554*a^7*b^10*f^3 - 1020*a^6*b^11
*f^3 + 465*a^5*b^12*f^3 - 140*a^4*b^13*f^3 + 25*a^3*b^14*f^3 - 2*a^2*b^15*f
^3)*e^(2*f*x + 2*e)/(a^16*b^2*f^4 - 12*a^15*b^3*f^4 + 66*a^14*b^4*f^4 - 220
*a^13*b^5*f^4 + 495*a^12*b^6*f^4 - 792*a^11*b^7*f^4 + 924*a^10*b^8*f^4 - 79
2*a^9*b^9*f^4 + 495*a^8*b^10*f^4 - 220*a^7*b^11*f^4 + 66*a^6*b^12*f^4 - 12*
a^5*b^13*f^4 + a^4*b^14*f^4) + 3*(8*a^14*b^3*f^3 - 89*a^13*b^4*f^3 + 452*a^
```

12*b^5*f^3 - 1385*a^11*b^6*f^3 + 2850*a^10*b^7*f^3 - 4146*a^9*b^8*f^3 + 436
8*a^8*b^9*f^3 - 3354*a^7*b^10*f^3 + 1860*a^6*b^11*f^3 - 725*a^5*b^12*f^3 +
188*a^4*b^13*f^3 - 29*a^3*b^14*f^3 + 2*a^2*b^15*f^3)/(a^16*b^2*f^4 - 12*a^1
5*b^3*f^4 + 66*a^14*b^4*f^4 - 220*a^13*b^5*f^4 + 495*a^12*b^6*f^4 - 792*a^1
1*b^7*f^4 + 924*a^10*b^8*f^4 - 792*a^9*b^9*f^4 + 495*a^8*b^10*f^4 - 220*a^7
*b^11*f^4 + 66*a^6*b^12*f^4 - 12*a^5*b^13*f^4 + a^4*b^14*f^4))*e^(2*f*x + 2
e) - 3(8*a^14*b^3*f^3 - 89*a^13*b^4*f^3 + 452*a^12*b^5*f^3 - 1385*a^11*b^
6*f^3 + 2850*a^10*b^7*f^3 - 4146*a^9*b^8*f^3 + 4368*a^8*b^9*f^3 - 3354*a^7*
b^10*f^3 + 1860*a^6*b^11*f^3 - 725*a^5*b^12*f^3 + 188*a^4*b^13*f^3 - 29*a^3
*b^14*f^3 + 2*a^2*b^15*f^3)/(a^16*b^2*f^4 - 12*a^15*b^3*f^4 + 66*a^14*b^4*f
^4 - 220*a^13*b^5*f^4 + 495*a^12*b^6*f^4 - 792*a^11*b^7*f^4 + 924*a^10*b^8*
f^4 - 792*a^9*b^9*f^4 + 495*a^8*b^10*f^4 - 220*a^7*b^11*f^4 + 66*a^6*b^12*f
^4 - 12*a^5*b^13*f^4 + a^4*b^14*f^4))*e^(2*f*x + 2*e) - (5*a^13*b^4*f^3 - 5
2*a^12*b^5*f^3 + 245*a^11*b^6*f^3 - 690*a^10*b^7*f^3 + 1290*a^9*b^8*f^3 - 1
680*a^8*b^9*f^3 + 1554*a^7*b^10*f^3 - 1020*a^6*b^11*f^3 + 465*a^5*b^12*f^3
- 140*a^4*b^13*f^3 + 25*a^3*b^14*f^3 - 2*a^2*b^15*f^3)/(a^16*b^2*f^4 - 12*a
^15*b^3*f^4 + 66*a^14*b^4*f^4 - 220*a^13*b^5*f^4 + 495*a^12*b^6*f^4 - 792*a
^11*b^7*f^4 + 924*a^10*b^8*f^4 - 792*a^9*b^9*f^4 + 495*a^8*b^10*f^4 - 220*a
^7*b^11*f^4 + 66*a^6*b^12*f^4 - 12*a^5*b^13*f^4 + a^4*b^14*f^4))/(b*e^(4*f*
x + 4*e) + 4*a*e^(2*f*x + 2*e) - 2*b*e^(2*f*x + 2*e) + b)^(3/2) + 2*arctan(
-1/2*(sqrt(b)*e^(2*f*x + 2*e) - sqrt(b*e^(4*f*x + 4*e) + 4*a*e^(2*f*x + 2*e
) - 2*b*e^(2*f*x + 2*e) + b) + sqrt(b))/sqrt(a - b))/((a^2*f - 2*a*b*f + b^
2*f)*sqrt(a - b))

3.395 $\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=330

$$\frac{(4a-b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^2b^2f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\left(8a^2-3ab-2b^2\right)\tanh(e+fx)\sqrt{}}{3a^2b^3f}$$

```
[Out] -((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(a - b)*(2*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt
[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e
 + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b^3*f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((4*a - b)*EllipticF[A
rcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
3*a^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*b^3*f)
```

Rubi [A] time = 0.331467, antiderivative size = 330, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3192, 413, 526, 531, 418, 492, 411}

$$\frac{\left(8a^2-3ab-2b^2\right)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2b^3f}-\frac{2(a-b)(2a+b)\sinh(e+fx)\cosh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}}+\frac{(4a-b)\text{sech}(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] -((a - b)*Cosh[e + f*x]^3*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(a - b)*(2*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*b^2*f*Sqrt
[a + b*Sinh[e + f*x]^2]) - ((8*a^2 - 3*a*b - 2*b^2)*EllipticE[ArcTan[Sinh[e
 + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b^3*f*
Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((4*a - b)*EllipticF[A
rcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(
3*a^2*b^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((8*a^2 -
3*a*b - 2*b^2)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*b^3*f)
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
```

```
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^6(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(1+x^2\right)^{5/2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+}}{}\right.}{3abf}$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} +$$

$$= -\frac{(a-b)\cosh^3(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-b)(2a+b)\cosh(e+fx)\sinh(e+fx)}{3a^2b^2f\sqrt{a+b\sinh^2(e+fx)}} -$$

Mathematica [C] time = 2.13341, size = 206, normalized size = 0.62

$$\frac{\frac{1}{2}(a-b)\left(-2\sqrt{2}b\sinh(2(e+fx))\left(8a^2+b(5a+2b)\cosh(2(e+fx))+ab-2b^2\right)+4ia^2(8a+b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\right.}{6a^2b^3f(2a+b\cosh(2(e+fx))-b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^6/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*I)*a^2*(8*a^2 - 3*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/
2)*EllipticE[I*(e + f*x), b/a] + ((a - b)*((4*I)*a^2*(8*a + b)*((2*a - b +
b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] - 2*Sqrt[2]*b*(8*
a^2 + a*b - 2*b^2 + b*(5*a + 2*b)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)]))/2)
/(6*a^2*b^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.133, size = 812, normalized size = 2.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((5*(-1/a*b)^(1/2)*a^2*b-3*(-1/a*b)^(1/2)*a*b^2-2*(-1/a*b)^(1/2)*b^3)*
sinh(f*x+e)*cosh(f*x+e)^4+(4*(-1/a*b)^(1/2)*a^3-6*(-1/a*b)^(1/2)*a^2*b+2*(-
1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
```

```
2)*(cosh(f*x+e)^2)^(1/2)*b*(4*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a^2-2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*Elliptic
F(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-8*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a^2+3*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b+2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e
)^2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-6*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a
^2*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3-8*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a^3+11*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(s
inh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)/a^2/(a+b*sinh(f*x+e)^2)^(3/2)/(
-1/a*b)^(1/2)/b^2/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{6}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\cosh\left(fx+e\right)^{6}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^6/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**6/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{6}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^6/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.396 $\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^2bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx}{\sqrt{a}}\right.\right.}{3a^{3/2}b^{3/2}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh}{a+b\sinh}}}$$

```
[Out] -((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) + (2*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqr
t[a]], 1 - a/b])/(3*a^(3/2)*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[
e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (EllipticF[ArcTan[Sinh[e + f*x]
], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b*f*Sqrt[(Sec
h[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rubi [A] time = 0.210649, antiderivative size = 223, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3192, 413, 525, 418, 411}

$$\frac{2(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}b^{3/2}f\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3a^2bf\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] -((a - b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) + (2*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqr
t[a]], 1 - a/b])/(3*a^(3/2)*b^(3/2)*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[
e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) - (EllipticF[ArcTan[Sinh[e + f*x]
], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*b*f*Sqrt[(Sec
h[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{a+2\ldots}{\sqrt{1+x\ldots}}\right.}{3abf}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\ldots}{\sqrt{1+x\ldots}}\right.}{3abf}$$

$$= -\frac{(a-b)\cosh(e+fx)\sinh(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\middle|1\ldots\right.}{3a^{3/2}b^{3/2}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+\ldots}}$$

Mathematica [C] time = 1.3781, size = 178, normalized size = 0.8

$$\frac{-ia^2(2a+b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(a^2+b(a+b)\cosh(2(e+fx))+2a\ldots\right.}{3a^2b^2f(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*(e
 + f*x), b/a] - I*a^2*(2*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*E
llipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(a^2 + 2*a*b - b^2 + b*(a + b)*Cosh[
2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*b^2*f*(2*a - b + b*Cosh[2*(e + f*x)
])^(3/2))
```

Maple [B] time = 0.253, size = 597, normalized size = 2.7

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((2*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+
((-1/a*b)^(1/2)*a^2+(-1/a*b)^(1/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*
sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(a*El
lipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))-2*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a-2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))*cosh(f*x+e)^2+
a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-2*(b/
a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f
*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+2*(b/a
*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*b^2)/a^2/(a+b*sinh(f*x+e)^2)^(3/2)/(-1/a*b)^(1/2
)/b/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\cosh\left(fx+e\right)^4}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.397 $\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=228

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^2f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right.}{3a^{3/2}\sqrt{b}f(a-b)\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}$$

```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((a -
 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 -
a/b])/(3*a^(3/2)*(a - b)*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e +
 f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + (EllipticF[ArcTan[Sinh[e + f*x]],
1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)*f*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rubi [A] time = 0.20429, antiderivative size = 228, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3192, 412, 525, 418, 411}

$$\frac{(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}\sqrt{b}f(a-b)\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{3a^2f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] (Cosh[e + f*x]*Sinh[e + f*x])/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((a -
 2*b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 -
a/b])/(3*a^(3/2)*(a - b)*Sqrt[b]*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e +
 f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + (EllipticF[ArcTan[Sinh[e + f*x]],
1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)*f*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```


Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\cosh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{-2-x^2}{\sqrt{1+x^2}\left(a+bx^2\right)}\right.}{3af}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{a+bx^2}}\right.}{3a(a-b)f}$$

$$= \frac{\cosh(e+fx)\sinh(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(a-2b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}(a-b)\sqrt{b}f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] time = 1.40191, size = 193, normalized size = 0.85

$$\frac{-2ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right)-\sqrt{2}b\sinh(2(e+fx))\left(-4a^2-b(a-2b)\cosh(2(e+fx))-\right.}{6a^2bf(a-b)(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(a - 2*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I*
(e + f*x), b/a] - (2*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/
2)*EllipticF[I*(e + f*x), b/a] - Sqrt[2]*b*(-4*a^2 + 7*a*b - 2*b^2 - (a - 2
*b)*b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(6*a^2*(a - b)*b*f*(2*a - b + b
*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.151, size = 662, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*a*b*sinh(f*x+e)^5-2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5+2*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2
)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b
^2*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^2+2*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*b^2*sinh(f*x+e)^2+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^3-2*(-1/
a*b)^(1/2)*a*b*sinh(f*x+e)^3-2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+2*a^2*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))-2*a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/
2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b-((a+b*sinh(f*x+e)^2)
/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))*a^2+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(
sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+2*(-1/a*b)^(1/2)*a^2*sinh(f*x+e
)-3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/a^2/(a-b)/(a+b*sinh(f*x+e)^2)^(3/2)/(-1
/a*b)^(1/2)/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\cosh\left(fx+e\right)^{2}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*cosh(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(fx+e)^2}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cosh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.398 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{3af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie\right.}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+}}$$

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.280601, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+fx}{3af(a-b)\left(a+b\sinh^2(e+fx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{-a(}{}}{}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{}{\sqrt{a}}}{}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2(2a\right.}{}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{2i(2a}{}$$

Mathematica [A] time = 1.26375, size = 190, normalized size = 0.76

$$\frac{ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(-5a^2+b(b-2a)\cosh(2(e+fx))+5\right.}{3a^2f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

Maple [A] time = 0., size = 406, normalized size = 1.6

$$\frac{1}{\cosh\left(fx+e\right)f}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\left(-\frac{\sinh\left(fx+e\right)}{3\,ab\left(a-b\right)}\sqrt{\left(a+b\left(\sinh\left(fx+e\right)\right)^{2}\right)\left(\cosh\left(fx+e\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/
2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a
*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*
x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh \left(fx + e\right)^2 + a\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

3.399 $\int \frac{\mathrm{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{b(9a-b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\mathrm{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^2f(a-b)^3\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\sqrt{b}\left(3a^2+7ab-2b^2\right)\cosh(e+fx}{3a^{3/2}f(a-b)^3\sqrt{a+b\sinh}}$$

```
[Out] (b*(3*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*(a + b*Sinh[e +
f*x]^2)^(3/2)) + (Sqrt[b]*(3*a^2 + 7*a*b - 2*b^2)*Cosh[e + f*x]*EllipticE[A
rcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*(a - b)^3*f*Sq
rt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]
) - ((9*a - b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sq
rt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*
Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3
/2))
```

Rubi [A] time = 0.313725, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3192, 414, 527, 525, 418, 411}

$$\frac{\sqrt{b}\left(3a^2+7ab-2b^2\right)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3a^{3/2}f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}-\frac{b(9a-b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}\right.}{3a^2f(a-b)^3\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (b*(3*a + b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)^2*f*(a + b*Sinh[e +
f*x]^2)^(3/2)) + (Sqrt[b]*(3*a^2 + 7*a*b - 2*b^2)*Cosh[e + f*x]*EllipticE[A
rcTan[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*a^(3/2)*(a - b)^3*f*Sq
rt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]
) - ((9*a - b)*b*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sq
rt[a + b*Sinh[e + f*x]^2])/(3*a^2*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*
Sinh[e + f*x]^2))/a]) + Tanh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3
/2))
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
```

```
&& !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] && !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\operatorname{sech}^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{b-3}{\sqrt{1+x^2}(a}\right.}{(-a+b)f}$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cosh^2(\right.}{}$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left((9a-b)b\right.}{}$$

$$= \frac{b(3a+b)\cosh(e+fx)\sinh(e+fx)}{3a(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\sqrt{b}\left(3a^2+7ab-2b^2\right)\cosh(e+fx)E\left(\tan^{-1}\left(\right.\right.}{3a^{3/2}(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\,\mathrm{s}}}$$

Mathematica [C] time = 3.45983, size = 260, normalized size = 0.89

$$\frac{-2ia^2\left(3a^2-2ab-b^2\right)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\frac{\tanh(e+fx)\left(4ab\left(6a^2+5ab-3b^2\right)\cosh(2(e+fx))+b^2\left(3a^2+7ab-2b^2\right)\right.}{\sqrt{2}}}{6a^2f(a-b)^3(2a+b\cosh(2(e+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*I)*a^2*(3*a^2 + 7*a*b - 2*b^2)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2
)*EllipticE[I*(e + f*x), b/a] - (2*I)*a^2*(3*a^2 - 2*a*b - b^2)*((2*a - b +
 b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticF[I*(e + f*x), b/a] + ((24*a^4 - 24*
a^3*b + 41*a^2*b^2 - 19*a*b^3 + 2*b^4 + 4*a*b*(6*a^2 + 5*a*b - 3*b^2)*Cosh[
2*(e + f*x)] + b^2*(3*a^2 + 7*a*b - 2*b^2)*Cosh[4*(e + f*x)])*Tanh[e + f*x]
)/Sqrt[2])/(6*a^2*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.166, size = 1002, normalized size = 3.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(3*(-1/a*b)^(1/2)*a^2*b^2*sinh(f*x+e)^5+7*(-1/a*b)^(1/2)*a*b^3*sinh(f*x
+e)^5-2*(-1/a*b)^(1/2)*b^4*sinh(f*x+e)^5-6*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b
^2*sinh(f*x+e)^2+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Elli
pticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3*sinh(f*x+e)^2-2*((a+b*s
inh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*b^4*sinh(f*x+e)^2-3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b^2*
sinh(f*x+e)^2-7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3*sinh(f*x+e)^2+2*((a+b*sinh
(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))*b^4*sinh(f*x+e)^2+6*(-1/a*b)^(1/2)*a^3*b*sinh(f*x+e)^3+8*(
-1/a*b)^(1/2)*a^2*b^2*sinh(f*x+e)^3+4*(-1/a*b)^(1/2)*a*b^3*sinh(f*x+e)^3-2*
(-1/a*b)^(1/2)*b^4*sinh(f*x+e)^3-6*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3*b+8*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*a^2*b^2-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^3-3*((a+b*si
nh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^
(1/2),(a/b)^(1/2))*a^3*b-7*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b^2+2*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))*a*b^3+3*(-1/a*b)^(1/2)*a^4*sinh(f*x+e)+8*(-1/a*b)^(1/2)*a^2
*b^2*sinh(f*x+e)-3*(-1/a*b)^(1/2)*a*b^3*sinh(f*x+e))/(-1/a*b)^(1/2)/(a+b*si
nh(f*x+e)^2)^(3/2)/a^2/(a-b)^3/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,\text{sech}\left(fx+e\right)^{2}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*sech(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\text{sech}\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sech(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.400 $\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=117

$$\frac{d\sinh(e+fx)\cosh^2(e+fx)^{\frac{1-m}{2}}(d\cosh(e+fx))^{m-1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};-\sinh^2\right.}{f}$$

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2
)/a)]*(d*Cosh[e + f*x])^(-1 + m)*(Cosh[e + f*x]^2)^((1 - m)/2)*Sinh[e + f*x
]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.110778, antiderivative size = 117, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3193, 430, 429}

$$\frac{d\sinh(e+fx)\cosh^2(e+fx)^{\frac{1-m}{2}}(d\cosh(e+fx))^{m-1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};-\sinh^2\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2
)/a)]*(d*Cosh[e + f*x])^(-1 + m)*(Cosh[e + f*x]^2)^((1 - m)/2)*Sinh[e + f*x
]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 3193

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(
ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Cos[e + f*x])^(2*FracPart[(m - 1)/2]))/(
f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/
2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f
, m, p}, x] &&  !IntegerQ[m]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p dx = \frac{\left(d(d\cosh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cosh^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int\left(1+x^2\right)^{\frac{1}{2}(-}\right.}{f}$$

$$= \frac{\left(d(d\cosh(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cosh^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\left(a+b\sinh^2(e+fx)\right)^p\right.}{}$$

$$= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)(d\cosh(e+fx))^{-1}}{}$$

Mathematica [F] time = 8.18999, size = 0, normalized size = 0.

$$\int (d\cosh(e+fx))^m \left(a+b\sinh^2(e+fx)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[(d*Cosh[e + f*x])^m*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.51, size = 0, normalized size = 0.

$$\int \left(d\cosh\left(fx+e\right)\right)^m \left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\cosh\left(fx+e\right)\right)^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\cosh\left(fx+e\right)\right)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))**m*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\left(d\cosh\left(fx+e\right)\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cosh(f*x+e))^m*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*cosh(f*x + e))^m, x)
```

3.401 $\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=214

$$\frac{\left(3a^2-2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin}{}\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] -(((3*a - b*(7 + 2*p))*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b^2*
f*(3 + 2*p)*(5 + 2*p))) + (Cosh[e + f*x]^2*Sinh[e + f*x]*(a + b*Sinh[e + f*
x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 2*a*b*(5 + 2*p) + b^2*(15 + 16*p
 + 4*p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e
 + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sinh[
e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.20449, antiderivative size = 214, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 416, 388, 246, 245}

$$\frac{\left(3a^2-2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin}{}\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a - b*(7 + 2*p))*Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b^2*
f*(3 + 2*p)*(5 + 2*p))) + (Cosh[e + f*x]^2*Sinh[e + f*x]*(a + b*Sinh[e + f*
x]^2)^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 - 2*a*b*(5 + 2*p) + b^2*(15 + 16*p
 + 4*p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e
 + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sinh[
e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
```

```
c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+bx^2\right)^p\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\cosh^2(e+fx)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(5+2p)} + \frac{\text{Subst}\left(\int \left(a+bx^2\right)\right.}{}$$

$$= -\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\cosh^2(e+fx)\,\text{s}}{}$$

$$= -\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\cosh^2(e+fx)\,\text{s}}{}$$

$$= -\frac{(3a-b(7+2p))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} + \frac{\cosh^2(e+fx)\,\text{s}}{}$$

Mathematica [F] time = 10.606, size = 0, normalized size = 0.

$$\int \cosh^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Cosh[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.645, size = 0, normalized size = 0.

$$\int \left(\cosh\left(fx+e\right)\right)^5\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{p} \cosh \left(fx+e\right)^{5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b \sinh \left(fx+e\right)^{2}+a\right)^{p} \cosh \left(fx+e\right)^{5}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**5*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{p} \cosh \left(fx+e\right)^{5} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^5, x)
```

3.402 $\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=125

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{p+1}}{bf(2p+3)}-\frac{(a-b(2p+3))\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-\right.}{bf(2p+3)}$$

```
[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) - ((a - b*(
3 + 2*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e
+ f*x]*(a + b*Sinh[e + f*x]^2)^p)/(b*f*(3 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a
)^p)
```

Rubi [A] time = 0.102418, antiderivative size = 119, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 388, 246, 245}

$$\frac{\left(1-\frac{a}{2bp+3b}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}+\frac{\sinh(e+fx)\left(a+b\sinh\right.}{bf(2p+}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p)) + ((1 - a/(
3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sin
h[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
```

```
GtQ[a, 0])
```

Rubi steps

$$\int \cosh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+bx^2\right)^p\,dx,x,\sinh(e+fx)\right)}{f}$$
$$=\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)}+\frac{\left(1-\frac{a}{3b+2bp}\right)\text{Subst}\left(\int\left(a+bx^2\right.\right.}{f}$$
$$=\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)}+\frac{\left(\left(1-\frac{a}{3b+2bp}\right)\left(a+b\sinh^2(e+\right.\right.}{}$$
$$=\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{1+p}}{bf(3+2p)}+\frac{\left(1-\frac{a}{3b+2bp}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\,\mathrm{s}}{}\right.}{}$$

Mathematica [A] time = 0.20196, size = 120, normalized size = 0.96

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\left((b(2p+3)-a)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)+\left(a+b\sinh^2(e\right.\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Sinh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p*((-a + b*(3 + 2*p))*Hypergeometric
2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)] + (a + b*Sinh[e + f*x]^2)*(1 +
(b*Sinh[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Maple [F] time = 0.453, size = 0, normalized size = 0.

$$\int\left(\cosh\left(fx+e\right)\right)^3\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^3, x)
```

3.403 $\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a
 + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0462517, antiderivative size = 67, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 246, 245}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a
 + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\int \cosh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(a+bx^2\right)^p\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^2}{a}\right)^p\,dx,x,\sinh\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(\right.}{a}}{f}$$

Mathematica [A] time = 0.0255693, size = 67, normalized size = 1.

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sinh[e + f*x]^2)/a)]*Sinh[e + f*x]*(a
 + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Maple [F] time = 0.341, size = 0, normalized size = 0.

$$\int \cosh\left(fx+e\right)\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(fx+e\right)^{2}+a\right)^{p} \cosh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e), x)
```

3.404 $\int \mathbf{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh
[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0757751, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 430, 429}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh
[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{1+x^2}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\operatorname{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1+x^2}\,dx, x, \sin\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e\right.}{f}$$

Mathematica [F] time = 3.3539, size = 0, normalized size = 0.

$$\int \operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sech[e + f*x]*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int \operatorname{sech}(fx+e)\left(a+b\left(\sinh(fx+e)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(fx+e)^2+a\right)^p\operatorname{sech}(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh(fx+e)^2+a\right)^p\operatorname{sech}(fx+e), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \operatorname{sech}\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e), x)
```

3.405 $\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=78

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh
[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0819552, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3190, 430, 429}

$$\frac{\sinh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sinh
[e + f*x]*(a + b*Sinh[e + f*x]^2)^p)/(f*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{(a+bx^2)^p}{(1+x^2)^2}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^2}{a}\right)^p}{(1+x^2)^2}\,dx,x,\sinh(\right.}{f}$$

$$=\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sinh(e+fx)\left(a+b\sinh^2(e+\right.}{f}$$

Mathematica [F] time = 5.45435, size = 0, normalized size = 0.

$$\int \operatorname{sech}^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sech[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.295, size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}\left(fx+e\right)\right)^3\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**3*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{p} \operatorname{sech}\left(fx+e\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^3*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^3, x)
```

3.406 $\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, -3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Si
nh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0856548, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Si
nh[e + f*x]^2)/a)^p)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^p\,dx,x,\mathrm{si}\right.}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\mathrm{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\right.}{f}$$

Mathematica [F] time = 9.8837, size = 0, normalized size = 0.

$$\int \cosh^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Cosh[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.482, size = 0, normalized size = 0.

$$\int \left(\cosh\left(fx+e\right)\right)^4\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{p} \cosh \left(fx+e\right)^{4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^4, x)
```

3.407 $\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+f\ldots}{a}\right.}{f}$$

```
[Out] (AppellF1[1/2, -1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Si
nh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0822861, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+f\ldots}{a}\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*S
qrt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Si
nh[e + f*x]^2)/a)^p)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\sqrt{1+x^2}\left(a+bx^2\right)^p\,dx,x,\sinh(e\right.}{f}$$
$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)}{f}$$
$$= \frac{F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh\right.}{f}$$

Mathematica [F] time = 10.0742, size = 0, normalized size = 0.

$$\int \cosh^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Cosh[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.365, size = 0, normalized size = 0.

$$\int \left(\cosh\left(fx+e\right)\right)^2\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\cosh\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^{2}+a\right)^{p}\cosh\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*cosh(f*x + e)^2, x)
```

3.408 $\int \left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0515143, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3185, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 3185

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f
*x]), Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^p}{\sqrt{1+x^2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{(}{}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh^2(e+fx)\right)^p\left(\right.}{f}$$

Mathematica [F] time = 2.13693, size = 0, normalized size = 0.

$$\int \left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.145, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p, x)
```

3.409 $\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0828345, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(a+bx^2)^p}{(1+x^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-p}\right)}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sinh^2\right.}{f}$$

Mathematica [F] time = 4.18865, size = 0, normalized size = 0.

$$\int \operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.257, size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}\left(fx+e\right)\right)^2\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**2*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(fx+e\right)^{2}+a\right)^{p} \operatorname{sech}\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^2*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^2, x)
```

3.410 $\int \mathrm{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=92

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

```
[Out] (AppellF1[1/2, 5/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.0818326, antiderivative size = 92, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cosh^2(e+fx)}\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 5/2, -p, 3/2, -Sinh[e + f*x]^2, -((b*Sinh[e + f*x]^2)/a)]*Sq
rt[Cosh[e + f*x]^2]*(a + b*Sinh[e + f*x]^2)^p*Tanh[e + f*x])/(f*(1 + (b*Sin
h[e + f*x]^2)/a)^p)
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int\frac{(a+bx^2)^p}{(1+x^2)^{5/2}}\,dx,x,\sinh(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh^2(e+fx)}{a}\right)^{-}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\sqrt{\cosh^2(e+fx)}\left(a+b\sin\right.}{f}$$

Mathematica [F] time = 7.95743, size = 0, normalized size = 0.

$$\int \operatorname{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p,x]
```

```
[Out] Integrate[Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^p, x]
```

Maple [F] time = 0.256, size = 0, normalized size = 0.

$$\int \left(\operatorname{sech}\left(fx+e\right)\right)^4\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

```
[Out] int(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\operatorname{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p\operatorname{sech}\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)**4*(a+b*sinh(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^p \operatorname{sech}\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(f*x+e)^4*(a+b*sinh(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*sech(f*x + e)^4, x)
```

3.411 $\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=259

$$\frac{2a^2\sinh^{\frac{7}{2}}(c+dx)}{7b^3d}-\frac{a^3\sinh^3(c+dx)}{3b^4d}+\frac{2\left(a^4+2b^4\right)\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}-\frac{a\left(a^4+2b^4\right)\sinh^2(c+dx)}{2b^6d}+\frac{2a^2\left(a^4+2b^4\right)\sin}{3b^7d}$$

```
[Out] (-2*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*(a^4 + b^
4)^2*Sqrt[Sinh[c + d*x]])/(b^9*d) - (a^3*(a^4 + 2*b^4)*Sinh[c + d*x])/(b^8*
d) + (2*a^2*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) - (a*(a^4 + 2*b^4)
*Sinh[c + d*x]^2)/(2*b^6*d) + (2*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2))/(5*b^5*
d) - (a^3*Sinh[c + d*x]^3)/(3*b^4*d) + (2*a^2*Sinh[c + d*x]^(7/2))/(7*b^3*d
) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + (2*Sinh[c + d*x]^(9/2))/(9*b*d)
```

Rubi [A] time = 0.296316, antiderivative size = 259, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3223, 1890, 1620}

$$\frac{2a^2\sinh^{\frac{7}{2}}(c+dx)}{7b^3d}-\frac{a^3\sinh^3(c+dx)}{3b^4d}+\frac{2\left(a^4+2b^4\right)\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}-\frac{a\left(a^4+2b^4\right)\sinh^2(c+dx)}{2b^6d}+\frac{2a^2\left(a^4+2b^4\right)\sin}{3b^7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*(a^4 + b^
4)^2*Sqrt[Sinh[c + d*x]])/(b^9*d) - (a^3*(a^4 + 2*b^4)*Sinh[c + d*x])/(b^8*
d) + (2*a^2*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) - (a*(a^4 + 2*b^4)
*Sinh[c + d*x]^2)/(2*b^6*d) + (2*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2))/(5*b^5*
d) - (a^3*Sinh[c + d*x]^3)/(3*b^4*d) + (2*a^2*Sinh[c + d*x]^(7/2))/(7*b^3*d
) - (a*Sinh[c + d*x]^4)/(4*b^2*d) + (2*Sinh[c + d*x]^(9/2))/(9*b*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1890

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+b\sqrt{x}}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^4)^2}{a+bx}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{(a^4+b^4)^2}{b^9} - \frac{a^3(a^4+2b^4)x}{b^8} + \frac{a^2(a^4+2b^4)x^2}{b^7} - \frac{a(a^4+2b^4)x^3}{b^6} + \frac{(a^4+2b^4)x^4}{b^5} - \frac{a^3x^5}{b^4} + \frac{a^2x^6}{b^3} - \frac{ax^7}{b^2} -\right.\right.}{d}$$
$$= -\frac{2a(a^4+b^4)^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d} + \frac{2(a^4+b^4)^2\sqrt{\sinh(c+dx)}}{b^9d} - \frac{a^3(a^4+2b^4)\sinh}{b^8d}$$

Mathematica [A] time = 0.443508, size = 220, normalized size = 0.85

$$\frac{360a^2b^7\sinh^{\frac{7}{2}}(c+dx) - 420a^3b^6\sinh^3(c+dx) + 504b^5(a^4+2b^4)\sinh^{\frac{5}{2}}(c+dx) - 630ab^4(a^4+2b^4)\sinh^2(c+dx) + 84}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2520*a*(a^4 + b^4)^2*Log[a + b*Sqrt[Sinh[c + d*x]]] + 2520*b*(a^4 + b^4)^
2*Sqrt[Sinh[c + d*x]] - 1260*a^3*b^2*(a^4 + 2*b^4)*Sinh[c + d*x] + 840*a^2*
b^3*(a^4 + 2*b^4)*Sinh[c + d*x]^(3/2) - 630*a*b^4*(a^4 + 2*b^4)*Sinh[c + d*
x]^2 + 504*b^5*(a^4 + 2*b^4)*Sinh[c + d*x]^(5/2) - 420*a^3*b^6*Sinh[c + d*x
]^3 + 360*a^2*b^7*Sinh[c + d*x]^(7/2) - 315*a*b^8*Sinh[c + d*x]^4 + 280*b^9
*Sinh[c + d*x]^(9/2))/(1260*b^10*d)
```

Maple [C] time = 0.105, size = 780, normalized size = 3.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x)
```

```
[Out] -a/d/b^2*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-1/4*a/
d/b^2/(tanh(1/2*d*x+1/2*c)+1)^4+a^7/d/b^8/(tanh(1/2*d*x+1/2*c)-1)+2*a^3/d/b
^4/(tanh(1/2*d*x+1/2*c)-1)+a^7/d/b^8/(tanh(1/2*d*x+1/2*c)+1)+1/2*a^5/d/b^6/
(tanh(1/2*d*x+1/2*c)+1)+2*a^3/d/b^4/(tanh(1/2*d*x+1/2*c)+1)+1/3*a^3/d/b^4/(
tanh(1/2*d*x+1/2*c)-1)^3-1/2*a^5/d/b^6/(tanh(1/2*d*x+1/2*c)-1)^2+1/2*a^3/d/
b^4/(tanh(1/2*d*x+1/2*c)-1)^2+a^9/d/b^10*ln(tanh(1/2*d*x+1/2*c)-1)+`int/ind
ef0`(-cosh(d*x+c)^4*b*sinh(d*x+c)^(1/2)/(-b^2*sinh(d*x+c)+a^2),sinh(d*x+c))
/d-9/8/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2*a-9/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^
2*a+7/8/d/b^2/(tanh(1/2*d*x+1/2*c)+1)*a-7/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a
+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+1/
2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^3*a-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3*a-1
/2*a^5/d/b^6/(tanh(1/2*d*x+1/2*c)-1)+2*a^5/d/b^6*ln(tanh(1/2*d*x+1/2*c)-1)+
1/3*a^3/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^3-1/2*a^5/d/b^6/(tanh(1/2*d*x+1/2*c)+
1)^2-1/2*a^3/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^2+a^9/d/b^10*ln(tanh(1/2*d*x+1/2
*c)+1)+2*a^5/d/b^6*ln(tanh(1/2*d*x+1/2*c)+1)-a^9/d/b^10*ln(a^2*tanh(1/2*d*x
```

```
+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-2*a^5/d/b^6*ln(a^2*tanh(1/2*d*x+1/
2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-1/4*a/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^4
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^5}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

Fricas [B] time = 8.19125, size = 6338, normalized size = 24.47

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/20160*(315*a*b^8*cosh(d*x + c)^8 + 315*a*b^8*sinh(d*x + c)^8 + 840*a^3*b
^6*cosh(d*x + c)^7 - 840*a^3*b^6*cosh(d*x + c) + 315*a*b^8 + 840*(3*a*b^8*c
osh(d*x + c) + a^3*b^6)*sinh(d*x + c)^7 + 1260*(2*a^5*b^4 + 3*a*b^8)*cosh(d
*x + c)^6 + 420*(21*a*b^8*cosh(d*x + c)^2 + 14*a^3*b^6*cosh(d*x + c) + 6*a^
5*b^4 + 9*a*b^8)*sinh(d*x + c)^6 + 2520*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x +
c)^5 + 2520*(7*a*b^8*cosh(d*x + c)^3 + 7*a^3*b^6*cosh(d*x + c)^2 + 4*a^7*b^
2 + 7*a^3*b^6 + 3*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c))*sinh(d*x + c)^5 - 20
160*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*cosh(d*x
+ c)^4 + 210*(105*a*b^8*cosh(d*x + c)^4 + 140*a^3*b^6*cosh(d*x + c)^3 - 96*
(a^9 + 2*a^5*b^4 + a*b^8)*d*x + 90*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^2 -
96*(a^9 + 2*a^5*b^4 + a*b^8)*c + 60*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c))*
sinh(d*x + c)^4 - 2520*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c)^3 + 840*(21*a*
b^8*cosh(d*x + c)^5 + 35*a^3*b^6*cosh(d*x + c)^4 - 12*a^7*b^2 - 21*a^3*b^6
+ 30*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^3 + 30*(4*a^7*b^2 + 7*a^3*b^6)*cos
h(d*x + c)^2 - 96*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8
)*c)*cosh(d*x + c))*sinh(d*x + c)^3 + 1260*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x +
 c)^2 + 1260*(7*a*b^8*cosh(d*x + c)^6 + 14*a^3*b^6*cosh(d*x + c)^5 + 2*a^5*
b^4 + 3*a*b^8 + 15*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^4 + 20*(4*a^7*b^2 +
7*a^3*b^6)*cosh(d*x + c)^3 - 96*((a^9 + 2*a^5*b^4 + a*b^8)*d*x + (a^9 + 2*a
^5*b^4 + a*b^8)*c)*cosh(d*x + c)^2 - 6*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c
))*sinh(d*x + c)^2 - 20160*((a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^4 + 4*(
a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^9 + 2*a^5*b^4
 + a*b^8)*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*(a^9 + 2*a^5*b^4 + a*b^8)*cos
h(d*x + c)*sinh(d*x + c)^3 + (a^9 + 2*a^5*b^4 + a*b^8)*sinh(d*x + c)^4)*log
((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2*cosh(d*x + c) - b^2 + 2
*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*cosh(d*x + c) + a*b*sinh(
d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 -
 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh(d*x + c))) +
20160*((a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)^4 + 4*(a^9 + 2*a^5*b^4 + a*b
^8)*cosh(d*x + c)^3*sinh(d*x + c) + 6*(a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x +
c)^2*sinh(d*x + c)^2 + 4*(a^9 + 2*a^5*b^4 + a*b^8)*cosh(d*x + c)*sinh(d*x +
 c)^3 + (a^9 + 2*a^5*b^4 + a*b^8)*sinh(d*x + c)^4)*log(2*(b^2*sinh(d*x + c)
 - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 840*(3*a*b^8*cosh(d*x + c)^7 + 7
```

```
*a^3*b^6*cosh(d*x + c)^6 - a^3*b^6 + 9*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c)^
5 + 15*(4*a^7*b^2 + 7*a^3*b^6)*cosh(d*x + c)^4 - 96*((a^9 + 2*a^5*b^4 + a*b
^8)*d*x + (a^9 + 2*a^5*b^4 + a*b^8)*c)*cosh(d*x + c)^3 - 9*(4*a^7*b^2 + 7*a
^3*b^6)*cosh(d*x + c)^2 + 3*(2*a^5*b^4 + 3*a*b^8)*cosh(d*x + c))*sinh(d*x +
 c) - 8*(35*b^9*cosh(d*x + c)^8 + 35*b^9*sinh(d*x + c)^8 + 90*a^2*b^7*cosh(
d*x + c)^7 - 90*a^2*b^7*cosh(d*x + c) + 35*b^9 + 10*(28*b^9*cosh(d*x + c) +
 9*a^2*b^7)*sinh(d*x + c)^7 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^6 + 14*
(70*b^9*cosh(d*x + c)^2 + 45*a^2*b^7*cosh(d*x + c) + 18*a^4*b^5 + 26*b^9)*s
inh(d*x + c)^6 + 30*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^5 + 2*(980*b^9*
cosh(d*x + c)^3 + 945*a^2*b^7*cosh(d*x + c)^2 + 420*a^6*b^3 + 705*a^2*b^7 +
 84*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c))*sinh(d*x + c)^5 + 42*(120*a^8*b + 2
28*a^4*b^5 + 101*b^9)*cosh(d*x + c)^4 + 2*(1225*b^9*cosh(d*x + c)^4 + 1575*
a^2*b^7*cosh(d*x + c)^3 + 2520*a^8*b + 4788*a^4*b^5 + 2121*b^9 + 210*(9*a^4
*b^5 + 13*b^9)*cosh(d*x + c)^2 + 75*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)
)*sinh(d*x + c)^4 - 30*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^3 + 2*(980*b
^9*cosh(d*x + c)^5 + 1575*a^2*b^7*cosh(d*x + c)^4 - 420*a^6*b^3 - 705*a^2*b
^7 + 280*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^3 + 150*(28*a^6*b^3 + 47*a^2*b^
7)*cosh(d*x + c)^2 + 84*(120*a^8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c))*
sinh(d*x + c)^3 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c)^2 + 2*(490*b^9*cosh
(d*x + c)^6 + 945*a^2*b^7*cosh(d*x + c)^5 + 126*a^4*b^5 + 182*b^9 + 210*(9*
a^4*b^5 + 13*b^9)*cosh(d*x + c)^4 + 150*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x
+ c)^3 + 126*(120*a^8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c)^2 - 45*(28*a
^6*b^3 + 47*a^2*b^7)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(140*b^9*cosh(d*x +
 c)^7 + 315*a^2*b^7*cosh(d*x + c)^6 - 45*a^2*b^7 + 84*(9*a^4*b^5 + 13*b^9)*
cosh(d*x + c)^5 + 75*(28*a^6*b^3 + 47*a^2*b^7)*cosh(d*x + c)^4 + 84*(120*a^
8*b + 228*a^4*b^5 + 101*b^9)*cosh(d*x + c)^3 - 45*(28*a^6*b^3 + 47*a^2*b^7)
*cosh(d*x + c)^2 + 28*(9*a^4*b^5 + 13*b^9)*cosh(d*x + c))*sinh(d*x + c))*sq
rt(sinh(d*x + c)))/(b^10*d*cosh(d*x + c)^4 + 4*b^10*d*cosh(d*x + c)^3*sinh(
d*x + c) + 6*b^10*d*cosh(d*x + c)^2*sinh(d*x + c)^2 + 4*b^10*d*cosh(d*x + c
)*sinh(d*x + c)^3 + b^10*d*sinh(d*x + c)^4)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^5}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sqrt(sinh(d*x + c)) + a), x)
```

3.412 $\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=136

$$\frac{2a^2\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}-\frac{a^3\sinh(c+dx)}{b^4d}+\frac{2\left(a^4+b^4\right)\sqrt{\sinh(c+dx)}}{b^5d}-\frac{2a\left(a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}-\frac{a\sinh^2(}{2b}$$

```
[Out] (-2*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*(a^4 + b^4)*
Sqrt[Sinh[c + d*x]])/(b^5*d) - (a^3*Sinh[c + d*x])/(b^4*d) + (2*a^2*Sinh[c
+ d*x]^(3/2))/(3*b^3*d) - (a*Sinh[c + d*x]^2)/(2*b^2*d) + (2*Sinh[c + d*x]^
(5/2))/(5*b*d)
```

Rubi [A] time = 0.156701, antiderivative size = 136, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3223, 1890, 1620}

$$\frac{2a^2\sinh^{\frac{3}{2}}(c+dx)}{3b^3d}-\frac{a^3\sinh(c+dx)}{b^4d}+\frac{2\left(a^4+b^4\right)\sqrt{\sinh(c+dx)}}{b^5d}-\frac{2a\left(a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}-\frac{a\sinh^2(}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*(a^4 + b^4)*
Sqrt[Sinh[c + d*x]])/(b^5*d) - (a^3*Sinh[c + d*x])/(b^4*d) + (2*a^2*Sinh[c
+ d*x]^(3/2))/(3*b^3*d) - (a*Sinh[c + d*x]^2)/(2*b^2*d) + (2*Sinh[c + d*x]^
(5/2))/(5*b*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1890

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+b\sqrt{x}}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^4)}{a+bx}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{a^4+b^4}{b^5} - \frac{a^3x}{b^4} + \frac{a^2x^2}{b^3} - \frac{ax^3}{b^2} + \frac{x^4}{b} - \frac{a(a^4+b^4)}{b^5(a+bx)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= -\frac{2a\left(a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d} + \frac{2\left(a^4+b^4\right)\sqrt{\sinh(c+dx)}}{b^5d} - \frac{a^3\sinh(c+dx)}{b^4d} + \frac{2a}{}$$

Mathematica [A] time = 0.150835, size = 117, normalized size = 0.86

$$\frac{20a^2b^3\sinh^{\frac{3}{2}}(c+dx) - 30a^3b^2\sinh(c+dx) + 60b\left(a^4+b^4\right)\sqrt{\sinh(c+dx)} - 60a\left(a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right) - 15}{30b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-60*a*(a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + 60*b*(a^4 + b^4)*Sqrt[S
inh[c + d*x]] - 30*a^3*b^2*Sinh[c + d*x] + 20*a^2*b^3*Sinh[c + d*x]^(3/2) -
 15*a*b^4*Sinh[c + d*x]^2 + 12*b^5*Sinh[c + d*x]^(5/2))/(30*b^6*d)
```

Maple [C] time = 0.1, size = 359, normalized size = 2.6

$$-\frac{a^5}{db^6}\ln\left(a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + 2\,b^2\tanh(1/2\,dx+c/2) - a^2\right) - \frac{a}{db^2}\ln\left(a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2 + 2\,b^2\tanh(1/2\,dx+c/2)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x)
```

```
[Out] -a^5/d/b^6*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-a/d/
b^2*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-1/2/d/b^2/(
tanh(1/2*d*x+1/2*c)+1)^2*a+a^3/d/b^4/(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b^2/(tan
h(1/2*d*x+1/2*c)+1)*a+a^5/d/b^6*ln(tanh(1/2*d*x+1/2*c)+1)+1/d*a/b^2*ln(tanh
(1/2*d*x+1/2*c)+1)-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2*a+a^3/d/b^4/(tanh(1/
2*d*x+1/2*c)-1)-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)*a+a^5/d/b^6*ln(tanh(1/2*d
*x+1/2*c)-1)+1/d*a/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+`int/indef0`(-cosh(d*x+c)^
2*b*sinh(d*x+c)^(1/2)/(-b^2*sinh(d*x+c)+a^2),sinh(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

Fricas [B] time = 7.32631, size = 2182, normalized size = 16.04

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] -1/120*(15*a*b^4*cosh(d*x + c)^4 + 15*a*b^4*sinh(d*x + c)^4 + 60*a^3*b^2*co
sh(d*x + c)^3 - 60*a^3*b^2*cosh(d*x + c) + 15*a*b^4 + 60*(a*b^4*cosh(d*x +
c) + a^3*b^2)*sinh(d*x + c)^3 - 120*((a^5 + a*b^4)*d*x + (a^5 + a*b^4)*c)*c
osh(d*x + c)^2 + 30*(3*a*b^4*cosh(d*x + c)^2 + 6*a^3*b^2*cosh(d*x + c) - 4*
(a^5 + a*b^4)*d*x - 4*(a^5 + a*b^4)*c)*sinh(d*x + c)^2 - 120*((a^5 + a*b^4)
*cosh(d*x + c)^2 + 2*(a^5 + a*b^4)*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a*b
^4)*sinh(d*x + c)^2)*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2
*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*c
osh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2
 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) -
 a^2)*sinh(d*x + c))) + 120*((a^5 + a*b^4)*cosh(d*x + c)^2 + 2*(a^5 + a*b^4
)*cosh(d*x + c)*sinh(d*x + c) + (a^5 + a*b^4)*sinh(d*x + c)^2)*log(2*(b^2*s
inh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 60*(a*b^4*cosh(d*x +
 c)^3 + 3*a^3*b^2*cosh(d*x + c)^2 - a^3*b^2 - 4*((a^5 + a*b^4)*d*x + (a^5 +
 a*b^4)*c)*cosh(d*x + c))*sinh(d*x + c) - 4*(3*b^5*cosh(d*x + c)^4 + 3*b^5*
sinh(d*x + c)^4 + 10*a^2*b^3*cosh(d*x + c)^3 - 10*a^2*b^3*cosh(d*x + c) + 3
*b^5 + 2*(6*b^5*cosh(d*x + c) + 5*a^2*b^3)*sinh(d*x + c)^3 + 6*(10*a^4*b +
9*b^5)*cosh(d*x + c)^2 + 6*(3*b^5*cosh(d*x + c)^2 + 5*a^2*b^3*cosh(d*x + c)
 + 10*a^4*b + 9*b^5)*sinh(d*x + c)^2 + 2*(6*b^5*cosh(d*x + c)^3 + 15*a^2*b^
3*cosh(d*x + c)^2 - 5*a^2*b^3 + 6*(10*a^4*b + 9*b^5)*cosh(d*x + c))*sinh(d*
x + c))*sqrt(sinh(d*x + c)))/(b^6*d*cosh(d*x + c)^2 + 2*b^6*d*cosh(d*x + c)
*sinh(d*x + c) + b^6*d*sinh(d*x + c)^2)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a), x)
```

3.413 $\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=43

$$\frac{2\sqrt{\sinh(c+dx)}}{bd}-\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

```
[Out] (-2*a*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*Sqrt[Sinh[c + d*x]])/(b*
d)
```

Rubi [A] time = 0.050232, antiderivative size = 43, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3223, 190, 43}

$$\frac{2\sqrt{\sinh(c+dx)}}{bd}-\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (-2*a*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*Sqrt[Sinh[c + d*x]])/(b*
d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+b\sqrt{x}}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{a+bx}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(\frac{1}{b}-\frac{a}{b(a+bx)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= -\frac{2a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2 d}+\frac{2\sqrt{\sinh(c+dx)}}{bd}$$

Mathematica [A] time = 0.0274741, size = 41, normalized size = 0.95

$$\frac{2\left(\frac{\sqrt{\sinh(c+dx)}}{b}-\frac{a\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (2*(-((a*Log[a + b*Sqrt[Sinh[c + d*x]]])/b^2) + Sqrt[Sinh[c + d*x]]/b))/d
```

Maple [B] time = 0.013, size = 89, normalized size = 2.1

$$2\,\frac{\sqrt{\sinh(dx+c)}}{bd}+\frac{a}{db^2}\ln\left(b\sqrt{\sinh(dx+c)}-a\right)-\frac{a}{db^2}\ln\left(a+b\sqrt{\sinh(dx+c)}\right)-\frac{a\ln\left(b^2\sinh(dx+c)-a^2\right)}{db^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x)
```

```
[Out] 2*sinh(d*x+c)^(1/2)/b/d+1/d/b^2*a*ln(b*sinh(d*x+c)^(1/2)-a)-a*ln(a+b*sinh(d
*x+c)^(1/2))/b^2/d-1/d*a*ln(b^2*sinh(d*x+c)-a^2)/b^2
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Fricas [B] time = 6.3547, size = 563, normalized size = 13.09

$$\frac{adx+a\log\left(\frac{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2+2\,a^2\cosh(dx+c)-b^2+2\left(b^2\cosh(dx+c)+a^2\right)\sinh(dx+c)-4\left(ab\cosh(dx+c)+ab\sinh(dx+c)\right)\sqrt{\sinh(dx+c)}}{b^2\cosh(dx+c)^2+b^2\sinh(dx+c)^2-2\,a^2\cosh(dx+c)-b^2+2\left(b^2\cosh(dx+c)-a^2\right)\sinh(dx+c)}\right)-a}{b^2 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] (a*d*x + a*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2*cosh(d*x
+ c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) - 4*(a*b*cosh(d*x +
c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2 + b^2*sin
h(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh
(d*x + c))) - a*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x +
 c))) + 2*b*sqrt(sinh(d*x + c)))/(b^2*d)
```

Sympy [A] time = 1.90048, size = 68, normalized size = 1.58

$$\begin{cases} \frac{x \cosh{(c)}}{a} & \text{for}\: b = 0 \wedge d = 0 \\ \frac{\sinh{(c + dx)}}{ad} & \text{for}\: b = 0 \\ \frac{x \cosh{(c)}}{a + b \sqrt{\sinh{(c)}}} & \text{for}\: d = 0 \\ - \frac{2a \log{\left(\frac{a}{b} + \sqrt{\sinh{(c + dx)}} \right)}}{b^{2} d} + \frac{2 \sqrt{\sinh{(c + dx)}}}{bd} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Piecewise((x*cosh(c)/a, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a*d), Eq(b, 0
)), (x*cosh(c)/(a + b*sqrt(sinh(c))), Eq(d, 0)), (-2*a*log(a/b + sqrt(sinh(
c + d*x)))/(b**2*d) + 2*sqrt(sinh(c + d*x))/(b*d), True))
```

Giac [A] time = 1.23697, size = 92, normalized size = 2.14

$$-\frac{2\,a \log\left(\left|b\sqrt{\frac{1}{2}\,e^{(dx+c)} - \frac{1}{2}\,e^{(-dx-c)}} + a\right|\right)}{b^{2}d} + \frac{2\,\sqrt{\frac{1}{2}\,e^{(dx+c)} - \frac{1}{2}\,e^{(-dx-c)}}}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] -2*a*log(abs(b*sqrt(1/2*e^(d*x + c) - 1/2*e^(-d*x - c)) + a))/(b^2*d) + 2*s
qrt(1/2*e^(d*x + c) - 1/2*e^(-d*x - c))/(b*d)
```

3.414 $\int \frac{\text{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx$

Optimal. Leaf size=286

$$-\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{d\left(a^4+b^4\right)}-\frac{b\left(a^2+b^2\right)\log\left(\sinh(c+dx)-\sqrt{2}\sqrt{\sinh(c+dx)}+1\right)}{2\sqrt{2}d\left(a^4+b^4\right)}+\frac{b\left(a^2+b^2\right)\log\left(\sinh(c+dx)\right.}{2\sqrt{2}d\left(a^4\right.}$$

```
[Out] (b*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4
)*d) - (b*(a^2 - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^
4 + b^4)*d) + (a^3*ArcTan[Sinh[c + d*x]])/((a^4 + b^4)*d) + (a*b^2*Log[Cosh
[c + d*x]])/((a^4 + b^4)*d) - (2*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^
4 + b^4)*d) - (b*(a^2 + b^2)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c +
 d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d) + (b*(a^2 + b^2)*Log[1 + Sqrt[2]*Sqrt[Sin
h[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d)
```

Rubi [A] time = 0.482466, antiderivative size = 286, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3223, 6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$-\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{d\left(a^4+b^4\right)}-\frac{b\left(a^2+b^2\right)\log\left(\sinh(c+dx)-\sqrt{2}\sqrt{\sinh(c+dx)}+1\right)}{2\sqrt{2}d\left(a^4+b^4\right)}+\frac{b\left(a^2+b^2\right)\log\left(\sinh(c+dx)\right.}{2\sqrt{2}d\left(a^4\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (b*(a^2 - b^2)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^4 + b^4
)*d) - (b*(a^2 - b^2)*ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]])/(Sqrt[2]*(a^
4 + b^4)*d) + (a^3*ArcTan[Sinh[c + d*x]])/((a^4 + b^4)*d) + (a*b^2*Log[Cosh
[c + d*x]])/((a^4 + b^4)*d) - (2*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^
4 + b^4)*d) - (b*(a^2 + b^2)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c +
 d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d) + (b*(a^2 + b^2)*Log[1 + Sqrt[2]*Sqrt[Sin
h[c + d*x]] + Sinh[c + d*x]])/(2*Sqrt[2]*(a^4 + b^4)*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
```

```
0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
```

```
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\text{sech}(c+dx)}{a+b\sqrt{\sinh(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)\left(1+x^2\right)}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{(a+bx)\left(1+x^4\right)}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{ab^3}{\left(a^4+b^4\right)(a+bx)}+\frac{b^3+a^3x-a^2bx^2+ab^2x^3}{\left(a^4+b^4\right)\left(1+x^4\right)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{2\,\text{Subst}\left(\int \frac{b^3+a^3x-a^2bx^2+ab^2x^3}{1+x^4}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}$$
$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{2\,\text{Subst}\left(\int \left(\frac{b^3-a^2bx^2}{1+x^4}+\frac{x\left(a^3+ab^2x^2\right)}{1+x^4}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}$$
$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{2\,\text{Subst}\left(\int \frac{b^3-a^2bx^2}{1+x^4}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{2\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{\text{Subst}\left(\int \frac{a^3+ab^2x}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{\left(a^4+b^4\right)d}-\frac{\left(b\left(a^2-b^2\right)\right)\text{Sul}}{}$$
$$= -\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}+\frac{a^3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \sinh(c+dx)\right)}{\left(a^4+b^4\right)d}+\frac{\left(ab^2\right)\text{Subst}\left(\int\right.}{(a}$$
$$= \frac{a^3\tan^{-1}(\sinh(c+dx))}{\left(a^4+b^4\right)d}+\frac{ab^2\log(\cosh(c+dx))}{\left(a^4+b^4\right)d}-\frac{2ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)d}-\frac{b\left(a^2+\right.}{}$$
$$= \frac{b\left(a^2-b^2\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\left(a^4+b^4\right)d}-\frac{b\left(a^2-b^2\right)\tan^{-1}\left(1+\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{\sqrt{2}\left(a^4+b^4\right)d}+\frac{a^3\,\text{t}}{}$$

Mathematica [C] time = 0.248221, size = 229, normalized size = 0.8

$$\frac{3\left(4a^3\tan^{-1}(\sinh(c+dx))-8ab^2\log\left(a+b\sqrt{\sinh(c+dx)}\right)+4ab^2\log(\cosh(c+dx))-\sqrt{2}b^3\log\left(\sinh(c+dx)-\sqrt{2}\sqrt{\text{si}}\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]]),x]
```

```
[Out] (3*(-2*Sqrt[2]*b^3*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 2*Sqrt[2]*b^3*
ArcTan[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]]] + 4*a^3*ArcTan[Sinh[c + d*x]] + 4*a
*b^2*Log[Cosh[c + d*x]] - 8*a*b^2*Log[a + b*Sqrt[Sinh[c + d*x]]] - Sqrt[2]*
b^3*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]] + Sqrt[2]*b^3*Log[
```

```
1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]]) - 8*a^2*b*Hypergeometric2
F1[3/4, 1, 7/4, -Sinh[c + d*x]^2]*Sinh[c + d*x]^(3/2))/(12*(a^4 + b^4)*d)
```

Maple [C] time = 0.108, size = 206, normalized size = 0.7

$$-\frac{ab^2}{d\left(a^4+b^4\right)}\ln\left(a^2\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+2\,b^2\tanh\left(1/2\,dx+c/2\right)-a^2\right)+\frac{ab^2}{d\left(a^4+b^4\right)}\ln\left(\left(\tanh\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2+1\right)+2\,\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x)
```

```
[Out] -a/d*b^2/(a^4+b^4)*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a
^2)+a/d/(a^4+b^4)*b^2*ln(tanh(1/2*d*x+1/2*c)^2+1)+2*a^3/d/(a^4+b^4)*arctan(
tanh(1/2*d*x+1/2*c))+`int/indef0`(b*sinh(d*x+c)^(1/2)*(-b^2*sinh(d*x+c)+a^2
)/(2*a^2*b^2*sinh(d*x+c)*cosh(d*x+c)^2-b^4*cosh(d*x+c)^4+(-a^4+b^4)*cosh(d*
x+c)^2),sinh(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(dx+c\right)}{b\sqrt{\sinh\left(dx+c\right)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}\left(c+dx\right)}{a+b\sqrt{\sinh\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2)),x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x))), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)}{b\sqrt{\sinh(dx+c)}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2)),x, algorithm="giac")
```

```
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a), x)
```

3.415 $\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=270

$$\frac{a^2\sinh^3(c+dx)}{b^4d}-\frac{8a^3\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}+\frac{\left(5a^4+2b^4\right)\sinh^2(c+dx)}{2b^6d}-\frac{4a\left(3a^4+2b^4\right)\sinh^{\frac{3}{2}}(c+dx)}{3b^7d}+\frac{a^2\left(7a^4+6b^4\right)\text{s}}{b^8d}$$

```
[Out] (2*(a^4 + b^4)*(9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*
a*(a^4 + b^4)^2)/(b^10*d*(a + b*Sqrt[Sinh[c + d*x]])) - (16*a^3*(a^4 + b^4)
*Sqrt[Sinh[c + d*x]])/(b^9*d) + (a^2*(7*a^4 + 6*b^4)*Sinh[c + d*x])/(b^8*d)
 - (4*a*(3*a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) + ((5*a^4 + 2*b^4)*S
inh[c + d*x]^2)/(2*b^6*d) - (8*a^3*Sinh[c + d*x]^(5/2))/(5*b^5*d) + (a^2*Si
nh[c + d*x]^3)/(b^4*d) - (4*a*Sinh[c + d*x]^(7/2))/(7*b^3*d) + Sinh[c + d*x
]^4/(4*b^2*d)
```

Rubi [A] time = 0.323083, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3223, 1890, 1620}

$$\frac{a^2\sinh^3(c+dx)}{b^4d}-\frac{8a^3\sinh^{\frac{5}{2}}(c+dx)}{5b^5d}+\frac{\left(5a^4+2b^4\right)\sinh^2(c+dx)}{2b^6d}-\frac{4a\left(3a^4+2b^4\right)\sinh^{\frac{3}{2}}(c+dx)}{3b^7d}+\frac{a^2\left(7a^4+6b^4\right)\text{s}}{b^8d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*(a^4 + b^4)*(9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^10*d) + (2*
a*(a^4 + b^4)^2)/(b^10*d*(a + b*Sqrt[Sinh[c + d*x]])) - (16*a^3*(a^4 + b^4)
*Sqrt[Sinh[c + d*x]])/(b^9*d) + (a^2*(7*a^4 + 6*b^4)*Sinh[c + d*x])/(b^8*d)
 - (4*a*(3*a^4 + 2*b^4)*Sinh[c + d*x]^(3/2))/(3*b^7*d) + ((5*a^4 + 2*b^4)*S
inh[c + d*x]^2)/(2*b^6*d) - (8*a^3*Sinh[c + d*x]^(5/2))/(5*b^5*d) + (a^2*Si
nh[c + d*x]^3)/(b^4*d) - (4*a*Sinh[c + d*x]^(7/2))/(7*b^3*d) + Sinh[c + d*x
]^4/(4*b^2*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1890

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
```

```
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^4)^2}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{8a^3(a^4+b^4)}{b^9}+\frac{a^2(7a^4+6b^4)x}{b^8}-\frac{2a(3a^4+2b^4)x^2}{b^7}+\frac{(5a^4+2b^4)x^3}{b^6}-\frac{4a^3x^4}{b^5}+\frac{3a^2x^5}{b^4}-\frac{2ax^6}{b^3}+\right.\right.}{d}$$

$$= \frac{2\left(a^4+b^4\right)\left(9a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^{10}d}+\frac{2a\left(a^4+b^4\right)^2}{b^{10}d\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{16a^3\left(a^4\right.}{}$$

Mathematica [A] time = 0.618676, size = 288, normalized size = 1.07

$$\frac{180a^2b^7\sinh^{\frac{7}{2}}(c+dx)-252a^3b^6\sinh^3(c+dx)+42b^5\left(9a^4+10b^4\right)\sinh^{\frac{5}{2}}(c+dx)-70ab^4\left(9a^4+10b^4\right)\sinh^2(c+dx)+}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (840*a*(a^4 + b^4)*(a^4 + b^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]
]) + 840*b*(a^4 + b^4)*(-8*a^4 + (9*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]
]])*Sqrt[Sinh[c + d*x]] - 420*a^3*b^2*(9*a^4 + 10*b^4)*Sinh[c + d*x] + 140*
a^2*b^3*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(3/2) - 70*a*b^4*(9*a^4 + 10*b^4)*Si
nh[c + d*x]^2 + 42*b^5*(9*a^4 + 10*b^4)*Sinh[c + d*x]^(5/2) - 252*a^3*b^6*S
inh[c + d*x]^3 + 180*a^2*b^7*Sinh[c + d*x]^(7/2) - 135*a*b^8*Sinh[c + d*x]^
4 + 105*b^9*Sinh[c + d*x]^(9/2))/(420*b^10*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

Maple [C] time = 0.218, size = 955, normalized size = 3.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] 9/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^2+9/8/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2-7/d
/b^8/(tanh(1/2*d*x+1/2*c)-1)*a^6+5/2/d/b^6/(tanh(1/2*d*x+1/2*c)-1)*a^4-6/d/
b^4/(tanh(1/2*d*x+1/2*c)-1)*a^2-1/d/b^4/(tanh(1/2*d*x+1/2*c)-1)^3*a^2+5/2/d
/b^6/(tanh(1/2*d*x+1/2*c)-1)^2*a^4-3/2/d/b^4/(tanh(1/2*d*x+1/2*c)-1)^2*a^2-
9/d/b^10*ln(tanh(1/2*d*x+1/2*c)-1)*a^8-10/d/b^6*ln(tanh(1/2*d*x+1/2*c)-1)*a
^4+9/d/b^10*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^8
+10/d/b^6*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4-7
/d/b^8/(tanh(1/2*d*x+1/2*c)+1)*a^6-5/2/d/b^6/(tanh(1/2*d*x+1/2*c)+1)*a^4-6/
d/b^4/(tanh(1/2*d*x+1/2*c)+1)*a^2-1/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^3*a^2+5/2
```

```
/d/b^6/(tanh(1/2*d*x+1/2*c)+1)^2*a^4+3/2/d/b^4/(tanh(1/2*d*x+1/2*c)+1)^2*a^
2-9/d/b^10*ln(tanh(1/2*d*x+1/2*c)+1)*a^8-10/d/b^6*ln(tanh(1/2*d*x+1/2*c)+1)
*a^4-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)-4/
d*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-
a^2)-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^3+1/4/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^
4+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^3+1/d/b^2*ln(a^2*tanh(1/2*d*x+1/2*c)^2+
2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+1/4/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^4-7/8/d/b^
2/(tanh(1/2*d*x+1/2*c)+1)+7/8/d/b^2/(tanh(1/2*d*x+1/2*c)-1)+`int/indef0`(-2
*cosh(d*x+c)^4*a*b*sinh(d*x+c)^(1/2)/(b^4*sinh(d*x+c)^2-2*a^2*b^2*sinh(d*x+
c)+a^4),sinh(d*x+c))/d-8/d/b^4*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)
^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4-4/d/b^8*tanh(1/2*d*x+1/2*c)/(a^2*tanh
(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^8
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [B] time = 8.8936, size = 12573, normalized size = 46.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/6720*(105*b^10*cosh(d*x + c)^10 + 105*b^10*sinh(d*x + c)^10 + 630*a^2*b^8
*cosh(d*x + c)^9 + 630*a^2*b^8*cosh(d*x + c) - 105*b^10 + 210*(5*b^10*cosh(
d*x + c) + 3*a^2*b^8)*sinh(d*x + c)^9 + 105*(24*a^4*b^6 + 11*b^10)*cosh(d*x
 + c)^8 + 105*(45*b^10*cosh(d*x + c)^2 + 54*a^2*b^8*cosh(d*x + c) + 24*a^4*
b^6 + 11*b^10)*sinh(d*x + c)^8 + 840*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c
)^7 + 840*(15*b^10*cosh(d*x + c)^3 + 27*a^2*b^8*cosh(d*x + c)^2 + 18*a^6*b^
4 + 17*a^2*b^8 + (24*a^4*b^6 + 11*b^10)*cosh(d*x + c))*sinh(d*x + c)^7 - 42
0*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d
*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^6 + 210*(105*b^10*
cosh(d*x + c)^4 + 252*a^2*b^8*cosh(d*x + c)^3 - 224*a^8*b^2 - 188*a^4*b^6 -
 6*b^10 - 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 14*(24*a^4*b^6 + 11*b^10
)*cosh(d*x + c)^2 - 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c + 28*(18*a^6*b^4 +
 17*a^2*b^8)*cosh(d*x + c))*sinh(d*x + c)^6 - 1680*(16*a^10 + 60*a^6*b^4 +
37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4
 + a^2*b^8)*c)*cosh(d*x + c)^5 + 420*(63*b^10*cosh(d*x + c)^5 + 189*a^2*b^8
*cosh(d*x + c)^4 - 64*a^10 - 240*a^6*b^4 - 148*a^2*b^8 + 14*(24*a^4*b^6 + 1
1*b^10)*cosh(d*x + c)^3 + 32*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x + 42*(18*a
^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^2 + 32*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c
 - 6*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10
)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c))*sinh(d*x + c)^
5 + 420*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b
^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^4 + 210*(105
*b^10*cosh(d*x + c)^6 + 378*a^2*b^8*cosh(d*x + c)^5 + 224*a^8*b^2 + 188*a^4
*b^6 + 6*b^10 + 35*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^4 + 140*(18*a^6*b^4
```

```
 + 17*a^2*b^8)*cosh(d*x + c)^3 + 32*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x - 3
0*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d
*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^2 + 32*(9*a^8*b^2
+ 10*a^4*b^6 + b^10)*c - 40*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a^10
+ 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c)*cosh(d*x
 + c))*sinh(d*x + c)^4 + 840*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^3 + 84
0*(15*b^10*cosh(d*x + c)^7 + 63*a^2*b^8*cosh(d*x + c)^6 + 18*a^6*b^4 + 17*a
^2*b^8 + 7*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^5 + 35*(18*a^6*b^4 + 17*a^2
*b^8)*cosh(d*x + c)^4 - 10*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b
^2 + 10*a^4*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*
x + c)^3 - 20*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 +
 a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c)*cosh(d*x + c)^2 + 2*(1
12*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x +
 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c))*sinh(d*x + c)^3 - 105
*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^2 + 105*(45*b^10*cosh(d*x + c)^8 + 21
6*a^2*b^8*cosh(d*x + c)^7 - 24*a^4*b^6 - 11*b^10 + 28*(24*a^4*b^6 + 11*b^10
)*cosh(d*x + c)^6 + 168*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^5 - 60*(112
*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 1
6*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^4 - 160*(16*a^10 + 60*a^
6*b^4 + 37*a^2*b^8 - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10
*a^6*b^4 + a^2*b^8)*c)*cosh(d*x + c)^3 + 24*(112*a^8*b^2 + 94*a^4*b^6 + 3*b
^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 +
 b^10)*c)*cosh(d*x + c)^2 + 24*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c))*sin
h(d*x + c)^2 + 6720*((9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^6 + (9*a
^8*b^2 + 10*a^4*b^6 + b^10)*sinh(d*x + c)^6 - 2*(9*a^10 + 10*a^6*b^4 + a^2*
b^8)*cosh(d*x + c)^5 - 2*(9*a^10 + 10*a^6*b^4 + a^2*b^8 - 3*(9*a^8*b^2 + 10
*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^5 - (9*a^8*b^2 + 10*a^4*b^6 +
 b^10)*cosh(d*x + c)^4 - (9*a^8*b^2 + 10*a^4*b^6 + b^10 - 15*(9*a^8*b^2 + 1
0*a^4*b^6 + b^10)*cosh(d*x + c)^2 + 10*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(5*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x
+ c)^3 - 5*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x + c)^2 - (9*a^8*b^2 + 1
0*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(9*a^8*b^2 + 10*a^4*
b^6 + b^10)*cosh(d*x + c)^4 - 20*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x +
 c)^3 - 6*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^2)*sinh(d*x + c)^2
+ 2*(3*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^5 - 5*(9*a^10 + 10*a^6
*b^4 + a^2*b^8)*cosh(d*x + c)^4 - 2*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*
x + c)^3)*sinh(d*x + c))*log(-(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 +
2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(
a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))) + 6720*((9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x
 + c)^6 + (9*a^8*b^2 + 10*a^4*b^6 + b^10)*sinh(d*x + c)^6 - 2*(9*a^10 + 10*
a^6*b^4 + a^2*b^8)*cosh(d*x + c)^5 - 2*(9*a^10 + 10*a^6*b^4 + a^2*b^8 - 3*(
9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^5 - (9*a^8*b^2
+ 10*a^4*b^6 + b^10)*cosh(d*x + c)^4 - (9*a^8*b^2 + 10*a^4*b^6 + b^10 - 15*
(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^2 + 10*(9*a^10 + 10*a^6*b^4 +
 a^2*b^8)*cosh(d*x + c))*sinh(d*x + c)^4 + 4*(5*(9*a^8*b^2 + 10*a^4*b^6 + b
^10)*cosh(d*x + c)^3 - 5*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x + c)^2 -
(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c))*sinh(d*x + c)^3 + (15*(9*a^8
*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^4 - 20*(9*a^10 + 10*a^6*b^4 + a^2*b
^8)*cosh(d*x + c)^3 - 6*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^2)*si
nh(d*x + c)^2 + 2*(3*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*cosh(d*x + c)^5 - 5*(9
*a^10 + 10*a^6*b^4 + a^2*b^8)*cosh(d*x + c)^4 - 2*(9*a^8*b^2 + 10*a^4*b^6 +
 b^10)*cosh(d*x + c)^3)*sinh(d*x + c))*log(2*(b^2*sinh(d*x + c) - a^2)/(cos
h(d*x + c) - sinh(d*x + c))) + 210*(5*b^10*cosh(d*x + c)^9 + 27*a^2*b^8*cos
h(d*x + c)^8 + 3*a^2*b^8 + 4*(24*a^4*b^6 + 11*b^10)*cosh(d*x + c)^7 + 28*(1
8*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^6 - 12*(112*a^8*b^2 + 94*a^4*b^6 + 3*
b^10 + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6
+ b^10)*c)*cosh(d*x + c)^5 - 40*(16*a^10 + 60*a^6*b^4 + 37*a^2*b^8 - 8*(9*a
```

```
^10 + 10*a^6*b^4 + a^2*b^8)*d*x - 8*(9*a^10 + 10*a^6*b^4 + a^2*b^8)*c)*cosh
(d*x + c)^4 + 8*(112*a^8*b^2 + 94*a^4*b^6 + 3*b^10 + 16*(9*a^8*b^2 + 10*a^4
*b^6 + b^10)*d*x + 16*(9*a^8*b^2 + 10*a^4*b^6 + b^10)*c)*cosh(d*x + c)^3 +
12*(18*a^6*b^4 + 17*a^2*b^8)*cosh(d*x + c)^2 - (24*a^4*b^6 + 11*b^10)*cosh(
d*x + c))*sinh(d*x + c) - 32*(15*a*b^9*cosh(d*x + c)^9 + 15*a*b^9*sinh(d*x
+ c)^9 + 54*a^3*b^7*cosh(d*x + c)^8 - 54*a^3*b^7*cosh(d*x + c)^2 + 15*a*b^9
*cosh(d*x + c) + 27*(5*a*b^9*cosh(d*x + c) + 2*a^3*b^7)*sinh(d*x + c)^8 + 4
*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^7 + 4*(135*a*b^9*cosh(d*x + c)^2 + 1
08*a^3*b^7*cosh(d*x + c) + 63*a^5*b^5 + 55*a*b^9)*sinh(d*x + c)^7 + 2*(1260
*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c)^6 + 2*(630*a*b^9*cosh(d*x + c)^3 + 7
56*a^3*b^7*cosh(d*x + c)^2 + 1260*a^7*b^3 + 1319*a^3*b^7 + 14*(63*a^5*b^5 +
 55*a*b^9)*cosh(d*x + c))*sinh(d*x + c)^6 - 2*(3780*a^9*b + 4452*a^5*b^5 +
655*a*b^9)*cosh(d*x + c)^5 + 2*(945*a*b^9*cosh(d*x + c)^4 + 1512*a^3*b^7*co
sh(d*x + c)^3 - 3780*a^9*b - 4452*a^5*b^5 - 655*a*b^9 + 42*(63*a^5*b^5 + 55
*a*b^9)*cosh(d*x + c)^2 + 6*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c))*si
nh(d*x + c)^5 - 2*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c)^4 + 2*(945*a*
b^9*cosh(d*x + c)^5 + 1890*a^3*b^7*cosh(d*x + c)^4 - 1260*a^7*b^3 - 1319*a^
3*b^7 + 70*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^3 + 15*(1260*a^7*b^3 + 131
9*a^3*b^7)*cosh(d*x + c)^2 - 5*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9)*cosh
(d*x + c))*sinh(d*x + c)^4 + 4*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^3 + 4*
(315*a*b^9*cosh(d*x + c)^6 + 756*a^3*b^7*cosh(d*x + c)^5 + 63*a^5*b^5 + 55*
a*b^9 + 35*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^4 + 10*(1260*a^7*b^3 + 131
9*a^3*b^7)*cosh(d*x + c)^3 - 5*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9)*cosh
(d*x + c)^2 - 2*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c))*sinh(d*x + c)^
3 + 2*(270*a*b^9*cosh(d*x + c)^7 + 756*a^3*b^7*cosh(d*x + c)^6 - 27*a^3*b^7
 + 42*(63*a^5*b^5 + 55*a*b^9)*cosh(d*x + c)^5 + 15*(1260*a^7*b^3 + 1319*a^3
*b^7)*cosh(d*x + c)^4 - 10*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9)*cosh(d*x
 + c)^3 - 6*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c)^2 + 6*(63*a^5*b^5 +
 55*a*b^9)*cosh(d*x + c))*sinh(d*x + c)^2 + (135*a*b^9*cosh(d*x + c)^8 + 43
2*a^3*b^7*cosh(d*x + c)^7 - 108*a^3*b^7*cosh(d*x + c) + 15*a*b^9 + 28*(63*a
^5*b^5 + 55*a*b^9)*cosh(d*x + c)^6 + 12*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(
d*x + c)^5 - 10*(3780*a^9*b + 4452*a^5*b^5 + 655*a*b^9)*cosh(d*x + c)^4 - 8
*(1260*a^7*b^3 + 1319*a^3*b^7)*cosh(d*x + c)^3 + 12*(63*a^5*b^5 + 55*a*b^9)
*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^12*d*cosh(d*x + c)
^6 + b^12*d*sinh(d*x + c)^6 - 2*a^2*b^10*d*cosh(d*x + c)^5 - b^12*d*cosh(d*
x + c)^4 + 2*(3*b^12*d*cosh(d*x + c) - a^2*b^10*d)*sinh(d*x + c)^5 + (15*b^
12*d*cosh(d*x + c)^2 - 10*a^2*b^10*d*cosh(d*x + c) - b^12*d)*sinh(d*x + c)^
4 + 4*(5*b^12*d*cosh(d*x + c)^3 - 5*a^2*b^10*d*cosh(d*x + c)^2 - b^12*d*cos
h(d*x + c))*sinh(d*x + c)^3 + (15*b^12*d*cosh(d*x + c)^4 - 20*a^2*b^10*d*co
sh(d*x + c)^3 - 6*b^12*d*cosh(d*x + c)^2)*sinh(d*x + c)^2 + 2*(3*b^12*d*cos
h(d*x + c)^5 - 5*a^2*b^10*d*cosh(d*x + c)^4 - 2*b^12*d*cosh(d*x + c)^3)*sin
h(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**(1/2))**2,x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.416 $\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2} dx$

Optimal. Leaf size=142

$$-\frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d}+\frac{3a^2\sinh(c+dx)}{b^4d}+\frac{2a\left(a^4+b^4\right)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\left(5a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}-\frac{4a\sin}{}$$

```
[Out] (2*(5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*a*(a^4 + b^4)
)/(b^6*d*(a + b*Sqrt[Sinh[c + d*x]])) - (8*a^3*Sqrt[Sinh[c + d*x]])/(b^5*d)
 + (3*a^2*Sinh[c + d*x])/(b^4*d) - (4*a*Sinh[c + d*x]^(3/2))/(3*b^3*d) + Si
nh[c + d*x]^2/(2*b^2*d)
```

Rubi [A] time = 0.163606, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3223, 1890, 1620}

$$-\frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d}+\frac{3a^2\sinh(c+dx)}{b^4d}+\frac{2a\left(a^4+b^4\right)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\left(5a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d}-\frac{4a\sin}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*(5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^6*d) + (2*a*(a^4 + b^4)
)/(b^6*d*(a + b*Sqrt[Sinh[c + d*x]])) - (8*a^3*Sqrt[Sinh[c + d*x]])/(b^5*d)
 + (3*a^2*Sinh[c + d*x])/(b^4*d) - (4*a*Sinh[c + d*x]^(3/2))/(3*b^3*d) + Si
nh[c + d*x]^2/(2*b^2*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1890

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{g = Denominator
[n]}, Dist[g, Subst[Int[x^(g - 1)*(Pq /. x -> x^g)*(a + b*x^(g*n))^p, x], x
, x^(1/g)], x]] /; FreeQ[{a, b, p}, x] && PolyQ[Pq, x] && FractionQ[n]
```

Rule 1620

```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2]) && GtQ[E
xpon[Px, x], 2]
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{\left(a+b\sqrt{x}\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \frac{x(1+x^4)}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{4a^3}{b^5}+\frac{3a^2x}{b^4}-\frac{2ax^2}{b^3}+\frac{x^3}{b^2}-\frac{a(a^4+b^4)}{b^5(a+bx)^2}+\frac{5a^4+b^4}{b^5(a+bx)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\left(5a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^6d} + \frac{2a\left(a^4+b^4\right)}{b^6d\left(a+b\sqrt{\sinh(c+dx)}\right)} - \frac{8a^3\sqrt{\sinh(c+dx)}}{b^5d}$$

Mathematica [A] time = 0.464117, size = 123, normalized size = 0.87

$$\frac{18a^2b^2\sinh(c+dx)+12\left(\frac{a(a^4+b^4)}{a+b\sqrt{\sinh(c+dx)}}+\left(5a^4+b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)\right)-48a^3b\sqrt{\sinh(c+dx)}-8ab^3\sinh^{\frac{3}{2}}(c+}{6b^6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (12*((5*a^4 + b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]] + (a*(a^4 + b^4))/(a + b*
Sqrt[Sinh[c + d*x]])) - 48*a^3*b*Sqrt[Sinh[c + d*x]] + 18*a^2*b^2*Sinh[c +
d*x] - 8*a*b^3*Sinh[c + d*x]^(3/2) + 3*b^4*Sinh[c + d*x]^2)/(6*b^6*d)
```

Maple [C] time = 0.207, size = 481, normalized size = 3.4

$$-4\,\frac{\tanh(1/2\,dx+c/2)\,a^4}{db^4\left(a^2\left(\tanh(1/2\,dx+c/2)\right)^2+2\,b^2\tanh(1/2\,dx+c/2)-a^2\right)}-4\,\frac{\tanh(1/2\,dx+c/2)}{d\left(a^2\left(\tanh(1/2\,dx+c/2)\right)^2+2\,b^2\tanh(1/2\,dx+c/2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] -4/d/b^4*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+
1/2*c)-a^2)*a^4-4/d*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*ta
nh(1/2*d*x+1/2*c)-a^2)+5/d/b^6*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*
d*x+1/2*c)-a^2)*a^4+1/d/b^2*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x
+1/2*c)-a^2)+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)^2-3/d/b^4/(tanh(1/2*d*x+1/2*
c)+1)*a^2-1/2/d/b^2/(tanh(1/2*d*x+1/2*c)+1)-5/d/b^6*ln(tanh(1/2*d*x+1/2*c)+
1)*a^4-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)+1)+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)^
2-3/d/b^4/(tanh(1/2*d*x+1/2*c)-1)*a^2+1/2/d/b^2/(tanh(1/2*d*x+1/2*c)-1)-5/d
/b^6*ln(tanh(1/2*d*x+1/2*c)-1)*a^4-1/d/b^2*ln(tanh(1/2*d*x+1/2*c)-1)+`int/i
ndef0`(-2*cosh(d*x+c)^2*a*b*sinh(d*x+c)^(1/2)/(b^4*sinh(d*x+c)^2-2*a^2*b^2*
sinh(d*x+c)+a^4),sinh(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3}{\left(b\sqrt{\sinh(dx+c)}+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Fricas [B] time = 7.24589, size = 5027, normalized size = 35.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] 1/24*(3*b^6*cosh(d*x + c)^6 + 3*b^6*sinh(d*x + c)^6 + 30*a^2*b^4*cosh(d*x +
 c)^5 + 30*a^2*b^4*cosh(d*x + c) - 3*b^6 + 6*(3*b^6*cosh(d*x + c) + 5*a^2*b
^4)*sinh(d*x + c)^5 - 3*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*
a^4*b^2 + b^6)*c)*cosh(d*x + c)^4 + 3*(15*b^6*cosh(d*x + c)^2 + 50*a^2*b^4*
cosh(d*x + c) - 24*a^4*b^2 - b^6 - 8*(5*a^4*b^2 + b^6)*d*x - 8*(5*a^4*b^2 +
 b^6)*c)*sinh(d*x + c)^4 - 24*(4*a^6 + 7*a^2*b^4 - 2*(5*a^6 + a^2*b^4)*d*x
- 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c)^3 + 12*(5*b^6*cosh(d*x + c)^3 + 25*a
^2*b^4*cosh(d*x + c)^2 - 8*a^6 - 14*a^2*b^4 + 4*(5*a^6 + a^2*b^4)*d*x + 4*(
5*a^6 + a^2*b^4)*c - (24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^4
*b^2 + b^6)*c)*cosh(d*x + c))*sinh(d*x + c)^3 + 3*(24*a^4*b^2 + b^6 + 8*(5*
a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c)^2 + 3*(15*b^6*cos
h(d*x + c)^4 + 100*a^2*b^4*cosh(d*x + c)^3 + 24*a^4*b^2 + b^6 + 8*(5*a^4*b^
2 + b^6)*d*x - 6*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2
 + b^6)*c)*cosh(d*x + c)^2 + 8*(5*a^4*b^2 + b^6)*c - 24*(4*a^6 + 7*a^2*b^4
- 2*(5*a^6 + a^2*b^4)*d*x - 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c))*sinh(d*x
+ c)^2 + 24*((5*a^4*b^2 + b^6)*cosh(d*x + c)^4 + (5*a^4*b^2 + b^6)*sinh(d*x
 + c)^4 - 2*(5*a^6 + a^2*b^4)*cosh(d*x + c)^3 - 2*(5*a^6 + a^2*b^4 - 2*(5*a
^4*b^2 + b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (5*a^4*b^2 + b^6)*cosh(d*x +
 c)^2 - (5*a^4*b^2 + b^6 - 6*(5*a^4*b^2 + b^6)*cosh(d*x + c)^2 + 6*(5*a^6 +
 a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(2*(5*a^4*b^2 + b^6)*cosh(d*x
+ c)^3 - 3*(5*a^6 + a^2*b^4)*cosh(d*x + c)^2 - (5*a^4*b^2 + b^6)*cosh(d*x +
 c))*sinh(d*x + c))*log(-(b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 + 2*a^2
*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(a*b*c
osh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x + c)^2
 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) -
 a^2)*sinh(d*x + c))) + 24*((5*a^4*b^2 + b^6)*cosh(d*x + c)^4 + (5*a^4*b^2
+ b^6)*sinh(d*x + c)^4 - 2*(5*a^6 + a^2*b^4)*cosh(d*x + c)^3 - 2*(5*a^6 + a
^2*b^4 - 2*(5*a^4*b^2 + b^6)*cosh(d*x + c))*sinh(d*x + c)^3 - (5*a^4*b^2 +
b^6)*cosh(d*x + c)^2 - (5*a^4*b^2 + b^6 - 6*(5*a^4*b^2 + b^6)*cosh(d*x + c)
^2 + 6*(5*a^6 + a^2*b^4)*cosh(d*x + c))*sinh(d*x + c)^2 + 2*(2*(5*a^4*b^2 +
 b^6)*cosh(d*x + c)^3 - 3*(5*a^6 + a^2*b^4)*cosh(d*x + c)^2 - (5*a^4*b^2 +
b^6)*cosh(d*x + c))*sinh(d*x + c))*log(2*(b^2*sinh(d*x + c) - a^2)/(cosh(d*
x + c) - sinh(d*x + c))) + 6*(3*b^6*cosh(d*x + c)^5 + 25*a^2*b^4*cosh(d*x +
 c)^4 + 5*a^2*b^4 - 2*(24*a^4*b^2 + b^6 + 8*(5*a^4*b^2 + b^6)*d*x + 8*(5*a^
4*b^2 + b^6)*c)*cosh(d*x + c)^3 - 12*(4*a^6 + 7*a^2*b^4 - 2*(5*a^6 + a^2*b^
4)*d*x - 2*(5*a^6 + a^2*b^4)*c)*cosh(d*x + c)^2 + (24*a^4*b^2 + b^6 + 8*(5*
a^4*b^2 + b^6)*d*x + 8*(5*a^4*b^2 + b^6)*c)*cosh(d*x + c))*sinh(d*x + c) -
16*(a*b^5*cosh(d*x + c)^5 + a*b^5*sinh(d*x + c)^5 + 10*a^3*b^3*cosh(d*x + c
)^4 - 10*a^3*b^3*cosh(d*x + c)^2 + a*b^5*cosh(d*x + c) + 5*(a*b^5*cosh(d*x
+ c) + 2*a^3*b^3)*sinh(d*x + c)^4 - 2*(15*a^5*b + 4*a*b^5)*cosh(d*x + c)^3
+ 2*(5*a*b^5*cosh(d*x + c)^2 + 20*a^3*b^3*cosh(d*x + c) - 15*a^5*b - 4*a*b^
5)*sinh(d*x + c)^3 + 2*(5*a*b^5*cosh(d*x + c)^3 + 30*a^3*b^3*cosh(d*x + c)^
2 - 5*a^3*b^3 - 3*(15*a^5*b + 4*a*b^5)*cosh(d*x + c))*sinh(d*x + c)^2 + (5*
```

```
a*b^5*cosh(d*x + c)^4 + 40*a^3*b^3*cosh(d*x + c)^3 - 20*a^3*b^3*cosh(d*x +
c) + a*b^5 - 6*(15*a^5*b + 4*a*b^5)*cosh(d*x + c)^2)*sinh(d*x + c))*sqrt(si
nh(d*x + c)))/(b^8*d*cosh(d*x + c)^4 + b^8*d*sinh(d*x + c)^4 - 2*a^2*b^6*d*
cosh(d*x + c)^3 - b^8*d*cosh(d*x + c)^2 + 2*(2*b^8*d*cosh(d*x + c) - a^2*b^
6*d)*sinh(d*x + c)^3 + (6*b^8*d*cosh(d*x + c)^2 - 6*a^2*b^6*d*cosh(d*x + c)
 - b^8*d)*sinh(d*x + c)^2 + 2*(2*b^8*d*cosh(d*x + c)^3 - 3*a^2*b^6*d*cosh(d
*x + c)^2 - b^8*d*cosh(d*x + c))*sinh(d*x + c))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.417 $\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=49

$$\frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

```
[Out] (2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*a)/(b^2*d*(a + b*Sqrt[Sinh[
c + d*x]]))
```

Rubi [A] time = 0.0545739, antiderivative size = 49, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3223, 190, 43}

$$\frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}+\frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*Log[a + b*Sqrt[Sinh[c + d*x]]])/(b^2*d) + (2*a)/(b^2*d*(a + b*Sqrt[Sinh[
c + d*x]]))
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 190

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[Int[x^(1/n
- 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b, p}, x] && FractionQ[n] &&
 IntegerQ[1/n]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\cosh(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{x}{(a+bx)^2}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= \frac{2\,\text{Subst}\left(\int \left(-\frac{a}{b(a+bx)^2}+\frac{1}{b(a+bx)}\right)dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$
$$= \frac{2\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{b^2d} + \frac{2a}{b^2d\left(a+b\sqrt{\sinh(c+dx)}\right)}$$

Mathematica [A] time = 0.0609125, size = 42, normalized size = 0.86

$$\frac{2\left(\frac{a}{a+b\sqrt{\sinh(c+dx)}}+\log\left(a+b\sqrt{\sinh(c+dx)}\right)\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (2*(Log[a + b*Sqrt[Sinh[c + d*x]]] + a/(a + b*Sqrt[Sinh[c + d*x]])))/(b^2*d
)
```

Maple [B] time = 0.021, size = 144, normalized size = 2.9

$$-2\,\frac{a^2}{d\left(b^2\sinh(dx+c)-a^2\right)b^2}+\frac{\ln\left(b^2\sinh(dx+c)-a^2\right)}{db^2}+\frac{a}{db^2}\left(b\sqrt{\sinh(dx+c)}-a\right)^{-1}-\frac{1}{db^2}\ln\left(b\sqrt{\sinh(dx+c)}-a\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] -2/d*a^2/(b^2*sinh(d*x+c)-a^2)/b^2+1/d*ln(b^2*sinh(d*x+c)-a^2)/b^2+1/d*a/b^
2/(b*sinh(d*x+c)^(1/2)-a)-1/d/b^2*ln(b*sinh(d*x+c)^(1/2)-a)+a/b^2/d/(a+b*si
nh(d*x+c)^(1/2))+ln(a+b*sinh(d*x+c)^(1/2))/b^2/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{\left(b\sqrt{\sinh(dx+c)}+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Fricas [B] time = 1.77643, size = 1370, normalized size = 27.96

$$b^2dx + b^2c - \left(b^2dx + b^2c\right)\cosh\left(dx + c\right)^2 - \left(b^2dx + b^2c\right)\sinh\left(dx + c\right)^2 + 2\left(a^2dx + a^2c - 2a^2\right)\cosh\left(dx + c\right) + \left(b^2\cos\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] (b^2*d*x + b^2*c - (b^2*d*x + b^2*c)*cosh(d*x + c)^2 - (b^2*d*x + b^2*c)*si
nh(d*x + c)^2 + 2*(a^2*d*x + a^2*c - 2*a^2)*cosh(d*x + c) + (b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))*log((b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 +
2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) + a^2)*sinh(d*x + c) + 4*(
a*b*cosh(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^2*cosh(d*x +
 c)^2 + b^2*sinh(d*x + c)^2 - 2*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x +
 c) - a^2)*sinh(d*x + c))) + (b^2*cosh(d*x + c)^2 + b^2*sinh(d*x + c)^2 - 2
*a^2*cosh(d*x + c) - b^2 + 2*(b^2*cosh(d*x + c) - a^2)*sinh(d*x + c))*log(2
*(b^2*sinh(d*x + c) - a^2)/(cosh(d*x + c) - sinh(d*x + c))) + 2*(a^2*d*x +
a^2*c - 2*a^2 - (b^2*d*x + b^2*c)*cosh(d*x + c))*sinh(d*x + c) + 4*(a*b*cos
h(d*x + c) + a*b*sinh(d*x + c))*sqrt(sinh(d*x + c)))/(b^4*d*cosh(d*x + c)^2
 + b^4*d*sinh(d*x + c)^2 - 2*a^2*b^2*d*cosh(d*x + c) - b^4*d + 2*(b^4*d*cos
h(d*x + c) - a^2*b^2*d)*sinh(d*x + c))
```

Sympy [A] time = 4.92605, size = 151, normalized size = 3.08

$$\begin{cases} \frac{x\cosh(c)}{a^2} & \text{for } b = 0 \wedge d = 0 \\ \frac{\sinh(c+dx)}{a^2d} & \text{for } b = 0 \\ \frac{x\cosh(c)}{\left(a+b\sqrt{\sinh(c)}\right)^2} & \text{for } d = 0 \\ \frac{2a\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} + \frac{2a}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} + \frac{2b\log\left(\frac{a}{b}+\sqrt{\sinh(c+dx)}\right)\sqrt{\sinh(c+dx)}}{ab^2d+b^3d\sqrt{\sinh(c+dx)}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Piecewise((x*cosh(c)/a**2, Eq(b, 0) & Eq(d, 0)), (sinh(c + d*x)/(a**2*d), E
q(b, 0)), (x*cosh(c)/(a + b*sqrt(sinh(c)))**2, Eq(d, 0)), (2*a*log(a/b + sq
rt(sinh(c + d*x)))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))) + 2*a/(a*b**2*d
+ b**3*d*sqrt(sinh(c + d*x))) + 2*b*log(a/b + sqrt(sinh(c + d*x)))*sqrt(sin
h(c + d*x))/(a*b**2*d + b**3*d*sqrt(sinh(c + d*x))), True))
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.418 $\int \frac{\text{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$

Optimal. Leaf size=384

$$\frac{2ab^2}{d\left(a^4+b^4\right)\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{ab\left(2a^2b^2+a^4-b^4\right)\log\left(\sinh(c+dx)-\sqrt{2}\sqrt{\sinh(c+dx)}+1\right)}{\sqrt{2}d\left(a^4+b^4\right)^2}+\frac{ab\left(2a^2b^2+a^4-b\right.}{}$$

```
[Out] (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]
])/((a^4 + b^4)^2*d) - (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 + Sqrt
[2]*Sqrt[Sinh[c + d*x]]])/((a^4 + b^4)^2*d) + (a^2*(a^4 - 3*b^4)*ArcTan[Sin
h[c + d*x]])/((a^4 + b^4)^2*d) + (b^2*(3*a^4 - b^4)*Log[Cosh[c + d*x]])/((a
^4 + b^4)^2*d) - (2*b^2*(3*a^4 - b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^4
 + b^4)^2*d) - (a*b*(a^4 + 2*a^2*b^2 - b^4)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d
*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 + b^4)^2*d) + (a*b*(a^4 + 2*a^2*b^2 -
b^4)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 +
b^4)^2*d) + (2*a*b^2)/((a^4 + b^4)*d*(a + b*Sqrt[Sinh[c + d*x]]))
```

Rubi [A] time = 0.630777, antiderivative size = 384, normalized size of antiderivative = 1., number of steps used = 19, number of rules used = 13, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.565, Rules used = {3223, 6725, 1876, 1168, 1162, 617, 204, 1165, 628, 1248, 635, 203, 260}

$$\frac{2ab^2}{d\left(a^4+b^4\right)\left(a+b\sqrt{\sinh(c+dx)}\right)}-\frac{ab\left(2a^2b^2+a^4-b^4\right)\log\left(\sinh(c+dx)-\sqrt{2}\sqrt{\sinh(c+dx)}+1\right)}{\sqrt{2}d\left(a^4+b^4\right)^2}+\frac{ab\left(2a^2b^2+a^4-b\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```


```
[Out] (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]
])/((a^4 + b^4)^2*d) - (Sqrt[2]*a*b*(a^4 - 2*a^2*b^2 - b^4)*ArcTan[1 + Sqrt
[2]*Sqrt[Sinh[c + d*x]]])/((a^4 + b^4)^2*d) + (a^2*(a^4 - 3*b^4)*ArcTan[Sin
h[c + d*x]])/((a^4 + b^4)^2*d) + (b^2*(3*a^4 - b^4)*Log[Cosh[c + d*x]])/((a
^4 + b^4)^2*d) - (2*b^2*(3*a^4 - b^4)*Log[a + b*Sqrt[Sinh[c + d*x]]])/((a^4
 + b^4)^2*d) - (a*b*(a^4 + 2*a^2*b^2 - b^4)*Log[1 - Sqrt[2]*Sqrt[Sinh[c + d
*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 + b^4)^2*d) + (a*b*(a^4 + 2*a^2*b^2 -
b^4)*Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]])/(Sqrt[2]*(a^4 +
b^4)^2*d) + (2*a*b^2)/((a^4 + b^4)*d*(a + b*Sqrt[Sinh[c + d*x]]))
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 6725

```
Int[(u_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = RationalFunctionE
xpand[u/(a + b*x^n), x]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && IGtQ
[n, 0]
```

Rule 1876

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> With[{v = Sum[(x^ii*(Coeff
[Pq, x, ii] + Coeff[Pq, x, n/2 + ii]*x^(n/2)))/(a + b*x^n), {ii, 0, n/2 - 1
}]}, Int[v, x] /; SumQ[v]] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n/2,
 0] && Expon[Pq, x] < n
```

Rule 1168

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
a*c, 2]}, Dist[(d*q + a*e)/(2*a*c), Int[(q + c*x^2)/(a + c*x^4), x], x] + D
ist[(d*q - a*e)/(2*a*c), Int[(q - c*x^2)/(a + c*x^4), x], x]] /; FreeQ[{a,
c, d, e}, x] && NeQ[c*d^2 + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && NegQ[-(a*
c)]
```

Rule 1162

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(2*d)/e, 2]}, Dist[e/(2*c), Int[1/Simp[d/e + q*x + x^2, x], x], x] + Dist[e
/(2*c), Int[1/Simp[d/e - q*x + x^2, x], x], x]] /; FreeQ[{a, c, d, e}, x] &
& EqQ[c*d^2 - a*e^2, 0] && PosQ[d*e]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 1165

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
(-2*d)/e, 2]}, Dist[e/(2*c*q), Int[(q - 2*x)/Simp[d/e + q*x - x^2, x], x],
x] + Dist[e/(2*c*q), Int[(q + 2*x)/Simp[d/e - q*x - x^2, x], x], x]] /; Fre
eQ[{a, c, d, e}, x] && EqQ[c*d^2 - a*e^2, 0] && NegQ[d*e]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 1248

```
Int[(x_)*((d_) + (e_.)*(x_)^2)^(q_.)*((a_) + (c_.)*(x_)^4)^(p_.), x_Symbol]
 :> Dist[1/2, Subst[Int[(d + e*x)^q*(a + c*x^2)^p, x], x, x^2], x] /; FreeQ
[{a, c, d, e, p, q}, x]
```

Rule 635

```
Int[((d_) + (e_.)*(x_))/((a_) + (c_.)*(x_)^2), x_Symbol] :> Dist[d, Int[1/(
a + c*x^2), x], x] + Dist[e, Int[x/(a + c*x^2), x], x] /; FreeQ[{a, c, d, e
}, x] &&  !NiceSqrtQ[-(a*c)]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx = \frac{\operatorname{Subst}\left(\int \frac{1}{\left(a+b\sqrt{x}\right)^2(1+x^2)}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{2\operatorname{Subst}\left(\int \frac{x}{(a+bx)^2(1+x^4)}\,dx, x, \sqrt{\sinh(c+dx)}\right)}{d}$$

$$= \frac{2\operatorname{Subst}\left(\int \left(-\frac{ab^3}{\left(a^4+b^4\right)(a+bx)^2} + \frac{-3a^4b^3+b^7}{\left(a^4+b^4\right)^2(a+bx)} + \frac{4a^3b^3+a^2\left(a^4-3b^4\right)x-2ab\left(a^4-b^4\right)x^2+b^2\left(3a^4-b^4\right)x^3}{\left(a^4+b^4\right)^2(1+x^4)}\right)dx,}{d}$$

$$= -\frac{2b^2\left(3a^4-b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)^2 d} + \frac{2ab^2}{\left(a^4+b^4\right)d\left(a+b\sqrt{\sinh(c+dx)}\right)} + \frac{2\operatorname{Subst}}{}$$

$$= -\frac{2b^2\left(3a^4-b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)^2 d} + \frac{2ab^2}{\left(a^4+b^4\right)d\left(a+b\sqrt{\sinh(c+dx)}\right)} + \frac{2\operatorname{Subst}}{}$$

$$= -\frac{2b^2\left(3a^4-b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)^2 d} + \frac{2ab^2}{\left(a^4+b^4\right)d\left(a+b\sqrt{\sinh(c+dx)}\right)} + \frac{2\operatorname{Subst}}{}$$

$$= -\frac{2b^2\left(3a^4-b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)^2 d} + \frac{2ab^2}{\left(a^4+b^4\right)d\left(a+b\sqrt{\sinh(c+dx)}\right)} + \frac{\operatorname{Subst}\Big(}{}$$

$$= -\frac{2b^2\left(3a^4-b^4\right)\log\left(a+b\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)^2 d} + \frac{2ab^2}{\left(a^4+b^4\right)d\left(a+b\sqrt{\sinh(c+dx)}\right)} + \frac{\left(a^2\left(a^4\right.\right.}{}$$

$$= \frac{a^2\left(a^4-3b^4\right)\tan^{-1}(\sinh(c+dx))}{\left(a^4+b^4\right)^2 d} + \frac{b^2\left(3a^4-b^4\right)\log(\cosh(c+dx))}{\left(a^4+b^4\right)^2 d} - \frac{2b^2\left(3a^4-b^4\right)\log}{\left(a^4\right.}$$

$$= \frac{\sqrt{2}ab\left(a^4-2a^2b^2-b^4\right)\tan^{-1}\left(1-\sqrt{2}\sqrt{\sinh(c+dx)}\right)}{\left(a^4+b^4\right)^2 d} - \frac{\sqrt{2}ab\left(a^4-2a^2b^2-b^4\right)\tan^{-1}\left(1\right.}{\left(a^4+b^4\right)^2 d}$$

Mathematica [C] time = 0.732946, size = 280, normalized size = 0.73

$$\frac{-4ab\left(a^4-b^4\right)\sinh^{\frac{3}{2}}(c+dx)\,{}_2F_1\left(\frac{3}{4},1;\frac{7}{4};-\sinh^2(c+dx)\right) + \frac{6ab^2\left(a^4+b^4\right)}{a+b\sqrt{\sinh(c+dx)}} - 3\sqrt{2}a^3b^3\left(\log\left(\sinh(c+dx)-\sqrt{2}\sqrt{\sinh(c+\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sech[c + d*x]/(a + b*Sqrt[Sinh[c + d*x]])^2,x]
```

```
[Out] (-6*Sqrt[2]*a^3*b^3*(ArcTan[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]] - ArcTan[1 + S
qrt[2]*Sqrt[Sinh[c + d*x]]]) + 3*a^2*(a^4 - 3*b^4)*ArcTan[Sinh[c + d*x]] -
3*b^2*(-3*a^4 + b^4)*Log[Cosh[c + d*x]] + 6*b^2*(-3*a^4 + b^4)*Log[a + b*Sq
rt[Sinh[c + d*x]]] - 3*Sqrt[2]*a^3*b^3*(Log[1 - Sqrt[2]*Sqrt[Sinh[c + d*x]]
 + Sinh[c + d*x]] - Log[1 + Sqrt[2]*Sqrt[Sinh[c + d*x]] + Sinh[c + d*x]]) +
 (6*a*b^2*(a^4 + b^4))/(a + b*Sqrt[Sinh[c + d*x]]) - 4*a*b*(a^4 - b^4)*Hype
rgeometric2F1[3/4, 1, 7/4, -Sinh[c + d*x]^2]*Sinh[c + d*x]^(3/2))/(3*(a^4 +
 b^4)^2*d)
```

Maple [C] time = 0.234, size = 567, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x)
```

```
[Out] -4/d*b^4/(a^4+b^4)^2*tanh(1/2*d*x+1/2*c)/(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*t
anh(1/2*d*x+1/2*c)-a^2)*a^4-4/d*b^8/(a^4+b^4)^2*tanh(1/2*d*x+1/2*c)/(a^2*ta
nh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)-3/d*b^2/(a^4+b^4)^2*ln(a
^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)*a^4+1/d*b^6/(a^4+b^
4)^2*ln(a^2*tanh(1/2*d*x+1/2*c)^2+2*b^2*tanh(1/2*d*x+1/2*c)-a^2)+3/d/(a^8+2
*a^4*b^4+b^8)*ln(tanh(1/2*d*x+1/2*c)^2+1)*a^4*b^2-1/d/(a^8+2*a^4*b^4+b^8)*l
n(tanh(1/2*d*x+1/2*c)^2+1)*b^6+2/d/(a^8+2*a^4*b^4+b^8)*arctan(tanh(1/2*d*x+
1/2*c))*a^6-6/d/(a^8+2*a^4*b^4+b^8)*arctan(tanh(1/2*d*x+1/2*c))*a^2*b^4+`in
t/indef0`(2*a*b*sinh(d*x+c)^(1/2)*(b^4*sinh(d*x+c)^2-2*a^2*b^2*sinh(d*x+c)+
a^4)/(4*a^2*b^6*sinh(d*x+c)*cosh(d*x+c)^4+(4*a^6*b^2-4*a^2*b^6)*cosh(d*x+c)
^2*sinh(d*x+c)-b^8*cosh(d*x+c)^6+(-6*a^4*b^4+2*b^8)*cosh(d*x+c)^4+(-a^8+6*a
^4*b^4-b^8)*cosh(d*x+c)^2),sinh(d*x+c))/d
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(dx+c)}{\left(b\sqrt{\sinh(dx+c)}+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="maxima")
```

```
[Out] integrate(sech(d*x + c)/(b*sqrt(sinh(d*x + c)) + a)^2, x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\operatorname{sech}(c+dx)}{\left(a+b\sqrt{\sinh(c+dx)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)**(1/2))**2,x)
```

```
[Out] Integral(sech(c + d*x)/(a + b*sqrt(sinh(c + d*x)))**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sech(d*x+c)/(a+b*sinh(d*x+c)^(1/2))^2,x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.419 $\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{\sinh^5(c+dx)\,{}_2F_1\left(1,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5ad}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}+\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1\right.}{ad}$$

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d) + (2*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^
n)/a)]*Sinh[c + d*x]^3)/(3*a*d) + (Hypergeometric2F1[1, 5/n, (5 + n)/n, -((
b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a*d)
```

Rubi [A] time = 0.140401, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^5(c+dx)\,{}_2F_1\left(1,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5ad}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}+\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1\right.}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n),x]
```


```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d) + (2*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^
n)/a)]*Sinh[c + d*x]^3)/(3*a*d) + (Hypergeometric2F1[1, 5/n, (5 + n)/n, -((
b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a*d)
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a+bx^n}+\frac{2x^2}{a+bx^n}+\frac{x^4}{a+bx^n}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^4}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{2\,\text{Subst}\left(\int \frac{x^2}{a+b\right.}}{}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad} + \frac{2\,{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3ad} +$$

Mathematica [A] time = 0.141965, size = 119, normalized size = 0.92

$$\frac{3\sinh^5(c+dx)\,{}_2F_1\left(1,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)+10\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)+15\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1\right.}{15ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (15*Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh
[c + d*x] + 10*Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a
)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[1, 5/n, (5 + n)/n, -((b*Sinh[c + d
*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a*d)
```

Maple [F] time = 0.254, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^5}{a+b(\sinh(dx+c))^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

```
[Out] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^5}{b\sinh(dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^{5}}{b\sinh\left(dx+c\right)^{n}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(dx+c\right)^{5}}{b\sinh\left(dx+c\right)^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a), x)
```

3.420 $\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=84

$$\frac{\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}+\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d) + (Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)
/a)]*Sinh[c + d*x]^3)/(3*a*d)
```

Rubi [A] time = 0.103662, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3ad}+\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d) + (Hypergeometric2F1[1, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)
/a)]*Sinh[c + d*x]^3)/(3*a*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{a+b\sinh^n(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a+bx^n}+\frac{x^2}{a+bx^n}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^n}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad} + \frac{{}_2F_1\left(1,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.0547093, size = 82, normalized size = 0.98

$$\frac{\frac{\sinh^3(c+dx)\,{}_2F_1\left(1,\frac{3}{n};1+\frac{3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a}+\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] ((Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/a + (Hypergeometric2F1[1, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)]*
Sinh[c + d*x]^3)/(3*a))/d
```

Maple [F] time = 0.22, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^3}{a+b(\sinh(dx+c))^n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

```
[Out] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)^3}{b\sinh(dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh(dx+c)^3}{b\sinh(dx+c)^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh(dx+c)^3}{b\sinh(dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a), x)
```

3.421 $\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx$

Optimal. Leaf size=37

$$\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d)
```

Rubi [A] time = 0.0434645, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3223, 245}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{a+b\sinh^n(c+dx)}\,dx &= \frac{\operatorname{Subst}\left(\int \frac{1}{a+bx^n}\,dx,x,\sinh(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.0088907, size = 37, normalized size = 1.

$$\frac{\sinh(c+dx)\,{}_2F_1\left(1,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n),x]
```

```
[Out] (Hypergeometric2F1[1, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a*d)
```

Maple [F] time = 0.224, size = 0, normalized size = 0.

$$\int \frac{\cosh (dx+c)}{a+b (\sinh (dx+c))^{n}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
```

```
[Out] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh (dx+c)}{b \sinh (dx+c)^{n}+a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="maxima")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh (dx+c)}{b \sinh (dx+c)^{n}+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh(dx+c)}{b\sinh(dx+c)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n),x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a), x)
```

3.422 $\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$

Optimal. Leaf size=130

$$\frac{\sinh^5(c+dx)\,{}_2F_1\left(2,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5a^2d}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}+\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n}\right.}{a^2d}$$

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d) + (2*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x
]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d) + (Hypergeometric2F1[2, 5/n, (5 + n)/n,
 -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a^2*d)
```

Rubi [A] time = 0.13324, antiderivative size = 130, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^5(c+dx)\,{}_2F_1\left(2,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{5a^2d}+\frac{2\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}+\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n}\right.}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]
```



```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d) + (2*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x
]^n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d) + (Hypergeometric2F1[2, 5/n, (5 + n)/n,
 -((b*Sinh[c + d*x]^n)/a)]*Sinh[c + d*x]^5)/(5*a^2*d)
```


Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
```

```
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\cosh^5(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+bx^n)^2}+\frac{2x^2}{(a+bx^n)^2}+\frac{x^4}{(a+bx^n)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}+\frac{\text{Subst}\left(\int \frac{x^4}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}+ 2\,\text{Su}$$

$$= \frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}+\frac{2\,{}_2F_1\left(2,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c}{3a^2d}$$

Mathematica [A] time = 0.110634, size = 119, normalized size = 0.92

$$\frac{3\sinh^5(c+dx)\,{}_2F_1\left(2,\frac{5}{n};\frac{n+5}{n};-\frac{b\sinh^n(c+dx)}{a}\right)+10\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)+15\sinh(c+dx)\,{}_2F_1\left(2,\right.}{15a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^5/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (15*Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh
[c + d*x] + 10*Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^n)/a
)]*Sinh[c + d*x]^3 + 3*Hypergeometric2F1[2, 5/n, (5 + n)/n, -((b*Sinh[c + d
*x]^n)/a)]*Sinh[c + d*x]^5)/(15*a^2*d)
```

Maple [F] time = 5.743, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^5}{(a+b(\sinh(dx+c))^n)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2^n e^{(cn+10\,dx+10\,c)}+3\cdot 2^n e^{(cn+8\,dx+8\,c)}+2^{n+1} e^{(cn+6\,dx+6\,c)}-2^{n+1} e^{(cn+4\,dx+4\,c)}-3\cdot 2^n e^{(cn+2\,dx+2\,c)}-2^n e^{(cn)}\right)e^{(dnx)}}{32\left(2^n a^2 dn e^{(dnx+cn+5\,dx+5\,c)}+abdn e^{\left(5\,dx+n\log\left(e^{(dx+c)}+1\right)+n\log\left(e^{(dx+c)}-1\right)+5\,c\right)}\right)}+\frac{1}{32}\int$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/32*(2^n*e^(c*n + 10*d*x + 10*c) + 3*2^n*e^(c*n + 8*d*x + 8*c) + 2^(n + 1)
*e^(c*n + 6*d*x + 6*c) - 2^(n + 1)*e^(c*n + 4*d*x + 4*c) - 3*2^n*e^(c*n + 2
*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 5*d*x +
5*c) + a*b*d*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 5*c)) + 1/32*integrate((2^n*n*e^(c*n) - 5*2^n*e^(c*n) + (2^n*n*e^(c*n) - 5
*2^n*e^(c*n))*e^(10*d*x + 10*c) + (5*2^n*n*e^(c*n) - 9*2^n*e^(c*n))*e^(8*d*
x + 8*c) + (5*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(6*d*x + 6*c) + (5
*2^(n + 1)*n*e^(c*n) - 2^(n + 1)*e^(c*n))*e^(4*d*x + 4*c) + (5*2^n*n*e^(c*n
) - 9*2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x + c*n + 5
*d*x + 5*c) + a*b*n*e^(5*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) -
 1) + 5*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^5}{b^2\sinh\left(dx+c\right)^{2n}+2\,ab\sinh\left(dx+c\right)^n+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^5/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n +
 a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**5/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(dx+c\right)^5}{\left(b\sinh\left(dx+c\right)^n+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^5/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^5/(b*sinh(d*x + c)^n + a)^2, x)
```

3.423 $\int \frac{\cosh^3(c+dx)}{(a+b\sinh^n(c+dx))^2}\,dx$

Optimal. Leaf size=84

$$\frac{\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}+\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d) + (Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^
n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d)
```

Rubi [A] time = 0.102242, antiderivative size = 84, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3223, 1893, 245, 364}

$$\frac{\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};\frac{n+3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2d}+\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d) + (Hypergeometric2F1[2, 3/n, (3 + n)/n, -((b*Sinh[c + d*x]^
n)/a)]*Sinh[c + d*x]^3)/(3*a^2*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rubi steps

$$\int \frac{\cosh^3(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+bx^n)^2}+\frac{x^2}{(a+bx^n)^2}\right)dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{x^2}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}$$
$$= \frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d} + \frac{{}_2F_1\left(2,\frac{3}{n};\frac{3+n}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh^3(c+dx)}{3a^2d}$$

Mathematica [A] time = 0.0516983, size = 82, normalized size = 0.98

$$\frac{\frac{\sinh^3(c+dx)\,{}_2F_1\left(2,\frac{3}{n};1+\frac{3}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{3a^2}+\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]^3/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] ((Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
 + d*x])/a^2 + (Hypergeometric2F1[2, 3/n, 1 + 3/n, -((b*Sinh[c + d*x]^n)/a)
]*Sinh[c + d*x]^3)/(3*a^2))/d
```

Maple [F] time = 5.335, size = 0, normalized size = 0.

$$\int \frac{(\cosh(dx+c))^3}{\left(a+b(\sinh(dx+c))^n\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2^n e^{(cn+6\,dx+6\,c)}+2^n e^{(cn+4\,dx+4\,c)}-2^n e^{(cn+2\,dx+2\,c)}-2^n e^{(cn)}\right)e^{(dnx)}}{8\left(2^n a^2 dn e^{(dnx+cn+3\,dx+3\,c)}+abdn e^{\left(3\,dx+n\log\left(e^{(dx+c)}+1\right)+n\log\left(e^{(dx+c)}-1\right)+3\,c\right)}\right)}+\frac{1}{8}\int \frac{\left(2^n n e^{(cn)}-3\cdot 2^n e^{(cn)}+\left(2^n n e^{(cn)}-3\cdot 2^n e^{(}\right.\right.}{2^n a^2 n e^{(dnx}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/8*(2^n*e^(c*n + 6*d*x + 6*c) + 2^n*e^(c*n + 4*d*x + 4*c) - 2^n*e^(c*n + 2
*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n*x + c*n + 3*d*x +
```

```
3*c) + a*b*d*n*e^(3*d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 3*c)) + 1/8*integrate((2^n*n*e^(c*n) - 3*2^n*e^(c*n) + (2^n*n*e^(c*n) - 3*
2^n*e^(c*n))*e^(6*d*x + 6*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(4*d*x + 4
*c) + (3*2^n*n*e^(c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n
*e^(d*n*x + c*n + 3*d*x + 3*c) + a*b*n*e^(3*d*x + n*log(e^(d*x + c) + 1) +
n*log(e^(d*x + c) - 1) + 3*c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\cosh\left(dx+c\right)^{3}}{b^{2}\sinh\left(dx+c\right)^{2n}+2\,ab\sinh\left(dx+c\right)^{n}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)^3/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n +
 a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)**3/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\cosh\left(dx+c\right)^{3}}{\left(b\sinh\left(dx+c\right)^{n}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)^3/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)^3/(b*sinh(d*x + c)^n + a)^2, x)
```

3.424 $\int \frac{\cosh(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2}\,dx$

Optimal. Leaf size=37

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d)
```

Rubi [A] time = 0.0439955, antiderivative size = 37, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3223, 245}

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d)
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\cosh(c+dx)}{\left(a+b\sinh^n(c+dx)\right)^2}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(a+bx^n)^2}\,dx, x, \sinh(c+dx)\right)}{d}\\ &= \frac{{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)\sinh(c+dx)}{a^2d}\end{aligned}$$

Mathematica [A] time = 0.0084442, size = 37, normalized size = 1.

$$\frac{\sinh(c+dx)\,{}_2F_1\left(2,\frac{1}{n};1+\frac{1}{n};-\frac{b\sinh^n(c+dx)}{a}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cosh[c + d*x]/(a + b*Sinh[c + d*x]^n)^2,x]
```

```
[Out] (Hypergeometric2F1[2, n^(-1), 1 + n^(-1), -((b*Sinh[c + d*x]^n)/a)]*Sinh[c
+ d*x])/(a^2*d)
```

Maple [F] time = 5.571, size = 0, normalized size = 0.

$$\int \frac{\cosh (dx + c)}{\left(a + b (\sinh (dx + c))^{n}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

```
[Out] int(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\frac{\left(2^{n} e^{(cn+2\,dx+2\,c)} - 2^{n} e^{(cn)}\right) e^{(dnx)}}{2 \left(2^{n} a^{2} d n e^{(dnx+cn+dx+c)} + abdne^{\left(dx+n \log\left(e^{(dx+c)}+1\right)+n \log\left(e^{(dx+c)}-1\right)+c\right)}\right)} + \frac{1}{2} \int \frac{\left(2^{n} n e^{(cn)} - 2^{n} e^{(cn)} + \left(2^{n} n e^{(cn)} - 2^{n} e^{(cn)}\right) e^{(}}{2^{n} a^{2} n e^{(dnx+cn+dx+c)} + abne^{\left(dx+n \log\left(e^{(dx+c)}+1\right)+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2^n*e^(c*n + 2*d*x + 2*c) - 2^n*e^(c*n))*e^(d*n*x)/(2^n*a^2*d*n*e^(d*n
*x + c*n + d*x + c) + a*b*d*n*e^(d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*
x + c) - 1) + c)) + 1/2*integrate((2^n*n*e^(c*n) - 2^n*e^(c*n) + (2^n*n*e^(
c*n) - 2^n*e^(c*n))*e^(2*d*x + 2*c))*e^(d*n*x)/(2^n*a^2*n*e^(d*n*x + c*n +
d*x + c) + a*b*n*e^(d*x + n*log(e^(d*x + c) + 1) + n*log(e^(d*x + c) - 1) +
 c)), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\cosh (dx + c)}{b^{2} \sinh (dx + c)^{2\,n} + 2\,ab \sinh (dx + c)^{n} + a^{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="fricas")
```

```
[Out] integral(cosh(d*x + c)/(b^2*sinh(d*x + c)^(2*n) + 2*a*b*sinh(d*x + c)^n + a
^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\cosh\left(dx+c\right)}{\left(b\sinh\left(dx+c\right)^{n}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cosh(d*x+c)/(a+b*sinh(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(cosh(d*x + c)/(b*sinh(d*x + c)^n + a)^2, x)
```

3.425 $\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx$

Optimal. Leaf size=17

$$\log(\sinh(x)) - \frac{1}{2}\log\left(1-\sinh^2(x)\right)$$

```
[Out] Log[Sinh[x]] - Log[1 - Sinh[x]^2]/2
```

Rubi [A] time = 0.0324463, antiderivative size = 17, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3194, 36, 31, 29}

$$\log(\sinh(x)) - \frac{1}{2}\log\left(1-\sinh^2(x)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/(1 - Sinh[x]^2),x]
```

```
[Out] Log[Sinh[x]] - Log[1 - Sinh[x]^2]/2
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(x)}{1-\sinh^2(x)}\,dx &= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{(1-x)x}\,dx, x, \sinh^2(x)\right)\\
&= \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{1-x}\,dx, x, \sinh^2(x)\right) + \frac{1}{2}\operatorname{Subst}\left(\int \frac{1}{x}\,dx, x, \sinh^2(x)\right)\\
&= \log(\sinh(x)) - \frac{1}{2}\log\left(1-\sinh^2(x)\right)
\end{aligned}$$

Mathematica [A] time = 0.013551, size = 23, normalized size = 1.35

$$-2\left(\frac{1}{4}\log\left(1-\sinh^2(x)\right)-\frac{1}{2}\log(\sinh(x))\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/(1 - Sinh[x]^2),x]
```

```
[Out] -2*(-Log[Sinh[x]]/2 + Log[1 - Sinh[x]^2]/4)
```

Maple [B] time = 0.029, size = 41, normalized size = 2.4

$$-\frac{1}{2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2-2\tanh(x/2)-1\right)-\frac{1}{2}\ln\left(\left(\tanh\left(\frac{x}{2}\right)\right)^2+2\tanh(x/2)-1\right)+\ln\left(\tanh\left(\frac{x}{2}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(1-sinh(x)^2),x)
```

```
[Out] -1/2*ln(tanh(1/2*x)^2-2*tanh(1/2*x)-1)-1/2*ln(tanh(1/2*x)^2+2*tanh(1/2*x)-1
)+ln(tanh(1/2*x))
```

Maxima [B] time = 1.109, size = 61, normalized size = 3.59

$$-\frac{1}{2}\log\left(2e^{(-x)}+e^{(-2x)}-1\right)+\log\left(e^{(-x)}+1\right)+\log\left(e^{(-x)}-1\right)-\frac{1}{2}\log\left(-2e^{(-x)}+e^{(-2x)}-1\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*log(2*e^(-x) + e^(-2*x) - 1) + log(e^(-x) + 1) + log(e^(-x) - 1) - 1/2
*log(-2*e^(-x) + e^(-2*x) - 1)
```

Fricas [B] time = 1.78169, size = 165, normalized size = 9.71

$$-\frac{1}{2}\log\left(\frac{2\left(\cosh(x)^2+\sinh(x)^2-3\right)}{\cosh(x)^2-2\cosh(x)\sinh(x)+\sinh(x)^2}\right)+\log\left(\frac{2\sinh(x)}{\cosh(x)-\sinh(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="fricas")
```

```
[Out] -1/2*log(2*(cosh(x)^2 + sinh(x)^2 - 3)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sin
h(x)^2)) + log(2*sinh(x)/(cosh(x) - sinh(x)))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$-\int\frac{\coth(x)}{\sinh^2(x)-1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)**2),x)
```

```
[Out] -Integral(coth(x)/(sinh(x)**2 - 1), x)
```

Giac [A] time = 1.1704, size = 34, normalized size = 2.

$$-\frac{1}{2}\log\left(\left|e^{(4x)} - 6e^{(2x)} + 1\right|\right) + \log\left(\left|e^{(2x)} - 1\right|\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(1-sinh(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*log(abs(e^(4*x) - 6*e^(2*x) + 1)) + log(abs(e^(2*x) - 1))
```

3.426 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx$

Optimal. Leaf size=63

$$-\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{2a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] -a^2/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) + (2*a)/(f*Sqrt[a*Cosh[e + f*x]^2]) +
Sqrt[a*Cosh[e + f*x]^2]/f
```

Rubi [A] time = 0.128025, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{2a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -a^2/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) + (2*a)/(f*Sqrt[a*Cosh[e + f*x]^2]) +
Sqrt[a*Cosh[e + f*x]^2]/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^5(e+fx)\,dx=\int \sqrt{a\cosh^2(e+fx)}\tanh^5(e+fx)\,dx$$
$$=\frac{\text{Subst}\left(\int \frac{(1-x)^2\sqrt{ax}}{x^3}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=\frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{5/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}}-\frac{2}{a(ax)^{3/2}}+\frac{1}{a^2\sqrt{ax}}\right)dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=-\frac{a^2}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}+\frac{2a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.100224, size = 51, normalized size = 0.81

$$\frac{\left(3\cosh^4(e+fx)+6\cosh^2(e+fx)-1\right)\text{sech}^4(e+fx)\sqrt{a\cosh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(-1 + 6*Cosh[e + f*x]^2 + 3*Cosh[e + f*x]^4)*Sech[
e + f*x]^4)/(3*f)
```

Maple [C] time = 0.145, size = 42, normalized size = 0.7

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^5 a}{\left(\cosh\left(fx+e\right)\right)^4}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5*a/cosh(f*x+e)^4/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [B] time = 1.79859, size = 394, normalized size = 6.25

$$\frac{6\sqrt{a}e^{(-2fx-2e)}}{f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)}+\frac{25\sqrt{a}e^{(-4fx-4e)}}{3f\left(e^{(-fx-e)}+3e^{(-3fx-3e)}+3e^{(-5fx-5e)}+e^{(-7fx-7e)}\right)}+\frac{}{f\left(e^{(-\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] 6*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*
f*x - 5*e) + e^(-7*f*x - 7*e))) + 25/3*sqrt(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x
 - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 6*sq
rt(a)*e^(-6*f*x - 6*e)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x
- 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(a)*e^(-8*f*x - 8*e)/(f*(e^(-f*x - e)
 + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x - 7*e))) + 1/2*sqrt(
a)/(f*(e^(-f*x - e) + 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) + e^(-7*f*x -
 7*e)))
```

Fricas [B] time = 1.8592, size = 2361, normalized size = 37.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] 1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x
+ e)^8 + 12*(7*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*cos
h(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cosh(f
*x + e)^4 + 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21*cos
h(f*x + e)^5 + 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh(f*x
+ e)^3 + 12*(7*cosh(f*x + e)^6 + 45*cosh(f*x + e)^4 + 25*cosh(f*x + e)^2 +
3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 + 27*cosh(f*x + e)^5
+ 25*cosh(f*x + e)^3 + 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (3*cosh
(f*x + e)^8 + 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 + 36*cosh(f*x + e)^2
+ 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x
 - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 7*(f*c
osh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6 + 3*f*cosh(
f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 + f)*e^(2*f*x +
2*e) + f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (7
*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 +
3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + (35*f*
cosh(f*x + e)^4 + 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2*e) + 3*f)*sinh(f
*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x +
e) + (7*f*cosh(f*x + e)^5 + 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*
f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^7 + 3*f*co
sh(f*x + e)^5 + 3*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (7
*f*cosh(f*x + e)^6 + 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + (7*f*cosh
(f*x + e)^6 + 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*
e) + f)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.30062, size = 108, normalized size = 1.71

$$\frac{\sqrt{a}\left(\frac{8\left(3e^{(5fx+5e)}+4e^{(3fx+3e)}+3e^{(fx+e)}\right)}{\left(e^{(2fx+2e)}+1\right)^{3}}+3e^{(fx+e)}+3e^{(-fx-e)}\right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] 1/6*sqrt(a)*(8*(3*e^(5*f*x + 5*e) + 4*e^(3*f*x + 3*e) + 3*e^(f*x + e))/(e^(
2*f*x + 2*e) + 1)^3 + 3*e^(f*x + e) + 3*e^(-f*x - e))/f
```

3.427 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx$

Optimal. Leaf size=38

$$\frac{a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] a/(f*Sqrt[a*Cosh[e + f*x]^2]) + Sqrt[a*Cosh[e + f*x]^2]/f
```

Rubi [A] time = 0.113288, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{f\sqrt{a\cosh^2(e+fx)}}+\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] a/(f*Sqrt[a*Cosh[e + f*x]^2]) + Sqrt[a*Cosh[e + f*x]^2]/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^3(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\tanh^3(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)\sqrt{ax}}{x^2}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{3/2}}-\frac{1}{a\sqrt{ax}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a}{f\sqrt{a\cosh^2(e+fx)}} + \frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.0874479, size = 29, normalized size = 0.76

$$\frac{a\left(\cosh^2(e+fx)+1\right)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] (a*(1 + Cosh[e + f*x]^2))/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] time = 0.105, size = 42, normalized size = 1.1

$$\frac{1}{f}\,\text{`int/indef0`}\left(\frac{(\sinh(fx+e))^3 a}{(\cosh(fx+e))^2}\frac{1}{\sqrt{a(\cosh(fx+e))^2}}, \sinh(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3*a/cosh(f*x+e)^2/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [B] time = 1.79138, size = 143, normalized size = 3.76

$$\frac{3\sqrt{a}e^{(-2fx-2e)}}{f\left(e^{(-fx-e)}+e^{(-3fx-3e)}\right)} + \frac{\sqrt{a}e^{(-4fx-4e)}}{2f\left(e^{(-fx-e)}+e^{(-3fx-3e)}\right)} + \frac{\sqrt{a}}{2f\left(e^{(-fx-e)}+e^{(-3fx-3e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 3*sqrt(a)*e^(-2*f*x - 2*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt
(a)*e^(-4*f*x - 4*e)/(f*(e^(-f*x - e) + e^(-3*f*x - 3*e))) + 1/2*sqrt(a)/(f
*(e^(-f*x - e) + e^(-3*f*x - 3*e)))
```

Fricas [B] time = 1.89306, size = 829, normalized size = 21.82

$$\frac{\left(4 \cosh\left(fx+e\right) e^{\left(fx+e\right)} \sinh\left(fx+e\right)^{3} + e^{\left(fx+e\right)} \sinh\left(fx+e\right)^{4} + 6\left(\cosh\left(fx+e\right)^{2}+1\right) e^{\left(fx+e\right)} \sinh\left(fx+e\right)^{2} + 4\left(\cos\right.\right.}{2\left(f \cosh\left(fx+e\right)^{3} + \left(f e^{\left(2fx+2e\right)}+f\right) \sinh\left(fx+e\right)^{3} + 3\left(f \cosh\left(fx+e\right) e^{\left(2fx+2e\right)} + f \cosh\left(fx+e\right)\right) \sinh\left(fx+e\right)^{2} + \right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e
)^4 + 6*(cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 + 6*cos
h(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
 + a)*e^(-f*x - e)/(f*cosh(f*x + e)^3 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x +
e)^3 + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^
2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e)
 + (3*f*cosh(f*x + e)^2 + (3*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*si
nh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)} \tanh^{3}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**3, x)
```

Giac [A] time = 1.25468, size = 72, normalized size = 1.89

$$\frac{\sqrt{a}\left(\frac{\left(5 e^{\left(2fx+2e\right)}+1\right) e^{\left(-e\right)}}{e^{\left(3fx+2e\right)}+e^{\left(fx\right)}} + e^{\left(fx+e\right)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*((5*e^(2*f*x + 2*e) + 1)*e^(-e)/(e^(3*f*x + 2*e) + e^(f*x)) + e
^(f*x + e))/f
```

3.428 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx$

Optimal. Leaf size=18

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] Sqrt[a*Cosh[e + f*x]^2]/f
```

Rubi [A] time = 0.0693726, antiderivative size = 18, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3176, 3205, 16, 32}

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] Sqrt[a*Cosh[e + f*x]^2]/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\tanh(e+fx)\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{x}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.0402495, size = 18, normalized size = 1.

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] Sqrt[a*Cosh[e + f*x]^2]/f
```

Maple [A] time = 0.026, size = 19, normalized size = 1.1

$$\frac{1}{f}\sqrt{a+a\left(\sinh\left(fx+e\right)\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x)
```

```
[Out] 1/f*(a+a*sinh(f*x+e)^2)^(1/2)
```

Maxima [A] time = 1.75767, size = 43, normalized size = 2.39

$$\frac{\sqrt{a}e^{(fx+e)}}{2f} + \frac{\sqrt{a}e^{(-fx-e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*e^(f*x + e)/f + 1/2*sqrt(a)*e^(-f*x - e)/f
```

Fricas [B] time = 1.80631, size = 365, normalized size = 20.28

$$\frac{\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right) + e^{(fx+e)}\sinh\left(fx+e\right)^2 + \left(\cosh\left(fx+e\right)^2+1\right)e^{(fx+e)}\right)\sqrt{ae^{(4fx+4e)} + 2ae^{(2fx+2e)} \cdots}}{2\left(f\cosh\left(fx+e\right)e^{(2fx+2e)} + f\cosh\left(fx+e\right) + \left(fe^{(2fx+2e)}+f\right)\sinh\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*
x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x +
e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x), x)
```

Giac [A] time = 1.15893, size = 35, normalized size = 1.94

$$\frac{\sqrt{a}\left(e^{\left(fx+e\right)}+e^{\left(-fx-e\right)}\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e))/f
```

3.429 $\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cosh[e + f
*x]^2]/f
```

Rubi [A] time = 0.0977005, antiderivative size = 50, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3176, 3205, 50, 63, 206}

$$\frac{\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cosh[e + f
*x]^2]/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \coth(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx &= \int \sqrt{a\cosh^2(e+fx)}\coth(e+fx)\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a\cosh^2(e+fx)}}{f} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{\sqrt{a\cosh^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{f}\\
&= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a\cosh^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [A] time = 0.0588644, size = 42, normalized size = 0.84

$$\frac{\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\cosh(e+fx)+\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(Cosh[e + f*x] + Log[Tanh[(e + f*x)/2]])*Sech[e +
f*x])/f
```

Maple [C] time = 0.089, size = 42, normalized size = 0.8

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{a\left(\cosh\left(fx+e\right)\right)^2}{\sinh\left(fx+e\right)}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(a*cosh(f*x+e)^2/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

Maxima [A] time = 1.781, size = 92, normalized size = 1.84

$$\frac{\left(\sqrt{a}e^{\left(-2\,fx-2\,e\right)}+\sqrt{a}\right)e^{\left(fx+e\right)}}{2\,f}-\frac{\sqrt{a}\log\left(e^{\left(-fx-e\right)}+1\right)}{f}+\frac{\sqrt{a}\log\left(e^{\left(-fx-e\right)}-1\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*e^(f*x + e)/f - sqrt(a)*log(e^(-f*
x - e) + 1)/f + sqrt(a)*log(e^(-f*x - e) - 1)/f
```

Fricas [B] time = 1.8532, size = 549, normalized size = 10.98

$$\frac{\left(2\,\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{2}+\left(\cosh\left(fx+e\right)^{2}+1\right)e^{\left(fx+e\right)}+2\left(\cosh\left(fx+e\right)e^{\left(fx+e\right)}+\right.\right.}{2\left(f\cosh\left(fx+e\right)e^{\left(2\,fx+2\,e\right)}+f\cosh\left(fx+e\right)+\left(fe^{\left(2\,fx\right.}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e) + 2*(cosh(f*x + e)*e^(f*x + e) + e^(f
*x + e)*sinh(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x +
e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)
*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e) + (f*e^(2*
f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\coth\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x), x)
```

Giac [A] time = 1.20647, size = 69, normalized size = 1.38

$$\frac{\sqrt{a}\left(e^{\left(fx+e\right)}+e^{\left(-fx-e\right)}-2\,\log\left(e^{\left(fx+e\right)}+1\right)+2\,\log\left(\left|e^{\left(fx+e\right)}-1\right|\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(e^(f*x + e) + e^(-f*x - e) - 2*log(e^(f*x + e) + 1) + 2*log(ab
s(e^(f*x + e) - 1)))/f
```

3.430 $\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=87

$$\frac{3\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\operatorname{csch}^2(e+fx)\left(a\cosh^2(e+fx)\right)^{3/2}}{2af}$$

```
[Out] (-3*Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/(2*f) + (3*Sqrt[a*Cos
h[e + f*x]^2])/(2*f) - ((a*Cosh[e + f*x]^2)^(3/2)*Csch[e + f*x]^2)/(2*a*f)
```

Rubi [A] time = 0.142148, antiderivative size = 87, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3176, 3205, 16, 47, 50, 63, 206}

$$\frac{3\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\operatorname{csch}^2(e+fx)\left(a\cosh^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a]*ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]])/(2*f) + (3*Sqrt[a*Cos
h[e + f*x]^2])/(2*f) - ((a*Cosh[e + f*x]^2)^(3/2)*Csch[e + f*x]^2)/(2*a*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \coth^3(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\coth^3(e+fx)\,dx$$

$$= \frac{\text{Subst}\left(\int \frac{x\sqrt{ax}}{(1-x)^2}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{(ax)^{3/2}}{(1-x)^2}\,dx, x, \cosh^2(e+fx)\right)}{2af}$$

$$= -\frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} - \frac{3\,\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cosh^2(e+fx)\right)}{4f}$$

$$= \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} - \frac{(3a)\,\text{Subst}\left(\int \frac{}{(1}\right.}{}$$

$$= \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+fx)\right)^{3/2}\operatorname{csch}^2(e+fx)}{2af} - \frac{3\,\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\right.}{}$$

$$= -\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{3\sqrt{a\cosh^2(e+fx)}}{2f} - \frac{\left(a\cosh^2(e+fx)\right)^{3/2}\text{c}}{2af}$$

Mathematica [A] time = 0.260867, size = 77, normalized size = 0.89

$$\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(8\cosh(e+fx)-\operatorname{csch}^2\left(\frac{1}{2}(e+fx)\right)-\operatorname{sech}^2\left(\frac{1}{2}(e+fx)\right)+12\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(8*Cosh[e + f*x] - Csch[(e + f*x)/2]^2 + 12*Log[Ta
nh[(e + f*x)/2]] - Sech[(e + f*x)/2]^2)*Sech[e + f*x])/(8*f)
```

Maple [C] time = 0.092, size = 54, normalized size = 0.6

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{a\left(\cosh\left(fx+e\right)\right)^{4}}{\sinh\left(fx+e\right)\left(\left(\cosh\left(fx+e\right)\right)^{2}-1\right)}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^{2}}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(a*cosh(f*x+e)^4/sinh(f*x+e)/(cosh(f*x+e)^2-1)/(a*cosh(f*x+e)^2
)^(1/2),sinh(f*x+e))/f
```

Maxima [A] time = 1.94735, size = 170, normalized size = 1.95

$$-\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)}{2f}+\frac{3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)}{2f}-\frac{3\sqrt{a}e^{(-2fx-2e)}+3\sqrt{a}e^{(-4fx-4e)}-\sqrt{a}e^{(-6fx-6e)}-\sqrt{a}}{2f\left(e^{(-fx-e)}-2e^{(-3fx-3e)}+e^{(-5fx-5e)}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -3/2*sqrt(a)*log(e^(-f*x - e) + 1)/f + 3/2*sqrt(a)*log(e^(-f*x - e) - 1)/f
- 1/2*(3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) - sqrt(a)*e^
(-6*f*x - 6*e) - sqrt(a))/(f*(e^(-f*x - e) - 2*e^(-3*f*x - 3*e) + e^(-5*f*x
 - 5*e)))
```

Fricas [B] time = 1.97631, size = 2056, normalized size = 23.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x + e
)^6 + 3*(5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f*x
 + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x + e)
^4 - 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x + e)^
5 - 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*
x + e)^6 - 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 + 1)*e^(f*x + e) + 3*(5*co
sh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5 + 2*(
5*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x + e)^3 -
 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 - 6*cosh
(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 - 2*cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e) - 1
)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f
*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^5 + (f*e^(2*f*x + 2*e) + f)*si
nh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(
f*x + e)^4 - 2*f*cosh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^2 + (5*f*cosh(f*x +
 e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)^3 + 2*(5*f*cosh(f*x + e)^3 -
```

```
3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e)^2 + f*cosh(f*x + e) + (f*cosh(f*x + e)^5 - 2*f*cosh(f*x +
 e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (5*f*cosh(f*x + e)^4 - 6*f*cosh(
f*x + e)^2 + (5*f*cosh(f*x + e)^4 - 6*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e
) + f)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.25896, size = 120, normalized size = 1.38

$$-\frac{\sqrt{a}\left(\frac{2\left(e^{\left(3\,fx+3\,e\right)}+e^{\left(fx+e\right)}\right)}{\left(e^{\left(2\,fx+2\,e\right)}-1\right)^{2}}-e^{\left(fx+e\right)}-e^{\left(-fx-e\right)}+3\,\log\left(e^{\left(fx+e\right)}+1\right)-3\,\log\left(\left|e^{\left(fx+e\right)}-1\right|\right)\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(2*(e^(3*f*x + 3*e) + e^(f*x + e))/(e^(2*f*x + 2*e) - 1)^2 - e
^(f*x + e) - e^(-f*x - e) + 3*log(e^(f*x + e) + 1) - 3*log(abs(e^(f*x + e)
- 1)))/f
```

3.431 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx$

Optimal. Leaf size=120

$$-\frac{\tanh^5(e+fx)\sqrt{a\cosh^2(e+fx)}}{4f}-\frac{5\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}+\frac{15\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}-\frac{15\text{secl}}{}$$

```
[Out] (-15*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(8*f) + (
15*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(8*f) - (5*Sqrt[a*Cosh[e + f*x]^2
]*Tanh[e + f*x]^3)/(8*f) - (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^5)/(4*f)
```

Rubi [A] time = 0.129995, antiderivative size = 120, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3176, 3207, 2592, 288, 321, 203}

$$-\frac{\tanh^5(e+fx)\sqrt{a\cosh^2(e+fx)}}{4f}-\frac{5\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}+\frac{15\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{8f}-\frac{15\text{secl}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]
```


```
[Out] (-15*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(8*f) + (
15*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(8*f) - (5*Sqrt[a*Cosh[e + f*x]^2
]*Tanh[e + f*x]^3)/(8*f) - (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x]^5)/(4*f)
```


Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a+a\sinh^2(e+fx)}\tanh^6(e+fx)\,dx &= \int \sqrt{a\cosh^2(e+fx)}\tanh^6(e+fx)\,dx \\
&= \left(\sqrt{a\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\int \sinh(e+fx)\tanh^5(e+fx)\,dx \\
&= \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \sinh(e+fx)\right)}{f} \\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\tanh^5(e+fx)}{4f} + \frac{\left(5\sqrt{a\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Sul}}{4f} \\
&= -\frac{5\sqrt{a\cosh^2(e+fx)}\tanh^3(e+fx)}{8f} - \frac{\sqrt{a\cosh^2(e+fx)}\tanh^5(e+fx)}{4f} + \\
&= \frac{15\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{8f} - \frac{5\sqrt{a\cosh^2(e+fx)}\tanh^3(e+fx)}{8f} - \\
&= -\frac{15\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)}{8f} + \frac{15\sqrt{a\cosh^2(e+}}{8}
\end{aligned}$$

Mathematica [A] time = 0.34292, size = 75, normalized size = 0.62

$$-\frac{\text{sech}^5(e+fx)\sqrt{a\cosh^2(e+fx)}\left(-5\sinh(e+fx)-15\sinh(3(e+fx))-2\sinh(5(e+fx))+60\cosh^4(e+fx)\tan^{-1}(\text{si}\right.}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^6,x]
```

```
[Out] -(Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]^5*(60*ArcTan[Sinh[e + f*x]]*Cosh[e
+ f*x]^4 - 5*Sinh[e + f*x] - 15*Sinh[3*(e + f*x)] - 2*Sinh[5*(e + f*x)]))/(
32*f)
```

Maple [A] time = 0.109, size = 85, normalized size = 0.7

$$-\frac{a\left(15\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh\left(fx+e\right)\right)^{4}-8\left(\cosh\left(fx+e\right)\right)^{4}\sinh\left(fx+e\right)-9\left(\cosh\left(fx+e\right)\right)^{2}\sinh\left(fx+e\right)\right.}{8\left(\cosh\left(fx+e\right)\right)^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x)
```

```
[Out] -1/8*a*(15*arctan(sinh(f*x+e))*cosh(f*x+e)^4-8*cosh(f*x+e)^4*sinh(f*x+e)-9*
cosh(f*x+e)^2*sinh(f*x+e)+2*sinh(f*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1
/2)/f
```

Maxima [B] time = 1.90429, size = 1203, normalized size = 10.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 315/128*sqrt(a)*arctan(e^(-f*x - e))/f + 1/128*(105*sqrt(a)*arctan(e^(-f*x
- e)) + (279*sqrt(a)*e^(-f*x - e) + 511*sqrt(a)*e^(-3*f*x - 3*e) + 385*sqrt
(a)*e^(-5*f*x - 5*e) + 105*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) +
6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f + 1/128*
(105*sqrt(a)*arctan(e^(-f*x - e)) - (105*sqrt(a)*e^(-f*x - e) + 385*sqrt(a)
*e^(-3*f*x - 3*e) + 511*sqrt(a)*e^(-5*f*x - 5*e) + 279*sqrt(a)*e^(-7*f*x -
7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8
*f*x - 8*e) + 1))/f - 5/256*(15*sqrt(a)*arctan(e^(-f*x - e)) - (15*sqrt(a)*
e^(-f*x - e) + 55*sqrt(a)*e^(-3*f*x - 3*e) + 73*sqrt(a)*e^(-5*f*x - 5*e) -
15*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e
^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f - 5/256*(15*sqrt(a)*arctan(e^(-f
*x - e)) - (15*sqrt(a)*e^(-f*x - e) - 73*sqrt(a)*e^(-3*f*x - 3*e) - 55*sqrt
(a)*e^(-5*f*x - 5*e) - 15*sqrt(a)*e^(-7*f*x - 7*e))/(4*e^(-2*f*x - 2*e) + 6
*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))/f + 5/64*(3
*sqrt(a)*arctan(e^(-f*x - e)) - (3*sqrt(a)*e^(-f*x - e) + 11*sqrt(a)*e^(-3*
f*x - 3*e) - 11*sqrt(a)*e^(-5*f*x - 5*e) - 3*sqrt(a)*e^(-7*f*x - 7*e))/(4*e
^(-2*f*x - 2*e) + 6*e^(-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e
) + 1))/f + 1/256*(837*sqrt(a)*e^(-2*f*x - 2*e) + 1533*sqrt(a)*e^(-4*f*x -
4*e) + 1155*sqrt(a)*e^(-6*f*x - 6*e) + 315*sqrt(a)*e^(-8*f*x - 8*e) + 128*s
qrt(a))/(f*(e^(-f*x - e) + 4*e^(-3*f*x - 3*e) + 6*e^(-5*f*x - 5*e) + 4*e^(-
7*f*x - 7*e) + e^(-9*f*x - 9*e))) - 1/256*(315*sqrt(a)*e^(-f*x - e) + 1155*
sqrt(a)*e^(-3*f*x - 3*e) + 1533*sqrt(a)*e^(-5*f*x - 5*e) + 837*sqrt(a)*e^(-
7*f*x - 7*e) + 128*sqrt(a)*e^(-9*f*x - 9*e))/(f*(4*e^(-2*f*x - 2*e) + 6*e^(
-4*f*x - 4*e) + 4*e^(-6*f*x - 6*e) + e^(-8*f*x - 8*e) + 1))
```

Fricas [B] time = 2.04515, size = 4517, normalized size = 37.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] 1/4*(20*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^9 + 2*e^(f*x + e)*sinh(f*x
+ e)^10 + 15*(6*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^8 + 120*(2*c
osh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^7 + 5*(84*cosh(f*
x + e)^4 + 84*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^6 + 6*(84*cosh
(f*x + e)^5 + 140*cosh(f*x + e)^3 + 5*cosh(f*x + e))*e^(f*x + e)*sinh(f*x +
 e)^5 + 5*(84*cosh(f*x + e)^6 + 210*cosh(f*x + e)^4 + 15*cosh(f*x + e)^2 -
1)*e^(f*x + e)*sinh(f*x + e)^4 + 20*(12*cosh(f*x + e)^7 + 42*cosh(f*x + e)^
5 + 5*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 15*(6*
cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 5*cosh(f*x + e)^4 - 2*cosh(f*x + e)^
2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 10*(2*cosh(f*x + e)^9 + 12*cosh(f*x +
e)^7 + 3*cosh(f*x + e)^5 - 2*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)
*sinh(f*x + e) - 15*(9*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + e^(f*x +
 e)*sinh(f*x + e)^9 + 4*(9*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^7
 + 28*(3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 6*(
21*cosh(f*x + e)^4 + 14*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^5 +
2*(63*cosh(f*x + e)^5 + 70*cosh(f*x + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*
sinh(f*x + e)^4 + 4*(21*cosh(f*x + e)^6 + 35*cosh(f*x + e)^4 + 15*cosh(f*x
+ e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 12*(3*cosh(f*x + e)^7 + 7*cosh(f*
x + e)^5 + 5*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 +
 (9*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 30*cosh(f*x + e)^4 + 12*cosh(f*x
 + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^9 + 4*cosh(f*x + e)
^7 + 6*cosh(f*x + e)^5 + 4*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e))*ar
ctan(cosh(f*x + e) + sinh(f*x + e)) + (2*cosh(f*x + e)^10 + 15*cosh(f*x + e
)^8 + 5*cosh(f*x + e)^6 - 5*cosh(f*x + e)^4 - 15*cosh(f*x + e)^2 - 2)*e^(f*
x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*c
osh(f*x + e)^9 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^9 + 9*(f*cosh(f*x +
e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^8 + 4*f*cosh(f*x + e)^7
 + 4*(9*f*cosh(f*x + e)^2 + (9*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*
sinh(f*x + e)^7 + 28*(3*f*cosh(f*x + e)^3 + f*cosh(f*x + e) + (3*f*cosh(f*x
 + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 6*f*cosh(f*x
+ e)^5 + 6*(21*f*cosh(f*x + e)^4 + 14*f*cosh(f*x + e)^2 + (21*f*cosh(f*x +
e)^4 + 14*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 2*(
63*f*cosh(f*x + e)^5 + 70*f*cosh(f*x + e)^3 + 15*f*cosh(f*x + e) + (63*f*co
sh(f*x + e)^5 + 70*f*cosh(f*x + e)^3 + 15*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^4 + 4*f*cosh(f*x + e)^3 + 4*(21*f*cosh(f*x + e)^6 + 35*f*cos
h(f*x + e)^4 + 15*f*cosh(f*x + e)^2 + (21*f*cosh(f*x + e)^6 + 35*f*cosh(f*x
 + e)^4 + 15*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^3 +
12*(3*f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 + f*cos
h(f*x + e) + (3*f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)
^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) +
(f*cosh(f*x + e)^9 + 4*f*cosh(f*x + e)^7 + 6*f*cosh(f*x + e)^5 + 4*f*cosh(f
*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e) + (9*f*cosh(f*x + e)^8 + 28*f*
cosh(f*x + e)^6 + 30*f*cosh(f*x + e)^4 + 12*f*cosh(f*x + e)^2 + (9*f*cosh(f
*x + e)^8 + 28*f*cosh(f*x + e)^6 + 30*f*cosh(f*x + e)^4 + 12*f*cosh(f*x + e
)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**6,x)
```

```
[Out] Timed out
```

Giac [A] time = 1.40308, size = 134, normalized size = 1.12

$$\frac{\sqrt{a}\left(\frac{9 e^{\left(7 fx+7 e\right)}+e^{\left(5 fx+5 e\right)}-e^{\left(3 fx+3 e\right)}-9 e^{\left(fx+e\right)}}{\left(e^{\left(2 fx+2 e\right)}+1\right)^{4}}-15 \arctan\left(e^{\left(fx+e\right)}\right)+2 e^{\left(fx+e\right)}-2 e^{\left(-fx-e\right)}\right)}{4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^6,x, algorithm="giac")
```

```
[Out] 1/4*sqrt(a)*((9*e^(7*f*x + 7*e) + e^(5*f*x + 5*e) - e^(3*f*x + 3*e) - 9*e^(
f*x + e))/(e^(2*f*x + 2*e) + 1)^4 - 15*arctan(e^(f*x + e)) + 2*e^(f*x + e)
- 2*e^(-f*x - e))/f
```

3.432 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx$

Optimal. Leaf size=91

$$-\frac{\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}+\frac{3\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\tan^{-1}(\sinh(e+$$

```
[Out] (-3*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(2*f) + (3
*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(2*f) - (Sqrt[a*Cosh[e + f*x]^2]*Ta
nh[e + f*x]^3)/(2*f)
```

Rubi [A] time = 0.124577, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3176, 3207, 2592, 288, 321, 203}

$$-\frac{\tanh^3(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}+\frac{3\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{2f}-\frac{3\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\tan^{-1}(\sinh(e+$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] (-3*ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/(2*f) + (3
*Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/(2*f) - (Sqrt[a*Cosh[e + f*x]^2]*Ta
nh[e + f*x]^3)/(2*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
```

```
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\tanh^4(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\int \sinh(e+fx)\tanh^3(e+fx)\,dx$$
$$= \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \sinh(e+fx)\right)}{f}$$
$$= -\frac{\sqrt{a\cosh^2(e+fx)}\tanh^3(e+fx)}{2f} + \frac{\left(3\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)}{2}$$
$$= \frac{3\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{2f} - \frac{\sqrt{a\cosh^2(e+fx)}\tanh^3(e+fx)}{2f} - $$
$$= -\frac{3\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)}{2f} + 3\sqrt{a\cosh^2(e+}$$

Mathematica [A] time = 0.194464, size = 55, normalized size = 0.6

$$\frac{a\left((\cosh(2(e+fx))+2)\tanh(e+fx)-3\cosh(e+fx)\tan^{-1}(\sinh(e+fx))\right)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] (a*(-3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (2 + Cosh[2*(e + f*x)])*Tanh[e
 + f*x]))/(2*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [A] time = 0.098, size = 69, normalized size = 0.8

$$-\frac{a\left(3\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh\left(fx+e\right)\right)^2-2\left(\cosh\left(fx+e\right)\right)^2\sinh\left(fx+e\right)-\sinh\left(fx+e\right)\right)}{2f\cosh\left(fx+e\right)}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x)
```

```
[Out] -1/2*a*(3*arctan(sinh(f*x+e))*cosh(f*x+e)^2-2*cosh(f*x+e)^2*sinh(f*x+e)-sin
h(f*x+e))/cosh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.67472, size = 522, normalized size = 5.74

$$\frac{15\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{8f}+\frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right)+\frac{5\sqrt{a}e^{(-fx-e)}+3\sqrt{a}e^{(-3fx-3e)}}{2e^{(-2fx-2e)}+e^{(-4fx-4e)}+1}}{4f}+\frac{3\sqrt{a}\arctan\left(e^{(-fx-e)}\right)-\frac{3\sqrt{a}e^{(-fx-e)}+5\sqrt{a}e^{(-3}}{2e^{(-2fx-2e)}+e^{(-4fx-4}}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] 15/8*sqrt(a)*arctan(e^(-f*x - e))/f + 1/4*(3*sqrt(a)*arctan(e^(-f*x - e)) +
 (5*sqrt(a)*e^(-f*x - e) + 3*sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e)
+ e^(-4*f*x - 4*e) + 1))/f + 1/4*(3*sqrt(a)*arctan(e^(-f*x - e)) - (3*sqrt(
a)*e^(-f*x - e) + 5*sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e) + e^(-4*f
*x - 4*e) + 1))/f - 3/8*(sqrt(a)*arctan(e^(-f*x - e)) - (sqrt(a)*e^(-f*x -
e) - sqrt(a)*e^(-3*f*x - 3*e))/(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))
/f + 1/16*(25*sqrt(a)*e^(-2*f*x - 2*e) + 15*sqrt(a)*e^(-4*f*x - 4*e) + 8*sq
rt(a))/(f*(e^(-f*x - e) + 2*e^(-3*f*x - 3*e) + e^(-5*f*x - 5*e))) - 1/16*(1
5*sqrt(a)*e^(-f*x - e) + 25*sqrt(a)*e^(-3*f*x - 3*e) + 8*sqrt(a)*e^(-5*f*x
- 5*e))/(f*(2*e^(-2*f*x - 2*e) + e^(-4*f*x - 4*e) + 1))
```

Fricas [B] time = 1.96191, size = 2003, normalized size = 22.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] 1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^5 + e^(f*x + e)*sinh(f*x + e
)^6 + 3*(5*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^4 + 4*(5*cosh(f*x
 + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 3*(5*cosh(f*x + e)
^4 + 6*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 6*(cosh(f*x + e)^
5 + 2*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) - 6*(5*cos
h(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + e^(f*x + e)*sinh(f*x + e)^5 + 2*(5
*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 2*(5*cosh(f*x + e)^3 +
3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (5*cosh(f*x + e)^4 + 6*cosh(
f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^5 + 2*cosh(f*x +
 e)^3 + cosh(f*x + e))*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) +
 (cosh(f*x + e)^6 + 3*cosh(f*x + e)^4 - 3*cosh(f*x + e)^2 - 1)*e^(f*x + e))
*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x
 + e)^5 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^5 + 5*(f*cosh(f*x + e)*e^(2
*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^4 + 2*f*cosh(f*x + e)^3 + 2*(5
*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*
x + e)^3 + 2*(5*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (5*f*cosh(f*x + e)^
3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + f*cosh(f*x + e) +
```

```
 (f*cosh(f*x + e)^5 + 2*f*cosh(f*x + e)^3 + f*cosh(f*x + e))*e^(2*f*x + 2*e
) + (5*f*cosh(f*x + e)^4 + 6*f*cosh(f*x + e)^2 + (5*f*cosh(f*x + e)^4 + 6*f
*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}\tanh^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**4, x)
```

Giac [A] time = 1.28528, size = 100, normalized size = 1.1

$$\frac{\sqrt{a}\left(\frac{2\left(e^{(3fx+3e)}-e^{(fx+e)}\right)}{\left(e^{(2fx+2e)}+1\right)^2}-6\arctan\left(e^{(fx+e)}\right)+e^{(fx+e)}-e^{(-fx-e)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] 1/2*sqrt(a)*(2*(e^(3*f*x + 3*e) - e^(f*x + e))/(e^(2*f*x + 2*e) + 1)^2 - 6*
arctan(e^(f*x + e)) + e^(f*x + e) - e^(-f*x - e))/f
```

3.433 $\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\tan^{-1}(\sinh(e+fx))}{f}$$

```
[Out] -((ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/f) + (Sqrt[
a*Cosh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rubi [A] time = 0.108551, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3176, 3207, 2592, 321, 203}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\tan^{-1}(\sinh(e+fx))}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] -((ArcTan[Sinh[e + f*x]]*Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x])/f) + (Sqrt[
a*Cosh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\int \sqrt{a+a\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \int \sqrt{a\cosh^2(e+fx)}\tanh^2(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\int \sinh(e+fx)\tanh(e+fx)\,dx$$
$$= \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \sinh(e+fx)\right)}{f}$$
$$= \frac{\sqrt{a\cosh^2(e+fx)}\tanh(e+fx)}{f} - \frac{\left(\sqrt{a\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Sub}}{f}$$
$$= -\frac{\tan^{-1}(\sinh(e+fx))\sqrt{a\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)}{f} + \frac{\sqrt{a\cosh^2(e+f}}{f}$$

Mathematica [A] time = 0.0526466, size = 40, normalized size = 0.7

$$\frac{\mathbf{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}\left(\sinh(e+fx)-\tan^{-1}(\sinh(e+fx))\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*Sech[e + f*x]*(-ArcTan[Sinh[e + f*x]] + Sinh[e + f
*x]))/f
```

Maple [A] time = 0.088, size = 41, normalized size = 0.7

$$-\frac{a\cosh\left(fx+e\right)\left(-\sinh\left(fx+e\right)+\arctan\left(\sinh\left(fx+e\right)\right)\right)}{f}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x)
```

```
[Out] -a*cosh(f*x+e)*(-sinh(f*x+e)+arctan(sinh(f*x+e)))/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [A] time = 1.76125, size = 68, normalized size = 1.19

$$\frac{2\sqrt{a}\arctan\left(e^{(-fx-e)}\right)}{f} + \frac{\sqrt{a}e^{(fx+e)}}{2f} - \frac{\sqrt{a}e^{(-fx-e)}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*arctan(e^(-f*x - e))/f + 1/2*sqrt(a)*e^(f*x + e)/f - 1/2*sqrt(a)*
e^(-f*x - e)/f
```

Fricas [B] time = 1.97095, size = 497, normalized size = 8.72

$$\frac{\left(2 \cosh\left(fx+e\right) e^{\left(fx+e\right)} \sinh\left(fx+e\right) + e^{\left(fx+e\right)} \sinh\left(fx+e\right)^{2} - 4 \left(\cosh\left(fx+e\right) e^{\left(fx+e\right)} + e^{\left(fx+e\right)} \sinh\left(fx+e\right)\right) \arctan\left(\right.}{2 \left(f \cosh\left(fx+e\right) e^{\left(2fx+2e\right)} + f \cosh\left(fx+e\right) + \left(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 - 4*(cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*arctan(cosh(f
*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^2 - 1)*e^(f*x + e))*sqrt(a*e^(4*f
*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)*e^(2*f*x
 + 2*e) + f*cosh(f*x + e) + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a \left(\sinh^{2}\left(e + fx\right) + 1\right)} \tanh^{2}\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*tanh(e + f*x)**2, x)
```

Giac [A] time = 1.26655, size = 51, normalized size = 0.89

$$-\frac{\sqrt{a}\left(4 \arctan\left(e^{\left(fx+e\right)}\right) - e^{\left(fx+e\right)} + e^{\left(-fx-e\right)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*(4*arctan(e^(f*x + e)) - e^(f*x + e) + e^(-f*x - e))/f
```

3.434 $\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=56

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] -((Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f) + (Sqrt[a*Cosh[e
 + f*x]^2]*Tanh[e + f*x])/f
```

Rubi [A] time = 0.113688, antiderivative size = 56, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2590, 14}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f) + (Sqrt[a*Cosh[e
 + f*x]^2]*Tanh[e + f*x])/f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$\int \coth^2(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\coth^2(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\int \cosh(e+fx)\coth^2(e+fx)\,dx$$
$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -i\sinh(e+fx)\right)}{f}$$
$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -i\sinh(e+fx$$
$$= -\frac{\sqrt{a\cosh^2(e+fx)}\operatorname{csch}(e+fx)\operatorname{sech}(e+fx)}{f} + \frac{\sqrt{a\cosh^2(e+fx)}\tanh(e+$$

Mathematica [A] time = 0.0754151, size = 35, normalized size = 0.62

$$-\frac{\tanh(e+fx)\left(\operatorname{csch}^2(e+fx)-1\right)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cosh[e + f*x]^2]*(-1 + Csch[e + f*x]^2)*Tanh[e + f*x])/f)
```

Maple [A] time = 0.085, size = 42, normalized size = 0.8

$$\frac{\cosh(fx+e)\,a\left(-1+(\sinh(fx+e))^2\right)}{\sinh(fx+e)\,f}\frac{1}{\sqrt{a(\cosh(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] cosh(f*x+e)*a*(-1+sinh(f*x+e)^2)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.73373, size = 169, normalized size = 3.02

$$\frac{\sqrt{a}e^{(-fx-e)}}{f\left(e^{(-2fx-2e)}-1\right)} - \frac{2\sqrt{a}e^{(-2fx-2e)}-\sqrt{a}}{2f\left(e^{(-fx-e)}-e^{(-3fx-3e)}\right)} + \frac{2\sqrt{a}e^{(-fx-e)}-\sqrt{a}e^{(-3fx-3e)}}{2f\left(e^{(-2fx-2e)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*e^(-f*x - e)/(f*(e^(-2*f*x - 2*e) - 1)) - 1/2*(2*sqrt(a)*e^(-2*f*x
 - 2*e) - sqrt(a))/(f*(e^(-f*x - e) - e^(-3*f*x - 3*e))) + 1/2*(2*sqrt(a)*e^
```

```
(-f*x - e) - sqrt(a)*e^(-3*f*x - 3*e))/(f*(e^(-2*f*x - 2*e) - 1))
```

Fricas [B] time = 1.78849, size = 829, normalized size = 14.8

$$\frac{\left(4 \cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{3}+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{4}+6\left(\cosh\left(fx+e\right)^{2}-1\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{2}+4\left(\right.}{2\left(f\cosh\left(fx+e\right)^{3}+\left(fe^{\left(2fx+2e\right)}+f\right)\sinh\left(fx+e\right)^{3}+3\left(f\cosh\left(fx+e\right)e^{\left(2fx+2e\right)}+f\cosh\left(fx+e\right)\right)\sinh\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*(4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e
)^4 + 6*(cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e
)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 - 6*cos
h(f*x + e)^2 + 1)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e)
 + a)*e^(-f*x - e)/(f*cosh(f*x + e)^3 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x +
e)^3 + 3*(f*cosh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^
2 - f*cosh(f*x + e) + (f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e)
 + (3*f*cosh(f*x + e)^2 + (3*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e) - f)*si
nh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}\coth^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sinh(e + f*x)**2 + 1))*coth(e + f*x)**2, x)
```

Giac [A] time = 1.29183, size = 77, normalized size = 1.38

$$-\frac{\sqrt{a}\left(\frac{\left(5e^{\left(2fx+2e\right)}-1\right)e^{\left(-e\right)}}{e^{\left(3fx+2e\right)}-e^{\left(fx\right)}}-e^{\left(fx+e\right)}\right)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a)*((5*e^(2*f*x + 2*e) - 1)*e^(-e)/(e^(3*f*x + 2*e) - e^(f*x)) -
e^(f*x + e))/f
```

3.435 $\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=91

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\text{csch}^3(e+fx)\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{3f}-\frac{2\text{csch}(e+fx)\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

```
[Out] (-2*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e
 + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/(3*f) + (Sqrt[a*Cosh[e + f*x]^2]*
Tanh[e + f*x])/f
```

Rubi [A] time = 0.121694, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2590, 270}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\text{csch}^3(e+fx)\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{3f}-\frac{2\text{csch}(e+fx)\text{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2],x]
```


```
[Out] (-2*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e
 + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/(3*f) + (Sqrt[a*Cosh[e + f*x]^2]*
Tanh[e + f*x])/f
```


Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \coth^4(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\coth^4(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\textbf{sech}(e+fx)\right)\int \cosh(e+fx)\coth^4(e+fx)\,dx$$
$$= \frac{\left(i\sqrt{a\cosh^2(e+fx)}\textbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}dx, x, -i\sinh(e+fx)\right.}{f}$$
$$= \frac{\left(i\sqrt{a\cosh^2(e+fx)}\textbf{sech}(e+fx)\right)\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, -i\sinh(e\right.}{f}$$
$$= -\frac{2\sqrt{a\cosh^2(e+fx)}\textbf{csch}(e+fx)\textbf{sech}(e+fx)}{f} - \frac{\sqrt{a\cosh^2(e+fx)}\textbf{csch}^3}{3f}$$

Mathematica [A] time = 0.0729251, size = 47, normalized size = 0.52

$$-\frac{\tanh(e+fx)\left(\text{csch}^4(e+fx)+6\text{csch}^2(e+fx)-3\right)\sqrt{a\cosh^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Sqrt[a*Cosh[e + f*x]^2]*(-3 + 6*Csch[e + f*x]^2 + Csch[e + f*x]^4)*Tanh[e
 + f*x])/(3*f)
```

Maple [A] time = 0.092, size = 55, normalized size = 0.6

$$\frac{\cosh(fx+e)\,a\left(3\left(\sinh(fx+e)\right)^4-6\left(\sinh(fx+e)\right)^2-1\right)}{3\left(\sinh(fx+e)\right)^3 f}\frac{1}{\sqrt{a\left(\cosh(fx+e)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*cosh(f*x+e)*a*(3*sinh(f*x+e)^4-6*sinh(f*x+e)^2-1)/sinh(f*x+e)^3/(a*cosh
(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.78277, size = 657, normalized size = 7.22

$$-\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-3\sqrt{a}\log\left(e^{(-fx-e)}-1\right)-\frac{2\left(9\sqrt{a}e^{(-fx-e)}-8\sqrt{a}e^{(-3fx-3e)}+3\sqrt{a}e^{(-5fx-5e)}\right)}{3e^{(-2fx-2e)}-3e^{(-4fx-4e)}+e^{(-6fx-6e)}-1}}{12f}+\frac{3\sqrt{a}\log\left(e^{(-fx-e)}+1\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/12*(3*sqrt(a)*log(e^(-f*x - e) + 1) - 3*sqrt(a)*log(e^(-f*x - e) - 1) -
2*(9*sqrt(a)*e^(-f*x - e) - 8*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*
x - 5*e))/(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*e) + e^(-6*f*x - 6*e) - 1))
/f + 1/12*(3*sqrt(a)*log(e^(-f*x - e) + 1) - 3*sqrt(a)*log(e^(-f*x - e) - 1
) + 2*(3*sqrt(a)*e^(-f*x - e) - 8*sqrt(a)*e^(-3*f*x - 3*e) + 9*sqrt(a)*e^(-
5*f*x - 5*e))/(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*e) + e^(-6*f*x - 6*e) -
 1))/f + sqrt(a)*e^(-3*f*x - 3*e)/(f*(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x - 4*
e) + e^(-6*f*x - 6*e) - 1)) - 1/12*(33*sqrt(a)*e^(-2*f*x - 2*e) - 40*sqrt(a
)*e^(-4*f*x - 4*e) + 15*sqrt(a)*e^(-6*f*x - 6*e) - 6*sqrt(a))/(f*(e^(-f*x -
 e) - 3*e^(-3*f*x - 3*e) + 3*e^(-5*f*x - 5*e) - e^(-7*f*x - 7*e))) + 1/12*(
15*sqrt(a)*e^(-f*x - e) - 40*sqrt(a)*e^(-3*f*x - 3*e) + 33*sqrt(a)*e^(-5*f*
x - 5*e) - 6*sqrt(a)*e^(-7*f*x - 7*e))/(f*(3*e^(-2*f*x - 2*e) - 3*e^(-4*f*x
 - 4*e) + e^(-6*f*x - 6*e) - 1))
```

Fricas [B] time = 1.97043, size = 2361, normalized size = 25.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*(24*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + 3*e^(f*x + e)*sinh(f*x
+ e)^8 + 12*(7*cosh(f*x + e)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^6 + 24*(7*cos
h(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 10*(21*cosh(f
*x + e)^4 - 54*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(21*cos
h(f*x + e)^5 - 90*cosh(f*x + e)^3 + 25*cosh(f*x + e))*e^(f*x + e)*sinh(f*x
+ e)^3 + 12*(7*cosh(f*x + e)^6 - 45*cosh(f*x + e)^4 + 25*cosh(f*x + e)^2 -
3)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(3*cosh(f*x + e)^7 - 27*cosh(f*x + e)^5
+ 25*cosh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (3*cosh
(f*x + e)^8 - 36*cosh(f*x + e)^6 + 50*cosh(f*x + e)^4 - 36*cosh(f*x + e)^2
+ 3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x
 - e)/(f*cosh(f*x + e)^7 + (f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 7*(f*c
osh(f*x + e)*e^(2*f*x + 2*e) + f*cosh(f*x + e))*sinh(f*x + e)^6 - 3*f*cosh(
f*x + e)^5 + 3*(7*f*cosh(f*x + e)^2 + (7*f*cosh(f*x + e)^2 - f)*e^(2*f*x +
2*e) - f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e) + (7
*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 +
3*f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 30*f*cosh(f*x + e)^2 + (35*f*
cosh(f*x + e)^4 - 30*f*cosh(f*x + e)^2 + 3*f)*e^(2*f*x + 2*e) + 3*f)*sinh(f
*x + e)^3 + 3*(7*f*cosh(f*x + e)^5 - 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x +
e) + (7*f*cosh(f*x + e)^5 - 10*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*e^(2*
f*x + 2*e))*sinh(f*x + e)^2 - f*cosh(f*x + e) + (f*cosh(f*x + e)^7 - 3*f*co
sh(f*x + e)^5 + 3*f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e) + (7
*f*cosh(f*x + e)^6 - 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 + (7*f*cosh
(f*x + e)^6 - 15*f*cosh(f*x + e)^4 + 9*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*
e) - f)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.27173, size = 108, normalized size = 1.19

$$-\frac{\sqrt{a}\left(\frac{8\left(3e^{(5fx+5e)}-4e^{(3fx+3e)}+3e^{(fx+e)}\right)}{\left(e^{(2fx+2e)}-1\right)^{3}}-3e^{(fx+e)}+3e^{(-fx-e)}\right)}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/6*sqrt(a)*(8*(3*e^(5*f*x + 5*e) - 4*e^(3*f*x + 3*e) + 3*e^(f*x + e))/(e^
(2*f*x + 2*e) - 1)^3 - 3*e^(f*x + e) + 3*e^(-f*x - e))/f
```

3.436 $\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}^5(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{5f}-\frac{\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e}{f}$$

```
[Out] (-3*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e
 + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/f - (Sqrt[a*Cosh[e + f*x]^2]*Csch
[e + f*x]^5*Sech[e + f*x])/(5*f) + (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/
f
```

Rubi [A] time = 0.124526, antiderivative size = 124, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2590, 270}

$$\frac{\tanh(e+fx)\sqrt{a\cosh^2(e+fx)}}{f}-\frac{\operatorname{csch}^5(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{5f}-\frac{\operatorname{csch}^3(e+fx)\operatorname{sech}(e+fx)\sqrt{a\cosh^2(e}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a*Cosh[e + f*x]^2]*Csch[e + f*x]*Sech[e + f*x])/f - (Sqrt[a*Cosh[e
 + f*x]^2]*Csch[e + f*x]^3*Sech[e + f*x])/f - (Sqrt[a*Cosh[e + f*x]^2]*Csch
[e + f*x]^5*Sech[e + f*x])/(5*f) + (Sqrt[a*Cosh[e + f*x]^2]*Tanh[e + f*x])/
f
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rubi steps

$$\int \coth^6(e+fx)\sqrt{a+a\sinh^2(e+fx)}\,dx = \int \sqrt{a\cosh^2(e+fx)}\coth^6(e+fx)\,dx$$
$$= \left(\sqrt{a\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\int \cosh(e+fx)\coth^6(e+fx)\,dx$$
$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}\,dx, x, -i\sinh(e+fx\right.}{f}$$
$$= -\frac{\left(i\sqrt{a\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)dx, x, -\right.}{f}$$
$$= -\frac{3\sqrt{a\cosh^2(e+fx)}\mathbf{csch}(e+fx)\mathbf{sech}(e+fx)}{f} - \frac{\sqrt{a\cosh^2(e+fx)}\mathbf{csch}^3}{f}$$

Mathematica [A] time = 0.193013, size = 67, normalized size = 0.54

$$\frac{(235\cosh(2(e+fx))-90\cosh(4(e+fx))+5\cosh(6(e+fx))-182)\mathbf{csch}^5(e+fx)\mathbf{sech}(e+fx)\sqrt{a\cosh^2(e+fx)}}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6*Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cosh[e + f*x]^2]*(-182 + 235*Cosh[2*(e + f*x)] - 90*Cosh[4*(e + f*x
)] + 5*Cosh[6*(e + f*x)])*Csch[e + f*x]^5*Sech[e + f*x])/(160*f)
```

Maple [A] time = 0.092, size = 65, normalized size = 0.5

$$\frac{\cosh(fx+e)\,a\left(5(\sinh(fx+e))^6-15(\sinh(fx+e))^4-5(\sinh(fx+e))^2-1\right)}{5(\sinh(fx+e))^5 f}\frac{1}{\sqrt{a(\cosh(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/5*cosh(f*x+e)*a*(5*sinh(f*x+e)^6-15*sinh(f*x+e)^4-5*sinh(f*x+e)^2-1)/sinh
(f*x+e)^5/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.78392, size = 1418, normalized size = 11.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/320*(105*sqrt(a)*log(e^(-f*x - e) + 1) - 105*sqrt(a)*log(e^(-f*x - e) -
1) - 2*(375*sqrt(a)*e^(-f*x - e) - 790*sqrt(a)*e^(-3*f*x - 3*e) + 896*sqrt(
a)*e^(-5*f*x - 5*e) - 490*sqrt(a)*e^(-7*f*x - 7*e) + 105*sqrt(a)*e^(-9*f*x
- 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5
*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/f + 1/320*(105*sqrt(a)*log(e^(
-f*x - e) + 1) - 105*sqrt(a)*log(e^(-f*x - e) - 1) + 2*(105*sqrt(a)*e^(-f*x
 - e) - 490*sqrt(a)*e^(-3*f*x - 3*e) + 896*sqrt(a)*e^(-5*f*x - 5*e) - 790*s
qrt(a)*e^(-7*f*x - 7*e) + 375*sqrt(a)*e^(-9*f*x - 9*e))/(5*e^(-2*f*x - 2*e)
 - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) + e^(-10*
f*x - 10*e) - 1))/f + 1/256*(15*sqrt(a)*log(e^(-f*x - e) + 1) - 15*sqrt(a)*
log(e^(-f*x - e) - 1) + 2*(15*sqrt(a)*e^(-f*x - e) + 250*sqrt(a)*e^(-3*f*x
- 3*e) - 128*sqrt(a)*e^(-5*f*x - 5*e) + 70*sqrt(a)*e^(-7*f*x - 7*e) - 15*sq
rt(a)*e^(-9*f*x - 9*e))/(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-
6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))/f - 1/256*(15*
sqrt(a)*log(e^(-f*x - e) + 1) - 15*sqrt(a)*log(e^(-f*x - e) - 1) + 2*(15*sq
rt(a)*e^(-f*x - e) - 70*sqrt(a)*e^(-3*f*x - 3*e) + 128*sqrt(a)*e^(-5*f*x -
5*e) - 250*sqrt(a)*e^(-7*f*x - 7*e) - 15*sqrt(a)*e^(-9*f*x - 9*e))/(5*e^(-2
*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e
) + e^(-10*f*x - 10*e) - 1))/f + 2*sqrt(a)*e^(-5*f*x - 5*e)/(f*(5*e^(-2*f*x
 - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5*e^(-8*f*x - 8*e) +
e^(-10*f*x - 10*e) - 1)) - 1/640*(2895*sqrt(a)*e^(-2*f*x - 2*e) - 7110*sqrt
(a)*e^(-4*f*x - 4*e) + 8064*sqrt(a)*e^(-6*f*x - 6*e) - 4410*sqrt(a)*e^(-8*f
*x - 8*e) + 945*sqrt(a)*e^(-10*f*x - 10*e) - 320*sqrt(a))/(f*(e^(-f*x - e)
- 5*e^(-3*f*x - 3*e) + 10*e^(-5*f*x - 5*e) - 10*e^(-7*f*x - 7*e) + 5*e^(-9*
f*x - 9*e) - e^(-11*f*x - 11*e))) + 1/640*(945*sqrt(a)*e^(-f*x - e) - 4410*
sqrt(a)*e^(-3*f*x - 3*e) + 8064*sqrt(a)*e^(-5*f*x - 5*e) - 7110*sqrt(a)*e^(
-7*f*x - 7*e) + 2895*sqrt(a)*e^(-9*f*x - 9*e) - 320*sqrt(a)*e^(-11*f*x - 11
*e))/(f*(5*e^(-2*f*x - 2*e) - 10*e^(-4*f*x - 4*e) + 10*e^(-6*f*x - 6*e) - 5
*e^(-8*f*x - 8*e) + e^(-10*f*x - 10*e) - 1))
```

Fricas [B] time = 2.11362, size = 4725, normalized size = 38.1

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/10*(60*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^11 + 5*e^(f*x + e)*sinh(f*
x + e)^12 + 30*(11*cosh(f*x + e)^2 - 3)*e^(f*x + e)*sinh(f*x + e)^10 + 100*
(11*cosh(f*x + e)^3 - 9*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^9 + 5*(495
*cosh(f*x + e)^4 - 810*cosh(f*x + e)^2 + 47)*e^(f*x + e)*sinh(f*x + e)^8 +
40*(99*cosh(f*x + e)^5 - 270*cosh(f*x + e)^3 + 47*cosh(f*x + e))*e^(f*x + e
)*sinh(f*x + e)^7 + 28*(165*cosh(f*x + e)^6 - 675*cosh(f*x + e)^4 + 235*cos
h(f*x + e)^2 - 13)*e^(f*x + e)*sinh(f*x + e)^6 + 8*(495*cosh(f*x + e)^7 - 2
835*cosh(f*x + e)^5 + 1645*cosh(f*x + e)^3 - 273*cosh(f*x + e))*e^(f*x + e)
*sinh(f*x + e)^5 + 5*(495*cosh(f*x + e)^8 - 3780*cosh(f*x + e)^6 + 3290*cos
h(f*x + e)^4 - 1092*cosh(f*x + e)^2 + 47)*e^(f*x + e)*sinh(f*x + e)^4 + 20*
(55*cosh(f*x + e)^9 - 540*cosh(f*x + e)^7 + 658*cosh(f*x + e)^5 - 364*cosh(
f*x + e)^3 + 47*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^3 + 10*(33*cosh(f*
x + e)^10 - 405*cosh(f*x + e)^8 + 658*cosh(f*x + e)^6 - 546*cosh(f*x + e)^4
 + 141*cosh(f*x + e)^2 - 9)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(15*cosh(f*x +
e)^11 - 225*cosh(f*x + e)^9 + 470*cosh(f*x + e)^7 - 546*cosh(f*x + e)^5 + 2
35*cosh(f*x + e)^3 - 45*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (5*cosh(
f*x + e)^12 - 90*cosh(f*x + e)^10 + 235*cosh(f*x + e)^8 - 364*cosh(f*x + e)
^6 + 235*cosh(f*x + e)^4 - 90*cosh(f*x + e)^2 + 5)*e^(f*x + e))*sqrt(a*e^(4
*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(f*cosh(f*x + e)^11 + (
```

```
f*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^11 + 11*(f*cosh(f*x + e)*e^(2*f*x + 2*
e) + f*cosh(f*x + e))*sinh(f*x + e)^10 - 5*f*cosh(f*x + e)^9 + 5*(11*f*cosh
(f*x + e)^2 + (11*f*cosh(f*x + e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e)
^9 + 15*(11*f*cosh(f*x + e)^3 - 3*f*cosh(f*x + e) + (11*f*cosh(f*x + e)^3 -
 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*f*cosh(f*x + e)^7
 + 10*(33*f*cosh(f*x + e)^4 - 18*f*cosh(f*x + e)^2 + (33*f*cosh(f*x + e)^4
- 18*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e) + f)*sinh(f*x + e)^7 + 14*(33*f
*cosh(f*x + e)^5 - 30*f*cosh(f*x + e)^3 + 5*f*cosh(f*x + e) + (33*f*cosh(f*
x + e)^5 - 30*f*cosh(f*x + e)^3 + 5*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(
f*x + e)^6 - 10*f*cosh(f*x + e)^5 + 2*(231*f*cosh(f*x + e)^6 - 315*f*cosh(f
*x + e)^4 + 105*f*cosh(f*x + e)^2 + (231*f*cosh(f*x + e)^6 - 315*f*cosh(f*x
 + e)^4 + 105*f*cosh(f*x + e)^2 - 5*f)*e^(2*f*x + 2*e) - 5*f)*sinh(f*x + e)
^5 + 10*(33*f*cosh(f*x + e)^7 - 63*f*cosh(f*x + e)^5 + 35*f*cosh(f*x + e)^3
 - 5*f*cosh(f*x + e) + (33*f*cosh(f*x + e)^7 - 63*f*cosh(f*x + e)^5 + 35*f*
cosh(f*x + e)^3 - 5*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 5*f
*cosh(f*x + e)^3 + 5*(33*f*cosh(f*x + e)^8 - 84*f*cosh(f*x + e)^6 + 70*f*co
sh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + (33*f*cosh(f*x + e)^8 - 84*f*cosh(f*
x + e)^6 + 70*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*e^(2*f*x + 2*e)
 + f)*sinh(f*x + e)^3 + 5*(11*f*cosh(f*x + e)^9 - 36*f*cosh(f*x + e)^7 + 42
*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e) + (11*f*cosh(
f*x + e)^9 - 36*f*cosh(f*x + e)^7 + 42*f*cosh(f*x + e)^5 - 20*f*cosh(f*x +
e)^3 + 3*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - f*cosh(f*x + e
) + (f*cosh(f*x + e)^11 - 5*f*cosh(f*x + e)^9 + 10*f*cosh(f*x + e)^7 - 10*f
*cosh(f*x + e)^5 + 5*f*cosh(f*x + e)^3 - f*cosh(f*x + e))*e^(2*f*x + 2*e) +
 (11*f*cosh(f*x + e)^10 - 45*f*cosh(f*x + e)^8 + 70*f*cosh(f*x + e)^6 - 50*
f*cosh(f*x + e)^4 + 15*f*cosh(f*x + e)^2 + (11*f*cosh(f*x + e)^10 - 45*f*co
sh(f*x + e)^8 + 70*f*cosh(f*x + e)^6 - 50*f*cosh(f*x + e)^4 + 15*f*cosh(f*x
 + e)^2 - f)*e^(2*f*x + 2*e) - f)*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6*(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.29864, size = 140, normalized size = 1.13

$$-\frac{\sqrt{a}\left(\frac{4\left(15\,e^{(9\,fx+9\,e)}-40\,e^{(7\,fx+7\,e)}+66\,e^{(5\,fx+5\,e)}-40\,e^{(3\,fx+3\,e)}+15\,e^{(fx+e)}\right)}{\left(e^{(2\,fx+2\,e)}-1\right)^5}-5\,e^{(fx+e)}+5\,e^{(-fx-e)}\right)}{10\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6*(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/10*sqrt(a)*(4*(15*e^(9*f*x + 9*e) - 40*e^(7*f*x + 7*e) + 66*e^(5*f*x + 5
*e) - 40*e^(3*f*x + 3*e) + 15*e^(f*x + e))/(e^(2*f*x + 2*e) - 1)^5 - 5*e^(f
*x + e) + 5*e^(-f*x - e))/f
```

3.437 $\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}+\frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -a^2/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) + (2*a)/(3*f*(a*Cosh[e + f*x]^2)^(3/2)
) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.129055, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}+\frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -a^2/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) + (2*a)/(3*f*(a*Cosh[e + f*x]^2)^(3/2)
) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^5(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{7/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}} - \frac{2}{a(ax)^{5/2}} + \frac{1}{a^2(ax)^{3/2}}\right)dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2}{5f\left(a\cosh^2(e+fx)\right)^{5/2}} + \frac{2a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}} - \frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.0896333, size = 43, normalized size = 0.65

$$\frac{-3\text{sech}^4(e+fx) + 10\text{sech}^2(e+fx) - 15}{15f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-15 + 10*Sech[e + f*x]^2 - 3*Sech[e + f*x]^4)/(15*f*Sqrt[a*Cosh[e + f*x]^2
])
```

Maple [C] time = 0.102, size = 41, normalized size = 0.6

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^5}{\left(\cosh\left(fx+e\right)\right)^6}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}, \sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^6/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

Maxima [B] time = 2.08391, size = 602, normalized size = 9.12

$$-\frac{2\,e^{(-fx-e)}}{\left(5\,\sqrt{a}e^{(-2\,fx-2\,e)} + 10\,\sqrt{a}e^{(-4\,fx-4\,e)} + 10\,\sqrt{a}e^{(-6\,fx-6\,e)} + 5\,\sqrt{a}e^{(-8\,fx-8\,e)} + \sqrt{a}e^{(-10\,fx-10\,e)} + \sqrt{a}\right)f} - \frac{}{3\left(5\,\sqrt{a}e^{(-2\,fx-2\,e)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e)
+ 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10
*f*x - 10*e) + sqrt(a))*f) - 8/3*e^(-3*f*x - 3*e)/((5*sqrt(a)*e^(-2*f*x - 2
*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)
*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e) + sqrt(a))*f) - 116/15*e^(-5
*f*x - 5*e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10
*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x
 - 10*e) + sqrt(a))*f) - 8/3*e^(-7*f*x - 7*e)/((5*sqrt(a)*e^(-2*f*x - 2*e)
+ 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(
-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e) + sqrt(a))*f) - 2*e^(-9*f*x - 9*
e)/((5*sqrt(a)*e^(-2*f*x - 2*e) + 10*sqrt(a)*e^(-4*f*x - 4*e) + 10*sqrt(a)*
e^(-6*f*x - 6*e) + 5*sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)*e^(-10*f*x - 10*e)
+ sqrt(a))*f)
```

Fricas [B] time = 1.89517, size = 3776, normalized size = 57.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh(
f*x + e)^9 + 20*(27*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^7 + 140*
(9*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945*co
sh(f*x + e)^4 + 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 + 10*
(189*cosh(f*x + e)^5 + 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 + 35*cosh(f*x + e)^4 + 29*cosh(f*x
+ e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 + 21*cosh(
f*x + e)^5 + 29*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(27*cosh(f*x + e)^8 + 28*cosh(f*x + e)^6 + 58*cosh(f*x + e)^4 + 12*
cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e)^9 + 20*c
osh(f*x + e)^7 + 58*cosh(f*x + e)^5 + 20*cosh(f*x + e)^3 + 15*cosh(f*x + e)
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^10 + 5
*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x
 + e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*cosh(f*x +
 e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 40
*(3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e)^3 + a*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*a*f*cosh(f*x + e)^
4 + 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^4 + 14*a*f*cosh(f*
x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 10*a*f*cosh(f*x + e)^4 +
 4*(63*a*f*cosh(f*x + e)^5 + 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e)
+ (63*a*f*cosh(f*x + e)^5 + 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e))*
e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 10*(21*a*f*cosh(f*x + e)^6 + 35*a*f*cosh
(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^6 + 35*a
*f*cosh(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*
x + e)^4 + 5*a*f*cosh(f*x + e)^2 + 40*(3*a*f*cosh(f*x + e)^7 + 7*a*f*cosh(f
*x + e)^5 + 5*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e
)^7 + 7*a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^
(2*f*x + 2*e))*sinh(f*x + e)^3 + 5*(9*a*f*cosh(f*x + e)^8 + 28*a*f*cosh(f*x
 + e)^6 + 30*a*f*cosh(f*x + e)^4 + 12*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*co
sh(f*x + e)^8 + 28*a*f*cosh(f*x + e)^6 + 30*a*f*cosh(f*x + e)^4 + 12*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x
 + e)^10 + 5*a*f*cosh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 10*a*f*cosh(f*x
 + e)^4 + 5*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 10*(a*f*cosh(f*x +
 e)^9 + 4*a*f*cosh(f*x + e)^7 + 6*a*f*cosh(f*x + e)^5 + 4*a*f*cosh(f*x + e)
```

```
^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^9 + 4*a*f*cosh(f*x + e)^7 + 6*a
*f*cosh(f*x + e)^5 + 4*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5(e + fx)}{\sqrt{a(\sinh^2(e + fx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.39911, size = 128, normalized size = 1.94

$$-\frac{2\left(15\sqrt{a}e^{(9fx+9e)} + 20\sqrt{a}e^{(7fx+7e)} + 58\sqrt{a}e^{(5fx+5e)} + 20\sqrt{a}e^{(3fx+3e)} + 15\sqrt{a}e^{(fx+e)}\right)}{15af\left(e^{(2fx+2e)} + 1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/15*(15*sqrt(a)*e^(9*f*x + 9*e) + 20*sqrt(a)*e^(7*f*x + 7*e) + 58*sqrt(a)
*e^(5*f*x + 5*e) + 20*sqrt(a)*e^(3*f*x + 3*e) + 15*sqrt(a)*e^(f*x + e))/(a*
f*(e^(2*f*x + 2*e) + 1)^5)
```

3.438 $\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] a/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.117641, antiderivative size = 42, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] a/(3*f*(a*Cosh[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx=\int \frac{\tanh^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$

$$=-\frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$

$$=-\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{5/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$

$$=-\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}}-\frac{1}{a(ax)^{3/2}}\right)dx,x,\cosh^2(e+fx)\right)}{2f}$$

$$=\frac{a}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.0701354, size = 31, normalized size = 0.74

$$\frac{\text{sech}^2(e+fx)-3}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (-3 + Sech[e + f*x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] time = 0.098, size = 41, normalized size = 1.

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^3}{\left(\cosh\left(fx+e\right)\right)^4}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^4/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

Maxima [B] time = 1.89918, size = 248, normalized size = 5.9

$$-\frac{2\,e^{(-fx-e)}}{\left(3\sqrt{a}e^{(-2fx-2e)}+3\sqrt{a}e^{(-4fx-4e)}+\sqrt{a}e^{(-6fx-6e)}+\sqrt{a}\right)f}-\frac{4\,e^{(-3fx-3e)}}{3\left(3\sqrt{a}e^{(-2fx-2e)}+3\sqrt{a}e^{(-4fx-4e)}+\sqrt{a}e^{(-6fx-6e)}+\sqrt{a}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) +
 sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a))*f) - 4/3*e^(-3*f*x - 3*e)/((3*sqrt(a)*
e^(-2*f*x - 2*e) + 3*sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)*e^(-6*f*x - 6*e) +
sqrt(a))*f) - 2*e^(-5*f*x - 5*e)/((3*sqrt(a)*e^(-2*f*x - 2*e) + 3*sqrt(a)*e
^(-4*f*x - 4*e) + sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a))*f)
```

Fricas [B] time = 1.76068, size = 1692, normalized size = 40.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*x
 + e)^5 + 2*(15*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*cos
h(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*x +
 e)^4 + 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x + e)
^5 + 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*
e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*f*e^(2
*f*x + 2*e) + a*f)*sinh(f*x + e)^6 + 3*a*f*cosh(f*x + e)^4 + 6*(a*f*cosh(f*
x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*a*f*cosh
(f*x + e)^2 + a*f + (5*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x
 + e) + (5*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh
(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 + 6*a*f*cosh(f*x + e)^2 + a*f + (5*a
*f*cosh(f*x + e)^4 + 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^2 + a*f + (a*f*cosh(f*x + e)^6 + 3*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f
*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 + 2*a*f*cosh(f*x
+ e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 + 2*a*f*cosh(f*x + e)^3 +
 a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3\left(e+fx\right)}{\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.34448, size = 88, normalized size = 2.1

$$-\frac{2\left(3\sqrt{a}e^{\left(5\,fx+5\,e\right)}+2\sqrt{a}e^{\left(3\,fx+3\,e\right)}+3\sqrt{a}e^{\left(fx+e\right)}\right)}{3\,af\left(e^{\left(2\,fx+2\,e\right)}+1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(a)*e^(5*f*x + 5*e) + 2*sqrt(a)*e^(3*f*x + 3*e) + 3*sqrt(a)*e^(
f*x + e))/(a*f*(e^(2*f*x + 2*e) + 1)^3)
```

3.439 $\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=19

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rubi [A] time = 0.0696189, antiderivative size = 19, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3176, 3205, 16, 32}

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.0317421, size = 19, normalized size = 1.

$$-\frac{1}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(1/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Maple [A] time = 0.024, size = 20, normalized size = 1.1

$$-\frac{1}{f}\frac{1}{\sqrt{a+a\left(\sinh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f/(a+a*sinh(f*x+e)^2)^(1/2)
```

Maxima [A] time = 1.77428, size = 45, normalized size = 2.37

$$-\frac{2\,e^{\left(-fx-e\right)}}{\left(\sqrt{a}e^{\left(-2\,fx-2\,e\right)}+\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*e^(-f*x - e)/((sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a))*f)
```

Fricas [B] time = 1.75363, size = 428, normalized size = 22.53

$$-\frac{2\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\left(\cosh(fx+e)e^{(fx+e)}+e^{(fx+e)}\sinh(fx+e)\right)e^{(-}}{af\cosh(fx+e)^2+\left(afe^{(2fx+2e)}+af\right)\sinh(fx+e)^2+af+\left(af\cosh(fx+e)^2+af\right)e^{(2fx+2e)}+2\left(af\cosh(fx+e)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x
+ e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*f*
e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^2 + a*f)*
e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))
*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(e+fx)}{\sqrt{a\left(\sinh^2(e+fx)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.25999, size = 39, normalized size = 2.05

$$-\frac{2\,e^{(fx+e)}}{\sqrt{a}f\left(e^{(2fx+2e)}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*e^(f*x + e)/(sqrt(a)*f*(e^(2*f*x + 2*e) + 1))
```

3.440 $\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rubi [A] time = 0.0856261, antiderivative size = 31, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3176, 3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\int \frac{\coth(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{af}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Mathematica [A] time = 0.053196, size = 49, normalized size = 1.58

$$\frac{\cosh(e+fx)\left(\log\left(\sinh\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cosh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (Cosh[e + f*x]*(-Log[Cosh[(e + f*x)/2]] + Log[Sinh[(e + f*x)/2]]))/(f*Sqrt[
a*Cosh[e + f*x]^2])
```

Maple [C] time = 0.07, size = 33, normalized size = 1.1

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{1}{\sinh(fx+e)}\frac{1}{\sqrt{a(\cosh(fx+e))^2}},\sinh(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [A] time = 1.86608, size = 54, normalized size = 1.74

$$-\frac{\log\left(e^{(-fx-e)}+1\right)}{\sqrt{a}f}+\frac{\log\left(e^{(-fx-e)}-1\right)}{\sqrt{a}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -log(e^(-f*x - e) + 1)/(sqrt(a)*f) + log(e^(-f*x - e) - 1)/(sqrt(a)*f)
```

Fricas [B] time = 1.84154, size = 455, normalized size = 14.68

$$\left[\frac{\sqrt{ae^{\left(4\,fx+4\,e\right)}+2\,ae^{\left(2\,fx+2\,e\right)}+a}\log\left(\frac{\cosh\left(fx+e\right)+\sinh\left(fx+e\right)-1}{\cosh\left(fx+e\right)+\sinh\left(fx+e\right)+1}\right)}{afe^{\left(2\,fx+2\,e\right)}+af},\ \frac{2\,\sqrt{-a}\arctan\left(\frac{\sqrt{ae^{\left(4\,fx+4\,e\right)}+2\,ae^{\left(2\,fx+2\,e\right)}+a}\sqrt{-a}}{a\cosh\left(fx+e\right)e^{\left(2\,fx+2\,e\right)}+a\cosh\left(fx+e\right)+\left(ae^{\left(2\,fx+2\,e\right)}+\right.}\right.}{af}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*log((cosh(f*x + e) + sin
h(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1))/(a*f*e^(2*f*x + 2*e) +
 a*f), 2*sqrt(-a)*arctan(sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*
sqrt(-a)/(a*cosh(f*x + e)*e^(2*f*x + 2*e) + a*cosh(f*x + e) + (a*e^(2*f*x +
 2*e) + a)*sinh(f*x + e)))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(e+fx\right)}{\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.32173, size = 49, normalized size = 1.58

$$-\frac{\frac{\log\left(e^{\left(fx+e\right)}+1\right)}{\sqrt{a}}-\frac{\log\left(\left|e^{\left(fx+e\right)}-1\right|\right)}{\sqrt{a}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -(log(e^(f*x + e) + 1)/sqrt(a) - log(abs(e^(f*x + e) - 1))/sqrt(a))/f
```

3.441 $\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2af}$$

```
[Out] -ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cosh[e +
f*x]^2]*Csch[e + f*x]^2)/(2*a*f)
```

Rubi [A] time = 0.129395, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3176, 3205, 16, 47, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cosh[e +
f*x]^2]*Csch[e + f*x]^2)/(2*a*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^3(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\coth^3(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{(1-x)^2}\,dx,x,\cosh^2(e+fx)\right)}{2af}\\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af}-\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx,x,\cosh^2(e+fx)\right)}{4f}\\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af}-\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx,x,\sqrt{a\cosh^2(e+fx)}\right)}{2af}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2af}
\end{aligned}
$$

Mathematica [A] time = 0.122417, size = 65, normalized size = 0.98

$$
-\frac{\cosh(e+fx)\left(\text{csch}^2\left(\frac{1}{2}(e+fx)\right)+\text{sech}^2\left(\frac{1}{2}(e+fx)\right)-4\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\sqrt{a\cosh^2(e+fx)}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Cosh[e + f*x]*(Csch[(e + f*x)/2]^2 - 4*Log[Tanh[(e + f*x)/2]] + Sech[(e +
 f*x)/2]^2))/(8*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] time = 0.086, size = 42, normalized size = 0.6

$$
\frac{1}{f}\text{` int/indef0`}\left(\left(\left(\sinh\left(fx+e\right)\right)^{-1}+\left(\sinh\left(fx+e\right)\right)^{-3}\right)\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a*cosh(f*x+e)^2)^(1/2),sinh(f
*x+e))/f
```

Maxima [A] time = 1.90444, size = 135, normalized size = 2.05

$$-\frac{\log\left(e^{(-fx-e)}+1\right)}{2\sqrt{a}f}+\frac{\log\left(e^{(-fx-e)}-1\right)}{2\sqrt{a}f}+\frac{e^{(-fx-e)}+e^{(-3fx-3e)}}{\left(2\sqrt{a}e^{(-2fx-2e)}-\sqrt{a}e^{(-4fx-4e)}-\sqrt{a}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*log(e^(-f*x - e) + 1)/(sqrt(a)*f) + 1/2*log(e^(-f*x - e) - 1)/(sqrt(a)
*f) + (e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) - sqrt
(a)*e^(-4*f*x - 4*e) - sqrt(a))*f)
```

Fricas [B] time = 1.94943, size = 1409, normalized size = 21.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x
+ e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x
 + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*x +
e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f
*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(
f*x + e) + sinh(f*x + e) - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)))*sqrt(a*
e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^
4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 + 4
*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^3 +
2*(3*a*f*cosh(f*x + e)^2 - a*f + (3*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2
*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^4 - 2*a*f*cosh(f*x + e)^2 +
 a*f)*e^(2*f*x + 2*e) + 4*(a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (a*f*c
osh(f*x + e)^3 - a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3\left(e+fx\right)}{\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.36714, size = 109, normalized size = 1.65

$$-\frac{\frac{\log\left(e^{(fx+e)}+1\right)}{\sqrt{a}}-\frac{\log\left(\left|e^{(fx+e)}-1\right|\right)}{\sqrt{a}}+\frac{2\left(\sqrt{a}e^{(3fx+3e)}+\sqrt{a}e^{(fx+e)}\right)}{a\left(e^{(2fx+2e)}-1\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*(log(e^(f*x + e) + 1)/sqrt(a) - log(abs(e^(f*x + e) - 1))/sqrt(a) + 2*
(sqrt(a)*e^(3*f*x + 3*e) + sqrt(a)*e^(f*x + e))/(a*(e^(2*f*x + 2*e) - 1)^2)
)/f
```

3.442 $\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$-\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}-\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}+\frac{3\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - (3*
Tanh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[e + f*x]^3/(4*f*Sqrt[a*
Cosh[e + f*x]^2])
```

Rubi [A] time = 0.144237, antiderivative size = 91, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2611, 3770}

$$-\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}-\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}+\frac{3\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - (3*
Tanh[e + f*x])/(8*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[e + f*x]^3/(4*f*Sqrt[a*
Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx &= \int \frac{\tanh^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx\\
&= \frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\tanh^4(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}+\frac{(3\cosh(e+fx))\int \operatorname{sech}(e+fx)\tanh^2(e+fx)\,dx}{4\sqrt{a\cosh^2(e+fx)}}\\
&= -\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}+\frac{(3\cosh(e+fx))\int \operatorname{sech}(e+fx)\,dx}{8\sqrt{a\cosh^2(e+fx)}}\\
&= \frac{3\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}-\frac{3\tanh(e+fx)}{8f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^3(e+fx)}{4f\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.132891, size = 66, normalized size = 0.73

$$\frac{\tanh(e+fx)\left(-8\tanh^2(e+fx)-6\operatorname{sech}^2(e+fx)+3\right)+3\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (3*ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + Tanh[e + f*x]*(3 - 6*Sech[e + f*x]
^2 - 8*Tanh[e + f*x]^2))/(8*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [A] time = 0.108, size = 68, normalized size = 0.8

$$\frac{3\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh\left(fx+e\right)\right)^4-5\left(\cosh\left(fx+e\right)\right)^2\sinh\left(fx+e\right)+2\sinh\left(fx+e\right)}{8\left(\cosh\left(fx+e\right)\right)^3 f}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/8*(3*arctan(sinh(f*x+e))*cosh(f*x+e)^4-5*cosh(f*x+e)^2*sinh(f*x+e)+2*sinh
(f*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.95509, size = 848, normalized size = 9.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/48*(15*arctan(e^(-f*x - e))/sqrt(a) - (15*e^(-f*x - e) + 55*e^(-3*f*x - 3
*e) + 73*e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/(4*sqrt(a)*e^(-2*f*x - 2*e
) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8
*f*x - 8*e) + sqrt(a)))/f + 1/48*(15*arctan(e^(-f*x - e))/sqrt(a) - (15*e^(
-f*x - e) - 73*e^(-3*f*x - 3*e) - 55*e^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e)
)/(4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-
6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a)))/f - 3/32*(3*arctan(e^(-
f*x - e))/sqrt(a) - (3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-5*f*x -
5*e) - 3*e^(-7*f*x - 7*e))/(4*sqrt(a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*
x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))
)/f - 35/32*arctan(e^(-f*x - e))/(sqrt(a)*f) - 1/192*(279*e^(-f*x - e) + 51
1*e^(-3*f*x - 3*e) + 385*e^(-5*f*x - 5*e) + 105*e^(-7*f*x - 7*e))/((4*sqrt(
a)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*
e) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))*f) + 1/192*(105*e^(-f*x - e) + 385
*e^(-3*f*x - 3*e) + 511*e^(-5*f*x - 5*e) + 279*e^(-7*f*x - 7*e))/((4*sqrt(a
)*e^(-2*f*x - 2*e) + 6*sqrt(a)*e^(-4*f*x - 4*e) + 4*sqrt(a)*e^(-6*f*x - 6*e
) + sqrt(a)*e^(-8*f*x - 8*e) + sqrt(a))*f)
```

Fricas [B] time = 2.04753, size = 3584, normalized size = 39.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*x
 + e)^7 + 3*(35*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + (175*cosh(f*
x + e)^4 - 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^3 + 3*(35*cosh
(f*x + e)^5 - 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x +
e)^2 + (35*cosh(f*x + e)^6 - 15*cosh(f*x + e)^4 + 9*cosh(f*x + e)^2 - 5)*e^
(f*x + e)*sinh(f*x + e) - 3*(8*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 +
e^(f*x + e)*sinh(f*x + e)^8 + 4*(7*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*
x + e)^6 + 8*(7*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^5 + 2*(35*cosh(f*x + e)^4 + 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x
+ e)^4 + 8*(7*cosh(f*x + e)^5 + 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^3 + 4*(7*cosh(f*x + e)^6 + 15*cosh(f*x + e)^4 + 9*cosh
(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(cosh(f*x + e)^7 + 3*cosh(
f*x + e)^5 + 3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) +
 (cosh(f*x + e)^8 + 4*cosh(f*x + e)^6 + 6*cosh(f*x + e)^4 + 4*cosh(f*x + e)
^2 + 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (5*cosh(f*x +
e)^7 - 3*cosh(f*x + e)^5 + 3*cosh(f*x + e)^3 - 5*cosh(f*x + e))*e^(f*x + e)
)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(
f*x + e)^8 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^8 + 4*a*f*cosh(f*x +
 e)^6 + 8*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x
+ e)^7 + 4*(7*a*f*cosh(f*x + e)^2 + a*f + (7*a*f*cosh(f*x + e)^2 + a*f)*e^(
2*f*x + 2*e))*sinh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 8*(7*a*f*cosh(f*x +
 e)^3 + 3*a*f*cosh(f*x + e) + (7*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))
*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 2*(35*a*f*cosh(f*x + e)^4 + 30*a*f*cosh
(f*x + e)^2 + 3*a*f + (35*a*f*cosh(f*x + e)^4 + 30*a*f*cosh(f*x + e)^2 + 3*
a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 4*a*f*cosh(f*x + e)^2 + 8*(7*a*f*co
sh(f*x + e)^5 + 10*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e) + (7*a*f*cosh(
f*x + e)^5 + 10*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 + 4*(7*a*f*cosh(f*x + e)^6 + 15*a*f*cosh(f*x + e)^4 + 9*a*
f*cosh(f*x + e)^2 + a*f + (7*a*f*cosh(f*x + e)^6 + 15*a*f*cosh(f*x + e)^4 +
 9*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f
```

```
*cosh(f*x + e)^8 + 4*a*f*cosh(f*x + e)^6 + 6*a*f*cosh(f*x + e)^4 + 4*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 8*(a*f*cosh(f*x + e)^7 + 3*a*f*cosh(
f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)
^7 + 3*a*f*cosh(f*x + e)^5 + 3*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(
2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4\left(e+fx\right)}{\sqrt{a\left(\sinh^2\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**4/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.37038, size = 130, normalized size = 1.43

$$\frac{\frac{3\arctan\left(e^{(fx+e)}\right)}{\sqrt{a}}-\frac{5\sqrt{a}e^{(7fx+7e)}-3\sqrt{a}e^{(5fx+5e)}+3\sqrt{a}e^{(3fx+3e)}-5\sqrt{a}e^{(fx+e)}}{a\left(e^{(2fx+2e)}+1\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/4*(3*arctan(e^(f*x + e))/sqrt(a) - (5*sqrt(a)*e^(7*f*x + 7*e) - 3*sqrt(a)
*e^(5*f*x + 5*e) + 3*sqrt(a)*e^(3*f*x + 3*e) - 5*sqrt(a)*e^(f*x + e))/(a*(e
^(2*f*x + 2*e) + 1)^4))/f
```

3.443 $\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{2f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(2*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[
e + f*x]/(2*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.120652, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2611, 3770}

$$\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{2f\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```


```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(2*f*Sqrt[a*Cosh[e + f*x]^2]) - Tanh[
e + f*x]/(2*f*Sqrt[a*Cosh[e + f*x]^2])
```


Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\tanh^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\tanh^2(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}} + \frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\,dx}{2\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}} - \frac{\tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.0574041, size = 44, normalized size = 0.71

$$\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx)) - \tanh(e+fx)}{2f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] - Tanh[e + f*x])/(2*f*Sqrt[a*Cosh[e +
f*x]^2])
```

Maple [A] time = 0.224, size = 51, normalized size = 0.8

$$\frac{1}{f\cosh(fx+e)}\left(\frac{\arctan(\sinh(fx+e))(\cosh(fx+e))^2}{2} - \frac{\sinh(fx+e)}{2}\right)\frac{1}{\sqrt{a(\cosh(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] (1/2*arctan(sinh(f*x+e))*cosh(f*x+e)^2-1/2*sinh(f*x+e))/cosh(f*x+e)/(a*cosh
(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.80806, size = 293, normalized size = 4.73

$$\frac{\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}} - \frac{e^{(-fx-e)}-e^{(-3fx-3e)}}{2\sqrt{a}e^{(-2fx-2e)}+\sqrt{a}e^{(-4fx-4e)}+\sqrt{a}}}{2f} - \frac{3\arctan\left(e^{(-fx-e)}\right)}{2\sqrt{a}f} - \frac{5e^{(-fx-e)}+3e^{(-3fx-3e)}}{4\left(2\sqrt{a}e^{(-2fx-2e)}+\sqrt{a}e^{(-4fx-4e)}+\sqrt{a}\right)f} + \frac{}{4(2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arctan(e^(-f*x - e))/sqrt(a) - (e^(-f*x - e) - e^(-3*f*x - 3*e))/(2*sq
rt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a)))/f - 3/2*arcta
```

```
n(e^(-f*x - e))/(sqrt(a)*f) - 1/4*(5*e^(-f*x - e) + 3*e^(-3*f*x - 3*e))/((2
*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*e^(-4*f*x - 4*e) + sqrt(a))*f) + 1/4*(3
*e^(-f*x - e) + 5*e^(-3*f*x - 3*e))/((2*sqrt(a)*e^(-2*f*x - 2*e) + sqrt(a)*
e^(-4*f*x - 4*e) + sqrt(a))*f)
```

Fricas [B] time = 1.84082, size = 1343, normalized size = 21.66

$$-\frac{\left(3\cosh\left(fx+e\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{2}+e^{\left(fx+e\right)}\sinh\left(fx+e\right)^{3}+\left(3\cosh\left(fx+e\right)^{2}-1\right)e^{\left(fx+e\right)}\sinh\left(fx+e\right)-\left(4\cosh\right.\right.}{af\cosh\left(fx+e\right)^{4}+\left(afe^{\left(2fx+2e\right)}+af\right)\sinh\left(fx+e\right)^{4}+2af\cosh\left(fx+e\right)^{2}+4\left(af\cosh\left(f\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -(3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x + e)^3
 + (3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e) - (4*cosh(f*x + e)*e^(
f*x + e)*sinh(f*x + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)
^2 + 1)*e^(f*x + e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*e
^(f*x + e)*sinh(f*x + e) + (cosh(f*x + e)^4 + 2*cosh(f*x + e)^2 + 1)*e^(f*x
 + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^3 - cosh(f*x
+ e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a*f*cosh(f*x + e)^4 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^4 +
 2*a*f*cosh(f*x + e)^2 + 4*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*
x + e))*sinh(f*x + e)^3 + 2*(3*a*f*cosh(f*x + e)^2 + a*f + (3*a*f*cosh(f*x
+ e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + a*f + (a*f*cosh(f*x + e)^4
 + 2*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e) + 4*(a*f*cosh(f*x + e)^3 +
a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh^{2}\left(e+fx\right)}{\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.31437, size = 85, normalized size = 1.37

$$\frac{\frac{\arctan\left(e^{\left(fx+e\right)}\right)}{\sqrt{a}}-\frac{\sqrt{a}e^{\left(3fx+3e\right)}-\sqrt{a}e^{\left(fx+e\right)}}{a\left(e^{\left(2fx+2e\right)}+1\right)^{2}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (arctan(e^(f*x + e))/sqrt(a) - (sqrt(a)*e^(3*f*x + 3*e) - sqrt(a)*e^(f*x +
e))/(a*(e^(2*f*x + 2*e) + 1)^2))/f
```

3.444 $\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rubi [A] time = 0.104443, antiderivative size = 25, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2606, 8}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth^2(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth(e+fx)\text{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{(i\cosh(e+fx))\,\text{Subst}(\int 1\,dx, x, -i\text{csch}(e+fx))}{f\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.0373885, size = 25, normalized size = 1.

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2]))
```

Maple [A] time = 0.07, size = 32, normalized size = 1.3

$$-\frac{\cosh(fx+e)}{\sinh(fx+e)f}\frac{1}{\sqrt{a(\cosh(fx+e))^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -cosh(f*x+e)/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.84918, size = 136, normalized size = 5.44

$$\frac{\frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}} + \frac{\sqrt{a}e^{(-fx-e)}}{ae^{(-2fx-2e)}-a}}{f} - \frac{\arctan\left(e^{(-fx-e)}\right)}{\sqrt{a}f} + \frac{\sqrt{a}e^{(-fx-e)}}{\left(ae^{(-2fx-2e)}-a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] (arctan(e^(-f*x - e))/sqrt(a) + sqrt(a)*e^(-f*x - e)/(a*e^(-2*f*x - 2*e) -
a))/f - arctan(e^(-f*x - e))/(sqrt(a)*f) + sqrt(a)*e^(-f*x - e)/((a*e^(-2*f
*x - 2*e) - a)*f)
```

Fricas [B] time = 1.87446, size = 428, normalized size = 17.12

$$-\frac{2\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\left(\cosh\left(fx+e\right)e^{(fx+e)}+e^{(fx+e)}\sinh\left(fx+e\right)\right)e^{(-}}{af\cosh\left(fx+e\right)^{2}+\left(afe^{(2fx+2e)}+af\right)\sinh\left(fx+e\right)^{2}-af+\left(af\cosh\left(fx+e\right)^{2}-af\right)e^{(2fx+2e)}+2\left(af\cosh\left(fx+e\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(cosh(f*x + e)*e^(f*x
+ e) + e^(f*x + e)*sinh(f*x + e))*e^(-f*x - e)/(a*f*cosh(f*x + e)^2 + (a*f*
e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x + e)^2 - a*f)*
e^(2*f*x + 2*e) + 2*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))
*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth^{2}\left(e+fx\right)}{\sqrt{a\left(\sinh^{2}\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/sqrt(a*(sinh(e + f*x)**2 + 1)), x)
```

Giac [A] time = 1.37947, size = 39, normalized size = 1.56

$$-\frac{2\,e^{(fx+e)}}{\sqrt{a}f\left(e^{(2fx+2e)}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2*e^(f*x + e)/(sqrt(a)*f*(e^(2*f*x + 2*e) - 1))
```

3.445 $\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=61

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (Coth[e + f*x]*Csch[e + f*x]
^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.114711, antiderivative size = 61, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3176, 3207, 2606}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (Coth[e + f*x]*Csch[e + f*x]
^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth^4(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth^3(e+fx)\mathrm{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{(i\cosh(e+fx))\,\mathrm{Subst}\left(\int \left(-1+x^2\right)\,dx, x, -i\mathrm{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.0614957, size = 37, normalized size = 0.61

$$-\frac{\coth(e+fx)\left(\mathrm{csch}^2(e+fx)+3\right)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/Sqrt[a + a*Sinh[e + f*x]^2],x]

[Out] -(Coth[e + f*x]*(3 + Csch[e + f*x]^2))/(3*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [A] time = 0.098, size = 44, normalized size = 0.7

$$-\frac{\cosh\left(fx+e\right)\left(3\left(\sinh\left(fx+e\right)\right)^2+1\right)}{3\left(\sinh\left(fx+e\right)\right)^3 f}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x)

[Out] -1/3*cosh(f*x+e)*(3*sinh(f*x+e)^2+1)/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/
f
```

Maxima [B] time = 2.00025, size = 751, normalized size = 12.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) + 3*log(e^(-f*x - e) + 1)/sqrt(a) - 3*
log(e^(-f*x - e) - 1)/sqrt(a) + 4*(3*sqrt(a)*e^(-f*x - e) - sqrt(a)*e^(-3*f
*x - 3*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e
) - a))/f + 1/12*(6*arctan(e^(-f*x - e))/sqrt(a) - 3*log(e^(-f*x - e) + 1)/
```

```
sqrt(a) + 3*log(e^(-f*x - e) - 1)/sqrt(a) - 4*(sqrt(a)*e^(-3*f*x - 3*e) - 3
*sqrt(a)*e^(-5*f*x - 5*e))/(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a
*e^(-6*f*x - 6*e) - a))/f - 1/4*(3*arctan(e^(-f*x - e))/sqrt(a) + (3*sqrt(a
)*e^(-f*x - e) - 10*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-5*f*x - 5*e))/
(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a))/f -
 1/4*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/24*(27*sqrt(a)*e^(-f*x - e) - 38*
sqrt(a)*e^(-3*f*x - 3*e) + 15*sqrt(a)*e^(-5*f*x - 5*e))/((3*a*e^(-2*f*x - 2
*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f) + 1/24*(15*sqrt(a)*
e^(-f*x - e) - 38*sqrt(a)*e^(-3*f*x - 3*e) + 27*sqrt(a)*e^(-5*f*x - 5*e))/(
(3*a*e^(-2*f*x - 2*e) - 3*a*e^(-4*f*x - 4*e) + a*e^(-6*f*x - 6*e) - a)*f)
```

Fricas [B] time = 1.75004, size = 1692, normalized size = 27.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/3*(15*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^4 + 3*e^(f*x + e)*sinh(f*x
 + e)^5 + 2*(15*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 6*(5*cos
h(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 3*(5*cosh(f*x +
 e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + (3*cosh(f*x + e)
^5 - 2*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*
e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a*f*cosh(f*x + e)^6 + (a*f*e^(2
*f*x + 2*e) + a*f)*sinh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 6*(a*f*cosh(f*
x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x + e))*sinh(f*x + e)^5 + 3*(5*a*f*cosh
(f*x + e)^2 - a*f + (5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^4 + 3*a*f*cosh(f*x + e)^2 + 4*(5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x
 + e) + (5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh
(f*x + e)^3 + 3*(5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f + (5*a
*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x
 + e)^2 - a*f + (a*f*cosh(f*x + e)^6 - 3*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f
*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 6*(a*f*cosh(f*x + e)^5 - 2*a*f*cosh(f*x
+ e)^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^5 - 2*a*f*cosh(f*x + e)^3 +
 a*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.49729, size = 88, normalized size = 1.44

$$-\frac{2\left(3\sqrt{a}e^{\left(5fx+5e\right)} - 2\sqrt{a}e^{\left(3fx+3e\right)} + 3\sqrt{a}e^{\left(fx+e\right)}\right)}{3af\left(e^{\left(2fx+2e\right)} - 1\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*sqrt(a)*e^(5*f*x + 5*e) - 2*sqrt(a)*e^(3*f*x + 3*e) + 3*sqrt(a)*e^(
f*x + e))/(a*f*(e^(2*f*x + 2*e) - 1)^3)
```

3.446 $\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\text{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (2*Coth[e + f*x]*Csch[e + f*
x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x]^4)/(5*f*
Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.122724, antiderivative size = 96, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2606, 194}

$$-\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\text{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]/(f*Sqrt[a*Cosh[e + f*x]^2])) - (2*Coth[e + f*x]*Csch[e + f*
x]^2)/(3*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*Csch[e + f*x]^4)/(5*f*
Sqrt[a*Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rubi steps

$$\int \frac{\coth^6(e+fx)}{\sqrt{a+a\sinh^2(e+fx)}}\,dx = \int \frac{\coth^6(e+fx)}{\sqrt{a\cosh^2(e+fx)}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth^5(e+fx)\operatorname{csch}(e+fx)\,dx}{\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{(i\cosh(e+fx))\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, -i\operatorname{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{(i\cosh(e+fx))\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, -i\operatorname{csch}(e+fx)\right)}{f\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)}{f\sqrt{a\cosh^2(e+fx)}} - \frac{2\coth(e+fx)\operatorname{csch}^2(e+fx)}{3f\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5f\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.0819224, size = 49, normalized size = 0.51

$$-\frac{\coth(e+fx)\left(3\operatorname{csch}^4(e+fx)+10\operatorname{csch}^2(e+fx)+15\right)}{15f\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6/Sqrt[a + a*Sinh[e + f*x]^2],x]
```

```
[Out] -(Coth[e + f*x]*(15 + 10*Csch[e + f*x]^2 + 3*Csch[e + f*x]^4))/(15*f*Sqrt[a
*Cosh[e + f*x]^2])
```

Maple [A] time = 0.113, size = 54, normalized size = 0.6

$$-\frac{\cosh\left(fx+e\right)\left(15\left(\sinh\left(fx+e\right)\right)^4+10\left(\sinh\left(fx+e\right)\right)^2+3\right)}{15\left(\sinh\left(fx+e\right)\right)^5 f}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15*cosh(f*x+e)*(15*sinh(f*x+e)^4+10*sinh(f*x+e)^2+3)/sinh(f*x+e)^5/(a*co
sh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 2.24241, size = 1662, normalized size = 17.31

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/256*(120*arctan(e^(-f*x - e))/sqrt(a) + 45*log(e^(-f*x - e) + 1)/sqrt(a)
 - 45*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(105*sqrt(a)*e^(-f*x - e) - 530*sqr
t(a)*e^(-3*f*x - 3*e) + 328*sqrt(a)*e^(-5*f*x - 5*e) - 110*sqrt(a)*e^(-7*f*
x - 7*e) + 15*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*
f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x -
10*e) - a))/f - 1/256*(120*arctan(e^(-f*x - e))/sqrt(a) - 45*log(e^(-f*x -
e) + 1)/sqrt(a) + 45*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(15*sqrt(a)*e^(-f*x
- e) - 110*sqrt(a)*e^(-3*f*x - 3*e) + 328*sqrt(a)*e^(-5*f*x - 5*e) - 530*sq
rt(a)*e^(-7*f*x - 7*e) + 105*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e
) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) +
a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arctan(e^(-f*x - e))/sqrt(a) + 75*
log(e^(-f*x - e) + 1)/sqrt(a) - 75*log(e^(-f*x - e) - 1)/sqrt(a) + 2*(105*s
qrt(a)*e^(-f*x - e) + 130*sqrt(a)*e^(-3*f*x - 3*e) - 284*sqrt(a)*e^(-5*f*x
- 5*e) + 190*sqrt(a)*e^(-7*f*x - 7*e) - 45*sqrt(a)*e^(-9*f*x - 9*e))/(5*a*e
^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8
*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/320*(60*arctan(e^(-f*x - e))
/sqrt(a) - 75*log(e^(-f*x - e) + 1)/sqrt(a) + 75*log(e^(-f*x - e) - 1)/sqrt
(a) - 2*(45*sqrt(a)*e^(-f*x - e) - 190*sqrt(a)*e^(-3*f*x - 3*e) + 284*sqrt(
a)*e^(-5*f*x - 5*e) - 130*sqrt(a)*e^(-7*f*x - 7*e) - 105*sqrt(a)*e^(-9*f*x
- 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*
e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f + 1/24*(15*arctan(
e^(-f*x - e))/sqrt(a) + (15*sqrt(a)*e^(-f*x - e) - 80*sqrt(a)*e^(-3*f*x - 3
*e) + 178*sqrt(a)*e^(-5*f*x - 5*e) - 80*sqrt(a)*e^(-7*f*x - 7*e) + 15*sqrt(
a)*e^(-9*f*x - 9*e))/(5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4*f*x - 4*e) + 10*a*e
^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x - 10*e) - a))/f - 1/1
6*arctan(e^(-f*x - e))/(sqrt(a)*f) + 1/1920*(2685*sqrt(a)*e^(-f*x - e) - 73
70*sqrt(a)*e^(-3*f*x - 3*e) + 8632*sqrt(a)*e^(-5*f*x - 5*e) - 4790*sqrt(a)*
e^(-7*f*x - 7*e) + 1035*sqrt(a)*e^(-9*f*x - 9*e))/((5*a*e^(-2*f*x - 2*e) -
10*a*e^(-4*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^
(-10*f*x - 10*e) - a)*f) + 1/1920*(1035*sqrt(a)*e^(-f*x - e) - 4790*sqrt(a)
*e^(-3*f*x - 3*e) + 8632*sqrt(a)*e^(-5*f*x - 5*e) - 7370*sqrt(a)*e^(-7*f*x
- 7*e) + 2685*sqrt(a)*e^(-9*f*x - 9*e))/((5*a*e^(-2*f*x - 2*e) - 10*a*e^(-4
*f*x - 4*e) + 10*a*e^(-6*f*x - 6*e) - 5*a*e^(-8*f*x - 8*e) + a*e^(-10*f*x -
 10*e) - a)*f)
```

Fricas [B] time = 1.89456, size = 3776, normalized size = 39.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -2/15*(135*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^8 + 15*e^(f*x + e)*sinh(
f*x + e)^9 + 20*(27*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^7 + 140*
(9*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^6 + 2*(945*co
sh(f*x + e)^4 - 210*cosh(f*x + e)^2 + 29)*e^(f*x + e)*sinh(f*x + e)^5 + 10*
(189*cosh(f*x + e)^5 - 70*cosh(f*x + e)^3 + 29*cosh(f*x + e))*e^(f*x + e)*s
inh(f*x + e)^4 + 20*(63*cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 29*cosh(f*x
+ e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^3 + 20*(27*cosh(f*x + e)^7 - 21*cosh(
f*x + e)^5 + 29*cosh(f*x + e)^3 - 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(27*cosh(f*x + e)^8 - 28*cosh(f*x + e)^6 + 58*cosh(f*x + e)^4 - 12*
cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e) + (15*cosh(f*x + e)^9 - 20*c
osh(f*x + e)^7 + 58*cosh(f*x + e)^5 - 20*cosh(f*x + e)^3 + 15*cosh(f*x + e)
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a*f*cosh(f*x + e)^10 + (a*f*e^(2*f*x + 2*e) + a*f)*sinh(f*x + e)^10 - 5
*a*f*cosh(f*x + e)^8 + 10*(a*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a*f*cosh(f*x
```

```
 + e))*sinh(f*x + e)^9 + 5*(9*a*f*cosh(f*x + e)^2 - a*f + (9*a*f*cosh(f*x +
 e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 + 40
*(3*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e)^3 - a*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^7 + 10*(21*a*f*cosh(f*x + e)^
4 - 14*a*f*cosh(f*x + e)^2 + a*f + (21*a*f*cosh(f*x + e)^4 - 14*a*f*cosh(f*
x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 - 10*a*f*cosh(f*x + e)^4 +
 4*(63*a*f*cosh(f*x + e)^5 - 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e)
+ (63*a*f*cosh(f*x + e)^5 - 70*a*f*cosh(f*x + e)^3 + 15*a*f*cosh(f*x + e))*
e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 10*(21*a*f*cosh(f*x + e)^6 - 35*a*f*cosh
(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 - a*f + (21*a*f*cosh(f*x + e)^6 - 35*a
*f*cosh(f*x + e)^4 + 15*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e))*sinh(f*
x + e)^4 + 5*a*f*cosh(f*x + e)^2 + 40*(3*a*f*cosh(f*x + e)^7 - 7*a*f*cosh(f
*x + e)^5 + 5*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e) + (3*a*f*cosh(f*x + e
)^7 - 7*a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)^3 - a*f*cosh(f*x + e))*e^
(2*f*x + 2*e))*sinh(f*x + e)^3 + 5*(9*a*f*cosh(f*x + e)^8 - 28*a*f*cosh(f*x
 + e)^6 + 30*a*f*cosh(f*x + e)^4 - 12*a*f*cosh(f*x + e)^2 + a*f + (9*a*f*co
sh(f*x + e)^8 - 28*a*f*cosh(f*x + e)^6 + 30*a*f*cosh(f*x + e)^4 - 12*a*f*co
sh(f*x + e)^2 + a*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 - a*f + (a*f*cosh(f*x
 + e)^10 - 5*a*f*cosh(f*x + e)^8 + 10*a*f*cosh(f*x + e)^6 - 10*a*f*cosh(f*x
 + e)^4 + 5*a*f*cosh(f*x + e)^2 - a*f)*e^(2*f*x + 2*e) + 10*(a*f*cosh(f*x +
 e)^9 - 4*a*f*cosh(f*x + e)^7 + 6*a*f*cosh(f*x + e)^5 - 4*a*f*cosh(f*x + e)
^3 + a*f*cosh(f*x + e) + (a*f*cosh(f*x + e)^9 - 4*a*f*cosh(f*x + e)^7 + 6*a
*f*cosh(f*x + e)^5 - 4*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(1/2),x)

[Out] Timed out
```

Giac [A] time = 1.56128, size = 128, normalized size = 1.33

$$-\frac{2\left(15\sqrt{a}e^{(9fx+9e)}-20\sqrt{a}e^{(7fx+7e)}+58\sqrt{a}e^{(5fx+5e)}-20\sqrt{a}e^{(3fx+3e)}+15\sqrt{a}e^{(fx+e)}\right)}{15af\left(e^{(2fx+2e)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] -2/15*(15*sqrt(a)*e^(9*f*x + 9*e) - 20*sqrt(a)*e^(7*f*x + 7*e) + 58*sqrt(a)
*e^(5*f*x + 5*e) - 20*sqrt(a)*e^(3*f*x + 3*e) + 15*sqrt(a)*e^(f*x + e))/(a*
f*(e^(2*f*x + 2*e) - 1)^5)
```

3.447 $\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] -a^2/(7*f*(a*Cosh[e + f*x]^2)^(7/2)) + (2*a)/(5*f*(a*Cosh[e + f*x]^2)^(5/2)
) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.141147, antiderivative size = 68, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3205, 16, 43}

$$-\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -a^2/(7*f*(a*Cosh[e + f*x]^2)^(7/2)) + (2*a)/(5*f*(a*Cosh[e + f*x]^2)^(5/2)
) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tanh^5(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$=\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(ax)^{3/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=\frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{9/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{9/2}}-\frac{2}{a(ax)^{7/2}}+\frac{1}{a^2(ax)^{5/2}}\right)dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=-\frac{a^2}{7f\left(a\cosh^2(e+fx)\right)^{7/2}}+\frac{2a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.112383, size = 51, normalized size = 0.75

$$\frac{\left(-35\cosh^4(e+fx)+42\cosh^2(e+fx)-15\right)\text{sech}^4(e+fx)}{105f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-15 + 42*Cosh[e + f*x]^2 - 35*Cosh[e + f*x]^4)*Sech[e + f*x]^4)/(105*f*(a
*Cosh[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.107, size = 44, normalized size = 0.7

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^5}{\left(\cosh\left(fx+e\right)\right)^8 a}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^8/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [B] time = 2.21789, size = 791, normalized size = 11.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x -
 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3
```

```
/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x
- 14*e) + a^(3/2))*f) + 32/15*e^(-5*f*x - 5*e)/((7*a^(3/2)*e^(-2*f*x - 2*e)
 + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e
^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12
*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) - 304/35*e^(-7*f*x - 7*e)/((
7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6
*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) +
 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f) +
32/15*e^(-9*f*x - 9*e)/((7*a^(3/2)*e^(-2*f*x - 2*e) + 21*a^(3/2)*e^(-4*f*x
- 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*e^(-8*f*x - 8*e) + 21*a^(
3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x
 - 14*e) + a^(3/2))*f) - 8/3*e^(-11*f*x - 11*e)/((7*a^(3/2)*e^(-2*f*x - 2*e
) + 21*a^(3/2)*e^(-4*f*x - 4*e) + 35*a^(3/2)*e^(-6*f*x - 6*e) + 35*a^(3/2)*
e^(-8*f*x - 8*e) + 21*a^(3/2)*e^(-10*f*x - 10*e) + 7*a^(3/2)*e^(-12*f*x - 1
2*e) + a^(3/2)*e^(-14*f*x - 14*e) + a^(3/2))*f)
```

Fricas [B] time = 2.14094, size = 6649, normalized size = 97.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/105*(385*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^10 + 35*e^(f*x + e)*sin
h(f*x + e)^11 + 7*(275*cosh(f*x + e)^2 - 4)*e^(f*x + e)*sinh(f*x + e)^9 + 2
1*(275*cosh(f*x + e)^3 - 12*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^8 + 6*
(1925*cosh(f*x + e)^4 - 168*cosh(f*x + e)^2 + 19)*e^(f*x + e)*sinh(f*x + e)
^7 + 42*(385*cosh(f*x + e)^5 - 56*cosh(f*x + e)^3 + 19*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^6 + 14*(1155*cosh(f*x + e)^6 - 252*cosh(f*x + e)^4 + 1
71*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 14*(825*cosh(f*x + e)
^7 - 252*cosh(f*x + e)^5 + 285*cosh(f*x + e)^3 - 10*cosh(f*x + e))*e^(f*x +
 e)*sinh(f*x + e)^4 + 7*(825*cosh(f*x + e)^8 - 336*cosh(f*x + e)^6 + 570*co
sh(f*x + e)^4 - 40*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(27
5*cosh(f*x + e)^9 - 144*cosh(f*x + e)^7 + 342*cosh(f*x + e)^5 - 40*cosh(f*x
 + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 7*(55*cosh(f*x +
e)^10 - 36*cosh(f*x + e)^8 + 114*cosh(f*x + e)^6 - 20*cosh(f*x + e)^4 + 15*
cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (35*cosh(f*x + e)^11 - 28*cosh
(f*x + e)^9 + 114*cosh(f*x + e)^7 - 28*cosh(f*x + e)^5 + 35*cosh(f*x + e)^3
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a^2*f*cosh(f*x + e)^14 + 7*a^2*f*cosh(f*x + e)^12 + (a^2*f*e^(2*f*x + 2
*e) + a^2*f)*sinh(f*x + e)^14 + 14*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a
^2*f*cosh(f*x + e))*sinh(f*x + e)^13 + 21*a^2*f*cosh(f*x + e)^10 + 7*(13*a^
2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x +
 2*e))*sinh(f*x + e)^12 + 28*(13*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x +
 e) + (13*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*s
inh(f*x + e)^11 + 35*a^2*f*cosh(f*x + e)^8 + 7*(143*a^2*f*cosh(f*x + e)^4 +
 66*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (143*a^2*f*cosh(f*x + e)^4 + 66*a^2*f
*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^10 + 14*(143*a^2
*f*cosh(f*x + e)^5 + 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (
143*a^2*f*cosh(f*x + e)^5 + 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x +
 e))*e^(2*f*x + 2*e))*sinh(f*x + e)^9 + 35*a^2*f*cosh(f*x + e)^6 + 7*(429*a
^2*f*cosh(f*x + e)^6 + 495*a^2*f*cosh(f*x + e)^4 + 135*a^2*f*cosh(f*x + e)^
2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^6 + 495*a^2*f*cosh(f*x + e)^4 + 135*
a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 8*(429*
a^2*f*cosh(f*x + e)^7 + 693*a^2*f*cosh(f*x + e)^5 + 315*a^2*f*cosh(f*x + e)
^3 + 35*a^2*f*cosh(f*x + e) + (429*a^2*f*cosh(f*x + e)^7 + 693*a^2*f*cosh(f
```

```
*x + e)^5 + 315*a^2*f*cosh(f*x + e)^3 + 35*a^2*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e)^7 + 21*a^2*f*cosh(f*x + e)^4 + 7*(429*a^2*f*cosh(f*x +
e)^8 + 924*a^2*f*cosh(f*x + e)^6 + 630*a^2*f*cosh(f*x + e)^4 + 140*a^2*f*co
sh(f*x + e)^2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^8 + 924*a^2*f*cosh(f*x +
 e)^6 + 630*a^2*f*cosh(f*x + e)^4 + 140*a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^
(2*f*x + 2*e))*sinh(f*x + e)^6 + 14*(143*a^2*f*cosh(f*x + e)^9 + 396*a^2*f*
cosh(f*x + e)^7 + 378*a^2*f*cosh(f*x + e)^5 + 140*a^2*f*cosh(f*x + e)^3 + 1
5*a^2*f*cosh(f*x + e) + (143*a^2*f*cosh(f*x + e)^9 + 396*a^2*f*cosh(f*x + e
)^7 + 378*a^2*f*cosh(f*x + e)^5 + 140*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh
(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 7*a^2*f*cosh(f*x + e)^2 + 7*(
143*a^2*f*cosh(f*x + e)^10 + 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*x
 + e)^6 + 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 + 3*a^2*f +
(143*a^2*f*cosh(f*x + e)^10 + 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*
x + e)^6 + 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*
e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 28*(13*a^2*f*cosh(f*x + e)^11 + 55*a^2*f
*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 + 70*a^2*f*cosh(f*x + e)^5 + 25
*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (13*a^2*f*cosh(f*x + e)^11
 + 55*a^2*f*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 + 70*a^2*f*cosh(f*x
+ e)^5 + 25*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 + a^2*f + 7*(13*a^2*f*cosh(f*x + e)^12 + 66*a^2*f*cosh(f*x
 + e)^10 + 135*a^2*f*cosh(f*x + e)^8 + 140*a^2*f*cosh(f*x + e)^6 + 75*a^2*f
*cosh(f*x + e)^4 + 18*a^2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x +
e)^12 + 66*a^2*f*cosh(f*x + e)^10 + 135*a^2*f*cosh(f*x + e)^8 + 140*a^2*f*c
osh(f*x + e)^6 + 75*a^2*f*cosh(f*x + e)^4 + 18*a^2*f*cosh(f*x + e)^2 + a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^14 + 7*a^2*f*cos
h(f*x + e)^12 + 21*a^2*f*cosh(f*x + e)^10 + 35*a^2*f*cosh(f*x + e)^8 + 35*a
^2*f*cosh(f*x + e)^6 + 21*a^2*f*cosh(f*x + e)^4 + 7*a^2*f*cosh(f*x + e)^2 +
 a^2*f)*e^(2*f*x + 2*e) + 14*(a^2*f*cosh(f*x + e)^13 + 6*a^2*f*cosh(f*x + e
)^11 + 15*a^2*f*cosh(f*x + e)^9 + 20*a^2*f*cosh(f*x + e)^7 + 15*a^2*f*cosh(
f*x + e)^5 + 6*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*
x + e)^13 + 6*a^2*f*cosh(f*x + e)^11 + 15*a^2*f*cosh(f*x + e)^9 + 20*a^2*f*
cosh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^5 + 6*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.73257, size = 132, normalized size = 1.94

$$-\frac{8\left(35\,\sqrt{a}e^{\left(11\,fx+11\,e\right)}-28\,\sqrt{a}e^{\left(9\,fx+9\,e\right)}+114\,\sqrt{a}e^{\left(7\,fx+7\,e\right)}-28\,\sqrt{a}e^{\left(5\,fx+5\,e\right)}+35\,\sqrt{a}e^{\left(3\,fx+3\,e\right)}\right)}{105\,a^{2}f\left(e^{\left(2\,fx+2\,e\right)}+1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/105*(35*sqrt(a)*e^(11*f*x + 11*e) - 28*sqrt(a)*e^(9*f*x + 9*e) + 114*sqr
t(a)*e^(7*f*x + 7*e) - 28*sqrt(a)*e^(5*f*x + 5*e) + 35*sqrt(a)*e^(3*f*x + 3
*e))/(a^2*f*(e^(2*f*x + 2*e) + 1)^7)
```

3.448 $\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] a/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.126134, antiderivative size = 44, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] a/(5*f*(a*Cosh[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\tanh^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$=-\frac{\text{Subst}\left(\int \frac{1-x}{x^2(ax)^{3/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=-\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{7/2}}\,dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=-\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{1}{a(ax)^{5/2}}\right)dx,x,\cosh^2(e+fx)\right)}{2f}$$
$$=\frac{a}{5f\left(a\cosh^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.122513, size = 34, normalized size = 0.77

$$\frac{a\left(3-5\cosh^2(e+fx)\right)}{15f\left(a\cosh^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(3 - 5*Cosh[e + f*x]^2))/(15*f*(a*Cosh[e + f*x]^2)^(5/2))
```

Maple [C] time = 0.098, size = 44, normalized size = 1.

$$\frac{1}{f}\,`\texttt{int/indef0}`\left(\frac{\left(\sinh\left(fx+e\right)\right)^3}{\left(\cosh\left(fx+e\right)\right)^6 a}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^6/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [B] time = 1.87989, size = 362, normalized size = 8.23

$$-\frac{8\,e^{\left(-3\,fx-3\,e\right)}}{3\left(5\,a^{\frac{3}{2}}e^{\left(-2\,fx-2\,e\right)}+10\,a^{\frac{3}{2}}e^{\left(-4\,fx-4\,e\right)}+10\,a^{\frac{3}{2}}e^{\left(-6\,fx-6\,e\right)}+5\,a^{\frac{3}{2}}e^{\left(-8\,fx-8\,e\right)}+a^{\frac{3}{2}}e^{\left(-10\,fx-10\,e\right)}+a^{\frac{3}{2}}\right)f}+\frac{}{15\left(5\,a^{\frac{3}{2}}e^{\left(-2\,fx-2\,e\right)}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((5*a^(3/2)*e^(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x -
 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
```

```
e^(-10*f*x - 10*e) + a^(3/2))*f) + 16/15*e^(-5*f*x - 5*e)/((5*a^(3/2)*e^(-2
*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x - 4*e) + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5
*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) + a^(3/2))*f) - 8/3*
e^(-7*f*x - 7*e)/((5*a^(3/2)*e^(-2*f*x - 2*e) + 10*a^(3/2)*e^(-4*f*x - 4*e)
 + 10*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-1
0*f*x - 10*e) + a^(3/2))*f)
```

Fricas [B] time = 1.89151, size = 3571, normalized size = 81.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*
x + e)^7 + (105*cosh(f*x + e)^2 - 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 - 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cosh(f
*x + e)^4 - 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*cosh
(f*x + e)^5 - 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(7*cosh(f*x + e)^6 - 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e^(f*x
+ e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 - 2*cosh(f*x + e)^5 + 5*cosh(f*x +
e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*e^(2*f*x
+ 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e)
+ a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^6 + 5*(9*a^
2*f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x +
2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) +
 (3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x
+ e)^7 + 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x + e)^4 + 14*a^2*f
*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 + 14*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2*f*cosh(f*x + e)^
5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (63*a^2*f*cosh(f*x
+ e)^5 + 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e)
)*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21*a^2*f*cosh(f*x + e)^6
+ 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*c
osh(f*x + e)^6 + 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 + a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 40*(3*a^2*f*cosh(f*x + e)^7 + 7*a^2*f
*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (3*a^2*f
*cosh(f*x + e)^7 + 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*f + 5*(9*a^2*f*cosh
(f*x + e)^8 + 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 + 12*a^2*
f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^8 + 28*a^2*f*cosh(f*x +
e)^6 + 30*a^2*f*cosh(f*x + e)^4 + 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*
x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^10 + 5*a^2*f*cosh(f*x + e)
^8 + 10*a^2*f*cosh(f*x + e)^6 + 10*a^2*f*cosh(f*x + e)^4 + 5*a^2*f*cosh(f*x
 + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 10*(a^2*f*cosh(f*x + e)^9 + 4*a^2*f*cosh
(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cos
h(f*x + e) + (a^2*f*cosh(f*x + e)^9 + 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cos
h(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(e+fx)}{\left(a\left(\sinh^2(e+fx)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [A] time = 1.53619, size = 92, normalized size = 2.09

$$-\frac{8\left(5\sqrt{a}e^{(7fx+7e)}-2\sqrt{a}e^{(5fx+5e)}+5\sqrt{a}e^{(3fx+3e)}\right)}{15a^2f\left(e^{(2fx+2e)}+1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/15*(5*sqrt(a)*e^(7*f*x + 7*e) - 2*sqrt(a)*e^(5*f*x + 5*e) + 5*sqrt(a)*e^
(3*f*x + 3*e))/(a^2*f*(e^(2*f*x + 2*e) + 1)^5)
```

3.449 $\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=21

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

```
[Out] -1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.0774936, antiderivative size = 21, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3176, 3205, 16, 32}

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tanh(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= \frac{a\,\text{Subst}\left(\int \frac{1}{(ax)^{5/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}$$
$$= -\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.0388074, size = 21, normalized size = 1.

$$-\frac{1}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.018, size = 20, normalized size = 1.

$$-\frac{1}{3f}\left(a+a\left(\sinh\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/f/(a+a*sinh(f*x+e)^2)^(3/2)
```

Maxima [B] time = 1.8976, size = 82, normalized size = 3.9

$$-\frac{8\,e^{\left(-3\,fx-3\,e\right)}}{3\left(3\,a^{\frac{3}{2}}e^{\left(-2\,fx-2\,e\right)}+3\,a^{\frac{3}{2}}e^{\left(-4\,fx-4\,e\right)}+a^{\frac{3}{2}}e^{\left(-6\,fx-6\,e\right)}+a^{\frac{3}{2}}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/3*e^(-3*f*x - 3*e)/((3*a^(3/2)*e^(-2*f*x - 2*e) + 3*a^(3/2)*e^(-4*f*x -
4*e) + a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2))*f)
```

Fricas [B] time = 1.84687, size = 1511, normalized size = 71.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x
+ e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x +
 e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*
f*cosh(f*x + e)^6 + 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) + a^2*
f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*
x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f*x + e
)^2 + a^2*f + (5*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x +
 e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (5*a^2*f*cosh(
f*x + e)^3 + 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*
f + 3*(5*a^2*f*cosh(f*x + e)^4 + 6*a^2*f*cosh(f*x + e)^2 + a^2*f + (5*a^2*f
*cosh(f*x + e)^4 + 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f
*x + e)^2 + (a^2*f*cosh(f*x + e)^6 + 3*a^2*f*cosh(f*x + e)^4 + 3*a^2*f*cosh
(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^5 + 2*a^2*f*c
osh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5 + 2*a^2*f*cos
h(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [A] time = 1.31568, size = 43, normalized size = 2.05

$$-\frac{8\,e^{\left(3\,fx+3\,e\right)}}{3\,a^{\frac{3}{2}}f\left(e^{\left(2\,fx+2\,e\right)}+1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/3*e^(3*f*x + 3*e)/(a^(3/2)*f*(e^(2*f*x + 2*e) + 1)^3)
```

3.450 $\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cos
h[e + f*x]^2])
```

Rubi [A] time = 0.107651, antiderivative size = 53, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3176, 3205, 51, 63, 206}

$$\frac{1}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] -(ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cos
h[e + f*x]^2])
```


Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
```

```
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx \\
&= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f} \\
&= \frac{1}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2af} \\
&= \frac{1}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{a^2f} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.0621527, size = 41, normalized size = 0.77

$$\frac{\cosh(e+fx)\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)+1}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (1 + Cosh[e + f*x]*Log[Tanh[(e + f*x)/2]])/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [C] time = 0.066, size = 44, normalized size = 0.8

$$\frac{1}{f}\,\text{`int/indef0`}\left(\frac{1}{\left(\cosh\left(fx+e\right)\right)^2\sinh\left(fx+e\right)a}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(1/cosh(f*x+e)^2/sinh(f*x+e)/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [A] time = 1.79287, size = 103, normalized size = 1.94

$$\frac{2\sqrt{a}e^{(-fx-e)}}{\left(a^2e^{(-2fx-2e)}+a^2\right)f}-\frac{\log\left(e^{(-fx-e)}+1\right)}{a^{\frac{3}{2}}f}+\frac{\log\left(e^{(-fx-e)}-1\right)}{a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 2*sqrt(a)*e^(-f*x - e)/((a^2*e^(-2*f*x - 2*e) + a^2)*f) - log(e^(-f*x - e)
+ 1)/(a^(3/2)*f) + log(e^(-f*x - e) - 1)/(a^(3/2)*f)
```

Fricas [B] time = 1.85301, size = 706, normalized size = 13.32

$$\frac{\sqrt{ae^{(4fx+4e)}+2ae^{(2fx+2e)}+a}\left(2\cosh\left(fx+e\right)e^{(fx+e)}+\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)\right.\right.}{a^2f\cosh\left(fx+e\right)^2+a^2f+\left(a^2fe^{(2fx+2e)}+a^2f\right)\sinh\left(fx+e\right)^2+\left(a^2f\cosh\left(fx+e\right)^2+a^2f\right)e^{(2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*(2*cosh(f*x + e)*e^(f*x +
 e) + (2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e
)^2 + (cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(f*x + e) + sinh(f*x + e)
 - 1)/(cosh(f*x + e) + sinh(f*x + e) + 1)) + 2*e^(f*x + e)*sinh(f*x + e))*e
^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 + a^2*f + (a^2*f*e^(2*f*x + 2*e) + a^2*f
)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 2*(a^
2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [A] time = 1.35105, size = 84, normalized size = 1.58

$$-\frac{\frac{\log\left(e^{(fx+e)}+1\right)}{a^{\frac{3}{2}}}-\frac{\log\left(\left|e^{(fx+e)}-1\right|\right)}{a^{\frac{3}{2}}}-\frac{2\,e^{(fx+e)}}{a^{\frac{3}{2}}\left(e^{(2\,fx+2\,e)}+1\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(log(e^(f*x + e) + 1)/a^(3/2) - log(abs(e^(f*x + e) - 1))/a^(3/2) - 2*e^(f
*x + e)/(a^(3/2)*(e^(2*f*x + 2*e) + 1)))/f
```

3.451 $\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2a^2f}$$

```
[Out] ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cosh[e + f
*x]^2]*Csch[e + f*x]^2)/(2*a^2*f)
```

Rubi [A] time = 0.139467, antiderivative size = 66, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3176, 3205, 16, 51, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\operatorname{csch}^2(e+fx)\sqrt{a\cosh^2(e+fx)}}{2a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a*Cosh[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cosh[e + f
*x]^2]*Csch[e + f*x]^2)/(2*a^2*f)
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^3(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^3(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\
&= \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(ax)^{3/2}}\,dx, x, \cosh^2(e+fx)\right)}{2f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{(1-x)^2\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{2af}\\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cosh^2(e+fx)\right)}{4af}\\
&= -\frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cosh^2(e+fx)}\right)}{2a^2f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{a\cosh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\sqrt{a\cosh^2(e+fx)}\,\text{csch}^2(e+fx)}{2a^2f}
\end{aligned}$$

Mathematica [A] time = 0.131715, size = 67, normalized size = 1.02

$$-\frac{\cosh^3(e+fx)\left(\text{csch}^2\left(\frac{1}{2}(e+fx)\right)+\text{sech}^2\left(\frac{1}{2}(e+fx)\right)+4\log\left(\tanh\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cosh[e + f*x]^3*(Csch[(e + f*x)/2]^2 + 4*Log[Tanh[(e + f*x)/2]] + Sech[(e
 + f*x)/2]^2))/(8*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.069, size = 36, normalized size = 0.6

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{1}{\left(\sinh(fx+e)\right)^3 a}\frac{1}{\sqrt{a\left(\cosh(fx+e)\right)^2}}, \sinh(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)^3/a/(a*cosh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [A] time = 2.11193, size = 135, normalized size = 2.05

$$\frac{e^{(-fx-e)}+e^{(-3fx-3e)}}{\left(2a^{\frac{3}{2}}e^{(-2fx-2e)}-a^{\frac{3}{2}}e^{(-4fx-4e)}-a^{\frac{3}{2}}\right)f}+\frac{\log\left(e^{(-fx-e)}+1\right)}{2a^{\frac{3}{2}}f}-\frac{\log\left(e^{(-fx-e)}-1\right)}{2a^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] (e^(-f*x - e) + e^(-3*f*x - 3*e))/((2*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^
(-4*f*x - 4*e) - a^(3/2))*f) + 1/2*log(e^(-f*x - e) + 1)/(a^(3/2)*f) - 1/2*
log(e^(-f*x - e) - 1)/(a^(3/2)*f)
```

Fricas [B] time = 1.87461, size = 1458, normalized size = 22.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*(6*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + 2*e^(f*x + e)*sinh(f*x
+ e)^3 + 2*(3*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e) + 2*(cosh(f*x
+ e)^3 + cosh(f*x + e))*e^(f*x + e) - (4*cosh(f*x + e)*e^(f*x + e)*sinh(f*x
 + e)^3 + e^(f*x + e)*sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*e^(f*x +
e)*sinh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f
*x + e) + (cosh(f*x + e)^4 - 2*cosh(f*x + e)^2 + 1)*e^(f*x + e))*log((cosh(
f*x + e) + sinh(f*x + e) + 1)/(cosh(f*x + e) + sinh(f*x + e) - 1)))*sqrt(a*
e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e
)^4 - 2*a^2*f*cosh(f*x + e)^2 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x +
e)^4 + 4*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f
*x + e)^3 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^2*f + (3*a^2*f*cosh(f*x
+ e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^4 -
 2*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e) + 4*(a^2*f*cosh(f*x + e)^
3 - a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*e^(
2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.452 $\int \frac{\tanh^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)\mathrm{sech}^2(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) + Tan
h[e + f*x]/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Sech[e + f*x]^2*Tanh[e + f*x]
)/(4*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.159819, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3176, 3207, 2611, 3768, 3770}

$$\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\tanh(e+fx)\mathrm{sech}^2(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) + Tan
h[e + f*x]/(8*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Sech[e + f*x]^2*Tanh[e + f*x]
)/(4*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
```

```
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tanh^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx \\
&= \frac{\cosh(e+fx)\int \operatorname{sech}^3(e+fx)\tanh^2(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\int \operatorname{sech}^3(e+fx)\,dx}{4a\sqrt{a\cosh^2(e+fx)}} \\
&= \frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\operatorname{sech}^2(e+fx)\tanh(e+fx)}{4af\sqrt{a\cosh^2(e+fx)}}+\frac{\cosh(e+fx)\int \operatorname{sech}(e+fx)\,dx}{8a\sqrt{a\cosh^2(e+fx)}} \\
&= \frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}+\frac{\tanh(e+fx)}{8af\sqrt{a\cosh^2(e+fx)}}-\frac{\operatorname{sech}^2(e+fx)\tanh(e+f}{4af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.093846, size = 58, normalized size = 0.55

$$\frac{\tanh(e+fx)\left(1-2\operatorname{sech}^2(e+fx)\right)+\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{8af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTan[Sinh[e + f*x]]*Cosh[e + f*x] + (1 - 2*Sech[e + f*x]^2)*Tanh[e + f*x
])/(8*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Maple [A] time = 0.142, size = 69, normalized size = 0.7

$$\frac{\arctan\left(\sinh\left(fx+e\right)\right)\left(\cosh\left(fx+e\right)\right)^4+\left(\cosh\left(fx+e\right)\right)^2\sinh\left(fx+e\right)-2\sinh\left(fx+e\right)}{8a\left(\cosh\left(fx+e\right)\right)^3 f}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/8/a*(arctan(sinh(f*x+e))*cosh(f*x+e)^4+cosh(f*x+e)^2*sinh(f*x+e)-2*sinh(f
*x+e))/cosh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.89891, size = 498, normalized size = 4.7

$$-\frac{\frac{3\,e^{(-fx-e)}+11\,e^{(-3\,fx-3\,e)}-11\,e^{(-5\,fx-5\,e)}-3\,e^{(-7\,fx-7\,e)}}{4\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+6\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+4\,a^{\frac{3}{2}}e^{(-6\,fx-6\,e)}+a^{\frac{3}{2}}e^{(-8\,fx-8\,e)}+a^{\frac{3}{2}}}-\frac{3\,\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{8\,f}+\frac{15\,e^{(-fx-e)}+55\,e^{(-3\,fx-3\,e)}+73\,e^{(}}{48\left(4\,a^{\frac{3}{2}}e^{(-2\,fx-2\,e)}+6\,a^{\frac{3}{2}}e^{(-4\,fx-4\,e)}+4\,a^{\frac{3}{2}}e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*((3*e^(-f*x - e) + 11*e^(-3*f*x - 3*e) - 11*e^(-5*f*x - 5*e) - 3*e^(-7
*f*x - 7*e))/(4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(3/2)*e^(-4*f*x - 4*e) + 4*a
^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)) - 3*arctan(e^
(-f*x - e))/a^(3/2))/f + 1/48*(15*e^(-f*x - e) + 55*e^(-3*f*x - 3*e) + 73*e
^(-5*f*x - 5*e) - 15*e^(-7*f*x - 7*e))/((4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(
3/2)*e^(-4*f*x - 4*e) + 4*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*
e) + a^(3/2))*f) + 1/48*(15*e^(-f*x - e) - 73*e^(-3*f*x - 3*e) - 55*e^(-5*f
*x - 5*e) - 15*e^(-7*f*x - 7*e))/((4*a^(3/2)*e^(-2*f*x - 2*e) + 6*a^(3/2)*e
^(-4*f*x - 4*e) + 4*a^(3/2)*e^(-6*f*x - 6*e) + a^(3/2)*e^(-8*f*x - 8*e) + a
^(3/2))*f) - 5/8*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Fricas [B] time = 2.00091, size = 3698, normalized size = 34.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*(7*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + e^(f*x + e)*sinh(f*x + e
)^7 + 7*(3*cosh(f*x + e)^2 - 1)*e^(f*x + e)*sinh(f*x + e)^5 + 35*(cosh(f*x
+ e)^3 - cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 7*(5*cosh(f*x + e)^4
- 10*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(3*cosh(f*x + e)^
5 - 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + (7*
cosh(f*x + e)^6 - 35*cosh(f*x + e)^4 + 21*cosh(f*x + e)^2 - 1)*e^(f*x + e)*
sinh(f*x + e) + (8*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^7 + e^(f*x + e)*
sinh(f*x + e)^8 + 4*(7*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^6 + 8
*(7*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^5 + 2*(35*
cosh(f*x + e)^4 + 30*cosh(f*x + e)^2 + 3)*e^(f*x + e)*sinh(f*x + e)^4 + 8*(
7*cosh(f*x + e)^5 + 10*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(
f*x + e)^3 + 4*(7*cosh(f*x + e)^6 + 15*cosh(f*x + e)^4 + 9*cosh(f*x + e)^2
+ 1)*e^(f*x + e)*sinh(f*x + e)^2 + 8*(cosh(f*x + e)^7 + 3*cosh(f*x + e)^5 +
 3*cosh(f*x + e)^3 + cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e) + (cosh(f*x +
 e)^8 + 4*cosh(f*x + e)^6 + 6*cosh(f*x + e)^4 + 4*cosh(f*x + e)^2 + 1)*e^(f
*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)) + (cosh(f*x + e)^7 - 7*cosh(
f*x + e)^5 + 7*cosh(f*x + e)^3 - cosh(f*x + e))*e^(f*x + e))*sqrt(a*e^(4*f*
x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^8 + 4
*a^2*f*cosh(f*x + e)^6 + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^8 +
8*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*x + e))*sinh(f*x + e)
^7 + 6*a^2*f*cosh(f*x + e)^4 + 4*(7*a^2*f*cosh(f*x + e)^2 + a^2*f + (7*a^2*
f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 8*(7*a^2*f*co
sh(f*x + e)^3 + 3*a^2*f*cosh(f*x + e) + (7*a^2*f*cosh(f*x + e)^3 + 3*a^2*f*
cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 4*a^2*f*cosh(f*x + e)^2 +
 2*(35*a^2*f*cosh(f*x + e)^4 + 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (35*a^2
*f*cosh(f*x + e)^4 + 30*a^2*f*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*s
```

```
inh(f*x + e)^4 + 8*(7*a^2*f*cosh(f*x + e)^5 + 10*a^2*f*cosh(f*x + e)^3 + 3*
a^2*f*cosh(f*x + e) + (7*a^2*f*cosh(f*x + e)^5 + 10*a^2*f*cosh(f*x + e)^3 +
 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 + a^2*f + 4*(7*a^2
*f*cosh(f*x + e)^6 + 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f*x + e)^2 + a
^2*f + (7*a^2*f*cosh(f*x + e)^6 + 15*a^2*f*cosh(f*x + e)^4 + 9*a^2*f*cosh(f
*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^
8 + 4*a^2*f*cosh(f*x + e)^6 + 6*a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e) + 8*(a^2*f*cosh(f*x + e)^7 + 3*a^2*f*cosh(f*x
 + e)^5 + 3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x +
 e)^7 + 3*a^2*f*cosh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x
+ e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [A] time = 1.42199, size = 126, normalized size = 1.19

$$\frac{\frac{\arctan\left(e^{(fx+e)}\right)}{a^{\frac{3}{2}}}+\frac{\sqrt{a}e^{(7fx+7e)}-7\sqrt{a}e^{(5fx+5e)}+7\sqrt{a}e^{(3fx+3e)}-\sqrt{a}e^{(fx+e)}}{a^2\left(e^{(2fx+2e)}+1\right)^4}}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/4*(arctan(e^(f*x + e))/a^(3/2) + (sqrt(a)*e^(7*f*x + 7*e) - 7*sqrt(a)*e^(
5*f*x + 5*e) + 7*sqrt(a)*e^(3*f*x + 3*e) - sqrt(a)*e^(f*x + e))/(a^2*(e^(2*
f*x + 2*e) + 1)^4))/f
```

3.453 $\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=64

$$-\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -((ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(a*f*Sqrt[a*Cosh[e + f*x]^2])) - Co
th[e + f*x]/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.131732, antiderivative size = 64, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3176, 3207, 2621, 321, 207}

$$-\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}-\frac{\cosh(e+fx)\tan^{-1}(\sinh(e+fx))}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((ArcTan[Sinh[e + f*x]]*Cosh[e + f*x])/(a*f*Sqrt[a*Cosh[e + f*x]^2])) - Co
th[e + f*x]/(a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\coth^2(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^2(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx \\
&= \frac{\cosh(e+fx)\int \operatorname{csch}^2(e+fx)\operatorname{sech}(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{(i\cosh(e+fx))\,\mathrm{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}} - \frac{(i\cosh(e+fx))\,\mathrm{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\tan^{-1}(\sinh(e+fx))\cosh(e+fx)}{af\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)}{af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.0787, size = 46, normalized size = 0.72

$$-\frac{\coth(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};-\sinh^2(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Coth[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, -Sinh[e + f*x]^2])/(a*f*Sq
rt[a*Cosh[e + f*x]^2]))
```

Maple [A] time = 0.093, size = 51, normalized size = 0.8

$$-\frac{\cosh\left(fx+e\right)\left(\arctan\left(\sinh\left(fx+e\right)\right)\sinh\left(fx+e\right)+1\right)}{a\sinh\left(fx+e\right)f}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/a*cosh(f*x+e)*(arctan(sinh(f*x+e))*sinh(f*x+e)+1)/sinh(f*x+e)/(a*cosh(f*
x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.71757, size = 433, normalized size = 6.77

$$-\frac{\frac{3\sqrt{a}e^{(-fx-e)}+2\sqrt{a}e^{(-3fx-3e)}+3\sqrt{a}e^{(-5fx-5e)}}{a^2e^{(-2fx-2e)}-a^2e^{(-4fx-4e)}-a^2e^{(-6fx-6e)}+a^2}-\frac{3\arctan\left(e^{(-fx-e)}\right)}{a^{\frac{3}{2}}}}{2f}-\frac{5\sqrt{a}e^{(-fx-e)}+6\sqrt{a}e^{(-3fx-3e)}-3\sqrt{a}e^{(-5fx-5e)}}{4\left(a^2e^{(-2fx-2e)}-a^2e^{(-4fx-4e)}-a^2e^{(-6fx-6e)}+a^2\right)f}+\frac{3}{4\left(a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((3*sqrt(a)*e^(-f*x - e) + 2*sqrt(a)*e^(-3*f*x - 3*e) + 3*sqrt(a)*e^(-
5*f*x - 5*e))/(a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x
- 6*e) + a^2) - 3*arctan(e^(-f*x - e))/a^(3/2))/f - 1/4*(5*sqrt(a)*e^(-f*x
- e) + 6*sqrt(a)*e^(-3*f*x - 3*e) - 3*sqrt(a)*e^(-5*f*x - 5*e))/((a^2*e^(-2
*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x - 6*e) + a^2)*f) + 1/4*(
3*sqrt(a)*e^(-f*x - e) - 6*sqrt(a)*e^(-3*f*x - 3*e) - 5*sqrt(a)*e^(-5*f*x -
 5*e))/((a^2*e^(-2*f*x - 2*e) - a^2*e^(-4*f*x - 4*e) - a^2*e^(-6*f*x - 6*e)
 + a^2)*f) + 1/2*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Fricas [B] time = 1.80605, size = 652, normalized size = 10.19

$$-\frac{2\left(\left(2\cosh\left(fx+e\right)e^{(fx+e)}\sinh\left(fx+e\right)+e^{(fx+e)}\sinh\left(fx+e\right)^2+\left(\cosh\left(fx+e\right)^2-1\right)e^{(fx+e)}\right)\arctan\left(\cosh\left(fx+\right.\right.\right.}{a^2f\cosh\left(fx+e\right)^2-a^2f+\left(a^2fe^{(2fx+2e)}+a^2f\right)\sinh\left(fx+e\right)^2+\left(a^2f\cosh\left(fx+e\right)^2-a^2\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -2*((2*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e) + e^(f*x + e)*sinh(f*x + e)^
2 + (cosh(f*x + e)^2 - 1)*e^(f*x + e))*arctan(cosh(f*x + e) + sinh(f*x + e)
) + cosh(f*x + e)*e^(f*x + e) + e^(f*x + e)*sinh(f*x + e))*sqrt(a*e^(4*f*x
+ 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*f*cosh(f*x + e)^2 - a^2
*f + (a^2*f*e^(2*f*x + 2*e) + a^2*f)*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)
^2 - a^2*f)*e^(2*f*x + 2*e) + 2*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*
f*cosh(f*x + e))*sinh(f*x + e))
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth^2\left(e+fx\right)}{\left(a\left(\sinh^2\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/(a*(sinh(e + f*x)**2 + 1))**(3/2), x)
```

Giac [A] time = 1.42971, size = 58, normalized size = 0.91

$$-\frac{2\left(\frac{\arctan\left(e^{\left(fx+e\right)}\right)}{a^{\frac{3}{2}}}+\frac{e^{\left(fx+e\right)}}{a^{\frac{3}{2}}\left(e^{\left(2\,fx+2\,e\right)}-1\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2*(arctan(e^(f*x + e))/a^(3/2) + e^(f*x + e)/(a^(3/2)*(e^(2*f*x + 2*e) - 1
)))/f
```

3.454 $\int \frac{\coth^4(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.126204, antiderivative size = 38, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2606, 30}

$$-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\coth^4(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\cosh(e+fx)\int \coth(e+fx)\text{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}$$
$$= \frac{(i\cosh(e+fx))\,\text{Subst}\left(\int x^2\,dx, x, -i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}$$
$$= -\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Mathematica [A] time = 0.0546857, size = 29, normalized size = 0.76

$$-\frac{\coth^3(e+fx)}{3f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -Coth[e + f*x]^3/(3*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.078, size = 35, normalized size = 0.9

$$-\frac{\cosh\left(fx+e\right)}{3\,a\left(\sinh\left(fx+e\right)\right)^3 f}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*cosh(f*x+e)/a/sinh(f*x+e)^3/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 1.98643, size = 1111, normalized size = 29.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*((21*e^(-f*x - e) - 16*e^(-3*f*x - 3*e) + 34*e^(-5*f*x - 5*e) + 8*e^(-
7*f*x - 7*e) - 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e
^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a
^(3/2)*e^(-10*f*x - 10*e) - a^(3/2)) + 3*arctan(e^(-f*x - e))/a^(3/2) + 9*l
og(e^(-f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/12*((
15*e^(-f*x - e) - 8*e^(-3*f*x - 3*e) - 34*e^(-5*f*x - 5*e) + 16*e^(-7*f*x -
```

```
7*e) - 21*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*
x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
e^(-10*f*x - 10*e) - a^(3/2)) - 3*arctan(e^(-f*x - e))/a^(3/2) + 9*log(e^(-
f*x - e) + 1)/a^(3/2) - 9*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8*((15*e^(-f
*x - e) - 20*e^(-3*f*x - 3*e) - 22*e^(-5*f*x - 5*e) - 20*e^(-7*f*x - 7*e) +
 15*e^(-9*f*x - 9*e))/(a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e
) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*
f*x - 10*e) - a^(3/2)) + 15*arctan(e^(-f*x - e))/a^(3/2))/f + 1/48*(45*e^(-
f*x - e) - 52*e^(-3*f*x - 3*e) - 74*e^(-5*f*x - 5*e) + 92*e^(-7*f*x - 7*e)
+ 21*e^(-9*f*x - 9*e))/((a^(3/2)*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4
*e) - 2*a^(3/2)*e^(-6*f*x - 6*e) - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-1
0*f*x - 10*e) - a^(3/2))*f) + 1/48*(21*e^(-f*x - e) + 92*e^(-3*f*x - 3*e) -
 74*e^(-5*f*x - 5*e) - 52*e^(-7*f*x - 7*e) + 45*e^(-9*f*x - 9*e))/((a^(3/2)
*e^(-2*f*x - 2*e) + 2*a^(3/2)*e^(-4*f*x - 4*e) - 2*a^(3/2)*e^(-6*f*x - 6*e)
 - a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - a^(3/2))*f) + 11
/8*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Fricas [B] time = 1.78055, size = 1511, normalized size = 39.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/3*(cosh(f*x + e)^3*e^(f*x + e) + 3*cosh(f*x + e)^2*e^(f*x + e)*sinh(f*x
+ e) + 3*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^2 + e^(f*x + e)*sinh(f*x +
 e)^3)*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x - e)/(a^2*
f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + (a^2*f*e^(2*f*x + 2*e) + a^2*
f)*sinh(f*x + e)^6 + 6*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a^2*f*cosh(f*
x + e))*sinh(f*x + e)^5 + 3*a^2*f*cosh(f*x + e)^2 + 3*(5*a^2*f*cosh(f*x + e
)^2 - a^2*f + (5*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e))*sinh(f*x +
 e)^4 + 4*(5*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e) + (5*a^2*f*cosh(
f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 - a^2*
f + 3*(5*a^2*f*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f + (5*a^2*f
*cosh(f*x + e)^4 - 6*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f
*x + e)^2 + (a^2*f*cosh(f*x + e)^6 - 3*a^2*f*cosh(f*x + e)^4 + 3*a^2*f*cosh
(f*x + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 6*(a^2*f*cosh(f*x + e)^5 - 2*a^2*f*c
osh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*x + e)^5 - 2*a^2*f*cos
h(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.65161, size = 43, normalized size = 1.13

$$-\frac{8\, e^{\left(3\, f x + 3\, e\right)}}{3\, a^{\frac{3}{2}} f {\left(e^{\left(2\, f x + 2\, e\right)} - 1\right)}^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/3*e^(3*f*x + 3*e)/(a^(3/2)*f*(e^(2*f*x + 2*e) - 1)^3)
```

3.455 $\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}} dx$

Optimal. Leaf size=77

$$-\frac{\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e
+ f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.143745, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2606, 14}

$$-\frac{\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e
+ f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\coth^6(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^6(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx \\
&= \frac{\cosh(e+fx)\int \coth^3(e+fx)\operatorname{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{(i\cosh(e+fx))\,\text{Subst}\left(\int x^2\left(-1+x^2\right)dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{(i\cosh(e+fx))\,\text{Subst}\left(\int \left(-x^2+x^4\right)dx, x, -i\operatorname{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}} \\
&= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}} - \frac{\coth(e+fx)\operatorname{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}
\end{aligned}
$$

Mathematica [A] time = 0.108842, size = 41, normalized size = 0.53

$$
-\frac{\coth^3(e+fx)\left(3\operatorname{csch}^2(e+fx)+5\right)}{15f\left(a\cosh^2(e+fx)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^6/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Coth[e + f*x]^3*(5 + 3*Csch[e + f*x]^2))/(15*f*(a*Cosh[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.132, size = 67, normalized size = 0.9

$$
-\frac{\cosh(fx+e)\left(5\left(\cosh(fx+e)\right)^2-2\right)}{15\left(\cosh(fx+e)-1\right)^2\left(\cosh(fx+e)+1\right)^2 a\sinh(fx+e)f}\frac{1}{\sqrt{a\left(\cosh(fx+e)\right)^2}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15*cosh(f*x+e)*(5*cosh(f*x+e)^2-2)/(cosh(f*x+e)-1)^2/(cosh(f*x+e)+1)^2/a
/sinh(f*x+e)/(a*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 2.32633, size = 2067, normalized size = 26.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -3/256*(2*(105*e^(-f*x - e) - 300*e^(-3*f*x - 3*e) + 81*e^(-5*f*x - 5*e) -
248*e^(-7*f*x - 7*e) + 51*e^(-9*f*x - 9*e) + 100*e^(-11*f*x - 11*e) - 45*e^
```

```
(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) -
5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*
x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(
3/2)) + 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)/a^(3/2)
- 75*log(e^(-f*x - e) - 1)/a^(3/2))/f + 1/48*((105*e^(-f*x - e) - 350*e^(-3
*f*x - 3*e) + 231*e^(-5*f*x - 5*e) + 412*e^(-7*f*x - 7*e) + 231*e^(-9*f*x -
 9*e) - 350*e^(-11*f*x - 11*e) + 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f
*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/
2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 1
2*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) + 105*arctan(e^(-f*x - e))/a^(
3/2))/f + 3/256*(2*(45*e^(-f*x - e) - 100*e^(-3*f*x - 3*e) - 51*e^(-5*f*x -
 5*e) + 248*e^(-7*f*x - 7*e) - 81*e^(-9*f*x - 9*e) + 300*e^(-11*f*x - 11*e)
 - 105*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x
- 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*
e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*x - 14
*e) - a^(3/2)) - 60*arctan(e^(-f*x - e))/a^(3/2) + 75*log(e^(-f*x - e) + 1)
/a^(3/2) - 75*log(e^(-f*x - e) - 1)/a^(3/2))/f - 3/320*(4*(45*e^(-f*x - e)
- 135*e^(-3*f*x - 3*e) + 54*e^(-5*f*x - 5*e) + 198*e^(-7*f*x - 7*e) - 211*e
^(-9*f*x - 9*e) - 15*e^(-11*f*x - 11*e))/(3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3
/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8
*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e
^(-14*f*x - 14*e) - a^(3/2)) + 90*arctan(e^(-f*x - e))/a^(3/2) + 45*log(e^(
-f*x - e) + 1)/a^(3/2) - 45*log(e^(-f*x - e) - 1)/a^(3/2))/f + 3/320*(4*(15
*e^(-3*f*x - 3*e) + 211*e^(-5*f*x - 5*e) - 198*e^(-7*f*x - 7*e) - 54*e^(-9*
f*x - 9*e) + 135*e^(-11*f*x - 11*e) - 45*e^(-13*f*x - 13*e))/(3*a^(3/2)*e^(
-2*f*x - 2*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a
^(3/2)*e^(-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x
 - 12*e) + a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2)) - 90*arctan(e^(-f*x - e))/
a^(3/2) + 45*log(e^(-f*x - e) + 1)/a^(3/2) - 45*log(e^(-f*x - e) - 1)/a^(3/
2))/f + 1/1920*(1155*e^(-f*x - e) + 1460*e^(-3*f*x - 3*e) - 4173*e^(-5*f*x
- 5*e) + 2024*e^(-7*f*x - 7*e) + 1857*e^(-9*f*x - 9*e) - 2140*e^(-11*f*x -
11*e) + 585*e^(-13*f*x - 13*e))/((3*a^(3/2)*e^(-2*f*x - 2*e) - a^(3/2)*e^(-
4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(-8*f*x - 8*e) + a^
(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) + a^(3/2)*e^(-14*f*
x - 14*e) - a^(3/2))*f) + 1/1920*(585*e^(-f*x - e) - 2140*e^(-3*f*x - 3*e)
+ 1857*e^(-5*f*x - 5*e) + 2024*e^(-7*f*x - 7*e) - 4173*e^(-9*f*x - 9*e) + 1
460*e^(-11*f*x - 11*e) + 1155*e^(-13*f*x - 13*e))/((3*a^(3/2)*e^(-2*f*x - 2
*e) - a^(3/2)*e^(-4*f*x - 4*e) - 5*a^(3/2)*e^(-6*f*x - 6*e) + 5*a^(3/2)*e^(
-8*f*x - 8*e) + a^(3/2)*e^(-10*f*x - 10*e) - 3*a^(3/2)*e^(-12*f*x - 12*e) +
 a^(3/2)*e^(-14*f*x - 14*e) - a^(3/2))*f) + 29/32*arctan(e^(-f*x - e))/(a^(
3/2)*f)
```

Fricas [B] time = 1.94361, size = 3571, normalized size = 46.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/15*(35*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^6 + 5*e^(f*x + e)*sinh(f*
x + e)^7 + (105*cosh(f*x + e)^2 + 2)*e^(f*x + e)*sinh(f*x + e)^5 + 5*(35*co
sh(f*x + e)^3 + 2*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^4 + 5*(35*cosh(f
*x + e)^4 + 4*cosh(f*x + e)^2 + 1)*e^(f*x + e)*sinh(f*x + e)^3 + 5*(21*cosh
(f*x + e)^5 + 4*cosh(f*x + e)^3 + 3*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e
)^2 + 5*(7*cosh(f*x + e)^6 + 2*cosh(f*x + e)^4 + 3*cosh(f*x + e)^2)*e^(f*x
+ e)*sinh(f*x + e) + (5*cosh(f*x + e)^7 + 2*cosh(f*x + e)^5 + 5*cosh(f*x +
```

```
e)^3)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*
x - e)/(a^2*f*cosh(f*x + e)^10 - 5*a^2*f*cosh(f*x + e)^8 + (a^2*f*e^(2*f*x
+ 2*e) + a^2*f)*sinh(f*x + e)^10 + 10*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e)
+ a^2*f*cosh(f*x + e))*sinh(f*x + e)^9 + 10*a^2*f*cosh(f*x + e)^6 + 5*(9*a^
2*f*cosh(f*x + e)^2 - a^2*f + (9*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x +
2*e))*sinh(f*x + e)^8 + 40*(3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e) +
 (3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x
+ e)^7 - 10*a^2*f*cosh(f*x + e)^4 + 10*(21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f
*cosh(f*x + e)^2 + a^2*f + (21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f*cosh(f*x +
e)^2 + a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^6 + 4*(63*a^2*f*cosh(f*x + e)^
5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (63*a^2*f*cosh(f*x
+ e)^5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e)
)*sinh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^2 + 10*(21*a^2*f*cosh(f*x + e)^6
- 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*f + (21*a^2*f*c
osh(f*x + e)^6 - 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 40*(3*a^2*f*cosh(f*x + e)^7 - 7*a^2*f
*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e) + (3*a^2*f
*cosh(f*x + e)^7 - 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^3 - a^2*f + 5*(9*a^2*f*cosh
(f*x + e)^8 - 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 - 12*a^2*
f*cosh(f*x + e)^2 + a^2*f + (9*a^2*f*cosh(f*x + e)^8 - 28*a^2*f*cosh(f*x +
e)^6 + 30*a^2*f*cosh(f*x + e)^4 - 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*e^(2*f*
x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^10 - 5*a^2*f*cosh(f*x + e)
^8 + 10*a^2*f*cosh(f*x + e)^6 - 10*a^2*f*cosh(f*x + e)^4 + 5*a^2*f*cosh(f*x
 + e)^2 - a^2*f)*e^(2*f*x + 2*e) + 10*(a^2*f*cosh(f*x + e)^9 - 4*a^2*f*cosh
(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cos
h(f*x + e) + (a^2*f*cosh(f*x + e)^9 - 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cos
h(f*x + e)^5 - 4*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e))*e^(2*f*x + 2*
e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**6/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.80338, size = 92, normalized size = 1.19

$$-\frac{8\left(5\sqrt{a}e^{(7fx+7e)}+2\sqrt{a}e^{(5fx+5e)}+5\sqrt{a}e^{(3fx+3e)}\right)}{15a^2f\left(e^{(2fx+2e)}-1\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^6/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/15*(5*sqrt(a)*e^(7*f*x + 7*e) + 2*sqrt(a)*e^(5*f*x + 5*e) + 5*sqrt(a)*e^
(3*f*x + 3*e))/(a^2*f*(e^(2*f*x + 2*e) - 1)^5)
```

3.456 $\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\coth(e+fx)\text{csch}^6(e+fx)}{7af\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (2*Coth[
e + f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*
Csch[e + f*x]^6)/(7*a*f*Sqrt[a*Cosh[e + f*x]^2])
```

Rubi [A] time = 0.149823, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3176, 3207, 2606, 270}

$$-\frac{\coth(e+fx)\text{csch}^6(e+fx)}{7af\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] -(Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (2*Coth[
e + f*x]*Csch[e + f*x]^4)/(5*a*f*Sqrt[a*Cosh[e + f*x]^2]) - (Coth[e + f*x]*
Csch[e + f*x]^6)/(7*a*f*Sqrt[a*Cosh[e + f*x]^2])
```


Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^8(e+fx)}{\left(a+a\sinh^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\coth^8(e+fx)}{\left(a\cosh^2(e+fx)\right)^{3/2}}\,dx\\ &= \frac{\cosh(e+fx)\int \coth^5(e+fx)\text{csch}^3(e+fx)\,dx}{a\sqrt{a\cosh^2(e+fx)}}\\ &= \frac{(i\cosh(e+fx))\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx,x,-i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}\\ &= \frac{(i\cosh(e+fx))\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)dx,x,-i\text{csch}(e+fx)\right)}{af\sqrt{a\cosh^2(e+fx)}}\\ &= -\frac{\coth(e+fx)\text{csch}^2(e+fx)}{3af\sqrt{a\cosh^2(e+fx)}}-\frac{2\coth(e+fx)\text{csch}^4(e+fx)}{5af\sqrt{a\cosh^2(e+fx)}}-\frac{\coth(e+fx)\text{csch}^6(e+}{7af\sqrt{a\cosh^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.136423, size = 51, normalized size = 0.44

$$-\frac{\coth^3(e+fx)\left(15\text{csch}^4(e+fx)+42\text{csch}^2(e+fx)+35\right)}{105f\left(a\cosh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^8/(a + a*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Coth[e + f*x]^3*(35 + 42*Csch[e + f*x]^2 + 15*Csch[e + f*x]^4))/(105*f*(a
*Cosh[e + f*x]^2)^(3/2))
```

Maple [A] time = 0.105, size = 57, normalized size = 0.5

$$-\frac{\cosh\left(fx+e\right)\left(35\left(\cosh\left(fx+e\right)\right)^4-28\left(\cosh\left(fx+e\right)\right)^2+8\right)}{105\,a\left(\sinh\left(fx+e\right)\right)^7 f}\frac{1}{\sqrt{a\left(\cosh\left(fx+e\right)\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/105*cosh(f*x+e)*(35*cosh(f*x+e)^4-28*cosh(f*x+e)^2+8)/a/sinh(f*x+e)^7/(a
*cosh(f*x+e)^2)^(1/2)/f
```

Maxima [B] time = 2.60066, size = 2992, normalized size = 26.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")

[Out] 1/3840*(2*(4095*e^(-f*x - e) - 20090*e^(-3*f*x - 3*e) + 31654*e^(-5*f*x - 5
*e) - 850*e^(-7*f*x - 7*e) - 51148*e^(-9*f*x - 9*e) + 51090*e^(-11*f*x - 11
*e) - 2646*e^(-13*f*x - 13*e) + 4410*e^(-15*f*x - 15*e) - 1155*e^(-17*f*x -
 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/
2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x
- 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/
2)) + 2940*arctan(e^(-f*x - e))/a^(3/2) + 2625*log(e^(-f*x - e) + 1)/a^(3/2
) - 2625*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/8960*(2*(4095*e^(-f*x - e) -
21630*e^(-3*f*x - 3*e) + 39354*e^(-5*f*x - 5*e) - 13830*e^(-7*f*x - 7*e) -
47848*e^(-9*f*x - 9*e) + 66950*e^(-11*f*x - 11*e) - 22106*e^(-13*f*x - 13*e
) - 18690*e^(-15*f*x - 15*e) + 3465*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*
x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^
(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*
f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) + 7560*arctan(e^(-f*x -
 e))/a^(3/2) + 315*log(e^(-f*x - e) + 1)/a^(3/2) - 315*log(e^(-f*x - e) - 1
)/a^(3/2))/f - 1/8960*(2*(3465*e^(-f*x - e) - 18690*e^(-3*f*x - 3*e) - 2210
6*e^(-5*f*x - 5*e) + 66950*e^(-7*f*x - 7*e) - 47848*e^(-9*f*x - 9*e) - 1383
0*e^(-11*f*x - 11*e) + 39354*e^(-13*f*x - 13*e) - 21630*e^(-15*f*x - 15*e)
+ 4095*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*
x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*
a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*
f*x - 18*e) - a^(3/2)) + 7560*arctan(e^(-f*x - e))/a^(3/2) - 315*log(e^(-f*
x - e) + 1)/a^(3/2) + 315*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/3840*(2*(115
5*e^(-f*x - e) - 4410*e^(-3*f*x - 3*e) + 2646*e^(-5*f*x - 5*e) - 51090*e^(-
7*f*x - 7*e) + 51148*e^(-9*f*x - 9*e) + 850*e^(-11*f*x - 11*e) - 31654*e^(-
13*f*x - 13*e) + 20090*e^(-15*f*x - 15*e) - 4095*e^(-17*f*x - 17*e))/(5*a^(
3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x -
 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^
(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) - 2940*arc
tan(e^(-f*x - e))/a^(3/2) + 2625*log(e^(-f*x - e) + 1)/a^(3/2) - 2625*log(e
^(-f*x - e) - 1)/a^(3/2))/f + 1/768*(2*(1155*e^(-f*x - e) - 5670*e^(-3*f*x
- 3*e) + 8946*e^(-5*f*x - 5*e) - 270*e^(-7*f*x - 7*e) + 4696*e^(-9*f*x - 9*
e) - 2930*e^(-11*f*x - 11*e) - 658*e^(-13*f*x - 13*e) + 1190*e^(-15*f*x - 1
5*e) - 315*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-
4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e)
+ 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(
-18*f*x - 18*e) - a^(3/2)) + 840*arctan(e^(-f*x - e))/a^(3/2) + 735*log(e^(
-f*x - e) + 1)/a^(3/2) - 735*log(e^(-f*x - e) - 1)/a^(3/2))/f - 1/768*(2*(3
15*e^(-f*x - e) - 1190*e^(-3*f*x - 3*e) + 658*e^(-5*f*x - 5*e) + 2930*e^(-7
*f*x - 7*e) - 4696*e^(-9*f*x - 9*e) + 270*e^(-11*f*x - 11*e) - 8946*e^(-13*
f*x - 13*e) + 5670*e^(-15*f*x - 15*e) - 1155*e^(-17*f*x - 17*e))/(5*a^(3/2)
*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e
) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2
)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e) - a^(3/2)) - 840*arctan(e
^(-f*x - e))/a^(3/2) + 735*log(e^(-f*x - e) + 1)/a^(3/2) - 735*log(e^(-f*x
- e) - 1)/a^(3/2))/f - 1/128*((315*e^(-f*x - e) - 1680*e^(-3*f*x - 3*e) + 3
108*e^(-5*f*x - 5*e) - 1200*e^(-7*f*x - 7*e) - 3646*e^(-9*f*x - 9*e) - 1200
*e^(-11*f*x - 11*e) + 3108*e^(-13*f*x - 13*e) - 1680*e^(-15*f*x - 15*e) + 3
15*e^(-17*f*x - 17*e))/(5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x -
4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3
/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x
- 18*e) - a^(3/2)) + 315*arctan(e^(-f*x - e))/a^(3/2))/f + 1/2688*(1155*e^(
-f*x - e) + 1393*e^(-3*f*x - 3*e) - 4865*e^(-5*f*x - 5*e) + 3965*e^(-7*f*x
- 7*e) + 825*e^(-9*f*x - 9*e) - 3245*e^(-11*f*x - 11*e) + 1925*e^(-13*f*x -
 13*e) - 385*e^(-15*f*x - 15*e))/((5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e
^(-4*f*x - 4*e) + 14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*
e) + 8*a^(3/2)*e^(-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*
```

```
e^(-18*f*x - 18*e) - a^(3/2))*f) - 1/2688*(385*e^(-3*f*x - 3*e) - 1925*e^(-
5*f*x - 5*e) + 3245*e^(-7*f*x - 7*e) - 825*e^(-9*f*x - 9*e) - 3965*e^(-11*f
*x - 11*e) + 4865*e^(-13*f*x - 13*e) - 1393*e^(-15*f*x - 15*e) - 1155*e^(-1
7*f*x - 17*e))/((5*a^(3/2)*e^(-2*f*x - 2*e) - 8*a^(3/2)*e^(-4*f*x - 4*e) +
14*a^(3/2)*e^(-8*f*x - 8*e) - 14*a^(3/2)*e^(-10*f*x - 10*e) + 8*a^(3/2)*e^(
-14*f*x - 14*e) - 5*a^(3/2)*e^(-16*f*x - 16*e) + a^(3/2)*e^(-18*f*x - 18*e)
 - a^(3/2))*f) + 55/128*arctan(e^(-f*x - e))/(a^(3/2)*f)
```

Fricas [B] time = 2.14295, size = 6649, normalized size = 57.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -8/105*(385*cosh(f*x + e)*e^(f*x + e)*sinh(f*x + e)^10 + 35*e^(f*x + e)*sin
h(f*x + e)^11 + 7*(275*cosh(f*x + e)^2 + 4)*e^(f*x + e)*sinh(f*x + e)^9 + 2
1*(275*cosh(f*x + e)^3 + 12*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^8 + 6*
(1925*cosh(f*x + e)^4 + 168*cosh(f*x + e)^2 + 19)*e^(f*x + e)*sinh(f*x + e)
^7 + 42*(385*cosh(f*x + e)^5 + 56*cosh(f*x + e)^3 + 19*cosh(f*x + e))*e^(f*
x + e)*sinh(f*x + e)^6 + 14*(1155*cosh(f*x + e)^6 + 252*cosh(f*x + e)^4 + 1
71*cosh(f*x + e)^2 + 2)*e^(f*x + e)*sinh(f*x + e)^5 + 14*(825*cosh(f*x + e)
^7 + 252*cosh(f*x + e)^5 + 285*cosh(f*x + e)^3 + 10*cosh(f*x + e))*e^(f*x +
 e)*sinh(f*x + e)^4 + 7*(825*cosh(f*x + e)^8 + 336*cosh(f*x + e)^6 + 570*co
sh(f*x + e)^4 + 40*cosh(f*x + e)^2 + 5)*e^(f*x + e)*sinh(f*x + e)^3 + 7*(27
5*cosh(f*x + e)^9 + 144*cosh(f*x + e)^7 + 342*cosh(f*x + e)^5 + 40*cosh(f*x
 + e)^3 + 15*cosh(f*x + e))*e^(f*x + e)*sinh(f*x + e)^2 + 7*(55*cosh(f*x +
e)^10 + 36*cosh(f*x + e)^8 + 114*cosh(f*x + e)^6 + 20*cosh(f*x + e)^4 + 15*
cosh(f*x + e)^2)*e^(f*x + e)*sinh(f*x + e) + (35*cosh(f*x + e)^11 + 28*cosh
(f*x + e)^9 + 114*cosh(f*x + e)^7 + 28*cosh(f*x + e)^5 + 35*cosh(f*x + e)^3
)*e^(f*x + e))*sqrt(a*e^(4*f*x + 4*e) + 2*a*e^(2*f*x + 2*e) + a)*e^(-f*x -
e)/(a^2*f*cosh(f*x + e)^14 - 7*a^2*f*cosh(f*x + e)^12 + (a^2*f*e^(2*f*x + 2
*e) + a^2*f)*sinh(f*x + e)^14 + 14*(a^2*f*cosh(f*x + e)*e^(2*f*x + 2*e) + a
^2*f*cosh(f*x + e))*sinh(f*x + e)^13 + 21*a^2*f*cosh(f*x + e)^10 + 7*(13*a^
2*f*cosh(f*x + e)^2 - a^2*f + (13*a^2*f*cosh(f*x + e)^2 - a^2*f)*e^(2*f*x +
 2*e))*sinh(f*x + e)^12 + 28*(13*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x +
 e) + (13*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))*s
inh(f*x + e)^11 - 35*a^2*f*cosh(f*x + e)^8 + 7*(143*a^2*f*cosh(f*x + e)^4 -
 66*a^2*f*cosh(f*x + e)^2 + 3*a^2*f + (143*a^2*f*cosh(f*x + e)^4 - 66*a^2*f
*cosh(f*x + e)^2 + 3*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^10 + 14*(143*a^2
*f*cosh(f*x + e)^5 - 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e) + (
143*a^2*f*cosh(f*x + e)^5 - 110*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x +
 e))*e^(2*f*x + 2*e))*sinh(f*x + e)^9 + 35*a^2*f*cosh(f*x + e)^6 + 7*(429*a
^2*f*cosh(f*x + e)^6 - 495*a^2*f*cosh(f*x + e)^4 + 135*a^2*f*cosh(f*x + e)^
2 - 5*a^2*f + (429*a^2*f*cosh(f*x + e)^6 - 495*a^2*f*cosh(f*x + e)^4 + 135*
a^2*f*cosh(f*x + e)^2 - 5*a^2*f)*e^(2*f*x + 2*e))*sinh(f*x + e)^8 + 8*(429*
a^2*f*cosh(f*x + e)^7 - 693*a^2*f*cosh(f*x + e)^5 + 315*a^2*f*cosh(f*x + e)
^3 - 35*a^2*f*cosh(f*x + e) + (429*a^2*f*cosh(f*x + e)^7 - 693*a^2*f*cosh(f
*x + e)^5 + 315*a^2*f*cosh(f*x + e)^3 - 35*a^2*f*cosh(f*x + e))*e^(2*f*x +
2*e))*sinh(f*x + e)^7 - 21*a^2*f*cosh(f*x + e)^4 + 7*(429*a^2*f*cosh(f*x +
e)^8 - 924*a^2*f*cosh(f*x + e)^6 + 630*a^2*f*cosh(f*x + e)^4 - 140*a^2*f*co
sh(f*x + e)^2 + 5*a^2*f + (429*a^2*f*cosh(f*x + e)^8 - 924*a^2*f*cosh(f*x +
 e)^6 + 630*a^2*f*cosh(f*x + e)^4 - 140*a^2*f*cosh(f*x + e)^2 + 5*a^2*f)*e^
(2*f*x + 2*e))*sinh(f*x + e)^6 + 14*(143*a^2*f*cosh(f*x + e)^9 - 396*a^2*f*
cosh(f*x + e)^7 + 378*a^2*f*cosh(f*x + e)^5 - 140*a^2*f*cosh(f*x + e)^3 + 1
5*a^2*f*cosh(f*x + e) + (143*a^2*f*cosh(f*x + e)^9 - 396*a^2*f*cosh(f*x + e
```

```
)^7 + 378*a^2*f*cosh(f*x + e)^5 - 140*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh
(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e)^5 + 7*a^2*f*cosh(f*x + e)^2 + 7*(
143*a^2*f*cosh(f*x + e)^10 - 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*x
 + e)^6 - 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 - 3*a^2*f +
(143*a^2*f*cosh(f*x + e)^10 - 495*a^2*f*cosh(f*x + e)^8 + 630*a^2*f*cosh(f*
x + e)^6 - 350*a^2*f*cosh(f*x + e)^4 + 75*a^2*f*cosh(f*x + e)^2 - 3*a^2*f)*
e^(2*f*x + 2*e))*sinh(f*x + e)^4 + 28*(13*a^2*f*cosh(f*x + e)^11 - 55*a^2*f
*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 - 70*a^2*f*cosh(f*x + e)^5 + 25
*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e) + (13*a^2*f*cosh(f*x + e)^11
 - 55*a^2*f*cosh(f*x + e)^9 + 90*a^2*f*cosh(f*x + e)^7 - 70*a^2*f*cosh(f*x
+ e)^5 + 25*a^2*f*cosh(f*x + e)^3 - 3*a^2*f*cosh(f*x + e))*e^(2*f*x + 2*e))
*sinh(f*x + e)^3 - a^2*f + 7*(13*a^2*f*cosh(f*x + e)^12 - 66*a^2*f*cosh(f*x
 + e)^10 + 135*a^2*f*cosh(f*x + e)^8 - 140*a^2*f*cosh(f*x + e)^6 + 75*a^2*f
*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*f + (13*a^2*f*cosh(f*x +
e)^12 - 66*a^2*f*cosh(f*x + e)^10 + 135*a^2*f*cosh(f*x + e)^8 - 140*a^2*f*c
osh(f*x + e)^6 + 75*a^2*f*cosh(f*x + e)^4 - 18*a^2*f*cosh(f*x + e)^2 + a^2*
f)*e^(2*f*x + 2*e))*sinh(f*x + e)^2 + (a^2*f*cosh(f*x + e)^14 - 7*a^2*f*cos
h(f*x + e)^12 + 21*a^2*f*cosh(f*x + e)^10 - 35*a^2*f*cosh(f*x + e)^8 + 35*a
^2*f*cosh(f*x + e)^6 - 21*a^2*f*cosh(f*x + e)^4 + 7*a^2*f*cosh(f*x + e)^2 -
 a^2*f)*e^(2*f*x + 2*e) + 14*(a^2*f*cosh(f*x + e)^13 - 6*a^2*f*cosh(f*x + e
)^11 + 15*a^2*f*cosh(f*x + e)^9 - 20*a^2*f*cosh(f*x + e)^7 + 15*a^2*f*cosh(
f*x + e)^5 - 6*a^2*f*cosh(f*x + e)^3 + a^2*f*cosh(f*x + e) + (a^2*f*cosh(f*
x + e)^13 - 6*a^2*f*cosh(f*x + e)^11 + 15*a^2*f*cosh(f*x + e)^9 - 20*a^2*f*
cosh(f*x + e)^7 + 15*a^2*f*cosh(f*x + e)^5 - 6*a^2*f*cosh(f*x + e)^3 + a^2*
f*cosh(f*x + e))*e^(2*f*x + 2*e))*sinh(f*x + e))
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**8/(a+a*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.94312, size = 132, normalized size = 1.15

$$-\frac{8\left(35\sqrt{a}e^{\left(11\,fx+11\,e\right)}+28\sqrt{a}e^{\left(9\,fx+9\,e\right)}+114\sqrt{a}e^{\left(7\,fx+7\,e\right)}+28\sqrt{a}e^{\left(5\,fx+5\,e\right)}+35\sqrt{a}e^{\left(3\,fx+3\,e\right)}\right)}{105\,a^{2}f\left(e^{\left(2\,fx+2\,e\right)}-1\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^8/(a+a*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -8/105*(35*sqrt(a)*e^(11*f*x + 11*e) + 28*sqrt(a)*e^(9*f*x + 9*e) + 114*sqr
t(a)*e^(7*f*x + 7*e) + 28*sqrt(a)*e^(5*f*x + 5*e) + 35*sqrt(a)*e^(3*f*x + 3
*e))/(a^2*f*(e^(2*f*x + 2*e) - 1)^7)
```

3.457 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx$

Optimal. Leaf size=187

$$\frac{\left(8a^2-24ab+15b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)^2}-\frac{\left(8a^2-24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{3/2}}-\frac{\text{sech}^4(e+fx)\left(a+b\sinh^2\right.}{4f(a-b)}$$

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]
])/(8*(a - b)^(3/2)*f) + ((8*a^2 - 24*a*b + 15*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2])/(8*(a - b)^2*f) + ((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
)^(3/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/
(4*(a - b)*f)
```

Rubi [A] time = 0.22461, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{\left(8a^2-24ab+15b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)^2}-\frac{\left(8a^2-24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{3/2}}-\frac{\text{sech}^4(e+fx)\left(a+b\sinh^2\right.}{4f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]
])/(8*(a - b)^(3/2)*f) + ((8*a^2 - 24*a*b + 15*b^2)*Sqrt[a + b*Sinh[e + f*x
]^2])/(8*(a - b)^2*f) + ((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2
)^(3/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/
(4*(a - b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{(1+x)^3}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-4a+3b)+2(a-b)x\right)\sqrt{a}}{(1+x)^2}\right.}{4(a-b}$$

$$= \frac{(8a-7b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8(a-b)^2f} - \frac{\text{sech}^4(e+fx)\left(a+b\text{s}\right.}{4(a-b)}$$

$$= \frac{\left(8a^2-24ab+15b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} + \frac{(8a-7b)\text{sech}^2(e+fx)\left(a-\right.}{8(a-b)^2}$$

$$= \frac{\left(8a^2-24ab+15b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2f} + \frac{(8a-7b)\text{sech}^2(e+fx)\left(a-\right.}{8(a-b)^2}$$

$$= -\frac{\left(8a^2-24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{3/2}f} + \frac{\left(8a^2-24ab+15b^2\right)\sqrt{}}{8(a-b}$$

Mathematica [A] time = 0.571949, size = 151, normalized size = 0.81

$$-\frac{\left(8a^2-24ab+15b^2\right)\left(\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)-\sqrt{a+b\sinh^2(e+fx)}\right)+2(a-b)\text{sech}^4(e+fx)\left(a+b\sinh^2\right.}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^5,x]
```

```
[Out] -(-((8*a - 7*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2)) + 2*(a - b)*
Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2) + (8*a^2 - 24*a*b + 15*b^2)*(
Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] - Sqrt[a + b*S
inh[e + f*x]^2]))/(8*(a - b)^2*f)
```

Maple [C] time = 0.341, size = 43, normalized size = 0.2

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^{5}}{\left(\cosh\left(fx+e\right)\right)^{6}}\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^5/cosh(f*x+e)^6,sinh(f*x
+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\tanh\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^5, x)
```

Fricas [B] time = 9.01519, size = 12141, normalized size = 64.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^9 + 9*(8*a^2 - 24*a*b + 15
*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 - 24*a*b + 15*b^2)*sinh(f*x +
e)^9 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 - 24*a*b +
 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^7 + 28*(3
*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh
(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 +
6*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 14*(8*a^2 - 24*a*b + 15*b
^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^5 + 2*(63*(8*a
^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 70*(8*a^2 - 24*a*b + 15*b^2)*cosh(f
*x + e)^3 + 15*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + 4
*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 - 24*a*b + 15*b^2
)*cosh(f*x + e)^6 + 35*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 15*(8*a^
```

2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^3 + 12*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 7*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e) + (9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^8 + 28*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^6 + 30*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 12*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*(2*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 16*(a^2 - 2*a*b + b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + 2*(a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^6 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b + 17*b^2)*sinh(f*x + e)^6 + 2*(56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^3 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^4 + (140*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^4 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + 20*a^2 - 42*a*b + 22*b^2)*sinh(f*x + e)^4 + 4*(28*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5 + 5*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^3 + 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^4 + 12*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^2 + 16*a^2 - 33*a*b + 17*b^2)*sinh(f*x + e)^2 + 2*a^2 - 4*a*b + 2*b^2 + 2*(8*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^7 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^5 + 4*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^8 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^9 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 4*(9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^7 + 6*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 28*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 14*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^5 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 2*(63*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e) + 12*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^7 + 7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 28*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^6 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 12*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)), -1/8*(((8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^9 + 9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^9 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 14*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^5 + 2*(63*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x

```
+ e)^5 + 70*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + 15*(8*a^2 - 24*a*b
+ 15*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^2 - 24*a*b + 15*b^2)*cosh
(f*x + e)^3 + 4*(21*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^6 + 35*(8*a^2 -
 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 15*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x +
 e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e)^3 + 12*(3*(8*a^2 - 24*a*b +
15*b^2)*cosh(f*x + e)^7 + 7*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^5 + 5*(
8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^3 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f
*x + e))*sinh(f*x + e)^2 + (8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e) + (9*(8*
a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^8 + 28*(8*a^2 - 24*a*b + 15*b^2)*cosh(
f*x + e)^6 + 30*(8*a^2 - 24*a*b + 15*b^2)*cosh(f*x + e)^4 + 12*(8*a^2 - 24*
a*b + 15*b^2)*cosh(f*x + e)^2 + 8*a^2 - 24*a*b + 15*b^2)*sinh(f*x + e))*sqr
t(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh
(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + s
inh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(
2)*(2*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^8 + 16*(a^2 - 2*a*b + b^2)*cosh(f*x
 + e)*sinh(f*x + e)^7 + 2*(a^2 - 2*a*b + b^2)*sinh(f*x + e)^8 + (16*a^2 - 3
3*a*b + 17*b^2)*cosh(f*x + e)^6 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^2 +
 16*a^2 - 33*a*b + 17*b^2)*sinh(f*x + e)^6 + 2*(56*(a^2 - 2*a*b + b^2)*cosh
(f*x + e)^3 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 +
 2*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^4 + (140*(a^2 - 2*a*b + b^2)*co
sh(f*x + e)^4 + 15*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^2 + 20*a^2 - 42
*a*b + 22*b^2)*sinh(f*x + e)^4 + 4*(28*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^5
+ 5*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^3 + 2*(10*a^2 - 21*a*b + 11*b^
2)*cosh(f*x + e))*sinh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e
)^2 + (56*(a^2 - 2*a*b + b^2)*cosh(f*x + e)^6 + 15*(16*a^2 - 33*a*b + 17*b^
2)*cosh(f*x + e)^4 + 12*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^2 + 16*a^2
 - 33*a*b + 17*b^2)*sinh(f*x + e)^2 + 2*a^2 - 4*a*b + 2*b^2 + 2*(8*(a^2 - 2
*a*b + b^2)*cosh(f*x + e)^7 + 3*(16*a^2 - 33*a*b + 17*b^2)*cosh(f*x + e)^5
+ 4*(10*a^2 - 21*a*b + 11*b^2)*cosh(f*x + e)^3 + (16*a^2 - 33*a*b + 17*b^2)
*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^9 + 9*(a^2 - 2*a*b + b^2)*f*cosh
(f*x + e)*sinh(f*x + e)^8 + (a^2 - 2*a*b + b^2)*f*sinh(f*x + e)^9 + 4*(a^2
- 2*a*b + b^2)*f*cosh(f*x + e)^7 + 4*(9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)
^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^7 + 6*(a^2 - 2*a*b + b^2)*f*cosh(
f*x + e)^5 + 28*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b
^2)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(21*(a^2 - 2*a*b + b^2)*f*cosh(f*x
 + e)^4 + 14*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)
*sinh(f*x + e)^5 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + 2*(63*(a^2 - 2
*a*b + b^2)*f*cosh(f*x + e)^5 + 70*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 +
15*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(21*(a^2 - 2*a*
b + b^2)*f*cosh(f*x + e)^6 + 35*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 15*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e
)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e) + 12*(3*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^7 + 7*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^5 + 5*(a^2 - 2*a*b + b
^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)^
2 + (9*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^8 + 28*(a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^6 + 30*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 12*(a^2 - 2*a*b +
 b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**5,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh \left(fx+e\right)^{2}+a} \tanh \left(fx+e\right)^{5}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^5, x)
```

3.458 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx$

Optimal. Leaf size=126

$$\frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f\sqrt{a-b}}+\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f(a-b)}$$

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*Sqrt[a -
 b]*f) + ((2*a - 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f) + (Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(2*(a - b)*f)
```

Rubi [A] time = 0.125283, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f\sqrt{a-b}}+\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*Sqrt[a -
 b]*f) + ((2*a - 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f) + (Sech[e
+ f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(2*(a - b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{(1+x)^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)f} + \frac{(2a-3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x\right.}{4(a-b)f}$$

$$= \frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)f}$$

$$= \frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2(a-b)f}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2\sqrt{a-b}f} + \frac{(2a-3b)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} +$$

Mathematica [A] time = 0.405799, size = 88, normalized size = 0.7

$$\frac{(\cosh(2(e+fx))+2)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)} - \frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3,x]
```

```
[Out] (-(((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/Sqrt[a -
b]) + (2 + Cosh[2*(e + f*x)])*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/
(2*f)
```

Maple [C] time = 0.102, size = 43, normalized size = 0.3

$$\frac{1}{f}`\ \texttt{int/indef0}`\left(\frac{\left(\sinh\left(fx+e\right)\right)^3}{\left(\cosh\left(fx+e\right)\right)^4}\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}, \sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)^3/cosh(f*x+e)^4,sinh(f*x
+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \tanh\left(fx+e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^3, x)
```

Fricas [B] time = 7.67511, size = 4352, normalized size = 34.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e)*sinh(f*x
+ e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x + e)^3 + 2*(5
*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5*(2*a - 3*b
)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a - 3
*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*b)*cosh(f*x
 + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*
b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*
x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*
sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e)
 + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(
f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x
 + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4
*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 2*sqrt(2)*((a - b)
*cosh(f*x + e)^4 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f
*x + e)^4 + 4*(a - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 + 2*a
- 2*b)*sinh(f*x + e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a - b)*cosh(f*x +
e))*sinh(f*x + e) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*
a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))
)/((a - b)*f*cosh(f*x + e)^5 + 5*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^4 +
(a - b)*f*sinh(f*x + e)^5 + 2*(a - b)*f*cosh(f*x + e)^3 + 2*(5*(a - b)*f*co
sh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^3 + (a - b)*f*cosh(f*x + e) + 2*(5
*(a - b)*f*cosh(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (
5*(a - b)*f*cosh(f*x + e)^4 + 6*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh
(f*x + e)), -1/2*(((2*a - 3*b)*cosh(f*x + e)^5 + 5*(2*a - 3*b)*cosh(f*x + e
)*sinh(f*x + e)^4 + (2*a - 3*b)*sinh(f*x + e)^5 + 2*(2*a - 3*b)*cosh(f*x +
e)^3 + 2*(5*(2*a - 3*b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e)^3 + 2*(5
*(2*a - 3*b)*cosh(f*x + e)^3 + 3*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^2
 + (2*a - 3*b)*cosh(f*x + e) + (5*(2*a - 3*b)*cosh(f*x + e)^4 + 6*(2*a - 3*
b)*cosh(f*x + e)^2 + 2*a - 3*b)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqr
```

```
t(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)
*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*((a - b)*cosh(f*x + e)^4
 + 4*(a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (a - b)*sinh(f*x + e)^4 + 4*(a
 - b)*cosh(f*x + e)^2 + 2*(3*(a - b)*cosh(f*x + e)^2 + 2*a - 2*b)*sinh(f*x
+ e)^2 + 4*((a - b)*cosh(f*x + e)^3 + 2*(a - b)*cosh(f*x + e))*sinh(f*x + e
) + a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cos
h(f*x + e)^5 + 5*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^4 + (a - b)*f*sinh(f
*x + e)^5 + 2*(a - b)*f*cosh(f*x + e)^3 + 2*(5*(a - b)*f*cosh(f*x + e)^2 +
(a - b)*f)*sinh(f*x + e)^3 + (a - b)*f*cosh(f*x + e) + 2*(5*(a - b)*f*cosh(
f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (5*(a - b)*f*cosh
(f*x + e)^4 + 6*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\tanh^3\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**3, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx + e\right)^2 + a}\tanh\left(fx + e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^3, x)
```

3.459 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx$

Optimal. Leaf size=62

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[
a + b*Sinh[e + f*x]^2]/f
```

Rubi [A] time = 0.0635905, antiderivative size = 62, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] -((Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + Sqrt[
a + b*Sinh[e + f*x]^2]/f
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf}$$
$$= -\frac{\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.0523067, size = 65, normalized size = 1.05

$$\frac{\sqrt{a+b\cosh^2(e+fx)-b} - \sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(e+fx)-b}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x],x]
```

```
[Out] (-(Sqrt[a - b]*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]) + Sqrt
[a - b + b*Cosh[e + f*x]^2])/f
```

Maple [C] time = 0.079, size = 41, normalized size = 0.7

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{\sinh(fx+e)}{(\cosh(fx+e))^2}\sqrt{a+b(\sinh(fx+e))^2}, \sinh(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x)
```

```
[Out] `int/indef0`((a+b*sinh(f*x+e)^2)^(1/2)*sinh(f*x+e)/cosh(f*x+e)^2,sinh(f*x+e
))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(fx+e)^2+a}\tanh(fx+e)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e), x)
```

Fricas [B] time = 7.16423, size = 1705, normalized size = 27.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a - b)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 +
4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(
f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2
)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(
f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x +
e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sin
h(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f
*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 +
 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*
cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*
x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e
)), -1/2*(2*sqrt(-a + b)*(cosh(f*x + e) + sinh(f*x + e))*arctan(-1/2*sqrt(2
)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*co
sh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^{2}\left(e + fx\right)} \tanh\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.460 $\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Si
nh[e + f*x]^2]/f
```

Rubi [A] time = 0.0742229, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Si
nh[e + f*x]^2]/f
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Mathematica [A] time = 0.0485787, size = 53, normalized size = 0.98

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)-\sqrt{a+b\sinh^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqrt[a + b*Sinh[e
 + f*x]^2])/f)
```

Maple [C] time = 0.085, size = 46, normalized size = 0.9

$$\frac{1}{f}\text{` int/indef0`}\left(\left(b\sinh\left(fx+e\right)+\frac{a}{\sinh\left(fx+e\right)}\right)\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((b*sinh(f*x+e)+a/sinh(f*x+e))/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f
*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\coth\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e), x)
```

Fricas [B] time = 3.9074, size = 1656, normalized size = 30.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*(cosh(f*x + e) + sinh(f*x + e))*log((b*cosh(f*x + e)^4 + 4*b*
cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*
x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b
)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*
(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
 e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e) + f*sinh(f*x + e)), 1/2*(2*sqr
t(-a)*(cosh(f*x + e) + sinh(f*x + e))*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))
) + sqrt(2)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x +
 e) + f*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sinh^2\left(e + fx\right)}\coth\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.461 $\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=106

$$\frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af}$$

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) + (
(2*a + b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*a*f) - (Csch[e + f*x]^2*(a + b*Si
nh[e + f*x]^2)^(3/2))/(2*a*f)
```

Rubi [A] time = 0.121803, antiderivative size = 106, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f}-\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) + (
(2*a + b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*a*f) - (Csch[e + f*x]^2*(a + b*Si
nh[e + f*x]^2)^(3/2))/(2*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)\sqrt{a+bx}}{x^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x,\right.}{4af}$$

$$= \frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af} - \frac{\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af} +$$

$$= \frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af} - \frac{\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{2af} +$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} + \frac{(2a+b)\sqrt{a+b\sinh^2(e+fx)}}{2af} - \text{cs}$$

Mathematica [A] time = 0.36274, size = 69, normalized size = 0.65

$$-\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}} + \left(\text{csch}^2(e+fx)-2\right)\sqrt{a+b\sinh^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/Sqrt[a] + (-2 +
Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f)
```

Maple [C] time = 0.214, size = 58, normalized size = 0.6

$$\frac{1}{f}\text{` int/indef0`}\left(\left(b\sinh\left(fx+e\right)+\frac{a+b}{\sinh\left(fx+e\right)}+\frac{a}{\left(\sinh\left(fx+e\right)\right)^3}\right)\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((b*sinh(f*x+e)+(a+b)/sinh(f*x+e)+a/sinh(f*x+e)^3)/(a+b*sinh(f*
x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^{2}+a} \coth\left(fx+e\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^3, x)
```

Fricas [B] time = 4.97737, size = 3903, normalized size = 36.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a + b)*cosh(f*x + e)^5 + 5*(2*a + b)*cosh(f*x + e)*sinh(f*x + e)^
4 + (2*a + b)*sinh(f*x + e)^5 - 2*(2*a + b)*cosh(f*x + e)^3 + 2*(5*(2*a + b
)*cosh(f*x + e)^2 - 2*a - b)*sinh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x + e)
^3 - 3*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a + b)*cosh(f*x + e) +
 (5*(2*a + b)*cosh(f*x + e)^4 - 6*(2*a + b)*cosh(f*x + e)^2 + 2*a + b)*sinh
(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)
^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)
^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e
)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh
(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sin
h(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh
(f*x + e) + 1)) + 2*sqrt(2)*(a*cosh(f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x
 + e)^3 + a*sinh(f*x + e)^4 - 4*a*cosh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2
- 2*a)*sinh(f*x + e)^2 + 4*(a*cosh(f*x + e)^3 - 2*a*cosh(f*x + e))*sinh(f*x
 + e) + a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x
+ e)^5 + 5*a*f*cosh(f*x + e)*sinh(f*x + e)^4 + a*f*sinh(f*x + e)^5 - 2*a*f*
cosh(f*x + e)^3 + 2*(5*a*f*cosh(f*x + e)^2 - a*f)*sinh(f*x + e)^3 + a*f*cos
h(f*x + e) + 2*(5*a*f*cosh(f*x + e)^3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^
2 + (5*a*f*cosh(f*x + e)^4 - 6*a*f*cosh(f*x + e)^2 + a*f)*sinh(f*x + e)), 1
/2*(((2*a + b)*cosh(f*x + e)^5 + 5*(2*a + b)*cosh(f*x + e)*sinh(f*x + e)^4
+ (2*a + b)*sinh(f*x + e)^5 - 2*(2*a + b)*cosh(f*x + e)^3 + 2*(5*(2*a + b)*
cosh(f*x + e)^2 - 2*a - b)*sinh(f*x + e)^3 + 2*(5*(2*a + b)*cosh(f*x + e)^3
 - 3*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a + b)*cosh(f*x + e) + (
5*(2*a + b)*cosh(f*x + e)^4 - 6*(2*a + b)*cosh(f*x + e)^2 + 2*a + b)*sinh(f
*x + e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + sqrt(2)*(a*cosh(
```

```
f*x + e)^4 + 4*a*cosh(f*x + e)*sinh(f*x + e)^3 + a*sinh(f*x + e)^4 - 4*a*co
sh(f*x + e)^2 + 2*(3*a*cosh(f*x + e)^2 - 2*a)*sinh(f*x + e)^2 + 4*(a*cosh(f
*x + e)^3 - 2*a*cosh(f*x + e))*sinh(f*x + e) + a)*sqrt((b*cosh(f*x + e)^2 +
 b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
 e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^5 + 5*a*f*cosh(f*x + e)*sinh(f*
x + e)^4 + a*f*sinh(f*x + e)^5 - 2*a*f*cosh(f*x + e)^3 + 2*(5*a*f*cosh(f*x
+ e)^2 - a*f)*sinh(f*x + e)^3 + a*f*cosh(f*x + e) + 2*(5*a*f*cosh(f*x + e)^
3 - 3*a*f*cosh(f*x + e))*sinh(f*x + e)^2 + (5*a*f*cosh(f*x + e)^4 - 6*a*f*c
osh(f*x + e)^2 + a*f)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.462 $\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=167

$$\frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}-\frac{(8a-b)\text{csch}^2(e+fx)\left(a+b\sinh^2\right.}{8a^2f}$$

```
[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*a^
(3/2)*f) + ((8*a^2 + 8*a*b - b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a^2*f) -
((8*a - b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csch
[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*a*f)
```

Rubi [A] time = 0.179832, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{(8a^2+8ab-b^2)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{(8a^2+8ab-b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}-\frac{(8a-b)\text{csch}^2(e+fx)\left(a+b\sinh^2\right.}{8a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*a^
(3/2)*f) + ((8*a^2 + 8*a*b - b^2)*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a^2*f) -
((8*a - b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csch
[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2))/(4*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
```

```
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth^5(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2\sqrt{a+bx}}{x^3}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(8a-b)+2ax\right)\sqrt{a+bx}}{x^2}\,dx$$

$$= -\frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{8a^2f} - \frac{\operatorname{csch}^4(e+fx)\left(a+b\sinh\right.}{4af}$$

$$= \frac{\left(8a^2+8ab-b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sin\right.}{8a^2f}$$

$$= \frac{\left(8a^2+8ab-b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f} - \frac{(8a-b)\operatorname{csch}^2(e+fx)\left(a+b\sin\right.}{8a^2f}$$

$$= -\frac{\left(8a^2+8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{\left(8a^2+8ab-b^2\right)\sqrt{a+b\sin}}{8a^2f}$$

Mathematica [A] time = 0.564118, size = 102, normalized size = 0.61

$$\frac{\left(-8a^2-8ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a}\sqrt{a+b\sinh^2(e+fx)}\left((8a+b)\operatorname{csch}^2(e+fx)+2a\operatorname{csch}^4(e+fx)-8a\right.}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 - 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] - Sqrt
[a]*(-8*a + (8*a + b)*Csch[e + f*x]^2 + 2*a*Csch[e + f*x]^4)*Sqrt[a + b*Sin
h[e + f*x]^2])/(8*a^(3/2)*f)
```

Maple [C] time = 0.131, size = 80, normalized size = 0.5

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\cosh\left(fx+e\right)\right)^{4}\left(a-b+b\left(\cosh\left(fx+e\right)\right)^{2}\right)}{\sinh\left(fx+e\right)\left(\left(\cosh\left(fx+e\right)\right)^{4}-2\left(\cosh\left(fx+e\right)\right)^{2}+1\right)}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4*
(a-b+b*cosh(f*x+e)^2)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\coth\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^5, x)
```

Fricas [B] time = 5.96195, size = 9559, normalized size = 57.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^9 + 9*(8*a^2 + 8*a*b - b^2)*co
sh(f*x + e)*sinh(f*x + e)^8 + (8*a^2 + 8*a*b - b^2)*sinh(f*x + e)^9 - 4*(8*
a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 + 8*a*b - b^2)*c
osh(f*x + e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(
8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 + 8*a*b - b^2)*cosh(f*x
 + e)^4 - 14*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*s
inh(f*x + e)^5 + 2*(63*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 - 70*(8*a^2 +
8*a*b - b^2)*cosh(f*x + e)^3 + 15*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh
(f*x + e)^4 - 4*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 + 8*a*
b - b^2)*cosh(f*x + e)^6 - 35*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 + 15*(8
*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^3
+ 12*(3*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 - 7*(8*a^2 + 8*a*b - b^2)*cos
```

```
h(f*x + e)^5 + 5*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b
^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 + 8*a*b - b^2)*cosh(f*x + e) +
(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^8 - 28*(8*a^2 + 8*a*b - b^2)*cosh(f*
x + e)^6 + 30*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 - 12*(8*a^2 + 8*a*b - b
^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x + e))*sqrt(a)*log((b*co
sh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(
4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^
2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b
)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cos
h(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e
))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 +
sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x +
e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*
(2*a^2*cosh(f*x + e)^8 + 16*a^2*cosh(f*x + e)*sinh(f*x + e)^7 + 2*a^2*sinh(
f*x + e)^8 - (16*a^2 + a*b)*cosh(f*x + e)^6 + (56*a^2*cosh(f*x + e)^2 - 16*
a^2 - a*b)*sinh(f*x + e)^6 + 2*(56*a^2*cosh(f*x + e)^3 - 3*(16*a^2 + a*b)*c
osh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2 + a*b)*cosh(f*x + e)^4 + (140*a^2
*cosh(f*x + e)^4 - 15*(16*a^2 + a*b)*cosh(f*x + e)^2 + 20*a^2 + 2*a*b)*sinh
(f*x + e)^4 + 4*(28*a^2*cosh(f*x + e)^5 - 5*(16*a^2 + a*b)*cosh(f*x + e)^3
+ 2*(10*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e)^3 - (16*a^2 + a*b)*cosh(f*x
 + e)^2 + (56*a^2*cosh(f*x + e)^6 - 15*(16*a^2 + a*b)*cosh(f*x + e)^4 + 12*
(10*a^2 + a*b)*cosh(f*x + e)^2 - 16*a^2 - a*b)*sinh(f*x + e)^2 + 2*a^2 + 2*
(8*a^2*cosh(f*x + e)^7 - 3*(16*a^2 + a*b)*cosh(f*x + e)^5 + 4*(10*a^2 + a*b
)*cosh(f*x + e)^3 - (16*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*f*cosh(f*x + e)^9 + 9*a^2*f*co
sh(f*x + e)*sinh(f*x + e)^8 + a^2*f*sinh(f*x + e)^9 - 4*a^2*f*cosh(f*x + e)
^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*(9*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x
 + e)^7 + 28*(3*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)^
6 - 4*a^2*f*cosh(f*x + e)^3 + 6*(21*a^2*f*cosh(f*x + e)^4 - 14*a^2*f*cosh(f
*x + e)^2 + a^2*f)*sinh(f*x + e)^5 + 2*(63*a^2*f*cosh(f*x + e)^5 - 70*a^2*f
*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e))*sinh(f*x + e)^4 + a^2*f*cosh(f*x
 + e) + 4*(21*a^2*f*cosh(f*x + e)^6 - 35*a^2*f*cosh(f*x + e)^4 + 15*a^2*f*c
osh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^3 + 12*(3*a^2*f*cosh(f*x + e)^7 - 7*a
^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh(
f*x + e)^2 + (9*a^2*f*cosh(f*x + e)^8 - 28*a^2*f*cosh(f*x + e)^6 + 30*a^2*f
*cosh(f*x + e)^4 - 12*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)), 1/8*((
(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^9 + 9*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
e)*sinh(f*x + e)^8 + (8*a^2 + 8*a*b - b^2)*sinh(f*x + e)^9 - 4*(8*a^2 + 8*a
*b - b^2)*cosh(f*x + e)^7 + 4*(9*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 - 8*
a^2 - 8*a*b + b^2)*sinh(f*x + e)^7 + 28*(3*(8*a^2 + 8*a*b - b^2)*cosh(f*x +
 e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 6*(8*a^2 + 8
*a*b - b^2)*cosh(f*x + e)^5 + 6*(21*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 -
 14*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x +
 e)^5 + 2*(63*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^5 - 70*(8*a^2 + 8*a*b - b
^2)*cosh(f*x + e)^3 + 15*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)
^4 - 4*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^2 + 8*a*b - b^2)*
cosh(f*x + e)^6 - 35*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 + 15*(8*a^2 + 8*
a*b - b^2)*cosh(f*x + e)^2 - 8*a^2 - 8*a*b + b^2)*sinh(f*x + e)^3 + 12*(3*(
8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^7 - 7*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e
)^5 + 5*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^3 - (8*a^2 + 8*a*b - b^2)*cosh(
f*x + e))*sinh(f*x + e)^2 + (8*a^2 + 8*a*b - b^2)*cosh(f*x + e) + (9*(8*a^2
 + 8*a*b - b^2)*cosh(f*x + e)^8 - 28*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^6
+ 30*(8*a^2 + 8*a*b - b^2)*cosh(f*x + e)^4 - 12*(8*a^2 + 8*a*b - b^2)*cosh(
f*x + e)^2 + 8*a^2 + 8*a*b - b^2)*sinh(f*x + e))*sqrt(-a)*arctan(1/2*sqrt(2
)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e
) + a*sinh(f*x + e))) + 2*sqrt(2)*(2*a^2*cosh(f*x + e)^8 + 16*a^2*cosh(f*x
+ e)*sinh(f*x + e)^7 + 2*a^2*sinh(f*x + e)^8 - (16*a^2 + a*b)*cosh(f*x + e)
```

```
^6 + (56*a^2*cosh(f*x + e)^2 - 16*a^2 - a*b)*sinh(f*x + e)^6 + 2*(56*a^2*co
sh(f*x + e)^3 - 3*(16*a^2 + a*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(10*a^2
 + a*b)*cosh(f*x + e)^4 + (140*a^2*cosh(f*x + e)^4 - 15*(16*a^2 + a*b)*cosh
(f*x + e)^2 + 20*a^2 + 2*a*b)*sinh(f*x + e)^4 + 4*(28*a^2*cosh(f*x + e)^5 -
 5*(16*a^2 + a*b)*cosh(f*x + e)^3 + 2*(10*a^2 + a*b)*cosh(f*x + e))*sinh(f*
x + e)^3 - (16*a^2 + a*b)*cosh(f*x + e)^2 + (56*a^2*cosh(f*x + e)^6 - 15*(1
6*a^2 + a*b)*cosh(f*x + e)^4 + 12*(10*a^2 + a*b)*cosh(f*x + e)^2 - 16*a^2 -
 a*b)*sinh(f*x + e)^2 + 2*a^2 + 2*(8*a^2*cosh(f*x + e)^7 - 3*(16*a^2 + a*b)
*cosh(f*x + e)^5 + 4*(10*a^2 + a*b)*cosh(f*x + e)^3 - (16*a^2 + a*b)*cosh(f
*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
 b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(
a^2*f*cosh(f*x + e)^9 + 9*a^2*f*cosh(f*x + e)*sinh(f*x + e)^8 + a^2*f*sinh(
f*x + e)^9 - 4*a^2*f*cosh(f*x + e)^7 + 6*a^2*f*cosh(f*x + e)^5 + 4*(9*a^2*f
*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^7 + 28*(3*a^2*f*cosh(f*x + e)^3 - a
^2*f*cosh(f*x + e))*sinh(f*x + e)^6 - 4*a^2*f*cosh(f*x + e)^3 + 6*(21*a^2*f
*cosh(f*x + e)^4 - 14*a^2*f*cosh(f*x + e)^2 + a^2*f)*sinh(f*x + e)^5 + 2*(6
3*a^2*f*cosh(f*x + e)^5 - 70*a^2*f*cosh(f*x + e)^3 + 15*a^2*f*cosh(f*x + e)
)*sinh(f*x + e)^4 + a^2*f*cosh(f*x + e) + 4*(21*a^2*f*cosh(f*x + e)^6 - 35*
a^2*f*cosh(f*x + e)^4 + 15*a^2*f*cosh(f*x + e)^2 - a^2*f)*sinh(f*x + e)^3 +
 12*(3*a^2*f*cosh(f*x + e)^7 - 7*a^2*f*cosh(f*x + e)^5 + 5*a^2*f*cosh(f*x +
 e)^3 - a^2*f*cosh(f*x + e))*sinh(f*x + e)^2 + (9*a^2*f*cosh(f*x + e)^8 - 2
8*a^2*f*cosh(f*x + e)^6 + 30*a^2*f*cosh(f*x + e)^4 - 12*a^2*f*cosh(f*x + e)
^2 + a^2*f)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \coth\left(fx+e\right)^5 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^5, x)
```

3.463 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx$

Optimal. Leaf size=292

$$\frac{(3a-4b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh^3(e+fx)\sqrt{a+b\sinh^2(e+}}{3f}$$

```
[Out] -((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[
e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[
e + f*x])/(3*(a - b)*f) - ((3*a - 4*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e +
 f*x])/(3*(a - b)*f) - (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.306428, antiderivative size = 292, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 467, 578, 531, 418, 492, 411}

$$-\frac{\tanh^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}-\frac{(3a-4b)\tanh(e+fx)\sqrt{a+}}{3f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```


```
[Out] -((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[
e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[
e + f*x])/(3*(a - b)*f) - ((3*a - 4*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e +
 f*x])/(3*(a - b)*f) - (Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^3)/(3*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^4(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(1+x^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh^3(e+fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)}{}$$

$$= -\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} - \frac{\sqrt{a+b\sinh^2(e+fx)}\,\text{t}}{3f}$$

$$= -\frac{(3a-4b)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{3(a-b)f} - \frac{\sqrt{a+b\sinh^2(e+fx)}\,\text{t}}{3f}$$

$$= \frac{(3a-4b)F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e+fx)\sqrt{a+b\sinh^2(e+f}}{3(a-b)f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

$$= -\frac{(7a-8b)E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+}}{3(a-b)f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 1.94724, size = 214, normalized size = 0.73

$$\frac{8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right) - \frac{\tanh(e+fx)\mathrm{sech}^2(e+fx)\left(4\left(4a^2-6ab+b^2\right)\cosh(2(e+fx))+8a^2+b(4a-5b)\cosh(4(e+}{2\sqrt{2}}}{6f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^4,x]
```

```
[Out] ((-2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*El
lipticF[I*(e + f*x), b/a] - ((8*a^2 - 12*a*b + b^2 + 4*(4*a^2 - 6*a*b + b^2
)*Cosh[2*(e + f*x)] + (4*a - 5*b)*b*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh
[e + f*x])/(2*Sqrt[2]))/(6*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.177, size = 369, normalized size = 1.3

$$-\frac{1}{3\left(\cosh\left(fx+e\right)\right)^3(a-b)f}\left(\left(4\sqrt{-\frac{b}{a}}ab-5\sqrt{-\frac{b}{a}}b^2\right)\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^4+\left(4\sqrt{-\frac{b}{a}}a^2-10\sqrt{-\frac{b}{a}}ab+6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x)
```

```
[Out] -1/3*((4*(-1/a*b)^(1/2)*a*b-5*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4
+(4*(-1/a*b)^(1/2)*a^2-10*(-1/a*b)^(1/2)*a*b+6*(-1/a*b)^(1/2)*b^2)*cosh(f*x
+e)^2*sinh(f*x+e)+(-(-1/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*
b^2)*sinh(f*x+e)-(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(3
```

```
*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-11*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8
*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/cosh
(f*x+e)^3/(a-b)/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \tanh\left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b \sinh\left(fx+e\right)^2 + a} \tanh\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b \sinh^2\left(e + fx\right)} \tanh^4\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**4, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b \sinh\left(fx+e\right)^2 + a} \tanh\left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4, x)
```

3.464 $\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx$

Optimal. Leaf size=168

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{2\text{se}}{}$$

```
[Out] (-2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Elli
pticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x
]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```

Rubi [A] time = 0.177173, antiderivative size = 168, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3196, 467, 531, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2\operatorname{sech}(e+f}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```


```
[Out] (-2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Elli
pticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x
]^2])/(f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*S
inh[e + f*x]^2]*Tanh[e + f*x])/f
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \sqrt{a+b\sinh^2(e+fx)}\tanh^2(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{\left(1+x^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Sub}}{}$$

$$= -\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f} + \frac{\left(a\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Su}}{}$$

$$= \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{\sqrt{a+b}}{}$$

$$= -\frac{2E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{F\left(\mathrm{ta}\right.}{}$$

Mathematica [C] time = 0.493876, size = 150, normalized size = 0.89

$$\frac{i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\tanh(e+fx)(-2a-b\cosh(2(e+fx))+b)-2i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))}{a}}}{f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x]^2,x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
f*x), b/a] + I*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[
I*(e + f*x), b/a] + (-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(f*Sqrt
[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.135, size = 233, normalized size = 1.4

$$-\frac{1}{f\cosh\left(fx+e\right)}\left(\sqrt{-\frac{b}{a}}b\left(\sinh\left(fx+e\right)\right)^{3}-a\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x)
```

```
[Out] -((-1/a*b)^(1/2)*b*sinh(f*x+e)^3-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+(-1/a*b)^(1/2)*a*sinh(f*x+
e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\tanh\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\tanh\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sinh^{2}\left(e+fx\right)}\tanh^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2)*tanh(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*tanh(e + f*x)**2, x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\tanh\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2, x)
```

3.465 $\int \sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rubi [A] time = 0.0380248, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[1 +
(b*Sinh[e + f*x]^2)/a])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sinh^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\\ &= -\frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.0841224, size = 69, normalized size = 1.15

$$-\frac{ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a]
)/(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0., size = 140, normalized size = 2.3

$$\frac{1}{\cosh\left(fx+e\right)f}\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\left(a\mathrm{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)-b\mathrm{EllipticF}\left(\sinh\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)))/(-1/a*b)^(1/2)/
cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{b\sinh\left(fx+e\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int\sqrt{a+b\sinh^{2}\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.466 $\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=202

$$\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}$$

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (2*EllipticE[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Se
ch[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*EllipticF[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/f
```

Rubi [A] time = 0.198898, antiderivative size = 202, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3196, 473, 531, 418, 492, 411}

$$\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F}{af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) - (2*EllipticE[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*Sqrt[(Se
ch[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((a + b)*EllipticF[ArcTan[Sinh
[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(
Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^
2]*Tanh[e + f*x])/f
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
```

```
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \coth^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}\sqrt{a+bx^2}}{x^2}\,dx, x, \sinh(e+fx)\right.}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(2b\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)}{}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(a+b)F\left(\tan^{-1}(\sinh(e+fx))|1-\right.}{af\sqrt{\frac{\text{sech}^2(e+}{}}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{2E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\text{se}}{f\sqrt{\frac{\text{sech}^2(e+fx)(}{}}}$$

Mathematica [C] time = 0.570496, size = 154, normalized size = 0.76

$$\frac{i\sqrt{2}(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)-2i\sqrt{2}a\sqrt{\frac{2a+b\cos}{}}}{f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*(a -
 b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(f
*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.142, size = 215, normalized size = 1.1

$$-\frac{1}{\cosh\left(fx+e\right)\sinh\left(fx+e\right)f}\left(-\sinh\left(fx+e\right)\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^{2}}{a}+\frac{a-b}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\left(a\text{EllipticF}\left(\sinh\left(fx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(-sinh(f*x+e)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(a*E
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2)))+(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*co
sh(f*x+e)^2)/sinh(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\coth\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\coth\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2, x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a+b\sinh^{2}\left(e+fx\right)}\coth^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(e + f*x)**2)*coth(e + f*x)**2, x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.467 $\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx$

Optimal. Leaf size=270

$$\frac{(3a+5b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2}}{3af}$$

```
[Out] -((3*a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e +
f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a + b)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqr
t[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[Ar
cTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3
*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a
 + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```

Rubi [A] time = 0.295758, antiderivative size = 270, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 473, 580, 531, 418, 492, 411}

$$\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af}-\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(}}{3af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] -((3*a + b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (Coth[e +
f*x]^3*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a + b)*EllipticE[ArcTan[Sin
h[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f*Sqr
t[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[Ar
cTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3
*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a
 + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \coth^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^{3/2}\sqrt{a+bx^2}}{x^4}dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Su}}{}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e}}{3f}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e}}{3f}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e}}{3f}$$

$$= -\frac{(3a+b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} - \frac{\coth^3(e+fx)\sqrt{a+b\sinh^2(e}}{3f}$$

Mathematica [C] time = 3.06738, size = 210, normalized size = 0.78

$$\frac{8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right) - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\left(4\left(4a^2-2ab-b^2\right)\cosh(2(e+fx))-8a^2+b(4a+b)\cosh(4(e+fx))+}{2\sqrt{2}}}{6af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-((-8*a^2 + 4*a*b + 3*b^2 + 4*(4*a^2 - 2*a*b - b^2)*Cosh[2*(e + f*x)] + b*
(4*a + b)*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/(2*Sqrt[2]) - (
2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f
*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cF[I*(e + f*x), b/a])/(6*a*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.17, size = 519, normalized size = 1.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6-3
*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x
+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3+2*b*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2)
)*a*sinh(f*x+e)^3+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3-7*((a+b*sinh
```

```
(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1
/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+
e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+6*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+
(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2+2*(-1/a
*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2)/a/sinh(f*x+e)^3/(-1/a*b)^(1
/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\coth\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\sqrt{b\sinh\left(fx+e\right)^{2}+a}\coth\left(fx+e\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh\left(fx+e\right)^{2}+a}\coth\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4, x)
```

3.468 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx$

Optimal. Leaf size=232

$$\frac{\left(8a^2-40ab+35b^2\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f(a-b)^2}+\frac{\left(8a^2-40ab+35b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)}-\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}}{8f\sqrt{a-b}}$$

```
[Out] -((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]
])/(8*Sqrt[a - b]*f) + ((8*a^2 - 40*a*b + 35*b^2)*Sqrt[a + b*Sinh[e + f*x]^
2])/(8*(a - b)*f) + ((8*a^2 - 40*a*b + 35*b^2)*(a + b*Sinh[e + f*x]^2)^(3/2
))/(24*(a - b)^2*f) + ((8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^
(5/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4
*(a - b)*f)
```

Rubi [A] time = 0.290255, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{\left(8a^2-40ab+35b^2\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f(a-b)^2}+\frac{\left(8a^2-40ab+35b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8f(a-b)}-\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}}{8f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]
```


```
[Out] -((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]
])/(8*Sqrt[a - b]*f) + ((8*a^2 - 40*a*b + 35*b^2)*Sqrt[a + b*Sinh[e + f*x]^
2])/(8*(a - b)*f) + ((8*a^2 - 40*a*b + 35*b^2)*(a + b*Sinh[e + f*x]^2)^(3/2
))/(24*(a - b)^2*f) + ((8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^
(5/2))/(8*(a - b)^2*f) - (Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4
*(a - b)*f)
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^5(e+fx)\,dx=\frac{\text{Subst}\left(\int\frac{x^2(a+bx)^{3/2}}{(1+x)^3}\,dx,x,\sinh^2(e+fx)\right)}{2f}$$

$$=-\frac{\text{sech}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4(a-b)f}+\frac{\text{Subst}\left(\int\frac{\left(\frac{1}{2}(-4a+5b)+2(a-b)x\right)}{(1+x)^2}\right.}{4(a}$$

$$=\frac{(8a-9b)\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8(a-b)^2f}-\frac{\text{sech}^4(e+fx)\left(a+\right.}{4(a-}$$

$$=\frac{\left(8a^2-40ab+35b^2\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24(a-b)^2f}+\frac{(8a-9b)\text{sech}^2(e+fx}{8(a}$$

$$=\frac{\left(8a^2-40ab+35b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)f}+\frac{\left(8a^2-40ab+35b^2\right)\left(a+\right.}{24(a-b}$$

$$=\frac{\left(8a^2-40ab+35b^2\right)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)f}+\frac{\left(8a^2-40ab+35b^2\right)\left(a+\right.}{24(a-b}$$

$$=-\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8\sqrt{a-b}f}+\frac{\left(8a^2-40ab+35b^2\right)}{8(a}$$

Mathematica [A] time = 1.63614, size = 169, normalized size = 0.73

$$-\frac{-\left(8a^2-40ab+35b^2\right)\left(\sqrt{a+b\sinh^2(e+fx)}\left(4a+b\sinh^2(e+fx)-3b\right)-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right)+6(a}{24f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^5,x]
```

```
[Out] -(-3*(8*a - 9*b)*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + 6*(a - b)*
Sech[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2) - (8*a^2 - 40*a*b + 35*b^2)*(
-3*(a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a
+ b*Sinh[e + f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/(24*(a - b)^2*f)
```

Maple [C] time = 0.122, size = 71, normalized size = 0.3

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^5\left(b^2\left(\sinh\left(fx+e\right)\right)^4+2ab\left(\sinh\left(fx+e\right)\right)^2+a^2\right)}{\left(\cosh\left(fx+e\right)\right)^6}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh
(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^5, x)
```

Fricas [B] time = 9.4306, size = 16471, normalized size = 71.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 - 40*a*b +
 35*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 - 40*a*b + 35*b^2)*sinh(f*
x + e)^11 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 - 40*a
*b + 35*b^2)*cosh(f*x + e)^2 + 32*a^2 - 160*a*b + 140*b^2)*sinh(f*x + e)^9
```

+ 3*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 12*(8*a^2 - 40*a*b + 35
*b^2)*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x
 + e)^7 + 6*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 24*(8*a^2 - 40*
a*b + 35*b^2)*cosh(f*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^7 +
42*(11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 8*(8*a^2 - 40*a*b + 35*b
^2)*cosh(f*x + e)^3 + (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e
)^6 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^6 + 252*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 +
 63*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 16*a^2 - 80*a*b + 70*b^2)*s
inh(f*x + e)^5 + 2*(165*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 + 252*(8*
a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 105*(8*a^2 - 40*a*b + 35*b^2)*cosh
(f*x + e)^3 + 10*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 +
 (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + (165*(8*a^2 - 40*a*b + 35*b^2)
*cosh(f*x + e)^8 + 336*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 + 210*(8*a
^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 40*(8*a^2 - 40*a*b + 35*b^2)*cosh(f
*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^9 + 144*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 +
 126*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 40*(8*a^2 - 40*a*b + 35*b^
2)*cosh(f*x + e)^3 + 3*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x +
e)^2 + (11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^10 + 36*(8*a^2 - 40*a*b
+ 35*b^2)*cosh(f*x + e)^8 + 42*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 +
20*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 40*a*b + 35*b^2)*
cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*co
sh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x +
e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt
(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + s
inh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x
+ e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e
)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(co
sh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*sqrt(2)*((a*b - b^2)
*cosh(f*x + e)^12 + 12*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^11 + (a*b -
b^2)*sinh(f*x + e)^12 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^10 + 2*(3
3*(a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x + e)^10 +
 20*(11*(a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x +
e))*sinh(f*x + e)^9 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^8 + (495*
(a*b - b^2)*cosh(f*x + e)^4 + 90*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2
+ 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^8 + 8*(99*(a*b - b^2)*cosh(f*x
 + e)^5 + 30*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b
 + 223*b^2)*cosh(f*x + e))*sinh(f*x + e)^7 + 8*(18*a^2 - 59*a*b + 41*b^2)*c
osh(f*x + e)^6 + 4*(231*(a*b - b^2)*cosh(f*x + e)^6 + 105*(8*a^2 - 25*a*b +
 17*b^2)*cosh(f*x + e)^4 + 7*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2
+ 36*a^2 - 118*a*b + 82*b^2)*sinh(f*x + e)^6 + 8*(99*(a*b - b^2)*cosh(f*x +
 e)^7 + 63*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^5 + 7*(112*a^2 - 335*a*b
 + 223*b^2)*cosh(f*x + e)^3 + 6*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e))*s
inh(f*x + e)^5 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^4 + (495*(a*b
- b^2)*cosh(f*x + e)^8 + 420*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^6 + 70
*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^4 + 120*(18*a^2 - 59*a*b + 41*
b^2)*cosh(f*x + e)^2 + 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^4 + 4*(55
*(a*b - b^2)*cosh(f*x + e)^9 + 60*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^7
 + 14*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^5 + 40*(18*a^2 - 59*a*b +
 41*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e))*sin
h(f*x + e)^3 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2 + 2*(33*(a*b - b
^2)*cosh(f*x + e)^10 + 45*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^8 + 14*(1
12*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^6 + 60*(18*a^2 - 59*a*b + 41*b^2)
*cosh(f*x + e)^4 + 3*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2 + 8*a^2
- 25*a*b + 17*b^2)*sinh(f*x + e)^2 + a*b - b^2 + 4*(3*(a*b - b^2)*cosh(f*x
+ e)^11 + 5*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^9 + 2*(112*a^2 - 335*a*
b + 223*b^2)*cosh(f*x + e)^7 + 12*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^

5 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x + e)^11 + 11*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^10 + (a - b)*f*sinh(f*x + e)^11 + 4*(a - b)*f*cosh(f*x + e)^9 + (55*(a - b)*f*cosh(f*x + e)^2 + 4*(a - b)*f)*sinh(f*x + e)^9 + 6*(a - b)*f*cosh(f*x + e)^7 + 3*(55*(a - b)*f*cosh(f*x + e)^3 + 12*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(55*(a - b)*f*cosh(f*x + e)^4 + 24*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^7 + 4*(a - b)*f*cosh(f*x + e)^5 + 42*(11*(a - b)*f*cosh(f*x + e)^5 + 8*(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(231*(a - b)*f*cosh(f*x + e)^6 + 252*(a - b)*f*cosh(f*x + e)^4 + 63*(a - b)*f*cosh(f*x + e)^2 + 2*(a - b)*f)*sinh(f*x + e)^5 + (a - b)*f*cosh(f*x + e)^3 + 2*(165*(a - b)*f*cosh(f*x + e)^7 + 252*(a - b)*f*cosh(f*x + e)^5 + 105*(a - b)*f*cosh(f*x + e)^3 + 10*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^4 + (165*(a - b)*f*cosh(f*x + e)^8 + 336*(a - b)*f*cosh(f*x + e)^6 + 210*(a - b)*f*cosh(f*x + e)^4 + 40*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)^3 + (55*(a - b)*f*cosh(f*x + e)^9 + 144*(a - b)*f*cosh(f*x + e)^7 + 126*(a - b)*f*cosh(f*x + e)^5 + 40*(a - b)*f*cosh(f*x + e)^3 + 3*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(a - b)*f*cosh(f*x + e)^10 + 36*(a - b)*f*cosh(f*x + e)^8 + 42*(a - b)*f*cosh(f*x + e)^6 + 20*(a - b)*f*cosh(f*x + e)^4 + 3*(a - b)*f*cosh(f*x + e)^2)*sinh(f*x + e)), -1/24*(3*((8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^11 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 32*a^2 - 160*a*b + 140*b^2)*sinh(f*x + e)^9 + 3*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 12*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 + 6*(55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 24*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^7 + 42*(11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 8*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 4*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 + 252*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 63*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 16*a^2 - 80*a*b + 70*b^2)*sinh(f*x + e)^5 + 2*(165*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 + 252*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 105*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 10*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + (8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + (165*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^8 + 336*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 + 210*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 40*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2 + 8*a^2 - 40*a*b + 35*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^9 + 144*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^7 + 126*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^5 + 40*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^10 + 36*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^8 + 42*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^6 + 20*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 40*a*b + 35*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*((a*b - b^2)*cosh(f*x + e)^12 + 12*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^11 + (a*b - b^2)*sinh(f*x + e)^12 + 2*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^10 + 2*(33*(a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x + e)^10 + 20*(11*(a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x + e)^9 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^8 + (495*(a*b - b^2)*cosh(f*x + e)^4 + 90*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^2 + 112*a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^8 + 8*(99*(a*b - b^2)*cosh(f*x + e)^5 + 30*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^3 + (112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e))*

```
sinh(f*x + e)^7 + 8*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^6 + 4*(231*(a*
b - b^2)*cosh(f*x + e)^6 + 105*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^4 +
7*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2 + 36*a^2 - 118*a*b + 82*b^2
)*sinh(f*x + e)^6 + 8*(99*(a*b - b^2)*cosh(f*x + e)^7 + 63*(8*a^2 - 25*a*b
+ 17*b^2)*cosh(f*x + e)^5 + 7*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^3
 + 6*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + (112*a^2 -
 335*a*b + 223*b^2)*cosh(f*x + e)^4 + (495*(a*b - b^2)*cosh(f*x + e)^8 + 42
0*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^6 + 70*(112*a^2 - 335*a*b + 223*b
^2)*cosh(f*x + e)^4 + 120*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^2 + 112*
a^2 - 335*a*b + 223*b^2)*sinh(f*x + e)^4 + 4*(55*(a*b - b^2)*cosh(f*x + e)^
9 + 60*(8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^7 + 14*(112*a^2 - 335*a*b +
223*b^2)*cosh(f*x + e)^5 + 40*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^3 +
(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(8*a^2 - 2
5*a*b + 17*b^2)*cosh(f*x + e)^2 + 2*(33*(a*b - b^2)*cosh(f*x + e)^10 + 45*(
8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e)^8 + 14*(112*a^2 - 335*a*b + 223*b^2)
*cosh(f*x + e)^6 + 60*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^4 + 3*(112*a
^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^2 + 8*a^2 - 25*a*b + 17*b^2)*sinh(f*x
 + e)^2 + a*b - b^2 + 4*(3*(a*b - b^2)*cosh(f*x + e)^11 + 5*(8*a^2 - 25*a*b
 + 17*b^2)*cosh(f*x + e)^9 + 2*(112*a^2 - 335*a*b + 223*b^2)*cosh(f*x + e)^
7 + 12*(18*a^2 - 59*a*b + 41*b^2)*cosh(f*x + e)^5 + (112*a^2 - 335*a*b + 22
3*b^2)*cosh(f*x + e)^3 + (8*a^2 - 25*a*b + 17*b^2)*cosh(f*x + e))*sinh(f*x
+ e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a - b)*f*cosh(f*x
 + e)^11 + 11*(a - b)*f*cosh(f*x + e)*sinh(f*x + e)^10 + (a - b)*f*sinh(f*x
 + e)^11 + 4*(a - b)*f*cosh(f*x + e)^9 + (55*(a - b)*f*cosh(f*x + e)^2 + 4*
(a - b)*f)*sinh(f*x + e)^9 + 6*(a - b)*f*cosh(f*x + e)^7 + 3*(55*(a - b)*f*
cosh(f*x + e)^3 + 12*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(55*(a -
b)*f*cosh(f*x + e)^4 + 24*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x +
 e)^7 + 4*(a - b)*f*cosh(f*x + e)^5 + 42*(11*(a - b)*f*cosh(f*x + e)^5 + 8*
(a - b)*f*cosh(f*x + e)^3 + (a - b)*f*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(2
31*(a - b)*f*cosh(f*x + e)^6 + 252*(a - b)*f*cosh(f*x + e)^4 + 63*(a - b)*f
*cosh(f*x + e)^2 + 2*(a - b)*f)*sinh(f*x + e)^5 + (a - b)*f*cosh(f*x + e)^3
 + 2*(165*(a - b)*f*cosh(f*x + e)^7 + 252*(a - b)*f*cosh(f*x + e)^5 + 105*(
a - b)*f*cosh(f*x + e)^3 + 10*(a - b)*f*cosh(f*x + e))*sinh(f*x + e)^4 + (1
65*(a - b)*f*cosh(f*x + e)^8 + 336*(a - b)*f*cosh(f*x + e)^6 + 210*(a - b)*
f*cosh(f*x + e)^4 + 40*(a - b)*f*cosh(f*x + e)^2 + (a - b)*f)*sinh(f*x + e)
^3 + (55*(a - b)*f*cosh(f*x + e)^9 + 144*(a - b)*f*cosh(f*x + e)^7 + 126*(a
 - b)*f*cosh(f*x + e)^5 + 40*(a - b)*f*cosh(f*x + e)^3 + 3*(a - b)*f*cosh(f
*x + e))*sinh(f*x + e)^2 + (11*(a - b)*f*cosh(f*x + e)^10 + 36*(a - b)*f*co
sh(f*x + e)^8 + 42*(a - b)*f*cosh(f*x + e)^6 + 20*(a - b)*f*cosh(f*x + e)^4
 + 3*(a - b)*f*cosh(f*x + e)^2)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**5,x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(fx+e\right)^2 + a\right)^{\frac{3}{2}} \tanh\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^5,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^5, x)
```

3.469 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx$

Optimal. Leaf size=156

$$\frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6f(a-b)}+\frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{(2a-5b)\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}+\text{se}$$

```
[Out] -((2*a - 5*b)*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])
/(2*f) + ((2*a - 5*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a - 5*b)*(a
+ b*Sinh[e + f*x]^2)^(3/2))/(6*(a - b)*f) + (Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2)^(5/2))/(2*(a - b)*f)
```

Rubi [A] time = 0.158917, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6f(a-b)}+\frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{(2a-5b)\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f}+\text{se}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```


```
[Out] -((2*a - 5*b)*Sqrt[a - b]*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])
/(2*f) + ((2*a - 5*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a - 5*b)*(a
+ b*Sinh[e + f*x]^2)^(3/2))/(6*(a - b)*f) + (Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2)^(5/2))/(2*(a - b)*f)
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^{3/2}}{(1+x)^2}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2(a-b)f} + \frac{(2a-5b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}\,dx, x\right.}{4(a-b)f}$$

$$= \frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6(a-b)f} + \frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5}}{2(a-b)f}$$

$$= \frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6(a-b)f} + \text{sec}$$

$$= \frac{(2a-5b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a-5b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6(a-b)f} + \text{sec}$$

$$= -\frac{(2a-5b)\sqrt{a-b}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f} + \frac{(2a-5b)\sqrt{a+b\sinh^2(e+}}{2f}$$

Mathematica [A] time = 0.54235, size = 122, normalized size = 0.78

$$\frac{(2a-5b)\left(\sqrt{a+b\sinh^2(e+fx)}\left(4a+b\sinh^2(e+fx)-3b\right)-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)\right)+3\text{sech}^2(e+fx)\left(a\right.}{6f(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3,x]
```

```
[Out] (3*Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2) + (2*a - 5*b)*(-3*(a - b)^
(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]] + Sqrt[a + b*Sinh[e
+ f*x]^2]*(4*a - 3*b + b*Sinh[e + f*x]^2)))/(6*(a - b)*f)
```

Maple [C] time = 0.125, size = 71, normalized size = 0.5

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^3\left(b^2\left(\sinh\left(fx+e\right)\right)^4+2ab\left(\sinh\left(fx+e\right)\right)^2+a^2\right)}{\left(\cosh\left(fx+e\right)\right)^4}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh
(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tanh \left(fx+e\right)^{3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^3, x)
```

Fricas [B] time = 7.38749, size = 6527, normalized size = 41.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [-1/24*(6*((2*a - 5*b)*cosh(f*x + e)^7 + 7*(2*a - 5*b)*cosh(f*x + e)*sinh(f
*x + e)^6 + (2*a - 5*b)*sinh(f*x + e)^7 + 2*(2*a - 5*b)*cosh(f*x + e)^5 + (
21*(2*a - 5*b)*cosh(f*x + e)^2 + 4*a - 10*b)*sinh(f*x + e)^5 + 5*(7*(2*a -
5*b)*cosh(f*x + e)^3 + 2*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a
- 5*b)*cosh(f*x + e)^3 + (35*(2*a - 5*b)*cosh(f*x + e)^4 + 20*(2*a - 5*b)*c
osh(f*x + e)^2 + 2*a - 5*b)*sinh(f*x + e)^3 + (21*(2*a - 5*b)*cosh(f*x + e)
^5 + 20*(2*a - 5*b)*cosh(f*x + e)^3 + 3*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x
 + e)^2 + (7*(2*a - 5*b)*cosh(f*x + e)^6 + 10*(2*a - 5*b)*cosh(f*x + e)^4 +
 3*(2*a - 5*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x
+ e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3
*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 +
 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a -
b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(co
sh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x
+ e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3
 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x
 + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - sqrt(2)
*(b*cosh(f*x + e)^8 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8
 + 8*(2*a - 3*b)*cosh(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a - 6*b)*sinh
(f*x + e)^6 + 8*(7*b*cosh(f*x + e)^3 + 6*(2*a - 3*b)*cosh(f*x + e))*sinh(f*
x + e)^5 + 2*(28*a - 37*b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(
2*a - 3*b)*cosh(f*x + e)^2 + 28*a - 37*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x
 + e)^5 + 20*(2*a - 3*b)*cosh(f*x + e)^3 + (28*a - 37*b)*cosh(f*x + e))*sin
h(f*x + e)^3 + 8*(2*a - 3*b)*cosh(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*
(2*a - 3*b)*cosh(f*x + e)^4 + 3*(28*a - 37*b)*cosh(f*x + e)^2 + 4*a - 6*b)*
sinh(f*x + e)^2 + 8*(b*cosh(f*x + e)^7 + 6*(2*a - 3*b)*cosh(f*x + e)^5 + (2
8*a - 37*b)*cosh(f*x + e)^3 + 2*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) +
b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2
```

```
- 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7
*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*sinh(f*x + e)^7 + 2*f*cosh(f*x + e)^5
+ (21*f*cosh(f*x + e)^2 + 2*f)*sinh(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 2
*f*cosh(f*x + e))*sinh(f*x + e)^4 + f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)
^4 + 20*f*cosh(f*x + e)^2 + f)*sinh(f*x + e)^3 + (21*f*cosh(f*x + e)^5 + 20
*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e
)^6 + 10*f*cosh(f*x + e)^4 + 3*f*cosh(f*x + e)^2)*sinh(f*x + e)), -1/24*(12
*((2*a - 5*b)*cosh(f*x + e)^7 + 7*(2*a - 5*b)*cosh(f*x + e)*sinh(f*x + e)^6
 + (2*a - 5*b)*sinh(f*x + e)^7 + 2*(2*a - 5*b)*cosh(f*x + e)^5 + (21*(2*a -
 5*b)*cosh(f*x + e)^2 + 4*a - 10*b)*sinh(f*x + e)^5 + 5*(7*(2*a - 5*b)*cosh
(f*x + e)^3 + 2*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a - 5*b)*co
sh(f*x + e)^3 + (35*(2*a - 5*b)*cosh(f*x + e)^4 + 20*(2*a - 5*b)*cosh(f*x +
 e)^2 + 2*a - 5*b)*sinh(f*x + e)^3 + (21*(2*a - 5*b)*cosh(f*x + e)^5 + 20*(
2*a - 5*b)*cosh(f*x + e)^3 + 3*(2*a - 5*b)*cosh(f*x + e))*sinh(f*x + e)^2 +
 (7*(2*a - 5*b)*cosh(f*x + e)^6 + 10*(2*a - 5*b)*cosh(f*x + e)^4 + 3*(2*a -
 5*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt
(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x
 + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*(b*cosh(f*x + e)^8 + 8*b*cosh(f*x
 + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a - 3*b)*cosh(f*x + e)^6 +
 4*(7*b*cosh(f*x + e)^2 + 4*a - 6*b)*sinh(f*x + e)^6 + 8*(7*b*cosh(f*x + e)
^3 + 6*(2*a - 3*b)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(28*a - 37*b)*cosh(f*
x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a - 3*b)*cosh(f*x + e)^2 + 28*a
- 37*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a - 3*b)*cosh(f*x
+ e)^3 + (28*a - 37*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a - 3*b)*cosh(
f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a - 3*b)*cosh(f*x + e)^4 + 3*(2
8*a - 37*b)*cosh(f*x + e)^2 + 4*a - 6*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x +
e)^7 + 6*(2*a - 3*b)*cosh(f*x + e)^5 + (28*a - 37*b)*cosh(f*x + e)^3 + 2*(2
*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 +
 f*sinh(f*x + e)^7 + 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 + 2*f)*sin
h(f*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 + 2*f*cosh(f*x + e))*sinh(f*x + e)^4
+ f*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 + 20*f*cosh(f*x + e)^2 + f)*sin
h(f*x + e)^3 + (21*f*cosh(f*x + e)^5 + 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x
+ e))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 + 10*f*cosh(f*x + e)^4 + 3*f*c
osh(f*x + e)^2)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^3, x)
```

3.470 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx$

Optimal. Leaf size=90

$$\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
 - b)*Sqrt[a + b*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)
```

Rubi [A] time = 0.090232, antiderivative size = 90, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}-\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]
```

```
[Out] -(((a - b)^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/f) + ((a
 - b)*Sqrt[a + b*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1+x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1+x}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)^2\,\text{Sub}}{}$$

$$= \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{(a-b)^2\,\text{Sub}}{}$$

$$= -\frac{(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f} + \frac{(a-b)\sqrt{a+b\sinh^2(e+fx)}}{f} + $$

Mathematica [A] time = 0.146198, size = 86, normalized size = 0.96

$$\frac{\left(4a+b\cosh^2(e+fx)-4b\right)\sqrt{a+b\cosh^2(e+fx)-b}-3(a-b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(e+fx)-b}}{\sqrt{a-b}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x],x]
```

```
[Out] (-3*(a - b)^(3/2)*ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]] + (4
*a - 4*b + b*Cosh[e + f*x]^2)*Sqrt[a - b + b*Cosh[e + f*x]^2])/(3*f)
```

Maple [C] time = 0.296, size = 69, normalized size = 0.8

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\sinh\left(fx+e\right)\left(b^2\left(\sinh\left(fx+e\right)\right)^4+2ab\left(\sinh\left(fx+e\right)\right)^2+a^2\right)}{\left(\cosh\left(fx+e\right)\right)^2}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/cosh(f
*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="maxima")

[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e), x)
```

Fricas [B] time = 6.38133, size = 2827, normalized size = 31.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="fricas")

[Out] [-1/24*(12*((a - b)*cosh(f*x + e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x +
e) + 3*(a - b)*cosh(f*x + e)*sinh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqr
t(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sin
h(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a
 - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^
3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh
(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sin
h(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh
(f*x + e) + 1)) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
 e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x
 + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - 7*b)*c
osh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^
2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f
*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3), -1/24*(24*((a - b)*cosh(f*x +
 e)^3 + 3*(a - b)*cosh(f*x + e)^2*sinh(f*x + e) + 3*(a - b)*cosh(f*x + e)*s
inh(f*x + e)^2 + (a - b)*sinh(f*x + e)^3)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*
sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f
*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh
(f*x + e) + (a - b)*sinh(f*x + e))) - sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh
(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - 7*b)*cosh(f*x + e)
^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - 7*b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x +
e)^3 + (8*a - 7*b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^
2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*
x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f
*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e),x)

[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e), x)
```

3.471 $\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b
*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)
```

Rubi [A] time = 0.0924509, antiderivative size = 78, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b
*Sinh[e + f*x]^2])/f + (a + b*Sinh[e + f*x]^2)^(3/2)/(3*f)
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+b}}\right.}{}$$
$$= \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$
$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{\left(a+b\sinh\right.}{3}$$

Mathematica [A] time = 0.122469, size = 69, normalized size = 0.88

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(4a+b\sinh^2(e+fx)\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sinh[
e + f*x]^2]*(4*a + b*Sinh[e + f*x]^2))/(3*f)
```

Maple [C] time = 0.078, size = 62, normalized size = 0.8

$$\frac{1}{f}\text{` int/indef0`}\left(\left(b^2\left(\sinh\left(fx+e\right)\right)^3+2ab\sinh\left(fx+e\right)+\frac{a^2}{\sinh\left(fx+e\right)}\right)\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(fx+\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^3+2*a*b*sinh(f*x+e)+a^2/sinh(f*x+e))/(a+b*sin
h(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e), x)
```

Fricas [B] time = 3.79683, size = 2696, normalized size = 34.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(12*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cosh
(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(a)*log((b*cosh(f*x + e)
^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*co
sh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(
2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
 + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e)
+ sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x
 + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(c
osh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*(b*cosh(f*x +
 e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - b)
*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 8*a - b)*sinh(f*x + e)^2 + 4*(b
*cosh(f*x + e)^3 + (8*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e
)^2*sinh(f*x + e) + 3*f*cosh(f*x + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3),
 1/24*(24*(a*cosh(f*x + e)^3 + 3*a*cosh(f*x + e)^2*sinh(f*x + e) + 3*a*cosh
(f*x + e)*sinh(f*x + e)^2 + a*sinh(f*x + e)^3)*sqrt(-a)*arctan(1/2*sqrt(2)*
sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e)
+ a*sinh(f*x + e))) + sqrt(2)*(b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f
*x + e)^3 + b*sinh(f*x + e)^4 + 2*(8*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f
*x + e)^2 + 8*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (8*a - b)*cos
h(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e)^2*sinh(f*x + e) + 3*f*cosh(f*x
 + e)*sinh(f*x + e)^2 + f*sinh(f*x + e)^3)]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.472 $\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$\frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af}+\frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\operatorname{csch}^2(e+$$

```
[Out] -(Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*f) +
 ((2*a + 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a + 3*b)*(a + b*Sinh
[e + f*x]^2)^(3/2))/(6*a*f) - (Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2
))/(2*a*f)
```

Rubi [A] time = 0.147753, antiderivative size = 140, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af}+\frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f}-\frac{\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{\operatorname{csch}^2(e+$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*f) +
 ((2*a + 3*b)*Sqrt[a + b*Sinh[e + f*x]^2])/(2*f) + ((2*a + 3*b)*(a + b*Sinh
[e + f*x]^2)^(3/2))/(6*a*f) - (Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2
))/(2*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(1+x)(a+bx)^{3/2}}{x^2}dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{2af} + \frac{(2a+3b)\,\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\right.}{4af}$$

$$= \frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af} - \frac{\operatorname{csch}^2(e+fx)\left(a+b\sinh^2(e+f\right.}{2af}$$

$$= \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af} -$$

$$= \frac{(2a+3b)\sqrt{a+b\sinh^2(e+fx)}}{2f} + \frac{(2a+3b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{6af} -$$

$$= -\frac{\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2f} + \frac{(2a+3b)\sqrt{a+b\sinh^2(e+\right.}{2f}$$

Mathematica [A] time = 0.44647, size = 90, normalized size = 0.64

$$\frac{\sqrt{a+b\sinh^2(e+fx)}\left(-3a\operatorname{csch}^2(e+fx)+8a+b\cosh(2(e+fx))+5b\right)-3\sqrt{a}(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*Sqrt[a]*(2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + (8*a
 + 5*b + b*Cosh[2*(e + f*x)] - 3*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x
]^2])/(6*f)
```

Maple [C] time = 0.124, size = 84, normalized size = 0.6

$$\frac{1}{f}\text{` int/indef0`}\left(\left(b^2\left(\sinh(fx+e)\right)^3+\left(2ab+b^2\right)\sinh(fx+e)+\frac{a^2+2ab}{\sinh(fx+e)}+\frac{a^2}{\left(\sinh(fx+e)\right)^3}\right)\frac{}{\sqrt{a+b\left(\mathrm{si}\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`((b^2*sinh(f*x+e)^3+(2*a*b+b^2)*sinh(f*x+e)+(a^2+2*a*b)/sinh(f*
x+e)+a^2/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \coth \left(fx+e\right)^{3} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^3, x)
```

Fricas [B] time = 5.47209, size = 6418, normalized size = 45.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/24*(6*((2*a + 3*b)*cosh(f*x + e)^7 + 7*(2*a + 3*b)*cosh(f*x + e)*sinh(f*
x + e)^6 + (2*a + 3*b)*sinh(f*x + e)^7 - 2*(2*a + 3*b)*cosh(f*x + e)^5 + (2
1*(2*a + 3*b)*cosh(f*x + e)^2 - 4*a - 6*b)*sinh(f*x + e)^5 + 5*(7*(2*a + 3*
b)*cosh(f*x + e)^3 - 2*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x + e)^4 + (2*a +
3*b)*cosh(f*x + e)^3 + (35*(2*a + 3*b)*cosh(f*x + e)^4 - 20*(2*a + 3*b)*cos
h(f*x + e)^2 + 2*a + 3*b)*sinh(f*x + e)^3 + (21*(2*a + 3*b)*cosh(f*x + e)^5
 - 20*(2*a + 3*b)*cosh(f*x + e)^3 + 3*(2*a + 3*b)*cosh(f*x + e))*sinh(f*x +
 e)^2 + (7*(2*a + 3*b)*cosh(f*x + e)^6 - 10*(2*a + 3*b)*cosh(f*x + e)^4 + 3
*(2*a + 3*b)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4
 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh
(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)
*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x +
 e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) +
sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x +
 e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)
^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cos
h(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + sqrt(2)*(b*cosh(f*x + e
)^8 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a + b)*c
osh(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^6 + 8*(7
*b*cosh(f*x + e)^3 + 6*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(28*a +
 9*b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a + b)*cosh(f*x + e
)^2 - 28*a - 9*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a + b)*c
osh(f*x + e)^3 - (28*a + 9*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a + b)*
cosh(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a + b)*cosh(f*x + e)^4 - 3
*(28*a + 9*b)*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x
+ e)^7 + 6*(2*a + b)*cosh(f*x + e)^5 - (28*a + 9*b)*cosh(f*x + e)^3 + 2*(2*
a + b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f
*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sin
```

```
h(f*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*
sinh(f*x + e)^7 - 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 - 2*f)*sinh(f
*x + e)^5 + 5*(7*f*cosh(f*x + e)^3 - 2*f*cosh(f*x + e))*sinh(f*x + e)^4 + f
*cosh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*sinh(f
*x + e)^3 + (21*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e
))*sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 - 10*f*cosh(f*x + e)^4 + 3*f*cosh
(f*x + e)^2)*sinh(f*x + e)), 1/24*(12*((2*a + 3*b)*cosh(f*x + e)^7 + 7*(2*a
 + 3*b)*cosh(f*x + e)*sinh(f*x + e)^6 + (2*a + 3*b)*sinh(f*x + e)^7 - 2*(2*
a + 3*b)*cosh(f*x + e)^5 + (21*(2*a + 3*b)*cosh(f*x + e)^2 - 4*a - 6*b)*sin
h(f*x + e)^5 + 5*(7*(2*a + 3*b)*cosh(f*x + e)^3 - 2*(2*a + 3*b)*cosh(f*x +
e))*sinh(f*x + e)^4 + (2*a + 3*b)*cosh(f*x + e)^3 + (35*(2*a + 3*b)*cosh(f*
x + e)^4 - 20*(2*a + 3*b)*cosh(f*x + e)^2 + 2*a + 3*b)*sinh(f*x + e)^3 + (2
1*(2*a + 3*b)*cosh(f*x + e)^5 - 20*(2*a + 3*b)*cosh(f*x + e)^3 + 3*(2*a + 3
*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (7*(2*a + 3*b)*cosh(f*x + e)^6 - 10*(2
*a + 3*b)*cosh(f*x + e)^4 + 3*(2*a + 3*b)*cosh(f*x + e)^2)*sinh(f*x + e))*s
qrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + sqrt(2)*(b*cosh(f*x + e)^8
 + 8*b*cosh(f*x + e)*sinh(f*x + e)^7 + b*sinh(f*x + e)^8 + 8*(2*a + b)*cosh
(f*x + e)^6 + 4*(7*b*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^6 + 8*(7*b*
cosh(f*x + e)^3 + 6*(2*a + b)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(28*a + 9*
b)*cosh(f*x + e)^4 + 2*(35*b*cosh(f*x + e)^4 + 60*(2*a + b)*cosh(f*x + e)^2
 - 28*a - 9*b)*sinh(f*x + e)^4 + 8*(7*b*cosh(f*x + e)^5 + 20*(2*a + b)*cosh
(f*x + e)^3 - (28*a + 9*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*(2*a + b)*cos
h(f*x + e)^2 + 4*(7*b*cosh(f*x + e)^6 + 30*(2*a + b)*cosh(f*x + e)^4 - 3*(2
8*a + 9*b)*cosh(f*x + e)^2 + 4*a + 2*b)*sinh(f*x + e)^2 + 8*(b*cosh(f*x + e
)^7 + 6*(2*a + b)*cosh(f*x + e)^5 - (28*a + 9*b)*cosh(f*x + e)^3 + 2*(2*a +
 b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
+ e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f
*x + e)^2)))/(f*cosh(f*x + e)^7 + 7*f*cosh(f*x + e)*sinh(f*x + e)^6 + f*sin
h(f*x + e)^7 - 2*f*cosh(f*x + e)^5 + (21*f*cosh(f*x + e)^2 - 2*f)*sinh(f*x
+ e)^5 + 5*(7*f*cosh(f*x + e)^3 - 2*f*cosh(f*x + e))*sinh(f*x + e)^4 + f*co
sh(f*x + e)^3 + (35*f*cosh(f*x + e)^4 - 20*f*cosh(f*x + e)^2 + f)*sinh(f*x
+ e)^3 + (21*f*cosh(f*x + e)^5 - 20*f*cosh(f*x + e)^3 + 3*f*cosh(f*x + e))*
sinh(f*x + e)^2 + (7*f*cosh(f*x + e)^6 - 10*f*cosh(f*x + e)^4 + 3*f*cosh(f*
x + e)^2)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.473 $\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=203

$$\frac{\left(8a^2+3b(8a+b)\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+b\sinh^2(e+fx)}}{8af}-\frac{\left(8a^2+3b(8a+b)\right)\tanh^{-1}\left(\cdots\right.}{8\sqrt{a}f}$$

```
[Out] -((8*a^2 + 3*b*(8*a + b))*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((8*a^2 + 3*b*(8*a + b))*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a*f)
+ ((8*a^2 + 3*b*(8*a + b))*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*a^2*f) - ((8*
a + b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csch[e +
 f*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4*a*f)
```

Rubi [A] time = 0.23248, antiderivative size = 199, normalized size of antiderivative = 0.98, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{\left(\frac{3b(8a+b)}{a^2}+8\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24f}+\frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+b\sinh^2(e+fx)}}{8af}-\frac{\left(8a^2+3b(8a+b)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\cdots}}{\cdots}\right.}{8\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 + 3*b*(8*a + b))*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((8*a^2 + 3*b*(8*a + b))*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a*f)
+ ((8 + (3*b*(8*a + b))/a^2)*(a + b*Sinh[e + f*x]^2)^(3/2))/(24*f) - ((8*a
+ b)*Csch[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csch[e + f
*x]^4*(a + b*Sinh[e + f*x]^2)^(5/2))/(4*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \coth^5(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2(a+bx)^{3/2}}{x^3}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(8a+b)+2ax\right)(a+bx)^{3/2}}{x^2}\right.}{4af}$$

$$= -\frac{(8a+b)\text{csch}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{5/2}}{8a^2f} - \frac{\text{csch}^4(e+fx)\left(a+b\,\text{si}\right.}{4af}$$

$$= \frac{\left(8a^2+3b(8a+b)\right)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{24a^2f} - \frac{(8a+b)\text{csch}^2(e+fx)\left(a+\right.}{8a^2f}$$

$$= \frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{\left(8a^2+3b(8a+b)\right)\left(a+b\,\text{sinl}\right.}{24a^2f}$$

$$= \frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+b\sinh^2(e+fx)}}{8af} + \frac{\left(8a^2+3b(8a+b)\right)\left(a+b\,\text{sinl}\right.}{24a^2f}$$

$$= -\frac{\left(8a^2+3b(8a+b)\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f} + \frac{\left(8a^2+3b(8a+b)\right)\sqrt{a+}}{8af}$$

Mathematica [A] time = 0.789817, size = 123, normalized size = 0.61

$$\frac{\sqrt{a}\sqrt{a+b\sinh^2(e+fx)}\left(8\left(4a+b\sinh^2(e+fx)+6b\right)-3(8a+5b)\text{csch}^2(e+fx)-6a\text{csch}^4(e+fx)\right)-3\left(8a^2+24a\right.}{24\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] +
 Sqrt[a]*Sqrt[a + b*Sinh[e + f*x]^2]*(-3*(8*a + 5*b)*Csch[e + f*x]^2 - 6*a*
Csch[e + f*x]^4 + 8*(4*a + 6*b + b*Sinh[e + f*x]^2)))/(24*Sqrt[a]*f)
```

Maple [C] time = 0.254, size = 113, normalized size = 0.6

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\cosh\left(fx+e\right)\right)^4\left(b^2\left(\cosh\left(fx+e\right)\right)^4+2\,ab\left(\cosh\left(fx+e\right)\right)^2-2\,b^2\left(\cosh\left(fx+e\right)\right)^2+a^2-2\,a\right.}{\sinh\left(fx+e\right)\left(\left(\cosh\left(fx+e\right)\right)^4-2\left(\cosh\left(fx+e\right)\right)^2+1\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(cosh(f*x+e)^4-2*cosh(f*x+e)^2+1)*cosh(f*x+e)^4*
(b^2*cosh(f*x+e)^4+2*a*b*cosh(f*x+e)^2-2*b^2*cosh(f*x+e)^2+a^2-2*a*b+b^2)/(
a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^5, x)
```

Fricas [B] time = 6.13721, size = 14438, normalized size = 71.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 + 24*a*b +
3*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 + 24*a*b + 3*b^2)*sinh(f*x +
 e)^11 - 4*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 + 24*a*b +
```

3*b^2)*cosh(f*x + e)^2 - 32*a^2 - 96*a*b - 12*b^2)*sinh(f*x + e)^9 + 3*(55
*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 12*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 +
6*(55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 - 24*(8*a^2 + 24*a*b + 3*b^2
)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3*b^2)*sinh(f*x + e)^7 + 42*(11*(8*a^2
 + 24*a*b + 3*b^2)*cosh(f*x + e)^5 - 8*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^3 + (8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 - 4*(8*a^2 +
 24*a*b + 3*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x
 + e)^6 - 252*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 + 63*(8*a^2 + 24*a*b
 + 3*b^2)*cosh(f*x + e)^2 - 16*a^2 - 48*a*b - 6*b^2)*sinh(f*x + e)^5 + 2*(1
65*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 - 252*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^5 + 105*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 10*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + (8*a^2 + 24*a*b + 3*b^2)
*cosh(f*x + e)^3 + (165*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 - 336*(8*a
^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^6 + 210*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*
x + e)^4 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3
*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 - 144*
(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 + 126*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e)^5 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 + 24*
a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^10 - 36*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 + 42*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e)^6 - 20*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^4 + 3*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(a)*l
og((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)
^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f
*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^
2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh
(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x +
 e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cos
h(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 2*
sqrt(2)*(a*b*cosh(f*x + e)^12 + 12*a*b*cosh(f*x + e)*sinh(f*x + e)^11 + a*b
*sinh(f*x + e)^12 + 2*(8*a^2 + 9*a*b)*cosh(f*x + e)^10 + 2*(33*a*b*cosh(f*x
 + e)^2 + 8*a^2 + 9*a*b)*sinh(f*x + e)^10 + 20*(11*a*b*cosh(f*x + e)^3 + (8
*a^2 + 9*a*b)*cosh(f*x + e))*sinh(f*x + e)^9 - (112*a^2 + 111*a*b)*cosh(f*x
 + e)^8 + (495*a*b*cosh(f*x + e)^4 + 90*(8*a^2 + 9*a*b)*cosh(f*x + e)^2 - 1
12*a^2 - 111*a*b)*sinh(f*x + e)^8 + 8*(99*a*b*cosh(f*x + e)^5 + 30*(8*a^2 +
 9*a*b)*cosh(f*x + e)^3 - (112*a^2 + 111*a*b)*cosh(f*x + e))*sinh(f*x + e)^
7 + 8*(18*a^2 + 23*a*b)*cosh(f*x + e)^6 + 4*(231*a*b*cosh(f*x + e)^6 + 105*
(8*a^2 + 9*a*b)*cosh(f*x + e)^4 - 7*(112*a^2 + 111*a*b)*cosh(f*x + e)^2 + 3
6*a^2 + 46*a*b)*sinh(f*x + e)^6 + 8*(99*a*b*cosh(f*x + e)^7 + 63*(8*a^2 + 9
*a*b)*cosh(f*x + e)^5 - 7*(112*a^2 + 111*a*b)*cosh(f*x + e)^3 + 6*(18*a^2 +
 23*a*b)*cosh(f*x + e))*sinh(f*x + e)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)
^4 + (495*a*b*cosh(f*x + e)^8 + 420*(8*a^2 + 9*a*b)*cosh(f*x + e)^6 - 70*(1
12*a^2 + 111*a*b)*cosh(f*x + e)^4 + 120*(18*a^2 + 23*a*b)*cosh(f*x + e)^2 -
 112*a^2 - 111*a*b)*sinh(f*x + e)^4 + 4*(55*a*b*cosh(f*x + e)^9 + 60*(8*a^2
 + 9*a*b)*cosh(f*x + e)^7 - 14*(112*a^2 + 111*a*b)*cosh(f*x + e)^5 + 40*(18
*a^2 + 23*a*b)*cosh(f*x + e)^3 - (112*a^2 + 111*a*b)*cosh(f*x + e))*sinh(f*
x + e)^3 + 2*(8*a^2 + 9*a*b)*cosh(f*x + e)^2 + 2*(33*a*b*cosh(f*x + e)^10 +
 45*(8*a^2 + 9*a*b)*cosh(f*x + e)^8 - 14*(112*a^2 + 111*a*b)*cosh(f*x + e)^
6 + 60*(18*a^2 + 23*a*b)*cosh(f*x + e)^4 - 3*(112*a^2 + 111*a*b)*cosh(f*x +
 e)^2 + 8*a^2 + 9*a*b)*sinh(f*x + e)^2 + a*b + 4*(3*a*b*cosh(f*x + e)^11 +
5*(8*a^2 + 9*a*b)*cosh(f*x + e)^9 - 2*(112*a^2 + 111*a*b)*cosh(f*x + e)^7 +
 12*(18*a^2 + 23*a*b)*cosh(f*x + e)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)^3
 + (8*a^2 + 9*a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 +
b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x +
e) + sinh(f*x + e)^2)))/(a*f*cosh(f*x + e)^11 + 11*a*f*cosh(f*x + e)*sinh(f
*x + e)^10 + a*f*sinh(f*x + e)^11 - 4*a*f*cosh(f*x + e)^9 + (55*a*f*cosh(f*
x + e)^2 - 4*a*f)*sinh(f*x + e)^9 + 6*a*f*cosh(f*x + e)^7 + 3*(55*a*f*cosh(

```
f*x + e)^3 - 12*a*f*cosh(f*x + e))*sinh(f*x + e)^8 + 6*(55*a*f*cosh(f*x + e
)^4 - 24*a*f*cosh(f*x + e)^2 + a*f)*sinh(f*x + e)^7 - 4*a*f*cosh(f*x + e)^5
 + 42*(11*a*f*cosh(f*x + e)^5 - 8*a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*
sinh(f*x + e)^6 + 2*(231*a*f*cosh(f*x + e)^6 - 252*a*f*cosh(f*x + e)^4 + 63
*a*f*cosh(f*x + e)^2 - 2*a*f)*sinh(f*x + e)^5 + a*f*cosh(f*x + e)^3 + 2*(16
5*a*f*cosh(f*x + e)^7 - 252*a*f*cosh(f*x + e)^5 + 105*a*f*cosh(f*x + e)^3 -
 10*a*f*cosh(f*x + e))*sinh(f*x + e)^4 + (165*a*f*cosh(f*x + e)^8 - 336*a*f
*cosh(f*x + e)^6 + 210*a*f*cosh(f*x + e)^4 - 40*a*f*cosh(f*x + e)^2 + a*f)*
sinh(f*x + e)^3 + (55*a*f*cosh(f*x + e)^9 - 144*a*f*cosh(f*x + e)^7 + 126*a
*f*cosh(f*x + e)^5 - 40*a*f*cosh(f*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x
 + e)^2 + (11*a*f*cosh(f*x + e)^10 - 36*a*f*cosh(f*x + e)^8 + 42*a*f*cosh(f
*x + e)^6 - 20*a*f*cosh(f*x + e)^4 + 3*a*f*cosh(f*x + e)^2)*sinh(f*x + e)),
 1/24*(3*((8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^11 + 11*(8*a^2 + 24*a*b +
3*b^2)*cosh(f*x + e)*sinh(f*x + e)^10 + (8*a^2 + 24*a*b + 3*b^2)*sinh(f*x +
 e)^11 - 4*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 + (55*(8*a^2 + 24*a*b +
 3*b^2)*cosh(f*x + e)^2 - 32*a^2 - 96*a*b - 12*b^2)*sinh(f*x + e)^9 + 3*(55
*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 12*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e))*sinh(f*x + e)^8 + 6*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 +
6*(55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 - 24*(8*a^2 + 24*a*b + 3*b^2
)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3*b^2)*sinh(f*x + e)^7 + 42*(11*(8*a^2
 + 24*a*b + 3*b^2)*cosh(f*x + e)^5 - 8*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^3 + (8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 - 4*(8*a^2 +
 24*a*b + 3*b^2)*cosh(f*x + e)^5 + 2*(231*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x
 + e)^6 - 252*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^4 + 63*(8*a^2 + 24*a*b
 + 3*b^2)*cosh(f*x + e)^2 - 16*a^2 - 48*a*b - 6*b^2)*sinh(f*x + e)^5 + 2*(1
65*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 - 252*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^5 + 105*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 - 10*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + (8*a^2 + 24*a*b + 3*b^2)
*cosh(f*x + e)^3 + (165*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 - 336*(8*a
^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^6 + 210*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*
x + e)^4 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 + 24*a*b + 3
*b^2)*sinh(f*x + e)^3 + (55*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^9 - 144*
(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^7 + 126*(8*a^2 + 24*a*b + 3*b^2)*cos
h(f*x + e)^5 - 40*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 + 24*
a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (11*(8*a^2 + 24*a*b + 3*b^2)*
cosh(f*x + e)^10 - 36*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^8 + 42*(8*a^2
+ 24*a*b + 3*b^2)*cosh(f*x + e)^6 - 20*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x +
e)^4 + 3*(8*a^2 + 24*a*b + 3*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt(-a)*
arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
)/(a*cosh(f*x + e) + a*sinh(f*x + e))) + sqrt(2)*(a*b*cosh(f*x + e)^12 + 12
*a*b*cosh(f*x + e)*sinh(f*x + e)^11 + a*b*sinh(f*x + e)^12 + 2*(8*a^2 + 9*a
*b)*cosh(f*x + e)^10 + 2*(33*a*b*cosh(f*x + e)^2 + 8*a^2 + 9*a*b)*sinh(f*x
+ e)^10 + 20*(11*a*b*cosh(f*x + e)^3 + (8*a^2 + 9*a*b)*cosh(f*x + e))*sinh(
f*x + e)^9 - (112*a^2 + 111*a*b)*cosh(f*x + e)^8 + (495*a*b*cosh(f*x + e)^4
 + 90*(8*a^2 + 9*a*b)*cosh(f*x + e)^2 - 112*a^2 - 111*a*b)*sinh(f*x + e)^8
+ 8*(99*a*b*cosh(f*x + e)^5 + 30*(8*a^2 + 9*a*b)*cosh(f*x + e)^3 - (112*a^2
 + 111*a*b)*cosh(f*x + e))*sinh(f*x + e)^7 + 8*(18*a^2 + 23*a*b)*cosh(f*x +
 e)^6 + 4*(231*a*b*cosh(f*x + e)^6 + 105*(8*a^2 + 9*a*b)*cosh(f*x + e)^4 -
7*(112*a^2 + 111*a*b)*cosh(f*x + e)^2 + 36*a^2 + 46*a*b)*sinh(f*x + e)^6 +
8*(99*a*b*cosh(f*x + e)^7 + 63*(8*a^2 + 9*a*b)*cosh(f*x + e)^5 - 7*(112*a^2
 + 111*a*b)*cosh(f*x + e)^3 + 6*(18*a^2 + 23*a*b)*cosh(f*x + e))*sinh(f*x +
 e)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)^4 + (495*a*b*cosh(f*x + e)^8 + 42
0*(8*a^2 + 9*a*b)*cosh(f*x + e)^6 - 70*(112*a^2 + 111*a*b)*cosh(f*x + e)^4
+ 120*(18*a^2 + 23*a*b)*cosh(f*x + e)^2 - 112*a^2 - 111*a*b)*sinh(f*x + e)^
4 + 4*(55*a*b*cosh(f*x + e)^9 + 60*(8*a^2 + 9*a*b)*cosh(f*x + e)^7 - 14*(11
2*a^2 + 111*a*b)*cosh(f*x + e)^5 + 40*(18*a^2 + 23*a*b)*cosh(f*x + e)^3 - (
112*a^2 + 111*a*b)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(8*a^2 + 9*a*b)*cosh(
f*x + e)^2 + 2*(33*a*b*cosh(f*x + e)^10 + 45*(8*a^2 + 9*a*b)*cosh(f*x + e)^
```

```
8 - 14*(112*a^2 + 111*a*b)*cosh(f*x + e)^6 + 60*(18*a^2 + 23*a*b)*cosh(f*x
+ e)^4 - 3*(112*a^2 + 111*a*b)*cosh(f*x + e)^2 + 8*a^2 + 9*a*b)*sinh(f*x +
e)^2 + a*b + 4*(3*a*b*cosh(f*x + e)^11 + 5*(8*a^2 + 9*a*b)*cosh(f*x + e)^9
- 2*(112*a^2 + 111*a*b)*cosh(f*x + e)^7 + 12*(18*a^2 + 23*a*b)*cosh(f*x + e
)^5 - (112*a^2 + 111*a*b)*cosh(f*x + e)^3 + (8*a^2 + 9*a*b)*cosh(f*x + e))*
sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a*f*cosh(
f*x + e)^11 + 11*a*f*cosh(f*x + e)*sinh(f*x + e)^10 + a*f*sinh(f*x + e)^11
- 4*a*f*cosh(f*x + e)^9 + (55*a*f*cosh(f*x + e)^2 - 4*a*f)*sinh(f*x + e)^9
+ 6*a*f*cosh(f*x + e)^7 + 3*(55*a*f*cosh(f*x + e)^3 - 12*a*f*cosh(f*x + e))
*sinh(f*x + e)^8 + 6*(55*a*f*cosh(f*x + e)^4 - 24*a*f*cosh(f*x + e)^2 + a*f
)*sinh(f*x + e)^7 - 4*a*f*cosh(f*x + e)^5 + 42*(11*a*f*cosh(f*x + e)^5 - 8*
a*f*cosh(f*x + e)^3 + a*f*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(231*a*f*cosh(
f*x + e)^6 - 252*a*f*cosh(f*x + e)^4 + 63*a*f*cosh(f*x + e)^2 - 2*a*f)*sinh
(f*x + e)^5 + a*f*cosh(f*x + e)^3 + 2*(165*a*f*cosh(f*x + e)^7 - 252*a*f*co
sh(f*x + e)^5 + 105*a*f*cosh(f*x + e)^3 - 10*a*f*cosh(f*x + e))*sinh(f*x +
e)^4 + (165*a*f*cosh(f*x + e)^8 - 336*a*f*cosh(f*x + e)^6 + 210*a*f*cosh(f*
x + e)^4 - 40*a*f*cosh(f*x + e)^2 + a*f)*sinh(f*x + e)^3 + (55*a*f*cosh(f*x
 + e)^9 - 144*a*f*cosh(f*x + e)^7 + 126*a*f*cosh(f*x + e)^5 - 40*a*f*cosh(f
*x + e)^3 + 3*a*f*cosh(f*x + e))*sinh(f*x + e)^2 + (11*a*f*cosh(f*x + e)^10
 - 36*a*f*cosh(f*x + e)^8 + 42*a*f*cosh(f*x + e)^6 - 20*a*f*cosh(f*x + e)^4
 + 3*a*f*cosh(f*x + e)^2)*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5*(a+b*sinh(f*x+e)**2)**(3/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] Exception raised: TypeError
```

3.474 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx$

Optimal. Leaf size=305

$$\frac{(3a-8b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh^3(e+fx)\left(a+b\sinh^2(e+\ldots\right.}{3f}$$

```
[Out] -((3*a - 8*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f
) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sq
rt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*
f) + ((a - 2*b)*Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/
f - ((a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3)/(3*f)
```

Rubi [A] time = 0.379704, antiderivative size = 305, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 467, 577, 582, 531, 418, 492, 411}

$$-\frac{\tanh^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-2b)\sinh^2(e+fx)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f}+\frac{8(a-2b)\tan\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

```
[Out] -((3*a - 8*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f
) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sq
rt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^
2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e +
 f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*
f) + ((a - 2*b)*Sinh[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/
f - ((a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^3)/(3*f)
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
```

```
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^4(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{5/2}}\,dx, x, \sinh(e+fx\right.}{f}$$

$$= -\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^3(e+fx)}{3f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+\right.}{}$$

$$= \frac{(a-2b)\sinh^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{f} - \frac{\left(a+b\sin\right.}{}$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(a-2b)}{}$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(a-2b)}{}$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(3a-8b}{}$$

$$= -\frac{(3a-8b)\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{8(a-2b}{}$$

Mathematica [C] time = 2.79562, size = 224, normalized size = 0.73

$$\frac{4ia(5a-8b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\left(\left(64a^2-160ab+17b^2\right)\cosh(2(e+fx))+32a^2+2b(6a-17\right.}{4\sqrt{2}}}{12f\sqrt{2a+b\cosh(2(e+fx))-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^4,x]
```

```
[Out] ((-32*I)*a*(a - 2*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
 + f*x), b/a] + (4*I)*a*(5*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]
*EllipticF[I*(e + f*x), b/a] - ((32*a^2 - 108*a*b + 18*b^2 + (64*a^2 - 160*
a*b + 17*b^2)*Cosh[2*(e + f*x)] + 2*(6*a - 17*b)*b*Cosh[4*(e + f*x)] - b^2*
Cosh[6*(e + f*x)])*Sech[e + f*x]^2*Tanh[e + f*x])/(4*Sqrt[2]))/(12*f*Sqrt[2
*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.167, size = 389, normalized size = 1.3

$$-\frac{1}{3\left(\cosh\left(fx+e\right)\right)^3 f}\left(-\sqrt{-\frac{b}{a}}b^2\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^6 + \left(3\sqrt{-\frac{b}{a}}ab - 7\sqrt{-\frac{b}{a}}b^2\right)\left(\cosh\left(fx+e\right)\right)^4\sinh\left(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x)
```

```
[Out] -1/3*(-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^6+(3*(-1/a*b)^(1/2)*a*b-7
*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4*sinh(f*x+e)+(4*(-1/a*b)^(1/2)*a^2-13*(-1
/a*b)^(1/2)*a*b+9*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+(-(-1/a*b)^
(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)-(b/a*cosh(f*
x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*a^2-16*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b+16*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+8*Elliptic
E(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-16*EllipticE(sinh(f*x+e)*(-1/
a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/(-1/a*b)^(1/2)/cosh(f*x+e)^3/(a
+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tanh\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^4, x)
```

3.475 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx$

Optimal. Leaf size=260

$$\frac{(3a-4b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)}{f}$$

```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a
 - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*
Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
 ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f) - ((a
+ b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/f
```

Rubi [A] time = 0.245338, antiderivative size = 260, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 467, 528, 531, 418, 492, 411}

$$-\frac{\tanh(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{4b\sinh(e+fx)\cosh(e+fx)\sqrt{}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2,x]
```


```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - ((7*a
 - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*
Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) +
 ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f) - ((a
+ b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x])/f
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh^2(e+fx)\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(1+x^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b\sinh^2(e+fx)\right)^{3/2}\tanh(e+fx)}{f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\left(a+b\sinh^2(e+fx\right.}{f}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\left(a+b\sinh^2(e+fx\right.}{f}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} + \frac{(3a-4b)F\left(\tan^{-1}(s\right.}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{(7a-8b)E\left(\tan^{-1}(s\right.}{}$$

Mathematica [C] time = 2.88685, size = 188, normalized size = 0.72

$$\frac{32ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}\tanh(e+fx)\left(-24a^2-4b(2a-3b)\cosh(2(e+fx))+40ab\right.}{24f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2)*Tanh[e + f*x]^2,x]
```

```
[Out] ((-8*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(
e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*E
llipticF[I*(e + f*x), b/a] + Sqrt[2]*(-24*a^2 + 40*a*b - 13*b^2 - 4*(2*a -
3*b)*b*Cosh[2*(e + f*x)] + b^2*Cosh[4*(e + f*x)])*Tanh[e + f*x])/(24*f*Sqrt
[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.148, size = 413, normalized size = 1.6

$$\frac{1}{3f\cosh\left(fx+e\right)}\left(\sqrt{-\frac{b}{a}}b^2\left(\sinh\left(fx+e\right)\right)^5-2\sqrt{-\frac{b}{a}}ab\left(\sinh\left(fx+e\right)\right)^3+4\sqrt{-\frac{b}{a}}b^2\left(\sinh\left(fx+e\right)\right)^3+3a^2\sqrt{\frac{a+b\left(\sin\right.}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)^5-2*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^3+4*
(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^3+3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-11*a*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a
```

```
*b)^(1/2),(a/b)^(1/2))*b+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1
/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+7*((a+b*sinh(f*x+
e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*a*b-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2-3*(-1/a*b)^(1/2)*a^2*sinh(
f*x+e)+4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e))/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tanh \left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tanh \left(fx+e\right)^{2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2)*tanh(f*x+e)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \tanh \left(fx+e\right)^{2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2)*tanh(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*tanh(f*x + e)^2, x)
```

3.476 $\int \left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=174

$$\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)\sqrt{a+b}}{3f}$$

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.182364, antiderivative size = 174, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$\frac{b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{ia(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}F\left(ie+ifx\middle|\frac{b}{a}\right)}{3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(}}{3f\sqrt{\frac{b\sinh^2(}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (((2*I)
/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(f*S
qrt[1 + (b*Sinh[e + f*x]^2)/a]) + ((I/3)*a*(a - b)*EllipticF[I*e + I*f*x, b
/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a-b)+2(2a-b)b\sinh}{\sqrt{a+b\sinh^2(e+f}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{1}{3}(a(a-b))\int \frac{1}{\sqrt{a+b\sinh^2(e+}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{\left(2(2a-b)\sqrt{a+b\sinh^2(e+fx)}\right.}{3\sqrt{1+\frac{b\sinh^2}{a}}}$$

$$= \frac{b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{2i(2a-b)E\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b}}{3f\sqrt{1+\frac{b\sinh^2(e+fx}{a}}}$$

Mathematica [A] time = 0.641269, size = 169, normalized size = 0.97

$$\frac{2i\sqrt{2}a(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+b\sinh(2(e+fx))(2a+b\cosh(2(e+fx))-b)-4i\sqrt{2}a(2a}{6f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-4*I)*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Ellipti
cE[I*(e + f*x), b/a] + (2*I)*Sqrt[2]*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e
+ f*x)])/a]*EllipticF[I*(e + f*x), b/a] + b*(2*a - b + b*Cosh[2*(e + f*x)])
*Sinh[2*(e + f*x)])/(6*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0., size = 416, normalized size = 2.4

$$\frac{1}{3\cosh(fx+e)f}\left(\sqrt{-\frac{b}{a}}b^2\sinh(fx+e)(\cosh(fx+e))^4+\left(\sqrt{-\frac{b}{a}}ab-\sqrt{-\frac{b}{a}}b^2\right)(\cosh(fx+e))^2\sinh(fx+e)+3\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*((-1/a*b)^(1/2)*b^2*sinh(f*x+e)*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a*b-(-1/a
*b)^(1/2)*b^2)*cosh(f*x+e)^2*sinh(f*x+e)+3*a^2*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-5*a*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)
*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^
2+4*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(
f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*
(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2
)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(fx + e\right)^2 + a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.477 $\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=256

$$\frac{(3a+5b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2}}{3f}$$

```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (Coth
[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/f - ((7*a + b)*EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*
f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a +
b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rubi [A] time = 0.271529, antiderivative size = 256, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 473, 528, 531, 418, 492, 411}

$$\frac{(7a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{4b\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth(e+fx)\left(a+b\sinh\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] (4*b*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f) - (Coth
[e + f*x]*(a + b*Sinh[e + f*x]^2)^(3/2))/f - ((7*a + b)*EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + 5*b)*EllipticF[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*
f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((7*a + b)*Sqrt[a +
b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \coth^2(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}\left(a+bx^2\right)^{3/2}}{x^2}\,dx, x, \sinh(e+f\right.}{f}$$

$$= -\frac{\coth(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx\right.}{}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+fx)\left(a+b\right.}{f}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+fx)\left(a+b\right.}{f}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+fx)\left(a+b\right.}{f}$$

$$= \frac{4b\cosh(e+fx)\sinh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f} - \frac{\coth(e+fx)\left(a+b\right.}{f}$$

Mathematica [C] time = 2.12637, size = 184, normalized size = 0.72

$$\frac{32ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}\coth(e+fx)\left(-24a^2-4b(2a+b)\cosh(2(e+fx))+8ab+b\right.}{24f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*(-24*a^2 + 8*a*b + 3*b^2 - 4*b*(2*a + b)*Cosh[2*(e + f*x)] + b^2*C
osh[4*(e + f*x)])*Coth[e + f*x] - (8*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[
2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (32*I)*a*(a - b)*Sqrt[(2*a -
 b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(24*f*Sqrt[2*a -
b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.156, size = 327, normalized size = 1.3

$$\frac{1}{3\cosh\left(fx+e\right)\sinh\left(fx+e\right)f}\left(\sinh\left(fx+e\right)\sqrt{\left(\cosh\left(fx+e\right)\right)^2}\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^2}{a}+\frac{a-b}{a}}\left(3\,\text{EllipticF}\left(\sinh\left(fx\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(3
*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-2*EllipticF(sinh(f*x
+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a
```

```
/b)^(1/2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+Elli
pticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)+(-1/a*b)^(1/2)*b^2*cosh(
f*x+e)^6+(-2*(-1/a*b)^(1/2)*a*b-2*(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^4+(-3*(-1
/a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*cosh(f*x+e)^2)/sin
h(f*x+e)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\coth\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\coth\left(fx+e\right)^2\sinh\left(fx+e\right)^2+a\coth\left(fx+e\right)^2\right)\sqrt{b\sinh\left(fx+e\right)^2+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*coth(f*x + e)^2*sinh(f*x + e)^2 + a*coth(f*x + e)^2)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.478 $\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=306

$$\frac{(3a+b)(a+3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{8(a+b)\tanh(e+fx)\sqrt{a+}}{3f}$$

```
[Out] -(((a + b)*Cosh[e + f*x]^2*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) +
((3*a + 5*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f)
 - (Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2))/(3*f) - (8*(a + b)*Ellip
ticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]
^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + b)*(
a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b
*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
) + (8*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```

Rubi [A] time = 0.358856, antiderivative size = 306, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 473, 580, 528, 531, 418, 492, 411}

$$\frac{8(a+b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}+\frac{(3a+5b)\sinh(e+fx)\cosh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f}-\frac{\coth^3(e+fx}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] -(((a + b)*Cosh[e + f*x]^2*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/f) +
((3*a + 5*b)*Cosh[e + f*x]*Sinh[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*f)
 - (Coth[e + f*x]^3*(a + b*Sinh[e + f*x]^2)^(3/2))/(3*f) - (8*(a + b)*Ellip
ticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]
^2])/(3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a + b)*(
a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b
*Sinh[e + f*x]^2])/(3*a*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]
) + (8*(a + b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \coth^4(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}{x^4}dx, x, \sinh(e+\right.}{f}$$

$$= -\frac{\coth^3(e+fx)\left(a+b\sinh^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(2\sqrt{\cosh^2(e+fx)}\mathrm{sech}(e+f\right.}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} - \frac{\coth^3(e+f}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\,\mathrm{co}}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\,\mathrm{co}}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\,\mathrm{co}}{}$$

$$= -\frac{(a+b)\cosh^2(e+fx)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{f} + \frac{(3a+5b)\,\mathrm{co}}{}$$

Mathematica [C] time = 4.70133, size = 229, normalized size = 0.75

$$\frac{4i\left(5a^2-2ab-3b^2\right)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right)-\frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\left(\left(64a^2+32ab-79b^2\right)\cosh(2(e+fx))-32a^2+2b(6\right.}{4\sqrt{2}}}{12f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4*(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((-32*a^2 - 44*a*b + 58*b^2 + (64*a^2 + 32*a*b - 79*b^2)*Cosh[2*(e + f*x)
] + 2*b*(6*a + 11*b)*Cosh[4*(e + f*x)] - b^2*Cosh[6*(e + f*x)])*Coth[e + f*
x]*Csch[e + f*x]^2)/(4*Sqrt[2]) - (32*I)*a*(a + b)*Sqrt[(2*a - b + b*Cosh[2
*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + (4*I)*(5*a^2 - 2*a*b - 3*b^2)
*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(12*f
*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.288, size = 540, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(-(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^8+3*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6+
3*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cos
```

```
h(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*
x+e)^3-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(si
nh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*sinh(f*x+e)^3+5*((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*b^2*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(
1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3-8*
((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+
e)^4+8*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^4+4*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+
5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2+5*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a
*b)^(1/2)*a^2)/(-1/a*b)^(1/2)/sinh(f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(fx+e\right)^2+a\right)^{\frac{3}{2}} \coth \left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(3/2)*coth(f*x + e)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\coth\left(fx+e\right)^4\sinh\left(fx+e\right)^2+a\coth\left(fx+e\right)^4\right)\sqrt{b\sinh\left(fx+e\right)^2+a}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*coth(f*x + e)^4*sinh(f*x + e)^2 + a*coth(f*x + e)^4)*sqrt(b*sin
h(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4*(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4*(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.479 $\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=142

$$-\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{5/2}}-\frac{\operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4f(a-b)}+\frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{a+b\cdots}}{8f(a-b)^2}$$

```
[Out] -((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])
/(8*(a - b)^(5/2)*f) + ((8*a - 5*b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x
]^2])/(8*(a - b)^2*f) - (Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*(a
 - b)*f)
```

Rubi [A] time = 0.193336, antiderivative size = 142, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 89, 78, 63, 208}

$$-\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{5/2}}-\frac{\operatorname{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4f(a-b)}+\frac{(8a-5b)\operatorname{sech}^2(e+fx)\sqrt{a+b\cdots}}{8f(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])
/(8*(a - b)^(5/2)*f) + ((8*a - 5*b)*Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x
]^2])/(8*(a - b)^2*f) - (Sech[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*(a
 - b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
```

```
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a+b)+2(a-b)x}{(1+x)^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{4(a-b)f}$$

$$= \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2 f} - \frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f} + \frac{(8a}{}$$

$$= \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8(a-b)^2 f} - \frac{\text{sech}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4(a-b)f} + \frac{(8a}{}$$

$$= -\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{5/2}f} + \frac{(8a-5b)\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+f}}{8(a-b)^2 f}$$

Mathematica [A] time = 0.462506, size = 116, normalized size = 0.82

$$\frac{\left(-8a^2+8ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right) + \sqrt{a-b}\,\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\left(-2(a-b)\text{sech}^2(e+fx)+8a\right.}{8f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]
+ Sqrt[a - b]*Sech[e + f*x]^2*(8*a - 5*b - 2*(a - b)*Sech[e + f*x]^2)*Sqrt[
a + b*Sinh[e + f*x]^2])/(8*(a - b)^(5/2)*f)
```

Maple [C] time = 0.125, size = 43, normalized size = 0.3

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^{5}}{\left(\cosh\left(fx+e\right)\right)^{6}}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^5/cosh(f*x+e)^6/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{5}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 3.74004, size = 9879, normalized size = 69.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 +
4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*co
sh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(
f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 +
24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^5 + 10*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(
f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 15*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 8
*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 -
 8*a*b + 3*b^2)*cosh(f*x + e)^7 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5
 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x
 + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 +
 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b
)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*
x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) +
```

```
b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 +
2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x
 + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*((8*a^2 - 13*a*b +
 5*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)*sinh(f*x
 + e)^4 + (8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2
)*cosh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + 4*a^2 -
 5*a*b + b^2)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)
^3 + 3*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 13*a
*b + 5*b^2)*cosh(f*x + e) + (5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^4 + 6
*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x +
 e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*f*cosh(f*x + e)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f
*x + e)*sinh(f*x + e)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*sinh(f*x + e)^8
 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*s
inh(f*x + e)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 8*(7
*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + 3*(a^3 - 3*a^2*b + 3*a
*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3 - 3*a^2*b + 3*a*b
^2 - b^3)*f*cosh(f*x + e)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x
 + e)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^4 + 4*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*f*cosh(f*x + e)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e
)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)^3 +
4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 15*(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*co
sh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^2 + (a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*f + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x
 + e)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^5 + 3*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*
f*cosh(f*x + e))*sinh(f*x + e)), -1/8*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x +
e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^8 + 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 +
 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh
(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e)^4 + 2*(35*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 30*(8*a^2 - 8
*a*b + 3*b^2)*cosh(f*x + e)^2 + 24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 +
8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 10*(8*a^2 - 8*a*b + 3*b^2)*c
osh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*c
osh(f*x + e)^6 + 15*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^2 + 8*a
^2 - 8*a*b + 3*b^2 + 8*((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^7 + 3*(8*a^2
- 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^
3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arct
an(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((8*a^2 - 1
3*a*b + 5*b^2)*cosh(f*x + e)^5 + 5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)*s
inh(f*x + e)^4 + (8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*
b + b^2)*cosh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 +
4*a^2 - 5*a*b + b^2)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f
*x + e)^3 + 3*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2
 - 13*a*b + 5*b^2)*cosh(f*x + e) + (5*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e
)^4 + 6*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3 - 3*a^2
*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^8 + 8*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f
```

```
*cosh(f*x + e)*sinh(f*x + e)^7 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*sinh(f*x
 + e)^8 + 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^3 -
 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*f)*sinh(f*x + e)^6 + 6*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^4
 + 8*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + 3*(a^3 - 3*a^2*
b + 3*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 30*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*c
osh(f*x + e)^2 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^4 + 4*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^3 - 3*a^2*b + 3*
a*b^2 - b^3)*f*cosh(f*x + e)^5 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(
f*x + e)^3 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x +
e)^3 + 4*(7*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^6 + 15*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^4 + 9*(a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*f*cosh(f*x + e)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f)*sinh(f*x + e)^2 +
 (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f + 8*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*c
osh(f*x + e)^7 + 3*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^5 + 3*(a
^3 - 3*a^2*b + 3*a*b^2 - b^3)*f*cosh(f*x + e)^3 + (a^3 - 3*a^2*b + 3*a*b^2
- b^3)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^5\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**5/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^5}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.480 $\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{3/2}}$$

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^(3
/2)*f) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f)
```

Rubi [A] time = 0.112257, antiderivative size = 89, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3194, 78, 63, 208}

$$\frac{\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2f(a-b)}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^(3
/2)*f) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*(a - b)*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx\right.}{4(a-b)f}$$

$$= \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e\right.}{2(a-b)bf}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{3/2}f} + \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2(a-b)f}$$

Mathematica [A] time = 0.116503, size = 85, normalized size = 0.96

$$-\frac{\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}} - \frac{\text{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a-b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(a - b)^(3/2
) - (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(a - b))/(2*f)
```

Maple [C] time = 0.129, size = 43, normalized size = 0.5

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{\left(\sinh\left(fx+e\right)\right)^3}{\left(\cosh\left(fx+e\right)\right)^4}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}}, \sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)^3/cosh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x
+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.74653, size = 3425, normalized size = 38.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^
3 + (2*a - b)*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b
)*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^3
 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a - b)*log((b*cos
h(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4
*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x +
e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 +
2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2
))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cos
h(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x
+ e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*co
sh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4
*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x +
 e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^4 + 4*
(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2 - 2*a*b + b^2)*f
*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + 2*(3*(a^2 - 2*
a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh(f*x + e)^2 + (a^
2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^3 + (a^2 - 2*a*
b + b^2)*f*cosh(f*x + e))*sinh(f*x + e)), -1/2*(((2*a - b)*cosh(f*x + e)^4
+ 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 + 2
*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 + 2*a - b)*sinh
(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(
f*x + e) + 2*a - b)*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*c
osh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
 + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*si
nh(f*x + e))) - 2*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2 - 2*a*b + b^2)*f*cos
h(f*x + e)^4 + 4*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2
 - 2*a*b + b^2)*f*sinh(f*x + e)^4 + 2*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2
 + 2*(3*(a^2 - 2*a*b + b^2)*f*cosh(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f)*sinh
(f*x + e)^2 + (a^2 - 2*a*b + b^2)*f + 4*((a^2 - 2*a*b + b^2)*f*cosh(f*x + e
)^3 + (a^2 - 2*a*b + b^2)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^3}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.481 $\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rubi [A] time = 0.0583869, antiderivative size = 41, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3194, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{\sqrt{a-b}f}$$

Mathematica [A] time = 0.0423987, size = 44, normalized size = 1.07

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\cosh^2(e+fx)-b}}{\sqrt{a-b}}\right)}{f\sqrt{a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a - b + b*Cosh[e + f*x]^2]/Sqrt[a - b]]/(Sqrt[a - b]*f))
```

Maple [C] time = 0.093, size = 41, normalized size = 1.

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\sinh(fx+e)}{(\cosh(fx+e))^2}\frac{1}{\sqrt{a+b(\sinh(fx+e))^2}}, \sinh(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(sinh(f*x+e)/cosh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e
))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.35449, size = 1168, normalized size = 28.49

$$\left[\frac{\log\left(\frac{b\cosh(fx+e)^4+4\,b\cosh(fx+e)\sinh(fx+e)^3+b\sinh(fx+e)^4+2\,(4\,a-3\,b)\cosh(fx+e)^2+2\left(3\,b\cosh(fx+e)^2+4\,a-3\,b\right)\sinh(fx+e)^2-4\,\sqrt{2}\sqrt{a-b}\sqrt{\frac{\cdots}{\cosh(fx\cdots}}\cdots}{\cosh(fx+e)^4+4\,\cosh(fx+e)\sinh(fx+e)^3+\sinh(fx+e)^4+2\left(3\,\cosh(fx+e)^2+1\right)\sinh(fx+e\cdots}\right)}{2\sqrt{a-b}f}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*
x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3
*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sin
h(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 +
(4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*
x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x
 + e) + 1))/(sqrt(a - b)*f), -sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)
*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 -
2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) +
(a - b)*sinh(f*x + e)))/((a - b)*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(e+fx\right)}{\sqrt{a+b\sinh^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.482 $\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rubi [A] time = 0.070531, antiderivative size = 33, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3194, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Mathematica [A] time = 0.0353481, size = 33, normalized size = 1.

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

Maple [C] time = 0.073, size = 35, normalized size = 1.1

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{1}{\sinh\left(fx+e\right)}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}}, \sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.22778, size = 1111, normalized size = 33.67

$$\left[\frac{\log\left(\frac{b\cosh\left(fx+e\right)^{4}+4\,b\cosh\left(fx+e\right)\sinh\left(fx+e\right)^{3}+b\sinh\left(fx+e\right)^{4}+2\,(4\,a-b)\cosh\left(fx+e\right)^{2}+2\,\left(3\,b\cosh\left(fx+e\right)^{2}+4\,a-b\right)\sinh\left(fx+e\right)^{2}-4\,\sqrt{2}\sqrt{a}\sqrt{\frac{b\cdots}{\cosh(fx+\cdots}}\cdots}{\cosh\left(fx+e\right)^{4}+4\,\cosh\left(fx+e\right)\sinh\left(fx+e\right)^{3}+\sinh\left(fx+e\right)^{4}+2\,\left(3\,\cosh\left(fx+e\right)^{2}-1\right)\sinh\left(fx+e\cdots}\right)}{2\,\sqrt{a}f}\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*
x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*
sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x
 + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b
)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh
(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 -
 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)
)/(sqrt(a)*f), sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2
 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x
 + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e)))/(a*f)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError
```

3.483 $\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=77

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - (
Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*a*f)
```

Rubi [A] time = 0.102198, antiderivative size = 77, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.16, Rules used = {3194, 78, 63, 208}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(3/2)*f) - (
Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(2*a*f)
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x}{x^2\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{4af}$$
$$= -\frac{\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af} + \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{2abf}$$
$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{2af}$$

Mathematica [A] time = 0.201515, size = 72, normalized size = 0.94

$$-\frac{\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((((2*a - b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/a^(3/2) + (Csch[
e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/a)/(2*f)
```

Maple [C] time = 0.107, size = 44, normalized size = 0.6

$$\frac{1}{f}\text{`int/indef0`}\left(\left(\left(\sinh(fx+e)\right)^{-1} + \left(\sinh(fx+e)\right)^{-3}\right)\frac{1}{\sqrt{a+b\left(\sinh(fx+e)\right)^2}}, \sinh(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((1/sinh(f*x+e)+1/sinh(f*x+e)^3)/(a+b*sinh(f*x+e)^2)^(1/2),sinh
(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)^3}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.49188, size = 2957, normalized size = 38.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f*x + e)*sinh(f*x + e)
^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*(2*a -
b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*a - b)*cosh(f*x + e)^
3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*sqrt(a)*log((b*cosh(f
*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a
- b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 +
4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(c
osh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*
x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*s
inh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh
(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2
 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(a*c
osh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2)))/(a^2*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2
*f*sinh(f*x + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x
+ e)^2 - a^2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x
 + e))*sinh(f*x + e)), 1/2*(((2*a - b)*cosh(f*x + e)^4 + 4*(2*a - b)*cosh(f
*x + e)*sinh(f*x + e)^3 + (2*a - b)*sinh(f*x + e)^4 - 2*(2*a - b)*cosh(f*x
+ e)^2 + 2*(3*(2*a - b)*cosh(f*x + e)^2 - 2*a + b)*sinh(f*x + e)^2 + 4*((2*
a - b)*cosh(f*x + e)^3 - (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + 2*a - b)*
sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) - 2*sqrt(2)*(a*cosh(f*x + e
) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)
/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2
*f*cosh(f*x + e)^4 + 4*a^2*f*cosh(f*x + e)*sinh(f*x + e)^3 + a^2*f*sinh(f*x
 + e)^4 - 2*a^2*f*cosh(f*x + e)^2 + a^2*f + 2*(3*a^2*f*cosh(f*x + e)^2 - a^
2*f)*sinh(f*x + e)^2 + 4*(a^2*f*cosh(f*x + e)^3 - a^2*f*cosh(f*x + e))*sinh
(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^{3}\left(e + fx\right)}{\sqrt{a + b\sinh^{2}\left(e + fx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth \left(fx+e\right)^3}{\sqrt{b\sinh \left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^3/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.484 $\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=126

$$-\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af}$$

```
[Out] -((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*
a^(5/2)*f) - ((8*a - 3*b)*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a
^2*f) - (Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*a*f)
```

Rubi [A] time = 0.147265, antiderivative size = 126, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 89, 78, 63, 208}

$$-\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((8*a^2 - 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(8*
a^(5/2)*f) - ((8*a - 3*b)*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(8*a
^2*f) - (Csch[e + f*x]^4*Sqrt[a + b*Sinh[e + f*x]^2])/(4*a*f)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \frac{\coth^5(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3\sqrt{a+bx}}\,dx,x,\sinh^2(e+fx)\right)}{2f}\\
&= -\frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af}+\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-3b)+2ax}{x^2\sqrt{a+bx}}\,dx,x,\sinh^2(e+fx)\right)}{4af}\\
&= -\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af}+\\
&= -\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{8a^2f}-\frac{\text{csch}^4(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{4af}+\\
&= -\frac{\left(8a^2-8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{(8a-3b)\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e}{8a^2f}\end{aligned}$$

Mathematica [A] time = 0.362039, size = 100, normalized size = 0.79

$$\frac{\left(-8a^2+8ab-3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\text{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\left(-2a\text{csch}^2(e+fx)-8a+3b\right)}{8a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 8*a*b - 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]] + Sq
rt[a]*Csch[e + f*x]^2*(-8*a + 3*b - 2*a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e
+ f*x]^2])/(8*a^(5/2)*f)
```

Maple [C] time = 0.102, size = 54, normalized size = 0.4

$$\frac{1}{f}\text{` int/indef0`}\left(\left(\left(\sinh\left(fx+e\right)\right)^{-1}+2\left(\sinh\left(fx+e\right)\right)^{-3}+\left(\sinh\left(fx+e\right)\right)^{-5}\right)\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}}},\sinh\left(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] `int/indef0`((1/sinh(f*x+e)+2/sinh(f*x+e)^3+1/sinh(f*x+e)^5)/(a+b*sinh(f*x+
e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^5}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [B] time = 2.84606, size = 7682, normalized size = 60.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)
*cosh(f*x + e)*sinh(f*x + e)^7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 -
4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*co
sh(f*x + e)^2 - 8*a^2 + 8*a*b - 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e)^3 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(
f*x + e)^5 + 6*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 - 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 +
24*a^2 - 24*a*b + 9*b^2)*sinh(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^5 - 10*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*
b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 - 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(
f*x + e)^2 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^6 - 15*(8*a^2 - 8*a
*b + 3*b^2)*cosh(f*x + e)^4 + 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8
*a^2 + 8*a*b - 3*b^2)*sinh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 -
 8*a*b + 3*b^2)*cosh(f*x + e)^7 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5
 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 - (8*a^2 - 8*a*b + 3*b^2)*cosh
(f*x + e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e
)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*
b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*co
sh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x
+ e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*
(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x
+ e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x
+ e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh
(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((8*a^2 - 3*a*b)*cosh(f*x + e)^5
 + 5*(8*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (8*a^2 - 3*a*b)*sinh(f
*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(8*a^2 - 3*a*b)*cosh(f
*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 3*a*b)*cosh(f*x
```

```
+ e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^2 - 3*a*b)
*cosh(f*x + e) + (5*(8*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cos
h(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/(a^3*f*cosh(f*x + e)^8 + 8*a^3*f*cosh(f*x + e)*sinh(f*x
 + e)^7 + a^3*f*sinh(f*x + e)^8 - 4*a^3*f*cosh(f*x + e)^6 + 6*a^3*f*cosh(f*
x + e)^4 + 4*(7*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x + e)^6 - 4*a^3*f*co
sh(f*x + e)^2 + 8*(7*a^3*f*cosh(f*x + e)^3 - 3*a^3*f*cosh(f*x + e))*sinh(f*
x + e)^5 + 2*(35*a^3*f*cosh(f*x + e)^4 - 30*a^3*f*cosh(f*x + e)^2 + 3*a^3*f
)*sinh(f*x + e)^4 + a^3*f + 8*(7*a^3*f*cosh(f*x + e)^5 - 10*a^3*f*cosh(f*x
+ e)^3 + 3*a^3*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*f*cosh(f*x + e)^
6 - 15*a^3*f*cosh(f*x + e)^4 + 9*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x +
e)^2 + 8*(a^3*f*cosh(f*x + e)^7 - 3*a^3*f*cosh(f*x + e)^5 + 3*a^3*f*cosh(f*
x + e)^3 - a^3*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((8*a^2 - 8*a*b + 3*b^
2)*cosh(f*x + e)^8 + 8*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)*sinh(f*x + e)^
7 + (8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^8 - 4*(8*a^2 - 8*a*b + 3*b^2)*cos
h(f*x + e)^6 + 4*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 8*a*b
 - 3*b^2)*sinh(f*x + e)^6 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 -
3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 6*(8*a^2 - 8*a*b
 + 3*b^2)*cosh(f*x + e)^4 + 2*(35*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 -
 30*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 24*a^2 - 24*a*b + 9*b^2)*sinh
(f*x + e)^4 + 8*(7*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 - 10*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^3 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sin
h(f*x + e)^3 - 4*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(8*a^2 - 8*
a*b + 3*b^2)*cosh(f*x + e)^6 - 15*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 +
 9*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 8*a*b - 3*b^2)*sinh(f*
x + e)^2 + 8*a^2 - 8*a*b + 3*b^2 + 8*((8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)
^7 - 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^5 + 3*(8*a^2 - 8*a*b + 3*b^2)*
cosh(f*x + e)^3 - (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e))*sqr
t(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)
^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x +
 e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) - 2*sqrt(2)*((8*a^2 - 3*a*b)*c
osh(f*x + e)^5 + 5*(8*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (8*a^2 -
 3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(8*a^2 -
 3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(8*a^2 - 3*
a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (
8*a^2 - 3*a*b)*cosh(f*x + e) + (5*(8*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^
2 - 3*a*b)*cosh(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2)))/(a^3*f*cosh(f*x + e)^8 + 8*a^3*f*cosh(f*x
 + e)*sinh(f*x + e)^7 + a^3*f*sinh(f*x + e)^8 - 4*a^3*f*cosh(f*x + e)^6 + 6
*a^3*f*cosh(f*x + e)^4 + 4*(7*a^3*f*cosh(f*x + e)^2 - a^3*f)*sinh(f*x + e)^
6 - 4*a^3*f*cosh(f*x + e)^2 + 8*(7*a^3*f*cosh(f*x + e)^3 - 3*a^3*f*cosh(f*x
 + e))*sinh(f*x + e)^5 + 2*(35*a^3*f*cosh(f*x + e)^4 - 30*a^3*f*cosh(f*x +
e)^2 + 3*a^3*f)*sinh(f*x + e)^4 + a^3*f + 8*(7*a^3*f*cosh(f*x + e)^5 - 10*a
^3*f*cosh(f*x + e)^3 + 3*a^3*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*f*
cosh(f*x + e)^6 - 15*a^3*f*cosh(f*x + e)^4 + 9*a^3*f*cosh(f*x + e)^2 - a^3*
f)*sinh(f*x + e)^2 + 8*(a^3*f*cosh(f*x + e)^7 - 3*a^3*f*cosh(f*x + e)^5 + 3
*a^3*f*cosh(f*x + e)^3 - a^3*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)^5}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^5/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.485 $\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=219

$$\frac{(3a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+}}{3f(a-b)}$$

```
[Out] (-2*(2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[
e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*
(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]
*Tanh[e + f*x])/(3*(a - b)*f)
```

Rubi [A] time = 0.196234, antiderivative size = 219, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3196, 470, 525, 418, 411}

$$\frac{\tanh(e+fx)\operatorname{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3f(a-b)}+\frac{(3a-b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(\sinh(e+fx))\right.}{3f(a-b)^2\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-2*(2*a - b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[
a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[
e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*(a - b)^2*f*Sqrt[(Sech[e + f*x]^2*
(a + b*Sinh[e + f*x]^2))/a]) + (Sech[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2]
*Tanh[e + f*x])/(3*(a - b)*f)
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^{5/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\mathrm{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subs}}{3(a}$$

$$= \frac{\mathrm{sech}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\tanh(e+fx)}{3(a-b)f} - \frac{\left(2(2a-b)\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+f\right.}{3(a}$$

$$= -\frac{2(2a-b)E\left(\tan^{-1}(\sinh(e+fx))\middle|1-\frac{b}{a}\right)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3(a-b)^2 f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{(3a-b)F\left(\right.}{}$$

Mathematica [C] time = 2.10477, size = 206, normalized size = 0.94

$$\frac{2ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right) - \frac{\tanh(e+fx)\mathrm{sech}^2(e+fx)\left(2\left(4a^2-3ab+b^2\right)\cosh(2(e+fx))+(2a-b)(2a+b\cosh(4(e+fx))+\right.}{\sqrt{2}}}{6f(a-b)^2\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-4*I)*a*(2*a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (2*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - ((2*(4*a^2 - 3*a*b + b^2)*Cosh[2*(e + f*x)] + (2*
a - b)*(2*a + b + b*Cosh[4*(e + f*x)]))*Sech[e + f*x]^2*Tanh[e + f*x])/Sqrt
[2])/(6*(a - b)^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.289, size = 366, normalized size = 1.7

$$\frac{1}{3\left(\cosh\left(fx+e\right)\right)^{3}(a-b)^{2}f}\left(\left(-4\sqrt{-\frac{b}{a}}ab+2\sqrt{-\frac{b}{a}}b^{2}\right)\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^{4}+\left(-4\sqrt{-\frac{b}{a}}a^{2}+7\sqrt{-\frac{b}{a}}ab-\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-4*(-1/a*b)^(1/2)*a*b+2*(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4
+(-4*(-1/a*b)^(1/2)*a^2+7*(-1/a*b)^(1/2)*a*b-3*(-1/a*b)^(1/2)*b^2)*cosh(f*x
+e)^2*sinh(f*x+e)+((-1/a*b)^(1/2)*a^2-2*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b
^2)*sinh(f*x+e)+(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(3*
EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-5*EllipticF(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(
a/b)^(1/2))*b^2+4*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-2*E
llipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/cosh(f
*x+e)^3/(a-b)^2/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\tanh\left(fx+e\right)^{4}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^{4}\left(e+fx\right)}{\sqrt{a+b\sinh^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**4/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.486 $\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=156

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan \right.}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sin\right.}{a}}}$$

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]))
 + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rubi [A] time = 0.181255, antiderivative size = 156, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3196, 471, 422, 418, 492, 411}

$$\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,E\left(\tan^{-1}(\sinh(\right.}{f(a-b)\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] -((EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[
e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]))
 + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh
[e + f*x]^2])/((a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
```

```
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx &= \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}\\
&= -\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1+x^2}}\,d\ldots\right.}{(-a+b)f}\\
&= -\frac{\sqrt{a+b\sinh^2(e+fx)}\tanh(e+fx)}{(a-b)f} - \frac{\left(a\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1+x^2}\sqrt{\ldots}}\right.}{(-a+b)f}\\
&= \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}\ldots\right.}{\ldots}\\
&= -\frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{(a-b)f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}} + \frac{F\left(\tan^{-1}(\sinh(e+f\ldots\right.}{(a-\ldots}
\end{aligned}
$$

Mathematica [C] time = 0.382078, size = 109, normalized size = 0.7

$$
\frac{\sqrt{2}\tanh(e+fx)(-2a-b\cosh(2(e+fx))+b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{2f(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] + Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/(2*(a - b)*f*S
qrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.161, size = 239, normalized size = 1.5

$$\frac{1}{(a-b)\cosh(fx+e)f}\left(-\sqrt{-\frac{b}{a}}b(\sinh(fx+e))^3+a\sqrt{\frac{a+b(\sinh(fx+e))^2}{a}}\sqrt{(\cosh(fx+e))^2}\text{EllipticF}\left(\sinh(f\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)^3+a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*((a+b*sinh(
f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/
2),(a/b)^(1/2))+b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellip
ticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-(-1/a*b)^(1/2)*a*sinh(f*x+e))/
(a-b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\tanh(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.487 $\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0365019, antiderivative size = 60, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$-\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}F\left(ie+ifx\middle|\frac{b}{a}\right)}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(f*Sqrt[
a + b*Sinh[e + f*x]^2])
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sinh^2(e+fx)}}$$

$$= -\frac{iF\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}{f\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [A] time = 0.0707229, size = 68, normalized size = 1.13

$$-\frac{i\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)}{f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] ((-I)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/
(f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.001, size = 86, normalized size = 1.4

$$\frac{1}{\cosh\left(fx+e\right)f}\sqrt{\frac{a+b\left(\sinh\left(fx+e\right)\right)^{2}}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\text{EllipticF}\left(\sinh\left(fx+e\right)\sqrt{-\frac{b}{a}},\sqrt{\frac{a}{b}}\right)\frac{1}{\sqrt{-\frac{b}{a}}}\frac{1}{\sqrt{a+b\left(\sinh\left(fx\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)
^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{1}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{a + b\sinh^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\sqrt{b\sinh\left(fx + e\right)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.488 $\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=207

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\coth($$

```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e +
 f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(a*f)
```

Rubi [A] time = 0.197208, antiderivative size = 207, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3196, 475, 422, 418, 492, 411}

$$\frac{\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}-\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\tan^{-1}(s\right.}{af\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+f\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] -((Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f)) - (EllipticE[ArcTan[Si
nh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt
[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e +
 f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (Sqrt[a + b*Sinh[e + f*x]^2]*Tan
h[e + f*x])/(a*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 422

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
a, Int[1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] + Dist[b, Int[x^2/(Sqrt[
a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && PosQ[d/c]
&& PosQ[b/a]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{a+bx}}{\sqrt{1+x}}\right.}{af}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathbf{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{}{\sqrt{1+x^2}}\right.}{f}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} + \frac{F\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e+fx)\sqrt{a}}{af\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

$$= -\frac{\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{af} - \frac{E\left(\tan^{-1}(\sinh(e+fx))|1-\frac{b}{a}\right)\mathbf{sech}(e+fx)\sqrt{a}}{af\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}$$

Mathematica [C] time = 0.388523, size = 105, normalized size = 0.51

$$\frac{\sqrt{2}\coth(e+fx)(-2a-b\cosh(2(e+fx))+b)-2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\left|\frac{b}{a}\right.\right)}{2af\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (Sqrt[2]*(-2*a + b - b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a])/(2*a*f*Sqrt[2*a
 - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.179, size = 217, normalized size = 1.1

$$-\frac{1}{a\sinh\left(fx+e\right)\cosh\left(fx+e\right)f}\left(-\sinh\left(fx+e\right)\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^{2}}{a}+\frac{a-b}{a}}\left(a\mathrm{EllipticF}\left(\sinh\left(f\right.\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -(-sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(a*E
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-b*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))+b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
)))+(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-(-1/a*b)^(1/2)*b)*cosh
(f*x+e)^2)/(-1/a*b)^(1/2)/a/sinh(f*x+e)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/
2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\coth\left(fx+e\right)^{2}}{\sqrt{b\sinh\left(fx+e\right)^{2}+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/sqrt(a + b*sinh(e + f*x)**2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)^2}{\sqrt{b\sinh(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^2/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.489 $\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx$

Optimal. Leaf size=285

$$\frac{(3a-b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\mathrm{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^2f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2}}{3a^2f}$$

```
[Out] (-2*(2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[
e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (2*(2*a - b
)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + (2*(2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)
```

Rubi [A] time = 0.304744, antiderivative size = 285, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 474, 583, 531, 418, 492, 411}

$$\frac{2(2a-b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}-\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f}+\frac{(3a-b)\mathrm{sech}(e+fx)\sqrt{a+}}{3a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```


```
[Out] (-2*(2*a - b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^2*f) - (Coth[
e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a*f) - (2*(2*a - b
)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e
 + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a +
b*Sinh[e + f*x]^2])/(3*a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))
/a]) + (2*(2*a - b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3*a^2*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4\sqrt{a+bx^2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Sub}}{}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(}}{3af}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(}}{3af}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(}}{3af}$$

$$= -\frac{2(2a-b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^2f} - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\sqrt{a+b\sinh^2(}}{3af}$$

Mathematica [C] time = 3.54866, size = 208, normalized size = 0.73

$$\frac{2ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\operatorname{EllipticF}\left(i(e+fx), \frac{b}{a}\right) - \frac{\coth(e+fx)\operatorname{csch}^2(e+fx)\left(2\left(4a^2-5ab+2b^2\right)\cosh(2(e+fx))-(2a-b)(2a-b\cosh(4(e+fx))}{\sqrt{2}}}{6a^2f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/Sqrt[a + b*Sinh[e + f*x]^2],x]
```

```
[Out] (-(((2*(4*a^2 - 5*a*b + 2*b^2)*Cosh[2*(e + f*x)] - (2*a - b)*(2*a - 3*b - b
*Cosh[4*(e + f*x)]))*Coth[e + f*x]*Csch[e + f*x]^2)/Sqrt[2]) - (4*I)*a*(2*a
 - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] +
 (2*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f
*x), b/a])/(6*a^2*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.187, size = 522, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6
-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3+5*b*((a+b*sinh(f*x+e)^2)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*sinh(f*x+e)^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
```

```
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3-4*((a+b*
sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b
)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cos
h(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sin
h(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-1/a*b)^(1/2)*a*b*sinh(f*x
+e)^4-2*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^2
-(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2)/(-1/a*b)^(1/2)/a^2/si
nh(f*x+e)^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\coth\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^4}{\sqrt{b\sinh\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(coth(f*x + e)^4/sqrt(b*sinh(f*x + e)^2 + a), x)
```

3.490 $\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=187

$$\frac{8a^2+8ab-b^2}{8f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}-\frac{\left(8a^2+8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}-\frac{\operatorname{sech}^4(e+fx)}{4f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{}{8f(}$$

```
[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(
8*(a - b)^(7/2)*f) + (8*a^2 + 8*a*b - b^2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e
 + f*x]^2]) + ((8*a - 3*b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*Sqrt[a + b*Sinh[
e + f*x]^2]) - Sech[e + f*x]^4/(4*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.251366, antiderivative size = 187, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2+8ab-b^2}{8f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}-\frac{\left(8a^2+8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{7/2}}-\frac{\operatorname{sech}^4(e+fx)}{4f(a-b)\sqrt{a+b\sinh^2(e+fx)}}+\frac{}{8f(}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 + 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(
8*(a - b)^(7/2)*f) + (8*a^2 + 8*a*b - b^2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e
 + f*x]^2]) + ((8*a - 3*b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*Sqrt[a + b*Sinh[
e + f*x]^2]) - Sech[e + f*x]^4/(4*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a-b)+2(a-b)x}{(1+x)^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{4(a-b)f}$$

$$= \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(8a^2+8ab-b^2\right)\text{S}}{}$$

$$= \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{sech}^4(e+}{4(a-b)f\sqrt{a+b\text{s}}}$$

$$= \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8a-3b)\text{sech}^2(e+fx)}{8(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{sech}^4(e+}{4(a-b)f\sqrt{a+b\text{s}}}$$

$$= -\frac{\left(8a^2+8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{7/2}f} + \frac{8a^2+8ab-b^2}{8(a-b)^3f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(8}{8(a-}$$

Mathematica [C] time = 0.467676, size = 113, normalized size = 0.6

$$\frac{(8a^2+8ab-b^2)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)+a}{a-b}\right)+\frac{1}{2}(a-b)\text{sech}^4(e+fx)((8a-3b)\cosh(2(e+fx))+4a+b)}{8f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((8*a^2 + 8*a*b - b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]
^2)/(a - b)] + ((a - b)*(4*a + b + (8*a - 3*b)*Cosh[2*(e + f*x)])*Sech[e +
f*x]^4)/2)/(8*(a - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] time = 0.225, size = 103, normalized size = 0.6

$$\frac{1}{f}\text{` int/indef0`}\left(-\frac{(\sinh(fx+e))^5(\cosh(fx+e))^4}{-b^2(\cosh(fx+e))^{14}+(-2ab+2b^2)(\cosh(fx+e))^{12}+(-a^2+2ab-b^2)(\cosh(fx+e))^{10}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^5*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^4/(-b^2*c
osh(f*x+e)^14+(-2*a*b+2*b^2)*cosh(f*x+e)^12+(-a^2+2*a*b-b^2)*cosh(f*x+e)^10
),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)^5}{\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 6.76644, size = 24184, normalized size = 129.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b + 8*a*b^2
- b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b + 8*a*b^2 - b^3)*sinh(f*x
```

```
+ e)^12 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^10 + 2*(16*a^
3 + 24*a^2*b + 6*a*b^2 - b^3 + 33*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^3 + (16
*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e)^9 + (128*a^3
+ 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^8 + (495*(8*a^2*b + 8*a*b^2 - b
^3)*cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 90*(16*a^3 + 2
4*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(8*a^2*b
+ 8*a*b^2 - b^3)*cosh(f*x + e)^5 + 30*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*c
osh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e))*sinh
(f*x + e)^7 + 4*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^6 + 4*(2
31*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^6 + 105*(16*a^3 + 24*a^2*b + 6*a
*b^2 - b^3)*cosh(f*x + e)^4 + 48*a^3 + 40*a^2*b - 14*a*b^2 + b^3 + 7*(128*a
^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(
8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^7 + 63*(16*a^3 + 24*a^2*b + 6*a*b^2
- b^3)*cosh(f*x + e)^5 + 7*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x
+ e)^3 + 3*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e
)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^4 + (495*(8*a^2*
b + 8*a*b^2 - b^3)*cosh(f*x + e)^8 + 420*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3
)*cosh(f*x + e)^6 + 70*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)
^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 60*(48*a^3 + 40*a^2*b - 14*a*b^
2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(8*a^2*b + 8*a*b^2 - b^3)
*cosh(f*x + e)^9 + 60*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^7 +
 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^5 + 20*(48*a^3 + 4
0*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2
 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b + 8*a*b^2 - b^3 + 2*(16*a^
3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(33*(8*a^2*b + 8*a*b^2 -
b^3)*cosh(f*x + e)^10 + 45*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e
)^8 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^6 + 30*(48*a^
3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^4 + 16*a^3 + 24*a^2*b + 6*a*b^
2 - b^3 + 3*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*
x + e)^2 + 4*(3*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^11 + 5*(16*a^3 + 24
*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^9 + 2*(128*a^3 + 120*a^2*b - 24*a*b^2
 + b^3)*cosh(f*x + e)^7 + 6*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x +
 e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 +
24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*
cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2
*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x
 + e)^2 - 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*
cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f
*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2
*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1))
+ 4*sqrt(2)*((8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^9 + 9*(8*a^3 - 9*a*b^2 +
 b^3)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^3 - 9*a*b^2 + b^3)*sinh(f*x + e)
^9 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^7 + 4*(16*a^3 -
19*a^2*b + 5*a*b^2 - 2*b^3 + 9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^2)*sin
h(f*x + e)^7 + 28*(3*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 - 19
*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 28*a
^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*co
sh(f*x + e)^4 + 40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3 + 42*(16*a^3 - 19*a^2*
b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*
b^2 + b^3)*cosh(f*x + e)^5 + 70*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(
f*x + e)^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e))*sinh(f
*x + e)^4 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + 4*(21
*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^6 + 35*(16*a^3 - 19*a^2*b + 5*a*b^2
- 2*b^3)*cosh(f*x + e)^4 + 16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3 + 5*(40*a^3
- 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(9*(8*a
^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^7 + 21*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b
```

^3)*cosh(f*x + e)^5 + 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e
)^3 + 3*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^
2 + (8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e) + (9*(8*a^3 - 9*a*b^2 + b^3)*cosh
(f*x + e)^8 + 28*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 10
*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^4 + 8*a^3 - 9*a*b^2 +
b^3 + 12*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x +
e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^
2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 4*a^3*b^2
+ 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^12 + 12*(a^4*b - 4*a^3*b^2 + 6
*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^4*b - 4*a^3
*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*sinh(f*x + e)^12 + 2*(2*a^5 - 7*a^4*b +
8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^10 + 2*(33*(a^4*b -
4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*
b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f)*sinh(f*x + e)^10 + (16*a^5 -
65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^8 + 2
0*(11*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (
2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*s
inh(f*x + e)^9 + (495*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos
h(f*x + e)^4 + 90*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)
*f*cosh(f*x + e)^2 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b
^4 - b^5)*f)*sinh(f*x + e)^8 + 4*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^
3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^6 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^
3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^5 + 30*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*
a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b
^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231
*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 105*(2
*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^4 +
7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f
*x + e)^2 + (6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f
)*sinh(f*x + e)^6 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^
4 - b^5)*f*cosh(f*x + e)^4 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4
+ b^5)*f*cosh(f*x + e)^7 + 63*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2
*a*b^4 + b^5)*f*cosh(f*x + e)^5 + 7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a
^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^3 + 3*(6*a^5 - 25*a^4*b + 40*a^3*b
^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*(
a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^8 + 420*(2*a
^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 7
0*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*
x + e)^4 + 60*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)
*f*cosh(f*x + e)^2 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b
^4 - b^5)*f)*sinh(f*x + e)^4 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 -
2*a*b^4 + b^5)*f*cosh(f*x + e)^2 + 4*(55*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 -
4*a*b^4 + b^5)*f*cosh(f*x + e)^9 + 60*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*
b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^7 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^
2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^5 + 20*(6*a^5 - 25*a^4*b +
40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (16*a^5 - 65
*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f
x + e)^3 + 2(33*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*
x + e)^10 + 45*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*
cosh(f*x + e)^8 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b
^4 - b^5)*f*cosh(f*x + e)^6 + 30*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^
3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^4 + 3*(16*a^5 - 65*a^4*b + 100*a^3*b^2
- 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3
*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f)*sinh(f*x + e)^2 + (a^4*b - 4*a^3*b^2 +
6*a^2*b^3 - 4*a*b^4 + b^5)*f + 4*(3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b
^4 + b^5)*f*cosh(f*x + e)^11 + 5*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 -
2*a*b^4 + b^5)*f*cosh(f*x + e)^9 + 2*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70
*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^7 + 6*(6*a^5 - 25*a^4*b + 40*a^3
*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^5 + (16*a^5 - 65*a^4*b

```
+ 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (2*a^5 - 7
*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x +
 e)), -1/8*(((8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b + 8*a
*b^2 - b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b + 8*a*b^2 - b^3)*sinh
(f*x + e)^12 + 2*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^10 + 2*(
16*a^3 + 24*a^2*b + 6*a*b^2 - b^3 + 33*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x +
 e)^2)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^3
+ (16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e)^9 + (128
*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^8 + (495*(8*a^2*b + 8*a*b^
2 - b^3)*cosh(f*x + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 90*(16*a^
3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(8*a
^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^5 + 30*(16*a^3 + 24*a^2*b + 6*a*b^2 - b
^3)*cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e))
*sinh(f*x + e)^7 + 4*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^6 +
 4*(231*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^6 + 105*(16*a^3 + 24*a^2*b
+ 6*a*b^2 - b^3)*cosh(f*x + e)^4 + 48*a^3 + 40*a^2*b - 14*a*b^2 + b^3 + 7*(
128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*
(99*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^7 + 63*(16*a^3 + 24*a^2*b + 6*a
*b^2 - b^3)*cosh(f*x + e)^5 + 7*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh
(f*x + e)^3 + 3*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e))*sinh(f*
x + e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^4 + (495*(8
*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^8 + 420*(16*a^3 + 24*a^2*b + 6*a*b^2
- b^3)*cosh(f*x + e)^6 + 70*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x
 + e)^4 + 128*a^3 + 120*a^2*b - 24*a*b^2 + b^3 + 60*(48*a^3 + 40*a^2*b - 14
*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(8*a^2*b + 8*a*b^2 -
 b^3)*cosh(f*x + e)^9 + 60*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e
)^7 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^5 + 20*(48*a^
3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^3 + (128*a^3 + 120*a^2*b - 24*
a*b^2 + b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b + 8*a*b^2 - b^3 + 2*(
16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^2 + 2*(33*(8*a^2*b + 8*a*b
^2 - b^3)*cosh(f*x + e)^10 + 45*(16*a^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*
x + e)^8 + 14*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^6 + 30*(
48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(f*x + e)^4 + 16*a^3 + 24*a^2*b + 6
*a*b^2 - b^3 + 3*(128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^2)*si
nh(f*x + e)^2 + 4*(3*(8*a^2*b + 8*a*b^2 - b^3)*cosh(f*x + e)^11 + 5*(16*a^3
 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e)^9 + 2*(128*a^3 + 120*a^2*b - 24*
a*b^2 + b^3)*cosh(f*x + e)^7 + 6*(48*a^3 + 40*a^2*b - 14*a*b^2 + b^3)*cosh(
f*x + e)^5 + (128*a^3 + 120*a^2*b - 24*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a
^3 + 24*a^2*b + 6*a*b^2 - b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*a
rctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e
)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((8*a^3
- 9*a*b^2 + b^3)*cosh(f*x + e)^9 + 9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)*
sinh(f*x + e)^8 + (8*a^3 - 9*a*b^2 + b^3)*sinh(f*x + e)^9 + 4*(16*a^3 - 19*
a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^7 + 4*(16*a^3 - 19*a^2*b + 5*a*b^2 -
 2*b^3 + 9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(3
*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^3 + (16*a^3 - 19*a^2*b + 5*a*b^2 - 2
*b^3)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*
b^3)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^4 + 40*a
^3 - 28*a^2*b - 19*a*b^2 + 7*b^3 + 42*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)
*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x
+ e)^5 + 70*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + 5*(40*a
^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(16*a^
3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^3 + 4*(21*(8*a^3 - 9*a*b^2 +
b^3)*cosh(f*x + e)^6 + 35*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x +
e)^4 + 16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3 + 5*(40*a^3 - 28*a^2*b - 19*a*b^
2 + 7*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(9*(8*a^3 - 9*a*b^2 + b^3)*
cosh(f*x + e)^7 + 21*(16*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^5
+ 5*(40*a^3 - 28*a^2*b - 19*a*b^2 + 7*b^3)*cosh(f*x + e)^3 + 3*(16*a^3 - 19
```

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*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^3 - 9*a*b^2
 + b^3)*cosh(f*x + e) + (9*(8*a^3 - 9*a*b^2 + b^3)*cosh(f*x + e)^8 + 28*(16
*a^3 - 19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^6 + 10*(40*a^3 - 28*a^2*b
- 19*a*b^2 + 7*b^3)*cosh(f*x + e)^4 + 8*a^3 - 9*a*b^2 + b^3 + 12*(16*a^3 -
19*a^2*b + 5*a*b^2 - 2*b^3)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^
4 + b^5)*f*cosh(f*x + e)^12 + 12*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 +
 b^5)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4
*a*b^4 + b^5)*f*sinh(f*x + e)^12 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b
^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^10 + 2*(33*(a^4*b - 4*a^3*b^2 + 6*a^2*b
^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^
2*b^3 - 2*a*b^4 + b^5)*f)*sinh(f*x + e)^10 + (16*a^5 - 65*a^4*b + 100*a^3*b
^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^8 + 20*(11*(a^4*b - 4*a^3
*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (2*a^5 - 7*a^4*b + 8*
a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e)^9 + (49
5*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^4 + 90*(2
*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^2 +
 (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f)*sinh(f*
x + e)^8 + 4*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*
f*cosh(f*x + e)^6 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f
*cosh(f*x + e)^5 + 30*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 +
b^5)*f*cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20
*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231*(a^4*b - 4*a^3*b^2
+ 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 105*(2*a^5 - 7*a^4*b + 8*a
^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^4 + 7*(16*a^5 - 65*a^4*
b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^2 + (6*a^5 -
 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f)*sinh(f*x + e)^6 +
(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x
+ e)^4 + 8*(99*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x +
 e)^7 + 63*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh
(f*x + e)^5 + 7*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 -
b^5)*f*cosh(f*x + e)^3 + 3*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10
*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*(a^4*b - 4*a^3*b^2 +
6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^8 + 420*(2*a^5 - 7*a^4*b + 8*a^3
*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 70*(16*a^5 - 65*a^4*b
 + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^4 + 60*(6*a^5
 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^2 +
 (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f)*sinh(f*
x + e)^4 + 2*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*co
sh(f*x + e)^2 + 4*(55*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cos
h(f*x + e)^9 + 60*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)
*f*cosh(f*x + e)^7 + 14*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*
a*b^4 - b^5)*f*cosh(f*x + e)^5 + 20*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2
*b^3 + 10*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (16*a^5 - 65*a^4*b + 100*a^3*b^2
 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(33*(a
^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x + e)^10 + 45*(2*a^
5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e)^8 + 14
*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x
 + e)^6 + 30*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 + 10*a*b^4 - b^5)*
f*cosh(f*x + e)^4 + 3*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*
b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2
*a*b^4 + b^5)*f)*sinh(f*x + e)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4
 + b^5)*f + 4*(3*(a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*f*cosh(f*x
 + e)^11 + 5*(2*a^5 - 7*a^4*b + 8*a^3*b^2 - 2*a^2*b^3 - 2*a*b^4 + b^5)*f*co
sh(f*x + e)^9 + 2*(16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a^2*b^3 + 20*a*b^4
- b^5)*f*cosh(f*x + e)^7 + 6*(6*a^5 - 25*a^4*b + 40*a^3*b^2 - 30*a^2*b^3 +
10*a*b^4 - b^5)*f*cosh(f*x + e)^5 + (16*a^5 - 65*a^4*b + 100*a^3*b^2 - 70*a
^2*b^3 + 20*a*b^4 - b^5)*f*cosh(f*x + e)^3 + (2*a^5 - 7*a^4*b + 8*a^3*b^2 -
```

```
2*a^2*b^3 - 2*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.491 $\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=122

$$\frac{2a+b}{2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}+\frac{\operatorname{sech}^2(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^(5
/2)*f) + (2*a + b)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + Sech[e + f
*x]^2/(2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.139628, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a+b}{2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{5/2}}+\frac{\operatorname{sech}^2(e+fx)}{2f(a-b)\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^(5
/2)*f) + (2*a + b)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + Sech[e + f
*x]^2/(2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{4(a-b)f}$$

$$= \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2(e+fx)}{2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a+b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{5/2}f} + \frac{2a+b}{2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\operatorname{sech}^2}{2(a-b)f\sqrt{a+}}$$

Mathematica [C] time = 0.117849, size = 79, normalized size = 0.65

$$\frac{(2a+b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)+a}{a-b}\right)+(a-b)\operatorname{sech}^2(e+fx)}{2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a + b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sinh[e + f*x]^2)/(a - b)]
 + (a - b)*Sech[e + f*x]^2)/(2*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] time = 0.195, size = 103, normalized size = 0.8

$$\frac{1}{f}\text{` int/indef0`}\left(-\frac{\left(\sinh(fx+e)\right)^3\left(\cosh(fx+e)\right)^2}{-b^2\left(\cosh(fx+e)\right)^{10}+\left(-2ab+2b^2\right)\left(\cosh(fx+e)\right)^8+\left(-a^2+2ab-b^2\right)\left(\cosh(fx+e)\right)^6}\sqrt{a}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^3*(a+b*sinh(f*x+e)^2)^(1/2)*cosh(f*x+e)^2/(-b^2*c
osh(f*x+e)^10+(-2*a*b+2*b^2)*cosh(f*x+e)^8+(-a^2+2*a*b-b^2)*cosh(f*x+e)^6),
sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh \left(fx+e\right)^{3}}{\left(b\sinh \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.54302, size = 9603, normalized size = 78.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a*b + b^2)*cosh(f*x + e)^8 + 8*(2*a*b + b^2)*cosh(f*x + e)*sinh(f
*x + e)^7 + (2*a*b + b^2)*sinh(f*x + e)^8 + 4*(2*a^2 + a*b)*cosh(f*x + e)^6
 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*sinh(f*x + e)^6 + 8*(7
*(2*a*b + b^2)*cosh(f*x + e)^3 + 3*(2*a^2 + a*b)*cosh(f*x + e))*sinh(f*x +
e)^5 + 2*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b + b^2)*cosh(f
*x + e)^4 + 30*(2*a^2 + a*b)*cosh(f*x + e)^2 + 8*a^2 + 2*a*b - b^2)*sinh(f*
x + e)^4 + 8*(7*(2*a*b + b^2)*cosh(f*x + e)^5 + 10*(2*a^2 + a*b)*cosh(f*x +
 e)^3 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 + a
*b)*cosh(f*x + e)^2 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^6 + 15*(2*a^2 + a*b)
*cosh(f*x + e)^4 + 3*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*s
inh(f*x + e)^2 + 2*a*b + b^2 + 8*((2*a*b + b^2)*cosh(f*x + e)^7 + 3*(2*a^2
+ a*b)*cosh(f*x + e)^5 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^3 + (2*a^2 + a
*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*
cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x
+ e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sq
rt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) +
 sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*
x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x +
 e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(
cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*((2*a^2 -
a*b - b^2)*cosh(f*x + e)^5 + 5*(2*a^2 - a*b - b^2)*cosh(f*x + e)*sinh(f*x +
 e)^4 + (2*a^2 - a*b - b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2)*cosh(
f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 5*a*b + b^2
)*sinh(f*x + e)^3 + 2*(5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^3 + 3*(4*a^2 - 5
*a*b + b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a^2 - a*b - b^2)*cosh(f*x +
```

```
e) + (5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^4 + 6*(4*a^2 - 5*a*b + b^2)*cosh
(f*x + e)^2 + 2*a^2 - a*b - b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b
*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e
) + sinh(f*x + e)^2)))/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)
^8 + 8*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7
+ (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*sinh(f*x + e)^8 + 4*(a^4 - 3*a^3*b
+ 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^6 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3
- b^4)*f*cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*x
+ e)^6 + 2*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e)^
4 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^3 + 3*(a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3*b
 - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^4 + 30*(a^4 - 3*a^3*b + 3*a^2
*b^2 - a*b^3)*f*cosh(f*x + e)^2 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3
+ b^4)*f)*sinh(f*x + e)^4 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*
x + e)^2 + 8*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^5 + 10*
(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^3 + (4*a^4 - 13*a^3*b +
 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^3*b
 - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^6 + 15*(a^4 - 3*a^3*b + 3*a^2
*b^2 - a*b^3)*f*cosh(f*x + e)^4 + 3*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^
3 + b^4)*f*cosh(f*x + e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*
x + e)^2 + (a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f + 8*((a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*f*cosh(f*x + e)^7 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*
cosh(f*x + e)^5 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*
x + e)^3 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x +
e)), -1/2*(((2*a*b + b^2)*cosh(f*x + e)^8 + 8*(2*a*b + b^2)*cosh(f*x + e)*s
inh(f*x + e)^7 + (2*a*b + b^2)*sinh(f*x + e)^8 + 4*(2*a^2 + a*b)*cosh(f*x +
 e)^6 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2 + a*b)*sinh(f*x + e)^6 +
 8*(7*(2*a*b + b^2)*cosh(f*x + e)^3 + 3*(2*a^2 + a*b)*cosh(f*x + e))*sinh(f
*x + e)^5 + 2*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^4 + 2*(35*(2*a*b + b^2)*c
osh(f*x + e)^4 + 30*(2*a^2 + a*b)*cosh(f*x + e)^2 + 8*a^2 + 2*a*b - b^2)*si
nh(f*x + e)^4 + 8*(7*(2*a*b + b^2)*cosh(f*x + e)^5 + 10*(2*a^2 + a*b)*cosh(
f*x + e)^3 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^
2 + a*b)*cosh(f*x + e)^2 + 4*(7*(2*a*b + b^2)*cosh(f*x + e)^6 + 15*(2*a^2 +
 a*b)*cosh(f*x + e)^4 + 3*(8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^2 + 2*a^2 + a
*b)*sinh(f*x + e)^2 + 2*a*b + b^2 + 8*((2*a*b + b^2)*cosh(f*x + e)^7 + 3*(2
*a^2 + a*b)*cosh(f*x + e)^5 + (8*a^2 + 2*a*b - b^2)*cosh(f*x + e)^3 + (2*a^
2 + a*b)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqr
t(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x
+ e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*
x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((2*a^2 - a*b - b^2)*cosh(f*x
+ e)^5 + 5*(2*a^2 - a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a^2 - a*b
 - b^2)*sinh(f*x + e)^5 + 2*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^3 + 2*(5*(2
*a^2 - a*b - b^2)*cosh(f*x + e)^2 + 4*a^2 - 5*a*b + b^2)*sinh(f*x + e)^3 +
2*(5*(2*a^2 - a*b - b^2)*cosh(f*x + e)^3 + 3*(4*a^2 - 5*a*b + b^2)*cosh(f*x
 + e))*sinh(f*x + e)^2 + (2*a^2 - a*b - b^2)*cosh(f*x + e) + (5*(2*a^2 - a*
b - b^2)*cosh(f*x + e)^4 + 6*(4*a^2 - 5*a*b + b^2)*cosh(f*x + e)^2 + 2*a^2
- a*b - b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
))/((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^8 + 8*(a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^3*b - 3*a^2*b^2
 + 3*a*b^3 - b^4)*f*sinh(f*x + e)^8 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)
*f*cosh(f*x + e)^6 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x +
e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^6 + 2*(4*a^4 -
13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e)^4 + 8*(7*(a^3*b - 3*
a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^3 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 -
 a*b^3)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3*b - 3*a^2*b^2 + 3*a*b
^3 - b^4)*f*cosh(f*x + e)^4 + 30*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh
(f*x + e)^2 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f)*sinh(f*x +
 e)^4 + 4*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^2 + 8*(7*(a^3
```

```
*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cosh(f*x + e)^5 + 10*(a^4 - 3*a^3*b + 3*a
^2*b^2 - a*b^3)*f*cosh(f*x + e)^3 + (4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^
3 + b^4)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^3*b - 3*a^2*b^2 + 3*a*b
^3 - b^4)*f*cosh(f*x + e)^6 + 15*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh
(f*x + e)^4 + 3*(4*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x
+ e)^2 + (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f)*sinh(f*x + e)^2 + (a^3*b -
3*a^2*b^2 + 3*a*b^3 - b^4)*f + 8*((a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*f*cos
h(f*x + e)^7 + 3*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e)^5 + (4
*a^4 - 13*a^3*b + 15*a^2*b^2 - 7*a*b^3 + b^4)*f*cosh(f*x + e)^3 + (a^4 - 3*
a^3*b + 3*a^2*b^2 - a*b^3)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^3\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.492 $\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=69

$$\frac{1}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + 1/(
(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0738817, antiderivative size = 69, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(3/2)*f)) + 1/(
(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}\\
&= \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2(a-b)f}\\
&= \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{(a-b)bf}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{3/2}f} + \frac{1}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}}
\end{aligned}$$

Mathematica [C] time = 0.0781408, size = 58, normalized size = 0.84

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\cosh^2(e+fx)}{a-b}+1\right)}{f(b-a)\sqrt{a+b\cosh^2(e+fx)-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/((-a + b
)*f*Sqrt[a - b + b*Cosh[e + f*x]^2]))
```

Maple [C] time = 0.123, size = 93, normalized size = 1.4

$$\frac{1}{f}\text{` int/indef0`}\left(-\frac{\sinh(fx+e)}{-b^2\left(\sinh(fx+e)\right)^6+\left(-2ab-b^2\right)\left(\sinh(fx+e)\right)^4+\left(-a^2-2ab\right)\left(\sinh(fx+e)\right)^2-a^2}\sqrt{a-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)*(a+b*sinh(f*x+e)^2)^(1/2)/(-b^2*sinh(f*x+e)^6+(-2
*a*b-b^2)*sinh(f*x+e)^4+(-a^2-2*a*b)*sinh(f*x+e)^2-a^2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)}{\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.67186, size = 3475, normalized size = 50.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x
+ e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*si
nh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x +
e) + b)*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e
)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
 e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cos
h(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)
^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)
^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x
 + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((a - b)*cosh(f*x + e) + (a - b)*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^2*b - 2*a
*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)*s
inh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f*sinh(f*x + e)^4 + 2*(2*a^3 - 5*a
^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 + 2*(3*(a^2*b - 2*a*b^2 + b^3)*f*co
sh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f)*sinh(f*x + e)^2 + (a^2
*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^3 + (2*a
^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e))*sinh(f*x + e)), -((b*cosh(f*
x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(2*a -
 b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4
*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)*sqrt(-a +
 b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x +
 e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*
x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*((a
 - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*si
nh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) +
 sinh(f*x + e)^2)))/((a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^4 + 4*(a^2*b -
 2*a*b^2 + b^3)*f*cosh(f*x + e)*sinh(f*x + e)^3 + (a^2*b - 2*a*b^2 + b^3)*f
*sinh(f*x + e)^4 + 2*(2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x + e)^2 +
2*(3*(a^2*b - 2*a*b^2 + b^3)*f*cosh(f*x + e)^2 + (2*a^3 - 5*a^2*b + 4*a*b^2
 - b^3)*f)*sinh(f*x + e)^2 + (a^2*b - 2*a*b^2 + b^3)*f + 4*((a^2*b - 2*a*b^
2 + b^3)*f*cosh(f*x + e)^3 + (2*a^3 - 5*a^2*b + 4*a*b^2 - b^3)*f*cosh(f*x +
 e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.493 $\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
 + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0840825, antiderivative size = 57, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
 + b*Sinh[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^2(e+fx)\right)}{2af}$$

$$= \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^2(e+fx)}\right)}{abf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Mathematica [C] time = 0.0595502, size = 46, normalized size = 0.81

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)}{af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(a*f*Sqrt[a + b*
Sinh[e + f*x]^2])
```

Maple [C] time = 0.063, size = 35, normalized size = 0.6

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{1}{\sinh\left(fx+e\right)}\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(1/sinh(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 2.3862, size = 2955, normalized size = 51.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x +
 e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 2*a - b)*sin
h(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x + e))*sinh(f*x + e
) + b)*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 +
 b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 +
 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*s
inh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
+ sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3
+ (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x
 + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*
x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x
 + e) + 1)) + 4*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*
x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*
sinh(f*x + e) + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 + 4*a^2*b*f*cos
h(f*x + e)*sinh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b*f + 2*(2*a^3 -
 a^2*b)*f*cosh(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 + (2*a^3 - a^2*b)*
f)*sinh(f*x + e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 - a^2*b)*f*cosh(f*
x + e))*sinh(f*x + e)), ((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x +
e)^3 + b*sinh(f*x + e)^4 + 2*(2*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x +
e)^2 + 2*a - b)*sinh(f*x + e)^2 + 4*(b*cosh(f*x + e)^3 + (2*a - b)*cosh(f*x
 + e))*sinh(f*x + e) + b)*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh
(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x +
e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) +
 2*sqrt(2)*(a*cosh(f*x + e) + a*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*
sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e)
 + sinh(f*x + e)^2)))/(a^2*b*f*cosh(f*x + e)^4 + 4*a^2*b*f*cosh(f*x + e)*si
nh(f*x + e)^3 + a^2*b*f*sinh(f*x + e)^4 + a^2*b*f + 2*(2*a^3 - a^2*b)*f*cos
h(f*x + e)^2 + 2*(3*a^2*b*f*cosh(f*x + e)^2 + (2*a^3 - a^2*b)*f)*sinh(f*x +
 e)^2 + 4*(a^2*b*f*cosh(f*x + e)^3 + (2*a^3 - a^2*b)*f*cosh(f*x + e))*sinh(
f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth \left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.494 $\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}$$

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) +
 (2*a - 3*b)/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Csch[e + f*x]^2/(2*a*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.135962, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\operatorname{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) +
 (2*a - 3*b)/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) - Csch[e + f*x]^2/(2*a*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
```

```
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x}{x^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{4af}$$

$$= \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\right.}{4a^2f}$$

$$= \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^2(e+fx)}{2af\sqrt{a+b\sinh^2(e+fx)}} + \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\right.}{2a}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} + \frac{2a-3b}{2a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^2(e+}{2af\sqrt{a+b\sinh}}$$

Mathematica [C] time = 0.102635, size = 69, normalized size = 0.63

$$\frac{(2a-3b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)-a\,\text{csch}^2(e+fx)}{2a^2f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Csch[e + f*x]^2) + (2*a - 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*
Sinh[e + f*x]^2)/a])/(2*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] time = 0.104, size = 43, normalized size = 0.4

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{\left(\cosh(fx+e)\right)^2}{\left(\sinh(fx+e)\right)^3}\left(a+b\left(\sinh(fx+e)\right)^2\right)^{-\frac{3}{2}}, \sinh(fx+e)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/sinh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x
+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 3.05572, size = 7960, normalized size = 72.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a*b - 3*b^2)*cosh(f*x + e)^8 + 8*(2*a*b - 3*b^2)*cosh(f*x + e)*s
inh(f*x + e)^7 + (2*a*b - 3*b^2)*sinh(f*x + e)^8 + 4*(2*a^2 - 5*a*b + 3*b^2
)*cosh(f*x + e)^6 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b +
3*b^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 -
5*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(8*a^2 - 18*a*b + 9*b^2)*
cosh(f*x + e)^4 + 2*(35*(2*a*b - 3*b^2)*cosh(f*x + e)^4 + 30*(2*a^2 - 5*a*b
 + 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 18*a*b - 9*b^2)*sinh(f*x + e)^4 + 8*(7*
(2*a*b - 3*b^2)*cosh(f*x + e)^5 + 10*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^
3 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 - 5*
a*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^6 + 15*(2
*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^4 - 3*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x
 + e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^2 + 2*a*b - 3*b^2 + 8*((2*a*
b - 3*b^2)*cosh(f*x + e)^7 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^5 - (8
*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^3 + (2*a^2 - 5*a*b + 3*b^2)*cosh(f*x +
 e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh
(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh
(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x
 + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*s
inh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cos
h(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4
 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2
 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x +
 e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*((2*a^2 - 3*a*b)*cosh(f*x + e)^5 + 5*(
2*a^2 - 3*a*b)*cosh(f*x + e)*sinh(f*x + e)^4 + (2*a^2 - 3*a*b)*sinh(f*x + e
)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e
)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^3
 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + (2*a^2 - 3*a*b)*cosh(
```

```
f*x + e) + (5*(2*a^2 - 3*a*b)*cosh(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cosh(f*x
+ e)^2 + 2*a^2 - 3*a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x
 + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(
f*x + e)^2)))/(a^3*b*f*cosh(f*x + e)^8 + 8*a^3*b*f*cosh(f*x + e)*sinh(f*x +
 e)^7 + a^3*b*f*sinh(f*x + e)^8 + 4*(a^4 - a^3*b)*f*cosh(f*x + e)^6 + 4*(7*
a^3*b*f*cosh(f*x + e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^6 - 2*(4*a^4 - 3*a
^3*b)*f*cosh(f*x + e)^4 + 8*(7*a^3*b*f*cosh(f*x + e)^3 + 3*(a^4 - a^3*b)*f*
cosh(f*x + e))*sinh(f*x + e)^5 + a^3*b*f + 2*(35*a^3*b*f*cosh(f*x + e)^4 +
30*(a^4 - a^3*b)*f*cosh(f*x + e)^2 - (4*a^4 - 3*a^3*b)*f)*sinh(f*x + e)^4 +
 4*(a^4 - a^3*b)*f*cosh(f*x + e)^2 + 8*(7*a^3*b*f*cosh(f*x + e)^5 + 10*(a^4
 - a^3*b)*f*cosh(f*x + e)^3 - (4*a^4 - 3*a^3*b)*f*cosh(f*x + e))*sinh(f*x +
 e)^3 + 4*(7*a^3*b*f*cosh(f*x + e)^6 + 15*(a^4 - a^3*b)*f*cosh(f*x + e)^4 -
 3*(4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^2 +
 8*(a^3*b*f*cosh(f*x + e)^7 + 3*(a^4 - a^3*b)*f*cosh(f*x + e)^5 - (4*a^4 -
3*a^3*b)*f*cosh(f*x + e)^3 + (a^4 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)),
 1/2*(((2*a*b - 3*b^2)*cosh(f*x + e)^8 + 8*(2*a*b - 3*b^2)*cosh(f*x + e)*si
nh(f*x + e)^7 + (2*a*b - 3*b^2)*sinh(f*x + e)^8 + 4*(2*a^2 - 5*a*b + 3*b^2)
*cosh(f*x + e)^6 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^2 + 2*a^2 - 5*a*b + 3
*b^2)*sinh(f*x + e)^6 + 8*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^3 + 3*(2*a^2 - 5
*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^5 - 2*(8*a^2 - 18*a*b + 9*b^2)*c
osh(f*x + e)^4 + 2*(35*(2*a*b - 3*b^2)*cosh(f*x + e)^4 + 30*(2*a^2 - 5*a*b
+ 3*b^2)*cosh(f*x + e)^2 - 8*a^2 + 18*a*b - 9*b^2)*sinh(f*x + e)^4 + 8*(7*(
2*a*b - 3*b^2)*cosh(f*x + e)^5 + 10*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^3
 - (8*a^2 - 18*a*b + 9*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a^2 - 5*a
*b + 3*b^2)*cosh(f*x + e)^2 + 4*(7*(2*a*b - 3*b^2)*cosh(f*x + e)^6 + 15*(2*
a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^4 - 3*(8*a^2 - 18*a*b + 9*b^2)*cosh(f*x
+ e)^2 + 2*a^2 - 5*a*b + 3*b^2)*sinh(f*x + e)^2 + 2*a*b - 3*b^2 + 8*((2*a*b
 - 3*b^2)*cosh(f*x + e)^7 + 3*(2*a^2 - 5*a*b + 3*b^2)*cosh(f*x + e)^5 - (8*
a^2 - 18*a*b + 9*b^2)*cosh(f*x + e)^3 + (2*a^2 - 5*a*b + 3*b^2)*cosh(f*x +
e))*sinh(f*x + e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt
(2)*((2*a^2 - 3*a*b)*cosh(f*x + e)^5 + 5*(2*a^2 - 3*a*b)*cosh(f*x + e)*sinh
(f*x + e)^4 + (2*a^2 - 3*a*b)*sinh(f*x + e)^5 - 2*(4*a^2 - 3*a*b)*cosh(f*x
+ e)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^2 - 4*a^2 + 3*a*b)*sinh(f*x + e
)^3 + 2*(5*(2*a^2 - 3*a*b)*cosh(f*x + e)^3 - 3*(4*a^2 - 3*a*b)*cosh(f*x + e
))*sinh(f*x + e)^2 + (2*a^2 - 3*a*b)*cosh(f*x + e) + (5*(2*a^2 - 3*a*b)*cos
h(f*x + e)^4 - 6*(4*a^2 - 3*a*b)*cosh(f*x + e)^2 + 2*a^2 - 3*a*b)*sinh(f*x
+ e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)
^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^3*b*f*cosh(f*x +
 e)^8 + 8*a^3*b*f*cosh(f*x + e)*sinh(f*x + e)^7 + a^3*b*f*sinh(f*x + e)^8 +
 4*(a^4 - a^3*b)*f*cosh(f*x + e)^6 + 4*(7*a^3*b*f*cosh(f*x + e)^2 + (a^4 -
a^3*b)*f)*sinh(f*x + e)^6 - 2*(4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^4 + 8*(7*a^
3*b*f*cosh(f*x + e)^3 + 3*(a^4 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)^5 +
a^3*b*f + 2*(35*a^3*b*f*cosh(f*x + e)^4 + 30*(a^4 - a^3*b)*f*cosh(f*x + e)^
2 - (4*a^4 - 3*a^3*b)*f)*sinh(f*x + e)^4 + 4*(a^4 - a^3*b)*f*cosh(f*x + e)^
2 + 8*(7*a^3*b*f*cosh(f*x + e)^5 + 10*(a^4 - a^3*b)*f*cosh(f*x + e)^3 - (4*
a^4 - 3*a^3*b)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*b*f*cosh(f*x + e
)^6 + 15*(a^4 - a^3*b)*f*cosh(f*x + e)^4 - 3*(4*a^4 - 3*a^3*b)*f*cosh(f*x +
 e)^2 + (a^4 - a^3*b)*f)*sinh(f*x + e)^2 + 8*(a^3*b*f*cosh(f*x + e)^7 + 3*(
a^4 - a^3*b)*f*cosh(f*x + e)^5 - (4*a^4 - 3*a^3*b)*f*cosh(f*x + e)^3 + (a^4
 - a^3*b)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^3\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**3/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^3}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.495 $\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\left(8a^2-24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}-\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\text{csch}}{4af\sqrt{a+}}$$

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(
8*a^(7/2)*f) + (8*a^2 - 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sinh[e + f*x]^
2]) - ((8*a - 5*b)*Csch[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) -
 Csch[e + f*x]^4/(4*a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.201072, antiderivative size = 167, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\left(8a^2-24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}-\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\text{csch}}{4af\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 - 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(
8*a^(7/2)*f) + (8*a^2 - 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sinh[e + f*x]^
2]) - ((8*a - 5*b)*Csch[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) -
 Csch[e + f*x]^4/(4*a*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
```

```
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-5b)+2ax}{x^2(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{4af}$$

$$= -\frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(8a^2-24ab+15b^2\right)\text{Subst}\,(\cdots}{1\cdots}$$

$$= \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \cdots$$

$$= \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5b)\text{csch}^2(e+fx)}{8a^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\text{csch}^4(e+fx)}{4af\sqrt{a+b\sinh^2(e+fx)}} + \cdots$$

$$= -\frac{\left(8a^2-24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{8a^2-24ab+15b^2}{8a^3f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(8a-5\cdots}{8a^2f\sqrt{a\cdots}}$$

Mathematica [C] time = 0.327152, size = 94, normalized size = 0.56

$$\frac{\left(8a^2-24ab+15b^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)+a\text{csch}^2(e+fx)\left(-2a\text{csch}^2(e+fx)-8a+5b\right)}{8a^3f\sqrt{a+b\sinh^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Csch[e + f*x]^2*(-8*a + 5*b - 2*a*Csch[e + f*x]^2) + (8*a^2 - 24*a*b + 1
5*b^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sinh[e + f*x]^2)/a])/(8*a^3*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

Maple [C] time = 0.09, size = 43, normalized size = 0.3

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{\left(\cosh\left(fx+e\right)\right)^4}{\left(\sinh\left(fx+e\right)\right)^5}\left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^{-\frac{3}{2}},\sinh\left(fx+e\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^4/sinh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),sinh(f*x
+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [B] time = 4.58169, size = 19003, normalized size = 113.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^12 + 12*(8*a^2*b - 24*a
*b^2 + 15*b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (8*a^2*b - 24*a*b^2 + 15*b^
3)*sinh(f*x + e)^12 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x +
```

```
e)^10 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 + 33*(8*a^2*b - 24*a*b^2
 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(8*a^2*b - 24*a*b^2 +
 15*b^3)*cosh(f*x + e)^3 + (16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*
x + e))*sinh(f*x + e)^9 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(
f*x + e)^8 + (495*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^4 - 128*a^3 +
 504*a^2*b - 600*a*b^2 + 225*b^3 + 90*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b
^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*
cosh(f*x + e)^5 + 30*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)
^3 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e))*sinh(f*x +
e)^7 + 4*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^6 + 4*(231
*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^6 + 105*(16*a^3 - 72*a^2*b + 1
02*a*b^2 - 45*b^3)*cosh(f*x + e)^4 + 48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^
3 - 7*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x
 + e)^6 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^7 + 63*(16*a^3
- 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^5 - 7*(128*a^3 - 504*a^2*b +
 600*a*b^2 - 225*b^3)*cosh(f*x + e)^3 + 3*(48*a^3 - 184*a^2*b + 210*a*b^2 -
 75*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2
- 225*b^3)*cosh(f*x + e)^4 + (495*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x +
e)^8 + 420*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^6 - 70*(1
28*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^4 - 128*a^3 + 504*a
^2*b - 600*a*b^2 + 225*b^3 + 60*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*c
osh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(
f*x + e)^9 + 60*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^7 -
14*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^5 + 20*(48*a^3
 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - (128*a^3 - 504*a^2*b +
 600*a*b^2 - 225*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b - 24*a*b^2 +
 15*b^3 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^2 + 2*(3
3*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^10 + 45*(16*a^3 - 72*a^2*b +
102*a*b^2 - 45*b^3)*cosh(f*x + e)^8 - 14*(128*a^3 - 504*a^2*b + 600*a*b^2 -
 225*b^3)*cosh(f*x + e)^6 + 30*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*co
sh(f*x + e)^4 + 16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 - 3*(128*a^3 - 504*a
^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(8*a^2*
b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^11 + 5*(16*a^3 - 72*a^2*b + 102*a*b^2
- 45*b^3)*cosh(f*x + e)^9 - 2*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*c
osh(f*x + e)^7 + 6*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^
5 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^3 + (16*a^3 -
 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a)*log((
b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 +
 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - b)*sinh(f*x +
 e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a
 - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*
(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - b)*cosh(f*x
 + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^
3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*
x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt
(2)*((8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^9 + 9*(8*a^3 - 24*a^2*b +
15*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^3 - 24*a^2*b + 15*a*b^2)*sin
h(f*x + e)^9 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 4*(16*a^3
 - 29*a^2*b + 15*a*b^2 - 9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*s
inh(f*x + e)^7 + 28*(3*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 - (16*
a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 92*
a^2*b + 45*a*b^2)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)*cos
h(f*x + e)^4 + 40*a^3 - 92*a^2*b + 45*a*b^2 - 42*(16*a^3 - 29*a^2*b + 15*a*
b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)
*cosh(f*x + e)^5 - 70*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 5*(4
0*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 - 4*(16*a^3 - 2
9*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^3 - 24*a^2*b + 15*a*b^2)*c
osh(f*x + e)^6 - 35*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^4 - 16*a^3
 + 29*a^2*b - 15*a*b^2 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^2)*
```

sinh(f*x + e)^3 + 4*(9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 21*(
16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 + 5*(40*a^3 - 92*a^2*b + 45*a
*b^2)*cosh(f*x + e)^3 - 3*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sin
h(f*x + e)^2 + (8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e) + (9*(8*a^3 - 24
*a^2*b + 15*a*b^2)*cosh(f*x + e)^8 - 28*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh
(f*x + e)^6 + 10*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^4 + 8*a^3 - 2
4*a^2*b + 15*a*b^2 - 12*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sin
h(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*
x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^4*b*f*cosh
(f*x + e)^12 + 12*a^4*b*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b*f*sinh(f*x
 + e)^12 + 2*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^10 + 2*(33*a^4*b*f*cosh(f*x
+ e)^2 + (2*a^5 - 3*a^4*b)*f)*sinh(f*x + e)^10 - (16*a^5 - 15*a^4*b)*f*cosh
(f*x + e)^8 + 20*(11*a^4*b*f*cosh(f*x + e)^3 + (2*a^5 - 3*a^4*b)*f*cosh(f*x
 + e))*sinh(f*x + e)^9 + (495*a^4*b*f*cosh(f*x + e)^4 + 90*(2*a^5 - 3*a^4*b
)*f*cosh(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*sinh(f*x + e)^8 + 4*(6*a^5 - 5
*a^4*b)*f*cosh(f*x + e)^6 + 8*(99*a^4*b*f*cosh(f*x + e)^5 + 30*(2*a^5 - 3*a
^4*b)*f*cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e
)^7 + 4*(231*a^4*b*f*cosh(f*x + e)^6 + 105*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e
)^4 - 7*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^2 + (6*a^5 - 5*a^4*b)*f)*sinh(f
*x + e)^6 + a^4*b*f - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^4 + 8*(99*a^4*b*f
*cosh(f*x + e)^7 + 63*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^5 - 7*(16*a^5 - 15*
a^4*b)*f*cosh(f*x + e)^3 + 3*(6*a^5 - 5*a^4*b)*f*cosh(f*x + e))*sinh(f*x +
e)^5 + (495*a^4*b*f*cosh(f*x + e)^8 + 420*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)
^6 - 70*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^4 + 60*(6*a^5 - 5*a^4*b)*f*cosh
(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*sinh(f*x + e)^4 + 2*(2*a^5 - 3*a^4*b)*
f*cosh(f*x + e)^2 + 4*(55*a^4*b*f*cosh(f*x + e)^9 + 60*(2*a^5 - 3*a^4*b)*f*
cosh(f*x + e)^7 - 14*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^5 + 20*(6*a^5 - 5*
a^4*b)*f*cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e))*sinh(f*x +
e)^3 + 2*(33*a^4*b*f*cosh(f*x + e)^10 + 45*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e
)^8 - 14*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^6 + 30*(6*a^5 - 5*a^4*b)*f*cos
h(f*x + e)^4 - 3*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^2 + (2*a^5 - 3*a^4*b)*
f)*sinh(f*x + e)^2 + 4*(3*a^4*b*f*cosh(f*x + e)^11 + 5*(2*a^5 - 3*a^4*b)*f*
cosh(f*x + e)^9 - 2*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^7 + 6*(6*a^5 - 5*a^
4*b)*f*cosh(f*x + e)^5 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^3 + (2*a^5 - 3
*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e)), 1/8*(((8*a^2*b - 24*a*b^2 + 15*b^3
)*cosh(f*x + e)^12 + 12*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)*sinh(f*
x + e)^11 + (8*a^2*b - 24*a*b^2 + 15*b^3)*sinh(f*x + e)^12 + 2*(16*a^3 - 72
*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^10 + 2*(16*a^3 - 72*a^2*b + 102*
a*b^2 - 45*b^3 + 33*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x
 + e)^10 + 20*(11*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^3 + (16*a^3 -
 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e))*sinh(f*x + e)^9 - (128*a^3 -
 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^8 + (495*(8*a^2*b - 24*a*b^
2 + 15*b^3)*cosh(f*x + e)^4 - 128*a^3 + 504*a^2*b - 600*a*b^2 + 225*b^3 + 9
0*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8
 + 8*(99*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^5 + 30*(16*a^3 - 72*a^
2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^3 - (128*a^3 - 504*a^2*b + 600*a*b^
2 - 225*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(48*a^3 - 184*a^2*b + 210*a
*b^2 - 75*b^3)*cosh(f*x + e)^6 + 4*(231*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(
f*x + e)^6 + 105*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^4 +
 48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3 - 7*(128*a^3 - 504*a^2*b + 600*a*b
^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(8*a^2*b - 24*a*b^2
+ 15*b^3)*cosh(f*x + e)^7 + 63*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cos
h(f*x + e)^5 - 7*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^
3 + 3*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e))*sinh(f*x + e
)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^4 + (495*(8
*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^8 + 420*(16*a^3 - 72*a^2*b + 102*
a*b^2 - 45*b^3)*cosh(f*x + e)^6 - 70*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225
*b^3)*cosh(f*x + e)^4 - 128*a^3 + 504*a^2*b - 600*a*b^2 + 225*b^3 + 60*(48*
a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*

(55*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^9 + 60*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^7 - 14*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^5 + 20*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 8*a^2*b - 24*a*b^2 + 15*b^3 + 2*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^2 + 2*(33*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^10 + 45*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^8 - 14*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^6 + 30*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + 16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3 - 3*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(8*a^2*b - 24*a*b^2 + 15*b^3)*cosh(f*x + e)^11 + 5*(16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e)^9 - 2*(128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^7 + 6*(48*a^3 - 184*a^2*b + 210*a*b^2 - 75*b^3)*cosh(f*x + e)^5 - (128*a^3 - 504*a^2*b + 600*a*b^2 - 225*b^3)*cosh(f*x + e)^3 + (16*a^3 - 72*a^2*b + 102*a*b^2 - 45*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*((8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^9 + 9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + (8*a^3 - 24*a^2*b + 15*a*b^2)*sinh(f*x + e)^9 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2 - 9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(3*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 - (16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^4 + 40*a^3 - 92*a^2*b + 45*a*b^2 - 42*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(63*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 - 70*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 - 4*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 4*(21*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^6 - 35*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^4 - 16*a^3 + 29*a^2*b - 15*a*b^2 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 - 21*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 + 5*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^3 - 3*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + (8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e) + (9*(8*a^3 - 24*a^2*b + 15*a*b^2)*cosh(f*x + e)^8 - 28*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^6 + 10*(40*a^3 - 92*a^2*b + 45*a*b^2)*cosh(f*x + e)^4 + 8*a^3 - 24*a^2*b + 15*a*b^2 - 12*(16*a^3 - 29*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^4*b*f*cosh(f*x + e)^12 + 12*a^4*b*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b*f*sinh(f*x + e)^12 + 2*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^10 + 2*(33*a^4*b*f*cosh(f*x + e)^2 + (2*a^5 - 3*a^4*b)*f)*sinh(f*x + e)^10 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^8 + 20*(11*a^4*b*f*cosh(f*x + e)^3 + (2*a^5 - 3*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e)^9 + (495*a^4*b*f*cosh(f*x + e)^4 + 90*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*sinh(f*x + e)^8 + 4*(6*a^5 - 5*a^4*b)*f*cosh(f*x + e)^6 + 8*(99*a^4*b*f*cosh(f*x + e)^5 + 30*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231*a^4*b*f*cosh(f*x + e)^6 + 105*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^4 - 7*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^2 + (6*a^5 - 5*a^4*b)*f)*sinh(f*x + e)^6 + a^4*b*f - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^4 + 8*(99*a^4*b*f*cosh(f*x + e)^7 + 63*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^5 - 7*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^3 + 3*(6*a^5 - 5*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*a^4*b*f*cosh(f*x + e)^8 + 420*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^6 - 70*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^4 + 60*(6*a^5 - 5*a^4*b)*f*cosh(f*x + e)^2 - (16*a^5 - 15*a^4*b)*f)*sinh(f*x + e)^4 + 2*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^2 + 4*(55*a^4*b*f*cosh(f*x + e)^9 + 60*(2*a^5 - 3*a^4*b)*f*cosh(f*x + e)^7 - 14*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^5 + 20*(6*a

```
^5 - 5*a^4*b)*f*cosh(f*x + e)^3 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e))*sinh
(f*x + e)^3 + 2*(33*a^4*b*f*cosh(f*x + e)^10 + 45*(2*a^5 - 3*a^4*b)*f*cosh(
f*x + e)^8 - 14*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^6 + 30*(6*a^5 - 5*a^4*b
)*f*cosh(f*x + e)^4 - 3*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^2 + (2*a^5 - 3*
a^4*b)*f)*sinh(f*x + e)^2 + 4*(3*a^4*b*f*cosh(f*x + e)^11 + 5*(2*a^5 - 3*a^
4*b)*f*cosh(f*x + e)^9 - 2*(16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^7 + 6*(6*a^5
 - 5*a^4*b)*f*cosh(f*x + e)^5 - (16*a^5 - 15*a^4*b)*f*cosh(f*x + e)^3 + (2*
a^5 - 3*a^4*b)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.496 $\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{(3a+5b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3f(a-b)^3\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{4a\tanh(e+fx)}{3f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+$$

```
[Out] -(Sqrt[a]*Sqrt[b]*(7*a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e
+ f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*
Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + 5*b)*EllipticF[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*
(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (4*a*Tanh[
e + f*x])/(3*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (Sech[e + f*x]^2*Ta
nh[e + f*x])/(3*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.275938, antiderivative size = 275, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3196, 470, 527, 525, 418, 411}

$$-\frac{4a\tanh(e+fx)}{3f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{3f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{\sqrt{a}\sqrt{b}(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right.}{3f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+f}{a+b\sinh^2(e+}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```



```
[Out] -(Sqrt[a]*Sqrt[b]*(7*a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e
+ f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*
Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + 5*b)*EllipticF[Arc
Tan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*
(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - (4*a*Tanh[
e + f*x])/(3*(a - b)^2*f*Sqrt[a + b*Sinh[e + f*x]^2]) + (Sech[e + f*x]^2*Ta
nh[e + f*x])/(3*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
```

```
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4}{\left(1+x^2\right)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\mathrm{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{a-3a}{\left(1+x^2\right)^{3/2}(}\right.}{3(a-b)f}$$

$$= -\frac{4a\tanh(e+fx)}{3(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\mathrm{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+\right.}{}$$

$$= -\frac{4a\tanh(e+fx)}{3(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\mathrm{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(ab(7a+b)\sqrt{\right.}{}$$

$$= -\frac{\sqrt{a}\sqrt{b}(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{3(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a+5b)F\left(\tan^{-1}(\sin\right.}{3(a}$$

Mathematica [C] time = 2.1843, size = 212, normalized size = 0.77

$$\frac{8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)-\frac{\tanh(e+fx)\text{sech}^2(e+fx)\left(4\left(4a^2+3ab+b^2\right)\cosh(2(e+fx))+8a^2+b(7a+b)\cosh(4(e+fx))\right.}{2\sqrt{2}}}{6f(a-b)^3\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*a*(7*a + b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e
+ f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - ((8*a^2 + 21*a*b - 5*b^2 + 4*(4*a^2 + 3*a*b + b^2
)*Cosh[2*(e + f*x)] + b*(7*a + b)*Cosh[4*(e + f*x)])*Sech[e + f*x]^2*Tanh[e
 + f*x])/(2*Sqrt[2]))/(6*(a - b)^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.204, size = 354, normalized size = 1.3

$$-\frac{1}{3\left(\cosh\left(fx+e\right)\right)^{3}(a-b)^{3}f}\left(\left(7\sqrt{-\frac{b}{a}}ab+\sqrt{-\frac{b}{a}}b^{2}\right)\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^{4}+\left(4\sqrt{-\frac{b}{a}}a^{2}-4\sqrt{-\frac{b}{a}}ab\right)\left(\cosh\left(\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*((7*(-1/a*b)^(1/2)*a*b+(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)*cosh(f*x+e)^4+(
4*(-1/a*b)^(1/2)*a^2-4*(-1/a*b)^(1/2)*a*b)*cosh(f*x+e)^2*sinh(f*x+e)+(-(-1/
a*b)^(1/2)*a^2+2*(-1/a*b)^(1/2)*a*b-(-1/a*b)^(1/2)*b^2)*sinh(f*x+e)-(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*EllipticF(sinh(f*x+e)*(
-1/a*b)^(1/2),(a/b)^(1/2))*a^2-2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*a*b-EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+7*Ellipti
cE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+EllipticE(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^2)/cosh(f*x+e)^3/(-1/a*b)^(1/2)/(a-b
)^3/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\tanh\left(fx+e\right)^{4}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^4\left(e+fx\right)}{\left(a+b\sinh^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**4/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.497 $\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{af(a-b)^2\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\sqrt{a}\cdots}{f(\cdots}$$

```
[Out] (-2*Sqrt[a]*Sqrt[b]*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/
Sqrt[a]], 1 - a/b])/((a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f
*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((a + b)*EllipticF[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*Sq
rt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - Tanh[e + f*x]/((a - b)*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.212858, antiderivative size = 217, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3196, 471, 525, 418, 411}

$$-\frac{\tanh(e+fx)}{f(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}}+\frac{(a+b)\text{sech}(e+fx)\sqrt{a+b\sinh\cdots}}{af(a-b)^2\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] (-2*Sqrt[a]*Sqrt[b]*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e + f*x])/
Sqrt[a]], 1 - a/b])/((a - b)^2*f*Sqrt[(a*Cosh[e + f*x]^2)/(a + b*Sinh[e + f
*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((a + b)*EllipticF[ArcTan[Sinh[e + f
*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)^2*f*Sq
rt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - Tanh[e + f*x]/((a - b)*f
*Sqrt[a + b*Sinh[e + f*x]^2])
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{a-bx}{\sqrt{1+x^2}(a+}\right.}{(-a+b)f}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2ab\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1}}{(a+bx}\right.}{(a-b)(-a+b)f}$$

$$= -\frac{2\sqrt{a}\sqrt{b}\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)|1-\frac{a}{b}\right)}{(a-b)^2 f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+fx)}} + \frac{(a+b)F\left(\tan^{-1}(\sinh(e+fx\right.}{a(a-b)^2 f}$$

Mathematica [C] time = 1.28214, size = 158, normalized size = 0.73

$$\frac{i\sqrt{2}(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right) - 2\tanh(e+fx)(a+b\cosh(2(e+fx))) - 2i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+}{a}}}{f(a-b)^2\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*I)*Sqrt[2]*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e +
f*x), b/a] + I*Sqrt[2]*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*Elli
pticF[I*(e + f*x), b/a] - 2*(a + b*Cosh[2*(e + f*x)])*Tanh[e + f*x])/((a -
b)^2*f*Sqrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.185, size = 256, normalized size = 1.2

$$-\frac{1}{(a-b)^2\cosh(fx+e)f}\left(2\sqrt{-\frac{b}{a}}b(\sinh(fx+e))^3-a\sqrt{\frac{a+b(\sinh(fx+e))^2}{a}}\sqrt{(\cosh(fx+e))^2}\mathrm{EllipticF}\left(\sinh(fx\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(2*(-1/a*b)^(1/2)*b*sinh(f*x+e)^3-a*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*
x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+b*((a+b*sin
h(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(
1/2),(a/b)^(1/2))-2*b*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*E
llipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))+(-1/a*b)^(1/2)*a*sinh(f*x+
e)+b*sinh(f*x+e)*(-1/a*b)^(1/2))/(a-b)^2/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*si
nh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)^2}{\left(b\sinh(fx+e)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh(fx+e)^2+a}\tanh(fx+e)^2}{b^2\sinh(fx+e)^4+2ab\sinh(fx+e)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tanh(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.498 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rubi [A] time = 0.067648, antiderivative size = 115, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.25, Rules used = {3184, 21, 3178, 3177}

$$-\frac{b\sinh(e+fx)\cosh(e+fx)}{af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{i\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{af(a-b)\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] -((b*Cosh[e + f*x]*Sinh[e + f*x])/(a*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
) - (I*EllipticE[I*e + I*f*x, b/a]*Sqrt[a + b*Sinh[e + f*x]^2])/(a*(a - b)*
f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{-a-b\sinh^2(e+fx)}{\sqrt{a+b\sinh^2(e+fx)}}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\int \sqrt{a+b\sinh^2(e+fx)}\,dx}{a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\sqrt{a+b\sinh^2(e+fx)}\int \sqrt{1+\frac{b\sinh^2(e+fx)}{a}}\,dx}{a(a-b)\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{a(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{iE\left(ie+ifx\middle|\frac{b}{a}\right)\sqrt{a+b\sinh^2(e+fx)}}{a(a-b)f\sqrt{1+\frac{b\sinh^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.149781, size = 100, normalized size = 0.87

$$\frac{-\sqrt{2}b\sinh(2(e+fx)) - 2ia\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}E\left(i(e+fx)\middle|\frac{b}{a}\right)}{2af(a-b)\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-3/2),x]
```

```
[Out] ((-2*I)*a*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/
a] - Sqrt[2]*b*Sinh[2*(e + f*x)])/(2*a*(a - b)*f*Sqrt[2*a - b + b*Cosh[2*(e
 + f*x)]])
```

Maple [A] time = 0., size = 252, normalized size = 2.2

$$\frac{1}{a\,(a-b)\cosh\left(fx+e\right)f}\left(-\sqrt{-\frac{b}{a}}b\sinh\left(fx+e\right)\left(\cosh\left(fx+e\right)\right)^{2}+a\sqrt{\frac{b\left(\cosh\left(fx+e\right)\right)^{2}}{a}+\frac{a-b}{a}}\sqrt{\left(\cosh\left(fx+e\right)\right)^{2}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-1/a*b)^(1/2)*b*sinh(f*x+e)*cosh(f*x+e)^2+a*(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))-(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b+(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh
(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b)/a/(a-
b)/(-1/a*b)^(1/2)/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^2*sinh(f*x + e)^4 + 2*a*b*sinh(f*x
+ e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-3/2), x)
```

3.499 $\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\operatorname{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}-2$$

```
[Out] Coth[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*Coth[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(a^2*f) - (2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]
*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sec
h[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2
*f)
```

Rubi [A] time = 0.270323, antiderivative size = 237, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 469, 583, 531, 418, 492, 411}

$$\frac{2\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}-\frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f}+\frac{\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}F\left(\text{t}\right.}{a^2f\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh\right.}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] Coth[e + f*x]/(a*f*Sqrt[a + b*Sinh[e + f*x]^2]) - (2*Coth[e + f*x]*Sqrt[a +
 b*Sinh[e + f*x]^2])/(a^2*f) - (2*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]
*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a
 + b*Sinh[e + f*x]^2))/a]) + (EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sec
h[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(a^2*f*Sqrt[(Sech[e + f*x]^2*(a + b
*Sinh[e + f*x]^2))/a]) + (2*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(a^2
*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
```

```
n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\,\mathrm{sech}(e+fx)\right)\mathrm{Subst}\left(\int \frac{-2-x^2}{x^2\sqrt{1+x^2}\sqrt{a+bx^2}}\,d\right.}{af}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\right.}{}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cosh^2(e+fx)}\right.}{}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} + \frac{F\left(\tan^{-1}(\sinh(e+\right.}{}$$

$$= \frac{\coth(e+fx)}{af\sqrt{a+b\sinh^2(e+fx)}} - \frac{2\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{a^2f} - \frac{2E\left(\tan^{-1}(\sinh(e\right.}{}$$

Mathematica [C] time = 0.835849, size = 153, normalized size = 0.65

$$\frac{i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\,\mathrm{EllipticF}\left(i(e+fx),\frac{b}{a}\right) - 2\coth(e+fx)(a+b\cosh(2(e+fx))-b) - 2i\sqrt{2}a\sqrt{\frac{2a+b\cosh(2(e+fx}{a}}}{a^2f\sqrt{4a+2b\cosh(2(e+fx))-2b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - b + b*Cosh[2*(e + f*x)])*Coth[e + f*x] - (2*I)*Sqrt[2]*a*Sqrt[(2*a
 - b + b*Cosh[2*(e + f*x)])/a]*EllipticE[I*(e + f*x), b/a] + I*Sqrt[2]*a*Sq
rt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticF[I*(e + f*x), b/a])/(a^2*f*S
qrt[4*a - 2*b + 2*b*Cosh[2*(e + f*x)]])
```

Maple [A] time = 0.282, size = 219, normalized size = 0.9

$$-\frac{1}{\sinh(fx+e)\,a^2\cosh(fx+e)\,f}\left(-\sinh(fx+e)\sqrt{(\cosh(fx+e))^2}\sqrt{\frac{b(\cosh(fx+e))^2}{a}+\frac{a-b}{a}}\left(a\mathrm{EllipticF}\left(\sin\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -(-sinh(f*x+e)*(cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(a*E
llipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2*b*EllipticF(sinh(f*x+e)*
```

```
(-1/a*b)^(1/2),(a/b)^(1/2))+2*b*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2)))+2*(-1/a*b)^(1/2)*b*cosh(f*x+e)^4+((-1/a*b)^(1/2)*a-2*(-1/a*b)^(1/2)
*b)*cosh(f*x+e)^2)/(-1/a*b)^(1/2)/sinh(f*x+e)/a^2/cosh(f*x+e)/(a+b*sinh(f*x
+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\coth\left(fx+e\right)^{2}}{b^{2}\sinh\left(fx+e\right)^{4}+2\,ab\sinh\left(fx+e\right)^{2}+a^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth^{2}\left(e+fx\right)}{\left(a+b\sinh^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(coth(e + f*x)**2/(a + b*sinh(e + f*x)**2)**(3/2), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")

[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.500 $\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=341

$$\frac{(3a-4b)\mathrm{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\mathrm{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^3f\sqrt{\frac{\mathrm{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh}}{3a^3f}$$

```
[Out] -(((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) - ((7*a - 8*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f) + (
(3*a - 4*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a
^2*b*f) - ((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*
Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(
a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh
[e + f*x])/(3*a^3*f)
```

Rubi [A] time = 0.40835, antiderivative size = 341, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.28, Rules used = {3196, 468, 583, 531, 418, 492, 411}

$$\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f}+\frac{(3a-4b)\coth(e+fx)\mathrm{csc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```


```
[Out] -(((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sinh[e + f*x]^2
])) - ((7*a - 8*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f) + (
(3*a - 4*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a
^2*b*f) - ((7*a - 8*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f
*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh
[e + f*x]^2))/a]) + ((3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*
Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*f*Sqrt[(Sech[e + f*x]^2*(
a + b*Sinh[e + f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh
[e + f*x])/(3*a^3*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
```

```
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4(a+bx^2)^{3/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\coth(e+fx)\mathbf{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\mathbf{sech}(e+fx)\right)\text{Subst}\left(\int \frac{3a-4b+(}{x^4\sqrt{1+x^2}}\right.}{abf}$$

$$= -\frac{(a-b)\coth(e+fx)\mathbf{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a-4b)\coth(e+fx)\mathbf{csch}^2(e+fx)\sqrt{a+b\sinh}}{3a^2bf}$$

$$= -\frac{(a-b)\coth(e+fx)\mathbf{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f} + \frac{(3}{}$$

$$= -\frac{(a-b)\coth(e+fx)\mathbf{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f} + \frac{(3}{}$$

$$= -\frac{(a-b)\coth(e+fx)\mathbf{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f} + \frac{(3}{}$$

$$= -\frac{(a-b)\coth(e+fx)\mathbf{csch}^2(e+fx)}{abf\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f} + \frac{(3}{}$$

Mathematica [C] time = 3.21993, size = 214, normalized size = 0.63

$$\frac{8ia(a-b)\sqrt{\frac{2a+b\cosh(2(e+fx))-b}{a}}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right) - \frac{\coth(e+fx)\mathrm{csch}^2(e+fx)\left(4\left(4a^2-11ab+8b^2\right)\cosh(2(e+fx))-8a^2+b(7a-8b)\cosh(4(e+f}{2\sqrt{2}}}{6a^3f\sqrt{2a+b\cosh(2(e+fx))-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(3/2),x]
```

```
[Out] (-((-8*a^2 + 37*a*b - 24*b^2 + 4*(4*a^2 - 11*a*b + 8*b^2)*Cosh[2*(e + f*x)]
 + (7*a - 8*b)*b*Cosh[4*(e + f*x)])*Coth[e + f*x]*Csch[e + f*x]^2)/(2*Sqrt[
2]) - (2*I)*a*(7*a - 8*b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a]*EllipticE
[I*(e + f*x), b/a] + (8*I)*a*(a - b)*Sqrt[(2*a - b + b*Cosh[2*(e + f*x)])/a
]*EllipticF[I*(e + f*x), b/a])/(6*a^3*f*Sqrt[2*a - b + b*Cosh[2*(e + f*x)]]
)
```

Maple [A] time = 0.195, size = 522, normalized size = 1.5

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(7*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^6-8*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^6
-3*a^2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f
*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*sinh(f*x+e)^3+11*b*((a+b*sinh(f*x+e)^2)/a
)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*a*sinh(f*x+e)^3-8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*
EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*sinh(f*x+e)^3-7*((a+b
*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*
b)^(1/2),(a/b)^(1/2))*a*b*sinh(f*x+e)^3+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(co
sh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2*si
nh(f*x+e)^3+4*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^4+3*(-1/a*b)^(1/2)*a*b*sinh(f*
x+e)^4-8*(-1/a*b)^(1/2)*b^2*sinh(f*x+e)^4+5*(-1/a*b)^(1/2)*a^2*sinh(f*x+e)^
2-4*(-1/a*b)^(1/2)*a*b*sinh(f*x+e)^2+(-1/a*b)^(1/2)*a^2)/(-1/a*b)^(1/2)/sin
h(f*x+e)^3/a^3/cosh(f*x+e)/(a+b*sinh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^4}{b^2\sinh\left(fx+e\right)^4+2\,ab\sinh\left(fx+e\right)^2+a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4/(b^2*sinh(f*x + e)^4 +
 2*a*b*sinh(f*x + e)^2 + a^2), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(3/2), x)
```

3.501 $\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=232

$$\frac{8a^2+24ab+3b^2}{8f(a-b)^4\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2+24ab+3b^2}{24f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{9/2}}$$

```
[Out] -((8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]
)/(8*(a - b)^(9/2)*f) + (8*a^2 + 24*a*b + 3*b^2)/(24*(a - b)^3*f*(a + b*Sin
h[e + f*x]^2)^(3/2)) + ((8*a - b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*(a + b*Si
nh[e + f*x]^2)^(3/2)) - Sech[e + f*x]^4/(4*(a - b)*f*(a + b*Sinh[e + f*x]^2
)^(3/2)) + (8*a^2 + 24*a*b + 3*b^2)/(8*(a - b)^4*f*Sqrt[a + b*Sinh[e + f*x]
^2])
```

Rubi [A] time = 0.315846, antiderivative size = 232, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2+24ab+3b^2}{8f(a-b)^4\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2+24ab+3b^2}{24f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8f(a-b)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] -((8*a^2 + 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]
)/(8*(a - b)^(9/2)*f) + (8*a^2 + 24*a*b + 3*b^2)/(24*(a - b)^3*f*(a + b*Sin
h[e + f*x]^2)^(3/2)) + ((8*a - b)*Sech[e + f*x]^2)/(8*(a - b)^2*f*(a + b*Si
nh[e + f*x]^2)^(3/2)) - Sech[e + f*x]^4/(4*(a - b)*f*(a + b*Sinh[e + f*x]^2
)^(3/2)) + (8*a^2 + 24*a*b + 3*b^2)/(8*(a - b)^4*f*Sqrt[a + b*Sinh[e + f*x]
^2])
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1+x)^3(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{sech}^4(e+fx)}{4(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-4a-3b)+2(a-b)x}{(1+x)^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{4(a-b)f}$$

$$= \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{sech}^4(e+fx)}{4(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a^2+24a}{}$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{}{4(a-b)}$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{}{4(a-b)}$$

$$= \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(8a-b)\text{sech}^2(e+fx)}{8(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{}{4(a-b)}$$

$$= -\frac{\left(8a^2+24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{8(a-b)^{9/2}f} + \frac{8a^2+24ab+3b^2}{24(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Mathematica [C] time = 0.58949, size = 114, normalized size = 0.49

$$\frac{2\left(8a^2+24ab+3b^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)+a}{a-b}\right)+3(a-b)\text{sech}^4(e+fx)((8a-b)\cosh(2(e+fx))+4a+3b)}{48f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*(8*a^2 + 24*a*b + 3*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e
+ f*x]^2)/(a - b)] + 3*(a - b)*(4*a + 3*b + (8*a - b)*Cosh[2*(e + f*x)])*Se
ch[e + f*x]^4)/(48*(a - b)^3*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.298, size = 213, normalized size = 0.9

$$\frac{1}{f}\text{` int/indef0`}\left(-\frac{\left(\sinh\left(fx+e\right)\right)^5\left(b^2\left(\sinh\left(f\right.\right.\right.}{-b^4\left(\cosh\left(fx+e\right)\right)^{18}+\left(-4\,ab^3+4\,b^4\right)\left(\cosh\left(fx+e\right)\right)^{16}+\left(-6\,a^2b^2+12\,ab^3-6\,b^4\right)\left(\cosh\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^5*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)*cos
h(f*x+e)^4/(-b^4*cosh(f*x+e)^18+(-4*a*b^3+4*b^4)*cosh(f*x+e)^16+(-6*a^2*b^2
```

```
+12*a*b^3-6*b^4)*cosh(f*x+e)^14+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cosh(f
*x+e)^12+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^10)/(a+b*sinh(f*x
+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{5}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{5}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.502 $\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=163

$$\frac{2a+3b}{2f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a+3b}{6f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}+\frac{}{2f(a-}$$

```
[Out] -((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^
(7/2)*f) + (2*a + 3*b)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + Sech
[e + f*x]^2/(2*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a + 3*b)/(2*(a
 - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.165729, antiderivative size = 163, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a+3b}{2f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a+3b}{6f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2f(a-b)^{7/2}}+\frac{}{2f(a-}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] -((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]])/(2*(a - b)^
(7/2)*f) + (2*a + 3*b)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + Sech
[e + f*x]^2/(2*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a + 3*b)/(2*(a
 - b)^3*f*Sqrt[a + b*Sinh[e + f*x]^2])
```


Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1+x)^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx\right.}{4(a-b)f}$$

$$= \frac{2a+3b}{6(a-b)^2 f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a+3b)\,\text{Sul}}{}$$

$$= \frac{2a+3b}{6(a-b)^2 f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2}{2(a-b)^3 f\sqrt{a}}$$

$$= \frac{2a+3b}{6(a-b)^2 f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)}{2(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2}{2(a-b)^3 f\sqrt{a}}$$

$$= -\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{2(a-b)^{7/2}f} + \frac{2a+3b}{6(a-b)^2 f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{s}}{2(a-b)f\left(\right.}$$

Mathematica [C] time = 0.122481, size = 82, normalized size = 0.5

$$\frac{(2a+3b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)+a}{a-b}\right)+3(a-b)\text{sech}^2(e+fx)}{6f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a + 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sinh[e + f*x]^2)/(a -
b)] + 3*(a - b)*Sech[e + f*x]^2)/(6*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(3/
```

```
2))
```

Maple [C] time = 0.266, size = 213, normalized size = 1.3

$$\frac{1}{f}\text{` int/indef0`}\left(-\frac{\left(\sinh\left(fx+e\right)\right)^{3}\left(b^{2}\left(\sinh\left(fx\right.\right.\right.}{-b^{4}\left(\cosh\left(fx+e\right)\right)^{14}+\left(-4\,ab^{3}+4\,b^{4}\right)\left(\cosh\left(fx+e\right)\right)^{12}+\left(-6\,a^{2}b^{2}+12\,ab^{3}-6\,b^{4}\right)\left(\cosh\left(f\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)^3*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)*cos
h(f*x+e)^2/(-b^4*cosh(f*x+e)^14+(-4*a*b^3+4*b^4)*cosh(f*x+e)^12+(-6*a^2*b^2
+12*a*b^3-6*b^4)*cosh(f*x+e)^10+(-4*a^3*b+12*a^2*b^2-12*a*b^3+4*b^4)*cosh(f
*x+e)^8+(-a^4+4*a^3*b-6*a^2*b^2+4*a*b^3-b^4)*cosh(f*x+e)^6)/(a+b*sinh(f*x+e
)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 6.75029, size = 24677, normalized size = 151.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 + 3*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 + 3*b^3)*cosh(f
*x + e)*sinh(f*x + e)^11 + (2*a*b^2 + 3*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b
+ 10*a*b^2 - 3*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3 + 33*(
2*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 + 3*b^
3)*cosh(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x +
 e)^9 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + (495*(2*a*b
^2 + 3*b^3)*cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 + 90*(8*a
^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(2*a*b^2
+ 3*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^3
+ (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^7 + 4*
(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 4*(231*(2*a*b^2 +
3*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^4 +
 16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3 + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*
```

b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^7 + 63*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + (495*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^8 + 420*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 70*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3 + 60*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^9 + 60*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 20*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^3 + 2*a*b^2 + 3*b^3 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^2 + 2*(33*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^10 + 45*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 30*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 8*a^2*b + 10*a*b^2 - 3*b^3 + 3*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^11 + 5*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^9 + 2*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 6*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a - b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) - 4*sqrt(2)*(3*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^9 + 27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^8 + 3*(2*a^2*b + a*b^2 - 3*b^3)*sinh(f*x + e)^9 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^7 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(9*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^6 + 2*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3 + 42*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^5 + 70*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + 4*(63*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 35*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 21*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e) + (27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 28*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 10*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 6*a^2*b + 3*a*b^2 - 9*b^3 + 12*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^12 + 12*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*sinh(f*x + e)^12 + 2*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^10 + 2*(33*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*

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b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f)*sinh(f*x + e)^10 + (16*a^6 - 64*a^5*b
+ 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^8 +
 20*(11*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^3
 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(
f*x + e))*sinh(f*x + e)^9 + (495*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
 + b^6)*f*cosh(f*x + e)^4 + 90*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*
b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 -
60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*sinh(f*x + e)^8 + 4*(8*a^6 - 36
*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x +
 e)^6 + 8*(99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x
+ e)^5 + 30*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6
)*f*cosh(f*x + e)^3 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2
*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231*(a^4*b^2 -
4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^6 + 105*(4*a^5*b - 1
7*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^4 + 7*
(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*
f*cosh(f*x + e)^2 + (8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^
4 - 8*a*b^5 + b^6)*f)*sinh(f*x + e)^6 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 6
0*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^4 + 8*(99*(a^4*b^2
- 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^7 + 63*(4*a^5*b -
17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^5 + 7
*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)
*f*cosh(f*x + e)^3 + 3*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2
*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*(a^4*b^2 - 4*
a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^8 + 420*(4*a^5*b - 17*
a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^6 + 70*(
16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f
*cosh(f*x + e)^4 + 60*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*
b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 -
60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*sinh(f*x + e)^4 + 2*(4*a^5*b -
17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^2 + 4
*(55*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^9 +
60*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(
f*x + e)^7 + 14*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 +
 4*a*b^5 - b^6)*f*cosh(f*x + e)^5 + 20*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*
a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^3 + (16*a^6 - 64*a^5*
b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e))*
sinh(f*x + e)^3 + 2*(33*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f
*cosh(f*x + e)^10 + 45*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*
a*b^5 - b^6)*f*cosh(f*x + e)^8 + 14*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^
3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^6 + 30*(8*a^6 - 36*a^5*
b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^4
 + 3*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 -
b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 +
8*a*b^5 - b^6)*f)*sinh(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*
b^5 + b^6)*f + 4*(3*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cos
h(f*x + e)^11 + 5*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5
 - b^6)*f*cosh(f*x + e)^9 + 2*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3
+ 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^7 + 6*(8*a^6 - 36*a^5*b + 65*
a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^5 + (16*
a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*co
sh(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 -
 b^6)*f*cosh(f*x + e))*sinh(f*x + e)), -1/6*(3*((2*a*b^2 + 3*b^3)*cosh(f*x
+ e)^12 + 12*(2*a*b^2 + 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^11 + (2*a*b^2 +
3*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^10 +
 2*(8*a^2*b + 10*a*b^2 - 3*b^3 + 33*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh
(f*x + e)^10 + 20*(11*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + (8*a^2*b + 10*a*b
^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^9 + (32*a^3 + 48*a^2*b - 2*a*b^2 -
 3*b^3)*cosh(f*x + e)^8 + (495*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 32*a^3 +
```

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 48*a^2*b - 2*a*b^2 - 3*b^3 + 90*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)
^2)*sinh(f*x + e)^8 + 8*(99*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b
 + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3
)*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)
*cosh(f*x + e)^6 + 4*(231*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b
+ 10*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3
+ 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6
+ 8*(99*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^7 + 63*(8*a^2*b + 10*a*b^2 - 3*b^3)
*cosh(f*x + e)^5 + 7*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3
+ 3*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 +
 (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + (495*(2*a*b^2 + 3*
b^3)*cosh(f*x + e)^8 + 420*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 7
0*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 32*a^3 + 48*a^2*b
 - 2*a*b^2 - 3*b^3 + 60*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e
)^2)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^9 + 60*(8*a^2*
b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^7 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3
*b^3)*cosh(f*x + e)^5 + 20*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x
+ e)^3 + (32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)
^3 + 2*a*b^2 + 3*b^3 + 2*(8*a^2*b + 10*a*b^2 - 3*b^3)*cosh(f*x + e)^2 + 2*(
33*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^10 + 45*(8*a^2*b + 10*a*b^2 - 3*b^3)*cos
h(f*x + e)^8 + 14*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^6 + 3
0*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 8*a^2*b + 10*a*b
^2 - 3*b^3 + 3*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(
f*x + e)^2 + 4*(3*(2*a*b^2 + 3*b^3)*cosh(f*x + e)^11 + 5*(8*a^2*b + 10*a*b^
2 - 3*b^3)*cosh(f*x + e)^9 + 2*(32*a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f
*x + e)^7 + 6*(16*a^3 + 16*a^2*b - 10*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + (32*
a^3 + 48*a^2*b - 2*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + (8*a^2*b + 10*a*b^2 - 3
*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-
a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e
)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x +
 e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*(3*(2*a^2*b + a*b^2 - 3*b^3)*cosh
(f*x + e)^9 + 27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)*sinh(f*x + e)^8 +
3*(2*a^2*b + a*b^2 - 3*b^3)*sinh(f*x + e)^9 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2
 + 3*b^3)*cosh(f*x + e)^7 + 4*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 27*(2*a
^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(9*(2*a^2*b + a
*b^2 - 3*b^3)*cosh(f*x + e)^3 + (8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f
*x + e))*sinh(f*x + e)^6 + 2*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*
x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 56*a^3 - 70*a
^2*b + 17*a*b^2 - 3*b^3 + 42*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x
+ e)^2)*sinh(f*x + e)^5 + 2*(189*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^5
+ 70*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + 5*(56*a^3 - 70*
a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^3 + 2*a^2
*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^3 + 4*(63*(2*a^2*b + a*b^2 - 3*b^3)*co
sh(f*x + e)^6 + 35*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^4 + 8
*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3 + 5*(56*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3
)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f
*x + e)^7 + 21*(8*a^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^5 + 5*(56
*a^3 - 70*a^2*b + 17*a*b^2 - 3*b^3)*cosh(f*x + e)^3 + 3*(8*a^3 + 2*a^2*b -
13*a*b^2 + 3*b^3)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(2*a^2*b + a*b^2 - 3*b
^3)*cosh(f*x + e) + (27*(2*a^2*b + a*b^2 - 3*b^3)*cosh(f*x + e)^8 + 28*(8*a
^3 + 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^6 + 10*(56*a^3 - 70*a^2*b +
17*a*b^2 - 3*b^3)*cosh(f*x + e)^4 + 6*a^2*b + 3*a*b^2 - 9*b^3 + 12*(8*a^3 +
 2*a^2*b - 13*a*b^2 + 3*b^3)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f
*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)
*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a
*b^5 + b^6)*f*cosh(f*x + e)^12 + 12*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*
b^5 + b^6)*f*cosh(f*x + e)*sinh(f*x + e)^11 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*
b^4 - 4*a*b^5 + b^6)*f*sinh(f*x + e)^12 + 2*(4*a^5*b - 17*a^4*b^2 + 28*a^3*
b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^10 + 2*(33*(a^4*b^2 - 4*a
```

```
^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b
^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f)*sinh(f*x + e)^10 + (16*a^6
 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(
f*x + e)^8 + 20*(11*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cos
h(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 -
b^6)*f*cosh(f*x + e))*sinh(f*x + e)^9 + (495*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b
^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^4 + 90*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b
^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 9
5*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*sinh(f*x + e)^8 + 4
*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*
f*cosh(f*x + e)^6 + 8*(99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)
*f*cosh(f*x + e)^5 + 30*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8
*a*b^5 - b^6)*f*cosh(f*x + e)^3 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*
b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e))*sinh(f*x + e)^7 + 4*(231
*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^6 + 105*
(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x
 + e)^4 + 7*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a
*b^5 - b^6)*f*cosh(f*x + e)^2 + (8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3
 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f)*sinh(f*x + e)^6 + (16*a^6 - 64*a^5*b + 95
*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^4 + 8*(
99*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^7 + 63
*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*
x + e)^5 + 7*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*
a*b^5 - b^6)*f*cosh(f*x + e)^3 + 3*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*
b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*(
a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f*x + e)^8 + 420*(4
*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x +
 e)^6 + 70*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*
b^5 - b^6)*f*cosh(f*x + e)^4 + 60*(8*a^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b
^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^2 + (16*a^6 - 64*a^5*b + 9
5*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f)*sinh(f*x + e)^4 + 2
*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*
x + e)^2 + 4*(55*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*f*cosh(f
*x + e)^9 + 60*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4 + 8*a*b^5 -
b^6)*f*cosh(f*x + e)^7 + 14*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 +
10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^5 + 20*(8*a^6 - 36*a^5*b + 65*a
^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x + e)^3 + (16*a
^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cos
h(f*x + e))*sinh(f*x + e)^3 + 2*(33*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*
b^5 + b^6)*f*cosh(f*x + e)^10 + 45*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*
a^2*b^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^8 + 14*(16*a^6 - 64*a^5*b + 95*a^4
*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^6 + 30*(8*a
^6 - 36*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cos
h(f*x + e)^4 + 3*(16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4
+ 4*a*b^5 - b^6)*f*cosh(f*x + e)^2 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 2
2*a^2*b^4 + 8*a*b^5 - b^6)*f)*sinh(f*x + e)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^
2*b^4 - 4*a*b^5 + b^6)*f + 4*(3*(a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5
+ b^6)*f*cosh(f*x + e)^11 + 5*(4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b
^4 + 8*a*b^5 - b^6)*f*cosh(f*x + e)^9 + 2*(16*a^6 - 64*a^5*b + 95*a^4*b^2 -
 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5 - b^6)*f*cosh(f*x + e)^7 + 6*(8*a^6 - 36
*a^5*b + 65*a^4*b^2 - 60*a^3*b^3 + 30*a^2*b^4 - 8*a*b^5 + b^6)*f*cosh(f*x +
 e)^5 + (16*a^6 - 64*a^5*b + 95*a^4*b^2 - 60*a^3*b^3 + 10*a^2*b^4 + 4*a*b^5
 - b^6)*f*cosh(f*x + e)^3 + (4*a^5*b - 17*a^4*b^2 + 28*a^3*b^3 - 22*a^2*b^4
 + 8*a*b^5 - b^6)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.503 $\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=99

$$\frac{1}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + 1/(
3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + 1/((a - b)^2*f*Sqrt[a + b*Sinh
[e + f*x]^2])
```

Rubi [A] time = 0.0929154, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}+\frac{1}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{f(a-b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a - b]]/((a - b)^(5/2)*f)) + 1/(
3*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + 1/((a - b)^2*f*Sqrt[a + b*Sinh
[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\tanh(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2(a-b)f}$$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1+x)\sqrt{a}}\right.}{2(}$$

$$= \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2 f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{a}{b}+\frac{x^2}{b}}\right.}{}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a-b}}\right)}{(a-b)^{5/2}f} + \frac{1}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{(a-b)^2 f\sqrt{a+b\sinh}}$$

Mathematica [C] time = 0.0865591, size = 60, normalized size = 0.61

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\cosh^2(e+fx)}{a-b}+1\right)}{3f(a-b)\left(a+b\cosh^2(e+fx)-b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Cosh[e + f*x]^2)/(a - b)]/(3*(a - b
)*f*(a - b + b*Cosh[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.218, size = 173, normalized size = 1.8

$$\frac{1}{f}\text{` int/indef0`}\left(-\frac{\sinh(fx+e)\left(b^2(\sinh(fx+e))^4+2ab(\sinh\right.}{-b^4(\sinh(fx+e))^{10}+(-4ab^3-b^4)(\sinh(fx+e))^8+(-6a^2b^2-4ab^3)(\sinh(fx+e))^6+(-4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(-sinh(f*x+e)*(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/(-b^4
*sinh(f*x+e)^10+(-4*a*b^3-b^4)*sinh(f*x+e)^8+(-6*a^2*b^2-4*a*b^3)*sinh(f*x+
e)^6+(-4*a^3*b-6*a^2*b^2)*sinh(f*x+e)^4+(-a^4-4*a^3*b)*sinh(f*x+e)^2-a^4)/(
```

```
a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 3.47963, size = 9900, normalized size = 100.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cosh(f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*si
nh(f*x + e)^8 + 4*(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2
+ 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)
*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4
 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)
*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 4*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b
^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b
^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f
*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*
x + e))*sinh(f*x + e))*sqrt(a - b)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x +
e)*sinh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - 3*b)*cosh(f*x + e)^2 + 2*
(3*b*cosh(f*x + e)^2 + 4*a - 3*b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a - b)*s
qrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*
cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x +
 e)) + 4*(b*cosh(f*x + e)^3 + (4*a - 3*b)*cosh(f*x + e))*sinh(f*x + e) + b)
/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(
3*cosh(f*x + e)^2 + 1)*sinh(f*x + e)^2 + 2*cosh(f*x + e)^2 + 4*(cosh(f*x +
e)^3 + cosh(f*x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(3*(a*b - b^2)*cosh(f
*x + e)^5 + 15*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + 3*(a*b - b^2)*si
nh(f*x + e)^5 + 2*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^3 + 2*(15*(a*b - b
^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^3 + 6*(5*(a*b -
 b^2)*cosh(f*x + e)^3 + (8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e))*sinh(f*x +
e)^2 + 3*(a*b - b^2)*cosh(f*x + e) + 3*(5*(a*b - b^2)*cosh(f*x + e)^4 + 2*(
8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f*x + e))*sqrt((b
*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f
*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4
- b^5)*f*cosh(f*x + e)^8 + 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f
*x + e)*sinh(f*x + e)^7 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*sinh(f*x
+ e)^8 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e
)^6 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^4
```

*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^4 + 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^3 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^4 + 30*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f)*sinh(f*x + e)^4 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2 + 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^5 + 10*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^6 + 15*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^4 + 3*(8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^2 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)^2 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f + 8*((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^7 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^5 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^3 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e)), -1/3*(3*(b^2*cosh(f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*sinh(f*x + e)^8 + 4*(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a + b)*arctan(-1/2*sqrt(2)*sqrt(-a + b)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2))/((a - b)*cosh(f*x + e) + (a - b)*sinh(f*x + e))) - 2*sqrt(2)*(3*(a*b - b^2)*cosh(f*x + e)^5 + 15*(a*b - b^2)*cosh(f*x + e)*sinh(f*x + e)^4 + 3*(a*b - b^2)*sinh(f*x + e)^5 + 2*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^3 + 2*(15*(a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 13*a*b + 5*b^2)*sinh(f*x + e)^3 + 6*(5*(a*b - b^2)*cosh(f*x + e)^3 + (8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(a*b - b^2)*cosh(f*x + e) + 3*(5*(a*b - b^2)*cosh(f*x + e)^4 + 2*(8*a^2 - 13*a*b + 5*b^2)*cosh(f*x + e)^2 + a*b - b^2)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/((a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^8 + 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)*sinh(f*x + e)^7 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*sinh(f*x + e)^8 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^6 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^2 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(f*x + e)^6 + 2*(8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^4 + 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^3 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(35*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^4 + 30*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f)*sinh(f*x + e)^4 + 4*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^2 + 8*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^5 + 10*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^3 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^6 + 15*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e

```
)^4 + 3*(8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a^2*b^3 + 17*a*b^4 - 3*b^5)*f*c
osh(f*x + e)^2 + (2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3 - 5*a*b^4 + b^5)*f)*sinh(
f*x + e)^2 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*f + 8*((a^3*b^2 - 3*a^2*
b^3 + 3*a*b^4 - b^5)*f*cosh(f*x + e)^7 + 3*(2*a^4*b - 7*a^3*b^2 + 9*a^2*b^3
 - 5*a*b^4 + b^5)*f*cosh(f*x + e)^5 + (8*a^5 - 32*a^4*b + 51*a^3*b^2 - 41*a
^2*b^3 + 17*a*b^4 - 3*b^5)*f*cosh(f*x + e)^3 + (2*a^4*b - 7*a^3*b^2 + 9*a^2
*b^3 - 5*a*b^4 + b^5)*f*cosh(f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh(fx+e)}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.504 $\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$\frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
 b*Sinh[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.0995702, antiderivative size = 83, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
 b*Sinh[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sinh^2(e+fx)\right)}{2af}$$

$$= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{si}\right.}{2a^2f}$$

$$= \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\ }\right.}{a^2bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sinh^2(e+f}}$$

Mathematica [C] time = 0.0653222, size = 49, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sinh[e + f*x]^2)/a]/(3*a*f*(a + b*S
inh[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.086, size = 65, normalized size = 0.8

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{1}{\left(b^2\left(\sinh\left(fx+e\right)\right)^4+2\,ab\left(\sinh\left(fx+e\right)\right)^2+a^2\right)\sinh\left(fx+e\right)}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(1/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh(f*x+e)/(a+b
*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 2.80733, size = 7507, normalized size = 90.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cosh(f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*si
nh(f*x + e)^8 + 4*(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2
+ 2*a*b - b^2)*sinh(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)
*cosh(f*x + e))*sinh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4
 + 2*(35*b^2*cosh(f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8
*a*b + 3*b^2)*sinh(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)
*cosh(f*x + e)^3 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 +
 4*(2*a*b - b^2)*cosh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b
^2)*cosh(f*x + e)^4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b
^2)*sinh(f*x + e)^2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f
*x + e)^5 + (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*
x + e))*sinh(f*x + e))*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*s
inh(f*x + e)^3 + b*sinh(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*c
osh(f*x + e)^2 + 4*a - b)*sinh(f*x + e)^2 - 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(
f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e
)*sinh(f*x + e) + sinh(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*
cosh(f*x + e)^3 + (4*a - b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e
)^4 + 4*cosh(f*x + e)*sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e
)^2 - 1)*sinh(f*x + e)^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*
x + e))*sinh(f*x + e) + 1)) + 4*sqrt(2)*(3*a*b*cosh(f*x + e)^5 + 15*a*b*cos
h(f*x + e)*sinh(f*x + e)^4 + 3*a*b*sinh(f*x + e)^5 + 2*(8*a^2 - 3*a*b)*cosh
(f*x + e)^3 + 2*(15*a*b*cosh(f*x + e)^2 + 8*a^2 - 3*a*b)*sinh(f*x + e)^3 +
3*a*b*cosh(f*x + e) + 6*(5*a*b*cosh(f*x + e)^3 + (8*a^2 - 3*a*b)*cosh(f*x +
 e))*sinh(f*x + e)^2 + 3*(5*a*b*cosh(f*x + e)^4 + 2*(8*a^2 - 3*a*b)*cosh(f*
x + e)^2 + a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2
+ 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)
^2)))/(a^3*b^2*f*cosh(f*x + e)^8 + 8*a^3*b^2*f*cosh(f*x + e)*sinh(f*x + e)^
7 + a^3*b^2*f*sinh(f*x + e)^8 + 4*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^6 + 4
*(7*a^3*b^2*f*cosh(f*x + e)^2 + (2*a^4*b - a^3*b^2)*f)*sinh(f*x + e)^6 + a^
3*b^2*f + 2*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^4 + 8*(7*a^3*b^2*
f*cosh(f*x + e)^3 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5
+ 2*(35*a^3*b^2*f*cosh(f*x + e)^4 + 30*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^
2 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e)^4 + 4*(2*a^4*b - a^3*b^2
)*f*cosh(f*x + e)^2 + 8*(7*a^3*b^2*f*cosh(f*x + e)^5 + 10*(2*a^4*b - a^3*b^
2)*f*cosh(f*x + e)^3 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e))*sinh(
f*x + e)^3 + 4*(7*a^3*b^2*f*cosh(f*x + e)^6 + 15*(2*a^4*b - a^3*b^2)*f*cosh
```

```
(f*x + e)^4 + 3*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)^2 + (2*a^4*b
- a^3*b^2)*f)*sinh(f*x + e)^2 + 8*(a^3*b^2*f*cosh(f*x + e)^7 + 3*(2*a^4*b -
 a^3*b^2)*f*cosh(f*x + e)^5 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f*x + e)
^3 + (2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/3*(3*(b^2*cosh(
f*x + e)^8 + 8*b^2*cosh(f*x + e)*sinh(f*x + e)^7 + b^2*sinh(f*x + e)^8 + 4*
(2*a*b - b^2)*cosh(f*x + e)^6 + 4*(7*b^2*cosh(f*x + e)^2 + 2*a*b - b^2)*sin
h(f*x + e)^6 + 8*(7*b^2*cosh(f*x + e)^3 + 3*(2*a*b - b^2)*cosh(f*x + e))*si
nh(f*x + e)^5 + 2*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^4 + 2*(35*b^2*cosh(
f*x + e)^4 + 30*(2*a*b - b^2)*cosh(f*x + e)^2 + 8*a^2 - 8*a*b + 3*b^2)*sinh
(f*x + e)^4 + 8*(7*b^2*cosh(f*x + e)^5 + 10*(2*a*b - b^2)*cosh(f*x + e)^3 +
 (8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(2*a*b - b^2)*c
osh(f*x + e)^2 + 4*(7*b^2*cosh(f*x + e)^6 + 15*(2*a*b - b^2)*cosh(f*x + e)^
4 + 3*(8*a^2 - 8*a*b + 3*b^2)*cosh(f*x + e)^2 + 2*a*b - b^2)*sinh(f*x + e)^
2 + b^2 + 8*(b^2*cosh(f*x + e)^7 + 3*(2*a*b - b^2)*cosh(f*x + e)^5 + (8*a^2
 - 8*a*b + 3*b^2)*cosh(f*x + e)^3 + (2*a*b - b^2)*cosh(f*x + e))*sinh(f*x +
 e))*sqrt(-a)*arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(
f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + si
nh(f*x + e)^2))/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*(3*a*b*cos
h(f*x + e)^5 + 15*a*b*cosh(f*x + e)*sinh(f*x + e)^4 + 3*a*b*sinh(f*x + e)^5
 + 2*(8*a^2 - 3*a*b)*cosh(f*x + e)^3 + 2*(15*a*b*cosh(f*x + e)^2 + 8*a^2 -
3*a*b)*sinh(f*x + e)^3 + 3*a*b*cosh(f*x + e) + 6*(5*a*b*cosh(f*x + e)^3 + (
8*a^2 - 3*a*b)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(5*a*b*cosh(f*x + e)^4 +
2*(8*a^2 - 3*a*b)*cosh(f*x + e)^2 + a*b)*sinh(f*x + e))*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh
(f*x + e) + sinh(f*x + e)^2)))/(a^3*b^2*f*cosh(f*x + e)^8 + 8*a^3*b^2*f*cos
h(f*x + e)*sinh(f*x + e)^7 + a^3*b^2*f*sinh(f*x + e)^8 + 4*(2*a^4*b - a^3*b
^2)*f*cosh(f*x + e)^6 + 4*(7*a^3*b^2*f*cosh(f*x + e)^2 + (2*a^4*b - a^3*b^2
)*f)*sinh(f*x + e)^6 + a^3*b^2*f + 2*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*cosh(f
*x + e)^4 + 8*(7*a^3*b^2*f*cosh(f*x + e)^3 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f
*x + e))*sinh(f*x + e)^5 + 2*(35*a^3*b^2*f*cosh(f*x + e)^4 + 30*(2*a^4*b -
a^3*b^2)*f*cosh(f*x + e)^2 + (8*a^5 - 8*a^4*b + 3*a^3*b^2)*f)*sinh(f*x + e)
^4 + 4*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^2 + 8*(7*a^3*b^2*f*cosh(f*x + e)
^5 + 10*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^3 + (8*a^5 - 8*a^4*b + 3*a^3*b^
2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 4*(7*a^3*b^2*f*cosh(f*x + e)^6 + 15*(
2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^4 + 3*(8*a^5 - 8*a^4*b + 3*a^3*b^2)*f*co
sh(f*x + e)^2 + (2*a^4*b - a^3*b^2)*f)*sinh(f*x + e)^2 + 8*(a^3*b^2*f*cosh(
f*x + e)^7 + 3*(2*a^4*b - a^3*b^2)*f*cosh(f*x + e)^5 + (8*a^5 - 8*a^4*b + 3
*a^3*b^2)*f*cosh(f*x + e)^3 + (2*a^4*b - a^3*b^2)*f*cosh(f*x + e))*sinh(f*x
 + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)**2)**(5/2),x)

[Out] Timed out
```

Giac [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: RuntimeError
```

3.505 $\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{\operatorname{csch}^2(e+f}{2af\left(a+b\sinh^2(e\right.}$$

```
[Out] -((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) +
 (2*a - 5*b)/(6*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - Csch[e + f*x]^2/(2*a
*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a - 5*b)/(2*a^3*f*Sqrt[a + b*Sinh[e
+ f*x]^2])
```

Rubi [A] time = 0.152358, antiderivative size = 143, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.2, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+fx)}}+\frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{\operatorname{csch}^2(e+f}{2af\left(a+b\sinh^2(e\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((2*a - 5*b)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) +
 (2*a - 5*b)/(6*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2)) - Csch[e + f*x]^2/(2*a
*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (2*a - 5*b)/(2*a^3*f*Sqrt[a + b*Sinh[e
+ f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
```

```
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^3(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1+x}{x^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a-5b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{4af}$$

$$= \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(2a-5b)\,\text{Subst}\left(\int \frac{1}{x(a+b\ldots}\right.}{4a\ldots}$$

$$= \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+f\ldots}}$$

$$= \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^2(e+fx)}{2af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{2a-5b}{2a^3f\sqrt{a+b\sinh^2(e+f\ldots}}$$

$$= -\frac{(2a-5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} + \frac{2a-5b}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^2(e+\ldots}{2af\left(a+b\sinh^2\ldots\right.}$$

Mathematica [C] time = 0.30026, size = 69, normalized size = 0.48

$$-\frac{(5b-2a)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)+3a\,\text{csch}^2(e+fx)}{6a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^3/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(3*a*Csch[e + f*x]^2 + (-2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (
b*Sinh[e + f*x]^2)/a])/(6*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Maple [C] time = 0.118, size = 73, normalized size = 0.5

$$\frac{1}{f}\text{` int/indef0`}\left(\frac{\left(\cosh\left(fx+e\right)\right)^{2}}{\left(b^{2}\left(\sinh\left(fx+e\right)\right)^{4}+2\,ab\left(\sinh\left(fx+e\right)\right)^{2}+a^{2}\right)\left(\sinh\left(fx+e\right)\right)^{3}}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^{2}}},\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^2/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh
(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^{3}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [B] time = 4.73291, size = 18669, normalized size = 130.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 - 5*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 - 5*b^3)*cosh(f
*x + e)*sinh(f*x + e)^11 + (2*a*b^2 - 5*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b
- 26*a*b^2 + 15*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3 + 33
*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 - 5*
b^3)*cosh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*
x + e)^9 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^8 + (495
*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^
3 + 90*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(
99*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b - 26*a*b^2 + 15*b^3)*cos
h(f*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e))*sin
h(f*x + e)^7 - 4*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^6 +
4*(231*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b - 26*a*b^2 + 15*b^3
)*cosh(f*x + e)^4 - 16*a^3 + 64*a^2*b - 70*a*b^2 + 25*b^3 + 7*(32*a^3 - 144
*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(2*a*
b^2 - 5*b^3)*cosh(f*x + e)^7 + 63*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x +
e)^5 + 7*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - 3*(16*
a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (32*a^
3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + (495*(2*a*b^2 - 5*b^3
```

)*cosh(f*x + e)^8 + 420*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^6 + 70*
(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^
2*b + 190*a*b^2 - 75*b^3 - 60*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(
f*x + e)^2)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^9 + 60*
(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^7 + 14*(32*a^3 - 144*a^2*b + 19
0*a*b^2 - 75*b^3)*cosh(f*x + e)^5 - 20*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b
^3)*cosh(f*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x +
e))*sinh(f*x + e)^3 + 2*a*b^2 - 5*b^3 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3)*cos
h(f*x + e)^2 + 2*(33*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^10 + 45*(8*a^2*b - 26*
a*b^2 + 15*b^3)*cosh(f*x + e)^8 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b
^3)*cosh(f*x + e)^6 - 30*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x +
e)^4 + 8*a^2*b - 26*a*b^2 + 15*b^3 + 3*(32*a^3 - 144*a^2*b + 190*a*b^2 - 7
5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(2*a*b^2 - 5*b^3)*cosh(f*x +
e)^11 + 5*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^9 + 2*(32*a^3 - 144*
a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^7 - 6*(16*a^3 - 64*a^2*b + 70*a*b
^2 - 25*b^3)*cosh(f*x + e)^5 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*co
sh(f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*x + e))
*sqrt(a)*log((b*cosh(f*x + e)^4 + 4*b*cosh(f*x + e)*sinh(f*x + e)^3 + b*sin
h(f*x + e)^4 + 2*(4*a - b)*cosh(f*x + e)^2 + 2*(3*b*cosh(f*x + e)^2 + 4*a -
b)*sinh(f*x + e)^2 + 4*sqrt(2)*sqrt(a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*
x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh
(f*x + e)^2))*(cosh(f*x + e) + sinh(f*x + e)) + 4*(b*cosh(f*x + e)^3 + (4*a
- b)*cosh(f*x + e))*sinh(f*x + e) + b)/(cosh(f*x + e)^4 + 4*cosh(f*x + e)*
sinh(f*x + e)^3 + sinh(f*x + e)^4 + 2*(3*cosh(f*x + e)^2 - 1)*sinh(f*x + e)
^2 - 2*cosh(f*x + e)^2 + 4*(cosh(f*x + e)^3 - cosh(f*x + e))*sinh(f*x + e)
+ 1)) - 4*sqrt(2)*(3*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^9 + 27*(2*a^2*b - 5*
a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + 3*(2*a^2*b - 5*a*b^2)*sinh(f*x + e)^
9 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^7 + 4*(8*a^3 - 26*a^2*b +
15*a*b^2 + 27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^7 + 28*(9
*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^3 + (8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f
*x + e))*sinh(f*x + e)^6 - 2*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^5
+ 2*(189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^4 - 56*a^3 + 98*a^2*b - 45*a*b^
2 + 42*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^5 + 2*(
189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^5 + 70*(8*a^3 - 26*a^2*b + 15*a*b^2)*
cosh(f*x + e)^3 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e))*sinh(f*x
+ e)^4 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 + 4*(63*(2*a^2*b -
5*a*b^2)*cosh(f*x + e)^6 + 35*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^
4 + 8*a^3 - 26*a^2*b + 15*a*b^2 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x
+ e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^7 + 21*(
8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^5 - 5*(56*a^3 - 98*a^2*b + 45*a*
b^2)*cosh(f*x + e)^3 + 3*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(
f*x + e)^2 + 3*(2*a^2*b - 5*a*b^2)*cosh(f*x + e) + (27*(2*a^2*b - 5*a*b^2)*
cosh(f*x + e)^8 + 28*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^6 - 10*(56
*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^4 + 6*a^2*b - 15*a*b^2 + 12*(8*a^
3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e))*sqrt((b*cosh(f*x +
e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sin
h(f*x + e) + sinh(f*x + e)^2)))/(a^4*b^2*f*cosh(f*x + e)^12 + 12*a^4*b^2*f*
cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b^2*f*sinh(f*x + e)^12 + 2*(4*a^5*b -
3*a^4*b^2)*f*cosh(f*x + e)^10 + 2*(33*a^4*b^2*f*cosh(f*x + e)^2 + (4*a^5*b
- 3*a^4*b^2)*f)*sinh(f*x + e)^10 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(
f*x + e)^8 + 20*(11*a^4*b^2*f*cosh(f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*cos
h(f*x + e))*sinh(f*x + e)^9 + (495*a^4*b^2*f*cosh(f*x + e)^4 + 90*(4*a^5*b
- 3*a^4*b^2)*f*cosh(f*x + e)^2 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f)*sinh(f
x + e)^8 - 4(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^6 + 8*(99*a^4*
b^2*f*cosh(f*x + e)^5 + 30*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^3 + (16*a^
6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^7 + a^4*b^2*f + 4
*(231*a^4*b^2*f*cosh(f*x + e)^6 + 105*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)
^4 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^2 - (8*a^6 - 12*a^5
*b + 5*a^4*b^2)*f)*sinh(f*x + e)^6 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cos

h(f*x + e)^4 + 8*(99*a^4*b^2*f*cosh(f*x + e)^7 + 63*(4*a^5*b - 3*a^4*b^2)*f
*cosh(f*x + e)^5 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^3 - 3
*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^5 + (495*a^4
*b^2*f*cosh(f*x + e)^8 + 420*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^6 + 70*(
16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^4 - 60*(8*a^6 - 12*a^5*b +
5*a^4*b^2)*f*cosh(f*x + e)^2 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f)*sinh(f*x
 + e)^4 + 2*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^2 + 4*(55*a^4*b^2*f*cosh(
f*x + e)^9 + 60*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^7 + 14*(16*a^6 - 32*a
^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^5 - 20*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*
cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e))*sinh(f*
x + e)^3 + 2*(33*a^4*b^2*f*cosh(f*x + e)^10 + 45*(4*a^5*b - 3*a^4*b^2)*f*co
sh(f*x + e)^8 + 14*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^6 - 30*
(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^4 + 3*(16*a^6 - 32*a^5*b + 1
5*a^4*b^2)*f*cosh(f*x + e)^2 + (4*a^5*b - 3*a^4*b^2)*f)*sinh(f*x + e)^2 + 4
*(3*a^4*b^2*f*cosh(f*x + e)^11 + 5*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^9
+ 2*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^7 - 6*(8*a^6 - 12*a^5*
b + 5*a^4*b^2)*f*cosh(f*x + e)^5 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(
f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)), 1/6*(3*
((2*a*b^2 - 5*b^3)*cosh(f*x + e)^12 + 12*(2*a*b^2 - 5*b^3)*cosh(f*x + e)*si
nh(f*x + e)^11 + (2*a*b^2 - 5*b^3)*sinh(f*x + e)^12 + 2*(8*a^2*b - 26*a*b^2
 + 15*b^3)*cosh(f*x + e)^10 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3 + 33*(2*a*b^2
- 5*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^10 + 20*(11*(2*a*b^2 - 5*b^3)*cosh(
f*x + e)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*x + e)^9 +
 (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^8 + (495*(2*a*b^2
- 5*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3 + 90*(8*
a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^8 + 8*(99*(2*a*b^
2 - 5*b^3)*cosh(f*x + e)^5 + 30*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)
^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e))*sinh(f*x + e)
^7 - 4*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^6 + 4*(231*(2*
a*b^2 - 5*b^3)*cosh(f*x + e)^6 + 105*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x
 + e)^4 - 16*a^3 + 64*a^2*b - 70*a*b^2 + 25*b^3 + 7*(32*a^3 - 144*a^2*b + 1
90*a*b^2 - 75*b^3)*cosh(f*x + e)^2)*sinh(f*x + e)^6 + 8*(99*(2*a*b^2 - 5*b^
3)*cosh(f*x + e)^7 + 63*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^5 + 7*(
32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^3 - 3*(16*a^3 - 64*a
^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e))*sinh(f*x + e)^5 + (32*a^3 - 144*a^
2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + (495*(2*a*b^2 - 5*b^3)*cosh(f*x
 + e)^8 + 420*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^6 + 70*(32*a^3 -
144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e)^4 + 32*a^3 - 144*a^2*b + 190*
a*b^2 - 75*b^3 - 60*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^2
)*sinh(f*x + e)^4 + 4*(55*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^9 + 60*(8*a^2*b -
 26*a*b^2 + 15*b^3)*cosh(f*x + e)^7 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 -
75*b^3)*cosh(f*x + e)^5 - 20*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f
*x + e)^3 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e))*sinh(f
*x + e)^3 + 2*a*b^2 - 5*b^3 + 2*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)
^2 + 2*(33*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^10 + 45*(8*a^2*b - 26*a*b^2 + 15
*b^3)*cosh(f*x + e)^8 + 14*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f
x + e)^6 - 30(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^3)*cosh(f*x + e)^4 + 8*
a^2*b - 26*a*b^2 + 15*b^3 + 3*(32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cos
h(f*x + e)^2)*sinh(f*x + e)^2 + 4*(3*(2*a*b^2 - 5*b^3)*cosh(f*x + e)^11 + 5
*(8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e)^9 + 2*(32*a^3 - 144*a^2*b + 19
0*a*b^2 - 75*b^3)*cosh(f*x + e)^7 - 6*(16*a^3 - 64*a^2*b + 70*a*b^2 - 25*b^
3)*cosh(f*x + e)^5 + (32*a^3 - 144*a^2*b + 190*a*b^2 - 75*b^3)*cosh(f*x + e
)^3 + (8*a^2*b - 26*a*b^2 + 15*b^3)*cosh(f*x + e))*sinh(f*x + e))*sqrt(-a)*
arctan(1/2*sqrt(2)*sqrt(-a)*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2
*a - b)/(cosh(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)
)/(a*cosh(f*x + e) + a*sinh(f*x + e))) + 2*sqrt(2)*(3*(2*a^2*b - 5*a*b^2)*c
osh(f*x + e)^9 + 27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)*sinh(f*x + e)^8 + 3*(
2*a^2*b - 5*a*b^2)*sinh(f*x + e)^9 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f
x + e)^7 + 4(8*a^3 - 26*a^2*b + 15*a*b^2 + 27*(2*a^2*b - 5*a*b^2)*cosh(f*

```
x + e)^2)*sinh(f*x + e)^7 + 28*(9*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^3 + (8*
a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^6 - 2*(56*a^3 - 98*
a^2*b + 45*a*b^2)*cosh(f*x + e)^5 + 2*(189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e
)^4 - 56*a^3 + 98*a^2*b - 45*a*b^2 + 42*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(
f*x + e)^2)*sinh(f*x + e)^5 + 2*(189*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^5 +
70*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^3 - 5*(56*a^3 - 98*a^2*b + 4
5*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^4 + 4*(8*a^3 - 26*a^2*b + 15*a*b^2)*c
osh(f*x + e)^3 + 4*(63*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^6 + 35*(8*a^3 - 26
*a^2*b + 15*a*b^2)*cosh(f*x + e)^4 + 8*a^3 - 26*a^2*b + 15*a*b^2 - 5*(56*a^
3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^2)*sinh(f*x + e)^3 + 4*(27*(2*a^2*b
- 5*a*b^2)*cosh(f*x + e)^7 + 21*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)
^5 - 5*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^3 + 3*(8*a^3 - 26*a^2*b
 + 15*a*b^2)*cosh(f*x + e))*sinh(f*x + e)^2 + 3*(2*a^2*b - 5*a*b^2)*cosh(f*
x + e) + (27*(2*a^2*b - 5*a*b^2)*cosh(f*x + e)^8 + 28*(8*a^3 - 26*a^2*b + 1
5*a*b^2)*cosh(f*x + e)^6 - 10*(56*a^3 - 98*a^2*b + 45*a*b^2)*cosh(f*x + e)^
4 + 6*a^2*b - 15*a*b^2 + 12*(8*a^3 - 26*a^2*b + 15*a*b^2)*cosh(f*x + e)^2)*
sinh(f*x + e))*sqrt((b*cosh(f*x + e)^2 + b*sinh(f*x + e)^2 + 2*a - b)/(cosh
(f*x + e)^2 - 2*cosh(f*x + e)*sinh(f*x + e) + sinh(f*x + e)^2)))/(a^4*b^2*f
*cosh(f*x + e)^12 + 12*a^4*b^2*f*cosh(f*x + e)*sinh(f*x + e)^11 + a^4*b^2*f
*sinh(f*x + e)^12 + 2*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^10 + 2*(33*a^4*
b^2*f*cosh(f*x + e)^2 + (4*a^5*b - 3*a^4*b^2)*f)*sinh(f*x + e)^10 + (16*a^6
 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^8 + 20*(11*a^4*b^2*f*cosh(f*x + e
)^3 + (4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^9 + (495*a^4*b^2
*f*cosh(f*x + e)^4 + 90*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^2 + (16*a^6 -
 32*a^5*b + 15*a^4*b^2)*f)*sinh(f*x + e)^8 - 4*(8*a^6 - 12*a^5*b + 5*a^4*b^
2)*f*cosh(f*x + e)^6 + 8*(99*a^4*b^2*f*cosh(f*x + e)^5 + 30*(4*a^5*b - 3*a^
4*b^2)*f*cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)
)*sinh(f*x + e)^7 + a^4*b^2*f + 4*(231*a^4*b^2*f*cosh(f*x + e)^6 + 105*(4*a
^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^4 + 7*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*
cosh(f*x + e)^2 - (8*a^6 - 12*a^5*b + 5*a^4*b^2)*f)*sinh(f*x + e)^6 + (16*a
^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^4 + 8*(99*a^4*b^2*f*cosh(f*x +
e)^7 + 63*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^5 + 7*(16*a^6 - 32*a^5*b +
15*a^4*b^2)*f*cosh(f*x + e)^3 - 3*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x
 + e))*sinh(f*x + e)^5 + (495*a^4*b^2*f*cosh(f*x + e)^8 + 420*(4*a^5*b - 3*
a^4*b^2)*f*cosh(f*x + e)^6 + 70*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x
 + e)^4 - 60*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^2 + (16*a^6 - 3
2*a^5*b + 15*a^4*b^2)*f)*sinh(f*x + e)^4 + 2*(4*a^5*b - 3*a^4*b^2)*f*cosh(f
*x + e)^2 + 4*(55*a^4*b^2*f*cosh(f*x + e)^9 + 60*(4*a^5*b - 3*a^4*b^2)*f*co
sh(f*x + e)^7 + 14*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^5 - 20*
(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^3 + (16*a^6 - 32*a^5*b + 15*
a^4*b^2)*f*cosh(f*x + e))*sinh(f*x + e)^3 + 2*(33*a^4*b^2*f*cosh(f*x + e)^1
0 + 45*(4*a^5*b - 3*a^4*b^2)*f*cosh(f*x + e)^8 + 14*(16*a^6 - 32*a^5*b + 15
*a^4*b^2)*f*cosh(f*x + e)^6 - 30*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x
+ e)^4 + 3*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^2 + (4*a^5*b -
3*a^4*b^2)*f)*sinh(f*x + e)^2 + 4*(3*a^4*b^2*f*cosh(f*x + e)^11 + 5*(4*a^5*
b - 3*a^4*b^2)*f*cosh(f*x + e)^9 + 2*(16*a^6 - 32*a^5*b + 15*a^4*b^2)*f*cos
h(f*x + e)^7 - 6*(8*a^6 - 12*a^5*b + 5*a^4*b^2)*f*cosh(f*x + e)^5 + (16*a^6
 - 32*a^5*b + 15*a^4*b^2)*f*cosh(f*x + e)^3 + (4*a^5*b - 3*a^4*b^2)*f*cosh(
f*x + e))*sinh(f*x + e))]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**3/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)^3}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^3/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^3/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.506 $\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{8a^2-40ab+35b^2}{8a^4f\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}-\frac{(8a-}{8a^2f\left(a+\right.}$$

```
[Out] -((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(
8*a^(9/2)*f) + (8*a^2 - 40*a*b + 35*b^2)/(24*a^3*f*(a + b*Sinh[e + f*x]^2)^
(3/2)) - ((8*a - 7*b)*Csch[e + f*x]^2)/(8*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - Csch[e + f*x]^4/(4*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (8*a^2 - 40*a
*b + 35*b^2)/(8*a^4*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.224905, antiderivative size = 208, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2-40ab+35b^2}{8a^4f\sqrt{a+b\sinh^2(e+fx)}}+\frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}-\frac{(8a-}{8a^2f\left(a+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -((8*a^2 - 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sinh[e + f*x]^2]/Sqrt[a]])/(
8*a^(9/2)*f) + (8*a^2 - 40*a*b + 35*b^2)/(24*a^3*f*(a + b*Sinh[e + f*x]^2)^
(3/2)) - ((8*a - 7*b)*Csch[e + f*x]^2)/(8*a^2*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - Csch[e + f*x]^4/(4*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + (8*a^2 - 40*a
*b + 35*b^2)/(8*a^4*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth^5(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x)^2}{x^3(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{csch}^4(e+fx)}{4af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(8a-7b)+2ax}{x^2(a+bx)^{5/2}}\,dx, x, \sinh^2(e+fx)\right)}{4af}$$

$$= -\frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^4(e+fx)}{4af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(8a^2-40ab+35b^2\right.}{}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^4(e+f}{4af\left(a+b\sinh^2(e\right.}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^4(e+f}{4af\left(a+b\sinh^2(e\right.}$$

$$= \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{(8a-7b)\text{csch}^2(e+fx)}{8a^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\text{csch}^4(e+f}{4af\left(a+b\sinh^2(e\right.}$$

$$= -\frac{\left(8a^2-40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f} + \frac{8a^2-40ab+35b^2}{24a^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - $$

Mathematica [C] time = 0.423643, size = 117, normalized size = 0.56

$$-\frac{\operatorname{csch}^2(e+fx)\left(\left(-8a^2+40ab-35b^2\right){}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sinh^2(e+fx)}{a}+1\right)+3a\operatorname{csch}^2(e+fx)\left(2a\operatorname{csch}^2(e+fx)+8a-7b\right)\right)}{24a^3f\sqrt{a+b\sinh^2(e+fx)}\left(a\operatorname{csch}^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^5/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Csch[e + f*x]^2*(3*a*Csch[e + f*x]^2*(8*a - 7*b + 2*a*Csch[e + f*x]^2) +
(-8*a^2 + 40*a*b - 35*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sinh[e +
 f*x]^2)/a]))/(24*a^3*f*(b + a*Csch[e + f*x]^2)*Sqrt[a + b*Sinh[e + f*x]^2]
)
```

Maple [C] time = 0.122, size = 73, normalized size = 0.4

$$\frac{1}{f}\text{`int/indef0`}\left(\frac{\left(\cosh\left(fx+e\right)\right)^4}{\left(b^2\left(\sinh\left(fx+e\right)\right)^4+2ab\left(\sinh\left(fx+e\right)\right)^2+a^2\right)\left(\sinh\left(fx+e\right)\right)^5}\frac{1}{\sqrt{a+b\left(\sinh\left(fx+e\right)\right)^2}},\sinh\left(\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] `int/indef0`(cosh(f*x+e)^4/(b^2*sinh(f*x+e)^4+2*a*b*sinh(f*x+e)^2+a^2)/sinh
(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(1/2),sinh(f*x+e))/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\coth\left(fx+e\right)^5}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**5/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^{5}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^5/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^5/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.507 $\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=333

$$\frac{(3a+b)(a+3b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af(a-b)^4\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{2(2a+b)\tanh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+\right.}$$

```
[Out] -(b*(5*a + 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^3*f*(a + b*Sinh[e +
 f*x]^2)^(3/2)) - (8*Sqrt[a]*Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan
[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^4*f*Sqrt[(a*Cosh[e
+ f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + b
)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a*(a - b)^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) - (2*(2*a + b)*Tanh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e +
 f*x]^2)^(3/2)) + (Sech[e + f*x]^2*Tanh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[
e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.399476, antiderivative size = 333, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3196, 470, 527, 525, 418, 411}

$$-\frac{2(2a+b)\tanh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{b(5a+3b)\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^3\left(a+b\sinh^2(e+fx)\right)^{3/2}}+\frac{\tanh(e+fx)\text{sech}^2(e+fx)}{3f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{8\sqrt{a}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] -(b*(5*a + 3*b)*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^3*f*(a + b*Sinh[e +
 f*x]^2)^(3/2)) - (8*Sqrt[a]*Sqrt[b]*(a + b)*Cosh[e + f*x]*EllipticE[ArcTan
[(Sqrt[b]*Sinh[e + f*x])/Sqrt[a]], 1 - a/b])/(3*(a - b)^4*f*Sqrt[(a*Cosh[e
+ f*x]^2)/(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + b
)*(a + 3*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a*(a - b)^4*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) - (2*(2*a + b)*Tanh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e +
 f*x]^2)^(3/2)) + (Sech[e + f*x]^2*Tanh[e + f*x])/(3*(a - b)*f*(a + b*Sinh[
e + f*x]^2)^(3/2))
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
```

```
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\tanh^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1+x^2)^{5/2}(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\text{sech}(e+fx)\right)\text{Subst}\left(\int \frac{a+(-3a-}{(1+x^2)^{3/2}(a}\right.}{3(a-b)f}$$

$$= -\frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}^2(e+fx)\tanh(e+fx)}{3(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e\right.}{}$$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}}{3(a-b)}$$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a+b)\tanh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\text{sech}}{3(a-b)}$$

$$= -\frac{b(5a+3b)\cosh(e+fx)\sinh(e+fx)}{3(a-b)^3f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{8\sqrt{a}\sqrt{b}(a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e}{\sqrt{a}}\right.\right.}{3(a-b)^4f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(}$$

Mathematica [C] time = 3.36955, size = 252, normalized size = 0.76

$$-\frac{i\left(2ab\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left(\left(-5a^2+2ab+3b^2\right)\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+8a(a+b)E\left(i(e+fx)\middle|\frac{b}{a}\right)\right)-i\sqrt{2}b\left(2ab(a-\right.\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-I/6)*(2*a*b*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*(8*a*(a + b)*Ellip
ticE[I*(e + f*x), b/a] + (-5*a^2 + 2*a*b + 3*b^2)*EllipticF[I*(e + f*x), b/
a]) - I*Sqrt[2]*b*(2*a*(a - b)*b*Sinh[2*(e + f*x)] + 4*b*(a + b)*(2*a - b +
 b*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)] + 4*(a + b)*(2*a - b + b*Cosh[2*(e
+ f*x)])^2*Tanh[e + f*x] - (a - b)*(2*a - b + b*Cosh[2*(e + f*x)])^2*Sech[e
 + f*x]^2*Tanh[e + f*x])))/((a - b)^4*b*f*(2*a - b + b*Cosh[2*(e + f*x)])^(
3/2))
```

Maple [A] time = 0.217, size = 661, normalized size = 2.

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((-8*(-1/a*b)^(1/2)*a*b^2-8*(-1/a*b)^(1/2)*b^3)*sinh(f*x+e)*cosh(f*x+e)
^6+(-13*(-1/a*b)^(1/2)*a^2*b+2*(-1/a*b)^(1/2)*a*b^2+11*(-1/a*b)^(1/2)*b^3)*
```

```
cosh(f*x+e)^4*sinh(f*x+e)+(-4*(-1/a*b)^(1/2)*a^3+6*(-1/a*b)^(1/2)*a^2*b-2*(
-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2*sinh(f*x+e)+((-1/a*b)^(1/2)*a^3-3*(-1/a*b)
^(1/2)*a^2*b+3*(-1/a*b)^(1/2)*a*b^2-(-1/a*b)^(1/2)*b^3)*sinh(f*x+e)+(b/a*co
sh(f*x+e)^2+(a-b)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*b*(3*EllipticF(sinh(f*x+e)
*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2+2*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/
b)^(1/2))*a*b-5*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2+8*Ell
ipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticE(sinh(f*x+e)*
(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*cosh(f*x+e)^4+(b/a*cosh(f*x+e)^2+(a-b)/a)^
(1/2)*(cosh(f*x+e)^2)^(1/2)*(3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(
1/2))*a^3-EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-7*Ellipti
cF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+5*EllipticF(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*b^3+8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^
(1/2))*a^2*b-8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3)*cosh(
f*x+e)^2)/cosh(f*x+e)^3/(-1/a*b)^(1/2)/(a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^4/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\tanh\left(fx+e\right)^{4}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.508 $\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=274

$$\frac{(3a+5b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3af(a-b)^3\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}-\frac{\tanh(e+fx)}{f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}$$

```
[Out] (-4*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (Sqrt[b]*(7*a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]], 1 - a/b])/(3*Sqrt[a]*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/
(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + 5*b)*Ellipt
icF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - T
anh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```

Rubi [A] time = 0.28018, antiderivative size = 274, normalized size of antiderivative = 1., number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.24, Rules used = {3196, 471, 527, 525, 418, 411}

$$-\frac{\tanh(e+fx)}{f(a-b)\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{4b\sinh(e+fx)\cosh(e+fx)}{3f(a-b)^2\left(a+b\sinh^2(e+fx)\right)^{3/2}}-\frac{\sqrt{b}(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e\cdots}{\sqrt{a}}\right.\right.}{3\sqrt{a}f(a-b)^3\sqrt{a+b\sinh^2(e+fx)}\sqrt{\frac{a\cdots}{a+b\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] (-4*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*(a - b)^2*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (Sqrt[b]*(7*a + b)*Cosh[e + f*x]*EllipticE[ArcTan[(Sqrt[b]*Sinh[e +
 f*x])/Sqrt[a]], 1 - a/b])/(3*Sqrt[a]*(a - b)^3*f*Sqrt[(a*Cosh[e + f*x]^2)/
(a + b*Sinh[e + f*x]^2)]*Sqrt[a + b*Sinh[e + f*x]^2]) + ((3*a + 5*b)*Ellipt
icF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a*(a - b)^3*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) - T
anh[e + f*x]/((a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/2))
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
```

```
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 525

```
Int[((e_) + (f_.)*(x_)^2)/(Sqrt[(a_) + (b_.)*(x_)^2]*((c_) + (d_.)*(x_)^2)^
(3/2)), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(Sqrt[a + b*x^2]*S
qrt[c + d*x^2]), x], x] - Dist[(d*e - c*f)/(b*c - a*d), Int[Sqrt[a + b*x^2]
/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && PosQ[b/a] &&
PosQ[d/c]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\tanh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{a-3bx}{\sqrt{1+x^2}(a+l}\right.}{(-a+b)f}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+\right.}{}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\tanh(e+fx)}{(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{\left(b(7a+b)\sqrt{c\right.}{}$$

$$= -\frac{4b\cosh(e+fx)\sinh(e+fx)}{3(a-b)^2f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\sqrt{b}(7a+b)\cosh(e+fx)E\left(\tan^{-1}\left(\frac{\sqrt{b}\sinh(e+fx)}{\sqrt{a}}\right)\right|}{3\sqrt{a}(a-b)^3f\sqrt{\frac{a\cosh^2(e+fx)}{a+b\sinh^2(e+fx)}}\sqrt{a+b\sinh^2(e+}}$$

Mathematica [C] time = 2.6007, size = 215, normalized size = 0.78

$$\frac{8ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)-\frac{\tanh(e+fx)\left(-4a^2b+24a^3+b^2(7a+b)\cosh(4(e+fx))+5ab^2+4ab(11a-3b)\cosh\right.}{\sqrt{2}}}{6af(a-b)^3(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tanh[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-2*I)*a^2*(7*a + b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3
/2)*EllipticF[I*(e + f*x), b/a] - ((24*a^3 - 4*a^2*b + 5*a*b^2 - b^3 + 4*a*
(11*a - 3*b)*b*Cosh[2*(e + f*x)] + b^2*(7*a + b)*Cosh[4*(e + f*x)])*Tanh[e
+ f*x])/Sqrt[2])/(6*a*(a - b)^3*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.192, size = 799, normalized size = 2.9

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(-7*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^5-(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^5
+3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*x+e)^2-2*((a+b*sinh(f*x+e)^2)/a)
^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*a*b^2*sinh(f*x+e)^2-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)
*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*sinh(f*x+e)^2+7*((a+
b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^2+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(c
osh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*s
inh(f*x+e)^2-11*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^3-4*(-1/a*b)^(1/2)*a*b^2*s
inh(f*x+e)^3-(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^3+3*((a+b*sinh(f*x+e)^2)/a)^(1/
2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*
a^3-2*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(
f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2+7*(
(a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-
1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^
2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2-3*(-1/a*b)
^(1/2)*a^3*sinh(f*x+e)-5*sinh(f*x+e)*b*a^2*(-1/a*b)^(1/2))/(-1/a*b)^(1/2)/(
a+b*sinh(f*x+e)^2)^(3/2)/(a-b)^3/a/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int\frac{\tanh\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^{2}+a}\tanh\left(fx+e\right)^{2}}{b^{3}\sinh\left(fx+e\right)^{6}+3\,ab^{2}\sinh\left(fx+e\right)^{4}+3\,a^{2}b\sinh\left(fx+e\right)^{2}+a^{3}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*tanh(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\tanh\left(fx+e\right)^{2}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tanh(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tanh(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.509 $\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=251

$$\frac{i\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}\,\text{EllipticF}\left(ie+ifx,\frac{b}{a}\right)}{3af(a-b)\sqrt{a+b\sinh^2(e+fx)}}-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(\right.}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}}}$$

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rubi [A] time = 0.288262, antiderivative size = 251, normalized size of antiderivative = 1., number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{2b(2a-b)\sinh(e+fx)\cosh(e+fx)}{3a^2f(a-b)^2\sqrt{a+b\sinh^2(e+fx)}}-\frac{2i(2a-b)\sqrt{a+b\sinh^2(e+fx)}E\left(ie+ifx\middle|\frac{b}{a}\right)}{3a^2f(a-b)^2\sqrt{\frac{b\sinh^2(e+fx)}{a}+1}}-\frac{b\sinh(e+fx)\cosh(e+}{3af(a-b)\left(a+b\sinh^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] -(b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a*(a - b)*f*(a + b*Sinh[e + f*x]^2)^(3/
2)) - (2*(2*a - b)*b*Cosh[e + f*x]*Sinh[e + f*x])/(3*a^2*(a - b)^2*f*Sqrt[a
 + b*Sinh[e + f*x]^2]) - (((2*I)/3)*(2*a - b)*EllipticE[I*e + I*f*x, b/a]*S
qrt[a + b*Sinh[e + f*x]^2])/(a^2*(a - b)^2*f*Sqrt[1 + (b*Sinh[e + f*x]^2)/a
]) + ((I/3)*EllipticF[I*e + I*f*x, b/a]*Sqrt[1 + (b*Sinh[e + f*x]^2)/a])/(a
*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2])
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a+2b+b\sinh^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{3/2}}\,dx}{3a(a-b)}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{-a(3a-}{}}{}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\int \frac{}{\sqrt{a+b\,\mathrm{s}}}}{3a(}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} + \frac{\left(2(2a-\right.}{}$$

$$= -\frac{b\cosh(e+fx)\sinh(e+fx)}{3a(a-b)f\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)b\cosh(e+fx)\sinh(e+fx)}{3a^2(a-b)^2f\sqrt{a+b\sinh^2(e+fx)}} - \frac{2i(2a-}{}$$

Mathematica [A] time = 1.25598, size = 190, normalized size = 0.76

$$\frac{ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\text{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+\sqrt{2}b\sinh(2(e+fx))\left(-5a^2+b(b-2a)\cosh(2(e+fx))+5ab\right.}{3a^2f(a-b)^2(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^(-5/2),x]
```

```
[Out] ((-2*I)*a^2*(2*a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*EllipticE[I
*(e + f*x), b/a] + I*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*
EllipticF[I*(e + f*x), b/a] + Sqrt[2]*b*(-5*a^2 + 5*a*b - b^2 + b*(-2*a + b
)*Cosh[2*(e + f*x)])*Sinh[2*(e + f*x)])/(3*a^2*(a - b)^2*f*(2*a - b + b*Cos
h[2*(e + f*x)])^(3/2))
```

Maple [A] time = 0., size = 406, normalized size = 1.6

$$\frac{1}{\cosh(fx+e)f}\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+e)\right)^2}\left(-\frac{\sinh(fx+e)}{3ab(a-b)}\sqrt{\left(a+b\left(\sinh(fx+e)\right)^2\right)\left(\cosh(fx+e\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] ((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*(-1/3/a/b/(a-b)*sinh(f*x+e)*((a+b
*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)/(sinh(f*x+e)^2+a/b)^2-2/3*b*cosh(f*x+e
)^2/a^2/(a-b)^2*sinh(f*x+e)*(2*a-b)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/
2)+(3*a-b)/(3*a^3-6*a^2*b+3*a*b^2)/(-1/a*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(
1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*x+e)^2)*cosh(f*x+e)^2)^(1/2)*Ellipt
icF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))-2/3*b*(2*a-b)/a^2/(a-b)^2/(-1/a
*b)^(1/2)*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)/((a+b*sinh(f*
x+e)^2)*cosh(f*x+e)^2)^(1/2)*(EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))-EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))))/cosh(f*x+e)/(a+b*s
inh(f*x+e)^2)^(1/2)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh(fx+e)^2+a}}{b^3\sinh(fx+e)^6+3ab^2\sinh(fx+e)^4+3a^2b\sinh(fx+e)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)/(b^3*sinh(f*x + e)^6 + 3*a*b^2*sinh(f*
x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{1}{\left(b \sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^(-5/2), x)
```

3.510 $\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=351

$$\frac{(3a-4b)\operatorname{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\,\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^3f(a-b)\sqrt{\frac{\operatorname{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\,\mathrm{s}}}{3a^3f(a-b)}$$

```
[Out] Coth[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((3*a - 4*b)*Coth[e +
 f*x])/(3*a^2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((7*a - 8*b)*Coth[e
+ f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f) - ((7*a - 8*b)*Ellipt
icE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3
*a^3*(a - b)*f)
```

Rubi [A] time = 0.4119, antiderivative size = 351, normalized size of antiderivative = 1., number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 469, 579, 583, 531, 418, 492, 411}

$$\frac{(7a-8b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)}-\frac{(7a-8b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^3f(a-b)}+\frac{(3a-4b)\coth(e}{3a^2f(a-b)\sqrt{a+b\sin}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] Coth[e + f*x]/(3*a*f*(a + b*Sinh[e + f*x]^2)^(3/2)) + ((3*a - 4*b)*Coth[e +
 f*x])/(3*a^2*(a - b)*f*Sqrt[a + b*Sinh[e + f*x]^2]) - ((7*a - 8*b)*Coth[e
+ f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f) - ((7*a - 8*b)*Ellipt
icE[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^
2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (
(3*a - 4*b)*EllipticF[ArcTan[Sinh[e + f*x]], 1 - b/a]*Sech[e + f*x]*Sqrt[a
+ b*Sinh[e + f*x]^2])/(3*a^3*(a - b)*f*Sqrt[(Sech[e + f*x]^2*(a + b*Sinh[e
+ f*x]^2))/a]) + ((7*a - 8*b)*Sqrt[a + b*Sinh[e + f*x]^2]*Tanh[e + f*x])/(3
*a^3*(a - b)*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
```

```
1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\coth^2(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1+x^2}}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{-4-3x^2}{x^2\sqrt{1+x^2}(a+b}\right.}{3af}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\right.}{}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e}{3}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e}{3}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e}{3}$$

$$= \frac{\coth(e+fx)}{3af\left(a+b\sinh^2(e+fx)\right)^{3/2}} + \frac{(3a-4b)\coth(e+fx)}{3a^2(a-b)f\sqrt{a+b\sinh^2(e+fx)}} - \frac{(7a-8b)\coth(e}{3}$$

Mathematica [C] time = 2.6774, size = 226, normalized size = 0.64

$$\frac{8ia^2(a-b)\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right) - \frac{\coth(e+fx)\left(4b\left(11a^2-19ab+8b^2\right)\cosh(2(e+fx))-68a^2b+24a^3+b^2(7a-8b)\cos\right.}{\sqrt{2}}}{6a^3f(a-b)(2a+b\cosh(2(e+fx))-b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^2/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(((24*a^3 - 68*a^2*b + 69*a*b^2 - 24*b^3 + 4*b*(11*a^2 - 19*a*b + 8*b^2)*
Cosh[2*(e + f*x)] + (7*a - 8*b)*b^2*Cosh[4*(e + f*x)])*Coth[e + f*x])/Sqrt[
2]) - (2*I)*a^2*(7*a - 8*b)*((2*a - b + b*Cosh[2*(e + f*x)])/a)^(3/2)*Ellip
ticE[I*(e + f*x), b/a] + (8*I)*a^2*(a - b)*((2*a - b + b*Cosh[2*(e + f*x)])
/a)^(3/2)*EllipticF[I*(e + f*x), b/a])/(6*a^3*(a - b)*f*(2*a - b + b*Cosh[2
*(e + f*x)])^(3/2))
```

Maple [A] time = 0.24, size = 640, normalized size = 1.8

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((cosh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*b*(3*EllipticF
(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2-11*EllipticF(sinh(f*x+e)*(-1/a
*b)^(1/2),(a/b)^(1/2))*a*b+8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/
2))*b^2+7*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b-8*EllipticE
(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^2)*sinh(f*x+e)*cosh(f*x+e)^2+(co
sh(f*x+e)^2)^(1/2)*(b/a*cosh(f*x+e)^2+(a-b)/a)^(1/2)*(7*EllipticE(sinh(f*x+
e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b-15*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2
),(a/b)^(1/2))*a*b^2+8*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^
3+3*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^3-14*EllipticF(sinh
(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b+19*EllipticF(sinh(f*x+e)*(-1/a*b)
^(1/2),(a/b)^(1/2))*a*b^2-8*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2
))*b^3)*sinh(f*x+e)+(-7*(-1/a*b)^(1/2)*a*b^2+8*(-1/a*b)^(1/2)*b^3)*cosh(f*x
+e)^6+(-11*(-1/a*b)^(1/2)*a^2*b+26*(-1/a*b)^(1/2)*a*b^2-16*(-1/a*b)^(1/2)*b
^3)*cosh(f*x+e)^4+(-3*(-1/a*b)^(1/2)*a^3+14*(-1/a*b)^(1/2)*a^2*b-19*(-1/a*b
)^(1/2)*a*b^2+8*(-1/a*b)^(1/2)*b^3)*cosh(f*x+e)^2)/(-1/a*b)^(1/2)/sinh(f*x+
e)/a^3/(a-b)/(a+b*sinh(f*x+e)^2)^(3/2)/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^2}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\,\coth\left(fx+e\right)^2}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^2/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**2/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(fx+e)^2}{\left(b\sinh(fx+e)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^2/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^2/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.511 $\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=385

$$\frac{(3a-8b)\text{sech}(e+fx)\sqrt{a+b\sinh^2(e+fx)}\text{EllipticF}\left(\tan^{-1}(\sinh(e+fx)),1-\frac{b}{a}\right)}{3a^4f\sqrt{\frac{\text{sech}^2(e+fx)\left(a+b\sinh^2(e+fx)\right)}{a}}}+\frac{8(a-2b)\tanh(e+fx)\sqrt{a+b\sinh}}{3a^4f}$$

```
[Out] -((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(a - 3*b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*S
inh[e + f*x]^2]) - (8*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*a^4*f) + ((3*a - 8*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e +
f*x]^2])/(3*a^3*b*f) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/
a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech[e + f*x]^
2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e +
 f*x]^2]*Tanh[e + f*x])/(3*a^4*f)
```

Rubi [A] time = 0.561806, antiderivative size = 385, normalized size of antiderivative = 1., number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.32, Rules used = {3196, 468, 579, 583, 531, 418, 492, 411}

$$\frac{8(a-2b)\tanh(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^4f}-\frac{8(a-2b)\coth(e+fx)\sqrt{a+b\sinh^2(e+fx)}}{3a^4f}+\frac{(3a-8b)\coth(e+fx)\text{csc}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```


```
[Out] -((a - b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a*b*f*(a + b*Sinh[e + f*x]^2)^(
3/2)) - (2*(a - 3*b)*Coth[e + f*x]*Csch[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*S
inh[e + f*x]^2]) - (8*(a - 2*b)*Coth[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/
(3*a^4*f) + ((3*a - 8*b)*Coth[e + f*x]*Csch[e + f*x]^2*Sqrt[a + b*Sinh[e +
f*x]^2])/(3*a^3*b*f) - (8*(a - 2*b)*EllipticE[ArcTan[Sinh[e + f*x]], 1 - b/
a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech[e + f*x]^
2*(a + b*Sinh[e + f*x]^2))/a]) + ((3*a - 8*b)*EllipticF[ArcTan[Sinh[e + f*x
]], 1 - b/a]*Sech[e + f*x]*Sqrt[a + b*Sinh[e + f*x]^2])/(3*a^4*f*Sqrt[(Sech
[e + f*x]^2*(a + b*Sinh[e + f*x]^2))/a]) + (8*(a - 2*b)*Sqrt[a + b*Sinh[e +
 f*x]^2]*Tanh[e + f*x])/(3*a^4*f)
```


Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 531

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Dist[e, Int[(a + b*x^n)^p*(c + d*x^n)^q, x],
x] + Dist[f, Int[x^n*(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c,
 d, e, f, n, p, q}, x]
```

Rule 418

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(Sqrt[a + b*x^2]*EllipticF[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(a*R
t[d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; Fre
eQ[{a, b, c, d}, x] && PosQ[d/c] && PosQ[b/a] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 492

```
Int[(x_)^2/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol]
 :> Simp[(x*Sqrt[a + b*x^2])/(b*Sqrt[c + d*x^2]), x] - Dist[c/b, Int[Sqrt[a
 + b*x^2]/(c + d*x^2)^(3/2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c -
a*d, 0] && PosQ[b/a] && PosQ[d/c] &&  !SimplerSqrtQ[b/a, d/c]
```

Rule 411

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/((c_) + (d_.)*(x_)^2)^(3/2), x_Symbol] :> Sim
p[(Sqrt[a + b*x^2]*EllipticE[ArcTan[Rt[d/c, 2]*x], 1 - (b*c)/(a*d)])/(c*Rt[
d/c, 2]*Sqrt[c + d*x^2]*Sqrt[(c*(a + b*x^2))/(a*(c + d*x^2))]), x] /; FreeQ
[{a, b, c, d}, x] && PosQ[b/a] && PosQ[d/c]
```

Rubi steps

$$\int \frac{\coth^4(e+fx)}{\left(a+b\sinh^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{(1+x^2)^{3/2}}{x^4(a+bx^2)^{5/2}}\,dx, x, \sinh(e+fx)\right)}{f}$$

$$= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cosh^2(e+fx)}\operatorname{sech}(e+fx)\right)\operatorname{Subst}\left(\int \frac{3(a-2b)-}{x^4\sqrt{1+x^2}}\right.}{3abf}$$

$$= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{\left(\sqrt{\cosh^2\right.}{}$$

$$= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} + \frac{(3a-8b)}{}$$

$$= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{}$$

$$= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{}$$

$$= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{}$$

$$= -\frac{(a-b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3abf\left(a+b\sinh^2(e+fx)\right)^{3/2}} - \frac{2(a-3b)\coth(e+fx)\operatorname{csch}^2(e+fx)}{3a^2bf\sqrt{a+b\sinh^2(e+fx)}} - \frac{8(a-2b)}{}$$

Mathematica [C] time = 2.78841, size = 247, normalized size = 0.64

$$-\frac{i\left(2a^2b\left(\frac{2a+b\cosh(2(e+fx))-b}{a}\right)^{3/2}\left((8b-5a)\operatorname{EllipticF}\left(i(e+fx),\frac{b}{a}\right)+8(a-2b)E\left(i(e+fx)\middle|\frac{b}{a}\right)\right)+\frac{ib\coth(e+fx)\operatorname{csch}^2(e+fx)(-2(}{}\right.}{6a^4bf(2a+b\cosh(2(e}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[e + f*x]^4/(a + b*Sinh[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-I/6)*((I*b*(8*a^3 - 63*a^2*b + 92*a*b^2 - 40*b^3 - 2*(8*a^3 - 38*a^2*b +
 63*a*b^2 - 30*b^3)*Cosh[2*(e + f*x)] - b*(13*a^2 - 36*a*b + 24*b^2)*Cosh[4
*(e + f*x)] - 2*a*b^2*Cosh[6*(e + f*x)] + 4*b^3*Cosh[6*(e + f*x)])*Coth[e +
 f*x]*Csch[e + f*x]^2)/Sqrt[2] + 2*a^2*b*((2*a - b + b*Cosh[2*(e + f*x)])/a
)^(3/2)*(8*(a - 2*b)*EllipticE[I*(e + f*x), b/a] + (-5*a + 8*b)*EllipticF[I
*(e + f*x), b/a])))/(a^4*b*f*(2*a - b + b*Cosh[2*(e + f*x)])^(3/2))
```

Maple [B] time = 0.226, size = 924, normalized size = 2.4

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(-8*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^8+16*(-1/a*b)^(1/2)*b^3*sinh(f*x+e
)^8+3*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*
x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*x+e)^5-16*((a+b*sinh(f*x+e)^2
)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)
^(1/2))*a*b^2*sinh(f*x+e)^5+16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2
)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*b^3*sinh(f*x+e)^5
+8*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e
)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^5-16*((a+b*sinh(f*x+e)^2)/a
)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1
/2))*b^3*sinh(f*x+e)^5-13*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^6+16*(-1/a*b)^(1
/2)*a*b^2*sinh(f*x+e)^6+16*(-1/a*b)^(1/2)*b^3*sinh(f*x+e)^6+3*((a+b*sinh(f*
x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2)
,(a/b)^(1/2))*a^3*sinh(f*x+e)^3-16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+
e)^2)^(1/2)*EllipticF(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a^2*b*sinh(f*
x+e)^3+16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticF(sin
h(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x+e)^3+8*((a+b*sinh(f*x+e
)^2)/a)^(1/2)*(cosh(f*x+e)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a
/b)^(1/2))*a^2*b*sinh(f*x+e)^3-16*((a+b*sinh(f*x+e)^2)/a)^(1/2)*(cosh(f*x+e
)^2)^(1/2)*EllipticE(sinh(f*x+e)*(-1/a*b)^(1/2),(a/b)^(1/2))*a*b^2*sinh(f*x
+e)^3-4*(-1/a*b)^(1/2)*a^3*sinh(f*x+e)^4-7*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)
^4+24*(-1/a*b)^(1/2)*a*b^2*sinh(f*x+e)^4-5*(-1/a*b)^(1/2)*a^3*sinh(f*x+e)^2
+6*(-1/a*b)^(1/2)*a^2*b*sinh(f*x+e)^2-(-1/a*b)^(1/2)*a^3)/(-1/a*b)^(1/2)/si
nh(f*x+e)^3/a^4/(a+b*sinh(f*x+e)^2)^(3/2)/cosh(f*x+e)/f
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^4}{\left(b\sinh\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\frac{\sqrt{b\sinh\left(fx+e\right)^2+a}\coth\left(fx+e\right)^4}{b^3\sinh\left(fx+e\right)^6+3\,ab^2\sinh\left(fx+e\right)^4+3\,a^2b\sinh\left(fx+e\right)^2+a^3},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*sinh(f*x + e)^2 + a)*coth(f*x + e)^4/(b^3*sinh(f*x + e)^6 +
 3*a*b^2*sinh(f*x + e)^4 + 3*a^2*b*sinh(f*x + e)^2 + a^3), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)**4/(a+b*sinh(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth\left(fx+e\right)^{4}}{\left(b\sinh\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(f*x+e)^4/(a+b*sinh(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(coth(f*x + e)^4/(b*sinh(f*x + e)^2 + a)^(5/2), x)
```

3.512 $\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$

Optimal. Leaf size=122

$$\frac{\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};-\sinh^2(e+f$$

$$df(m+1)$$

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, -Sinh[e + f*x]^2, -((b*Sinh[
e + f*x]^2)/a)]*(Cosh[e + f*x]^2)^((1 + m)/2)*(a + b*Sinh[e + f*x]^2)^p*(d*
Tanh[e + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rubi [A] time = 0.129349, antiderivative size = 122, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.12, Rules used = {3197, 511, 510}

$$\frac{\cosh^2(e+fx)^{\frac{m+1}{2}}(d\tanh(e+fx))^{m+1}\left(a+b\sinh^2(e+fx)\right)^p\left(\frac{b\sinh^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};-\sinh^2(e+f$$

$$df(m+1)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, -Sinh[e + f*x]^2, -((b*Sinh[
e + f*x]^2)/a)]*(Cosh[e + f*x]^2)^((1 + m)/2)*(a + b*Sinh[e + f*x]^2)^p*(d*
Tanh[e + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sinh[e + f*x]^2)/a)^p)
```

Rule 3197

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(
ff*(d*Tan[e + f*x])^(m + 1)*(Cos[e + f*x]^2)^((m + 1)/2))/(d*f*Sin[e + f*x]
^(m + 1)), Subst[Int[((ff*x)^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/
2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !In
tegerQ[m]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx = \frac{\left(i\cosh^2(e+fx)^{\frac{1+m}{2}}(i\sinh(e+fx))^{-1-m}(d\tanh(e+fx))^{1+m}\right)\text{Subst}\left(\int\right.}{df}$$

$$= \frac{\left(i\cosh^2(e+fx)^{\frac{1+m}{2}}(i\sinh(e+fx))^{-1-m}\left(a+b\sinh^2(e+fx)\right)^p\left(1+\frac{b\sinh}{}\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};-\sinh^2(e+fx),-\frac{b\sinh^2(e+fx)}{a}\right)\cosh^2(e+fx)^{\frac{1+m}{2}}}{df(1+m}$$

Mathematica [F] time = 9.49784, size = 0, normalized size = 0.

$$\int \left(a+b\sinh^2(e+fx)\right)^p (d\tanh(e+fx))^m\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m,x]
```

```
[Out] Integrate[(a + b*Sinh[e + f*x]^2)^p*(d*Tanh[e + f*x])^m, x]
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sinh\left(fx+e\right)\right)^2\right)^p \left(d\tanh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
```

```
[Out] int((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\tanh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh\left(fx+e\right)^2+a\right)^p \left(d\tanh\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)**2)**p*(d*tanh(f*x+e))**m,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(fx+e\right)^2+a\right)^p\left(d\tanh\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(f*x+e)^2)^p*(d*tanh(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(f*x + e)^2 + a)^p*(d*tanh(f*x + e))^m, x)
```

3.513 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx$

Optimal. Leaf size=110

$$\frac{\operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2d(a-b)}-\frac{(a-b(p+1))\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)^2}$$

```
[Out] -((a - b*(1 + p))*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2
)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)^2*d*(1 + p)) + (Sech
[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d)
```

Rubi [A] time = 0.120445, antiderivative size = 110, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3194, 78, 68}

$$\frac{\operatorname{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2d(a-b)}-\frac{(a-b(p+1))\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]
```

```
[Out] -((a - b*(1 + p))*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2
)/(a - b)]*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)^2*d*(1 + p)) + (Sech
[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d)
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{(1+x)^2}\,dx, x, \sinh^2(c+dx)\right)}{2d}$$

$$= \frac{\text{sech}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d} - \frac{(a-b(1+p))\,\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\right.}{2(-a+b)}$$

$$= -\frac{(a-b(1+p))\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+dx)\right)^{1}}{2(a-b)^2d(1+p)}$$

Mathematica [A] time = 0.238538, size = 90, normalized size = 0.82

$$\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\left((-a+bp+b)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)+(p+1)(a-b)\text{sech}^2(c+dx)\right)}{2d(p+1)(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^3,x]
```

```
[Out] (((-a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)
/(a - b)] + (a - b)*(1 + p)*Sech[c + d*x]^2)*(a + b*Sinh[c + d*x]^2)^(1 + p
))/(2*(a - b)^2*d*(1 + p))
```

Maple [F] time = 0.36, size = 0, normalized size = 0.

$$\int \left(a+b(\sinh(dx+c))^2\right)^p (\tanh(dx+c))^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(dx+c)^2+a\right)^p \tanh(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p \tanh(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**3,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\tanh\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^3, x)
```

3.514 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx$

Optimal. Leaf size=63

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)}$$

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*(a +
b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d*(1 + p))
```

Rubi [A] time = 0.0547016, antiderivative size = 63, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3194, 68}

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)+a}{a-b}\right)}{2d(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x],x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sinh[c + d*x]^2)/(a - b)]*(a +
b*Sinh[c + d*x]^2)^(1 + p))/(2*(a - b)*d*(1 + p))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sinh^2(c+dx)\right)^p \tanh(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1+x}\,dx, x, \sinh^2(c+dx)\right)}{2d}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sinh^2(c+dx)}{a-b}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2(a-b)d(1+p)}\end{aligned}$$

Mathematica [A] time = 0.070719, size = 65, normalized size = 1.03

$$-\frac{\left(a+b\cosh^2(c+dx)-b\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\cosh^2(c+dx)}{a-b}+1\right)}{2d(p+1)(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x],x]
```

```
[Out] -((a - b + b*Cosh[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
+ (b*Cosh[c + d*x]^2)/(a - b)])/(2*(a - b)*d*(1 + p))
```

Maple [F] time = 0.255, size = 0, normalized size = 0.

$$\int \left(a+b\left(\sinh \left(dx+c\right)\right)^{2}\right)^{p}\tanh \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh \left(dx+c\right)^{2}+a\right)^{p}\tanh \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh \left(dx+c\right)^{2}+a\right)^{p}\tanh \left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c), x)
```

3.515 $\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh
[c + d*x]^2)^(1 + p))/(2*a*d*(1 + p))
```

Rubi [A] time = 0.0632654, antiderivative size = 54, normalized size of antiderivative = 1., number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3194, 65}

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh
[c + d*x]^2)^(1 + p))/(2*a*d*(1 + p))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\begin{aligned}\int \coth(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sinh^2(c+dx)\right)}{2d}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sinh^2(c+dx)}{a}\right)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad(1+p)}\end{aligned}$$

Mathematica [A] time = 0.0602517, size = 54, normalized size = 1.

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh
[c + d*x]^2)^(1 + p))/(2*a*d*(1 + p))
```

Maple [F] time = 0.279, size = 0, normalized size = 0.

$$\int \coth(dx+c)\left(a+b(\sinh(dx+c))^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)
```

```
[Out] int(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(dx+c)^2+a\right)^p \coth(dx+c)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p \coth(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(dx+c\right)^2+a\right)^p \coth \left(dx+c\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c), x)
```

3.516 $\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=94

$$-\frac{(a+bp)\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2ad}$$

```
[Out] -(Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a*d) - ((a + b*p)*Hyp
ergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c +
d*x]^2)^(1 + p))/(2*a^2*d*(1 + p))
```

Rubi [A] time = 0.0907626, antiderivative size = 94, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3194, 78, 65}

$$-\frac{(a+bp)\left(a+b\sinh^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{p+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -(Csch[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a*d) - ((a + b*p)*Hyp
ergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sinh[c + d*x]^2)/a]*(a + b*Sinh[c +
d*x]^2)^(1 + p))/(2*a^2*d*(1 + p))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rubi steps

$$\int \coth^3(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int\frac{(1+x)(a+bx)^p}{x^2}dx,x,\sinh^2(c+dx)\right)}{2d}$$
$$= -\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad} + \frac{(a+bp)\,\text{Subst}\left(\int\frac{(a+bx)^p}{x}dx,x,\right.}{2ad}$$
$$= -\frac{\operatorname{csch}^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^{1+p}}{2ad} - \frac{(a+bp)\,{}_2F_1\left(1,1+p;2+p;1+\right.}{2a^2}$$

Mathematica [A] time = 0.379479, size = 71, normalized size = 0.76

$$-\frac{\left(a+b\sinh^2(c+dx)\right)^{p+1}\left(\frac{(a+bp)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sinh^2(c+dx)}{a}+1\right)}{p+1}+a\operatorname{csch}^2(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[c + d*x]^3*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -((a*Csch[c + d*x]^2 + ((a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b
*Sinh[c + d*x]^2)/a])/(1 + p))*(a + b*Sinh[c + d*x]^2)^(1 + p))/(2*a^2*d)
```

Maple [F] time = 0.265, size = 0, normalized size = 0.

$$\int (\coth(dx+c))^3\left(a+b(\sinh(dx+c))^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

```
[Out] int(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(dx+c)^2+a\right)^p\coth(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p\coth(dx+c)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**3*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(dx+c\right)^{2}+a\right)^{p}\coth\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^3*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^3, x)
```

3.517 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{\sinh^4(c+dx)\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(c+dx),-\frac{b}{}\right.}{5d}$$

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sq
rt[Cosh[c + d*x]^2]*Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]
)/(5*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rubi [A] time = 0.12086, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3196, 511, 510}

$$\frac{\sinh^4(c+dx)\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(c+dx),-\frac{b}{}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]
```

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sq
rt[Cosh[c + d*x]^2]*Sinh[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]
)/(5*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\mathbf{sech}(c+dx)\right)\text{Subst}\left(\int \frac{x^4(a+bx^2)^p}{(1+x^2)^{5/2}}\,dx, x, \sinh(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\mathbf{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sinh^2(c+dx)}{a}\right)^{-p}\right.}{d}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\sinh^4(c+}{5d}$$

Mathematica [F] time = 7.10546, size = 0, normalized size = 0.

$$\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4,x]
```

```
[Out] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^4, x]
```

Maple [F] time = 0.227, size = 0, normalized size = 0.

$$\int \left(a+b(\sinh(dx+c))^2\right)^p (\tanh(dx+c))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(dx+c)^2+a\right)^p \tanh(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p \tanh(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**4,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh \left(dx+c\right)^{2}+a\right)^{p} \tanh \left(dx+c\right)^{4} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^4, x)
```

3.518 $\int \left(a+b\sinh^2(c+dx)\right)^p \tanh^2(c+dx)\,dx$

Optimal. Leaf size=103

$$\frac{\sinh^2(c+dx)\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+dx),\right.}{3d}$$

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sq
rt[Cosh[c + d*x]^2]*Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]
)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rubi [A] time = 0.107292, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3196, 511, 510}

$$\frac{\sinh^2(c+dx)\sqrt{\cosh^2(c+dx)}\tanh(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+dx),\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]
```

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)]*Sq
rt[Cosh[c + d*x]^2]*Sinh[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]
)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \left(a+b\sinh^2(c+dx)\right)^p\tanh^2(c+dx)\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\mathrm{sech}(c+dx)\right)\mathrm{Subst}\left(\int\frac{x^2(a+bx^2)^p}{(1+x^2)^{3/2}}\,dx,x,\sinh(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sinh^2(c+dx)}{a}\right)^{-p}\right)\mathrm{S}}{d}$$

$$= \frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\sinh^2(c+dx}{3d}$$

Mathematica [F] time = 5.42295, size = 0, normalized size = 0.

$$\int \left(a+b\sinh^2(c+dx)\right)^p\tanh^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2,x]
```

```
[Out] Integrate[(a + b*Sinh[c + d*x]^2)^p*Tanh[c + d*x]^2, x]
```

Maple [F] time = 0.222, size = 0, normalized size = 0.

$$\int \left(a+b(\sinh(dx+c))^2\right)^p(\tanh(dx+c))^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
```

```
[Out] int((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(dx+c)^2+a\right)^p\tanh(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p\tanh(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)**2)**p*tanh(d*x+c)**2,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh\left(dx+c\right)^2+a\right)^p \tanh\left(dx+c\right)^2 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sinh(d*x+c)^2)^p*tanh(d*x+c)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*tanh(d*x + c)^2, x)
```

3.519 $\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{\cosh^2(c+dx)}\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh^2(c+dx),\right.}{d}$$

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)
]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)
^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p))
```

Rubi [A] time = 0.101677, antiderivative size = 99, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3196, 511, 510}

$$-\frac{\sqrt{\cosh^2(c+dx)}\operatorname{csch}(c+dx)\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh^2(c+dx),\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)
]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]*Sech[c + d*x]*(a + b*Sinh[c + d*x]^2)
^p)/(d*(1 + (b*Sinh[c + d*x]^2)/a)^p))
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\mathbf{sech}(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt{1+x^2}(a+bx^2)^p}{x^2}dx, x, \sinh(c+d\right.}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\mathbf{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sinh^2(c+dx)}{a}\right)^{-}\right.}{d}$$

$$= -\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\mathbf{csch}}{d}$$

Mathematica [F] time = 6.36908, size = 0, normalized size = 0.

$$\int \coth^2(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] Integrate[Coth[c + d*x]^2*(a + b*Sinh[c + d*x]^2)^p, x]
```

Maple [F] time = 0.218, size = 0, normalized size = 0.

$$\int (\coth(dx+c))^2\left(a+b(\sinh(dx+c))^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
```

```
[Out] int(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(dx+c)^2+a\right)^p\coth(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\text{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p\coth(dx+c)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**2*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh\left(dx+c\right)^2+a\right)^p\coth\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^2*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^2, x)
```

3.520 $\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=103

$$-\frac{\sqrt{\cosh^2(c+dx)}\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh^2(c+ \right.}{3d}$$

```
[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)
]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]^3*Sech[c + d*x]*(a + b*Sinh[c + d*x]^
2)^p)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rubi [A] time = 0.101907, antiderivative size = 103, normalized size of antiderivative = 1., number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.13, Rules used = {3196, 511, 510}

$$-\frac{\sqrt{\cosh^2(c+dx)}\operatorname{csch}^3(c+dx)\operatorname{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(\frac{b\sinh^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh^2(c+ \right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, -Sinh[c + d*x]^2, -((b*Sinh[c + d*x]^2)/a)
]*Sqrt[Cosh[c + d*x]^2]*Csch[c + d*x]^3*Sech[c + d*x]*(a + b*Sinh[c + d*x]^
2)^p)/(3*d*(1 + (b*Sinh[c + d*x]^2)/a)^p)
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[c, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rubi steps

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p dx = \frac{\left(\sqrt{\cosh^2(c+dx)}\mathrm{sech}(c+dx)\right)\mathrm{Subst}\left(\int \frac{(1+x^2)^{3/2}(a+bx^2)^p}{x^4}dx, x, \sinh(c+dx\right.}{d}$$

$$= \frac{\left(\sqrt{\cosh^2(c+dx)}\mathrm{sech}(c+dx)\left(a+b\sinh^2(c+dx)\right)^p\left(1+\frac{b\sinh^2(c+dx)}{a}\right)^{-p}\right)\mathrm{S}}{d}$$

$$= -\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};-\sinh^2(c+dx),-\frac{b\sinh^2(c+dx)}{a}\right)\sqrt{\cosh^2(c+dx)}\mathrm{csch}^3(}{3d}$$

Mathematica [F] time = 6.63468, size = 0, normalized size = 0.

$$\int \coth^4(c+dx)\left(a+b\sinh^2(c+dx)\right)^p dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p,x]
```

```
[Out] Integrate[Coth[c + d*x]^4*(a + b*Sinh[c + d*x]^2)^p, x]
```

Maple [F] time = 0.216, size = 0, normalized size = 0.

$$\int (\coth(dx+c))^4\left(a+b(\sinh(dx+c))^2\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

```
[Out] int(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b\sinh(dx+c)^2+a\right)^p \coth(dx+c)^4 dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

Fricas [F] time = 0., size = 0, normalized size = 0.

$$\mathrm{integral}\left(\left(b\sinh(dx+c)^2+a\right)^p \coth(dx+c)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)**4*(a+b*sinh(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \left(b \sinh (dx+c)^2+a\right)^p \coth (dx+c)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(d*x+c)^4*(a+b*sinh(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sinh(d*x + c)^2 + a)^p*coth(d*x + c)^4, x)
```

3.521 $\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx$

Optimal. Leaf size=152

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sinh(x)+b^{2/3}\sinh^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sinh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{\log\left(a+b\sin\right.}{3a}$$

```
[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sinh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(5/3)) - Csch[x]^2/(2*a) + Log[Sinh[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3
)*Sinh[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sinh[x] +
b^(2/3)*Sinh[x]^2])/(6*a^(5/3)) - Log[a + b*Sinh[x]^3]/(3*a)
```

Rubi [A] time = 0.251657, antiderivative size = 152, normalized size of antiderivative = 1., number of steps used = 12, number of rules used = 11, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.733, Rules used = {3230, 1834, 1871, 12, 200, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sinh(x)+b^{2/3}\sinh^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sinh(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}a^{5/3}}-\frac{\log\left(a+b\sin\right.}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]^3/(a + b*Sinh[x]^3),x]
```


```
[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sinh[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*
a^(5/3)) - Csch[x]^2/(2*a) + Log[Sinh[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3
)*Sinh[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sinh[x] +
b^(2/3)*Sinh[x]^2])/(6*a^(5/3)) - Log[a + b*Sinh[x]^3]/(3*a)
```


Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rubi steps

$$\int \frac{\coth^3(x)}{a+b\sinh^3(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{x^3\left(a+bx^3\right)}\,dx, x, \sinh(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{ax^3}+\frac{1}{ax}+\frac{-b-bx^2}{a\left(a+bx^3\right)}\right)dx, x, \sinh(x)\right)$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}+\frac{\text{Subst}\left(\int \frac{-b-bx^2}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\text{Subst}\left(\int \frac{b}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}-\frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sinh(x)\right)}{a}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \sinh(x)\right)}{3a^{5/3}}-\frac{b\,\text{Subs}}{}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}-\frac{\log\left(a+b\sinh^3(x)\right)}{3a}+\frac{b^{2/3}\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sinh(x)+b^{2/3}\text{si}\right.}{6a^{5/3}}$$

$$= \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sinh(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}a^{5/3}}-\frac{\text{csch}^2(x)}{2a}+\frac{\log(\sinh(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}\right.}{}$$

Mathematica [A] time = 0.338182, size = 136, normalized size = 0.89

$$-\frac{\left(a^{2/3}+(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sinh(x)\right)+\left(a^{2/3}+b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sinh(x)\right)+\left(a^{2/3}-\sqrt[3]{-1}b^{2/3}\right)\log\left(\sqrt[3]{a}+(\right.}{3a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]^3/(a + b*Sinh[x]^3),x]
```

```
[Out] -Csch[x]^2/(2*a) + Log[Sinh[x]]/a - ((a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-((
-1)^(2/3)*a^(1/3)) - b^(1/3)*Sinh[x]] + (a^(2/3) + b^(2/3))*Log[a^(1/3) + b
^(1/3)*Sinh[x]] + (a^(2/3) - (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b
^(1/3)*Sinh[x]])/(3*a^(5/3))
```

Maple [C] time = 0.06, size = 132, normalized size = 0.9

$$-\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^2-\frac{1}{8a}\left(\tanh\left(\frac{x}{2}\right)\right)^{-2}+\frac{1}{a}\ln\left(\tanh\left(\frac{x}{2}\right)\right)+\frac{1}{3a}\sum_{\_R=\text{RootOf}\left(a\_Z^6-3a\_Z^4-8b\_Z^3+3a\_Z^2-a\right)}\frac{-\_R^5a-\_R^4b+2\_}{\_R^5a-2\_R^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)^3/(a+b*sinh(x)^3),x)
```

```
[Out] -1/8/a*tanh(1/2*x)^2-1/8/a/tanh(1/2*x)^2+1/a*ln(tanh(1/2*x))+1/3/a*sum((-_R
^5*a-_R^4*b+2*_R^3*a+4*_R^2*b-_R*a+b)/(_R^5*a-2*_R^3*a-4*_R^2*b+_R*a)*ln(ta
```

```
nh(1/2*x)-_R),_R=RootOf(_Z^6*a-3*_Z^4*a-8*_Z^3*b+3*_Z^2*a-a))
```

Maxima [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] time = 11.7566, size = 2762, normalized size = 18.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(12*sqrt(1/3)*(a*e^(4*x) - 2*a*e^(2*x) + a)*sqrt((((1/2)^(1/3)*(I*sqr
t(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a +
 4)/a^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3
+ b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^4*e^(2*x) + b^2*e^(4*x) + 2*a
*b*e^(3*x) - 2*a*b*e^x - (a^2*b*e^(3*x) + 4*a^3*e^(2*x) - a^2*b*e^x)*((1/2)
^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) + b
^2 + 2*(2*a^2 - b^2)*e^(2*x))*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^
5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a^3 - 2*a^2)*sqrt((((1/2)^(1/3)*(I*sqrt(3
) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1
/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a + 4)
/a^2) + sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b
^2)/a^5)^(1/3) + 2/a)^2*a^5*e^x + 4*a^2*b*e^(2*x) + 4*a^3*e^x - 4*a^2*b - 2
*(a^3*b*e^(2*x) + 2*a^4*e^x - a^3*b)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 +
b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)^2*a^2 - 4*((1/2)^(1/3)*(I
*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a + 4)/a^2))
*e^(-x)/b^2) + 2*(a*e^(4*x) - 2*a*e^(2*x) + a)*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*log(((1/2)^(1/3)*(I*sqrt(
3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a)*a^2*e^x + b*e^(2*x
) - 2*a*e^x - b) - ((a*e^(4*x) - 2*a*e^(2*x) + a)*((1/2)^(1/3)*(I*sqrt(3) +
 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) - 6*e^(4*x) + 12*e^(2*
x) - 6)*log(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a^3 + b^2/a^5 - (a^2 + b^2)/a^5
)^(1/3) + 2/a)^2*a^4*e^(2*x) + b^2*e^(4*x) + 2*a*b*e^(3*x) - 2*a*b*e^x - (a
^2*b*e^(3*x) + 4*a^3*e^(2*x) - a^2*b*e^x)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/a
^3 + b^2/a^5 - (a^2 + b^2)/a^5)^(1/3) + 2/a) + b^2 + 2*(2*a^2 - b^2)*e^(2*x
)) - 12*(e^(4*x) - 2*e^(2*x) + 1)*log(e^(2*x) - 1) + 24*e^(2*x))/(a*e^(4*x)
 - 2*a*e^(2*x) + a)
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)**3/(a+b*sinh(x)**3),x)
```

```
[Out] Timed out
```

Giac [A] time = 1.23755, size = 282, normalized size = 1.86

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-e^{(-x)}+e^{x}\right|\right)}{3\,a^{2}}-\frac{\log\left(\left|-b\left(e^{(-x)}-e^{x}\right)^{3}+8\,a\right|\right)}{3\,a}+\frac{\log\left(\left|-e^{(-x)}+e^{x}\right|\right)}{a}-\frac{\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}\cdots}{3\left(-\cdots\right.}\right)}{3\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)^3/(a+b*sinh(x)^3),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(1/3)*log(abs(-2*(-a/b)^(1/3) - e^(-x) + e^x))/a^2 - 1/3*log(a
bs(-b*(e^(-x) - e^x)^3 + 8*a))/a + log(abs(-e^(-x) + e^x))/a - 1/3*sqrt(3)*
(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) - e^(-x) + e^x)/(-a/b)^(1/3
))/a^2 - 1/6*(-a*b^2)^(1/3)*log((e^(-x) - e^x)^2 - 2*(-a/b)^(1/3)*(e^(-x) -
 e^x) + 4*(-a/b)^(2/3))/a^2 - 1/2*(3*(e^(-x) - e^x)^2 + 4)/(a*(e^(-x) - e^x
)^2)
```

3.522 $\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rubi [A] time = 0.0763304, antiderivative size = 28, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^3(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx, x, \sinh(x)\right)$$
$$= \frac{1}{3}\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^3(x)\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^3(x)}\right)}{3b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Mathematica [A] time = 0.0235918, size = 28, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

Maple [A] time = 0.214, size = 21, normalized size = 0.8

$$-\frac{2}{3}\text{Artanh}\left(\sqrt{a+b\left(\sinh(x)\right)^3}\frac{1}{\sqrt{a}}\right)\frac{1}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)^3)^(1/2),x)
```

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

Fricas [F(-2)] time = 0., size = 0, normalized size = 0.

Exception raised: UnboundLocalError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")

[Out] Exception raised: UnboundLocalError
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b\sinh^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)**3)**(1/2),x)

[Out] Integral(coth(x)/sqrt(a + b*sinh(x)**3), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)^3 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")

[Out] integrate(coth(x)/sqrt(b*sinh(x)^3 + a), x)
```

3.523 $\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}\sqrt{a+b\sinh^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh[
x]^3])/3
```

Rubi [A] time = 0.0822686, antiderivative size = 45, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a+b\sinh^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]
```


```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh[
x]^3])/3
```


Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
```

```
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}\int \coth(x)\sqrt{a+b\sinh^3(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \sinh(x)\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sinh^3(x)\right)\\ &= \frac{2}{3}\sqrt{a+b\sinh^3(x)}+\frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^3(x)\right)\\ &= \frac{2}{3}\sqrt{a+b\sinh^3(x)}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^3(x)}\right)}{3b}\\ &= -\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)+\frac{2}{3}\sqrt{a+b\sinh^3(x)}\end{aligned}$$

Mathematica [A] time = 0.0225346, size = 45, normalized size = 1.

$$\frac{2}{3}\sqrt{a+b\sinh^3(x)}-\frac{2}{3}\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sinh[
x]^3])/3
```

Maple [A] time = 0.042, size = 34, normalized size = 0.8

$$-\frac{2}{3}\text{Artanh}\left(\sqrt{a+b\left(\sinh(x)\right)^3}\frac{1}{\sqrt{a}}\right)\sqrt{a}+\frac{2}{3}\sqrt{a+b\left(\sinh(x)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x)^3)^(1/2),x)
```

```
[Out] -2/3*arctanh((a+b*sinh(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sinh(x)^3)^(1/
2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(x)^3+a}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)
```

Fricas [B] time = 15.5511, size = 5184, normalized size = 115.2

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/6*(sqrt(a)*(cosh(x) + sinh(x))*log(-(b^2*cosh(x)^12 + 12*b^2*cosh(x)*sin
h(x)^11 + b^2*sinh(x)^12 - 6*b^2*cosh(x)^10 + 64*a*b*cosh(x)^9 + 6*(11*b^2*
cosh(x)^2 - b^2)*sinh(x)^10 + 15*b^2*cosh(x)^8 + 4*(55*b^2*cosh(x)^3 - 15*b
^2*cosh(x) + 16*a*b)*sinh(x)^9 - 192*a*b*cosh(x)^7 + 3*(165*b^2*cosh(x)^4 -
 90*b^2*cosh(x)^2 + 192*a*b*cosh(x) + 5*b^2)*sinh(x)^8 + 24*(33*b^2*cosh(x)
^5 - 30*b^2*cosh(x)^3 + 96*a*b*cosh(x)^2 + 5*b^2*cosh(x) - 8*a*b)*sinh(x)^7
 + 192*a*b*cosh(x)^5 + 4*(128*a^2 - 5*b^2)*cosh(x)^6 + 4*(231*b^2*cosh(x)^6
 - 315*b^2*cosh(x)^4 + 1344*a*b*cosh(x)^3 + 105*b^2*cosh(x)^2 - 336*a*b*cos
h(x) + 128*a^2 - 5*b^2)*sinh(x)^6 + 15*b^2*cosh(x)^4 + 24*(33*b^2*cosh(x)^7
 - 63*b^2*cosh(x)^5 + 336*a*b*cosh(x)^4 + 35*b^2*cosh(x)^3 - 168*a*b*cosh(x
)^2 + 8*a*b + (128*a^2 - 5*b^2)*cosh(x))*sinh(x)^5 - 64*a*b*cosh(x)^3 + 3*(
165*b^2*cosh(x)^8 - 420*b^2*cosh(x)^6 + 2688*a*b*cosh(x)^5 + 350*b^2*cosh(x
)^4 - 2240*a*b*cosh(x)^3 + 320*a*b*cosh(x) + 20*(128*a^2 - 5*b^2)*cosh(x)^2
 + 5*b^2)*sinh(x)^4 - 6*b^2*cosh(x)^2 + 4*(55*b^2*cosh(x)^9 - 180*b^2*cosh(
x)^7 + 1344*a*b*cosh(x)^6 + 210*b^2*cosh(x)^5 - 1680*a*b*cosh(x)^4 + 480*a*
b*cosh(x)^2 + 20*(128*a^2 - 5*b^2)*cosh(x)^3 + 15*b^2*cosh(x) - 16*a*b)*sin
h(x)^3 + 6*(11*b^2*cosh(x)^10 - 45*b^2*cosh(x)^8 + 384*a*b*cosh(x)^7 + 70*b
^2*cosh(x)^6 - 672*a*b*cosh(x)^5 + 320*a*b*cosh(x)^3 + 10*(128*a^2 - 5*b^2)
*cosh(x)^4 + 15*b^2*cosh(x)^2 - 32*a*b*cosh(x) - b^2)*sinh(x)^2 + b^2 - 16*
(b*cosh(x)^8 + 8*b*cosh(x)*sinh(x)^7 + b*sinh(x)^8 - 3*b*cosh(x)^6 + (28*b*
cosh(x)^2 - 3*b)*sinh(x)^6 + 16*a*cosh(x)^5 + 2*(28*b*cosh(x)^3 - 9*b*cosh(
x) + 8*a)*sinh(x)^5 + 3*b*cosh(x)^4 + (70*b*cosh(x)^4 - 45*b*cosh(x)^2 + 80
*a*cosh(x) + 3*b)*sinh(x)^4 + 4*(14*b*cosh(x)^5 - 15*b*cosh(x)^3 + 40*a*cos
h(x)^2 + 3*b*cosh(x))*sinh(x)^3 - b*cosh(x)^2 + (28*b*cosh(x)^6 - 45*b*cosh
(x)^4 + 160*a*cosh(x)^3 + 18*b*cosh(x)^2 - b)*sinh(x)^2 + 2*(4*b*cosh(x)^7
- 9*b*cosh(x)^5 + 40*a*cosh(x)^4 + 6*b*cosh(x)^3 - b*cosh(x))*sinh(x))*sqrt
(a)*sqrt((b*sinh(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*c
osh(x)*sinh(x) + sinh(x)^2)) + 12*(b^2*cosh(x)^11 - 5*b^2*cosh(x)^9 + 48*a*
b*cosh(x)^8 + 10*b^2*cosh(x)^7 - 112*a*b*cosh(x)^6 + 80*a*b*cosh(x)^4 + 2*(
128*a^2 - 5*b^2)*cosh(x)^5 + 5*b^2*cosh(x)^3 - 16*a*b*cosh(x)^2 - b^2*cosh(
x))*sinh(x))/(cosh(x)^12 + 12*cosh(x)*sinh(x)^11 + sinh(x)^12 + 6*(11*cosh(
x)^2 - 1)*sinh(x)^10 - 6*cosh(x)^10 + 20*(11*cosh(x)^3 - 3*cosh(x))*sinh(x)
^9 + 15*(33*cosh(x)^4 - 18*cosh(x)^2 + 1)*sinh(x)^8 + 15*cosh(x)^8 + 24*(33
*cosh(x)^5 - 30*cosh(x)^3 + 5*cosh(x))*sinh(x)^7 + 4*(231*cosh(x)^6 - 315*c
osh(x)^4 + 105*cosh(x)^2 - 5)*sinh(x)^6 - 20*cosh(x)^6 + 24*(33*cosh(x)^7 -
 63*cosh(x)^5 + 35*cosh(x)^3 - 5*cosh(x))*sinh(x)^5 + 15*(33*cosh(x)^8 - 84
*cosh(x)^6 + 70*cosh(x)^4 - 20*cosh(x)^2 + 1)*sinh(x)^4 + 15*cosh(x)^4 + 20
*(11*cosh(x)^9 - 36*cosh(x)^7 + 42*cosh(x)^5 - 20*cosh(x)^3 + 3*cosh(x))*si
nh(x)^3 + 6*(11*cosh(x)^10 - 45*cosh(x)^8 + 70*cosh(x)^6 - 50*cosh(x)^4 + 1
5*cosh(x)^2 - 1)*sinh(x)^2 - 6*cosh(x)^2 + 12*(cosh(x)^11 - 5*cosh(x)^9 + 1
0*cosh(x)^7 - 10*cosh(x)^5 + 5*cosh(x)^3 - cosh(x))*sinh(x) + 1)) + 2*sqrt(
(b*sinh(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*si
nh(x) + sinh(x)^2)))/(cosh(x) + sinh(x)), 1/3*(sqrt(-a)*(cosh(x) + sinh(x))
```

```
*arctan(8*(cosh(x)^2 + 2*cosh(x)*sinh(x) + sinh(x)^2)*sqrt(-a)*sqrt((b*sinh
(x)^3 + 3*(b*cosh(x)^2 - b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) +
 sinh(x)^2))/(b*cosh(x)^6 + 6*b*cosh(x)*sinh(x)^5 + b*sinh(x)^6 - 3*b*cosh(
x)^4 + 3*(5*b*cosh(x)^2 - b)*sinh(x)^4 + 16*a*cosh(x)^3 + 4*(5*b*cosh(x)^3
- 3*b*cosh(x) + 4*a)*sinh(x)^3 + 3*b*cosh(x)^2 + 3*(5*b*cosh(x)^4 - 6*b*cos
h(x)^2 + 16*a*cosh(x) + b)*sinh(x)^2 + 6*(b*cosh(x)^5 - 2*b*cosh(x)^3 + 8*a
*cosh(x)^2 + b*cosh(x))*sinh(x) - b)) + sqrt((b*sinh(x)^3 + 3*(b*cosh(x)^2
- b)*sinh(x) + 4*a)/(cosh(x)^2 - 2*cosh(x)*sinh(x) + sinh(x)^2)))/(cosh(x)
+ sinh(x))]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{a + b\sinh^3(x)} \coth(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sinh(x)**3)*coth(x), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(x)^3 + a} \coth(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^3 + a)*coth(x), x)
```

3.524 $\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rubi [A] time = 0.0865435, antiderivative size = 29, normalized size of antiderivative = 1., number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]/Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\int \frac{\coth(x)}{\sqrt{a+b\sinh^n(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \sinh(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sinh^n(x)\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sinh^n(x)}\right)}{bn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Mathematica [A] time = 0.0237434, size = 29, normalized size = 1.

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]/Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Maple [A] time = 0.033, size = 24, normalized size = 0.8

$$-2\,\frac{1}{n\sqrt{a}}\text{Artanh}\left(\frac{\sqrt{a+b\,(\sinh(x))^n}}{\sqrt{a}}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)/(a+b*sinh(x)^n)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)^n + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)
```

Fricas [A] time = 1.83506, size = 390, normalized size = 13.45

$$\left[\frac{\log\left(\frac{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))-2\sqrt{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))+a}\sqrt{a}+2\,a}{\cosh(n\log(\sinh(x)))+\sinh(n\log(\sinh(x)))}\right)}{\sqrt{a}n}, \frac{2\sqrt{-a}\arctan\left(\frac{\sqrt{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))+a}\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) - 2*sqrt(b*cosh(n*log
(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*log(sinh(x)
)) + sinh(n*log(sinh(x)))))/(sqrt(a)*n), 2*sqrt(-a)*arctan(sqrt(b*cosh(n*lo
g(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(-a)/a)/(a*n)]
```

Sympy [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{a + b\sinh^{n}(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)**n)**(1/2),x)
```

```
[Out] Integral(coth(x)/sqrt(a + b*sinh(x)**n), x)
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \frac{\coth(x)}{\sqrt{b\sinh(x)^{n} + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)/(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(coth(x)/sqrt(b*sinh(x)^n + a), x)
```

3.525 $\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{a+b\sinh^n(x)}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Sinh[
x]^n])/n
```

Rubi [A] time = 0.0919908, antiderivative size = 47, normalized size of antiderivative = 1., number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b\sinh^n(x)}}{n}-\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Sinh[
x]^n])/n
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rubi steps

$$\begin{aligned}
\int \coth(x)\sqrt{a+b\sinh^n(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx,x,\sinh(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx,x,\sinh^n(x)\right)}{n}\\
&= \frac{2\sqrt{a+b\sinh^n(x)}}{n}+\frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx,x,\sinh^n(x)\right)}{n}\\
&= \frac{2\sqrt{a+b\sinh^n(x)}}{n}+\frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx,x,\sqrt{a+b\sinh^n(x)}\right)}{bn}\\
&= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}+\frac{2\sqrt{a+b\sinh^n(x)}}{n}
\end{aligned}$$

Mathematica [A] time = 0.0208877, size = 45, normalized size = 0.96

$$\frac{2\sqrt{a+b\sinh^n(x)}-2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sinh^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Coth[x]*Sqrt[a + b*Sinh[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sinh[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sinh[x]^n
])/n
```

Maple [A] time = 0.016, size = 38, normalized size = 0.8

$$\frac{1}{n}\left(2\sqrt{a+b\left(\sinh(x)\right)^n}-2\sqrt{a}\text{Artanh}\left(\frac{\sqrt{a+b\left(\sinh(x)\right)^n}}{\sqrt{a}}\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(coth(x)*(a+b*sinh(x)^n)^(1/2),x)
```

```
[Out] 1/n*(2*(a+b*sinh(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sinh(x)^n)^(1/2)/a^(1/2
)))
```

Maxima [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(x)^n+a}\coth(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```

Fricas [A] time = 1.81846, size = 552, normalized size = 11.74

$$\left[\frac{\sqrt{a}\log\left(\frac{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))-2\sqrt{b\cosh(n\log(\sinh(x)))+b\sinh(n\log(\sinh(x)))+a}\sqrt{a}+2a}{\cosh(n\log(\sinh(x)))+\sinh(n\log(\sinh(x)))}\right)+2\sqrt{b\cosh\left(n\log\left(\sinh\right.\right.}}{n}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*cosh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) - 2*sqrt(b*c
osh(n*log(sinh(x))) + b*sinh(n*log(sinh(x))) + a)*sqrt(a) + 2*a)/(cosh(n*lo
g(sinh(x))) + sinh(n*log(sinh(x))))) + 2*sqrt(b*cosh(n*log(sinh(x))) + b*si
nh(n*log(sinh(x))) + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*cosh(n*log(sinh(x)))
+ b*sinh(n*log(sinh(x))) + a)*sqrt(-a)/a) + sqrt(b*cosh(n*log(sinh(x))) + b
*sinh(n*log(sinh(x))) + a))/n]
```

Sympy [F(-1)] time = 0., size = 0, normalized size = 0.

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)**n)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F] time = 0., size = 0, normalized size = 0.

$$\int \sqrt{b\sinh(x)^n + a}\coth(x)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(coth(x)*(a+b*sinh(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sinh(x)^n + a)*coth(x), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
```

```
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
    debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
    ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
           print("ExpnType_result<=ExpnType_optimal");
      fi;
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
```

```
                print("both result and optimal complex");
             fi;
             #both result and optimal complex
             if leaf_count_result<=2*leaf_count_optimal then
                return "A";
             else
                return "B";
             end if
          else #result contains complex but optimal is not
             if debug then
                print("result contains complex but optimal is not");
             fi;
             return "C";
          end if
       else # result do not contain complex
            # this assumes optimal do not as well
            if debug then
                print("result do not contain complex, this assumes optimal do
    not as well");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
               if debug then
                  print("leaf_count_result<=2*leaf_count_optimal");
               fi;
               return "A";
            else
               if debug then
                  print("leaf_count_result>2*leaf_count_optimal");
               fi;
               return "B";
            end if
       end if
  else #ExpnType(result) > ExpnType(optimal)
       if debug then
          print("ExpnType(result) > ExpnType(optimal)");
       fi;
       return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
```

```
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
```

```
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
```

```
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
```

```
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

def tree(expr):
    debug=False;
    if debug:
         print ("Enter tree(expr), expr=",expr)
         print ("expr.operator()=",expr.operator())
         print ("expr.operands()=",expr.operands())
         print ("map(tree, expr.operands()=",map(tree, expr.operands()))

    if expr.operator() is None:
        return expr
    else:
        return [expr.operator()]+list(map(tree, expr.operands()))

def leaf_count(anti):
    debug=False;

    if debug: print ("Enter leaf_count, anti=", anti, " len(anti)=", len(anti))

    if len(anti) == 0: #special check for optimal being 0 for some test cases.
       if debug: print ("len(anti) == 0")
       return 1
    else:
        if debug: print ("round(1.35*len(flatten(tree(anti))))=",round(1.35*len
    (flatten(tree(anti)))))
        return round(1.35*len(flatten(tree(anti)))) #fudge factor
                         #since this estimate of leaf count is bit lower than
```

```
                     #what it should be compared to Mathematica's

def is_sqrt(expr):
    debug=False;
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug = False

    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug = False

    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

```

```
def is_atom(expn):

    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):
    debug=False

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
```

```
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
    debug = False;

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```